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Theoretical aspects of the relation between unemployment and vacancies

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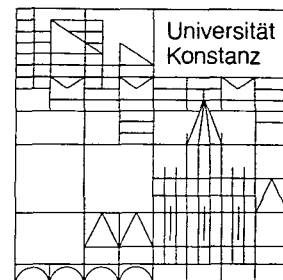
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Diskussionsbeiträge

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**Theoretical Aspects of the Relation
between Unemployment and Vacancies**

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**THEORETICAL ASPECTS OF THE RELATION BETWEEN
UNEMPLOYMENT AND VACANCIES**

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Theoretical Aspects of the Relation between Unemployment and Vacancies

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Abstract: This paper is devoted to a theoretical analysis of the Beveridge curve, i. e. , the relation between unemployment and vacancies. Most empirical studies of the $u - v$ curve for various countries find an outward shift thus indicating higher structural unemployment due to increased maladjustments on the labor market. The theory consists of three elements: the search process seen from both the employer's and the unemployed seeker's viewpoint and the matching technology governing the labor market. Major explanatory variables of possible shifts of the $u - v$ curve are variables which determine the contact and contract probability, respectively, such as unemployment benefits, long-term unemployment, minimum hiring standards set by the firm, costs of regional mobility, training costs, turnover rate, perceived wage distribution. Many of these variables, however, have an ambiguous effect on the location of the Beveridge curve thus rendering this relationship a less straightforward tool to analyse structural unemployment and to draw policy implications to combat this type of joblessness.

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1 Introduction

Unemployment in the Federal Republic of Germany has remained at a permanent high level for years. The explanation for this persistence of joblessness has shifted both in academia and in the public towards considerations which blame "structural factors" as the major source of this problem. The prerequisite for an economic analysis of these presumptions is a theoretical framework which is able both to capture factors which may be viewed as determining structural unemployment and to allow for an empirical test of the relevance of these determinants. The Beveridge curve, i. e. , the relation between vacancies and unemployment, is often used as an analytical instrument to identify the extent and the causes of structural unemployment. In recent years several theoretical and empirical studies employing the Beveridge curve have been carried out such as Christl (1987) for Austria, Börsch-Supan (1988) and Franz (1987) for the Federal Republic of Germany and Jackman, Layard, and Pissarides (1983), Jackman and Roper (1985) and Budd, Levine, and Smith (1988) for Great Britain, to name a few only. Most if not all studies of this type wind up with the conclusion that the Beveridge curve has shifted outwards thus indicating a higher degree of malfunctioning of labor markets. For Germany, as in some other countries, this result is also obtained by an estimation of a disequilibrium macro-model as is shown in the study by Entorf, Franz, König, and Smolny (1989).

The aim of this paper is to investigate on theoretical grounds whether the Beveridge curve is indeed a straightforward tool to analyse the extent and the causes of structural unemployment. Some skepticism about this appropriateness is raised by the observation that while the studies mentioned above are quite certain about the identification of a Beveridge curve and its possible shifts, they are more speculative about the determinants of the shifts. Prominent candidates are the rising share of long-term unemployed persons and various indicators for labor market maladjustments such as a regional and/or qualifications mismatch. However, while plausible, these variables are far from being unambiguous in determining the sign of the shift of the Beveridge curve. For the share of long-term unemployment, for example, this is recognised clearly in the study by Budd, Levine, Smith (1988, p. 1082). In order to see why this is and whether other explanatory factors have an ambiguous effect too, requires a theory of the unemployment/vacancy-relationship. Such a theory should also identify additional determinants (if any) for the location and curvature of the Beveridge curve.

Our paper attempts to make a contribution to a theoretical derivation of the Beveridge curve based on a microeconomic analysis of the search process seen both from the employer's viewpoint and from the perspective of an unemployed searcher. It also includes a treatment of the matching technology governing the labor market. Our primary goal is to evaluate whether such a theory can provide unambiguous explanations for the observed outward shift of the Beveridge curve.

If it turns out, however, that most candidates which explain an outward shift may also cause the Beveridge curve to shift inwards, this would mean that the Beveridge curve is a less straightforward guide for economic policy to combat structural unemployment.

The paper is organized as follows. In the next section we sketch the basic idea of the Beveridge curve in order to provide a guide to those readers unfamiliar with this relationship. Moreover, this section results in questions which have to be answered by the theoretical framework in subsequent considerations. Section (3) gives a detailed description of the theoretical model. It starts with an overview of the model (3.1) and goes on with a formulation of the search and hiring process seen from the firm's viewpoint (3.2). The search process seen from the applicant's viewpoint is outlined in subsection (3.3), while section (3.4) shows under which conditions and circumstances a match is formed. Subsection (3.5) derives the Beveridge curve and highlights several sources for shifts which may direct further empirical research. The paper ends with a conclusion.

2 Basic Concept of the Beveridge Curve

In this brief section we illustrate the basic idea of the unemployment/vacancy-relationship often called the Beveridge curve.² For any given structure of the labor market, vacancies and unemployed persons may be related in a manner indicated by the stylized curve B_0B_0 presented in figure 1. Locations on the 45°-ray represent situations in which the number of unemployed equals the number of vacancies. This means that unemployment is due to labor maladjustment since, in principle, there is a job for each unemployed. All positions on the Beveridge curve at which the number of unemployed exceeds the number of vacancies (i. e., all positions to the right of the 45°-ray) indicate that there is demand deficiency or that inflexible wages are too high. Hence, a movement on the Beveridge curve from, say, X to Y means that the increase in unemployment is mainly due to classical and/or Keynesian determinants. Worsening of the functioning of the labor market causes an outward shift of the Beveridge curve to, say, B_1B_1 . Moving from X to W indicates therefore, that the higher unemployment associated with this shift is the result of greater labor maladjustment rather than demand deficiency or classical factors. As has been mentioned, locations on the ray from the origin represent situations in which the number of unemployed equal the number of vacancies. In the present context, this amount of unemployment (such as \overline{OU}_W for B_1B_1) is defined as structural/frictional

²William H. Lord Beveridge (1879–1963) was a British economist and statistician and served as an advisor to the British government. The "Beveridge plan" made several proposals concerning labor market policy especially about the social insurance system. His main book is "Full Employment in a Free Society", London 1944 (1st. ed.).

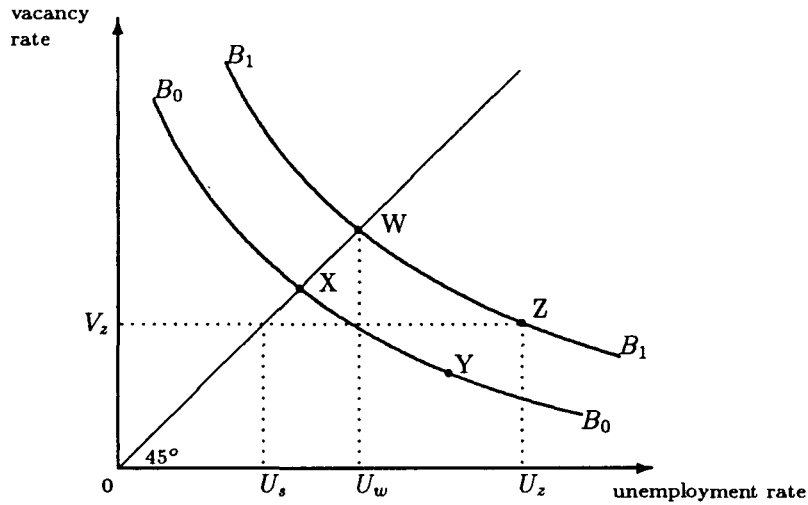


Figure 1: The Beveridge Curve

unemployment.³ This is due to the notion that the labor market is not able to match the unemployed to the existing unfilled job openings. Hence, if B_1B_1 is the relevant Beveridge curve and we are in a situation marked by Z and if classical unemployment is absent for the sake of simplicity, $\overline{U_W U_Z}$ gives the amount of Keynesian unemployment. An alternative view would be to regard the amount $\overline{U_S U_Z}$ as unemployment due to demand deficiency. However, this view would probably overstate Keynesian unemployment: an expansionary demand policy creating jobs implies graphically a move on B_1B_1 from Z to, say, W . Equality between unemployment and vacancies is thus reached at unemployment $\overline{O U_W}$ and not at $\overline{O U_S}$. The reason is that the match between newly created jobs and existing unemployment is not perfect as long as aggregate demand policy is not supported by policies which improve the matching process.

It should be pointed out that combinations on the 45°-ray are not necessarily optimal. If policy-makers are free to choose any point on the Beveridge curve, the optimal vacancy/unemployment-relation is where the marginal costs associated with another unemployed person (such as the output losses) equal those associated with another unfilled job (such as the costs of waiting in a longer queue or some inflationary pressure).⁴

³There are other possible definitions of structural unemployment such as the one proposed by Armstrong and Taylor (1981). They define structural unemployment as $u_s = \min(u - u_f, v - v_f)$ where u_f and v_f are frictional unemployment and vacancies, respectively, with $u_f = \sum_i \min(u_i, v_i)$ and i referring to sectors. As has been discussed by Jackman and Roper (1985), this measure suffers from several deficiencies namely that it is very sensitive to cyclical fluctuations.

⁴See Abraham (1983) and Hamermesh and Rees (1988).

3 Theoretical Aspects of Job Matching

3.1 Overview of the Model

The model considers the matching process to be influenced firstly by the decisions made by the firm which opens a vacancy, secondly, by the search process undertaken by the job seeker, and thirdly, by a technology function which governs the matching process on the labor market.

To begin with, we assume a profit maximizing firm under perfect competition which determines its optimal employment level, the offered wage rate and a minimum hiring standard of newly hired workers (if any). Leaving aside standard aspects of an optimal level of production and employment, the firm faces the following problem. The firm is uncertain about the abilities of each applicant (which determine the worker's efficiency) but it knows the density function of these abilities prevailing on a suitably defined labor market. Moreover, there is a minimum hiring standard to be met by the applicant due to specific requirements for the job under consideration or due to legal restrictions. The firm is allowed to train workers but it has to incur training costs. In sorting out workers, the firm sets its minimum hiring standard endogenously, evaluates expected training costs, and makes a wage offer. From this viewpoint two aspects are important for the matching process. First, the minimum hiring standard which may or may not be met by the job seeker, and, second, the wage offer made by the firm which may or may not be accepted by the applicant.

The applicant's decision is based on a conventional job search model. The job seeker maximizes expected wealth by accepting a wage offer which is not lower than the reservation wage. The individual contacts several employers submitting wage offers. The distribution of wage offers is the source of uncertainty: Although its parameters are known to the searcher, each offer is a realization of a random variable. Determinants of the reservation wage are the search costs, the unemployment benefits, the density function of wage offers and the discount rate.

The third element of the model is the matching process. The probability that a vacancy is filled can be decomposed into two probabilities, namely that an unemployed person contacts an employer with a vacancy, and the probability that a match is formed conditional on a contact between employer and searcher. Factors influencing the first probability are the number of unemployed persons and vacancies and the availability of information about both groups. The probability that a match is formed depends on the probability that the applicant meets the minimum hiring standard and that the reservation wage does not exceed the wage offered by the firm.

The Beveridge curve can then be derived by making use of the identity that the change of the number of unemployed persons equals the difference between inflows into unemployment and outflows from joblessness. The foregoing analysis concerns the outflows from unemployment to employment which is the number

of vacancies times the probability that a vacancy is filled with an unemployed applicant. The latter relationship constitutes the Beveridge curve and various sources for possible shifts of the $u - v$ curve can be identified.

3.2 The Firm's Decisions

Under the assumption of profit maximization, the firm determines its optimal employment level, the offered wage rate and the minimum hiring standard of newly hired workers.

A decreasing returns production function is given by

$$y = y(\varepsilon \cdot n) \quad \text{with} \quad y'_i > 0, y''_i < 0, i = \varepsilon, n, \quad (1)$$

where ε denotes the efficiency per worker n measured in efficiency units. The latter are determined by

$$\varepsilon = \varepsilon(x) \quad \text{with} \quad \varepsilon' > 0, \varepsilon'' < 0, \quad (2)$$

where x represents the abilities of the applicant. Although the firm is uncertain about each applicant's abilities, their density function $f(x - \beta)$ including an exogenous shift parameter β is objectively known to the firm. The minimum hiring standard required by the firm is x^0 which is to be determined endogeneously.⁵ Let $F(x - \beta)$ denote the distribution function of all abilities. Then the conditional density of all abilities which are not below the minimum standard x^0 is given by

$$f(x - \beta) / [1 - F(x^0 - \beta)]. \quad (3)$$

Since the firm does not know each applicant's abilities, it follows that the firm is also uncertain about his or her efficiency and productivity. More specifically, what does that mean for the production process? As a first step, the expected value of production provided that $x \geq x^0$ may be written as

$$\int_{x^0}^{\infty} y[\varepsilon(x) \cdot n] \cdot f(x - \beta) / [1 - F(x^0 - \beta)] dx, \quad (4)$$

that is production multiplied with the probability that $x \geq x^0$ and integrated over all abilities $x \geq x^0$. However, expression (4) neglects the firm's search and screening process. The condition $x \geq x^0$ means that the job market relevant for the firm is limited in the sense that only applicants with $x \geq x^0$ are considered for hiring. Therefore, the firm has to search for such applicants and has to screen them in order to be sure that they fulfill the minimum hiring requirement. For simplicity, assume that the firm is able to screen immediately whether $x \geq x^0$ is met by each applicant. For concreteness, such an immediate screening may be

⁵See Ohashi (1987) for similar considerations within the context of cyclical variations in wage differentials.

based on the applicant's diplomas, evaluation of former employers, past work experience and the like. The probability to find an applicant with $x \geq x^0$ is given by $[1 - F(x^0 - \beta)]$. Multiplying the integral in eq. (4) with this probability gives an expression for expected production conditional on a successful search and screening process undertaken by the firm. Hence, expected production for $x \geq x^0$ is obtained by the following equation:

$$y^* \equiv E(y | x \geq x^0) = \int_{x^0}^{\infty} y[\varepsilon(x) \cdot n] \cdot f(x - \beta) dx. \quad (5)$$

As has been mentioned, x^0 is a minimum hiring standard. The abilities of an applicant with $x \geq x^0$ may be improved by some training within the firm. We assume that training is undertaken at the outset of employment and the training costs T expressed in money terms and per employee depend on abilities:

$$T = T(x) \quad \text{with} \quad T' < 0, T'' > 0. \quad (6)$$

Uncertainty about the individual's abilities implies uncertainty about training costs. The expected value of T , i. e. , T^* , is obtained by similar considerations as for y^* , namely:⁶

$$T^* \equiv E(T(x) | x \geq x^0) = \int_{x^0}^{\infty} T(x) \cdot f(x - \beta) dx. \quad (7)$$

In order to decide whether it pays to train an applicant the firm has to take into account job quitting behavior. Let q denote the percentage of employees quitting the firm at any time. Hence, the expected duration of employment is $1/q$. The decision to quit is based on a comparison of two wage rates: the wage rate w paid by the firm, and the negotiated wage rate \bar{w} which is fixed in collective bargaining. It is assumed that the firm has no influence on \bar{w} , but that it is determined by observable characteristics such as school education, vocational training, work experience and the like. These characteristics are summarized in a vector z , so that $\bar{w} = \bar{w}(z)$. The firm is allowed to pay more than $\bar{w}(z)$ but w may not fall short of $\bar{w}(z)$.

$$q = q(w / \bar{w}) \quad \text{with} \quad q' < 0, q'' > 0 \quad \text{and} \quad w / \bar{w} \geq 1. \quad (8)$$

The expected training costs per employee at any time are, therefore, $q \cdot T^*$. If w is added, we obtain total costs per employee, i. e. , $w + q \cdot T^*$. Expected profits G^* are then given by (P denotes the output price):

$$G^* = P \cdot y^* - (w + qT^*) \cdot n$$

⁶Training costs $T(x)$ may fall sharply with higher x and soon reach a value close to zero so that the infinite upper border of the integral ($x \rightarrow \infty$) may not be relevant.

⁷This means that the quit rate reaches its maximum for $q(1)$.

$$\begin{aligned}
&= P \cdot E(y \mid x \geq x^0) - [w + q \cdot E(T \mid x \geq x^0)] \cdot n \\
&= P \cdot \int_{x^0}^{\infty} y[\varepsilon(x) \cdot n] \cdot f(x - \beta) dx \\
&\quad - \left[w + q(w/\bar{w}) \cdot \int_{x^0}^{\infty} T(x) \cdot f(x - \beta) dx \right] \cdot n. \tag{9}
\end{aligned}$$

The aim of the firm is to maximize expected profits subject to $w / \bar{w} \geq 1$. Properties of this simple model can be illustrated firstly by evaluating the first order conditions.

$$\frac{\partial G^*}{\partial n} = P \cdot \frac{\partial y^*}{\partial n} - w - q(w/\bar{w}) \cdot T^* = 0. \tag{10}$$

From equation (4) we obtain for the marginal product of labor:

$$\begin{aligned}
\frac{\partial y^*}{\partial n} &= \frac{\partial E(y \mid x \geq x^0)}{\partial n} \\
&= \int_{x^0}^{\infty} \varepsilon(x) \cdot y'(\varepsilon(x) \cdot n) \cdot f(x - \beta) dx \\
&= E[\varepsilon(x) \cdot y'(\varepsilon(x) \cdot n) \mid x \geq x^0] > 0. \tag{11}
\end{aligned}$$

Since $\varepsilon(x)$ as well as $y'(\cdot)$ are positive and the expected value of a positive variable is also positive, the expected marginal product of labor given by the last expression is positive. The expected marginal product of labor decreases with higher employment. This can be verified easily by inspection of the second order condition:

$$\begin{aligned}
\frac{\partial^2 y^*}{\partial n^2} &= \int_{x^0}^{\infty} \varepsilon^2(x) \cdot y''(\varepsilon(x) \cdot n) \cdot f(x - \beta) dx \\
&= E[\varepsilon^2(x) \cdot y''(\varepsilon(x) \cdot n) \mid x \geq x^0] < 0. \tag{12}
\end{aligned}$$

Having evaluated the expected marginal product of labor, we are now in a position to state the optimal demand for labor more explicitly. From eq. (7), (10) and (11) we have:

$$\begin{aligned}
&P \cdot \int_{x^0}^{\infty} \varepsilon(x) \cdot y'(\varepsilon(x) \cdot n) \cdot f(x - \beta) dx \\
&= w + q(w/\bar{w}) \cdot \int_{x^0}^{\infty} T(x) \cdot f(x - \beta) dx. \tag{13}
\end{aligned}$$

Eq. (13) reiterates the well-known condition that employment has reached its optimal level if the marginal revenue of labor equals its marginal costs. The latter are split into pure wage costs and expected training costs. Higher marginal costs of labor may therefore be caused by higher wages, higher expected training

costs per employee and/or by a higher quit rate. If so, eq. (13) postulates that the optimal level of employment declines with increased values of these variables.

What determines the firm's wage offer? Differentiating eq. (9) with respect to w / \bar{w} subject to the side condition:

$$w / \bar{w} - S - 1 = 0, \quad (14)$$

where S is a nonnegative shadow variable, gives

$$\bar{w} = \lambda / n - q'(w / \bar{w}) \cdot T^*, \quad (15)$$

with λ as the Lagrange variable associated with the side condition (14). In what follows we assume that this condition is fulfilled, i. e. , $\lambda = 0$.

$$\bar{w} = -q'(w / \bar{w}) \cdot \int_{x^0}^{\infty} T(x) \cdot f(x - \beta) dx. \quad (16)$$

Solving for w yields a relationship between the wage offer w and the negotiated wage rate, the marginal quit rate, and expected training costs per worker. Put differently, eq. (16) implies a condition for an optimal wage drift. Since the firm is able to control quitting by varying the wage drift, in the optimum the wage drift should equal marginal expected training costs per worker. On the other hand, if \bar{w} increases (due to collective bargaining, for example) then $q'(\cdot)$ decreases (i. e. , $q(\cdot)$ increases). Given expected training costs per worker (T^*) the firm must increase the wage drift in order to prevent employees from quitting and to avoid a loss due to additional training. By the same token, if T^* increases then the firm again has to pay a higher wage drift in order to offset the higher training costs by longer expected job duration, i. e. , a lower quit rate.

The model can also be solved for an optimal value of the minimum hiring standard x^0 . Differentiating eq. (9) with respect to x^0 yields:

$$\frac{\partial G^*}{\partial x^0} = P \cdot \frac{\partial y^*}{\partial x^0} - q(w / \bar{w}) \cdot n \cdot \frac{\partial T^*}{\partial x^0} = 0. \quad (17)$$

In eq. (17) we have to evaluate the terms $\partial y^* / \partial x^0$ and $\partial T^* / \partial x^0$. To begin with, using eq. (5) we obtain:

$$\frac{\partial y^*}{\partial x^0} = -f(x^0 - \beta) \cdot y(\varepsilon(x^0)) \cdot n < 0, \quad (18)$$

and

$$\begin{aligned} \frac{\partial^2 y^*}{\partial (x^0)^2} = & - [f'(x^0 - \beta) \cdot y(\varepsilon(x^0)) \cdot n \\ & + f(x^0 - \beta) \cdot y'(\varepsilon(x^0)) \cdot n \cdot \varepsilon'(x^0)]. \end{aligned} \quad (19)$$

As can be seen from eq. (18) the expression $\partial y^* / \partial x^0$ is negative. To understand this, recall that $\partial y^* / \partial x^0$ consists of (i) the probability to find a worker who

meets the minimum qualification $x = x^0$ and (ii) the production obtained with standard $x = x^0$. If x^0 increases the probability mentioned in (i) decreases, hence, $\partial y^* / \partial x^0 < 0$. Whether the r.h.s. expression in eq. (19) is positive or negative depends on the slope of the density function at $x = x^0$. If x^0 is chosen at a positive slope (i. e. , if $f'(\cdot)$ is positive), then the change of the expected marginal product of a higher minimum standard (i. e. , $\partial^2 y^* / \partial (x^0)^2$) is negative. From eq. (7) we obtain:

$$\frac{\partial T^*}{\partial x^0} = -f(x^0 - \beta) \cdot T(x^0) < 0 \quad (20)$$

and

$$\frac{\partial^2 T^*}{\partial (x^0)^2} = -[f'(x^0 - \beta) \cdot T(x^0) + f(x^0 - \beta) \cdot T'(x^0)]. \quad (21)$$

The change of the expected training costs T^* caused by an increase of x^0 is unambiguously negative. This is the result of two negative effects. First, the probability to find a worker with $x = x^0$ decreases if x^0 increases, second, less training is necessary if x^0 becomes higher. Referring to eq.(21) the r.h.s. term is unambiguously positive if $f'(\cdot)$ is negative. If x^0 is chosen, however, so that $f'(\cdot) > 0$, the r.h.s. term in eq.(21) remains positive only if $-f'(\cdot)/f(\cdot)$ outweighs $T'(\cdot)/T(\cdot)$ or, in words, if the change of the probability of having a higher x (relative to this probability) is lower than the negative value of the change in training costs due to a higher x (relative to the training costs), i. e. , if $f'(\cdot)/f(\cdot) < -T'(\cdot)/T(\cdot)$.

Inserting eqs. (18) and (20) into eq. (17) yields an implicit expression for an optimal hiring standard x^0 :

$$\begin{aligned} \frac{\partial G^*}{\partial x^0} = & - P \cdot f(x^0 - \beta) \cdot y(\varepsilon(x^0) \cdot n) \\ & + q(w / \bar{w}) \cdot f(x^0 - \beta) \cdot T(x^0) \cdot n = 0 \end{aligned} \quad (22)$$

or (for $x^0 < \infty$ and therefore $f(x^0 - \beta) > 0$):

$$P \cdot y(\varepsilon(x^0) \cdot n) / n = q(w / \bar{w}) \cdot T(x^0). \quad (23)$$

The minimum hiring standard x^0 is chosen optimally if the average revenue product of a worker with qualifications $x = x^0$ equals his training costs. In other words, the training costs of a worker with $x = x^0$ must not exceed his average revenue product.

What, if anything, can be learned from these considerations so far? Assume that for given functions $T(\cdot)$ and $f(\cdot)$, eqs. (13), (14) and (23) can be solved for the unknown variables n , w , and x^0 .

$$\begin{aligned} n &= n [\bar{w}(z), \beta, P] \\ w &= w [\bar{w}(z), \beta, P] \\ x^0 &= x^0 [\bar{w}(z), \beta, P]. \end{aligned} \quad (24)$$

Then the system is tractable, because the endogenous variables depend - albeit in a complicated manner - only on the exogenous variables z , the shift parameter β and the product price P . Now, if the optimal level of n exceeds its actual value then the firm will announce vacancies. Moreover, the firm makes a wage offer w and sets a minimum hiring standard x^0 . When an applicant shows up, he or she is quickly screened whether the minimum hiring standard x^0 is fulfilled. If so, the firm offers a wage rate which depends both on obvious characteristics condensed in z such as diplomas, work experience, and the like and of the probability that the applicant has certain abilities x . The latter information is described by the density function $f(\cdot)$ with the exogenous shift parameter β .

Note that an existing vacancy may not be filled even if an applicant shows up. First, the job seeker may not meet the minimum hiring standard due to several imperfections. He or she may not have the profession required for the job in question, his or her work experience is too short or is evaluated badly by the former employers. In short, this is called a "qualifications mismatch" in the sense that a vacancy is not filled by an (unemployed) applicant because his or her qualifications are inadequate compared to the requirements for the work place under consideration.

There may be another source for a qualifications mismatch. As has been argued the minimum hiring standard x^0 is evaluated by the firm using quickly accessible sources. If firms use unemployment experience itself as a screening device, then applicants with a long duration of unemployment are viewed as less promising candidates. A long duration of unemployment may result in a depreciation of human capital including the skills of the individual. Moreover, it is a criterion that may be easily and costlessly applied by the firm. Such a choosiness by firms is therefore rational and may be the more likely to be applied the longer the queue of applicants for the vacancy under consideration. Taken together a qualifications mismatch will increase with higher proportions of long-term unemployed. A second reason why a match will not be formed is due to the possibility that the reservation wage of the job seeker exceeds the wage offered by the firm. To be more specific on this point requires knowledge of the determinants of the reservation wage. This analysis is relegated to the next section.

The model may be extended in several ways. An obvious modification is that the firm may be unwilling to hire overqualified workers for a job with more or less given skill requirements. This may happen even if the applicant agrees upon a rather low wage rate compared with his abilities. At first glance, this violates the assumption of a profit maximizing firm. However, the marginal product of such an overqualified worker may not be higher than that of a worker who meets the requirements approximately. First, there may be a kind of dissatisfaction with the present situation. Second, other employees may withdraw cooperation with such a worker. Third, this worker will still be searching for another job which values his abilities more approximately, hence, this instability may reduce his incentives for working. Formally, this possibility can be introduced into the

model by an upper limit of abilities required for the job under consideration. Then the upper border of the integral in eqs. (5) and (7), for example, is denoted by a value x^1 rather than by infinity. This value x^1 has, of course, to be determined endogenously and this requires specific assumptions about $\varepsilon(x)$.

3.3 The Job Seeker's Decisions

The basic idea of job search models is that an individual maximizes expected wealth if he or she accepts a wage offer (combined with a job offer) which is not lower than his or her reservation wage.⁸ Let w^0 denote the reservation wage and $H_2(w - \delta)$ the known distribution of wage offers, with δ representing an exogenous shift parameter and where $h_2(w - \delta)$ is the corresponding density function. The probability h_1 of receiving a job offer, depends on personal characteristics denoted by x such as abilities and on the wage rate w . The wage rate w is a determinant of h_1 because the wage rate offered is not only associated with the applicant but also –as has been shown in the previous section– with the job in question.⁹ The higher the wage rate the longer the queue of applicants for this job who will be offered the job due to higher abilities and who will accept it. This implies a decreasing chance for a given searcher to receive a job offer, i. e., $\frac{\partial h_1}{\partial w} < 0$. Thus the probability of receiving and accepting a job offer in any time period H is given by

$$H(x, w^0, \delta) = \int_{w^0}^{\infty} h_1(x, w) h_2(w - \delta) dw. \quad (25)$$

Although the individual's abilities are assumed to be invariant during the search process, there is a distribution of wage offers since prospective employers do not value them equally. The individual contacts several employers submitting different job offers. The distribution of job offers is the source of uncertainty: Although its parameters are known to the searcher, each offer is a realization of a random variable. The conditional density function of receiving a job offer with a wage rate higher than w^0 is

$$\frac{h_1(x, w) \cdot h_2(w - \delta)}{H(w^0, x, \delta)}.$$

The present value of earnings from an expected accepted job offer is:

$$\frac{1}{r} E(w \mid w \geq w^0) = \frac{1}{r} \int_{w^0}^{\infty} w \cdot h_1(x, w) \cdot h_2(w - \delta) dw / H(w^0, x, \delta), \quad (26)$$

⁸See Hübler (1988), König (1979), McKenna (1987) and Mortensen (1986) for surveys. The following considerations draw on Franz (1982).

⁹Recall from (3.2) that w is also determined by the negotiated wage rate \bar{w} which depends on several job requirements to be met by the applicant. Moreover, the higher w the lower the quit rate and, hence, the lower the amount of job offers.

where the right hand side is the conditional mean of w given that $w \geq w^0$. Since an infinite time horizon is assumed, r is the constant discount rate. The reservation wage is optimal if the present value of accepting the reservation wage equals the present value of the gain from continuing search. The present value of the reservation wage is:¹⁰

$$\sum_{t=0}^{\infty} \frac{w^0}{(1+r)^t} = \frac{w^0(1+r)}{r}. \quad (27)$$

The expected net worth from continuing search until a wage offer equal to or better than w^0 is received consists of the unemployment compensation uc and the expected future wage (conditional on w^0) provided that a job offer with a wage equal to or better than w^0 is received and accepted, i. e.

$$\begin{aligned} & uc + \sum_{t=1}^{\infty} \frac{H(x, w^0, \delta) \cdot E(w \mid w \geq w^0)}{(1+r)^t} \\ & \qquad \qquad \qquad \text{(first period)} \\ & + [1 - H(x, w^0, \delta)] \cdot \left\{ \frac{uc}{1+r} + \sum_{t=2}^{\infty} \frac{H(x, w^0, \delta) \cdot E(w \mid w \geq w^0)}{(1+r)^t} \right\} \\ & \qquad \qquad \qquad \text{(second period)} \\ & + [1 - H(x, w^0, \delta)]^2 \cdot \left\{ \frac{uc}{(1+r)^2} + \sum_{t=3}^{\infty} \frac{H(x, w^0, \delta) \cdot E(w \mid w \geq w^0)}{(1+r)^t} \right\} \\ & \qquad \qquad \qquad \text{(third period)} \end{aligned} \quad (28)$$

and so forth. This can be rearranged to:

$$\frac{uc \cdot (1+r)}{r + H(x, w^0, \delta)} + H(x, w^0, \delta) \cdot E(w \mid w \geq w^0) \cdot \frac{1+r}{r \cdot [r + H(x, w^0, \delta)]}. \quad (29)$$

The optimal reservation wage must satisfy the relationship:

$$\frac{w^0 \cdot (1+r)}{r} = \frac{uc \cdot (1+r)}{r + H(x, w^0, \delta)} + \frac{H(x, w^0, \delta) \cdot E(w \mid w \geq w^0) \cdot (1+r)}{r \cdot [r + H(x, w^0, \delta)]} \quad (30)$$

or

$$w^0 = \frac{r \cdot uc + H(x, w^0, \delta) \cdot E(w \mid w \geq w^0)}{r + H(x, w^0, \delta)}. \quad (31)$$

Eq. (31) reveals some basic properties of the reservation wage. An increasing unemployment compensation implies a higher reservation wage since the costs of search become lower. Increasing the present value of earnings from an accepted

¹⁰See Addison and Siebert (1979), p. 197 for a similar observation.

wage offer $[E(w | w \geq w^0)]/r$ has the same effect. On the other hand, a lower probability of receiving and accepting a job offer due to poor or inadequate abilities tends to reduce the reservation wage. Finally, the search process can be viewed as an investment decision negatively related to the rate of discount, if unemployment compensation falls short of the expected wage offer.

Search costs are now introduced more specifically. Let c denote search costs. Then eq. (31) changes to:

$$w^0 = \frac{r \cdot uc + H(x, w^0, \delta) \cdot E(w | w \geq w^0) - r \cdot c}{r + H(x, w^0, \delta)} \quad (32)$$

or

$$w^0 = uc + \frac{1}{r} \cdot H(x, w^0, \delta) \cdot [E(w | w \geq w^0) - w^0] - c, \quad (33)$$

which corresponds with the results stated in other studies on job search models.¹¹ Search costs c may consist of a fixed part such as an advertisement in a newspaper but may also depend on the regional dispersion of the unemployed and the vacancies. Besides travelling costs (which may or may not be refunded by the firm), the job seeker must devote time to contact the firm which is more if the firm is located at a greater distance.

Besides their influence on search costs, regional imbalances between job seekers and vacancies affect the search process more importantly when considering different regional levels of the costs of living. These costs consist not only of items such as rents or house prices (including the loss when selling the home in a depressed area), but also non-pecuniary costs such as a break down of personal relationships and the like. Moreover, in the case of married job seekers the spouse, who has to give up her or his job, may not find other (suitable) work in the new region. Of course, these costs have to be balanced against the gain provided by the new area, so that only the net effect is relevant. To simplify matters, we change the expression $E(w | w \geq w^0)$ in eq. (33) to $E(\tilde{w} | \tilde{w} \geq \tilde{w}^0)$ where

$$\tilde{w} = w + \varphi(\sigma_r) \quad \text{with} \quad \varphi' \geq 0 \text{ if } \varphi \geq 0, \quad (34)$$

and where σ_r denotes the regional dispersion between vacancies and unemployed. The term $\varphi(\sigma_r)$ may be positive or negative. A positive value of $\varphi(\sigma_r)$ means that the job seeker corrects the wage rate offered by the firm upwards in order to take into account the gain due to his or her regional mobility. On the other hand, $\varphi(\cdot) < 0$ expressed in money terms takes into account unfavorable characteristics (seen from the job seeker's viewpoint) of the new region and/or the workplace.¹² The reservation wage \tilde{w}^0 is now decomposed into the monetary part w^0 , which accords with the reservation wage in the analysis above;

¹¹ See Kiefer and Neumann (1979), and Nickell (1979), for example.

¹² Alternatively it is possible to formulate the problem by

$$H(x, w^0, \delta, M^0) = \int_{w^0}^{\infty} \int_{M^0}^{\infty} h_1(x, w) \cdot h_2(w - \delta, M) dw \cdot dM, \quad (35)$$

this wage rate has to be compared with the money wage rate offered by the firm. The second component of \tilde{w}^0 are the known non-pecuniary benefits term φ :

$$\tilde{w}^0 = w^0 + \varphi(\sigma_r). \quad (36)$$

The resulting decision rule for accepting a job offer is $\tilde{w} \geq \tilde{w}^0$. With respect to the monetary comparison the decision rule remains unaffected. The conditional probability of accepting a job offer with a wage rate higher than $w^0 = \tilde{w}^0 - \varphi(\sigma_r)$ rises with growing positive regional benefits and vice versa.¹³ The condition which determines the optimal reservation wage \tilde{w}^0 changes to:

$$\tilde{w}^0 = \frac{r[uc - c(\sigma_r)] + \int_{\tilde{w}^0 - \varphi(\sigma_r)}^{\infty} [w + \varphi(\sigma_r)] \cdot h_1(x, w) \cdot h_2(w - \delta) dw}{r + H(x, w^0, \delta)}. \quad (40)$$

By differentiating equation (40) with respect to σ_r we obtain the effects of a change in the regional dispersion on the reservation wage and, more importantly, on its monetary term w^0 :

$$\frac{d\tilde{w}^0}{d\sigma_r} = \frac{\int_{\tilde{w}^0 - \varphi(\sigma_r)}^{\infty} \varphi'(\sigma_r) \cdot h_1 \cdot h_2 dw - r c'(\sigma_r)}{r + H(x, w^0, \delta)}. \quad (41)$$

A growing regional dispersion has two effects. Firstly it increases the search costs ($c' > 0$). Secondly it affects the regional benefits. If the regional dispersion causes a non-monetary loss ($\varphi' < 0$), \tilde{w}^0 unambiguously declines with a growing regional dispersion. In contrast if the regional dispersion induces a gain, it depends on whether the increase in search costs dominates the increase in the non-monetary benefits. The effects of σ_r on w^0 turns out to be:

$$\frac{dw^0}{d\sigma_r} = \frac{d\tilde{w}^0}{d\sigma_r} - \varphi' = -\frac{r \cdot \varphi'(\sigma_r) + r \cdot c'(\sigma_r)}{r + H(x, w^0, \delta)}. \quad (42)$$

Growing dispersions and therefore growing search costs reduce the reservation wage rate and its monetary component w^0 . The reactions of w^0 and \tilde{w}^0 to

where M indicates (non-pecuniary) attributes of the job or region in question. The expected value of w in eq. (33) is then replaced by $E(w \mid w \geq w^0, M \geq M^0)$.

¹³The conditional probability of receiving and accepting a job offer with a wage rate higher than w^0 is:

$$H(x, w^0, \delta) = \int_{w^0 = \tilde{w}^0 - \varphi(\sigma_r)}^{\infty} h_1(x, w) \cdot h_2(w - \delta) dw. \quad (37)$$

it follows:

$$\frac{\partial H}{\partial \sigma_r} = \varphi' \cdot h_1(x, w^0) \cdot h_2(w^0, \delta) \gtrless 0 \text{ for } \varphi' \gtrless 0. \quad (38)$$

The searcher calculates the present value of earnings including the non-monetary benefits:

$$\frac{1}{r} E(w + \varphi(\sigma_r) \mid w + \varphi(\sigma_r) \geq \tilde{w}^0) = \frac{1}{r} \int_{w^0 = \tilde{w}^0 - \varphi(\sigma_r)}^{\infty} [w + \varphi(\sigma_r)] \cdot h_1 \cdot h_2 dw. \quad (39)$$

changes in non-monetary benefits exhibit opposite directions. If σ_r causes non-monetary gains ($\varphi' > 0$), w^0 tends to decline whereas the reservation wage tends to rise. An increase in the non-monetary gain (loss) has a negative (positive) impact on w^0 that wage rate which is to be compared with the wage offer. Therefore the probability of getting and accepting a job offer rises.

Determining the parameters and the forms of the distribution functions h_1 and h_2 gives a solution for w^0 (if it exists):

$$w^0 = w^0[\delta, x, r, \sigma_r, uc]. \quad (43)$$

This reduced form of the monetary component of the reservation wage is the equation we are looking for. The searcher with abilities x contacts a firm with a vacancy. If $x \geq x^0$ and $w \geq w^0$ a match is formed.¹⁴

It should be stressed that the search model presented so far is extremely simple. For constant values of the explanatory variables in eq. (43) w^0 remains unchanged over time *ceteris paribus*. This may, however, be an oversimplification. Several hypotheses have been offered to justify a changing reservation wage over time.

(i) If a finite time horizon (the retirement age, for example) is introduced, a longer search duration may still lead to a higher job offer, but as a contrary effect the remaining time in the labor force becomes shorter. Hence, due to wealth considerations the reservation wage may decline with duration.

(ii) If the distribution of job offers is unknown, the reservation wage may be a function of the searcher's beliefs. It changes according to the revisions the individual makes in the light of his or her experience. In the case of an overestimated mean of the wage distribution, there may be a decreasing reservation wage and vice versa.

(iii) A further complication is the possibility of being laid off. The job search theory's explanation may be called into question if the tenure of jobs is rather short. Given the possible brevity of tenure in some jobs, an unemployed person may raise his or her total return from search by accepting less attractive jobs from the beginning.

(iv) Finally, another reason why optimal reservation wages may decline over time has been explained by introducing risk aversion into the model. Assume that the individual maximizes the expected utility of lifetime consumption (instead of the present value of income net of search costs, as assumed in most job search models). The utility function may indicate decreasing absolute risk aversion. Since wealth decreases during the search process, due to search costs, and because the expected working life shortens, it follows that risk aversion increases and hence the reservation wage

¹⁴Recall from (3.2) that x^0 denotes the minimum hiring standard set by the firm.

declines. Risk aversion can be influenced not only by wealth considerations, but also by personal characteristics such as age or marital status, for example. A married searcher who is the breadwinner for his or her family may be more risk averse than a single one.

Especially from point (i) and (iv) we may conclude that the longer the search process the lower the reservation wage. If search is undertaken while unemployed this implies that the longer the duration of unemployment the more likely the reservation wage meets the condition $\tilde{w} \geq \tilde{w}^0$ and, hence, the more likely a match is formed. From this follows that in the aggregate the matching is the more likely the higher the share of long-term unemployed. This conclusion stands in marked contrast to the findings of the previous section. Thus the effect of long-term unemployment on the matching process is ambiguous: on the one hand, a match becomes more unlikely if long-term unemployment is used by the firm as a screening device; on the other hand, it facilitates matching due to a declining reservation wage.

3.4 The Matching Process

We are now in a position to model the probability that a vacancy is filled by an unemployed person. This probability is denoted by π and can be decomposed into

- (i) the technology governing the labor market, i. e., the probability π_1 that an unemployed person contacts an employer with a vacancy,
- (ii) the conditional probability π_2 that a match is formed provided that a contact between job seeker and a firm with a vacancy has taken place.

Factors influencing the probability π_1 of a job contact are considered first. Following Hall (1977), the probability that a given worker will come into contact with a particular employer is $1/v$; where v denotes the number of vacancies on the labor market. The probability that an employer with a vacancy (each firm has only one vacancy at the most) will not be visited by any unemployed worker is

$$\left(1 - \frac{1}{v}\right)^u = \left[\left(1 - \frac{1}{v}\right)^{-v}\right]^{-u/v} \simeq e^{-u/v}, \quad (44)$$

where u stands for the number of unemployed persons. Vacancies v are taken to be large enough to justify the above transformation.¹⁵ Then the proportion of vacancies which are contacted, π'_1 , is

$$\pi'_1 = 1 - e^{-u/v}. \quad (45)$$

¹⁵The minimum number of vacancies in the FRG between 1950 and 1988 was 76,000 (in 1983). Hence, $(1 - \frac{1}{v})^{-v} \simeq 2.7183$ for $v = 76,000$.

In this formulation the contact probability depends only on the pure number of vacancies and unemployed persons. This may be an oversimplification since the technology governing the contact process on the labor market is disregarded. Information on each vacancy is usually not available nationwide. This concerns both vacancies registered at the labor office and those announced without intervention by the labor office. As a crude measure of such a regional mismatch due to a lack of information we define a variable

$$\sigma_r = \sum_i^R |u_i - v_i|, \quad (46)$$

where u_i is the proportion of unemployed located in region i ($i = 1, \dots, R$) and v_i is the corresponding series for vacancies.¹⁶ The more u_i approaches v_i , the smaller σ_r , i. e., the less is a mismatch due to the fact that the unemployed and the vacancies are located in different regions. The greater the dispersion σ_r , however, the more relevant is a potential information gap. In order to take into account this possibility we redefine π_1' in eq. (45) in the following way:

$$\pi_1 = 1 - e^{-\gamma} \quad (47)$$

where $\gamma = \gamma_1(\sigma_r) \cdot (u/v)$ and $\gamma_1' < 0$, $\gamma_1(0) = 1$.

In the absence of any regional mismatch (i. e., for $\sigma_r = 0$), we obtain the previous formula (45). On the other hand, the higher this type of a mismatch, the smaller the chance that a vacancy is contacted.¹⁷

We now turn to the conditional probability π_2 that a match is formed provided that a contact has taken place. As has been derived in sections 3.2 and 3.3, a match is formed if

(i) the abilities of the job seeker do not fall short of the minimum hiring standards, i. e., if $x \geq x^0$, and

(ii) the reservation wage \tilde{w}^0 does not exceed the wage offer w by the firm corrected for losses or gains of regional mobility $\varphi(\sigma_r)$, i. e., if $\tilde{w}^0 \leq w + \varphi(\sigma_r)$.

To simplify matters, let D be a dummy variable which equals one if (i) and (ii) are satisfied and zero otherwise. More formally,

$$D = \begin{cases} 1 & \text{if } x \geq x^0 \text{ and } \tilde{w}^0 \leq w + \varphi(\sigma_r) \\ 0 & \text{otherwise.} \end{cases} \quad (48)$$

¹⁶See Jackman, Layard, and Pissarides (1983).

¹⁷An explicit formula for γ_1 which satisfies this statement is, for example, $\gamma_1 = e^{-\gamma_2 \sigma_r}$, where γ_2 is a normalizing constant. For $\sigma_r \rightarrow 0$ we have $\gamma_1 \rightarrow 1$, for $\sigma_r \rightarrow \infty$ we obtain $\gamma_1 \rightarrow 0$ and, hence, $\pi_1 \rightarrow 0$.

Then the conditional probability π_2 that a match is formed is

$$\begin{aligned}\pi_2 &= Pr(D = 1) \\ &= Pr\{x \geq x^0 \text{ and } \tilde{w}^0 \leq w + \varphi(\sigma_r)\}.\end{aligned}\quad (49)$$

Recall from eqs. (24) and (43) that

$$\left. \begin{aligned}w &= w[\bar{w}(z), \beta, P] \\ x^0 &= x^0[\bar{w}(z), \beta, P] \\ w^0 &= w^0[\delta, x, r, c(\sigma_r), \varphi(\sigma_r), uc].\end{aligned}\right\} \quad (50)$$

Inserting eq. (50) into eq. (49) gives

$$\pi_2 = Pr[x \geq x^0[\bar{w}(z), \beta, P] \text{ and} \quad (51)$$

$$w[\bar{w}(z), \beta, P] \geq w^0[\delta, x, r, c(\sigma_r), \varphi(\sigma_r), uc]].$$

$$\pi_2 = Pr\{x \geq x^0(\beta, P, \bar{w}(z)) \wedge w(\beta, P, \bar{w}(z)) \geq w^0(uc, \sigma_r, x, r, \delta)\}.\quad (52)$$

The probability of a contract, π_2 , consists of two interdependent probabilities, $\pi_{21} = Pr[x \geq x^0]$ and $\pi_{22} = Pr[w \geq w^0]$. The joint probability of π_{21} and π_{22} is for $x = x_1$:

$$\pi_2 = Pr[x \geq x^0 | x = x_1] \cdot Pr[w \geq w^0 | x = x_1], \quad (53)$$

$$\text{with } \pi_{21} = \begin{cases} 0 & \text{for } x_1 < x^0 \\ 1 & \text{for } x_1 \geq x^0 \end{cases}.$$

If $\pi_{21} = 1$ we get for π_2 and all abilities $x_1 \geq x^0$:

$$\pi_2 = \int_{x^0}^{\infty} Pr\{w(P, \delta, \bar{w}(z)) \geq w^0(uc, \sigma_r, r, \delta, x_1)\} \cdot f(x_1 - \delta) dx_1 \quad (54)$$

with $f(x_1 - \delta)$ the conditional density function for all $x_1 \geq x^0$.

$Pr\{w \geq w^0(uc, \sigma_r, r, \delta, x_1)\}$ can be expressed using the conditional distribution function (\tilde{H}) of wage rates at $w = w^0$, conditional on $x_1 \geq x^0$, i. e.:

$$1 - \tilde{H}(w^0(uc, \sigma_r, r, \delta, x_1)).$$

For π_2 we get:

$$\pi_2 = \int_{x^0(P, \delta, \bar{w}(z))}^{\infty} \{1 - \tilde{H}[w^0(uc, \sigma_r, r, \delta, x_1)]\} \cdot f(x_1 - \beta) dx_1. \quad (55)$$

The above considerations suggest that the probability π that a vacancy is filled by an unemployed person, i. e.,

$$\pi = \pi_1 \cdot \pi_2, \quad (56)$$

increases unambiguously if

- the ratio of unemployed persons to vacancies (u / v) increases [eq. (47)],
- the unemployment benefits uc decrease [eq. (50)].

The effect of the following variables on the probability π is ambiguous, however:

- abilities x increase: this increases the likelihood that $x \geq x^0$, but the reaction of the reservation wage is ambiguous,
- the share of long-term unemployed becomes higher: if unemployment experience is used as a screening device by the firm this lowers the probability that $x \geq x^0$, but it decreases the reservation wage if the long-term unemployed run out of unemployment benefits,
- the regional mismatch σ_r increases: this lowers the probability that a contact is made; it increases the probability that a match is formed if regional probability is a gain for the job seeker [eqs. (50), 53]].

3.5 Derivation of the Beveridge Curve

As a starting point we use the identity

$$u_{t+1} - u_t \equiv i_t - o_t \quad (57)$$

where

- u_t = number of unemployed persons at the beginning of period t ,
- i_t = inflow into unemployment during period t (in persons),
- o_t = outflow from unemployment during period t (in persons).

Inflows as well as outflows can be decomposed according to where they come from and where they go to, respectively.¹⁸ Let i^e denote inflows into unemployment from employment and o^e outflows from unemployment to employment. If i^{ne} and o^{ne} stand for flows other than from or to employment such as new entrants or drop outs, respectively, total inflows and outflows can be described by

$$i_t = i_t^e + i_t^{ne} \quad (58)$$

$$o_t = o_t^e + o_t^{ne}. \quad (59)$$

Focussing on the flows from or to employment, i^e can be formalized as the separation rate s from employment times employment n . On the other hand, o^e is defined as the probability π that a vacancy is filled by an unemployed person times the number of vacancies:

$$i_t^e = s_t \cdot n_t \quad (60)$$

¹⁸See also Budd, Levine and Smith (1987, 1988), Jackman, Layard and Pissarides (1983).

$$o_t^e = \pi_t \cdot v_t. \quad (61)$$

Inserting eqs. (58)–(61) into eq. (57) gives:

$$u_{t+1} - u_t = i_t^{ne} + s_t \cdot n_t - o_t^{ne} - \pi_t \cdot v_t. \quad (62)$$

A steady state situation is characterized by $u_{t+1} = u_t$, $i_t^{ne} = o_t^{ne}$, and $n = \bar{n}$ and is analyzed first.

$$\pi_t \cdot v_t = s_t \cdot \bar{n}. \quad (63)$$

Recall that $\pi_t = \pi_{1t} \cdot \pi_{2t}$ (subsection 3.4) and from eq. (49):

$$\pi_1 = 1 - e^{-\gamma} = 1 - \exp[-\gamma_1(\sigma_r) \cdot (u / v)], \quad (64)$$

where $\partial\pi_1/\partial\sigma_r < 0$ and $\partial\pi_1/\partial(u/v) > 0$. Then for a given probability π_2 we obtain (suppressing time subscripts):

$$\left\{ 1 - \exp[-\gamma_1(\sigma_r) \cdot (u / v)] \right\} \cdot \pi_2 \cdot v = s \cdot \bar{n}. \quad (65)$$

Eq. (65) establishes a long-run Beveridge curve. To see this rewrite eq. (65) as:

$$\frac{s}{v/\bar{n}} = \pi_2 \cdot \left\{ 1 - \exp[-\gamma_1(\sigma_r) \cdot \frac{u/\bar{n}}{v/\bar{n}}] \right\}. \quad (66)$$

From eq. (66) we get the following derivatives:

$$\frac{d(u/\bar{n})}{d(v/\bar{n})} = \frac{e^{-\gamma}(1 + \gamma) - 1}{\gamma_1(\sigma_r) \cdot e^{-\gamma}} \quad (67)$$

$$\frac{d^2(u/\bar{n})}{d(v/\bar{n})^2} = \frac{u/\bar{n}}{(v/\bar{n})^2} (e^\gamma - 1). \quad (68)$$

The first derivative [eq.(67)] is negative if $(1 + \gamma) < e^\gamma$ which is fulfilled for realistic values for u and v . Hence, the $u-v$ curve is negatively sloped. Moreover, the differential on the left hand side of eq. (67) approaches zero slope for small values of u/v and a negative infinite slope for large values of u/v [for given values of $\gamma_1(\sigma_r)$]. Finally, the second order derivative [eq. (68)] is positive for positive values of all variables indicating that this $u - v$ curve is convex to the origin.

Eq. (65) enables us to identify already three important sources for a possible outward shift of the Beveridge curve. First, the separation rate s from employment into unemployment increases. This may be caused by a higher turnover rate if this is associated with a spell of unemployment. To the extent, however, to which changes of s are caused by variations in the u/v ratio (such as higher quits if u/v decreases), this does not mean a shift of the Beveridge curve but influences its slope. Second, the effect of a higher regional dispersion σ_r on the location of the $u - v$ curve is ambiguous: it shifts outwards due to a lower contact probability, but a contrary effect stems from a lower reservation wage due

to higher search costs. Such an effect is mitigated or even offset if the matching technology improves due to measures undertaken by the labor office such as computerized information about vacancies accessible by each unemployed person. Third, the location of the $u-v$ curve changes for different values of the matching probability π . More specifically, two important sources can be identified. On the one hand, the choosiness of employers may rise which means that the minimum hiring standard x^0 exceeds the abilities x of applicants. As has been discussed in previous sections, if employers use unemployment experience as a screening device the firm may view the abilities of long-term unemployed as falling short of x^0 . From this we would conclude that higher long-term unemployment shifts the Beveridge curve outwards. Improved abilities due to training and schooling have the opposite effect. On the other hand, if the reservation wage increases relatively to the wage rate offered by the firm the Beveridge curve shifts outwards. Reasons for this may be a higher replacement ratio for unemployment benefits or better abilities. Hence, the effect of improving abilities by training (financed by the labor office, for example) is ambiguous: They provide a higher probability that the minimum hiring standard is met, but they also raise the individual's reservation wage. The same argument holds for the effect of increases of long-term unemployment: It lowers the probability that $x \geq x^0$, but it decreases w^0 if the individual runs out of unemployment benefits.

More insight into possible shifts of the Beveridge curve and its dynamics can be gained from relaxing the steady state assumptions. Recall from eq. (62) that

$$\Delta u_{t+1} = i_t^{ne} - o_t^{ne} - \pi_t \cdot v_t + s_t \cdot n_t. \quad (69)$$

As a further source of a shift of the Beveridge curve the net inflow $i_t^{ne} - o_t^{ne}$ into unemployment from outside the employment pool may change. Explanations for such a phenomenon may be found in a higher female labor force participation, in an increasing number of young people entering the labor market or in higher (early) retirement of old persons. These examples hold as long as these people enter or leave the unemployment pool and their decisions are not influenced by the u/v -ratio. Labor force effects can be taken into account more generally. Recognize that by definition employment n is the difference between labor supply and unemployment and that in turn labor supply is population in working age, pop , times the labor force participation rate k :

$$n_t = k_t \cdot pop_t - u_t. \quad (70)$$

Inserting eq. (70) into eq. (69) gives

$$\Delta u_{t+1} = i_t^{ne} - o_t^{ne} - \pi_t \cdot v_t + s_t \cdot k_t \cdot pop_t - s_t \cdot u_t. \quad (71)$$

A higher labor force participation rate as well as a growing population in the working age cause an outward shift of the Beveridge curve (via separations from employment).

The difference equation (71) also shows the dynamics around the long-run Beveridge curve [eq. (63)]. As can be seen this dynamic path traces out anti-clockwise loops: For any given number of vacancies, when u falls, the left hand side of eq. (71) increases, i. e. , the level of u will become greater. Put differently, vacancies lead unemployment.¹⁹

4 Conclusion

The aim of this paper was to investigate in a theoretical model whether the Beveridge curve is a straightforward tool to identify the nature and causes of structural unemployment. Our tentative answer to this question is ambiguous: It is a tool, but it is far from being a straightforward one. Like other well-known economic relationships such as the Phillips curve, it offers at first glance an intuitively plausible diagnosis of an economic phenomenon, but at a second look it suffers from shortcomings which stem from a rather complicated theoretical foundation which makes it difficult if not impossible to make clear predictions and to draw policy implications. The effect of many variables on the location of the Beveridge curve is ambiguous: A higher share of long-term unemployment or a greater regional mismatch may cause an outward shift of the $u - v$ relationship, but on theoretical grounds the Beveridge curve may move inwards either, to give two examples. Moreover, although our model is fairly simple it soon yields relationships which are not tractable, let alone suitable for an econometric analysis.

¹⁹See Jackman, Layard, and Pissarides (1983), p. 6.

Appendix

A Decisions of the firm: Second-order conditions and comparative-static results

Firstly we have to differentiate the first order conditions eqs. (10), (16) and (22) of the main text with respect to n , x^0 , w .

The resulting second-order derivatives are:²⁰

$$\frac{\partial^2 G^*}{(\partial n)^2} = p \cdot \int_{x^0}^{\infty} y'' \cdot \varepsilon^2 \cdot f \, dx < 0, \quad (72)$$

$$\frac{\partial^2 G^*}{\partial n \partial w} = \frac{\partial^2 G^*}{\partial w \partial n} = -1 - q' \cdot \frac{1}{\bar{w}} \cdot T^* = 0, \quad (73)$$

$$\frac{\partial^2 G^*}{\partial n \partial x^0} = \frac{\partial^2 G^*}{\partial x^0 \partial n} = f_0 \cdot (q \cdot T_0 - p \cdot \varepsilon_0 \cdot y'_0). \quad (74)$$

In the optimum training costs $q \cdot T_0$ equal the average product $n^{-1} \cdot p \cdot y_0$, therefore eq. (74) changes to:

$$\frac{\partial^2 G^*}{\partial n \partial x^0} = \frac{\partial^2 G^*}{\partial x^0 \partial n} = f_0 \cdot (n^{-1} \cdot p \cdot y_0 - p \cdot \varepsilon_0 \cdot y'_0) \quad (75)$$

Since the marginal product ($\varepsilon_0 y_0$) is smaller than the average product ($n^{-1} y_0$) in the optimum, the r.h.s. of eq. (75) turns out to be positive:

$$\frac{\partial^2 G^*}{\partial n \partial x^0} = \frac{\partial^2 G^*}{\partial x^0 \partial n} > 0$$

$$\frac{\partial^2 G^*}{(\partial w)^2} = -q'' \cdot \frac{1}{\bar{w}^2} \cdot T^* \cdot n < 0, \quad (76)$$

$$\begin{aligned} \frac{\partial^2 G^*}{\partial w \partial x^0} = \frac{\partial^2 G^*}{\partial x^0 \partial w} &= -q' \cdot \frac{1}{\bar{w}} \cdot n \cdot \frac{\partial T^*}{\partial x^0} \\ &= q' \frac{1}{\bar{w}} \cdot n \cdot T_0 \cdot f_0 < 0, \end{aligned} \quad (77)$$

$$\frac{\partial^2 G^*}{(\partial x^0)^2} = f_0 \cdot (q \cdot T'_0 \cdot n - p \cdot y'_0 \cdot n \cdot \varepsilon'_0) < 0. \quad (78)$$

²⁰In order to simplify notation in the following we write T_0 for $T(x^0)$, y_0 for $y(\varepsilon(x^0), n)$, ε_0 for $\varepsilon(x^0)$ and f_0 for $f(x^0 - \beta)$.

The Hessian matrix (\mathcal{H}) of second-order derivatives is:

$$\mathcal{H} = \begin{bmatrix} p \int_{x^0}^{\infty} y'' \varepsilon^2 f dx & 0 & f_0(qT_0 - p\varepsilon_0 y'_0) \\ 0 & -q'' T^* \frac{1}{\bar{w}^2} n & q' n T_0 f_0 \frac{1}{\bar{w}} \\ f_0(qT_0 - p\varepsilon_0 y'_0) & q' n T_0 f_0 \frac{1}{\bar{w}} & f_0(qT'_0 n - p y'_0 \varepsilon'_0 n) \end{bmatrix}. \quad (79)$$

Due to the assumptions that y'' , $T'_0 < 0$ and q'' , y'_0 , $\varepsilon'_0 > 0$ the diagonal elements are negative. The second-order principal minors are:

$$\mathcal{H}_{12} = -p \int_{x^0}^{\infty} y'' \cdot \varepsilon^2 \cdot f dx \cdot q'' \cdot \frac{1}{\bar{w}^2} \cdot T^* \cdot n > 0, \quad (80)$$

$$\mathcal{H}_{13} = p \int_{x^0}^{\infty} y'' \varepsilon^2 f dx f_0(qT'_0 n - p y'_0 n \varepsilon'_0) - [f_0(qT_0 - p\varepsilon_0 y'_0)]^2, \quad (81)$$

$$\mathcal{H}_{23} = -q'' \frac{1}{\bar{w}^2} T^* n \cdot f_0(qT'_0 n - p y'_0 n \varepsilon'_0) - (q' \frac{1}{\bar{w}} n T_0 f_0)^2. \quad (82)$$

The r.h.s. of eqs. (81) and (82) are positive if we assume that the product of the direct second-order derivatives of the profit function is greater than the product of cross derivatives. The determinant of the Hessian is:

$$\det \mathcal{H} = \frac{\partial^2 G^*}{(\partial n)^2} \mathcal{H}_{23} + [f_0(q \cdot T_0 - p \cdot \varepsilon_0 \cdot y'_0)]^2 \cdot q'' \cdot \frac{1}{\bar{w}^2} \cdot T^* \cdot n \quad (83)$$

The determinant is negative if the first term on the r.h.s. dominates the second one.

In order to calculate the effects of the exogenous parameters \bar{w} (or z), p and β on the decisions of the firm, the first-order conditions are differentiated with respect to these variables:

$$\frac{\partial^2 G^*}{\partial n \partial z} = q' \cdot \frac{w}{\bar{w}^2} \frac{\partial \bar{w}}{\partial z} \cdot T^*; \quad (84)$$

The sign of $\frac{\partial^2 G^*}{\partial n \partial z}$ depends on the impact of z on \bar{w} .

$$\frac{\partial^2 G^*}{\partial n \partial \bar{w}} = q' \cdot \frac{w}{\bar{w}^2} \cdot T^* < 0; \quad (85)$$

$$\frac{\partial^2 G^*}{\partial n \partial p} = \int_{x^0}^{\infty} y' \cdot \varepsilon \cdot f dx > 0; \quad (86)$$

$$\frac{\partial^2 G^*}{\partial n \partial \beta} = \int_{x^0}^{\infty} f'(q \cdot T - p \cdot y' \cdot \varepsilon) dx. \quad (87)$$

The sign of $\frac{\partial^2 G^*}{\partial n \partial \beta}$ is indeterminate since f' can be positive or negative and, moreover, the term in brackets (training costs per worker minus marginal product) is ambiguous. After some calculations eq. (87) is replaced by :

$$\begin{aligned} \frac{\partial^2 G^*}{\partial n \partial \beta} = & p \cdot \int_{x^0}^{\infty} y' \varepsilon \varepsilon' n f dx + p \cdot \int_{x^0}^{\infty} y' \varepsilon' f dx \\ & - q \cdot \int_{x^0}^{\infty} T' f dx - f_0(qT_0 - py'_0 \varepsilon_0), \end{aligned} \quad (88)$$

where f' is eliminated. Nonetheless the sign of this expression is indeterminate.

$$\frac{\partial^2 G^*}{\partial w \partial z} = \left[q'' \frac{w}{\bar{w}} + q' \right] \cdot \frac{1}{\bar{w}^2} \frac{\partial \bar{w}}{\partial z} \cdot T^* n; \quad (89)$$

$$\begin{aligned} \frac{\partial^2 G^*}{\partial w \partial \bar{w}} &= \left[q'' \frac{w}{\bar{w}} + q' \right] \cdot \frac{1}{\bar{w}^2} \cdot T^* n \\ &\geq 0 \quad \text{if} \quad q'' \frac{w}{\bar{w}} + q' \geq 0; \end{aligned} \quad (90)$$

$$\frac{\partial^2 G^*}{\partial w \partial p} = 0; \quad (91)$$

$$\begin{aligned} \frac{\partial^2 G^*}{\partial w \partial \beta} &= -n \cdot q' \cdot \frac{1}{\bar{w}} \cdot \frac{\partial T^*}{\partial \beta} \\ &= n \cdot q' \cdot \frac{1}{\bar{w}} \cdot \int_{x^0}^{\infty} T \cdot f' dx \\ &= -n \cdot q' \cdot \frac{1}{\bar{w}} \left[T_0 \cdot f_0 + \int_{x^0}^{\infty} T' \cdot f dx \right] \\ &\geq 0 \quad \text{with} \quad T_0 f_0 + \int_{x^0}^{\infty} T' f dx \geq 0. \end{aligned} \quad (92)$$

$$\frac{\partial^2 G^*}{\partial x^0 \partial z} = -f_0 \cdot n \cdot T_0 \cdot q' \cdot \frac{w}{\bar{w}^2} \cdot \frac{\partial \bar{w}}{\partial z}; \quad (94)$$

$$\frac{\partial^2 G^*}{\partial x^0 \partial \bar{w}} = -f_0 \cdot n \cdot T_0 \cdot q' \cdot \frac{w}{\bar{w}^2} > 0; \quad (95)$$

$$\frac{\partial^2 G^*}{\partial x^0 \partial p} = -f_0 \cdot y_0 < 0; \quad (96)$$

$$\frac{\partial^2 G^*}{\partial x^0 \partial \beta} = -f'_0 \cdot (q \cdot T_0 \cdot n - p \cdot y_0) = 0. \quad (97)$$

The effects of the parameters \bar{w} , p and β are calculated via Cramer's rule. For the influences of \bar{w} we obtain:

$$\frac{\partial n}{\partial \bar{w}} = \frac{1}{\det \mathcal{H}} \cdot \left[\frac{w}{\bar{w}} \mathcal{H}_{23} + q' n^2 \frac{1}{\bar{w}^2} T_0 f_0^2 (qT_0 - py'_0 \varepsilon_0) \right], \quad (98)$$

$$\begin{aligned} \frac{\partial w^f}{\partial \bar{w}} = & - \frac{1}{\det \mathcal{H}} \cdot q' \frac{1}{\bar{w}^2} n \left[T^{**} \mathcal{H}_{13} + \frac{w}{\bar{w}} f_0 T_0 q' \left(T^{**} f_0 (qT_0 - py'_0 \varepsilon_0) \right. \right. \\ & \left. \left. + f_0 T_0 n \cdot p \int_{x^0}^{\infty} y'' \varepsilon^2 f dx \right) \right], \end{aligned} \quad (99)$$

$$\begin{aligned} \frac{\partial x^0}{\partial \bar{w}} = & \frac{1}{\det \mathcal{H}} \cdot n \frac{1}{\bar{w}^2} \left[\frac{w}{\bar{w}} q'' T^{**} f_0 (qT_0 - py'_0 \varepsilon_0) \right. \\ & \left. - f_0 n q' T_0 \cdot p \int_{x^0}^{\infty} y'' \varepsilon^2 f dx \right]. \end{aligned} \quad (100)$$

The reaction of the firm to a change in the exogenous wage rate cannot be determined unambiguously. The influences of z result from multiplication eqs. (98) - (100) with $\partial \bar{w} / \partial z$.

For the impact of p we find:

$$\frac{\partial n}{\partial p} = - \frac{1}{\det \mathcal{H}} \cdot \left[\int_{x^0}^{\infty} y' \varepsilon f dx \cdot \mathcal{H}_{23} + f_0^2 y_0 (qT_0 - p \varepsilon_0 y'_0) q'' \frac{1}{\bar{w}^2} T^{**} n \right] \quad (101)$$

$$\frac{\partial w}{\partial p} = - \frac{1}{\det \mathcal{H}} \cdot q' \frac{1}{\bar{w}} n f_0 T_0 \left[\int_{x^0}^{\infty} y' \varepsilon f dx f_0 (qT_0 - py'_0 \varepsilon_0) + f_0 y_0 \cdot p \int_{x^0}^{\infty} y'' \varepsilon^2 f dx \right]. \quad (102)$$

$$\frac{\partial x^0}{\partial p} = - \frac{1}{\det \mathcal{H}} \cdot q'' \frac{1}{\bar{w}^2} T^{**} n \left[\int_{x^0}^{\infty} y' \varepsilon f dx \cdot f_0 (qT_0 - p \varepsilon_0 y'_0) + f_0 y_0 \cdot p \int_{x^0}^{\infty} y'' \varepsilon^2 f dx \right]. \quad (103)$$

The effects depend on the expression in squared brackets. These terms are the same in eqs. (102) and (103). Since q' is negative and q'' is positive, the impacts of p on w and x^0 exhibit opposite signs.

The effects of the shift variable β are ambiguous since the derivatives of the first-order conditions with respect to β are indeterminate.

$$\begin{aligned} \frac{\partial n}{\partial \beta} = & \frac{1}{\det \mathcal{H}} \cdot \left\{ p \int_{x^0}^{\infty} \varepsilon y' f' dx \mathcal{H}_{23} + \int_{x^0}^{\infty} T f' dx \cdot n \frac{1}{\bar{w}^2} \cdot \right. \\ & \left. [q'' q T^{**} f_0 (qn T'_0 - py'_0 n \varepsilon'_0) + q'^2 n f_0^2 T_0 p y'_0 \varepsilon_0] \right\}. \end{aligned} \quad (104)$$

$$\frac{\partial w}{\partial \beta} = \frac{1}{\det \mathcal{H}} \cdot \left(p \int_{x^0}^{\infty} \varepsilon y' f' dx - q \int_{x^0}^{\infty} T f' dx \right) \cdot \left(f_0 (qT_0 - p\varepsilon_0 y'_0) q' n T_0 f_0 \frac{1}{\bar{w}} \right) - q' \frac{1}{\bar{w}} n \int_{x^0}^{\infty} T f' dx \cdot \mathcal{H}_{13} \quad (105)$$

$$\frac{\partial x^0}{\partial \beta} = \frac{1}{\det \mathcal{H}} \cdot \frac{1}{\bar{w}^2} \cdot n \left\{ \left(p \int_{x^0}^{\infty} y' \varepsilon f' dx - q \int_{x^0}^{\infty} T f' dx \right) f_0 (qT_0 - p\varepsilon_0 y'_0) q'' T^* + q'^2 n T_0 f_0 \int_{x^0}^{\infty} T f' dx \cdot p \int_{x^0}^{\infty} y'' \varepsilon^2 f dx \right\} \quad (106)$$

If β is changed, the distribution function is also changed. The effects depend on whether the firm chooses a level of abilities x^0 below or above the mean of the distribution function ($\bar{x} - \beta$). If $x^0 < \bar{x}$, f' changes from a positive value to $f' < 0$ on the interval $[x^0, \infty)$.

B The effects on the reservation wage

$$\tilde{w}^0 = \frac{r(uc - c(\sigma_r)) + \int_{w^0}^{\infty} [w + \varphi(\sigma_r)] \cdot h_1(w, x) \cdot h_2(w - \delta) dw}{r + H(x, w^0, \delta)} \quad (107)$$

$$\frac{\partial \tilde{w}^0}{\partial uc} = \frac{\partial w^0}{\partial uc} = \frac{r}{r + H(\cdot)} > 0, \quad (108)$$

$$\frac{\partial \tilde{w}^0}{\partial \sigma_r} = \frac{-rc' + \int_{w^0}^{\infty} \varphi' h_1 h_2 dw}{r + H(\cdot)}. \quad (109)$$

Higher unemployment compensation increases the reservation wage. The reaction of the reservation wage on changed regional dispersions is given by eq. (41) in the main text. The effect of σ_r on the monetary term is:

$$\begin{aligned} \frac{dw^0}{d\sigma_r} &= \frac{d\tilde{w}^0}{d\sigma_r} - \varphi' = \frac{\int_{w^0}^{\infty} \varphi' h_1 h_2 dw - \varphi' \cdot \int_{w^0}^{\infty} h_1 h_2 dw - rc' - r\varphi'}{r + H(\cdot)} \\ &= -\frac{r \cdot \varphi' + r \cdot c'}{r + H(\cdot)}. \end{aligned} \quad (110)$$

The reservation wage \tilde{w}^0 and its monetary term w^0 are changed in opposite directions if the regional dispersion varies.

$$\begin{aligned} \frac{\partial \tilde{w}^0}{\partial \delta} &= \frac{\partial w^0}{\partial \delta} = -\frac{(r + H(\cdot)) \cdot \int_{w^0}^{\infty} [\cdot \cdot] \cdot h_1 \cdot h_2'(w - \delta) dw}{[r + H(\cdot)]^2} \\ &\quad - \frac{(r(uc - c) + \int_{w^0}^{\infty} [\cdot \cdot] \cdot h_1 \cdot h_2 dw) \cdot \frac{\partial H}{\partial \delta}}{[r + H(\cdot)]^2}. \end{aligned} \quad (111)$$

This effect depends on $\frac{\partial H}{\partial \delta}$ and h'_2 . For $\frac{\partial H}{\partial \delta}$ we have:

$$\frac{\partial H}{\partial \delta} = - \int_{w^0}^{\infty} h_1(x, w) \cdot h'_2(w - \delta) dw. \quad (112)$$

It follows from $h'_2 \lesssim 0$ that $\frac{\partial H}{\partial \delta} \gtrsim 0$. Therefore the sign of the r.h.s. of eq. (111) is not determined unambiguously.

$$\begin{aligned} \frac{\partial \tilde{w}^0}{\partial x} = \frac{\partial w^0}{\partial x} = & \frac{(r + H(\cdot)) \cdot \int_{w^0}^{\infty} [\cdot \cdot] \cdot \frac{\partial h_1}{\partial x} \cdot h_2 dw}{[r + H(\cdot)]^2} \\ & - \frac{(r(uc - c) + \int_{w^0}^{\infty} [\cdot \cdot] \cdot h_1 \cdot h_2 dw) \cdot \frac{\partial H}{\partial x}}{[r + H(\cdot)]^2}. \end{aligned} \quad (113)$$

Derivation of H with respect to x leads to

$$\frac{\partial H}{\partial x} = \int_{w^0}^{\infty} \frac{\partial h_1}{\partial x} \cdot h_2(w - \delta) dw \begin{matrix} \geq \\ < \end{matrix} 0 \text{ if } \frac{\partial h_1}{\partial x} \begin{matrix} \geq \\ < \end{matrix} 0. \quad (114)$$

Therefore the sign of this effect is indeterminate too.

$$\frac{\partial \tilde{w}^0}{\partial r} = \frac{\partial w^0}{\partial r} = \frac{(uc - c) \cdot H(\cdot) - \int_{w^0}^{\infty} [\cdot \cdot] \cdot h_1 \cdot h_2 dw}{[r + H(\cdot)]^2}. \quad (115)$$

The reaction of the reservation wage on a change in the discount rate depends on whether the unemployment compensation dominates the sum of search costs and the expected net gain from accepting a job offer.

C Reactions of the probability of a contract, π_2 , with changing determinants of x^0 and w^0

$$\pi_2 = \int_{x^0(P, \beta, \tilde{w}(z))}^{\infty} \{1 - \tilde{H}[w^0(uc, \sigma_r, r, \delta, x_1)]\} \cdot f(x_1 - \beta) dx_1 \quad (116)$$

In order to simplify notation the parameters determining the reservation wages \tilde{w}^0 resp. w^0 are represented by a vector τ . The determinants of x^0 except β are components of the vector k . Then eq. (116) changes to:

$$\pi_2 = \int_{x^0(k, \beta)}^{\infty} \{1 - \tilde{H}[w^0(\tau)]\} \cdot f(x_1 - \beta) dx_1. \quad (117)$$

The impact of k turns out to be:

$$\frac{\partial \pi_2}{\partial k} = - \frac{\partial x^0}{\partial k} \cdot \{1 - \tilde{H}[w^0(\tau)]\} \cdot f(x^0 - \beta). \quad (118)$$

Since we could not find an unambiguous reaction of x^0 on changes of k , the impact on the probability π_2 is ambiguous too.

$$\frac{\partial \pi_2}{\partial \beta} = -\frac{\partial x^0}{\partial \beta} \cdot \{1 - \tilde{H}[w^0(\tau)]\} \cdot f(x^0 - \beta) dx_1 - \int_{x^0(k, \beta)}^{\infty} \{1 - \tilde{H}[w^0(\tau)]\} \cdot f'(x_1 - \beta) dx_1. \quad (119)$$

This result is also indeterminate.

For the effect of τ we obtain:

$$\frac{\partial \pi_2}{\partial \tau} = - \int_{x^0(k, \beta)}^{\infty} -\tilde{H}'[w^0(\tau)] \cdot \frac{\partial w^0}{\partial \tau} \cdot f(x_1 - \beta) dx_1. \quad (120)$$

The sign of this expression depends on whether \tilde{H}' is positive or negative.

D The probability π that a vacancy is filled by an unemployed person

$$\pi = \left[1 - e^{-\gamma_1(\sigma_r) \cdot \frac{u}{v}}\right] \cdot \int_{x^0}^{\infty} \left\{1 - \tilde{H}[w^0(uc, \sigma_r, r, \delta, x_1)]\right\} \cdot f(x_1 - \beta) dx_1 \quad (121)$$

$$\frac{\partial \pi}{\partial u} = \frac{\gamma_1}{v} e^{-\gamma_1 \frac{u}{v}} \cdot \pi_2 > 0 \quad (122)$$

$$\frac{\partial \pi}{\partial v} = -\frac{\gamma_1 u}{v^2} e^{-\gamma_1 \frac{u}{v}} \cdot \pi_2 < 0 \quad (123)$$

$$\frac{\partial \pi}{\partial \sigma_r} = \frac{\partial \pi_1}{\partial \sigma_r} \cdot \pi_2 + \frac{\partial \pi_2}{\partial \sigma_r} \cdot \pi_1, \quad (124)$$

$$\frac{\partial \pi}{\partial \tau} = \frac{\partial \pi_2}{\partial \tau} \cdot \pi_1 = -\pi_1 \cdot \int_{x^0}^{\infty} \tilde{H}' \cdot \frac{\partial w^0}{\partial \tau} f(x_1 - \beta) dx_1 \quad (125)$$

$$\frac{\partial \pi}{\partial k} = \pi_1 \cdot \frac{\partial \pi_2}{\partial k} \quad (126)$$

$$= -\pi_1 \cdot \frac{\partial x^0}{\partial k} \left\{1 - \tilde{H}[w^0(\tau)]\right\} f(x^0 - \beta) dx;$$

$$\frac{\partial \pi}{\partial \beta} = \pi_1 \cdot \frac{\partial \pi_2}{\partial \beta} \quad (127)$$

$$= -\pi_1 \cdot \left[\frac{\partial x^0}{\partial \beta} \left\{1 - \tilde{H}[w^0(uc, \sigma_r, r, \delta, x^0)]\right\} f(x^0 - \beta) \right]$$

$$- \int_{x^0}^{\infty} \{1 - \tilde{H}\} \cdot f'(x_1 - \beta) dx_1 \Big].$$

Due to the fact that eqs. (124) - (127) contain several terms which have already shown to be indeterminate these effects are also ambiguous.

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