

Lengnick, Matthias; Wohltmann, Hans-Werner

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## Optimal monetary policy in a new Keynesian model with animal spirits and financial markets

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# optimal monetary policy in a new keynesian model with animal spirits and financial markets

by Matthias Lengnick and Hans-Werner Wohltmann

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# Optimal Monetary Policy in a New Keynesian Model with Animal Spirits and Financial Markets

Matthias Lengnick\*

*University of Kiel*

Hans-Werner Wohltmann\*\*

*University of Kiel*

## Abstract

This paper relates to the literature on macro-finance-interaction models. We modify the boundedly rational New Keynesian model of De Grauwe (2010a) using a completely micro-founded IS equation, and combine it with the agent-based financial market model of Westerhoff (2008). For this purpose we derive four interactive channels between the financial and real sector where two channels are strictly microfounded. We analyze the impact of the different channels on economic stability and derive optimal (simple) monetary policy rules. We find that coefficients of optimal simple Taylor rules do not significantly change if financial market stabilization becomes part of the central bank's objective function. Additionally, we show that rule-based, backward-looking monetary policy creates huge instabilities if expectations are boundedly rational.

*JEL classification: E03, E5, G02*

*Keywords: Agent-based financial markets; New Keynesian macroeconomics; microfoundation; optimal monetary policy; unconventional monetary policy*

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\* *E-mail:* lengnick@economics.uni-kiel.de, *Tel:* +49 431 188 1446, *Fax:* +49 621 181 1774

\*\* *E-mail:* wohltmann@economics.uni-kiel.de

# 1. Introduction

The financial crisis of 2008 has put new issues on the economics research agenda. Recently, a growing literature investigates how speculative phenomena in financial markets spill over to the real economy and whether or not real market developments feed back on financial speculation. One straightforward way to answer such questions is to integrate the standard New Keynesian macroeconomic (NKM) model with those of the agent-based computational (ACE) finance literature.

Early attempts in this area are Kontonikas and Ioannidis (2005) and Kontonikas and Montagnoli (2006) who connect a New Keynesian Macroeconomic (NKM) model with a financial market (FM) model where stock prices result from two different sources: a momentum-effect and a reversal towards the fundamental value. Those models are clearly inspired by the agent-based (chartist/fundamentalist) literature<sup>1</sup> on financial markets. A similar approach can also be found in Bask (2011). The major drawback of these models is the lack of a consistent approach of expectation formation. The Rational expectations (RE) hypothesis which is standard in macroeconomics is kept for the NKM part while financial markets are driven by non-rational expectations that are implicitly contained in the behavior of chartists / fundamentalists (compare Brock and Hommes (1998) for example).

Some interesting work in the macro-finance-interaction literature that does not build upon NKM for the description of the real sector has been done by Westerhoff (2012) and Naimzada and Pireddu (2013). The authors employ a classical Keynesian demand function only to represent the real sector. The advantage of this approach is simplicity. Models are typically of small scale, so that analytical solutions are tractable. This simplicity however comes at the cost of a non-microfounded, ad-hoc real economy.<sup>2</sup>

In a series of papers Paul DeGrauwe<sup>3</sup> has proposed to replace the assumption of rational expectations in standard NKMs by an evolutionary learning approach. Following the ACE-FM literature (e.g. Beja and Goldman (1980)), agents in his model apply different forecasting heuristics and adjust their believes by ex post evaluation. His approach provides a perfect real sector submodel to an integrated (i.e. macro-finance-interaction) model framework because

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<sup>1</sup> A literature overview can be found in Samanidou et al. (2007), an empirical model contest in Franke and Westerhoff (2012a). For illustrative examples on exchange rate modeling consult Grauwe and Grimaldi (2005) and Bauer et al. (2009).

<sup>2</sup> A new literature designs agent-based macroeconomic models as object oriented simulations without the need for any equation system. Consult Lengnick (2013) for a simple example and Dosi et al. (2010) for a very elaborate, policy oriented one.

<sup>3</sup> De Grauwe (2010a), De Grauwe (2010c), De Grauwe (2010b).

it allows to state both submodels using identical expectations hypothesis. A first approach to integrate NKM of the DeGrauwe type with ACE financial markets has been proposed by Lengnick and Wohltmann (2013). The authors put a Westerhoff (2008) model alongside the DeGrauwe NKM and introduce two different interaction channels.

In the paper at hand we will further develop this approach in the following way. In section 2 we will derive an extended version of the IS-curve that gives rise to new interactive channels with the financial sector. In section 3 we will adjust the expectations heuristics of the real sector subsystem and define the macro-finance-interaction model. The role of the different channels on (in)stability is evaluated in section 4. In section 5 we derive optimal simple monetary policy rules of the Taylor-type and discuss the question whether they should be forward- or backward-looking. Section 6 concludes.

## 2. Microfounding an Extended IS-Curve

One important aspect on the research agenda to integrate NKM with ACE finance is the identification of the most important channels through which the different sectors influence each other. Several channels have been proposed, but all of them share two common problems: First, the interactive channels are not microfounded or empirically identified but assumed ad hoc. Second, the literature has not agreed upon which channels are most important.

Typical assumptions for possible channels which affect the real sector from within the financial one are, (1) the existence of wealth effects (Kontonikas and Montagnoli (2006), Bask (2011), Westerhoff (2012), Naimzada and Pireddu (2013)), (2) a collateral based cost effect (Lengnick and Wohltmann (2013)) or (3) a balance-sheet based leverage targeting effect (Scheffknecht and Geiger (2011)). Typical examples for channels going in the opposite direction are (1) a misperception effect (Kontonikas and Montagnoli (2006), Westerhoff (2012), DeGrauwe and Kaltwasser (2012), Lengnick and Wohltmann (2013), Naimzada and Pireddu (2013)), (2) a negative dependence on the (real) interest rate (Kontonikas and Montagnoli (2006)), or (3) a mixture of both (Bask (2011)).

But even if the same type of channel is applied, its formalization is often very different. The wealth effect, for example, is formalized in Kontonikas and Montagnoli (2006) and Westerhoff (2012) by adding  $+c_1 s_q$  to aggregate demand, where  $c_1$  is a positive parameter and  $s_q$  the (log) stock price in period  $q$ . On the other hand, Bask (2011) adds real stock price changes  $+c_1(\Delta s_q - \pi_q)$ , where  $\pi_q$  is the inflation rate. Naimzada and Pireddu (2013) add a weighted average of the current and fundamental stock price  $+c_1 [(1 - \omega)s^f + \omega s_q]$  where  $\omega$  is the

weight and  $s^f$  the fundamental value of  $s_q$ .<sup>4</sup> In the remainder of this section, we will derive channels that follow from a strict microfoundation approach, to check which of the above mentioned channels and formalizations are in line with first order principles.

In NKM, money is typically introduced by assuming that holding money generates utility for the household. To introduce stocks within the NKM microfoundation framework, we proceed analogously and assume that holding stocks creates utility in just the same way. Following Gali (2008, p. 27-32) the household's period utility function is given by

$$U(C_q, D_q, N_q) = \frac{Z_q^{1-\sigma}}{1-\sigma} - \frac{N_q^{1+\eta}}{1+\eta} \quad (1)$$

with  $Z_q$  being a composite index defined as:

$$Z_q = [\alpha_1 C_q^{1-\nu} + \alpha_2 D_q^{1-\nu}]^{\frac{1}{1-\nu}} \quad (2)$$

In the remainder of this paper we will call this approach *stock in non-separable utility* [SINU].

Consumption is given by  $C_q$ , labor supply by  $N_q$  and the amount of stock demand by  $D_q$ . Utility is maximized with respect to the budget constraint:

$$C_q + \frac{S_q D_q}{P_q} + \frac{T_q^n}{P_q} + \frac{B_q}{P_q} = \frac{W_q}{P_q} N_q + \left( \tilde{d}_{q-1} + S_{q-1} \right) \frac{D_{q-1}}{P_q} + (1 + i_{q-1}) \frac{B_{q-1}}{P_q} \quad (3)$$

$S_q$  denotes the stock price,  $T_q^n$  nominal taxes,  $B_q$  bonds demand,  $P_q$  the goods price and  $\tilde{d}_{q-1}$  is the dividend payment per stock. The costs of current (real) stock demand  $\frac{S_q D_q}{P_q}$  appears on the left hand side of (3) while the (real) worth of past stock demand plus (real) dividend receipts  $\left( \tilde{d}_{q-1} + S_{q-1} \right) \frac{D_{q-1}}{P_q}$  is added to the right hand side. It is assumed that dividend payments  $\tilde{d}_{q-1}$  are earned by firms in  $q-1$  and distributed to households at the beginning of period  $q$ .

Solving the above optimization problem for an infinitely lived household yields the stock demand function<sup>5</sup>

$$d_q = x_q - c_3(s_q - p_q) - c_4 i_q \quad (4)$$

where lower case letters denote log differences, i.e. relative deviations from steady state. In-

<sup>4</sup> Compare Kontonikas and Montagnoli (2006), eq. (3); Westerhoff (2012), eq. (2.3); Bask (2011), eq. (1); Naimzada and Pireddu (2013), section 2.1.

<sup>5</sup> Compare appendix A for a detailed derivation.

terpretation of the dependencies of  $d_q$  is straightforward: (1) The demand for stocks increases if an agent can afford higher consumption (which results in a higher output gap  $x_q$ ). (2) The higher the real price of stocks  $s_q - p_q$ , the lesser its demand. (3)  $d_q$  also depends on bond yields  $i_q$ , because bonds are a substitute for stocks: If bonds demand becomes more profitable, stock demand would decrease. Note that stock demand does not (directly) depend on the expected stock price change between  $q$  and  $q + 1$  because households' behavior is not driven by a speculative motive of stock demand.

The extended IS curve becomes

$$x_q = \mathbb{E}_q [x_{q+1}] - \frac{1}{\sigma} (i_q - \mathbb{E}_q [\pi_{q+1}]) + c_1 \cdot \mathbb{E}_q [\Delta s_{q+1} - \pi_{q+1}] + c_2 \cdot \mathbb{E}_q [\Delta i_{q+1}] + \epsilon_q^x \quad (5)$$

with the two new (positive) constants  $c_1$  and  $c_2$ . A detailed derivation can be found in appendix A. The interpretation of (5) is again straightforward and closely follows Galí (2008, chapter 2.5.2). In the case of expected (real) stock price increases ( $\mathbb{E}_q [\Delta s_{q+1} - \pi_{q+1}] > 0$ ), households expect future (real) stock prices to be higher than today. Hence, they expect lower stock demand for the future compared to today ( $d_{q+1} < d_q$ , see eq. (4)). Consequently, marginal utility of future consumption is lower than of current consumption. To smooth marginal utility of consumption in  $q$  and  $q + 1$ , current consumption is increased. The same rationale holds for the expected change in government bond yields. If  $i_q$  is expected to rise ( $\mathbb{E}_q [\Delta i_{q+1}] > 0$ ) future stock demand is expected to be lower than today ( $d_{q+1} < d_q$ ) which (as above) leads to increased current consumption and output.

Given the assumptions on the wealth effect of other authors (discussed above), we can conclude from this section that Bask (2011) was closest to a channel that is in line with utility optimizing behavior although he had a slightly different timing ( $+c_1(\Delta s_q - \pi_q)$  instead of  $+c_1 \cdot \mathbb{E}_q [\Delta s_{q+1} - \pi_{q+1}]$ ).

### 3. The Model

One problem that has to be solved when joining a NKM model with an ACE-FM is that both are developed to run on different time scales because transactions in financial markets take

place in much smaller time intervals (compared to the real market).<sup>6</sup> Time indices in the NKM represent quarters, while in ACE-FM they are interpreted as days.

To allow for a meaningful integrated model, we have to make sure that both submodels still run on the time scale they are designed for. For this purpose we assume that the financial market performs 64 increments of the time index  $t$  within one increment of the real market's time index  $q$  (figure 1). Quarter  $q$  consists of the days<sup>7</sup>  $t = 64(q - 1) + 1, \dots, 64q$ .<sup>8</sup>

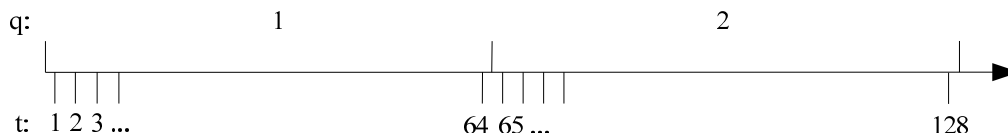


Figure 1: Time scale; indexed by days  $t$  and quarters  $q$

### 3.1. Real Sector

The real sector of our integrated model consists of a Taylor rule, an inflation equation of the Phillips-type and our extended IS-Curve (5). To be able to compare the model to others in the literature<sup>9</sup> we also allow for a cost effect ( $-\kappa s_q$ ) in the Phillips Curve:

$$\pi_q = \beta \tilde{E}_q [\pi_{q+1}] + \gamma x_q - \kappa s_q + \epsilon_q^\pi \quad (6)$$

The Taylor rule is depending on expected future inflation rate and output gap:<sup>10</sup>

$$i_q = \delta_\pi \left( \tilde{E}_q [\pi_{q+1}] - \pi^* \right) + \delta_x \tilde{E}_q [x_{q+1}] + \epsilon_q^i \quad (7)$$

The quarterly value of stock prices is given by the average of the corresponding daily values:<sup>11</sup>

$$s_q = \frac{1}{64} \sum_{t=64(q-1)+1}^{64q} s_t \quad (8)$$

<sup>6</sup> Algorithmic trading, for example, which is typically of a very short-term intra-daily nature already accounts for up to 60% of financial market transactions (Matheson (2011)). Such high frequencies are unusual for transactions of the real economy, e.g. labor is bought for at least one month. Consult also the approach of Franke and Sacht (2014) and Ahrens and Sacht (2014).

<sup>7</sup> It is assumed that trading does not take place on weekends.

<sup>8</sup> Lengnick and Wohltmann (2013), section 2.

<sup>9</sup> Compare literature overview in section 1.

<sup>10</sup> We relax this assumption in section 5.

<sup>11</sup> Compare Lengnick and Wohltmann (2013), eq. (24)



Alternatively, one could assume decaying weights to account for the fact that recent information has a stronger influence on traders than older information. It has been shown that the model outcome is robust against this assumption.<sup>12</sup>

Expectations are formed in a boundedly rational way using discrete choice learning. For the output gap the set of heuristics is given by:

$$\text{Targeters: } \tilde{E}_q^{\text{tar}} [x_{q+1}] = \bar{x} \quad (9)$$

$$\text{Static exp.: } \tilde{E}_q^{\text{sta}} [x_{q+1}] = x_{q-1} \quad (10)$$

$$\text{Extrapolaters: } \tilde{E}_q^{\text{ext}} [x_{q+1}] = x_q + \alpha_x \cdot [x_q - x_{q-1}] \quad (\alpha_x > 0) \quad (11)$$

Compared to the original De Grauwe model, we assume a different set of expectations that is more in line with those typically assumed in ACE-FM.<sup>13</sup> Additionally, we do not assume a hybrid form for the IS- and Phillips-Curve, because the boundedly rational expectations approach (especially static exp.) already gives rise to persistence in line with the rule-of-thumb idea. A hybrid form is therefore not necessary any more.<sup>14</sup>

A further advantage of our specification is that the special case of full price flexibility is still included in the model, while the original DeGrauwe model becomes explosive for high degrees of price flexibility. The NKM is derived by introducing real rigidities into the Real Business Cycle (RBC) model. Therefore, the RBC model is still incorporated in the NKM as a special case. This aspect is important conceptually and should also hold for a boundedly rational version of the NKM (compare appendix E for further details on this and other advantages of our specification).

For inflation, heuristics are given by:

$$\text{Targeters: } \tilde{E}_q^{\text{tar}} [\pi_{q+1}] = \pi_q^* \quad (12)$$

<sup>12</sup> Compare Lengnick and Wohltmann (2013), section 2 of online appendix.

<sup>13</sup> Compare, for example, the model of Westerhoff (2008) or De Grauwe and Grimaldi (2006) which is based on Brock and Hommes (1998). For an alternative approach where chartism is based on a moving average rule consult Chiarella et al. (2006).

<sup>14</sup> Introduction of hybridity into the baseline NKM is typically justified (microfounded) by assuming *habit formation* (Ravn et al. (2010), Smets and Wouters (2007)) or *rule-of-thumb* (Amato and Laubach (2003)) behavior. The BR expectations of DeGrauwe clearly fall in the second category because, first, they yield the same result on the aggregate level (i.e. persistence) and, second, they follow the four criteria (compare Amato and Laubach (2003) and Menz (2008)) for rule-of-thumb behavior: (1) They are applied if RE induce too high costs. (2) The orientation variable should be easily observable by the agents. (3) Calculating forecasts should involve virtually no computational burden. (4) Agents should learn, and learning algorithms should make sure that individual choices have converged once a steady state is reached.

$$\text{Static exp.: } \tilde{\mathbf{E}}_q^{\text{sta}} [\pi_{q+1}] = \pi_{q-1} \quad (13)$$

$$\text{Extrapolaters: } \tilde{\mathbf{E}}_q^{\text{ext}} [\pi_{q+1}] = \pi_q + \alpha_\pi \cdot [\pi_q - \pi_{q-1}] \quad (\alpha_\pi > 0) \quad (14)$$

Depending on their past performance, measured by the mean squared forecast error (MSFE), each forecasting heuristic  $j \in \{\text{tar}, \text{sta}, \text{ext}\}$  is ascribed a level of *attractivity*

$$A_q^{y,j} = - \left( y_{q-1} - \tilde{\mathbf{E}}_{q-2}^j [y_{q-1}] \right)^2 + \zeta A_{q-1}^{y,j} \quad y \in \{x, \pi\} \quad (15)$$

with the memory parameter  $\zeta$ . The fraction of agents  $\omega_q^{y,i}$  applying heuristic  $i$  is given by a discrete choice model

$$\omega_q^{y,j} = \frac{\exp\{\phi A_q^{y,j}\}}{\exp\{\phi A_q^{y,\text{tar}}\} + \exp\{\phi A_q^{y,\text{sta}}\} + \exp\{\phi A_q^{y,\text{ext}}\}} \quad (16)$$

and *market expectations* are given by the weighted average:

$$\tilde{\mathbf{E}}_q [y_{q+1}] = \sum_j \omega_q^{y,j} \tilde{\mathbf{E}}_q^j [y_{q+1}] \quad (17)$$

De Grauwe points out<sup>15</sup>, that agents do not use heuristics (instead of RE) *"because they are irrational, but rather because the complexity of the world is overwhelming"* that ex ante calculation of mean time paths is impossible. Therefore, *"heuristics [are] a rational response of agents who are aware of their limited capacity to understand the world"*. In the remainder of the paper we will denote this response *boundedly rational* [BR] to distinguish it from strict RE.

To keep the model simple, we do not add a set of heuristics and a discrete choice learning model for interest rate expectations of eq. (5), but only use static expectations:  $\tilde{\mathbf{E}}_q [i_{q+1}] = i_q$ . The solution of our real sector model is then given by (compare appendix B)

$$\begin{pmatrix} x_q \\ \pi_q \end{pmatrix} = \mathbf{A}_q^{-1} \mathbf{C}_q \begin{pmatrix} x_{q-1} \\ \pi_{q-1} \end{pmatrix} + \mathbf{A}_q^{-1} \mathbf{D}_q \cdot s_q + \mathbf{A}_q^{-1} \mathbf{F}_q \cdot s_{q-1} + \mathbf{A}_q^{-1} \begin{pmatrix} \sigma \epsilon_q^x - \epsilon_q^i \\ \epsilon_q^\pi \end{pmatrix} \quad (18)$$

with the time dependent matrices:

$$\mathbf{A}_q = \begin{pmatrix} \sigma - (\sigma - \delta_x) \omega_{x,q}^{\text{ext}} (1 + \alpha_x) - \sigma c_1 \omega_{s,q}^{\text{tar}} h & -(1 - \delta_\pi - \sigma c_1) \omega_{\pi,q}^{\text{ext}} (1 + \alpha_\pi) \\ -\gamma & 1 - \beta \omega_{\pi,q}^{\text{ext}} (1 + \alpha_\pi) \end{pmatrix}$$

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<sup>15</sup>De Grauwe (2010b), p. 415.

$$\mathbf{C}_q = \begin{pmatrix} (\sigma - \delta_x)(\omega_{x,q}^{\text{hab}} - \alpha_x \omega_{x,q}^{\text{ext}}) & (1 - \delta_\pi - \sigma c_1)(\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}}) \\ 0 & \beta (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}}) \end{pmatrix}$$

$$\mathbf{D}_q = \begin{pmatrix} c_1 \sigma (\omega_{s,q}^{\text{ext}} (1 + \alpha_s) - 1) \\ -\kappa \end{pmatrix} \quad \mathbf{F}_q = \begin{pmatrix} c_1 \sigma (\omega_{s,q}^{\text{sta}} - \alpha_s \omega_{s,q}^{\text{ext}}) \\ 0 \end{pmatrix}$$

### 3.2. Financial Sector

We use the model of Westerhoff (2008) for the financial sector of our economy.<sup>16</sup> In this section, we will shortly describe the original Westerhoff model. Afterwards it will be adjusted to allow for interactions with the real economy.

In this model, agents learn from a set of two different rules:

$$\text{Chartists: } \tilde{\mathbf{E}}_t^C [s_{t+1}] = s_t + k^C [s_t - s_{t-1}] \quad (19)$$

$$\text{Fundamentalists: } \tilde{\mathbf{E}}_t^F [s_{t+1}] = s_t + k^F [s_t^f - s_{t-1}] \quad (20)$$

Chartists believe in a continuation of the recently observed stock price trend while fundamentalists expect a reversal towards the fundamental value  $s_t^f$ . For both groups  $j$ , the excess demand for stocks  $D_t^j$  positively depends on the direction of the expected stock price change:

$$D_t^j = \ell \left( \tilde{\mathbf{E}}_t^j [s_{t+1}] - s_t \right) + \epsilon_t^j \quad j \in \{C, F\} \quad (21)$$

Note that the above equation denotes excess demand of institutional investors of the financial market, while the completely microfounded eq. (4) denotes households' demand. The fractions of agents  $W_t^j$  applying the different strategies  $j$  are determined by a discrete choice model. In addition to strategies  $C$  and  $F$ , Westerhoff (2008) also allows a 'no trading'-strategy:

$$W_t^j = \frac{\exp\{eA_t^j\}}{\exp\{eA_t^C\} + \exp\{eA_t^F\} + \exp\{eA_t^0\}} \quad j \in \{C, F, 0\} \quad (22)$$

$A_t^j$  is the attractivity of strategy  $i$  that is determined as a function of past profits (cf. eq. (15)):

$$A_t^j = (\exp\{s_t\} - \exp\{s_{t-1}\}) D_{t-2}^j + mA_{t-1}^j \quad (23)$$

<sup>16</sup>We decided to use this model because of its straightforward assumptions and implementation. The model is also empirically validated and has successfully been used for policy analysis. For alternative models compare Dieci and Westerhoff (2010) or Tramontana et al. (2013). An interesting example on a much debated policy issue can be found in Westerhoff and Dieci (2006).

The parameter  $m \in (0, 1)$  determines the memory of traders and the attractivity of no trading is normalized to  $A_t^0 = 0$  (i.e. no profits). Price adjustment is given by a price impact function

$$s_{t+1} = s_t + a (W_t^C D_t^C + W_t^F D_t^F) + \epsilon_t^s \quad (24)$$

that relates stock price changes positively to excess demand  $(W_t^C D_t^C + W_t^F D_t^F)$ . The random term  $\epsilon_t^s$  denotes the influence of trading strategies other than  $j \in \{C, F, 0\}$ .

**Impacts from the Real Sector** For the first interactive channel, we follow Kontonikas and Montagnoli (2006), Westerhoff (2012), DeGrauwe and Kaltwasser (2012), Lengnick and Wohltmann (2013)<sup>17</sup> and Naimzada and Pireddu (2013) by assuming, that the perceived fundamental value  $s_t^f$  is biased in the direction of the recent real economic development:<sup>18</sup>

$$s_t^f = h \cdot x_q \quad q = \text{floor} \left( \frac{t-1}{64} \right), \quad h \geq 0 \quad (25)$$

The completely microfounded *stock in non-separable utility* [SINU] approach gives rise to a second channel because households' demand for stocks (eq. (4)) has to be added to the demand of institutional investors of the financial market model (eq. (24)). If we assume that households' (quarterly) demand for stocks is distributed evenly among the 64 days of the quarter, we have to add  $\frac{1}{64} \Delta d_q$  to stock demand such that the price impact function becomes:<sup>19</sup>

$$s_{t+1} = s_t + a \left( W_t^C D_t^C + W_t^F D_t^F + \frac{k}{64} \Delta d_q \right) + \epsilon_t^s \quad (26)$$

The parameter  $k$  is introduced as a generalization that allows to vary the intensity of the channel. For  $k = 0$  the channel disappears.

### 3.3. Financial and Real Sector Interaction

In total, we have four channels through which the financial and real sector could impact each other (fig. 2): Channels I and II that are in line with the literature, but assumed ad hoc, and channels IIIa and IIIb that are newly introduced by the microfounded SINU approach.

<sup>17</sup> Compare eq. (26) in Lengnick and Wohltmann (2013).

<sup>18</sup> The floor function in eq. (25) rounds a number down to the next integer.

<sup>19</sup> Note that stock prices react to *excess* demand which is given by  $\Delta d_q$ , not  $d_q$ .

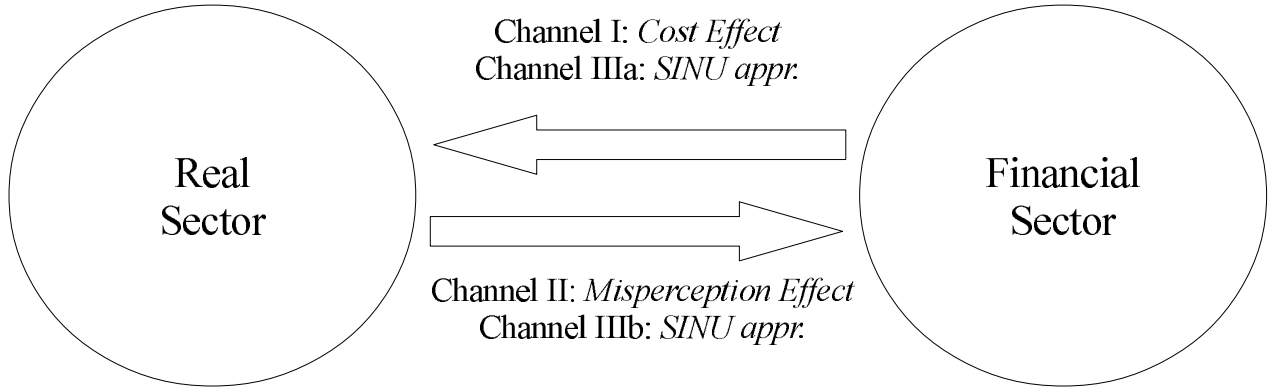


Figure 2: Real- and Financial Sector Interactions (Channels)

The economic rationale of channel I is that the nominal value of financial assets owned by firms increases when stock prices are rising. Firms' production is largely financed by credit. If their asset side of the balance sheet increases this leads to a rise in their credit worthiness and credit rating. Consequently, they have access to cheaper credits (compare Minsky (1986)). Hence, their costs of production fall which leads to lower prices. Channel IIIa results from intertemporal utility optimization of households. If households expect increasing real stock prices, they also expect falling marginal utility of consumption for the next period. Intertemporal utility smoothing makes them increase consumption today.

Channel II goes in the opposite direction and can be interpreted as follows. The *true* fundamental value of a given stock is hard to identify in reality (compare Rudebusch (2005), Bernanke et al. (1999)). If the true value of  $s_t^f$  is unknown, agents have to form assumptions about it. In our model they use proxies like the recent economic development. If output is high, they assume the fundamental value to be high and adjust their demand for stocks accordingly. The fourth channel, ch. IIIb, also results from the microfounded SINU approach. According to eq. (4), stock demand increases if (1) output increases, (2) the real stock price decreases, or (3) the nominal interest rate decreases. In all three cases, increasing demand will drive stock prices upward.

The intensity of each channel is given by the corresponding interaction parameters  $\kappa$  (Channel I),  $h$  (II),  $c_1$  (IIIa) and  $k$  (IIIb). In the special case of  $\kappa = h = c_1 = k = 0$  the two submodels operate in isolation.

## 4. Stability Analysis

In this section we are going to determine for each channel whether it is stabilizing or destabilizing the economy. For this purpose, we will vary the interaction parameters  $\kappa$ ,  $h$ ,  $c_1$  and  $k$  on an interval from zero (i.e. no interaction) upwards. All other parameters are kept constant. We report them in table 1 and keep them as our baseline parameterization throughout the entire paper. The impact on (in)stability of the real sector is measured by a typical central banks' loss function which is given as a weighted sum of the unconditional variances of inflation and output:<sup>20</sup>

$$\text{Real Sector: } L_r = \text{var}(\pi) + \frac{1}{2}\text{var}(x) \quad (27)$$

The interpretation of (27) is that volatile goods price inflation and production are associated with utility losses, where output stabilization is weighted half as much as price stability. Similarly, we set up a loss function for the financial sector

$$\text{Financial Sector: } L_f = \text{var}(s) \quad (28)$$

that associates volatile daily<sup>21</sup> stock prices with losses.

Table 1: Baseline parameterization

Real sector			Financial sector		
Structure	Learning	Noise	Structure	Learning	Noise
$\sigma = 1.0$	$\zeta = 0.5$	$\sigma_\epsilon = 0.15$	$\ell = 1$	$k^C = 0.04$	$\sigma_C = 0.05$
$\beta = 0.99$	$\phi = 10$		$a = 1$	$k^F = 0.04$	$\sigma_F = 0.01$
$\gamma = 0.33$	$\alpha_x = \alpha_\pi = 0.2$			$e = 300$	$\sigma_s = 0.01$
$\delta_x = 0.5$				$m = 0.975$	
$\delta_\pi = 1.5$					

Source: The Financial market parameterization is identical to Westerhoff (2008). The structural parameters of the real sector are standard in NKM (compare Gali (2008)):  $\sigma = 1.0$  gives rise to log utility,  $\beta = 0.99$  yields a steady state interest rate of about 4%.  $\gamma = 0.33$  follows if a unitary Frisch elasticity, a markup of 20%, constant returns to scale and price stickiness of  $\theta = 0.67$  are assumed. For the NKM learning parameters we follow De Grauwe (2010a,b,c) and Lengnick and Wohltmann (2013). For the new extrapolative heuristic, we assume a positive but mild trend extrapolation of 0.2.

<sup>20</sup> Compare e.g. Svensson (2003).

<sup>21</sup> Recall that daily stock prices are given by  $s_t$ , while quarterly are given by  $s_q$ .

To evaluate the effect of the four different interactive channels on economic stability, we proceed as follows. First, we pick pairs of two interaction parameters with opposed direction.<sup>22</sup> Second, we run the model for different values of the interactive parameters and for different realizations of the noise terms  $(\epsilon_q^i, \epsilon_q^x, \epsilon_q^\pi, \epsilon_t^s) \forall q, t$ . Finally, we compute the average loss values (eq. (27) and (28)) for a given parameterization which yields (approximately) the theoretical values  $L_r$  and  $L_f$ .<sup>23</sup>

In fig. 3 we illustrate the stabilization impact on the real sector. White areas (in the north-east) denote parameterizations for which no stable solution exists, i.e. the generated trajectories diverge/explode. Gray areas denote parameterizations that are non-explosive and the darkness indicates the corresponding loss value. The darker a region, the lower the associated loss  $L_r$ . The corresponding results for financial sector stabilization ( $L_f$ ) are displayed in figure 4.

**Channel I ( $\kappa$ )** The influence of  $\kappa$  on welfare is clearly negative for both, the real and financial subsystem. For a given  $h$  or  $k$ , an increase of  $\kappa$  leads to higher loss values. For  $h > 0.45$  or  $k > 0.14$  it even gives rise to explosive developments. E.g. if, in the top left panel of fig. 3, we fix  $h = 0.2$  and let  $\kappa$  increase from 0 upwards, we successively reach areas of higher  $L_r$  (i.e. higher loss/volatility, lower welfare). If we fix  $h = 0.8$ , the system even becomes explosive (i.e. no stable trajectory) as soon as  $\kappa > 0.12$ .

The economic explanation for this explosive behavior is straightforward. If stock prices increase, inflation will fall due to (negative) cost effects (eq. (6)). Lower inflation leads to lower inflation expectations (eq. (14)) and therefore also to an increase in output (if the extended Taylor principle  $\delta_\pi > 1 + \sigma c_1$  holds). The rising output creates a feedback mechanism that drives stock prices up further, no matter which of the opposing channels (II or IIIb) is active. If channel II is active ( $h > 0$ ), a higher perceived fundamental value leads to higher demand for stocks (eq. 25), while if channel IIIb is active ( $k > 0$ ), households directly demand more stocks (eq. 4) which drives prices up.

**Channel II ( $h$ )** For parameter  $h$  we find somewhat ambiguous results. Financial markets are always destabilized (for I and IIIa being the opposing channel). For sufficiently large  $\kappa$ ,

<sup>22</sup>For example, the parameter pair  $(\kappa, h)$  constitutes one channel that effects the financial sector from within the real sector and one channel of the opposite direction.

<sup>23</sup>This procedure is related to the approach of Naimzada and Pireddu (2013) who also vary the interaction parameter ( $\omega$  in their paper) to analyze stability. But instead of a loss function, the authors use bifurcation plots to illustrate stability impacts.

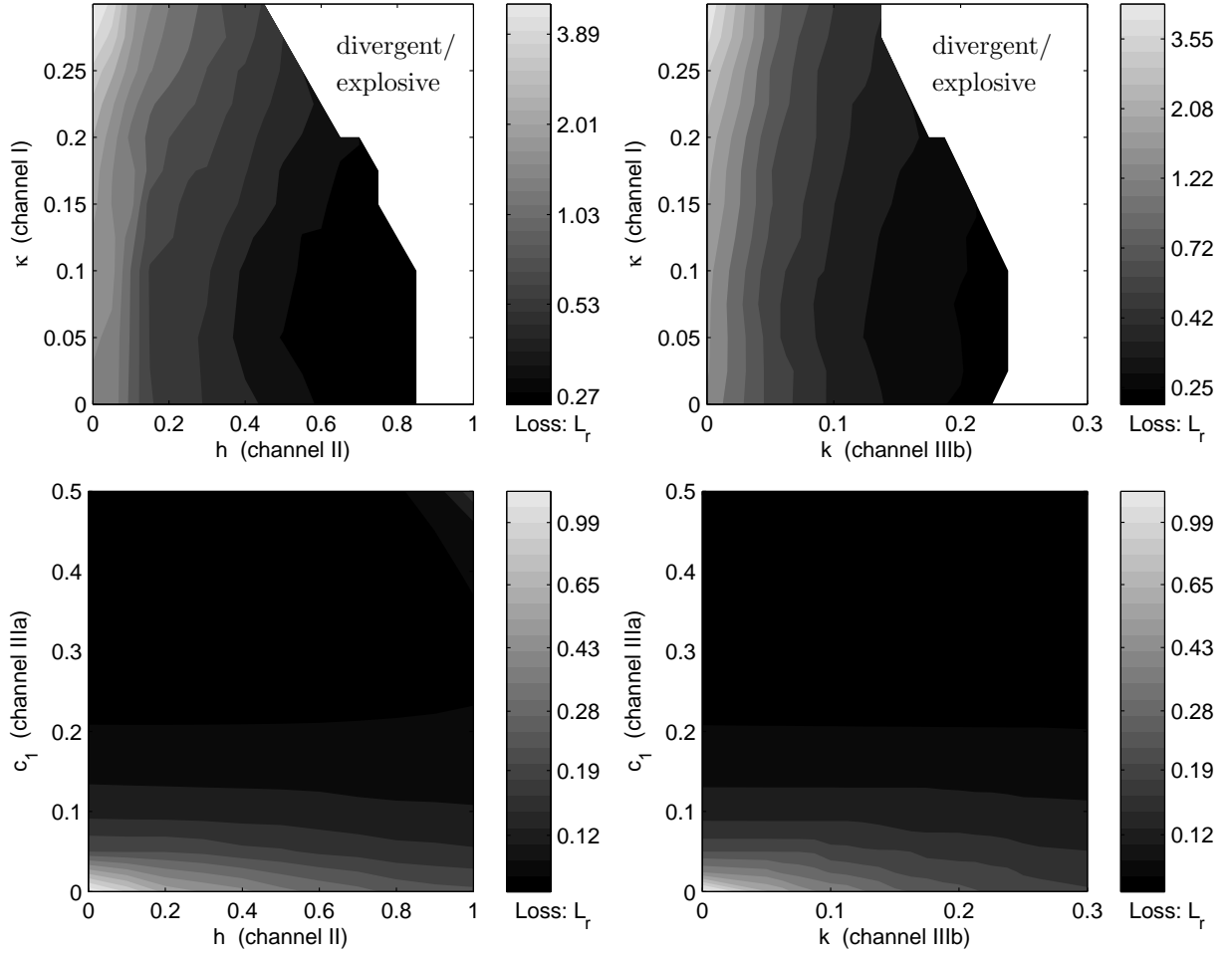


Figure 3: (De)stabilization of real submodel. Light gray areas denote parameterizations that yield high loss values (i.e. low welfare), while dark gray areas denote low losses. White areas, on the contrary, denote explosiveness.

an increase in  $h$  could even lead to explosive behavior. With respect to the real sector the results are not as clear. In combination with a significant strength of channel IIIa ( $c_1 > 0.1$ ), increasing values of  $h$  are stability neutral w.r.t.  $L_r$ , while for  $c_1 < 0.1$  a rise in  $h$  increases stability. In combination with channel I, higher  $h$  lead to more stable developments at first. If, however,  $h$  is increased above a certain threshold, the model suddenly becomes explosive.

**Channel IIIa ( $c_1$ )** Results for  $c_1$  are again ambiguous. When combined with channel IIIb, the impact of higher  $c_1$  is stabilizing for both, the real and the financial sector. When combined



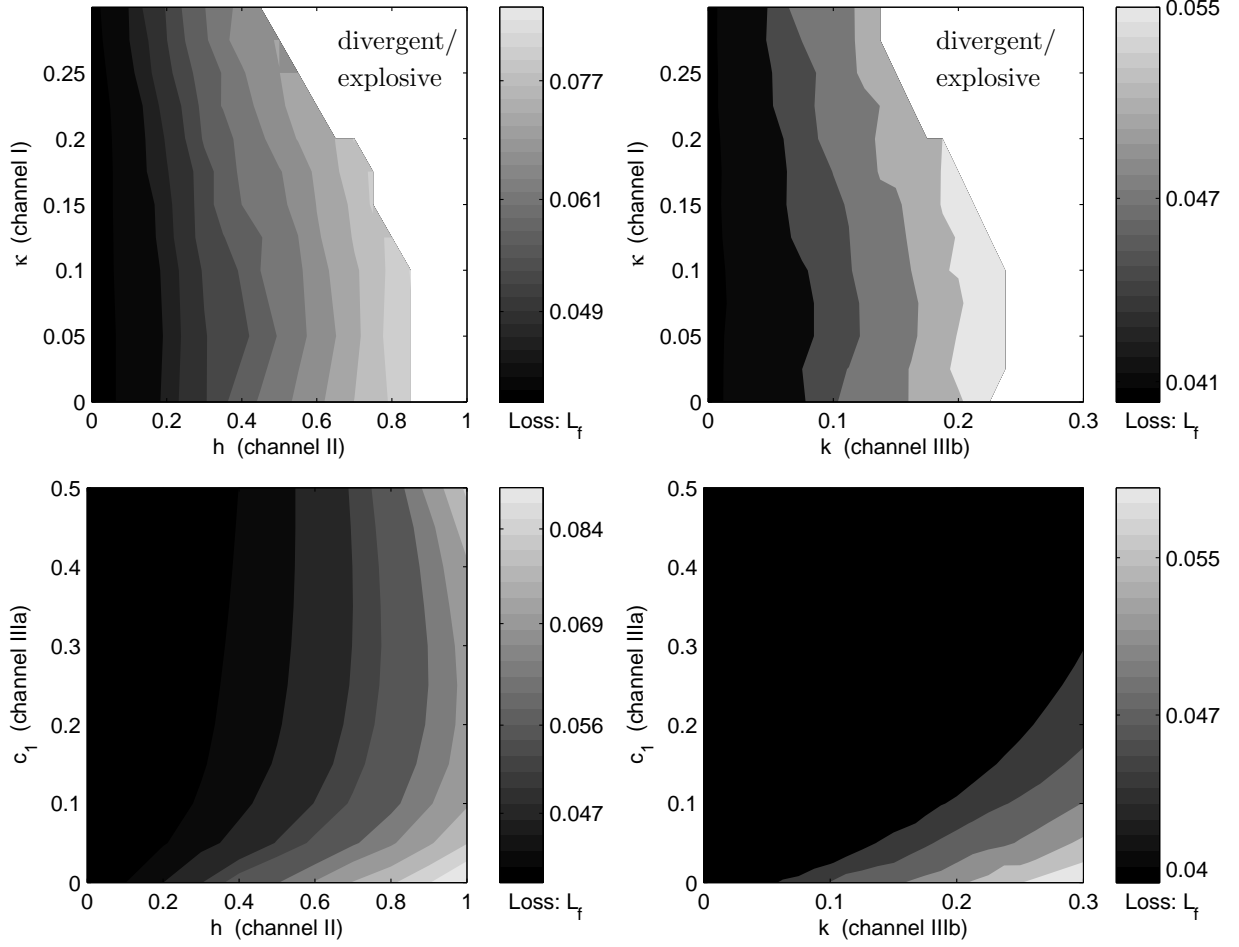


Figure 4: (De)stabilization of financial submodel. Light gray areas denote parameterizations that yield high loss values (i.e. low welfare), while dark gray areas denote low losses. White areas, on the contrary, denote explosiveness.

with channel II, higher values of  $c_1$  are only stabilizing the real sector but are almost neutral with respect to the financial one.

The economic intuition behind these results is the following. If channels IIIa and IIIb are active, an increase in output leads to higher stock demand of households (eq. (4)) and therefore higher stock prices. Through channel IIIa (eq. (5)), output depends positively on the expected future change in stock prices ( $\tilde{E}_q[\Delta s_{q+1}] = \tilde{E}_q[s_{q+1}] - s_q$ ). Higher stock prices ( $s_q$ ), therefore negatively effect output which dampens the original effect and stabilizes the economy.

If IIIa is combined with II one would expect the same results, since channel II (just as IIIb) positively relates stock prices to output development (eq. (25)). Channel II, however,

depends on market sentiments: Only if the fraction of fundamentalists in the financial market is significantly high, we could expect the misperception effect (channel II) to have a significant impact. Obviously, this dependence on market sentiments weakens the stabilizing effect of larger values of  $c_1$ : Only for  $c_1 < 0.12$  we find a positive stabilization for both markets, while for  $c_1 > 0.12$  only the real sector is stabilized by further increases of  $c_1$ .

**Channel IIIb ( $k$ )** The stability impact of  $k$  also depends on the active channels: In combination with channel I, channel IIIb has a stabilizing effect on the real market as long as  $k$  is sufficiently small. At the same time, it has a destabilizing effect on the financial sector. If  $k$  is increased by too much, however, the model dynamics become explosive (compare explanation under paragraph 'channel I ( $\kappa$ )').

In combination with channel IIIa, results are very different. Higher values of  $k$  have a positive impact on real sector stability but only for low values of  $c_1$ . Financial market stability, in contrast, is monotonically decreasing.

In a last step, we check whether the stabilizing effect of one channel could counteract the destabilizing effect of other channels by so much that a formerly explosive parametrization becomes unexplosive. As an example, we pick the parameter combinations ( $h = 0.8$ ,  $\kappa = 0.25$ ) or ( $k = 0.25$ ,  $\kappa = 0.25$ ) which both yield explosive dynamics (figure 4, top left and top right). If, additionally to these two channels, we set  $c_1 = 0.15$  the model becomes stable again in both cases. Increasing  $c_1$  therefore shifts the unstable (white) region outwards.

From this section we can conclude that there is no easy answer to the question whether interaction between financial markets and the real economy is stabilizing or destabilizing. The results depend strongly on the channels under consideration. Hence, future research has to clarify which of the proposed channels is most relevant empirically.<sup>24</sup>

## 5. Optimal Monetary Policy

In this section we will derive simple optimal policy rules for the central bank. In subsection 5.1 we derive optimal values for the Taylor parameters  $\delta_\pi$  and  $\delta_x$  under different types of policy rules and for different objective functions. In subsection 5.2 we analyze whether monetary policy should optimally be forward- or backward-looking.

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<sup>24</sup>Estimation of ACE models is relatively involved so that we have to leave this issue for future research. Compare Franke (2009) and Franke and Westerhoff (2012b) on the estimation of ACEs.

## 5.1. Optimal Simple Rule

We use the Taylor rule (7) and define the optimal simple rule [OSR] as the central banks' reaction  $(\delta_\pi^*, \delta_x^*)$  that yields the minimal loss value. In analogy to the previous section, we derive  $(\delta_\pi^*, \delta_x^*)$  as the minimizers of the average loss value over different realizations of the noise vector  $(\epsilon_q^i, \epsilon_q^x, \epsilon_q^\pi, \epsilon_t^s) \forall q, t$ :

$$(\delta_\pi^*, \delta_x^*) = \arg \min_{\delta_\pi, \delta_x} L_r \quad (29)$$

In this context, two straightforward questions arise in our interactive model: Does the presence of a financial sector change the optimal policy rule? If it does, to what extent do the different interactive channels matter?

To answer these questions, we are going to perform the optimization (29) for different cases: On the one hand, we assume different objective functions for the central bank. In a first setting we assume the typical form of a loss function where only the stability of the real subsystem is taken into account:

$$\text{Real sector only: } L_r = \text{var}(\pi) + \frac{1}{2} \text{var}(x) \quad (30)$$

In a second scenario we also add financial market stability  $\text{var}(s)$  with a relatively smaller weight:

$$\text{Real \& fin. sector: } L_{r+f} = \text{var}(\pi) + \frac{1}{2} \text{var}(x) + \frac{1}{10} \text{var}(s) \quad (31)$$

Given this loss function the central bank tries to stabilize inflation with highest priority, followed by output and by stock prices with least priority.

On the other hand, we also vary the set of interactive channels that are operating: we start with no channels, continue with all possible pairs of two channels of opposite direction, and end by activating all channels simultaneously. The resulting optimal values  $(\delta_\pi^*, \delta_x^*)$  are given in table 2.

The first interesting result is that, if the central bank additionally aims to stabilize financial markets, it should less strongly react to variations in inflation  $(\delta_\pi^*)$  while reaction to variations in output is unchanged. This result is closer examined in table 3 which shows the percentage change in both, the policy coefficients  $(\delta_\pi^*, \delta_x^*)$  and in the volatility measures  $(\text{var}(\pi), \text{var}(x), \text{var}(s))$  that occur in the OSR if the CB minimizes  $L_{r+f}$  instead of  $L_r$ . For

Table 2: Optimal simple rules for different channels and objective functions.  
Parameterization of channels:  $\kappa = 0.100$ ,  $h = 0.500$ ,  $c_1 = 0.200$ ,  $k = 0.200$ .

Channels	Real Market		Real & Fin. Market	
none	$\delta_\pi^* = 3.47$	$\delta_x^* = 1.29$	$\delta_\pi^* = 3.47$	$\delta_x^* = 1.29$
I & II	$\delta_\pi^* = 3.44$	$\delta_x^* = 1.28$	$\delta_\pi^* = 3.42$	$\delta_x^* = 1.28$
IIIa & II	$\delta_\pi^* = 3.18$	$\delta_x^* = 1.25$	$\delta_\pi^* = 3.16$	$\delta_x^* = 1.25$
I & IIIb	$\delta_\pi^* = 3.45$	$\delta_x^* = 1.27$	$\delta_\pi^* = 3.43$	$\delta_x^* = 1.27$
IIIa & IIIb	$\delta_\pi^* = 3.26$	$\delta_x^* = 1.27$	$\delta_\pi^* = 3.24$	$\delta_x^* = 1.27$
all	$\delta_\pi^* = 3.10$	$\delta_x^* = 1.18$	$\delta_\pi^* = 3.06$	$\delta_x^* = 1.18$

all channel parameterizations, the CB achieves a decrease in the volatility of  $x$  and  $s$  by accepting an increase in  $\text{var}(\pi)$ . The reason is that both channels (II & IIIb) that affect the financial sector are directly related to the output gap  $x$ . Stabilizing  $x$  therefore also indirectly stabilizes the financial market. This explains the CB's higher interest for output stabilization (which also decreases  $\text{var}(s)$ ).

To bring these results about, the CB has to lessen its reaction towards inflation (decrease in  $\delta_\pi^*$ ; table 2, second column) no matter which channels are active. The change in optimal reaction towards output is ambiguous: If the strongly destabilizing channel I (compare section 4) is active,  $\delta_x^*$  increases (table 2, third column). If channel IIIb (that weakly stabilizes the real sector; sec. 4) is active,  $\delta_x^*$  is decreased instead. This change in optimal reaction parameters, however, is very small and irrelevant for most practical considerations.

Table 3: Percentage change in OSR policy coefficients and in volatility measures if CB switches from  $L_r$  to  $L_{r+f}$ .

Channels	Policy Coefficients		Volatility Measures		
	$\delta_\pi^*$	$\delta_x^*$	$\text{var}(\pi)$	$\text{var}(x)$	$\text{var}(s)$
none	0.00%	0.00%	0.00%	0.00%	0.00%
I & II	-0.56%	+0.05%	+0.46%	-0.50%	-0.08%
IIIa & II	-0.60%	+0.41%	+0.51%	-0.53%	-0.08%
I & IIIb	-0.58%	-0.04%	+0.47%	-0.51%	-0.08%
IIIa & IIIb	-0.55%	-0.12%	+0.40%	-0.43%	-0.07%
all	-1.39%	+0.40%	+1.14%	-1.18%	-0.37%

Another robust finding is, that the central bank's reaction becomes weaker, the more

interactive channels exist. Policy reaction is strongest, if no channel is active at all. If two channels of opposite direction are added, the policy reaction becomes weaker. If all channels are active simultaneously, the central bank's intervention is decreased further. This is not caused by the interactive channels alone stabilizing the economy and making stabilization policy by the CB superfluous. On contrast, the loss values monotonically increase the more channels are taken into account (compare first column of tab. 4). Instead, the interactive channels make the economy harder to control, therefore CB policy is less efficient and has to be reduced. This finding is robust across both objective functions ( $L_r$  and  $L_{r+f}$ ) as well as different weights<sup>25</sup> within the objective function.

## 5.2. History-dependent or Forward-looking?

It is known that an inverse relationship exists between the forward-/backward-lookingness of optimal monetary policy and that of the underlying model: The more forward-looking the model becomes, the more backward-looking monetary policy should be and vice versa.<sup>26</sup>

Our boundedly rational model was originally composed in a forward-looking way (eq. (6) and (5)). However, the boundedly rational character of expectations makes the model depending on past variables (eq. (18)) and therefore backward-looking. The question therefore arises whether monetary policy should optimally be forward- or backward-looking, or something in between.

To answer this question, we derive the (expected) loss values  $L_r$  that correspond to the optimal simple rule under three different scenarios: In scenario one, monetary policy depends on expectations only (compare eq. (7)):

$$i_q = \delta_\pi \left( \tilde{\mathbb{E}}_q [\pi_{q+1}] - \pi^* \right) + \delta_x \tilde{\mathbb{E}}_q [x_{q+1}] + \epsilon_q^i$$

In the second scenario it depends on contemporaneous values

$$i_q = \delta_\pi (\pi_q - \pi^*) + \delta_x x_q + \epsilon_q^i \tag{32}$$

<sup>25</sup>If we change, for example, the weight of output stabilization from  $\frac{1}{2}$  to 1 (compare Wollmershäuser (2006)), our results remain qualitatively identical.

<sup>26</sup>This issue has been extensively discussed in macroeconomics. Consult, for example, Svensson (1997), Carlstrom and Fuerst (2000), Benhabib et al. (2003), Svensson and Woodford (2003), Eusepi (2005) and Leitemo (2008).

and in a third one it depends on the most recent past:

$$i_q = \delta_\pi (\pi_{q-1} - \pi^*) + \delta_x x_{q-1} + \epsilon_q^i \quad (33)$$

The resulting minimal loss values  $L_r$  are given in table 4. The alternative loss definition  $L_{r+f}$  is given in parenthesis. In analogy to the previous subsection we report values for different sets of interactive channels.

Table 4: Comparison of minimal loss values  $L_r$  for forward-looking, contemporaneous and backward-looking optimal simple rules. Loss values  $L_{r+f}$  in parentheses.

Channels	Policy Rule depending on		
	$\tilde{E}_q [\pi_{q+1}], \tilde{E}_q [x_{q+1}]$	$\pi_q, x_q$	$\pi_{q-1}, x_{q-1}$
none	0.086 (0.090)	0.039 (0.043)	2.60 (2.61)
I & II	0.088 (0.093)	0.041 (0.046)	$\rightarrow \infty$ ( $\rightarrow \infty$ )
IIIa & II	0.090 (0.094)	0.039 (0.044)	$\rightarrow \infty$ ( $\rightarrow \infty$ )
I & IIIb	0.088 (0.092)	0.040 (0.044)	$\rightarrow \infty$ ( $\rightarrow \infty$ )
IIIa & IIIb	0.087 (0.091)	0.039 (0.043)	$\rightarrow \infty$ ( $\rightarrow \infty$ )
all	0.093 (0.098)	0.041 (0.046)	$\rightarrow \infty$ ( $\rightarrow \infty$ )

Smallest loss values result if monetary policy reacts to contemporaneous values of output and inflation. If it reacts to agent's expectations instead, loss values increase by about 50%. This result is again robust across all combinations of interactive channels and different loss functions. If policy becomes backward-looking, the loss value increases dramatically in the case of no interaction between the financial and real sector. If interaction is taken into account, the model even becomes explosive (i.e.  $L_r \rightarrow \infty$  and  $L_{r+f} \rightarrow \infty$ ).

The standard results, mentioned in the beginning of this subsection, are generally confirmed in our boundedly rational NKM: The model is *de facto* backward-looking. Therefore, monetary policy should depend on information as recent as possible. Since computation of rational expectations is (by assumption) not possible, the most up-to-date information the CB can use is given by contemporaneous values. An interesting new aspect is that wrongly conducted (backward-looking) monetary policy could cause high volatility (large loss values) although it is strictly rule-based. In the presence of financial markets, such policy could even create explosive behavior.

To verify these results, we compare the system matrices of the real market subsystem ( $h = c_1 = 0$ ) for the three policy rules (7), (32) and (33). All system matrices are time

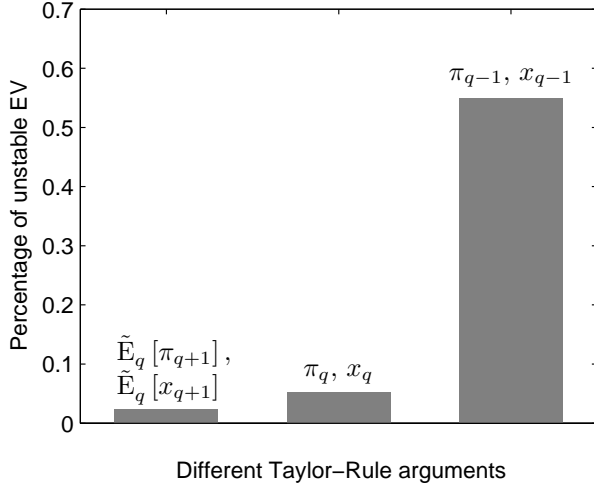


Figure 5: Stability of system matrix  $\mathbf{A}_q^{-1}\mathbf{C}_q$  for different Taylor arguments

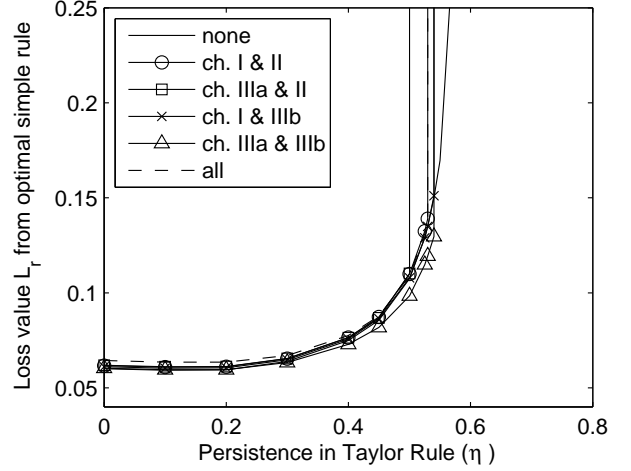


Figure 6: Effect of interest rate persistence

dependent. E.g. for rule (7) the system matrix is given by  $\mathbf{A}_q^{-1}\mathbf{C}_q$  with  $\mathbf{A}_q$  and  $\mathbf{C}_q$  defined in eq. (18).<sup>27</sup> In each time step  $q$  the fractions of agents  $\omega_q^{y,j}$  using the different heuristics (i.e. the state of the learning algorithm; (15) and (16)) are determined by recent economic development. To get an idea of how often the learning algorithm creates instabilities, we calculate the eigenvalues [EV] of  $\mathbf{A}_q^{-1}\mathbf{C}_q$  for 47916 different realizations over the entire possible range of learning states. In figure 5 we report the percentage of learning states that result in an unstable system matrix. Obviously, the probability of becoming explosive is much lower for Taylor rules depending on contemporaneous or expected future values of  $\pi$  and  $x$ . If the central bank reacts to past values, we observe an increase from about 5% to 55%.

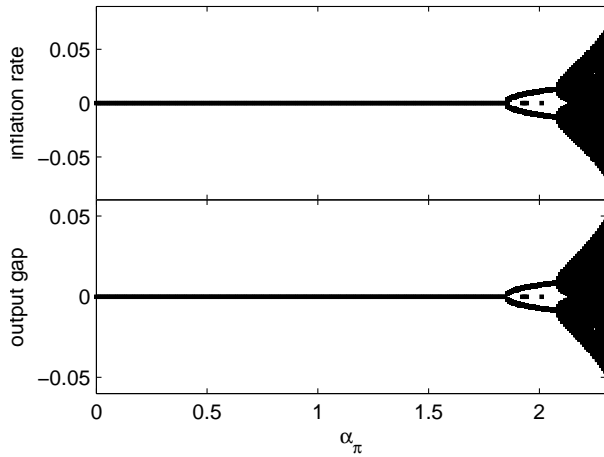
As a second verification, we introduce persistence (interest rate smoothing) into the Taylor rule

$$i_q = \eta i_{q-1} + (1 - \eta) \left\{ \delta_\pi \left( \tilde{E}_q[\pi_{q+1}] - \pi^* \right) + \delta_x \tilde{E}_q[x_{q+1}] \right\} + \epsilon_q^i \quad (34)$$

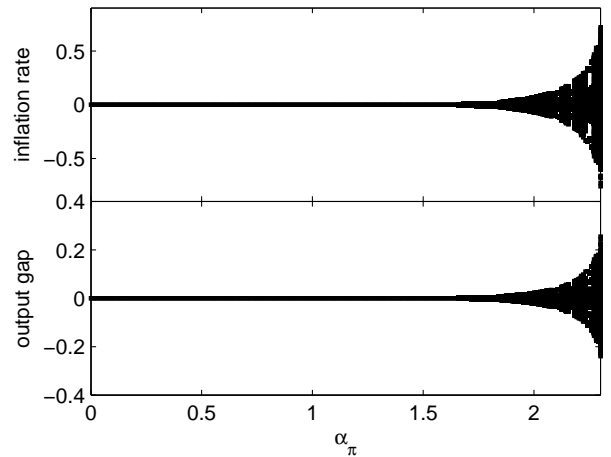
where  $\eta \in (0, 1)$  determines the degree of persistence or smoothness.<sup>28</sup> In figure 6 we illustrate the (expected) loss value  $L_r$  as a function of  $\eta$  for different interactive channels. Smallest loss values result for  $\eta \in (0, 0.2)$ . For higher  $\eta$  we find exponentially increasing losses. The result, that backward-looking monetary policy destabilizes the economy is therefore again confirmed. If a financial sector is active, losses even approach infinity.

<sup>27</sup> Compare appendices C and D for system matrices of rules (32) and (33).

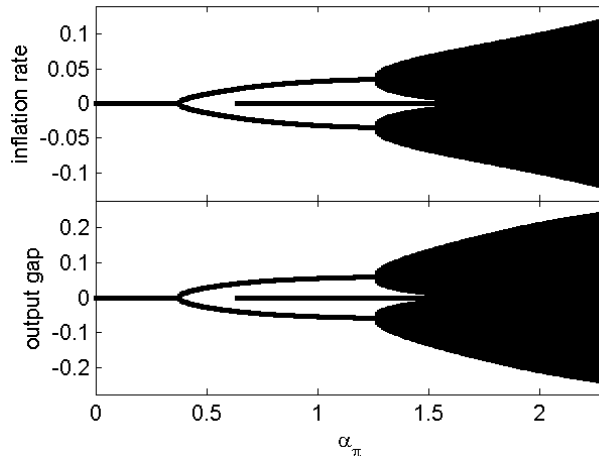
<sup>28</sup> This approach is common in the literature. Consult e.g. Clarida et al. (1998) and Clarida et al. (1999).



(a) Variables:  $\tilde{E}_q[\pi_{q+1}], \tilde{E}_q[x_{q+1}]$



(b) Variables:  $\pi_q, x_q$



(c) Variables:  $\pi_{q-1}, x_{q-1}$

Figure 7: Bifurcation plot of parameter  $\alpha_\pi$  for different monetary policy rules depending on different variables

As a third verification, we follow Naimzada and Pireddu (2013) by checking if, in a bifurcation plot<sup>29</sup>, the system loses stability earlier if the TR depends on  $\pi_{q-1}$  and  $x_{q-1}$ . Exemplary<sup>30</sup>, we show the bifurcation plots of  $\alpha_\pi$  (compare eq. (14) ) in figure 7 for the three different Taylor rules (7), (32) and (33). For rules depending on expected and contemporaneous values (panel (a) and (b)) the system loses the unique steady state at  $\alpha_\pi \approx 1.8$ .

<sup>29</sup>In the bifurcation plots we show the long run developments of the deterministic core, i.e. all stochastic terms are set to  $\epsilon_q^i = \epsilon_q^x = \epsilon_q^\pi = \epsilon_t^s = 0 \quad \forall q, t$ .

<sup>30</sup>A bifurcation analysis for all relevant parameters can be found in appendix E.2. Here, we consider  $\alpha_\pi$  only, because all other parameters do not give rise to bifurcations, except for  $\alpha_x$ , which produces very similar results to  $\alpha_\pi$  (compare fig. 10 in appendix).



If monetary policy becomes backward-looking (panel (c)) the first bifurcation takes place already at  $\alpha_\pi \approx 0.37$ .

### 5.3. Optimal Unconventional Monetary Policy

Another question that has been debated in the literature<sup>31</sup> on optimal monetary policy is, whether or not the CB should react to over-/under-valuation of financial assets (i.e. bubbles). This is typically done by adding a stock price reaction term to the Taylor rule, e.g.  $+\delta_s \cdot s_q$ . Of course, we could proceed in a similar way and simply derive the optimal  $\delta_s^*$ . But the results of Wollmershäuser (2006) who uses a similar<sup>32</sup> NKM without bounded rationality already suggest that such a welfare increase takes place but is very small.

In light of the policy recently performed by central banks in several advanced countries, we are going to analyze a slightly different question. Instead of reacting with the conventional instrument only (i.e. the interest rate), we equip the CB with another, *unconventional* instrument (i.e. direct purchases of financial assets) and derive the optimal mixture of both.

If the CB's direct purchases (given by  $\Delta d_q^{CB}$ ) are added to the price impact function (26) in just the same way as the excess demand of households, we get:

$$s_{t+1} = s_t + a \left( W_t^C D_t^C + W_t^F D_t^F + \frac{k}{64} \cdot \{ \Delta d_q + \Delta d_q^{CB} \} \right) + \epsilon_t^s \quad (35)$$

The first (conventional) instrument is given by eq. (7) while for the second (unconventional) we assume that direct purchases are proportional to stock price misalignment:

$$\begin{aligned} \text{Conventional instrument:} \quad i_q &= \delta_\pi \left( \tilde{\mathbb{E}}_q [\pi_{q+1}] - \pi^* \right) + \delta_x \tilde{\mathbb{E}}_q [x_{q+1}] + \epsilon_q^i \\ \text{Unconventional instrument:} \quad \Delta d_q^{CB} &= \delta_s^d \cdot (s_t - s^f) \end{aligned} \quad (36)$$

If, for example, the CB buys assets when prices are undervalued and sells when they are

<sup>31</sup>Rudebusch (2005), Kontonikas and Ioannidis (2005), Wollmershäuser (2006), Kontonikas and Montagnoli (2006) and Castelnovo and Nistico (2010).

<sup>32</sup>The model of Wollmershäuser consists of the typical three equation NKM extended by a nominal exchange rate, where the development of nominal exchange rates is modeled in several alternative ways. In one case it is given by a simplified chartist-fundamentalist model (eq. (3.4) and footnote 6) similar to our stock market.

overvalued, we have  $\delta_s^d < 0$ . The optimal (simple) mixture of conventional and unconventional instruments is then defined as (compare (29)):

$$(\delta_\pi^*, \delta_x^*, \delta_s^{d,*}) = \arg \min_{\delta_\pi, \delta_x, \delta_s^d} L_j \quad j \in \{r, r+f\} \quad (37)$$

The results of the optimization problem (37) for conventional versus unconventional policy are contrasted in table 5. We distinguish between the case where the CB only cares for real sector stability ( $L_r$ ) and the case where it also cares for financial market stability ( $L_{r+f}$ ).

Table 5: Conventional versus unconventional monetary policy  
Parameterization of channels:  $\kappa = 0.1$ ,  $h = 0.5$ ,  $c_1 = 0.2$ ,  $k = 0.2$

CB Target	Conventional $\delta_x, \delta_\pi$	Unconventional $\delta_x, \delta_\pi, \delta_s^d$
$L_r$	$L_r = 0.064$	$L_r = 0.062$
$L_{r+f}$	$L_{r+f} = 0.069$	$L_{r+f} = 0.062$

As expected, the loss value increases in the case of conventional policy, if  $\text{var}(s)$  is added to the welfare measure. Compared to conventional monetary policy, unconventional policy leads to a welfare gain for both welfare measures. This gain is larger, if financial market stability explicitly enters the loss function. In both cases ( $L_r$  &  $L_{r+f}$ ), however, the gain of using unconventional instruments is very small.

For simplicity we assumed in eq. (36) that the CB knows the true fundamental stock price  $s^f$ .<sup>33</sup> In more realistic settings, where the CB also has to form beliefs about the  $s^f$ , welfare gains might be even lower. Furthermore, we did not consider the presence of zero lower bound for the interest rate  $i_q$ . Unconventional instruments might be more influential if the conventional measures fail.

## 6. Conclusion

This paper extends the work of Kontonikas and Montagnoli (2006), Bask (2011), Scheffknecht and Geiger (2011), Bask (2011), Westerhoff (2012), Naimzada and Pireddu (2013), and Lengnick and Wohltmann (2013). It combines the macroeconomic BR-NKM of De Grauwe (2010a) with the financial ACE of Westerhoff (2008) by deriving a generalized IS curve that originates

<sup>33</sup>Recall, that private agents have to form beliefs about  $s^f$ . Compare eq. (25).

from a non-separable utility function including stocks. This approach gives rise to additional completely microfounded interaction channels with the financial sector.

Once the model is set up, we perform a stability analysis with ambiguous results. The cost channel is clearly destabilizing both, the real and financial sector. For the other channels, results either differ for both sectors (i.e. stabilizing one while destabilizing the other) or they change significantly with the opposing channel. In some regimes, the interactive channels strongly feed back on each other and yield explosive dynamics.

We derive optimal monetary policy rules under a set of different regimes. We find that the central bank's response to inflation decreases slightly if financial market stability enters the central bank's objective function. Another interesting result is that the optimal central bank reaction to deviations of inflation to their target becomes weaker, the higher the degree of interaction between the financial and real sector.

Finally, we test if the standard results that monetary policy should be backward-looking if the system is forward-looking (and vice versa), can be confirmed for our boundedly rational NKM. We have shown that because the backward-looking nature of the expectations algorithm turns the forward-looking model into a backward-looking one, monetary policy should optimally depend on contemporaneous variables. If, instead, the policy rule becomes backward-looking, the economy is strongly destabilized. Additionally we analyze the importance of unconventional monetary policy instruments and find that they increase welfare only marginally in comparison to conventional policy.

While a number of research questions have been answered in this paper, others had to remain open. For example, we did only focus on simple rules when deriving optimal monetary policy. A detailed treatment of optimal unrestricted policy rules in case of boundedly rational expectations should be conducted in future research.

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## Appendix

### A. Microfoundation of IS Curve with SINU Approach

The household's period utility is determined by (1)-(3) and reads

$$U(C_q, D_q, N_q) = \frac{Z_q^{1-\sigma}}{1-\sigma} - \frac{N_q^{1+\eta}}{1+\eta} \quad (38)$$

$$\text{with: } Z_q = [\alpha_1 C_q^{1-\nu} + \alpha_2 D_q^{1-\nu}]^{\frac{1}{1-\nu}} \quad (0 < \alpha_1, \alpha_2 < 1) \quad (39)$$

$$\text{s.t. } C_q + \frac{S_q D_q}{P_q} + \frac{T_q^n}{P_q} + \frac{B_q}{P_q} = \frac{W_q}{P_q} N_q + \left( \tilde{d}_{q-1} + S_{q-1} \right) \frac{D_{q-1}}{P_q} + (1 + i_{q-1}) \frac{B_{q-1}}{P_q} \quad (40)$$

$$C_q = \underbrace{-\frac{S_q D_q}{P_q} - \frac{T_q^n}{P_q} - \frac{B_q}{P_q} + \frac{W_q}{P_q} N_q + \left( \tilde{d}_{q-1} + S_{q-1} \right) \frac{D_{q-1}}{P_q} + (1 + i_{q-1}) \frac{B_{q-1}}{P_q}}_{=: Y_q} \quad (41)$$

The partial derivatives of  $U$  and  $Z$  are:

$$\frac{\partial U}{\partial Z_q} = Z_q^{-\sigma} \quad \frac{\partial Z}{\partial C_q} = \alpha_1 Z_q^\nu C_q^{-\nu} \quad \frac{\partial Z}{\partial D_q} = \alpha_2 Z_q^\nu D_q^{-\nu} \quad (42)$$

$$\frac{\partial U}{\partial C_q} = \frac{\partial U}{\partial Z_q} \frac{\partial Z_q}{\partial C_q} = \alpha_1 Z_q^{\nu-\sigma} C_q^{-\nu} \quad \frac{\partial U}{\partial D_q} = \frac{\partial U}{\partial Z_q} \frac{\partial Z_q}{\partial D_q} = \alpha_2 Z_q^{\nu-\sigma} D_q^{-\nu} \quad (43)$$

An infinitely lived household maximizes the expected infinite sum of a discounted utility stream for  $q = 0, 1, \dots, \infty$ . The Lagrangian of the given optimization problem reads:

$$\ell = \mathbb{E}_q \left[ \sum_{k=0}^{\infty} \beta^k \{ U(C_{q+k}, D_{q+k}, N_{q+k}) + \lambda_{q+k} (Y_{q+k} - C_{q+k}) \} \right] \quad (44)$$

$$\text{FOC 1: } \frac{\partial \ell}{\partial C_q} = \frac{\partial U}{\partial C_q} - \lambda_q \stackrel{!}{=} 0 \quad \Leftrightarrow \quad \lambda_q = \frac{\partial U}{\partial C_q} = \alpha_1 Z_q^{\nu-\sigma} C_q^{-\nu} \quad (45)$$

$$\text{FOC 2: } \frac{\partial \ell}{\partial B_q} = -\lambda_q \frac{1}{P_q} + \beta(1 + i_q) \cdot \mathbb{E}_q \left[ \frac{\lambda_{q+1}}{P_{q+1}} \right] \stackrel{!}{=} 0 \quad (46)$$



Plugging FOC 1 into FOC 2 yields the Euler equation

$$\alpha_1 Z_q^{\nu-\sigma} C_q^{-\nu} \frac{1}{P_q} = \beta \alpha_1 (1 + i_q) \cdot \mathbb{E}_q \left[ \frac{Z_{q+1}^{\nu-\sigma} C_{q+1}^{-\nu}}{P_{q+1}} \right] \quad (47)$$

$$\beta \cdot \mathbb{E}_q \left[ \left( \frac{Z_{q+1}}{Z_q} \right)^{\nu-\sigma} \left( \frac{C_{q+1}}{C_q} \right)^{-\nu} \left( \frac{P_q}{P_{q+1}} \right) \right] = \frac{1}{1 + i_q} \quad (48)$$

$$\text{FOC 3: } \frac{\partial \ell}{\partial D_q} = \frac{\partial U}{\partial D_q} + \lambda_q \frac{\partial Y_q}{\partial D_q} + \beta \mathbb{E}_q \left[ \lambda_{q+1} \frac{\partial Y_{q+1}}{\partial D_q} \right] \stackrel{!}{=} 0 \quad (49)$$

$$\Leftrightarrow \alpha_2 Z_q^{\nu-\sigma} D_q^{-\nu} - \alpha_1 Z_q^{\nu-\sigma} C_q^{-\nu} \frac{S_q}{P_q} + \beta \alpha_1 \mathbb{E}_q \left[ \frac{Z_{q+1}^{\nu-\sigma} C_{q+1}^{-\nu}}{P_{q+1}} \right] (\tilde{d}_q + S_q) = 0 \quad (50)$$

$$\Leftrightarrow \alpha_2 Z_q^{\nu-\sigma} D_q^{-\nu} + \beta \alpha_1 \mathbb{E}_q \left[ \frac{Z_{q+1}^{\nu-\sigma} C_{q+1}^{-\nu}}{P_{q+1}} \right] (\tilde{d}_q + S_q) = \alpha_1 Z_q^{\nu-\sigma} C_q^{-\nu} \frac{S_q}{P_q} \quad (51)$$

$$\Leftrightarrow \frac{\alpha_2}{\alpha_1} \left( \frac{D_q}{C_q} \right)^{-\nu} + \beta \mathbb{E}_q \left[ \left( \frac{Z_{q+1}}{Z_q} \right)^{\nu-\sigma} \left( \frac{C_{q+1}}{C_q} \right)^{-\nu} \frac{1}{P_{q+1}} \right] (\tilde{d}_q + S_q) = \frac{S_q}{P_q} \quad (52)$$

$$\Leftrightarrow \frac{\alpha_2}{\alpha_1} \left( \frac{D_q}{C_q} \right)^{-\nu} \frac{P_q}{S_q} + \underbrace{\frac{\tilde{d}_q + S_q}{S_q} \beta \mathbb{E}_q \left[ \left( \frac{Z_{q+1}}{Z_q} \right)^{\nu-\sigma} \left( \frac{C_{q+1}}{C_q} \right)^{-\nu} \frac{P_q}{P_{q+1}} \right]}_{\frac{1}{1+i_q}} = 1 \quad (53)$$

$$\Leftrightarrow \frac{\alpha_2}{\alpha_1} \left( \frac{D_q}{C_q} \right)^{-\nu} = \frac{S_q}{P_q} \left[ 1 - \frac{\tilde{d}_q + S_q}{S_q} \frac{1}{1 + i_q} \right] \quad (54)$$

$$\Leftrightarrow \frac{\alpha_2}{\alpha_1} \left( \frac{D_q}{C_q} \right)^{-\nu} = \frac{S_q}{P_q} \left[ \frac{S_q i_q - \tilde{d}_q}{S_q (1 + i_q)} \right] \quad (55)$$

$$\Leftrightarrow \left( \frac{D_q}{C_q} \right)^{-\nu} = \frac{\alpha_1}{\alpha_2} \frac{1}{P_q} \left[ \frac{S_q i_q - \tilde{d}_q}{1 + i_q} \right] \quad (56)$$

In the most simple case, the stock price  $s_q$  should relate to the discounted sum of future dividends (compare Campbell et al. (1997) chapter 7 for this equation and more general versions):

$$s_q = \sum_{k=0}^{\infty} \tilde{\beta}^k \mathbb{E}_q \left[ \tilde{d}_{q+k} \right] \quad (57)$$

For the sake of simplicity we do not model the expectation of future dividends  $\mathbb{E}_q \left[ \tilde{d}_{q+k} \right]$

in a way similar to the boundedly rational expectations (12)-(11) but simply assume static expectations

$$E_q \left[ \tilde{d}_{q+k} \right] = \tilde{d}_q \quad \forall k = 0, 1, \dots \quad (58)$$

which imply for (57)

$$s_q = \tilde{d}_q \frac{1}{1 - \tilde{\beta}} \quad (59)$$

$$\Gamma s_q = \tilde{d}_q \quad \text{with: } \Gamma = 1 - \tilde{\beta} \quad (60)$$

For the size of  $\Gamma$  there are three cases worth mentioning. First, in the case of riskless dividend streams and zero inflation (i.e. the central bank's target  $i^* = 0$  always fulfilled),  $\tilde{\beta}$  should be equal to the discount factor in the utility function (38) and  $\Gamma$  would be less than  $\bar{i}$  because:

$$\tilde{\beta} = \beta : \quad (\Gamma =) \quad 1 - \beta < \frac{1}{\beta} - 1 \quad (= \bar{i}) \quad \Leftrightarrow \quad (61)$$

$$1 - \beta < \frac{1 - \beta}{\beta} \quad \Leftrightarrow \quad (62)$$

$$(1 - \beta)\beta < 1 - \beta \quad \Leftrightarrow \quad (63)$$

$$\beta < 1 \quad \text{holds by assumption} \quad (64)$$

This implies  $\bar{i} - \Gamma > 0$ . Given the standard parameterization of  $\beta$  we have numerically  $\bar{i} - \Gamma \approx 0.0001$ . Second, in the case of risky dividend payments agents will discount dividend payments stronger than the save interest payments from bonds (still no inflation risk) and the discount factor  $\tilde{\beta}$  becomes smaller than  $\beta$  which implies  $\bar{i} - \Gamma < 0$ . In the third case, where inflation risk is taken into account, agents could discount payments from stocks less than from bonds which yields  $\bar{i} - \Gamma > 0$ . We focus on the last case because  $\bar{i} - \Gamma < 0$  would lead to a negative radicand in expression (103) and (104) and therefore to output levels  $x_q$  being complex numbers.

We substitute  $\tilde{d}_q$  in FOC 3 by  $\Gamma s_q^f$  (where  $\Gamma$  depends on the strength of discounting of bond yields compared to stock yields) to arrive at:

$$\left( \frac{D_q}{C_q} \right)^{-\nu} = \frac{\alpha_1}{\alpha_2} \frac{1}{P_q} \left[ \frac{S_q(i_q - \Gamma)}{1 + i_q} \right] \quad (65)$$

$$= \frac{\alpha_1}{\alpha_2} \frac{S_q}{P_q} \frac{i_q - \Gamma}{1 + i_q} \quad (66)$$

Taking logs (small letters denote log values):

$$-\nu[\log D_q - \log C_q] = \log \left\{ \frac{\alpha_1}{\alpha_2} \right\} + \log S_q - \log P_q + \log \left\{ \frac{i_q - \Gamma}{1 + i_q} \right\} \quad (67)$$

$$-\nu d_q + \nu c_q = \log \left\{ \frac{\alpha_1}{\alpha_2} \right\} + s_q - p_q + \log \left\{ \frac{i_q - \Gamma}{1 + i_q} \right\} \quad (68)$$

$$d_q = c_q - \frac{1}{\nu} \log \left\{ \frac{\alpha_1}{\alpha_2} \right\} - \frac{1}{\nu} (s_q - p_q) - \frac{1}{\nu} \log \left\{ \frac{i_q - \Gamma}{1 + i_q} \right\} \quad (69)$$

Note that the expression above is only defined for  $i_q > \Gamma$ .<sup>34</sup> Let

$$f(i_q) := \frac{1}{\nu} \log \left\{ \frac{i_q - \Gamma}{1 + i_q} \right\} \quad (i_q > \Gamma) . \quad (70)$$

Then the first order Taylor approximation around the steady state  $\bar{i}$  without the constant term  $f(\bar{i})$  is given by

$$f(i_q) \approx f'(\bar{i}) \cdot (i_q - \bar{i}) \quad (71)$$

$$\approx \frac{1}{\nu} \frac{1 + \bar{i}}{\bar{i} - \Gamma} \frac{1 + \bar{i} - (\bar{i} - \Gamma)}{(1 + \bar{i})^2} \cdot (i_q - \bar{i}) \quad (72)$$

$$\approx \frac{1}{\nu} \frac{1 + \Gamma}{(\bar{i} - \Gamma)(1 + \bar{i})} \cdot (i_q - \bar{i}) \quad (73)$$

Thus a linearization of (69) around the steady state yields the log-linear stock demand function

$$d_q \approx c_q - \frac{1}{\nu} (s_q - p_q) - \frac{1}{\nu} \frac{1 + \Gamma}{(\bar{i} - \Gamma)(1 + \bar{i})} \cdot (i_q - \bar{i}) \quad (74)$$

which was reported in eq. (4) in section 2. Next, the Euler equation has to be linearized (where we follow Galí (2008)). After rearranging (47) a bit

$$\beta \mathbf{E}_q \left[ \left( \frac{Z_{q+1}}{Z_q} \right)^{\nu - \sigma} \left( \frac{C_{q+1}}{C_q} \right)^{\sigma - \nu} \left( \frac{C_q}{C_{q+1}} \right)^\sigma \left( \frac{P_q}{P_{q+1}} \right) \right] = \frac{1}{1 + i_q} \quad (75)$$

$$\beta(1 + i_q) \mathbf{E}_q \left[ \left( \frac{Z_{q+1}}{Z_q} \right)^{\nu - \sigma} \left( \frac{C_q}{C_{q+1}} \right)^{\nu - \sigma} \left( \frac{C_q}{C_{q+1}} \right)^\sigma \left( \frac{P_q}{P_{q+1}} \right) \right] = 1 \quad (76)$$

<sup>34</sup>If percentage deviations from steady state are used for linearization (instead of a log-linearization approach)  $i_q < \Gamma$  would also be allowed. This linearization, however, would be mathematically more involved.

$$\beta(1+i_q)\mathbf{E}_q \left[ \left( \frac{Z_{q+1}}{C_{q+1}} \right)^{\nu-\sigma} \left( \frac{C_q}{Z_q} \right)^{\nu-\sigma} \right] = \mathbf{E}_q \left[ \left( \frac{C_{q+1}}{C_q} \right)^\sigma \underbrace{\left( \frac{P_{q+1}}{P_q} \right)}_{1+\pi_{q+1}} \right] \quad (77)$$

we take logs

$$\begin{aligned} \underbrace{\log \beta}_{-\bar{r}} + \underbrace{\log(1+i_q)}_{\approx i_q} + (\nu-\sigma)\mathbf{E}_q[x_{q+1}-c_{q+1}] + (\nu-\sigma)(c_q-x_q) \\ = \sigma \cdot \mathbf{E}_q[c_{q+1}-c_q] + \underbrace{\log \mathbf{E}_q[1+\pi_{q+1}]}_{\approx \mathbf{E}_q[\pi_{q+1}]} \end{aligned} \quad (78)$$

and solve for  $c_q$  to arrive at:

$$\sigma c_q = \sigma \mathbf{E}_q[c_{q+1}] - (i_q - \mathbf{E}_q[\pi_{q+1}] - \bar{r}) + (\nu-\sigma) \{ \mathbf{E}_q[c_{q+1}-x_{q+1}] - (c_q-x_q) \} \quad (79)$$

$$c_q = \mathbf{E}_q[c_{q+1}] - \frac{1}{\sigma} (i_q - \mathbf{E}_q[\pi_{q+1}] - \bar{r}) + \frac{\nu-\sigma}{\sigma} \{ \mathbf{E}_q[\Delta(c_{q+1}-x_{q+1})] \} \quad (80)$$

The expressions  $\mathbf{E}_q[c_{q+1}-x_{q+1}]$  and  $(c_q-x_q)$  can be replaced if we, first, linearize the composite index (39) around the steady state

$$dZ_q = \alpha_1 \bar{Z}^\nu \bar{C}^{-\nu} dC_q + \alpha_2 \bar{Z}^\nu \bar{D}^{-\nu} dD_q \quad (81)$$

$$\frac{dZ_q}{\bar{Z}} = \alpha_1 \bar{Z}^\nu \bar{C}^{-\nu} \frac{1}{\bar{Z}} \frac{\bar{C}}{\bar{C}} dC_q + \alpha_2 \bar{Z}^\nu \frac{1}{\bar{Z}} \frac{\bar{C}}{\bar{C}} \bar{D}^{-\nu} dD_q \quad (82)$$

$$z_q = \alpha_1 \left( \frac{\bar{C}}{\bar{Z}} \right)^{1-\nu} c_q + \alpha_2 \left( \frac{\bar{D}}{\bar{Z}} \right)^{1-\nu} d_q \quad (83)$$

where  $z_q = \frac{dZ_q}{\bar{Z}}$ ,  $c_q = \frac{dC_q}{\bar{C}}$ ,  $d_q = \frac{dD_q}{\bar{D}}$  denote percentage deviations from steady state and  $dZ_q$ ,  $dC_q$ ,  $dD_q$  absolute deviations. The steady state of (39) is given by:

$$\bar{Z}^{1-\nu} = \alpha_1 \bar{C}^{1-\nu} + \alpha_2 \bar{D}^{1-\nu} \quad (84)$$

$$1 - \alpha_1 \frac{\bar{C}^{1-\nu}}{\bar{Z}^{1-\nu}} = \alpha_2 \frac{\bar{D}^{1-\nu}}{\bar{Z}^{1-\nu}} \quad (85)$$

By subtracting (83) from  $c_q$

$$c_q - z_q = \left[ 1 - \alpha_1 \left( \frac{\bar{C}}{\bar{Z}} \right)^{1-\nu} \right] c_q - \alpha_2 \left( \frac{\bar{D}}{\bar{Z}} \right)^{1-\nu} d_q \quad (86)$$

plugging (85) into (86)

$$c_q - z_q = \alpha_2 \left( \frac{\bar{D}}{\bar{Z}} \right)^{1-\nu} c_q - \alpha_2 \left( \frac{\bar{D}}{\bar{Z}} \right)^{1-\nu} d_q \quad (87)$$

$$= \alpha_2 \left( \frac{\bar{D}}{\bar{Z}} \right)^{1-\nu} (c_q - d_q) \quad (88)$$

and plugging (74) into (88) we arrive at

$$c_q - z_q = \frac{\alpha_2}{\nu} \left( \frac{\bar{D}}{\bar{Z}} \right)^{1-\nu} \left\{ (s_q - p_q) + \frac{1 + \Gamma}{(\bar{i} - \Gamma)(1 + \bar{i})} \cdot (i_q - \bar{i}) \right\} \quad (89)$$

which implies for the expected change of  $c_q - z_q$ :

$$\mathbb{E}_q [\Delta(c_{q+1} - z_{q+1})] = \mathbb{E}_q [c_{q+1} - z_{q+1}] - (c_q - z_q) \quad (90)$$

$$= \frac{\alpha_2}{\nu} \left( \frac{\bar{D}}{\bar{Z}} \right)^{1-\nu} \left\{ \mathbb{E}_q [\Delta(s_{q+1} - p_{q+1})] + \frac{1 + \Gamma}{(\bar{i} - \Gamma)(1 + \bar{i})} \cdot \mathbb{E}_q [\Delta i_{q+1}] \right\} \quad (91)$$

Equation (80) then becomes

$$c_q = \mathbb{E}_q [c_{q+1}] - \frac{1}{\sigma} (i_q - \mathbb{E}_q [\pi_{q+1}] - \bar{r}) + \frac{\nu - \sigma}{\sigma} \frac{\alpha_2}{\nu} \left( \frac{\bar{D}}{\bar{Z}} \right)^{1-\nu} \cdot \left\{ \mathbb{E}_q [\Delta(s_{q+1} - p_{q+1})] + \frac{1 + \Gamma}{(\bar{i} - \Gamma)(1 + \bar{i})} \cdot \mathbb{E}_q [\Delta i_{q+1}] \right\} \quad (92)$$

or, if simplified further

$$c_q = \mathbb{E}_q [c_{q+1}] - \frac{1}{\sigma} (i_q - \mathbb{E}_q [\pi_{q+1}] - \bar{r}) + c_1 \cdot \mathbb{E}_q [\Delta(s_{q+1} - p_{q+1})] + c_2 \cdot \mathbb{E}_q [\Delta i_{q+1}] \quad (93)$$

with  $c_1 = \frac{\nu - \sigma}{\sigma} \frac{\alpha_2}{\nu} \left( \frac{\bar{D}}{\bar{Z}} \right)^{1-\nu}$  and  $c_2 = c_1 \cdot \frac{1 + \Gamma}{(\bar{i} - \Gamma)(1 + \bar{i})}$ . We can identify  $\left( \frac{\bar{D}}{\bar{Z}} \right)^{1-\nu}$  by first rewriting (84) to

$$\frac{\bar{Z}^{1-\nu}}{\bar{D}^{1-\nu}} = \alpha_1 \frac{\bar{C}^{1-\nu}}{\bar{D}^{1-\nu}} + \alpha_2 \quad (94)$$

and then rewriting (66) to get:

$$\left( \frac{\bar{D}}{\bar{C}} \right)^{1-\nu} = \left( \frac{\alpha_1 \bar{S} \bar{i} - \Gamma}{\alpha_2 \bar{P} 1 + \bar{i}} \right)^{\frac{1-\nu}{-\nu}} \quad (95)$$

$$\left(\frac{\bar{C}}{\bar{D}}\right)^{1-\nu} = \left(\frac{\alpha_1 \bar{S} \bar{i} - \Gamma}{\alpha_2 \bar{P} 1 + \bar{i}}\right)^{\frac{1-\nu}{\nu}} \quad (96)$$

Plugging (96) into (94) we get

$$\frac{\bar{Z}^{1-\nu}}{\bar{D}^{1-\nu}} = \alpha_1 \left(\frac{\alpha_1 \bar{S} \bar{i} - \Gamma}{\alpha_2 \bar{P} 1 + \bar{i}}\right)^{\frac{1-\nu}{\nu}} + \alpha_2 \quad (97)$$

$$\left(\frac{\bar{D}}{\bar{Z}}\right)^{1-\nu} = \frac{1}{\alpha_1 \left(\frac{\alpha_1 \bar{S} \bar{i} - \Gamma}{\alpha_2 \bar{P} 1 + \bar{i}}\right)^{\frac{1-\nu}{\nu}} + \alpha_2 \frac{\alpha_1}{\alpha_1}} \quad (98)$$

$$\left(\frac{\bar{D}}{\bar{Z}}\right)^{1-\nu} = \frac{1}{\alpha_1 \left[ \left(\frac{\alpha_1}{\alpha_2}\right)^{\frac{1-\nu}{\nu}} \left(\frac{\bar{S} \bar{i} - \Gamma}{\bar{P} 1 + \bar{i}}\right)^{\frac{1-\nu}{\nu}} + \frac{\alpha_2}{\alpha_1} \right]} \quad (99)$$

$$\left(\frac{\bar{D}}{\bar{Z}}\right)^{1-\nu} = \frac{1}{\alpha_1 \frac{\alpha_2}{\alpha_1} \left[ \left(\frac{\alpha_2}{\alpha_1}\right)^{-\frac{1}{\nu}} \left(\frac{\bar{S} \bar{i} - \Gamma}{\bar{P} 1 + \bar{i}}\right)^{\frac{1-\nu}{\nu}} + 1 \right]} \quad (100)$$

and the constants  $c_1$  and  $c_2$  become:

$$c_1 = \frac{\nu - \sigma}{\sigma} \frac{\alpha_2}{\nu} \frac{1}{\alpha_1 \frac{\alpha_2}{\alpha_1} \left[ \left(\frac{\alpha_2}{\alpha_1}\right)^{-\frac{1}{\nu}} \left(\frac{\bar{S} \bar{i} - \Gamma}{\bar{P} 1 + \bar{i}}\right)^{\frac{1-\nu}{\nu}} + 1 \right]} \quad (101)$$

$$= \frac{\nu - \sigma}{\sigma \nu} \frac{1}{\left(\frac{\alpha_2}{\alpha_1}\right)^{-\frac{1}{\nu}} \left(\frac{\bar{S} \bar{i} - \Gamma}{\bar{P} 1 + \bar{i}}\right)^{\frac{1-\nu}{\nu}} + 1} \quad (102)$$

$$= \frac{\nu - \sigma}{\sigma \nu} \frac{\left(\frac{\bar{S} \bar{i} - \Gamma}{\bar{P} 1 + \bar{i}}\right)^{\frac{\nu-1}{\nu}}}{\left(\frac{\alpha_1}{\alpha_2}\right)^{\frac{1}{\nu}} + \left(\frac{\bar{S} \bar{i} - \Gamma}{\bar{P} 1 + \bar{i}}\right)^{\frac{\nu-1}{\nu}}} \quad (103)$$

$$c_2 = \frac{\nu - \sigma}{\sigma \nu} \frac{\left(\frac{\bar{S} \bar{i} - \Gamma}{\bar{P} 1 + \bar{i}}\right)^{\frac{\nu-1}{\nu}}}{\left(\frac{\alpha_1}{\alpha_2}\right)^{\frac{1}{\nu}} + \left(\frac{\bar{S} \bar{i} - \Gamma}{\bar{P} 1 + \bar{i}}\right)^{\frac{\nu-1}{\nu}}} \frac{1 + \Gamma}{(\bar{i} - \Gamma)(1 + \bar{i})} \quad (104)$$

To be in line with the notation of (7)-(6) we drop  $\bar{r}$  in eq. (93) so that the nominal interest rate  $i_q$  is now interpreted as the difference to steady state:

$$c_q = \mathbb{E}_q [c_{q+1}] - \frac{1}{\sigma} (i_q - \mathbb{E}_q [\pi_{q+1}]) + c_1 \cdot \mathbb{E}_q [\Delta s_{q+1} - \pi_{q+1}] + c_2 \cdot \mathbb{E}_q [\Delta i_{q+1}] \quad (105)$$

In equilibrium, consumption equals production so that  $c_q$  is replaced by output  $x_q$ . Finally, a

noise term  $\epsilon_x$  is added and expectations are assumed to be given by the boundedly rational heuristic  $\tilde{\mathbb{E}}_q[\cdot]$ :

$$x_q = \mathbb{E}_q[x_{q+1}] - \frac{1}{\sigma} (i_q - \mathbb{E}_q[\pi_{q+1}]) + c_1 \cdot \mathbb{E}_q[\Delta s_{q+1} - \pi_{q+1}] + c_2 \cdot \mathbb{E}_q[\Delta i_{q+1}] + \epsilon_q^x \quad (106)$$

With stock demand of period  $q$  given as

$$d_q = x_q - c_3(s_q - p_q) - c_4 i_q \quad \text{with: } c_3 = \frac{1}{\nu}, \quad c_4 = \frac{1}{\nu} \frac{1 + \Gamma}{(\bar{i} - \Gamma)(1 + \bar{i})} \quad (107)$$

## B. Derive Solution of NKM

The three model equations read

$$i_q = \delta_\pi \left( \tilde{\mathbb{E}}_q[\pi_{q+1}] - \pi_q^* \right) + \delta_x \tilde{\mathbb{E}}_q[x_{q+1}] + \epsilon_q^i \quad (108)$$

$$x_q = \tilde{\mathbb{E}}_q[x_{q+1}] - \frac{1}{\sigma} \left( i_q - \tilde{\mathbb{E}}_q[\pi_{q+1}] \right) + c_1 \cdot \tilde{\mathbb{E}}_q[\Delta s_{q+1} - \pi_{q+1}] + c_2 \cdot \tilde{\mathbb{E}}_q[\Delta i_{q+1}] + \epsilon_q^x \quad (109)$$

$$\pi_q = \beta \tilde{\mathbb{E}}_q[\pi_{q+1}] + \gamma x_q - \kappa s_q + \epsilon_q^\pi \quad (110)$$

where the target inflation level  $\pi^*$  is again set to zero and market expectations are given by

$$\tilde{\mathbb{E}}_q[x_{q+1}] = \omega_{x,q}^{\text{tar}} \bar{x} + \omega_{x,q}^{\text{hab}} x_{q-1} + \omega_{x,q}^{\text{ext}} (x_q + \alpha_x \cdot [x_q - x_{q-1}]) \quad (111)$$

$$= \omega_{x,q}^{\text{ext}} (1 + \alpha_x) x_q + (\omega_{x,q}^{\text{hab}} - \alpha_x \omega_{x,q}^{\text{ext}}) x_{q-1} \quad (112)$$

$$\tilde{\mathbb{E}}_q[\pi_{q+1}] = \omega_{\pi,q}^{\text{tar}} \pi^* + \omega_{\pi,q}^{\text{rof}} \pi_{q-1} + \omega_{\pi,q}^{\text{ext}} (\pi_q + \alpha_\pi \cdot [\pi_q - \pi_{q-1}]) \quad (113)$$

$$= \omega_{\pi,q}^{\text{ext}} (1 + \alpha_\pi) \pi_q + (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}}) \pi_{q-1} \quad (114)$$

$$\tilde{\mathbb{E}}_q[s_{q+1}] = \omega_{s,q}^{\text{tar}} h x_q + \omega_{s,q}^{\text{sta}} s_{q-1} + \omega_{s,q}^{\text{ext}} (s_q + \alpha_s [s_q - s_{q-1}]) \quad (115)$$

$$= \omega_{s,q}^{\text{ext}} (1 + \alpha_s) s_q + (\omega_{s,q}^{\text{sta}} - \alpha_s \omega_{s,q}^{\text{ext}}) s_{q-1} + \omega_{s,q}^{\text{tar}} h x_q \quad (116)$$

$$\tilde{\mathbb{E}}_q[i_{q+1}] = i_q \quad (117)$$

where –following Lengnick and Wohltmann (2013) and Westerhoff (2012)– the perceived fundamental value is set to  $s_q^f = h \cdot x_q$ . Plugging (108) into (109) we get:

$$x_q = \tilde{\mathbb{E}}_q[x_{q+1}] - \frac{1}{\sigma} \left( \delta_\pi \tilde{\mathbb{E}}_q[\pi_{q+1}] + \delta_x \tilde{\mathbb{E}}_q[x_{q+1}] + \epsilon_q^i - \tilde{\mathbb{E}}_q[\pi_{q+1}] \right) + c_1 \cdot \tilde{\mathbb{E}}_q[\Delta s_{q+1} - \pi_{q+1}] + \epsilon_q^x \quad (118)$$

$$x_q = \frac{\sigma - \delta_x}{\sigma} \tilde{\mathbf{E}}_q [x_{q+1}] + \frac{1 - \delta_\pi - \sigma c_1}{\sigma} \tilde{\mathbf{E}}_q [\pi_{q+1}] + c_1 \tilde{\mathbf{E}}_q [s_{q+1}] - c_1 s_q + \epsilon_q^x - \frac{1}{\sigma} \epsilon_q^i \quad (119)$$

Plugging expectations into (119) and (110) gives

$$\begin{aligned} x_q &= \frac{\sigma - \delta_x}{\sigma} \underbrace{\left( \omega_{x,q}^{\text{ext}}(1 + \alpha_x)x_q + (\omega_{x,q}^{\text{hab}} - \alpha_x \omega_{x,q}^{\text{ext}})x_{q-1} \right)}_{\tilde{\mathbf{E}}_q[x_{q+1}]} \\ &+ \frac{1 - \delta_\pi - \sigma c_1}{\sigma} \underbrace{\left( \omega_{\pi,q}^{\text{ext}}(1 + \alpha_\pi)\pi_q + (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}})\pi_{q-1} \right)}_{\tilde{\mathbf{E}}_q[\pi_{q+1}]} \\ &+ c_1 \underbrace{\left( \omega_{s,q}^{\text{ext}}(1 + \alpha_s)s_q + (\omega_{s,q}^{\text{sta}} - \alpha_s \omega_{s,q}^{\text{ext}})s_{q-1} + \omega_{s,q}^{\text{tar}}h x_q \right)}_{\tilde{\mathbf{E}}_q[s_{q+1}]} - c_1 s_q + \epsilon_q^x - \frac{1}{\sigma} \epsilon_q^i \end{aligned} \quad (120)$$

$$\pi_q = \beta \underbrace{\left( \omega_{\pi,q}^{\text{ext}}(1 + \alpha_\pi)\pi_q + (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}})\pi_{q-1} \right)}_{\tilde{\mathbf{E}}_q[\pi_{q+1}]} + \gamma x_q - \kappa s_q + \epsilon_q^\pi \quad (121)$$

After some rearranging

$$\begin{aligned} &(\sigma - (\sigma - \delta_x)\omega_{x,q}^{\text{ext}}(1 + \alpha_x) - \sigma c_1 \omega_{s,q}^{\text{tar}}h) x_q - (1 - \delta_\pi - \sigma c_1)\omega_{\pi,q}^{\text{ext}}(1 + \alpha_\pi)\pi_q \\ &= (\sigma - \delta_x)(\omega_{x,q}^{\text{hab}} - \alpha_x \omega_{x,q}^{\text{ext}})x_{q-1} + (1 - \delta_\pi - \sigma c_1)(\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}})\pi_{q-1} \\ &\quad + c_1 \sigma (\omega_{s,q}^{\text{ext}}(1 + \alpha_s) - 1) s_q + c_1 \sigma (\omega_{s,q}^{\text{sta}} - \alpha_s \omega_{s,q}^{\text{ext}}) s_{q-1} + \sigma \epsilon_q^x - \epsilon_q^i \end{aligned} \quad (122)$$

$$-\gamma x_q + (1 - \beta \omega_{\pi,q}^{\text{ext}}(1 + \alpha_\pi)) \pi_q = \beta (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}}) \pi_{q-1} - \kappa s_q + \epsilon_q^\pi \quad (123)$$

we arrive at the matrix form

$$\mathbf{A}_q \begin{pmatrix} x_q \\ \pi_q \end{pmatrix} = \mathbf{C}_q \begin{pmatrix} x_{q-1} \\ \pi_{q-1} \end{pmatrix} + \mathbf{D}_q \cdot s_q + \mathbf{E}_q \cdot s_{q-1} + \begin{pmatrix} \sigma \epsilon_q^x - \epsilon_q^i \\ \epsilon_q^\pi \end{pmatrix} \quad (124)$$

with the *time dependent* system matrices:

$$\begin{aligned} \mathbf{A}_q &= \begin{pmatrix} \sigma - (\sigma - \delta_x)\omega_{x,q}^{\text{ext}}(1 + \alpha_x) - \sigma c_1 \omega_{s,q}^{\text{tar}}h & -(1 - \delta_\pi - \sigma c_1)\omega_{\pi,q}^{\text{ext}}(1 + \alpha_\pi) \\ -\gamma & 1 - \beta \omega_{\pi,q}^{\text{ext}}(1 + \alpha_\pi) \end{pmatrix} \\ \mathbf{C}_q &= \begin{pmatrix} (\sigma - \delta_x)(\omega_{x,q}^{\text{hab}} - \alpha_x \omega_{x,q}^{\text{ext}}) & (1 - \delta_\pi - \sigma c_1)(\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}}) \\ 0 & \beta (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}}) \end{pmatrix} \\ \mathbf{D}_q &= \begin{pmatrix} c_1 \sigma (\omega_{s,q}^{\text{ext}}(1 + \alpha_s) - 1) \\ -\kappa \end{pmatrix} \quad \mathbf{E}_q = \begin{pmatrix} c_1 \sigma (\omega_{s,q}^{\text{sta}} - \alpha_s \omega_{s,q}^{\text{ext}}) \\ 0 \end{pmatrix} \end{aligned}$$



Multiplying with  $\mathbf{A}_q^{-1}$  we arrive at the solution:

$$\begin{pmatrix} x_q \\ \pi_q \end{pmatrix} = \mathbf{A}_q^{-1} \mathbf{C}_q \begin{pmatrix} x_{q-1} \\ \pi_{q-1} \end{pmatrix} + \mathbf{A}_q^{-1} \mathbf{D}_q \cdot s_q + \mathbf{A}_q^{-1} \mathbf{E}_q \cdot s_{q-1} + \mathbf{A}_q^{-1} \begin{pmatrix} \sigma \epsilon_q^x - \epsilon_q^i \\ \epsilon_q^\pi \end{pmatrix} \quad (125)$$

## C. Derive Solution of NKM (Version 2)

In this version, the Taylor rule depends on contemporaneous values. The three model equations are given by:

$$i_q = \delta_\pi (\pi_q - \pi_q^*) + \delta_x x_q + \epsilon_q^i \quad (126)$$

$$x_q = \tilde{\mathbf{E}}_q [x_{q+1}] - \frac{1}{\sigma} \left( i_q - \tilde{\mathbf{E}}_q [\pi_{q+1}] \right) + c_1 \cdot \tilde{\mathbf{E}}_q [\Delta s_{q+1} - \pi_{q+1}] + c_2 \cdot \tilde{\mathbf{E}}_q [\Delta i_{q+1}] + \epsilon_q^x \quad (127)$$

$$\pi_q = \beta \tilde{\mathbf{E}}_q [\pi_{q+1}] + \gamma x_q - \kappa s_q + \epsilon_q^\pi \quad (128)$$

Plugging (126) into (127) we get:

$$x_q = \tilde{\mathbf{E}}_q [x_{q+1}] - \frac{1}{\sigma} \left( \{ \delta_\pi \pi_q + \delta_x x_q + \epsilon_q^i \} - \tilde{\mathbf{E}}_q [\pi_{q+1}] \right) + c_1 \cdot \tilde{\mathbf{E}}_q [\Delta s_{q+1} - \pi_{q+1}] + \epsilon_q^x \quad (129)$$

$$x_q = \tilde{\mathbf{E}}_q [x_{q+1}] + \frac{1}{\sigma} \tilde{\mathbf{E}}_q [\pi_{q+1}] - \frac{\delta_\pi}{\sigma} \pi_q - \frac{\delta_x}{\sigma} x_q + c_1 \cdot \tilde{\mathbf{E}}_q [\Delta s_{q+1} - \pi_{q+1}] + \epsilon_q^x - \frac{1}{\sigma} \epsilon_q^i \quad (130)$$

Plugging in expectations:

$$\begin{aligned} x_q = & \underbrace{\omega_{x,q}^{\text{ext}}(1 + \alpha_x)x_q + (\omega_{x,q}^{\text{hab}} - \alpha_x \omega_{x,q}^{\text{ext}})x_{q-1}}_{\tilde{\mathbf{E}}_q [x_{q+1}]} + \frac{1 - \sigma c_1}{\sigma} \underbrace{(\omega_{\pi,q}^{\text{ext}}(1 + \alpha_\pi)\pi_q + (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}})\pi_{q-1})}_{\tilde{\mathbf{E}}_q [\pi_{q+1}]} \\ & - \frac{\delta_\pi}{\sigma} \pi_q - \frac{\delta_x}{\sigma} x_q + c_1 \cdot \underbrace{(\omega_{s,q}^{\text{ext}}(1 + \alpha_s)s_q + (\omega_{s,q}^{\text{sta}} - \alpha_s \omega_{s,q}^{\text{ext}})s_{q-1} + \omega_{s,q}^{\text{tar}} h x_q)}_{\tilde{\mathbf{E}}_q [s_{q+1}]} - c_1 s_q + \epsilon_q^x - \frac{1}{\sigma} \epsilon_q^i \end{aligned} \quad (131)$$

$$\pi_q = \beta \underbrace{(\omega_{\pi,q}^{\text{ext}}(1 + \alpha_\pi)\pi_q + (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}})\pi_{q-1})}_{\tilde{\mathbf{E}}_q [\pi_{q+1}]} + \gamma x_q - \kappa s_q + \epsilon_q^\pi \quad (132)$$

After some rearranging

$$\begin{aligned} & (\sigma - \sigma \omega_{x,q}^{\text{ext}}(1 + \alpha_x) + \delta_x - c_1 \sigma \omega_{s,q}^{\text{tar}} h) x_q - [(1 - \sigma c_1) \omega_{\pi,q}^{\text{ext}}(1 + \alpha_\pi) + \delta_\pi] \pi_q \\ & = \sigma (\omega_{x,q}^{\text{hab}} - \alpha_x \omega_{x,q}^{\text{ext}}) x_{q-1} + (1 - \sigma c_1) (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}}) \pi_{q-1} \end{aligned}$$

$$+ c_1 \sigma (\omega_{s,q}^{\text{ext}} (1 + \alpha_s) s_q + (\omega_{s,q}^{\text{sta}} - \alpha_s \omega_{s,q}^{\text{ext}}) s_{q-1}) - c_1 \sigma s_q + \sigma \epsilon_q^x - \epsilon_q^i \quad (133)$$

$$-\gamma x_q + (1 - \beta \omega_{\pi,q}^{\text{ext}} (1 + \alpha_\pi)) \pi_q = \beta (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}}) \pi_{q-1} - \kappa s_q + \epsilon_q^\pi \quad (134)$$

we arrive at the matrix form

$$\mathbf{A}_q \begin{pmatrix} x_q \\ \pi_q \end{pmatrix} = \mathbf{C}_q \begin{pmatrix} x_{q-1} \\ \pi_{q-1} \end{pmatrix} + \mathbf{D}_q \cdot s_q + \mathbf{E}_q \cdot s_{q-1} + \begin{pmatrix} \sigma \epsilon_q^x - \epsilon_q^i \\ \epsilon_q^\pi \end{pmatrix} \quad (135)$$

with the *time dependent* system matrices:

$$\begin{aligned} \mathbf{A}_q &= \begin{pmatrix} \sigma - \sigma \omega_{x,q}^{\text{ext}} (1 + \alpha_x) + \delta_x - \sigma c_1 \omega_{s,q}^{\text{tar}} h & -(1 - \sigma c_1) \omega_{\pi,q}^{\text{ext}} (1 + \alpha_\pi) - \delta_\pi \\ -\gamma & 1 - \beta \omega_{\pi,q}^{\text{ext}} (1 + \alpha_\pi) \end{pmatrix} \\ \mathbf{C}_q &= \begin{pmatrix} \sigma (\omega_{x,q}^{\text{hab}} - \alpha_x \omega_{x,q}^{\text{ext}}) & (1 - \sigma c_1) (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}}) \\ 0 & \beta (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}}) \end{pmatrix} \\ \mathbf{D}_q &= \begin{pmatrix} c_1 \sigma (\omega_{s,q}^{\text{ext}} (1 + \alpha_s) - 1) \\ -\kappa \end{pmatrix} \quad \mathbf{E}_q = \begin{pmatrix} c_1 \sigma (\omega_{s,q}^{\text{sta}} - \alpha_s \omega_{s,q}^{\text{ext}}) \\ 0 \end{pmatrix} \end{aligned}$$

Multiplying with  $\mathbf{A}_q^{-1}$  we arrive at the solution:

$$\begin{pmatrix} x_q \\ \pi_q \end{pmatrix} = \mathbf{A}_q^{-1} \mathbf{C}_q \begin{pmatrix} x_{q-1} \\ \pi_{q-1} \end{pmatrix} + \mathbf{A}_q^{-1} \mathbf{D}_q \cdot s_q + \mathbf{A}_q^{-1} \mathbf{E}_q \cdot s_{q-1} + \mathbf{A}_q^{-1} \begin{pmatrix} \sigma \epsilon_q^x - \epsilon_q^i \\ \epsilon_q^\pi \end{pmatrix} \quad (136)$$

## D. Derive Solution of NKM (Version 3)

In this version, the Taylor rule depends on past values. The three model equations are given by:

$$i_q = \delta_\pi (\pi_{q-1} - \pi_q^*) + \delta_x x_{q-1} + \epsilon_q^i \quad (137)$$

$$x_q = \tilde{\mathbf{E}}_q [x_{q+1}] - \frac{1}{\sigma} \left( i_q - \tilde{\mathbf{E}}_q [\pi_{q+1}] \right) + c_1 \cdot \tilde{\mathbf{E}}_q [\Delta s_{q+1} - \pi_{q+1}] + c_2 \cdot \tilde{\mathbf{E}}_q [\Delta i_{q+1}] + \epsilon_q^x \quad (138)$$

$$\pi_q = \beta \tilde{\mathbf{E}}_q [\pi_{q+1}] + \gamma x_q - \kappa s_q + \epsilon_q^\pi \quad (139)$$

Plugging (137) into (138) we get:

$$x_q = \tilde{\mathbf{E}}_q [x_{q+1}] - \frac{1}{\sigma} \left( \left\{ \delta_\pi \pi_{q-1} + \delta_x x_{q-1} + \epsilon_q^i \right\} - \tilde{\mathbf{E}}_q [\pi_{q+1}] \right) + c_1 \cdot \tilde{\mathbf{E}}_q [\Delta s_{q+1} - \pi_{q+1}] + \epsilon_q^x \quad (140)$$

$$x_q = \tilde{\mathbf{E}}_q [x_{q+1}] + \frac{1}{\sigma} \tilde{\mathbf{E}}_q [\pi_{q+1}] - \frac{\delta_\pi}{\sigma} \pi_{q-1} - \frac{\delta_x}{\sigma} x_{q-1} + c_1 \cdot \tilde{\mathbf{E}}_q [\Delta s_{q+1} - \pi_{q+1}] + \epsilon_q^x - \frac{1}{\sigma} \epsilon_q^i \quad (141)$$

Plugging in expectations:

$$\begin{aligned} x_q &= \underbrace{\omega_{x,q}^{\text{ext}}(1 + \alpha_x)x_q + (\omega_{x,q}^{\text{hab}} - \alpha_x \omega_{x,q}^{\text{ext}})x_{q-1}}_{\tilde{\mathbf{E}}_q[x_{q+1}]} + \frac{1 - \sigma c_1}{\sigma} \underbrace{\{\omega_{\pi,q}^{\text{ext}}(1 + \alpha_\pi)\pi_q + (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}})\pi_{q-1}\}}_{\tilde{\mathbf{E}}_q[\pi_{q+1}]} \\ &\quad - \frac{\delta_\pi}{\sigma} \pi_{q-1} - \frac{\delta_x}{\sigma} x_{q-1} + c_1 \underbrace{\{\omega_{s,q}^{\text{ext}}(1 + \alpha_s)s_q + (\omega_{s,q}^{\text{sta}} - \alpha_s \omega_{s,q}^{\text{ext}})s_{q-1} + \omega_{s,q}^{\text{tar}} h x_q\}}_{\tilde{\mathbf{E}}_q[s_{q+1}]} \\ &\quad - c_1 \cdot s_q + \epsilon_q^x - \frac{1}{\sigma} \epsilon_q^i \end{aligned} \quad (142)$$

$$\pi_q = \beta \underbrace{\{\omega_{\pi,q}^{\text{ext}}(1 + \alpha_\pi)\pi_q + (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}})\pi_{q-1}\}}_{\tilde{\mathbf{E}}_q[\pi_{q+1}]} + \gamma x_q - \kappa s_q + \epsilon_q^\pi \quad (143)$$

After some rearranging

$$\begin{aligned} &\sigma (1 - \omega_{x,q}^{\text{ext}}(1 + \alpha_x) - c_1 \omega_{s,q}^{\text{tar}} h) x_q - (1 - \sigma c_1) \omega_{\pi,q}^{\text{ext}}(1 + \alpha_\pi) \pi_q \\ &\quad = (\sigma \omega_{x,q}^{\text{hab}} - \sigma \alpha_x \omega_{x,q}^{\text{ext}} - \delta_x) x_{q-1} + \{(1 - \sigma c_1)(\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}}) - \delta_\pi\} \pi_{q-1} \\ &\quad \quad + \sigma c_1 \{\omega_{s,q}^{\text{ext}}(1 + \alpha_s) - 1\} s_q + \sigma c_1 (\omega_{s,q}^{\text{sta}} - \alpha_s \omega_{s,q}^{\text{ext}}) s_{q-1} + \sigma \epsilon_q^x - \epsilon_q^i \end{aligned} \quad (144)$$

$$-\gamma x_q + (1 - \beta \omega_{\pi,q}^{\text{ext}}(1 + \alpha_\pi)) \pi_q = \beta (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}}) \pi_{q-1} - \kappa s_q + \epsilon_q^\pi \quad (145)$$

we arrive at the matrix form

$$\mathbf{A}_q \begin{pmatrix} x_q \\ \pi_q \end{pmatrix} = \mathbf{C}_q \begin{pmatrix} x_{q-1} \\ \pi_{q-1} \end{pmatrix} + \mathbf{D}_q \cdot s_q + \mathbf{E}_q \cdot s_{q-1} + \begin{pmatrix} \sigma \epsilon_q^x - \epsilon_q^i \\ \epsilon_q^\pi \end{pmatrix} \quad (146)$$

with the *time dependent* system matrices:

$$\begin{aligned} \mathbf{A}_q &= \begin{pmatrix} \sigma - \sigma \omega_{x,q}^{\text{ext}}(1 + \alpha_x) - \sigma c_1 \omega_{s,q}^{\text{tar}} h & -(1 - \sigma c_1) \omega_{\pi,q}^{\text{ext}}(1 + \alpha_\pi) \\ -\gamma & 1 - \beta \omega_{\pi,q}^{\text{ext}}(1 + \alpha_\pi) \end{pmatrix} \\ \mathbf{C}_q &= \begin{pmatrix} \sigma (\omega_{x,q}^{\text{hab}} - \alpha_x \omega_{x,q}^{\text{ext}}) - \delta_x & (1 - \sigma c_1) (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}}) - \delta_\pi \\ 0 & \beta (\omega_{\pi,q}^{\text{rof}} - \alpha_\pi \omega_{\pi,q}^{\text{ext}}) \end{pmatrix} \\ \mathbf{D}_q &= \begin{pmatrix} c_1 \sigma (\omega_{s,q}^{\text{ext}}(1 + \alpha_s) - 1) \\ -\kappa \end{pmatrix} \quad \mathbf{E}_q = \begin{pmatrix} c_1 \sigma (\omega_{s,q}^{\text{sta}} - \alpha_s \omega_{s,q}^{\text{ext}}) \\ 0 \end{pmatrix} \end{aligned}$$

Multiplying with  $\mathbf{A}_q^{-1}$  we arrive at the solution:

$$\begin{pmatrix} x_q \\ \pi_q \end{pmatrix} = \mathbf{A}_q^{-1} \mathbf{C}_q \begin{pmatrix} x_{q-1} \\ \pi_{q-1} \end{pmatrix} + \mathbf{A}_q^{-1} \mathbf{D}_q \cdot s_q + \mathbf{A}_q^{-1} \mathbf{E}_q \cdot s_{q-1} + \mathbf{A}_q^{-1} \begin{pmatrix} \sigma \epsilon_q^x - \epsilon_q^i \\ \epsilon_q^\pi \end{pmatrix} \quad (147)$$

## E. Further Comparison with the Original De Grauwe Model

In this section we compare the properties of our real sector submodel (appendix B; no extension) to those of the original DeGrauwe model (which was also used in Lengnick and Wohltmann (2013)).

### E.1. Expectations Heuristics

In both, the real and financial subsystem, agents make use of boundedly rational heuristics to form expectations. Unfortunately, those heuristics assumed in De Grauwe (2010a,b,c) and Westerhoff (2008) are of a different character. E.g.: Westerhoff assumes an extrapolation of the recently observed stock price ( $s_t$ ) movement<sup>35</sup> while DeGrauwe simply takes the past value as the extrapolators' rule:

$$\text{Westerhoff:} \quad \tilde{\mathbf{E}}[s_{t+1}] = s_t + k \cdot (s_t - s_{t-1}) \quad (148)$$

$$\text{DeGrauwe:} \quad \tilde{\mathbf{E}}[\pi_{q+1}] = \pi_{q-1} \quad (149)$$

For consistency, and because  $\tilde{\mathbf{E}}[x_{t+1}] = x_{t-1}$  is typically called *static* expectations (instead of extrapolative), we apply

$$\tilde{\mathbf{E}}_q^{\text{ext}}[\pi_{q+1}] = \pi_q + \alpha_\pi \cdot [\pi_q - \pi_{q-1}] \quad (150)$$

as the extrapolators' heuristic in the real sector. At the same time we keep the static expectations of DeGrauwe

$$\tilde{\mathbf{E}}_q^{\text{sta}}[\pi_{q+1}] = \pi_{q-1} \quad (151)$$

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<sup>35</sup>Westerhoff (2008) does only implicitly use expectations formation. Compare Lengnick and Wohltmann (2013), section 2.1.

because they give rise to hybridity (similar to the rule-of-thumb or habit formation arguments). The targeters' rule corresponds to that of fundamentalists in the financial sector:

$$\tilde{E}_q^{\text{tar}} [\pi_{q+1}] = \pi_q^* \quad (152)$$

Just as fundamentalists believe that stock prices are going to return to the steady state, targeters believe that inflation is going to return to its steady state.

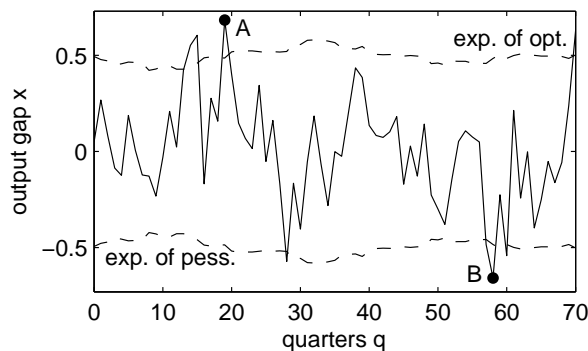


Figure 8: Output gap and its expectations in DeGrauwe NKM

Besides consistency issues, these assumptions also solve a problem that can occur in the original De Grauwe expectations. In his model agents' expectations are either given by a positive (optimists) or a negative value (pessimists).

$$\text{Optimists: } \tilde{E}_q^{\text{opt}} [x_{q+1}] = g_q \quad \text{with: } 2g_q = \mu + \nu \cdot \text{std}[x_q] \quad (153)$$

$$\text{Pessimists: } \tilde{E}_q^{\text{opt}} [x_{q+1}] = -g_q \quad \left( = -\tilde{E}_q^{\text{opt}} [x_{q+1}] \right) \quad (154)$$

Figure 8 illustrates the development of output (solid line) and the corresponding expectations of optimists and pessimists (dashed lines). For the parameterization consult table 6. Output fluctuates in a range between -0.6 and 0.6 while expectations are almost constant at -0.5 and 0.5. Sometimes (e.g. point A) we find the somewhat strange result that optimists expect falling output, a behavior that can hardly be called optimistic any more. Additionally, the pessimists would expect a fall of output from 0.6 down to -0.5. Such a huge change is never observed through the entire series and thus seems a bit too unreasonable to expect. Similar issues can also arise for pessimists (point B). This problem disappears when heuristics (9)-(14) are applied.

## E.2. Bifurcation Analysis

The De Grauwe model does also give rise to bifurcations and stable limit cycles. Bifurcation diagrams of the deterministic core for  $\pi$  and  $x$  with respect to the learning algorithm parameters  $\mu$ ,  $\phi$  and  $\zeta$  are shown in figure 9. The corresponding parameterization is given in table 6. Small levels of belief bias  $\mu \in (0; 0.575)$  guarantee a unique steady state. At  $\mu = 0.575$  a pitchfork bifurcation takes place and above 1.425 stable limit cycles emerge. Note, however, that these results exploit the problematic expectations heuristic described above.

Table 6: Parameterization of original DeGrauwe model

NKM	Hybridity	Policy	Learning	Noise
$\sigma = 1.0$	$\chi = 0.80$	$\delta_x = 0.5$	$\zeta = 0.5$	$\sigma_\epsilon = 0.15$
$\beta = 0.99$	$\psi = 0.80$	$\delta_\pi = 1.5$	$\phi = 10$	
$\gamma = 0.33$			$\mu = 0.5$	
			$\nu = 2$	

The parameter  $\phi$  does not give rise to similar phenomenon while the memory parameter  $\zeta$  gives rise to a bifurcation at  $\zeta = 0.63$ . If agents' memory is very high ( $\zeta \in [0.967, 0.983]$ ), the system displays stable limit cycles on the intervals  $x \in (-0.062, 0.062)$ ,  $\pi \in (-0.15, 0.15)$ . For  $\zeta \in [0.983, 1[$  the system either converges against a positive or a negative steady state, for perfect memory  $\zeta = 1$  the model explodes.

In contrast, our adjusted NKM displays bifurcation and stable limit cycles for the parameters  $\alpha_x$  and  $\alpha_\pi$  (figure 10). Like the original De Grauwe model the intensity of choice parameter  $\phi$  does not give rise to any bifurcations. In contrast to De Grauwe our version does also not give rise to bifurcations or limit cycles for high values of the memory parameter. Even for perfect memory  $\zeta = 1$  our model is solvable.

Table 7: Parameterization of our model

NKM	Hybridity	Policy	Learning	Noise
$\sigma = 1.0$	-	$\delta_x = 0.5$	$\zeta = 0.5$	$\sigma_\epsilon = 0.15$
$\beta = 0.99$		$\delta_\pi = 1.5$	$\phi = 10$	
$\gamma = 0.33$			$\alpha_x = \alpha_\pi = 0.2$	

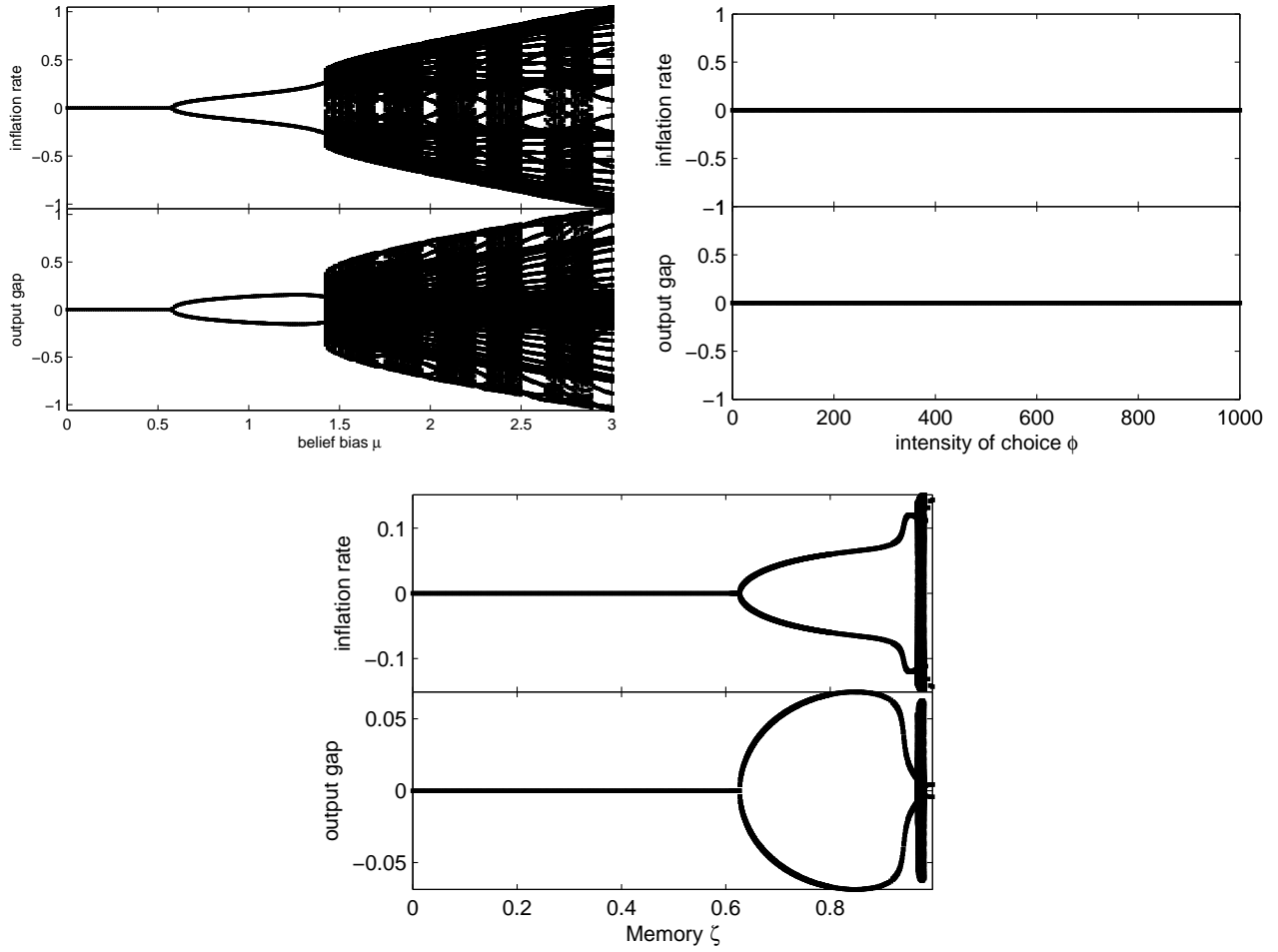


Figure 9: Bifurcation plots of original De Grauwe (2010a,b,c) NKM

### E.3. The Special Case of Full Price Flexibility

In NKMs, price stickiness is given by the Calvo parameter  $\theta$ . Perfect price flexibility is archived if  $(1 - \theta) \rightarrow 1$  while for  $\theta \rightarrow 1$  prices are fully rigid. The Calvo parameter is one of the deep parameters that affect  $\gamma$  in the Phillips curve:

$$\gamma = \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \frac{1 - \alpha}{1 - \alpha + \alpha\epsilon} \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \quad (155)$$

In the following we set  $\alpha = 0$  (constant returns to scale production function),  $\epsilon = 6$  (induces a markup of 20%),  $\varphi = 1.0$  (a unitary Frisch elasticity of labor supply),  $\sigma = 1.0$  (log utility). Compare Gali (2008) on the microfoundation of  $\gamma$  and our parameterization. To test the effect

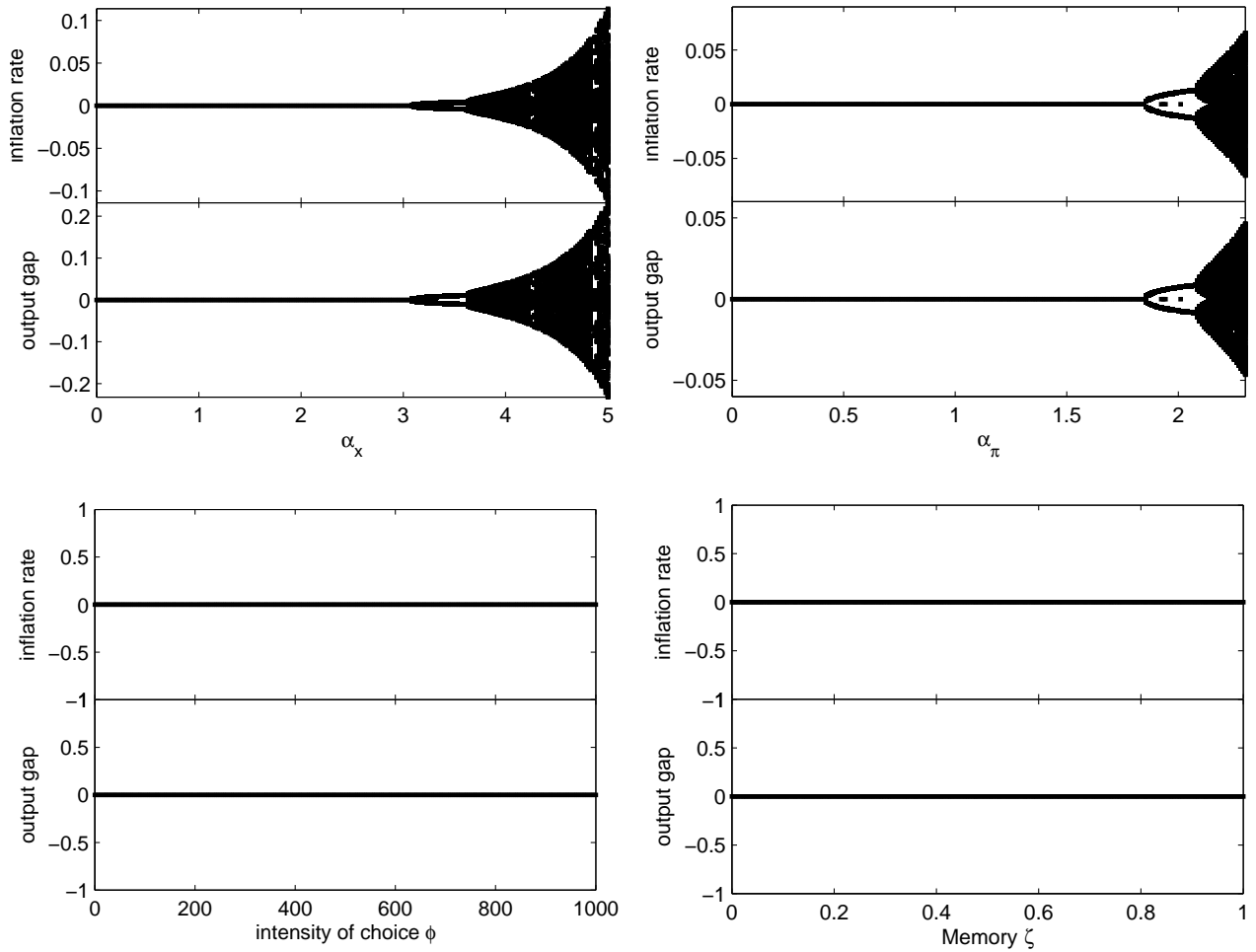
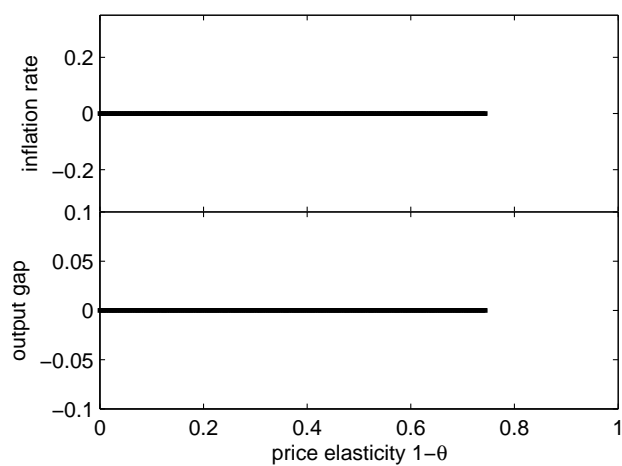


Figure 10: Bifurcation plots of adjusted NKM

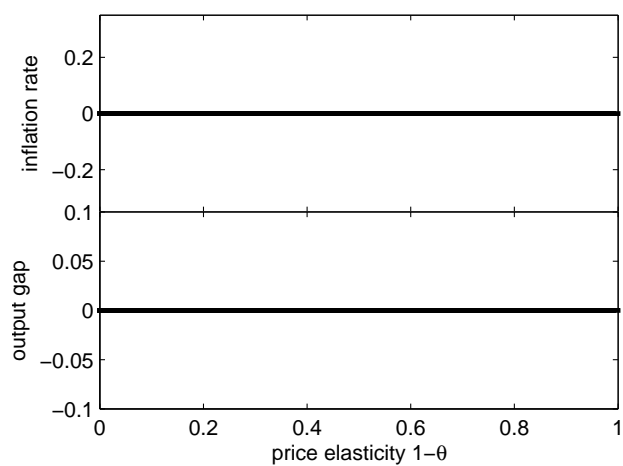
of price flexibility on our model, we vary  $1 - \theta$  on the interval from 0 to 1. The calibration of all other is given in table 6 for the original DeGrauwe model and in table 7 for our model.

To illustrate the impact of changing price flexibility on the system, we plot the (deterministic) steady state of inflation and output for different values of  $(1 - \theta)$  in figure 11. In the original DeGrauwe model (panel (a)), stability of the steady state is only guaranteed for values of  $(1 - \theta)$  up to 0.74. If price flexibility is increased above that value, the model yields explosive dynamics. This problem does not occur in our model (panel (b)). Even for full price flexibility, the model has convergent (i.e. stable) solutions.





(a) Original DeGrauwe



(b) Lengnick, Wohltmann model

Figure 11: Steady state of deterministic core for different degrees of price flexibility