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Working Paper

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Thünen-Series of Applied Economic Theory - Working Paper, No. 140

Provided in Cooperation with:

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Suggested Citation: Kersting, Stefan; Hüttel, Silke; Odening, Martin (2015) : Structural change in agriculture under capacity constraints: An equilibrium approach, Thünen-Series of Applied Economic Theory - Working Paper, No. 140, Universität Rostock, Institut für Volkswirtschaftslehre, Rostock

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Thünen-Series of Applied Economic Theory
Thünen-Reihe Angewandter Volkswirtschaftstheorie

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Structural change in agriculture under capacity constraints – an
equilibrium approach

by

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2015

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Stefan Kersting, Silke Hüttel and Martin Odening

Abstract

This paper analyses the impact of a tradable production quota on firm entry and exit in the agricultural industry. We develop a dynamic stochastic equilibrium framework considering that a release of production capacity by exiting firms affects the investment options for entrants. Firms' investment and exit decisions depend on future output and quota prices, which in turn will be affected by the evolution of industry structures themselves. Contrary to many static models we find that introducing a quota system may foster structural change. A tradable quota increases the liquidation value and makes inefficient firms cease production despite higher output prices.

Keywords: dynamic stochastic equilibrium, production quota, firm entry and exit, dairy sector

JEL codes: D41, L11, Q11

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1 Introduction

Structural change, that is changes in the composition of the elements of an industry such as number and size of firms over time, is a fundamental phenomenon of market economies. Such change can be understood as an aggregated outcome of firms' adjustments to a changing economic environment such as price or policy changes, and technical progress (cf. Dunne et al., 2013). Firms' decisions that affect structural change are, for example, market entries and exits, growth and shrinkage, change of the production structure or the adoption of new key technologies (cf. Caves, 1998). Understanding these adjustment processes is of great interest because structural change determines a sector's competitiveness (Jorgenson and Timmer, 2011). Moreover, these processes have consequences for distributional issues, regional development and rural employment and other policies (Piet et al., 2012).

A coherent analysis of structural change is challenging since firms' decisions have to be made in a dynamic and stochastic environment under possible capacity constraints, which lead to interdependent decisions. This is particularly true for the agricultural sector because the availability of crucial production factors such as land is usually limited. This shortage, in combination with immobility as in the case of farmland, cause a strong interdependence of firms' decisions within a region (Chavas, 2001). That is, farms usually cannot grow in size unless other farms¹ exit since only the capacity of ceasing firms provides new factor supply like land or other limited inputs (cf. Balmann et al., 2006). Hence, the price for production capacity strongly depends on the exit/shrinking rate determining the amount of free capacity, as well as on expansion activities of the other firms (e.g. Weiss, 1999; Zepeda, 1995; Richards and Jeffrey, 1997). The impact of scarcity on firms' decision making and the resulting competition for a production factor may further increase if some firms expect to benefit from economies of size. Given that production capacity as such represents a valuable asset, a firm's liquidation value may increase under binding capacity constraints.

The European dairy sector is particularly exposed to limited capacity. Besides the scarcity of farmland for primary production, the downstream market was highly regulated by production quotas (1984-2015). That is, overall milk supply was limited and attached to a production right. Our paper addresses the research question of how this specific capacity constraint – production quota – affects the dynamics of structural change. Particularly farms' investment/disinvestment decisions have been heavily influenced by the milk quota regime and it is frequently hypothesised that introducing a production

¹Farms and firms will be used interchangeably throughout the paper.

quota such as those in the European Union or Canada may have slowed down structural change and hindered efficient adjustment processes (Colman, 2000). But is this also true if quotas are traded and thus increase the liquidation value of exiting farms (Barichello, 1995)?

To assess the impact that a production quota has on structural dynamics, we employ a dynamic stochastic equilibrium model (DSGE)² that exhibits three important characteristics: First, entry and exit decisions of farms, as well as prices and production output are determined endogenously. That is, firms take the prospective entry/exit of other firms into consideration. Second, decisions are made in a dynamic framework. This approach allows us to track changes in the sector's composition. Third, the model is driven by a stochastic component.

The modelling framework rests on earlier work by Jovanovic (1982) and Hopenhayn (1992), who analyse the dynamics of an industry with endogenous entry and exit. The model considers a perfectly competitive but heterogeneous industry, where firms differ with respect to their productivity level, which is stochastic and assumed to follow a Markov process. Firms' entry and exit decisions are based on rational expectations of profits and are thus affected by the development of the output price as well as by the current state and evolution of the productivity. We enhance the basic model of Hopenhayn (1992) by introducing capacity constraints at the sector level. Entry costs, as well as the liquidation value of exiting firms should correlate with the industry structure. The overall limitation of capacities needed to expand in production is modelled through entry cost strongly increasing with the size of the sector. In contrast to Hopenhayn (1992) we analyse the sector dynamics in a finite time horizon which allows us to keep track of farms' adjustment processes in each period.

We apply our theoretic model to the West German dairy sector. To determine whether the milk quota as such and the possibility of trading slows down or accelerates structural change, we investigate a situation with and without a tradable quota. As one would expect, the output prices are higher if investment or entry of new firms is constrained by a production quota. Surprisingly, we find that the critical productivity threshold for staying in the industry is also higher in this case. This means that even relatively efficient farms are now better off selling their quota rights instead of sustaining production.

The contribution of this paper is twofold. First, we provide a dynamic stochastic framework that incorporates a limited production capacity into the firms' entry/exit decision. Instead of analysing steady state properties, which many other research studies do, we

²DSGE models are often used, particularly for macro-economic questions. See, for instance, Dewachter and Wouters (2014) for modelling endogenous financial risk, or Matsumoto et al. (2011) for studying the role of shocks on asset price volatility.

consider a finite dynamic equilibrium. The finite nature of the model may not fully represent rational firm behavior in later periods, though it allows us to keep track of structural changes in greater detail to explore the effect of a tradable production quota on the farm size distribution and the investment/disinvestment behavior of firms. Second, we illustrate how to calibrate a dynamic stochastic entry/exit-model using farm-level panel data, as well as how to calculate a finite equilibrium numerically.

The remainder of this article is structured as follows. In section 2 we present the dynamic stochastic framework and show how an explicit equilibrium can be computed. The focus will be on incorporating limited sectoral capacity into the farms' value function. The theoretic model is applied to the West German dairy sector in section 3. After describing the development of the industry over recent decades, we calibrate model parameters and calculate the dynamic market equilibrium. Based on the equilibrium outcome we will discuss the effect of a milk quota on farms' entry and exit decisions. Section 4 concludes and provides an outlook on further research.

2 Modelling structural change under capacity constraints

2.1 Overview and classification

The analysis of structural change is a well-established topic in agricultural economics and much effort has been spent on modelling farm adjustment processes theoretically and empirically (Balmann et al., 2006). However, most existing models consider either single farms (eg. Foltz, 2004), adopt an aggregated view of the sector (Wolf and Sumner, 2001), or compare only cross country differences (Adamopoulos and Restuccia, 2014). Individual farm models allow for dynamic stochastic adjustment behaviour, but they have to assume exogenous price processes and thus do not ensure rational expectations equilibria on product and factor markets. In contrast, sectoral equilibrium models leave little room for a micro-economic foundation of decision making (Féménia and Gohin, 2011). Only few modelling approaches take the interdependencies of growing and shrinking farms into account when describing the adjustment processes within a sector. These modelling approaches encompass multi-agent models, real options models and game theoretic models.

Multi-agent models consist of sets of rules defining how agents make individual decisions and how they interact with each other and with their environment. Agent-based models have been quite successful in explaining stylised empirical facts such as the path

dependency of systems, which are not well accounted for by existing representative-agent equilibrium models (Balmann et al., 2013). An advantage of this modelling framework is its flexibility. Heterogeneity of the agents with regard to their production capacities and constraints can be easily implemented. The usefulness of MAM, however, is not unchallenged; they have been criticised because the outcome of complex dynamic simulations is difficult to interpret and generalise (Leombruni and Richiardi, 2005). Moreover, it is practically impossible to implement the concept of a rational expectations equilibrium.

Real options models have been developed to derive optimal investment and disinvestment strategies for firms facing uncertainty and sunk costs while having some managerial flexibility with regard to the timing of (dis)investments. Initially, real options models consider single firms, however, Leahy (1993) shows that the optimal (dis)investment strategies derived for individual firms are also valid in a competitive environment with free market entry and homogeneous firms. Recently, Feil and Musshoff (2013) have utilised the real options framework for an evaluation of agricultural policy schemes in a dynamic stochastic context. By applying heuristic solution procedures these authors are able to relax simplifying assumptions of previous models that strive for closed form solutions of the dynamic equilibrium in a sector.

Game theoretic approaches have been applied to analyse the relationship between the dynamics of market structure and competition. The seminal work by Ericson and Pakes (1995) defines a dynamic stochastic game to describe the development of an oligopolistic market structure with heterogeneous firms. These models are particularly useful for explaining the emergence of asymmetric industry structures (Besanko and Doraszelski, 2004). Game theoretic models are capable of modelling growth and shrinkage of firms in a given market with endogenous supply or constrained capacities, but they are difficult to handle, particularly if there are more than two firms within the market.

Dynamic stochastic equilibrium models can sometimes provide a framework to overcome this shortcoming. Authors like Hopenhayn (1992) or Asplund and Nocke (2006) model a heterogeneous industry by a continuum of firms. Idiosyncratic uncertainty, which every firm faces in its production process, thus cancels out at the aggregate level. As a consequence, the evolution of the industry follows deterministic paths, and changes in the industry structure can be pursued in greater detail.

Hopenhayn's (1992) work is based on the concept of a stationary equilibrium and idiosyncratic uncertainty, and investigates high turnover rates within industries. The dynamic stochastic model takes into account endogenous exit and possible subsequent entry attached to sunk cost that are induced by exogenous firm-specific productivity shocks.³

³See also Bento (2014), who takes up the idea of Hopenhayn in modelling entry costs.

Exit as a precondition for entry takes place as soon as a firm's productivity shock falls below a reservation value. This enables the reallocation of resources between the firms. In the stationary equilibrium, entry and exit occur. Together with the productivity shocks and the respective production decisions entry and exit determine the firm size and profit distribution within that industry. Further, Hopenhayn's findings reveal that the size distribution is stochastically increasing with age, meaning that larger firms have a higher probability of survival.

Melitz (2003) extends the framework of Hopenhayn to consider monopolistic competition and analyses intra-industry effects of international trade. This author shows that the least productive firms are forced to leave the market, while the most productive produce for the export market. From a more global perspective this will lead to an international re-allocation towards the more productive firms. The core model assumption that the patterns of entry and exit are systematically related to productivity differences among firms is confirmed by Fariñas and Ruano (2005). The authors further show that sunk costs are one source of persistent heterogeneity in productivity, that is, in markets with high and sunk entry costs, a lower productivity becomes more likely.

2.2 The formal model

The basic setup of our model draws closely upon the seminal papers of Jovanovic (1982) and Hopenhayn (1992), whose respective approaches explicitly allow for endogenous entry and exit of the firms, which is crucial for analysing structural change in agriculture under capacity constraints.

We consider a perfectly competitive industry with a continuum of firms producing a homogeneous good. Each firm takes the output price as given and chooses its optimal output quantity. The output price will be determined by market clearance. The inverse demand function $D(Q) > 0$ should be continuously differentiable and strictly decreasing. We assume that $\lim_{Q \rightarrow +\infty} D(Q) = 0$. The time horizon $T < \infty$ is finite and competition takes place in discrete time ($t = 0, \dots, T$).

All firms have the same production technology but they differ with respect to their productivity level. That is, we account for firm-specific productivity differences which may be through farm size, capital stock, feed management, livestock management or natural conditions. The firm-specific productivity is supposed to be the only source of uncertainty faced by the firms.

We model the firm's individual productivity as a stochastic parameter $\varphi_t \in \mathbb{R}$, which

follows the AR(1)-process

$$\varphi_{t+1} = \rho\varphi_t + \varepsilon_{t+1}, \quad \rho \in (0, 1] \text{ and } \varepsilon_{t+1} \stackrel{\text{iid}}{\sim} N(\nu_\varepsilon, \sigma_\varepsilon^2). \quad (1)$$

The stochastic process defined in (1) describes the evolution of a firm's productivity and is the same for all incumbents. Nevertheless, the realisation of the error term ε_{t+1} is independent across firms and over time. The process as given in (1) inherits the Markov property and is time-homogeneous. Under the hypothesis $\varphi_t = \varphi$, it follows that $\varphi_{t+1} \sim N(\rho\varphi + \nu_\varepsilon, \sigma_\varepsilon^2)$. The density of this normal distribution is denoted by

$$f(z, \varphi) := \frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}} \exp\left(-\frac{(z - (\rho\varphi + \nu_\varepsilon))^2}{2\sigma_\varepsilon^2}\right), \quad (2)$$

and the conditional cumulative distribution function $F(\varphi'|\varphi) = \text{Prob}(\varphi_{t+1} \leq \varphi'|\varphi_t = \varphi)$ is given by

$$F(\varphi'|\varphi) = \int_{-\infty}^{\varphi'} f(z, \varphi) dz. \quad (3)$$

The function $F(\varphi'|\varphi)$ constitutes a probability kernel and is continuous with respect to both arguments. Moreover, it is strictly decreasing in φ if we keep φ' fixed.⁴ That is, all active firms can be explicitly distinguished by their current productivity level φ_t . The distribution of these values across all firms thus expresses the state of the industry in period t , which is denoted by the measure $\mu_t : \mathcal{B}(\mathbb{R}) \rightarrow \mathbb{R}_+$ defined on the Borel sets of the real numbers.⁵ Hence, any changes of the industry structure, caused by the stochastic productivity process as well as entry/exit of firms, translate into changes of μ_t .

We further proceed upon the assumption that firms with higher productivity levels are able to produce any amount of output q at lower costs. This property is represented by a twice continuously differentiable cost function $c(\varphi, q)$, which is monotonically decreasing in φ with the limits

$$\lim_{\varphi \rightarrow +\infty} c(\varphi, q) = 0 \quad \text{and} \quad \lim_{\varphi \rightarrow -\infty} c(\varphi, q) = \infty, \quad \forall q \geq 0. \quad (4)$$

Further, function $c : \mathbb{R}_+^0 \times \mathbb{R} \rightarrow \mathbb{R}_+^0$ should satisfy $c(\varphi, 0) = 0$, and

$$\frac{\partial c}{\partial q} > 0 \quad \text{with} \quad \frac{\partial c}{\partial q}(0, \varphi) = 0, \quad \frac{\partial^2 c}{\partial q^2} > 0, \quad \frac{\partial^2 c}{\partial \varphi \partial q} \leq 0, \quad \lim_{\bar{q} \rightarrow +\infty} \frac{\partial c}{\partial q}(\bar{q}, \varphi) = \infty. \quad (5)$$

⁴If $\varphi_1 < \varphi_2$, the distribution $F(\cdot|\varphi_2)$ stochastically dominates $F(\cdot|\varphi_1)$.

⁵ μ_t does not need to be a probability measure. The total mass $\mu_t(\mathbb{R})$ may be smaller or bigger than one, indicating the size of industry.

In each period t of the planning horizon all active firms have to choose their own optimal production output. These firms take the output price $p_t \geq 0$, as well as their current productivity level φ_t , as given and maximise:

$$\max_{q_t \geq 0} p_t q_t - c(\varphi_t, q_t). \quad (6)$$

The first-order condition for a maximum in (6) implies that the optimal firm-specific output q_t^* satisfies

$$p_t \leq \frac{\partial c}{\partial q}(\varphi_t, q_t), \quad \text{with equality if } q_t > 0. \quad (7)$$

The imposed restrictions on the cost function guarantee that for all valid combinations of p_t and φ_t , a unique solution $q_t^* = q^*(\varphi_t, p_t)$ to (7) exists. The firm-specific optimal output is thus a continuous function of the output price p and the productivity φ . Moreover, it is (strictly) monotonically increasing in both arguments, and every firm will produce a positive amount of output as long as the price is larger than zero.

The aggregate industry output $Q_t = Q^s(p_t, \mu_t)$ depends on the structure of the industry and is given by

$$Q^s(p_t, \mu_t) = \int_{\mathbb{R}} q^*(\varphi, p_t) d\mu_t(\varphi). \quad (8)$$

In case the integral on the right-hand side exists for any output price, we infer from q^* 's properties that $Q^s(p, \mu)$ is continuous and increasing with respect to p .

Production incurs a fixed cost $c_f > 0$, which is the same for all firms and has to be paid at the beginning of each period before a new productivity level is revealed to incumbents according to the Markov process (1). Hence, the fixed costs are sunk by the time firms get to know their new realisation of φ_t and choose the production output. A firm's profit per period is then

$$\pi(p_t, \varphi_t) := p_t q_t^* - c(\varphi_t, q_t^*) - c_f, \quad (9)$$

with $q_t^* = q^*(\varphi_t, p_t)$ being the optimal firm-specific output level. The properties of $c(\varphi, q)$ and q^* imply that $\pi(\varphi, p)$ is a continuous function as well, is strictly increasing in p and, in case the output price is positive, also in φ . Furthermore, the period profits will tend to the negative fixed costs $\pi(\varphi, p) \rightarrow -c_f$ whenever $p \rightarrow 0$ or $\varphi \rightarrow -\infty$.

At the end of each period firms have the option to leave or enter the market. New firms entering the market must acquire production capacity and have to pay entry costs $k_t > 0$. After paying both the entry and fixed production costs, each new firm is assigned with a productivity level, which is drawn from the common distribution function G . As this is the only kind of investment available to any firm, the same applies to established firms willing to expand their production capacity. An extension of production capacity can be

regarded as setting up new production units. Therefore, we do not distinguish between new firms and expanding firms explicitly, but refer to both groups as entering firms.

When a production quota exists the entry costs will be directly related to the number of firms leaving the industry. In this situation every incumbent possesses production capacity which can be sold in case of exit. If a firm decides to leave the industry it will release its capacity and receive a compensation payment r_t . This liquidation value in turn will depend on the demand for capacity generated by entering firms. In order to capture this interdependency between entry costs and liquidation value we model both as a function of the total industry mass $\mu_t(\mathbb{R})$. The industry mass depends on the number of firms leaving or entering the market and describes growth/shrinkage of the industry; it can be interpreted as a proxy for the availability of the production capacity at the sector level.

We introduce a continuous and non-decreasing function k and define the entry costs $k_t := c_e + k(\mu_t(\mathbb{R}))$. The entry costs are composed of two parts: Entering firms have to pay a constant part c_e , which is sunk afterwards, and a variable part $k(\mu_t(\mathbb{R}))$, which reflects the additional costs for quota rights. As the exit premium should be proportional to the quota costs at time t , we define $r_t := k(\mu_t(\mathbb{R}))$. If a production quota does not exist the variable part drops out of the entry costs. Hence, we model a scenario without limited capacity supply (free access) by setting the entry costs constant $k_t = c_e$ and the compensation payment equal to zero $r_t = 0$.

A firm bases its entry/exit decision on the expected discounted future profits. The discount rate for all firms is supposed to be $0 \leq \beta < 1$. If the output prices for all periods are known and denoted by the vector $\mathbf{p} = (p_0, \dots, p_T)$, the value of an incumbent with productivity φ at time t can be defined recursively by

$$v_t(\varphi, \mathbf{p}) = \pi(\varphi, p_t) + \beta \max \left\{ r_{t+1}, \int_{\mathbb{R}} v_{t+1}(\varphi', \mathbf{p}) dF(\varphi'|\varphi) \right\}, \quad \forall t = 0, \dots, T-1, \quad (10)$$

which is composed of the current profits plus the optional liquidation or continuation value. Since we assume a finite planning horizon, this definition holds true for all periods but the last one. The value at the end of competition is just equal to the profits generated in the final period $v_T(\varphi, \mathbf{p}) = \pi(\varphi, p_T)$.

A firm stays in the industry as long as its continuation value offsets the exit premium r_{t+1} . The continuation value indicates the expected future profits conditioned on the firm's current productivity level. The exit-point x_t describes the critical threshold for being indifferent between staying in or leaving the market,

$$x_t := \inf \left\{ \varphi \in \mathbb{R} : \int_{\mathbb{R}} v_{t+1}(\varphi', \mathbf{p}) dF(\varphi'|\varphi) \geq r_{t+1} \right\}. \quad (11)$$

The assumptions made on the stochastic process and the period profits imply that all firms with a productivity above the exit-point $\varphi_t \geq x_t$ stay in the industry, while all firms with a lower productivity $\varphi_t < x_t$ take the exit premium and quit. If the infimum in (11) does not exist, we are in a situation where no exit occurs in period t and we formally set $x_t = -\infty$.

The expected profits of a firm willing to enter the industry at the end of period t are given by

$$v_{t+1}^e(\mathbf{p}) = \int_{\mathbb{R}} v_{t+1}(\varphi, \mathbf{p}) dG(\varphi). \quad (12)$$

We denote the mass of firms which decide to enter at time t and start production in the following period by M_t . One should keep in mind that this may also include established firms building up additional production units. An increasing number of active firms will lead to a higher aggregate industry output and result in a lower market price. New firms will be entering the industry as long as their expected future profits cover the entry costs, i.e., in an equilibrium we have $v_{t+1}^e \leq k_{t+1}$. This condition must hold with equality if $M_t > 0$.

Due to the large number of firms in the industry (recall that firms are assumed to constitute a continuum), we do not have to deal with aggregate uncertainty. The frequency distribution of productivity levels in upcoming periods is completely specified by the stochastic productivity process and the entry/exit behaviour of firms.⁶ For a given exit-point x_t and entry-mass M_t , the industry structure in period $t + 1$ is

$$\mu_{t+1}((-\infty, \varphi']) = \int_{\varphi \geq x_t} F(\varphi'|\varphi) d\mu_t(\varphi) + M_t G(\varphi'). \quad (13)$$

If both μ_t and G have Lebesgue densities $m_t(z)$ and $g(z)$, the state of the sector μ_{t+1} can also be characterised by its density

$$m_{t+1}(z) = \int_{\varphi \geq x_t} f(z, \varphi) m_t(\varphi) d\varphi + M_t g(z). \quad (14)$$

2.3 Equilibrium analysis

As a direct consequence of (13), both industry output and market price follow deterministic sequences. Firms are atomistic and cannot affect price by the choice of their output quantity. However, firms have perfect information about the strategic decisions of others and are thus able to foresee the development of output prices. In a dynamic equilibrium

⁶A deterministic development of the industry structure is justified by the law of large numbers. Evidence can be found in Judd (1985) or Feldman and Gilles (1985)

they adjust their output as well as their entry/exit decisions to the anticipated prices. These output prices must be reinforced by the strategic behaviour of firms. Keeping this in mind, we define a dynamic stochastic equilibrium as follows:

Definition 1 (Dynamic Equilibrium). Given a starting distribution μ_0 , a dynamic equilibrium consists of a finite sequence of measures $\{\mu_t^*\}$ and vectors $\mathbf{p}^*, \mathbf{Q}^*, \mathbf{x}^*, \mathbf{M}^*$ containing the market prices, aggregate industry output, exit-points and entry-masses for each period such that for all times $t = 1, \dots, T$ the following conditions are satisfied:

- (i) the output market is cleared

$$\begin{aligned}\mathbf{p}_t^* &= D(\mathbf{Q}_t^*), \\ \mathbf{Q}_t^* &= Q^s(\mathbf{p}_t^*, \mu_t^*),\end{aligned}$$
- (ii) the exit-rule (11) holds with \mathbf{x}_t^* ,
- (iii) no more firms have an incentive to enter the industry, i.e. $v_t^e(\mathbf{p}^*) \leq k_t$, and
- (iv) μ_t^* is determined recursively by (13).

The question arises in which situations a dynamic equilibrium exists, and how it can be detected?⁷ The challenge here is to find values $\mathbf{p}^*, \mathbf{x}^*, \mathbf{M}^*$ such that the four equilibrium conditions are fulfilled. We illustrate in the Appendix A.2 that explicit expressions for the exit/entry conditions

$$\int_{\mathbb{R}} v_{t+1}(\varphi, \mathbf{p}^*) dF(\varphi|x_t) - r_{t+1} = 0, \quad \text{and} \quad (15)$$

$$\int_{\mathbb{R}} v_{t+1}(\varphi, \mathbf{p}^*) dG(\varphi) - k_{t+1} \leq 0, \quad \text{with equality if } M_t > 0 \quad (16)$$

can be derived if μ_0 and G are normal distributions, and the cost function is related to a Cobb Douglas production technology. In this particular case, the $2T$ exit/entry conditions, as well as the market clearing output price \mathbf{p}^* can be written as functions of the exit-points $\mathbf{x} = (x_0, \dots, x_{T-1})$ and entry-masses $\mathbf{M} = (M_0, \dots, M_{T-1})$. Any solution $(\mathbf{x}^*, \mathbf{M}^*)$ to this system of equations thus constitutes a dynamic equilibrium.

To some extent, the equilibrium outcome will be affected by the assumed length of the planning horizon. Since we consider a finite time horizon, the function $v_t(\varphi, \mathbf{p}^*)$ is essentially a discounted sum of expected future profits. The value of a firm at time t will therefore depend on the number of time periods which are still to come. At the beginning

⁷A proof for the existence of a dynamic equilibrium is provided in the Appendix A.1. More details regarding the computation of an equilibrium can be found in Appendix A.2.

firms take the industry development over the whole time span into consideration, while they base their entry/exit decision on just a few upcoming periods at the end of competition. An extension of the time horizon by one period may thus have a strong impact on the value of a firm in the final periods. As firms discount future profits by the factor $\beta < 1$, however, the impact on a firm's value in the first periods is less harsh and will possibly diminish in the long run. For this reason, we expect results to stabilise if the time horizon tends to infinity. But the numeric effort to calculate an equilibrium in this case will be enormous.

3 Structural change in the Western German dairy sector

3.1 Stylised facts: agricultural policy and development of farm size distribution

In 1984, the EU introduced the milk quota system, which limited farms' milk production and used intervention prices as a price minimum. In the initial years of the program, the production right was not transferable. This restriction has been relaxed over time, from family transfers and regional but rental transfers to official sales within auctions for Eastern and Western German Federal States separately. Within the 2003 Common Agricultural Policy (CAP) reform, the de-coupling of direct payments from the production levels and the further reduction of price regulation induced higher price volatility and lowered the certainty level of expectations. More recently, within the 2008 check of the CAP reform, further stages of the milk market reform (decided in 2003) have been implemented, which might have contributed to the falling milk prices at that time (cf. Figure 7 in Appendix A.3). Also, it was finally decided to end the milk quota scheme in 2015. Hence, farms were exposed to further pressure to adjust.

A frequently used measure for structural change is the development of the farm size distribution over time. Basically, two variables may be utilised as a proxy for farm size: the number of cows or the milk output per farm. Here, we start with the number of cows per farm: in Western Germany, the number of dairy farms declined from 1,216,700 in 1960 to 90,200 in 2010 (Statistisches Bundesamt), while the average farm size increased, viz. from an average of 5 cows per farm in 1960 to 43 cows per farm in 2010 (Statistisches Bundesamt), with considerable increases in farm productivity at the same time. The average milk yield per cow increased from 3.6 tons per cow and year in 1964 to 6.9 in 2009.

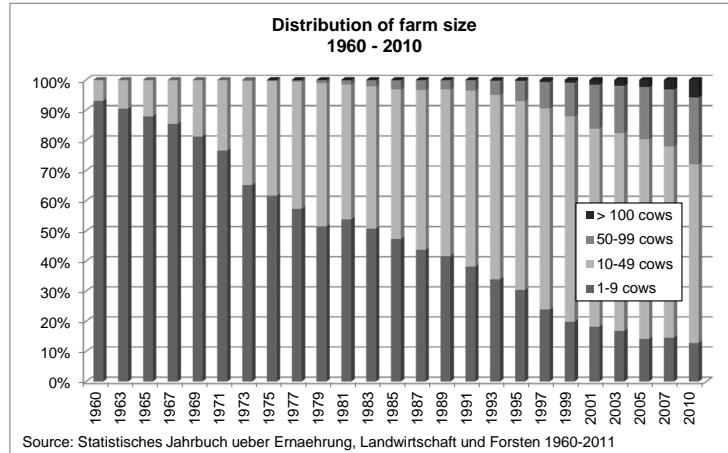


Figure 1: Dairy Farm Size Distribution western Germany 1960-2010

Source: Statistisches Jahrbuch über Ernährung, Landwirtschaft und Forsten 1960-2011

The rather strong consolidation process goes along with an altered farm size distribution as visualised in Figure 1. While the share of the small farms (less than 10 cows per farm) sharply declined over time, the medium (10-49 cows) and large (more than 50 cows) increased in numbers and shares of total number of dairy farms. The share of the large and very large farms particularly increased in more recent years (starting in the mid 1990s).

Figure 2 illustrates changes in the farm size distribution between the years 2000 and 2008 based on data from the Farm Accountancy Data Network (FADN) for both measures: number of cows (left-hand side) and milk output (right-hand side). Here we refer to a kernel density estimation (KDE) of the distribution in natural logarithms based on specialised dairy farms only. We opted for a Gaussian kernel. The right-shift of both distributions confirms the growth in average farm size and the industry's consolidation process, also for the shorter time period. Since the market has been influenced by the milk quota for more than 30 years, this begs the following questions: To what extent has the development been provoked by the quota, and how would the distribution look without quota limitations?

The empirical literature does not yet provide a clear answer on how the quota affected structural change, or what will happen in the nearer future. Some argue that structural change in the dairy sector might be accelerated after the quota removal, where this effect is expected to be stronger the tighter the transfer rules of the milk quota in the quota period are (Bailey, 2002). Nevertheless, even in EU Member States where the quota trade scheme is rather well organised, e.g. the United Kingdom (UK), the milk quota scheme could have been proven to foster inefficient production structures (Colman, 2000; Colman et al., 2002). Moreover, as Oskam and Speijers (1992) show, considerable increases in the capital costs of farms that bought or leased quota might hinder investments in efficient

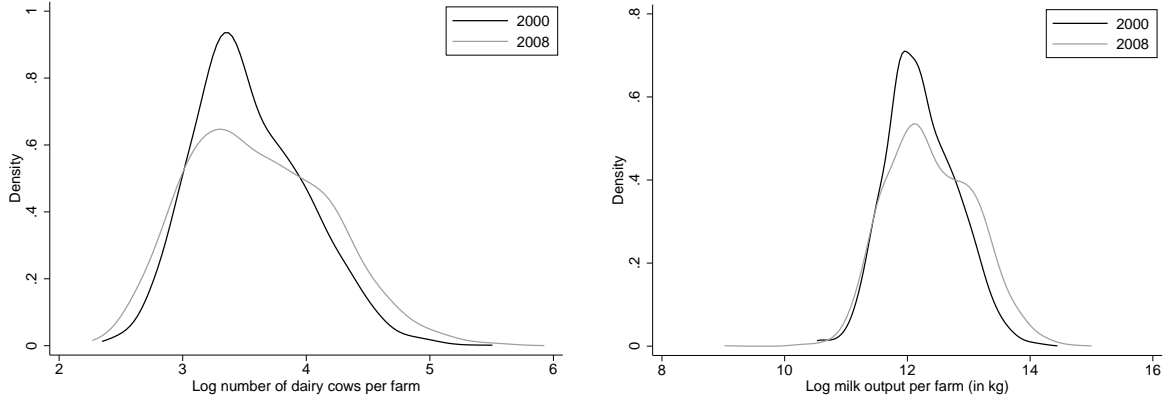


Figure 2: Dairy farm size distribution in 2000 and 2008 for Western Germany
Source: EU-FADN-DG AGRI 1997-2011

production structure. Richards (1995) and Richards and Jeffrey (1997) even find evidence that the milk quota scheme reduces the investment rate of dairy farms in Canada, hindering farm growth and necessary adaptations of technical progress. Thus, it is undisputed that even tradable quotas have an impact on the dairy production industry dynamics and the removal will influence farmers' decision-making.

In what follows, we will examine the effect in greater detail, and calculate and compare the dynamic equilibrium for two scenarios: the equilibrium of a capacity-constrained sector, that is, with production quotas, and an unconstrained sector, which reflects a potential situation after the production quota scheme. For this, we first need to calibrate the benchmark dairy production sector to German data as a base.

3.2 Model calibration

We calibrate our model to the Western German dairy sector in the year 2003. At this point the milk quota was already tradable among all western farms. The data is provided by the EU-FADN-DG AGRI 1997-2011, and the sample contains information on 1,500 specialised dairy farms between 1997 and 2011.

Cost function

We assume a single output and multiple input production technology. Output q is raw milk and assumed to be Cobb-Douglas in inputs n_1, \dots, n_k with a stochastic productivity component denoted by φ :

$$q = c n_1^{\alpha_1} \dots n_k^{\alpha_k} e^{\varphi}. \quad (17)$$

We directly use the estimates of Petrick and Kloss (2012) to calibrate the production function for western German dairy farmers with the inputs labour, land, working capital,

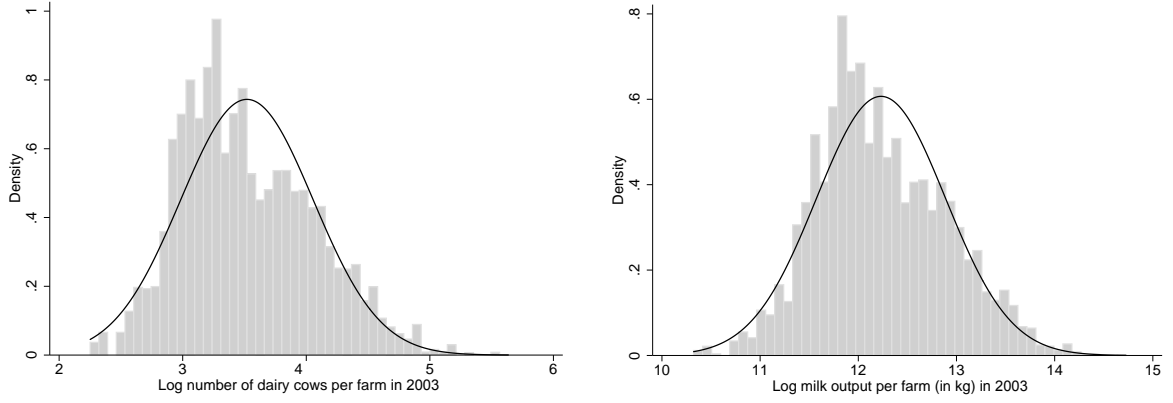


Figure 3: Distribution of dairy cows (left) and milk output (right) across farms in 2003 with adjusted normal distributions
Source: EU-FADN-DG AGRI 1997-2011

fixed capital and number of cows. Based on those results we derive the cost function as follows:

$$c(\varphi, q) = h \left(\frac{q}{\exp(\varphi)} \right)^{\frac{1}{\alpha}}, \quad (18)$$

with constant term h and $\alpha = \sum \alpha_j$. For a given productivity φ the optimal firm-specific output level in period t is thus given by

$$q^*(\varphi, p_t) = \left(\frac{\alpha p_t}{h} \right)^{\frac{\alpha}{1-\alpha}} e^{\frac{\varphi}{1-\alpha}}. \quad (19)$$

This functional form implies that the optimal output level q^* follows a log-normal distribution if φ is normally distributed, that is

$$\mu_t^\varphi = N(\nu_\varphi, \sigma_\varphi^2) \quad \Leftrightarrow \quad \mu_t^q = LN(\nu_q, \sigma_q^2). \quad (20)$$

Starting distribution of productivity levels across farms

Based on this relationship we determine a starting distribution μ_0^φ using the farm-level output distribution in 2003. Figures 3 and 4 suggest that milk output is close to being log-normally distributed across farms. To fit a log-normal distribution μ_0^q to the observed firm-specific output values q_{it} (for farms $i = 1, \dots, n$), we refer to equation (19), where the cost function parameter h is adjusted such that the corresponding distribution of productivity values φ_{it} is centered-normal: $\mu_0^\varphi = N(0, \sigma_\varphi^2)$.

Distribution of new farms

The nature of the farm accountancy data does not allow us to clearly define new firms. Therefore, we assume that the group of new firms also included investing firms (cf. also

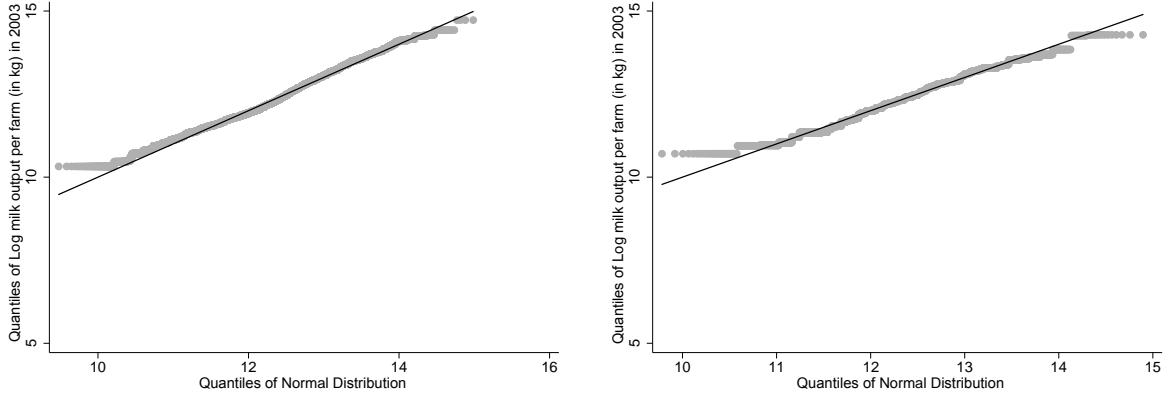


Figure 4: Quantiles of a normal distribution plotted against the quantiles of farms' log milk output (in kg) in 2003. The left side contains all farms while the right side displays just new farms, ie., those who have increased their number of cows by at least 20%.

Source: EU-FADN-DG AGRI 1997-2011

section 2.2), and select those farms from the sample that have increased their number of cows by at least 20% from 2002 until 2003. These farms' milk output distribution is similarly approximated, that is, by a log-normal one (see right side of Figure 4). In a second step, the output distribution is used for the normal distribution of productivity levels: $G = N(\nu_g, \sigma_g^2)$

Demand function

The demand function for milk is supposed to have constant price elasticity $\eta = -\frac{y'(p)p}{y(p)}$. Generally speaking, an isoelastic demand function is given by $y(p) = b p^{-\eta}$, or the corresponding inverse demand function $D(Q) = \left(\frac{Q}{b}\right)^{-\frac{1}{\eta}}$. Here, we refer to Thiele (2008) to calibrate the demand for dairy products in Germany; this author reports a price elasticity of about $\eta = 1.00$. Parameter b will be adjusted such that the market clearing output price matches the observed one in 2003. Given that the average milk price is $p_{2003} = 0.32$ €/kg and the observed distribution of milk output denoted by μ_{2003}^q , the 2003 calibrated demand function is given by

$$p_{2003} = \left(\frac{1}{b} \int_0^\infty y d\mu_{2003}^q(y) \right)^{-\frac{1}{\eta}}. \quad (21)$$

Productivity process

According to the formal model, a farm's productivity is assumed to follow an AR(1)-process:

$$\varphi_{it} = \rho \varphi_{i,t-1} + \varepsilon_{it}, \quad \text{with } \rho \in (0, 1] \quad \text{and} \quad \varepsilon_{it} \sim N(\nu_\varepsilon, \sigma_\varepsilon^2). \quad (22)$$

As the output levels are optimally chosen with respect to a farm's productivity level φ_{it} ,

the process parameter ρ may be estimated by means of the milk output per farm. Let q_{it} denote the milk yield per farm i and year t ; we normalise the values by the average farm level production over time $\bar{q}_t = \frac{1}{n} \sum_{j=1}^n q_{jt}$ as follows:

$$\tilde{q}_{it} = \frac{q_{it}}{\bar{q}_t} = \frac{\exp\left(\frac{\varphi_{it}}{1-\alpha}\right)}{\frac{1}{n} \sum_{j=1}^n \exp\left(\frac{\varphi_{jt}}{1-\alpha}\right)}. \quad (23)$$

For simplicity we let $\bar{e}_t = \frac{1}{n} \sum_{j=1}^n \exp\left(\frac{\varphi_{jt}}{1-\alpha}\right)$. Taking logs on both sides yields

$$\begin{aligned} \log(\tilde{q}_{it}) &= \log\left[\frac{\exp\left(\frac{\varphi_{it}}{1-\alpha}\right)}{\bar{e}_t}\right] \\ &= \frac{\varphi_{it}}{1-\alpha} - \log(\bar{e}_t) \\ &= \frac{\rho\varphi_{i,t-1} + \varepsilon_{it}}{1-\alpha} - \log(\bar{e}_t) + \rho\log(\bar{e}_{t-1}) - \rho\log(\bar{e}_{t-1}) \\ &= \rho\log(\bar{e}_{t-1}) - \log(\bar{e}_t) + \rho\left[\frac{\varphi_{i,t-1}}{1-\alpha} - \log(\bar{e}_{t-1})\right] + \frac{\varepsilon_{it}}{1-\alpha} \\ &= \tilde{a} + \rho\log(\widetilde{q_{i,t-1}}) + \tilde{\varepsilon}_{it}, \end{aligned}$$

with $\tilde{\varepsilon}_{it} \sim N\left(0, \frac{\sigma_\varepsilon^2}{(1-\alpha)^2}\right)$. We apply this log-log specification to the FADN data and estimate the parameters \tilde{a} , ρ and σ_ε^2 using a dynamic panel estimator in line with Arellano and Bond (1991).⁸

Fixed costs

We infer the fixed costs c_f from the development of the output distribution over time. For simplicity we presume the costs to be constant over incumbents. Given that the transition from μ_t^q to μ_{t+1}^q is induced by passing the exit-point x_t , our idea to estimate fixed costs is to estimate the critical productivity level as a threshold. By definition (11), each firm at the critical productivity level $\varphi = x_t$ must be indifferent between staying in or leaving the industry. Fixed costs are derived such that the continuation value of the indifferent firms exactly offsets the liquidation value, which is supposed to be zero.

First, we need to calibrate the critical threshold level. Given that our base year is 2002, we log-normally approximate the distribution of milk output in 2002, denoted by μ_{2002}^q . Accordingly, the 2002 productivity levels follow a normal distribution μ_{2002}^φ . Second, according to (13) and (14) the distribution of productivity levels across farms is uniquely determined by the current distribution, the stochastic productivity process, and entry/exit

⁸The dynamic model applied to panel data causes the lagged dependent to be endogenous because of unobserved farm-specific effects. Accordingly, we use the second- and higher-order lags of the output variables as instruments and estimate the model by the generalized method of moments (cf. Bond (2002) for a similar model).

of farms. If $\mu_{2002}^\varphi = N(\nu, \sigma^2)$, then μ_{2003}^φ is given by the density function

$$m_{2003}^\varphi(z) = \frac{1}{\sqrt{2\pi(\sigma_\varepsilon^2 + \sigma^2\rho^2)}} e^{-\frac{(z - (\rho\nu + \nu_\varepsilon))^2}{2(\sigma_\varepsilon^2 + \sigma^2\rho^2)}} (1 - \hat{F}(x_{2002})) + \frac{M_{2002}}{\sqrt{2\pi\sigma_g^2}} e^{-\frac{(z - \nu_g)^2}{2\sigma_g^2}}, \quad (24)$$

with \hat{F} being the density function of a $N\left(\frac{\sigma^2\rho(z - \nu_\varepsilon) + \sigma_\varepsilon^2\nu}{\sigma_\varepsilon^2 + \sigma^2\rho^2}, \frac{\sigma_\varepsilon^2\sigma^2}{\sigma_\varepsilon^2 + \sigma^2\rho^2}\right)$ random variable. The function constitutes a probability density as long as the mass of entry equals the mass of exit. Presuming that $M_{2002} = \mu_{2002}^\varphi((-\infty, x_{2002}))$, the density function depends solely on the critical exit threshold x_{2002} for the given parameters.

Third, we will estimate the critical threshold level x_{2002} using the method of maximum likelihood (ML). To set up the likelihood function we use the observed and independent farm-specific output values in 2003 ($q_{1,2003}, \dots, q_{n,2003}$) and the corresponding productivity levels $\varphi_{1,2003}, \dots, \varphi_{n,2003}$ based on equation (19):

$$L(x_{2002}) = \prod_{k=1}^n m_{2003}^\varphi(\varphi_{k,2003}; x_{2002}). \quad (25)$$

The unknown threshold level is estimated by maximising the likelihood function in (25).

The continuation value of a firm with productivity level $\varphi = x_{2002}$ must coincide with its liquidation value. If no tradable quota exists, firms giving up production will not be able to generate any positive liquidation value. That is why $r_{t+1} = 0$ must hold for all periods. Further, according to (11) the following equality must hold:

$$\int_{\mathbb{R}} v_{2003}(\varphi', \mathbf{p}) dF(\varphi' | x_{2002}) = 0. \quad (26)$$

Since the continuation value contains the expected future profits, which in turn include fixed costs and depend on future output prices, we choose the level of fixed costs c_f such that (26) is satisfied. For simplicity, we assume constant prices.

Entry costs

Given that the data contains wide range of investment levels, determining a single value for entry costs remains a challenge. The entry costs differ by scenario. In a situation with free access, that is, no entry costs, and assuming a constant price vector \mathbf{p} , the expected value of a new firm is about 14 times the fixed costs. To ensure that new firms truly have an incentive to invest and enter the industry, the constant entry costs c_e should be smaller than this upper boundary. Here we choose $c_e = 8 c_f$.

If the industry is constrained by a quota, new firms must buy production rights in addition to the constant entry costs. Here we suppose the production rights to be dis-

tributed among all active farms. As a result, the quota costs coincide with the revenue that farms withdrawing from production can gain. Referring to the theoretical model, the total industry mass $\mu_t(\mathbb{R})$ is determined by the balance of farms leaving and entering the industry. For this reason, we define the quota costs as an upward-sloping function of the industry mass

$$k(\mu_t(\mathbb{R})) = \frac{c_f}{2} \exp[100 (\mu_t(\mathbb{R}) - 1)]. \quad (27)$$

The function k rises dramatically if the mass of new farms outweighs the exiting ones, and the total industry mass exceeds one. At this point, entry is no longer profitable, and hence $\mu_t(\mathbb{R}) = 1$ serves as a rough upper boundary to the industry's size. Under production quotas, entry costs are composed of the quota costs and the constant part c_e , that is, $k_t = c_e + k(\mu_t(\mathbb{R}))$. The exit premium r_t , which is the value that firms receive while selling their production rights in case of exiting the sector, equals (27). By this definition we guarantee that the quota confines the total size of the industry.

3.3 Findings

	Functional form	Specified parameters
Demand function	$D(Q) = \left(\frac{Q}{b}\right)^{-\frac{1}{\eta}}$	$b = 81,470$; $\eta = 1.00$
Cost function	$c(\varphi, q) = h \left(\frac{q}{\exp(\varphi)}\right)^{\frac{1}{\alpha}}$	$h = 0.0376$; $\alpha = 0.86$
Productivity process	$\varphi_{t+1} = \rho \varphi_t + \varepsilon_{t+1}$, with $\varepsilon_{t+1} \sim N(\nu_\varepsilon, \sigma_\varepsilon^2)$	$\rho = 0.99$; $\nu_\varepsilon = -0.0027$; $\sigma_\varepsilon^2 = 0.0001$
Starting distribution	$\mu_0 = N(\nu_0, \sigma_0^2)$	$\nu_0 = 0.0000$; $\sigma_0^2 = 0.0085$
Distribution of new firms	$G = N(\nu_g, \sigma_g^2)$	$\nu_g = 0.0150$; $\sigma_g^2 = 0.0105$
Discount factor	–	$\beta = 0.9$
Fixed costs	–	$c_f = 3,938 \text{ €}$
Entry costs (free access)	–	$c_e = 31,500 \text{ €}$

Table 1: Utilized functional forms and estimated parameters
Source: Estimations based on Farm Accountancy Data Network

Table 1 summarises the calibrated parameter values. The estimated stochastic productivity process is almost a random walk, with $\rho = 0.99$ being rather close to one. This value, together with the rather low volatility $\sigma_\varepsilon^2 = 0.0001$, indicates that a farm's productivity is rather stable over time. A farm with productivity $\varphi_t = \bar{\varphi}$ in period t will likely achieve a similar productivity level in the consecutive period $t + 1$. Here, one time period represents one year.

The fixed costs value c_f captures all non-variable costs in the production process during one year. This may include, for example, fixed insurance rates, expenditures for rents, as

well as depreciation costs of machinery that result from previous investments. Considering this, the specified fixed costs seem to be on a relatively low level. Still, every farm not able to cover this absolute value with their production profits is forced to leave the industry. One should keep in mind that those fixed costs are also paid by new farms once they have entered the industry. The constant entry costs c_e , thus comprise all additional investments that are obligatory to set up production but are not covered by the fixed costs.

Based on the calibration values as given in Table 1, we now assess the effect of a tradable milk quota on entry and exit of farms by comparing the dynamic equilibria of a scenario with quota (constrained case) to the one without quota (unconstrained case). Given the numerical efforts needed to derive those equilibria, we restrict the planning horizon to $T = 15$ time periods.

Table 2 contains the equilibrium values for the exit-points x_t^* and entry-masses M_t^* , which were found numerically by solving the corresponding system of exit/entry equations in (15) and (16). The exit rate describes the share (in percentage) of farms leaving the industry in relation to the total industry mass $\mu_t(\mathbb{R})$. If no entry occurs, this figure illustrates the shrinkage of the industry in terms of production units. The rather low volatility of output prices over time is a result of the absence of aggregate demand shocks, which we do not consider in our analysis. To make the minimum productivity threshold for staying active more comprehensible to the reader, we have translated the productivity values back into their respective output levels. Hence, the column labelled as $\ln(q_x^*)$ describes the log milk output that a farm with productivity level $\varphi_t = x_t^*$ expects to produce in the forthcoming period.

In a scenario without a milk quota we observe that all new farms enter the industry in the first period. Hence, farms have no incentive to postpone investment when the access to production capacity is unrestricted.⁹ This changes, however, if a quota is introduced, and investing into new farms requires disinvestment by withdrawing farms. In this case, entry takes place not just in the first but during the first seven periods. Furthermore, the mass of entry is bounded (as the total size of the industry is) by construction. As long as farms are entering the industry, the mass of new farms indeed matches the amount of exiting ones.

The continuously lower total industry mass $\mu_t(\mathbb{R})$ in the constrained scenario leads to a smaller aggregate milk output, and thus brings on higher output prices p_t^* . However, irrespective of the higher output prices, the exit-points in the first periods are higher under a quota regime, that is, the critical productivity threshold for staying active increases

⁹Recall that we do not account for limited availability of agricultural land. In practice, the entry of new farms will be confined by this production factor.

Period t	Scenario without milk quota						Scenario with milk quota						Quota costs
	x_t^*	M_t^*	$\mu_t(\mathbb{R})$	Exit rate	p_t^*	$\ln(q_x^*)$	x_t^*	M_t^*	$\mu_t(\mathbb{R})$	Exit rate	p_t^*	$\ln(q_x^*)$	
0	-0.079	0.85	1.00	19.3	32.0	11.12	-0.035	0.37	1.00	35.2	32.0	11.77	6.4
1	-0.081	0.00	1.66	11.6	29.2	11.13	-0.042	0.15	1.02	15.1	30.9	11.66	5.4
2	-0.082	0.00	1.46	4.7	29.3	11.14	-0.048	0.08	1.02	8.4	30.6	11.58	4.4
3	-0.082	0.00	1.40	4.2	29.4	11.15	-0.054	0.06	1.02	5.9	30.4	11.52	3.5
4	-0.082	0.00	1.34	3.9	29.5	11.16	-0.060	0.04	1.01	4.5	30.3	11.46	2.5
5	-0.082	0.00	1.29	3.8	29.5	11.17	-0.069	0.03	1.01	3.4	30.2	11.39	1.5
6	-0.081	0.00	1.24	3.7	29.6	11.19	-0.083	0.01	1.00	2.1	30.2	11.29	0.6
7	-0.080	0.00	1.19	3.6	29.6	11.21	-0.094	0.00	1.00	1.6	30.2	11.22	0.1
8	-0.079	0.00	1.15	3.7	29.7	11.23	-0.095	0.00	0.98	1.9	30.2	11.22	0.0
9	-0.078	0.00	1.10	3.6	29.7	11.25	-0.094	0.00	0.96	2.3	30.2	11.24	0.0
10	-0.076	0.00	1.06	3.7	29.8	11.27	-0.091	0.00	0.94	2.5	30.3	11.26	0.0
11	-0.074	0.00	1.02	3.8	29.9	11.30	-0.089	0.00	0.91	2.7	30.3	11.29	0.0
12	-0.071	0.00	0.99	3.9	29.9	11.34	-0.085	0.00	0.89	2.8	30.4	11.33	0.0
13	-0.066	0.00	0.95	4.2	30.0	11.38	-0.080	0.00	0.86	3.0	30.4	11.37	0.0
14	-0.060	0.00	0.91	4.8	30.1	11.44	-0.073	0.00	0.83	3.5	30.5	11.44	0.0
15	-	-	0.86		30.1	-	-	-	0.79		30.5	-	-

The quota costs and output prices p_t^* are displayed in ct/kg. The exit rate is the share of farms leaving the industry. The column labelled as $\ln(q_x^*)$ describes the log milk output that a farm with productivity level $\varphi_t = x_t^*$ expects to produce in the next period.

Table 2: Dynamic equilibrium outcome for the calibration shown in Table 1

Source: Authors' own calculations

compared to a situation without constraints (no quota scenario). As a consequence, we observe higher exit rates under the quota scenario. Apparently, even relatively productive farms are better off selling their quota capacity instead of pursuing production. We infer that the liquidation value outweighs the additional variable profits generated by higher output prices at the beginning of the planning horizon.

The finding that a milk quota regime speeds up structural change, however, holds only in the first periods after its implementation. The exit values x_t^* are smaller compared with the unconstrained scenario as long as entering farms create a positive demand for quota. The effect reverses as time proceeds. When the farms' incentive to invest diminishes in later periods, the quota value declines and the constrained exit-points become smaller than the unconstrained ones. This implies that less productive farms particularly benefit from higher output prices in the long run.

The output levels belonging to the optimal exit-points show a similar behaviour. While the constrained values $\ln(q_x^*)$ exceed the unconstrained ones at the beginning, they converge at the end of the planning horizon. Since farms act as price-takers in the output market, their optimal output is determined by their productivity level as well as by the price. This explains why higher milk prices under the quota regime lead to a higher farm-specific milk output. Marginal producers, that is, farms, which are indifferent between staying in or leaving the industry, produce the same amount of output in both scenarios, though at lower productivity levels in the constrained case.

The development of the industry structure μ_t^φ resulting from farms' entry and exit

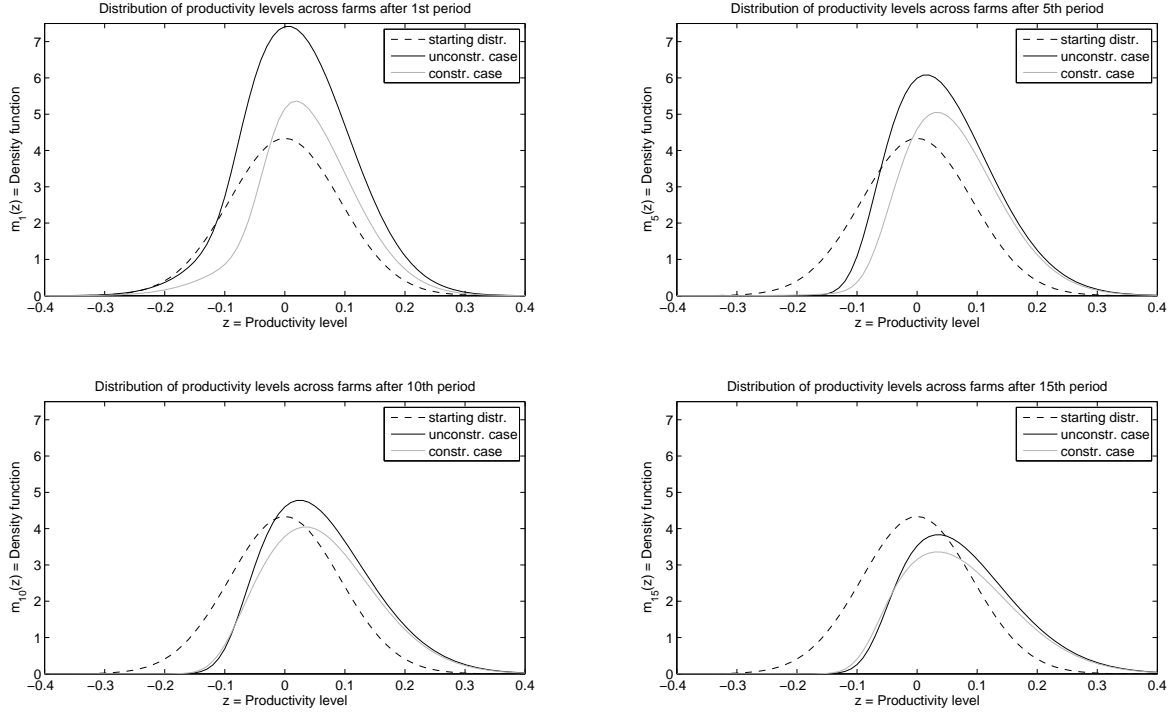


Figure 5: Distribution of productivity levels φ after 1, 5, 10 and 15 time periods
Source: Authors' own calculations

decisions, is depicted in Figure 5. We compare the starting distribution (dashed line) with the distribution of productivity levels across farms after 1, 5, 10, and 15 time periods, respectively, for both scenarios. The area below the curves equals the value $\mu_t(\mathbb{R})$ given in Table 2. Both the constrained and the unconstrained distribution shift to the right in the course of time, implying that the average productivity level increases. Furthermore, the findings reveal that the industry shrinks over time in both scenarios, and the total size in terms of $\mu_t(\mathbb{R})$ is always larger in the unconstrained case. According to (8) the aggregate industry output Q^s is defined as the integral of $q^*(\varphi, p)$ with respect to the measure μ^φ . The shape of the displayed density function μ_t^φ in Figure 5 thus provides an intuitive explanation of why the aggregate output is larger in the unconstrained case and the output price is lower than in the constrained scenario.

Figure 6 shows the distribution of milk output across farms that is induced by the underlying industry structure μ_t^φ . In contrast to Figure 5, the density functions here constitute probability measures and can be directly compared to the empirical densities in Figure 2. The frequency distributions of output levels are derived in three steps: First, we simulate a sequence of random numbers distributed according to the normalised measure $\frac{\mu_t^\varphi}{\mu_t(\mathbb{R})}$. Afterwards, we calculate the milk output referring to those randomly-chosen productivity values, taking into account that the firm-specific output is also affected by

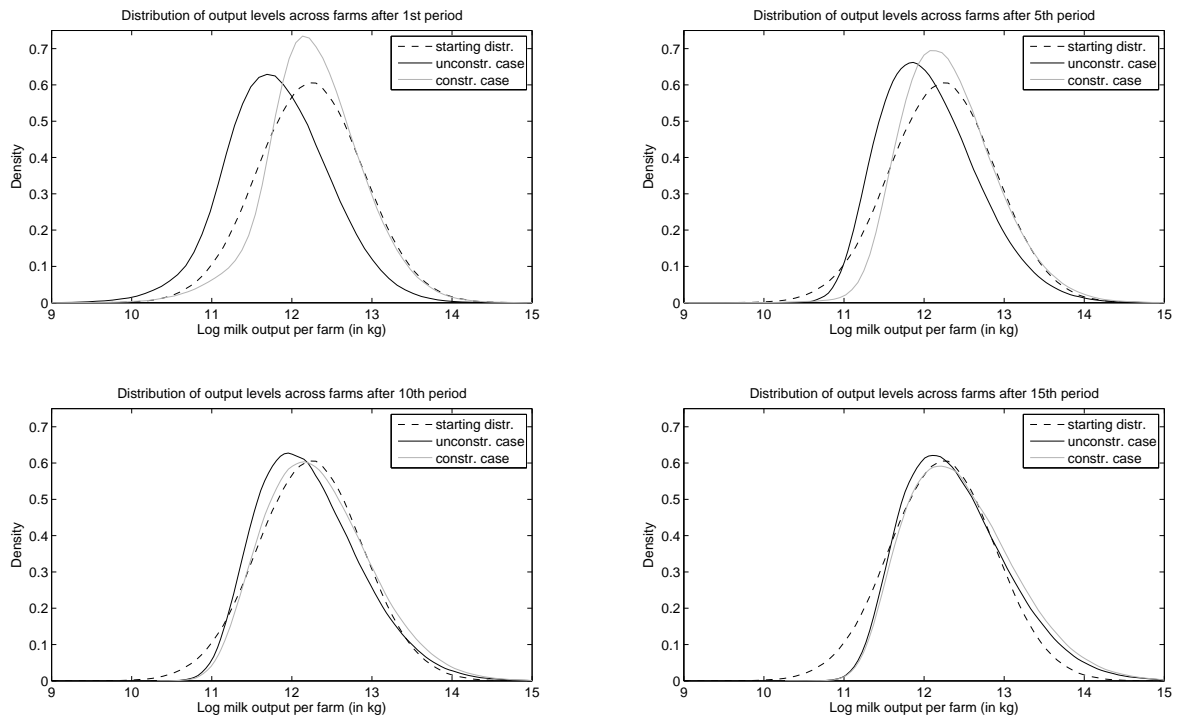


Figure 6: Distribution of output levels $\log(q)$ after 1, 5, 10 and 15 time periods
Source: Authors' own calculations

price. Finally, we apply a kernel density estimation to the log-output values.

Somewhat surprisingly, the unconstrained output distribution is located left of the starting distribution after the first period. This is because the optimal farm-specific output depends on the price, and the dropping milk price in the first period causes all farms to produce less output. In later periods, however, both the constrained and the unconstrained output distribution shift to the right. Note that the upper tails of the density functions spread out. This mimics the empirical distribution in Figure 2 and implies that the share of farms with a high milk yield grows over time. Figure 6 illustrates, moreover, that the constrained density function is always located to the right of the unconstrained one, meaning that the ratio of farms producing a high milk output is always bigger in the constrained case compared to the unconstrained situation. This indicates that the industry is more concentrated under a quota regime.

4 Conclusions

This article has examined how a constrained sectoral production capacity in the agricultural industry affects farms' entry and exit decisions. We have presented a method

to incorporate tradable production quotas into a dynamic stochastic framework with endogenous entry and exit of firms. Our model considers firm-specific uncertainty. Due to the large number of heterogeneous firms in the industry uncertainty washes out on the aggregate level. As a result, the industry structure and output price follow a deterministic pattern.

In contrast to the majority of comparable models, we conduct our analysis in a finite time framework. The common approach for analysing industry dynamics is to derive a stationary equilibrium. However, the concept of a stationary equilibrium does not seem capable of capturing the dynamics of an industry. In the agricultural industry, for example, we found a permanent change of the farm size distribution over recent decades, and the question arises whether a stationary equilibrium will be reached at all. The proposed finite time framework is more flexible since it allows us to trace changes of the productivity distribution over time in great detail. This flexibility comes at the cost that the equilibrium outcome depends on the length of the planning horizon. In particular, the last periods of competition may be biased.

Our results have important implications for the economic appraisal of production quotas. Quotas have not only been criticised for negative welfare effects due to price distortions. It has also been argued that the introduction of production quotas hinders adjustment processes in an industry and thus retains inefficient production structures (Colman, 2000). Our results demonstrate that this simple view of the effect of production quotas needs to be qualified. We find that the establishment of a tradable milk quota may lead to a higher exit rate of farms, at least as long as competitors seek to expand their production capacity and create a high demand for quota. If the farms' incentive to invest declines, however, less productive farms benefit from higher output prices and stay longer in the industry. Furthermore, the simulated output distributions imply that the share of farms with high milk output is always larger if the industry is constrained by a production quota. It should be noted that these findings are based on a set of specific assumptions that were required to calibrate the model. A further robustness analysis is needed to assess the sensitivity of the model outcome with respect to parameter changes.

The work presented here can be extended in several directions. First, the market for tradable production quota could be modelled explicitly. Thus far, the entry costs have been specified by an exogenous function of the industry size. Alternatively, one could determine demand and supply functions for the quota as well as its equilibrium price endogenously. Second, our model abstracts from output price stochasticity, which seems unrealistic considering the volatility of milk markets. Aggregate uncertainty can be introduced through demand shocks. This, however would generate real options effects and

would complicate the firms' decision problem considerably. Finally, it could be interesting to apply our modelling approach to other production factors that are bounded at the sector level and tradable amongst firms. An example of this is agricultural land, for which supply is very inelastic. Although regional differences and soil quality play an important role in land markets, we conjecture that its impact on farm entry and exit may be comparable to a quota regime.

A Appendix

A.1 Existence of a finite dynamic equilibrium

Lemma 1 summarises some basic properties of the optimal output q^* , which follow directly from the first order condition (7) and the assumed structure of the cost function in (4), (5). The implications for the period profits π are given in Lemma 2.

Lemma 1. *(i) The function $q^*(\varphi, p)$ is continuous and (strictly) monotonic increasing in p and φ . (ii) For all $\varphi \in \mathbb{R}$, we have $q^*(\varphi, p) > 0$ if $p > 0$ and $q^*(\varphi, 0) = 0$. (iii) $q(\varphi, p) \rightarrow \infty$ if either $p \rightarrow +\infty$ or $\varphi \rightarrow +\infty$.*

Lemma 2. *(i) π is continuous in φ and p . (ii) π is strictly increasing in p , and if $p > 0$, it is strictly increasing in φ . (iii) $\pi(\varphi, p) \rightarrow \infty$ if either $p \rightarrow +\infty$ or $\varphi \rightarrow +\infty$. (iv) $\pi(\varphi, p) \rightarrow -c_f$ if either $p \rightarrow 0$ or $\varphi \rightarrow -\infty$.*

A necessary condition for the existence of a finite dynamic equilibrium is the integrability of $q^*(\varphi, p)$ and $\pi(\varphi, p)$ with respect to the relevant measures. If both functions are integrable with respect to any Normal distribution, the properties in Lemma 2 translate one-to-one to the value function $v_t(\varphi, \mathbf{p})$, and also to the continuation value

$$v_t^C(\varphi, \mathbf{p}) = \int_{\mathbb{R}} v_t(\varphi', \mathbf{p}) dF(\varphi'|\varphi). \quad (\text{A.1})$$

Hence, the continuation value is continuous and strictly increasing in φ . Furthermore, the limits

$$\lim_{\varphi \rightarrow +\infty} v_t^C(\varphi, \mathbf{p}) = \infty \quad \text{and} \quad \lim_{\varphi \rightarrow -\infty} v_t^C(\varphi, \mathbf{p}) = -c_f \quad (\text{A.2})$$

hold for every price vector $\mathbf{p} > 0$.

Theorem 1 (Existence of a dynamic equilibrium). *Let μ_0, G be continuous distributions with compact support, and both q^* and π be integrable with respect to Normal distributions. A dynamic equilibrium exists in the finite framework if $D(0) =: p^{\max} < \infty$.*

The additional assumptions made in Theorem 1 imply that all integrals, which will be considered in the following proof, exist.

Proof. (Without capacity constraints)

We show the existence of a dynamic equilibrium in the case without capacity constraints first. This implies $r_t = 0$, $k_t = c_e$ for all t , and

$$v_t(\varphi, \mathbf{p}) = \pi(\varphi, p_t) + \beta \max \left\{ 0, \int_{\mathbb{R}} v_{t+1}(\varphi', \mathbf{p}) dF(\varphi'|\varphi) \right\}. \quad (\text{A.3})$$

The basic idea behind this proof is to apply Brouwer's Fixed Point Theorem. The theorem states that every continuous function, mapping a compact space into itself, has a fixed point. Hence, we will define a continuous mapping and illustrate, in the first step, that a fixed point of this mapping constitutes a dynamic equilibrium. In a second step, we will specify a compact subset and show that the previously defined function maps this set into itself.

Step I. By condition (i) of Definition 1, the equilibrium prices p_t^* must clear the output market in every single period, ie. the equality

$$p_t = D(Q^s(p_t, \mu_t)) = D \left(\int_{\mathbb{R}} q^*(\varphi, p_t) d\mu_t(\varphi) \right) \quad (\text{A.4})$$

must be satisfied. The properties of $D(Q)$ and $q^*(\varphi, p)$ imply that for any given industry structure μ_t a unique solution p_t^* to (A.4) exists (Intermediate Value Theorem). Considering the industry dynamics (13), p_t^* will be a continuous function of all previous exit-points x_0, \dots, x_{t-1} and entry-masses M_0, \dots, M_{t-1} .¹⁰ Furthermore, the market clearing output price p_t^* is upward sloping in x_j and downward sloping with respect to M_j (this holds for any $j \in \{0, \dots, t-1\}$).

Now, we construct an operator $T : \mathbb{R}^{2T} \rightarrow \mathbb{R}^{2T}$ that maps a given vector (\mathbf{x}, \mathbf{M}) of exit-points and entry-masses to another vector $(\tilde{\mathbf{x}}, \tilde{\mathbf{M}})$. For any vector $(\mathbf{x}, \mathbf{M}) \in \mathbb{R}^{2T}$, containing a number of exit-points x_t and entry-masses M_t , we can derive the resulting industry structures μ_t according to the industry dynamics (13). The market clearing output prices p_t^* are then determined by the equality (A.4). Given this output price vector \mathbf{p}^* , we can define the mapping $T(\mathbf{x}, \mathbf{M}) = (\tilde{\mathbf{x}}, \tilde{\mathbf{M}})$ by:

$$\tilde{x}_t := \inf \left\{ \varphi \in \mathbb{R} : \int_{\mathbb{R}} v_{t+1}(\varphi', \mathbf{p}^*) dF(\varphi'|\varphi) \geq 0 \right\}, \quad (\text{A.5})$$

$$\tilde{M}_t := \min \left\{ M_t^{max}, \max \left\{ 0, v_{t+1}^e(\mathbf{p}^*) - c_e + M_t \right\} \right\}. \quad (\text{A.6})$$

¹⁰Recall that $\mu_0, F(\cdot|\varphi)$ and G are all supposed to be continuous distributions

The value M_t^{max} determines an upper boundary to the entry-mass, which can possibly arise in an equilibrium and will be specified later on. It is trivial that (A.5) coincides with the exit-rule of our equilibrium definition, and equation (A.6) rephrases the entry condition (iii). Hence, the values \tilde{x}_t represent the critical productivity thresholds under the price vector \mathbf{p}^* . According to (A.2), the function $v_t^C(\varphi, \mathbf{p})$ tends to infinity for $\varphi \rightarrow +\infty$. Therefore, the infimum in (A.5) does indeed exist for any output price vector. The solution \tilde{x}_t will be a continuous function of \mathbf{p}^* as the continuation value is continuous and strictly increasing with respect to φ . It is evident that the same applies to the values \widetilde{M}_t . Hence, the mapping T , which is a composition of continuous functions, must be continuous itself. The way we have constructed T implies, moreover, that a fixed point of this mapping describes an equilibrium.

Step II. In the remainder we will specify a compact subset $S \subset \mathbb{R}^{2T}$ such that $T(S) \subseteq S$, i.e. T maps S into itself. First, recall that the output prices, which may occur in an equilibrium, are bounded by $p_t^* \in (0, p^{max}]$. This allows us to determine lower boundaries for the exit-points. We define the price vector $\mathbf{p}^{max} := (p^{max}, \dots, p^{max})$, which would for instance arise in an empty industry. The corresponding solutions of the exit-rule with respect to \mathbf{p}^{max} determine the minimum attainable values x_t^{min} .

Next, we define the upper boundary M_t^{max} for the entry-masses that can materialize in an equilibrium. Note first, that the expected value of entrants can be written as

$$v_t^e(\mathbf{p}) = \sum_{j=t}^T \beta^{j-1} \int_{\mathbb{R}} \pi(\varphi, p_j) d\bar{\mu}_j(\varphi), \quad (\text{A.7})$$

with $\bar{\mu}_t \equiv G$ and $\bar{\mu}_j$ being the distribution of a firm's productivity in period j . These measures depend implicitly on the firm's optimal exit decisions (with respect to any given price vector \mathbf{p}), and the total mass $\mu_j(\mathbb{R})$ displays the probability of still being active in period j . In an equilibrium, the firm's exit policy must coincide exactly with the exit-points x_t, \dots, x_{T-1} .

Now, we turn this around and define a firm's productivity distribution in period j as an explicit function of given exit-points x_t, \dots, x_{j-1} . The distribution is denoted by the measure

$$\lambda_j((-\infty, \varphi']) = \int_{x_t}^{\infty} \cdots \int_{x_{j-1}}^{\infty} F(\varphi' | \varphi_{j-1}) dF(\varphi_{j-1} | \varphi_{j-2}) \cdots dF(\varphi_{t+1} | \varphi_t) dG(\varphi_t). \quad (\text{A.8})$$

This implies $\lambda_j = \bar{\mu}_j$ whenever the exit-points x_t, \dots, x_{j-1} represent an optimal exit policy (as they do, for instance, in an equilibrium).

For any given vector of exit-points $\mathbf{x} = (x_0, \dots, x_{T-1})$ there exists an entry-mass M_0

such that

$$v_1^e(\mathbf{p}^*, \mathbf{x}) = \sum_{j=1}^T \beta^{j-1} \int_{\mathbb{R}} \pi(\varphi, p_j^*) d\lambda_j(\varphi) \leq c_e. \quad (\text{A.9})$$

The reason for this is simply that all output prices p_1^*, \dots, p_T^* tend to zero if the exit points are fixed and $M_0 \rightarrow \infty$. Hence, we define $\bar{M}_x := \inf \{M_0 \geq 0 : v_1^e(\mathbf{p}^*, \mathbf{x}) \leq c_e\}$ as the smallest entry-mass satisfying (A.9). Furthermore, $\lambda_j((-\infty, \varphi']) \rightarrow 0$ if any exit-point $x_k \rightarrow \infty$ and $k \in \{t, \dots, j-1\}$. Therefore, we can find exit values \bar{x}_{j-1} such that

$$\int_{\mathbb{R}} \pi(\varphi, p^{max}) d\lambda_j(\varphi) \leq \frac{c_e}{T} \quad (\text{A.10})$$

for any $x_{j-1} \geq \bar{x}_{j-1}$.

Recall that the market clearing output price in period t is a continuous function of all previous exit-points and entry-masses. Hence, the function $v_1^e(\mathbf{p}^*, \mathbf{x})$ is also continuous with respect to M_0 and \mathbf{x} . On the compact subset $X := [x_0^{min}, \bar{x}_0] \times \dots \times [x_{T-1}^{min}, \bar{x}_{T-1}]$, there must be a maximum value $M_0^{max} = \sup_{\mathbf{x} \in X} \bar{M}_x < \infty$ that satisfies the inequality in (A.9) for every exit vector $\mathbf{x} \in [x_0^{min}, \infty) \times \dots \times [x_{T-1}^{min}, \infty)$. The values $M_1^{max}, \dots, M_{T-1}^{max}$ are determined by exactly the same procedure.

To determine maximum attainable exit values x_t^{max} , we need to calculate the minimum output prices first. It is clear that the output prices are minimised if the aggregate output is maximised. This is the case if no exit takes place and the maximum amount of firms M_t^{max} enters the industry in each period t . Hence, the entry/exit-vector $(\mathbf{x}, \mathbf{M}) = (-\infty, \dots, -\infty, M_0^{max}, \dots, M_{T-1}^{max})$ yields the minimum justifiable output prices $(p_1^{min}, \dots, p_T^{min})$, which are given by market clearance (A.4). By taking the minimum over all $t = 1, \dots, T$ we can also determine an absolute minimum price $p^{min} > 0$ that serves as a lower boundary to all market clearing output prices. Solving the exit-rule for the constant price vector $\mathbf{p}^{min} = (p^{min}, \dots, p^{min})$ thus yields the maximum possible exit values x_t^{max} .

With all those exit and entry values at hand, we define the subset $S \subset \mathbb{R}^{2T}$ by the cartesian product

$$S := [x_0^{min}, x_0^{max}] \times \dots \times [x_{T-1}^{min}, x_{T-1}^{max}] \times [0, M_0^{max}] \times \dots \times [0, M_{T-1}^{max}]. \quad (\text{A.11})$$

Obviously, this set is compact and mapped into itself by the operator T . If we take any $(\mathbf{x}, \mathbf{M}) \in S$, the resulting output prices p_t^* will be in $[p^{min}, p^{max}]$. Due to the construction of S and the monotonicity of the continuation value $v_t^C(\varphi, \mathbf{p})$, which was mentioned right at the beginning of the proof, the resulting exit-points \tilde{x}_t must lie inside the interval $[x_t^{min}, x_t^{max}]$. Furthermore, the calculated entry values \tilde{M}_t are surely between 0 and M_t^{max} . This implies $(\tilde{\mathbf{x}}, \tilde{\mathbf{M}}) \in S$, and we have indeed $T(S) \subseteq S$.

Summing up, we have argued that $T : S \rightarrow S$ is a continuous mapping on a compact space. In compliance with Brouwer's Theorem this mapping possesses a fixed point. The fixed point essentially represents a finite dynamic equilibrium and, thus, the theorem is proven. \square

Proof. (With capacity constraints)

Now, we turn to the scenario where the total production capacity is limited to the sector level, and show the existence of a dynamic equilibrium in this case. The proof is carried out in the same fashion as the previous one. But, we request that $k_t = c_e + k(\mu_t(\mathbb{R}))$ and $r_t = k(\mu_t(\mathbb{R}))$, with $k(x)$ being an upward sloping continuous function satisfying $k(0) = 0$ and $\lim_{x \rightarrow \infty} k(x) = \infty$. Let the vector $\mathbf{r} = (r_1, \dots, r_T)$ comprise the exit premiums. The value function thus alters into

$$v_t(\varphi, \mathbf{p}, \mathbf{r}) = \pi(\varphi, p_t) + \beta \max \left\{ r_{t+1}, \int_{\mathbb{R}} v_{t+1}(\varphi', \mathbf{p}, \mathbf{r}) dF(\varphi'|\varphi) \right\}. \quad (\text{A.12})$$

Note, that the continuity and monotonicity properties with respect to φ and \mathbf{p} remain unchanged. Moreover, the firm's value at t increases with respect to all subsequent exit premiums r_{t+1}, \dots, r_T .

Step I. We construct a continuous mapping $T : \mathbb{R}^{2T} \rightarrow \mathbb{R}^{2T}$. For any entry/exit-vector (\mathbf{x}, \mathbf{M}) the industry structures μ_t and market clearing output prices p_t^* are derived as before. This time, however, we also calculate the capacity values k_t and r_t based on $\mu_t(\mathbb{R})$. The operator $T(\mathbf{x}, \mathbf{M}) = (\tilde{\mathbf{x}}, \tilde{\mathbf{M}})$ is then determined by the exit/entry-rules

$$\tilde{x}_t := \inf \left\{ \varphi \in \mathbb{R} : \int_{\mathbb{R}} v_{t+1}(\varphi', \mathbf{p}^*, \mathbf{r}) dF(\varphi'|\varphi) \geq r_{t+1} \right\} \quad (\text{A.13})$$

$$\tilde{M}_t := \min \left\{ M_t^{max}, \max \{ 0, v_{t+1}^e(\mathbf{p}^*, \mathbf{r}) - k_{t+1} + M_t \} \right\} \quad (\text{A.14})$$

The constant M_t^{max} is the maximum amount of firms that will possibly enter the industry by the end of period t . The exact value M_t^{max} will be specified later on. Recall that T is again a continuous mapping, and the dynamic equilibrium is characterized as a fixed point of T .

Step II. The challenge is once more to specify a compact subset $S \subset \mathbb{R}^{2T}$ such that $T(S) \subseteq S$. Utilizing the constant price vector \mathbf{p}^{max} allows us to compute a M_t^{max} such that

$$v_{t+1}^e(\mathbf{p}^{max}, \mathbf{r}) = \int_{\mathbb{R}} v_{t+1}(\varphi, \mathbf{p}^{max}, \mathbf{r}) dG(\varphi) \leq c_e + k(M_t^{max}). \quad (\text{A.15})$$

Here, we presume that new firms enter an empty industry, and $r_j = k(M_t^{max})$ for all $j = 1, \dots, T$. Firms having entered the industry in period t and paid capacity costs $k(M_t^{max})$ can, thus, recapture the same (discounted) value as exit premium in prospective periods. The discount factor $\beta < 1$ guarantees that a solution to equation (A.15) exists. Because $v_t^e(\mathbf{p}, \mathbf{r}) \geq v_{t+1}^e(\mathbf{p}, \mathbf{r})$ for any constant vectors $\mathbf{p}, \mathbf{r} \geq 0$, we will have $M_{t-1}^{max} \geq M_t^{max}$.

By the same approach as in the unconstrained case, we compute minimum justifiable output prices with the entry/exit-vector $(\mathbf{x}, \mathbf{M}) = (-\infty, \dots, -\infty, M_0^{max}, \dots, M_{T-1}^{max})$. The minimum of the market clearing output prices constitutes the lower boundary p^{min} . We can also determine an upper boundary for the exit premium by

$$r^{max} = k \left(\sum_{j=1}^T M_j^{max} \right). \quad (\text{A.16})$$

This implies minimum and maximum values for the exit-points by

$$x_t^{min} := \inf \left\{ \varphi \in \mathbb{R} : \int_{\mathbb{R}} v_{t+1}(\varphi', \mathbf{p}^{max}, \mathbf{r}^{max}) dF(\varphi'|\varphi) \geq 0 \right\}, \quad (\text{A.17})$$

$$x_t^{max} := \inf \left\{ \varphi \in \mathbb{R} : \int_{\mathbb{R}} v_{t+1}(\varphi', \mathbf{p}^{min}, \mathbf{0}) dF(\varphi'|\varphi) \geq r^{max} \right\}. \quad (\text{A.18})$$

The compact subset $S \subseteq \mathbb{R}^{2T}$ is set up in the same way as in the unconstrained proof:

$$S := [x_0^{min}, x_0^{max}] \times \dots \times [x_{T-1}^{min}, x_{T-1}^{max}] \times [0, M_0^{max}] \times \dots \times [0, M_{T-1}^{max}]. \quad (\text{A.19})$$

For any entry/exit-vector $(\mathbf{x}, \mathbf{M}) \in S$, the resulting output prices p_t^* are in $[p^{min}, p^{max}]$, and the exit premium satisfies $r_t \in [0, r^{max}]$. Consequently, the exit points \tilde{x}_t , which are related to \mathbf{p} and \mathbf{r} , will lie inside the interval $[x_t^{min}, x_t^{max}]$. Due to the definition of \tilde{M}_t , the vector $(\tilde{\mathbf{x}}, \tilde{\mathbf{M}}) = T(\mathbf{x}, \mathbf{M})$ will indeed be an element of the set S . Hence, we have $T : S \rightarrow S$, and can apply Brouwer's Fixed Point Theorem to conclude that a finite dynamic equilibrium exists in the constrained case. \square

A.2 Derivation of model solutions

In the following we will illustrate how the problem of finding a dynamic equilibrium reduces to a system of equations that needs to be solved simultaneously with respect to \mathbf{x}^* and \mathbf{M}^* . The structure of the industry at the beginning of competition μ_0 is supposed to be known. According to the industry dynamics (13), the structure at time t depends on the whole history of exit/entry decisions made by the firms up to this point. This

means that distribution μ_t is uniquely determined by the starting distribution μ_0 and all previous exit-points x_0, \dots, x_{t-1} , and entry-masses M_0, \dots, M_{t-1} .

Under certain conditions an explicit formulation for the measure μ_t can be derived. If both $\mu_0 = N(\nu_0, \sigma_0^2)$ and $G = N(\nu_g, \sigma_g^2)$ are Normal distributions, for instance, the industry structure in period t is given by the density function

$$m_t(z) = \phi \left(z \middle| \nu_0 \rho^t + \nu_\varepsilon \sum_{j=0}^{t-1} \rho^j, \sigma_0^2 \rho^{2t} + \sigma_\varepsilon^2 \sum_{j=0}^{t-1} \rho^{2j} \right) \hat{F}_t \begin{pmatrix} -x_0 \\ \vdots \\ -x_{t-1} \end{pmatrix} \\ + \sum_{k=1}^t M_{k-1} \phi \left(z \middle| \nu_g \rho^{t-k} + \nu_\varepsilon \sum_{j=0}^{t-k-1} \rho^j, \sigma_g^2 \rho^{2(t-k)} + \sigma_\varepsilon^2 \sum_{j=0}^{t-k-1} \rho^{2j} \right) \hat{F}_{t-k} \begin{pmatrix} -x_k \\ \vdots \\ -x_{t-1} \end{pmatrix}. \quad (\text{A.20})$$

Here, the function $\phi(z|\nu, \sigma^2)$ denotes the pdf of a Normal distribution with mean ν and variance σ^2 . The function \hat{F}_t is the cdf of a t -dim Normal distribution $N(\lambda, \Sigma)$ that is subject to

$$\Sigma^{-1} = \begin{bmatrix} \frac{\sigma_0^2 \rho^2 + \sigma_\varepsilon^2}{\sigma_\varepsilon^2 \sigma_0^2} & -\frac{\rho}{\sigma_\varepsilon^2} & 0 & \dots & 0 \\ -\frac{\rho}{\sigma_\varepsilon^2} & \frac{\rho^2 + 1}{\sigma_\varepsilon^2} & -\frac{\rho}{\sigma_\varepsilon^2} & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & -\frac{\rho}{\sigma_\varepsilon^2} & \frac{\rho^2 + 1}{\sigma_\varepsilon^2} & -\frac{\rho}{\sigma_\varepsilon^2} \\ 0 & \dots & 0 & -\frac{\rho}{\sigma_\varepsilon^2} & \frac{\rho^2 + 1}{\sigma_\varepsilon^2} \end{bmatrix} \quad \text{and} \quad \Sigma^{-1} \lambda = \begin{bmatrix} \frac{\sigma_0^2 \nu_\varepsilon \rho - \sigma_\varepsilon^2 \nu_0}{\sigma_0^2 \sigma_\varepsilon^2} \\ \frac{(\rho-1)\nu_\varepsilon}{\sigma_\varepsilon^2} \\ \vdots \\ \frac{(\rho-1)\nu_\varepsilon}{\sigma_\varepsilon^2} \\ \frac{(\rho-1)\nu_\varepsilon - \rho z}{\sigma_\varepsilon^2} \end{bmatrix}. \quad (\text{A.21})$$

In the same manner, each \hat{F}_{t-k} constitutes the cdf of a $(t-k)$ -dim Normal distribution. The variables ν_0, σ_0^2 in (A.21) need to be replaced by ν_g and σ_g^2 , however.

Due to condition (i), the equilibrium output price in period t is implicitly defined by

$$p_t = D(Q^s(p_t, \mu_t)). \quad (\text{A.22})$$

The properties of the demand function D and the aggregate industry output Q^s ensure that for any given industry structure μ_t a unique solution $p_t^* > 0$ to (A.22) exists. Therefore, the equilibrium output price is a function of μ_t , and it may be expressed in terms of $p_t^* = s_t(x_0, \dots, x_{t-1}, M_0, \dots, M_{t-1})$. If μ_0 represents a continuous measure, $s_t : \mathbb{R}^{2t} \rightarrow \mathbb{R}_+$ is a continuously differentiable function, and the partial derivatives satisfy

$$\frac{\partial s_t}{\partial x_j} \geq 0, \quad \frac{\partial s_t}{\partial M_j} \leq 0 \quad \forall j = 0, \dots, t-1. \quad (\text{A.23})$$

In general, equation (A.22) defines the function $s_t(\cdot)$ only implicitly. When we apply the model in section 3, we refer to an isoelastic demand function $D(Q) = \left(\frac{Q}{b}\right)^{-\frac{1}{\eta}}$ and a Cobb Douglas cost function $c(\varphi, q) = h \left(\frac{q}{\exp(\varphi)}\right)^{\frac{1}{\alpha}}$ with constants $h > 0$ and $\alpha \in (0, 1)$. Those functional forms, in combination with the normality assumptions on μ_0 and G , enable us to deduce an explicit formulation for the equilibrium output price by hand.

As firms are supposed to be price-takers, their optimal output under those assumptions is $q^*(\varphi, p_t) = \left(\frac{\alpha p_t}{h}\right)^{\frac{\alpha}{1-\alpha}} \exp\left(\frac{\varphi}{1-\alpha}\right)$. Hence, the aggregate industry output is given by

$$Q^s(p_t, \mu_t) = \int_{\mathbb{R}} q^*(\varphi, p_t) d\mu_t(\varphi) \quad (\text{A.24})$$

$$= \left(\frac{\alpha p_t}{h}\right)^{\frac{\alpha}{1-\alpha}} \int_{-\infty}^{\infty} e^{\frac{z}{1-\alpha}} m_t(z) dz. \quad (\text{A.25})$$

In an equilibrium the output price clears the market, i.e., the equality $p_t^* = D(Q^s(p_t^*, \mu_t))$ must hold. Keeping this in mind we can derive the equilibrium price

$$p_t^* = \left[\frac{1}{b} \left(\frac{\alpha}{h}\right)^{\frac{\alpha}{1-\alpha}} \int_{-\infty}^{\infty} e^{\frac{z}{1-\alpha}} m_t(z) dz \right]^{\frac{1-\alpha}{\eta\alpha - \eta - \alpha}} \quad (\text{A.26})$$

$$=: s_t(x_0, \dots, x_{t-1}, M_0, \dots, M_{t-1}), \quad (\text{A.27})$$

which is indeed a function of all previous exit-points and entry-masses.

The equilibrium values for all x_t and M_t are determined by the exit and entry conditions. In each period $t = 0, \dots, T-1$ the following pair of equations has to be satisfied

$$\int_{\mathbb{R}} v_{t+1}(\varphi', \mathbf{p}^*) dF(\varphi'|x_t) - r_{t+1} = 0 \quad (\text{A.28})$$

$$\int_{\mathbb{R}} v_{t+1}(\varphi', \mathbf{p}^*) dG(\varphi') - k_{t+1} \leq 0, \quad \text{with equality if } M_t > 0. \quad (\text{A.29})$$

Since we consider a finite time horizon, the value function $v_{t+1}(\varphi', \mathbf{p}^*)$ is essentially a discounted sum of expected future profits. The final exit condition, which must hold in a dynamic equilibrium, is therefore

$$\int_{\mathbb{R}} \pi(\varphi', p_T^*) dF(\varphi'|x_{T-1}) - r_T = 0. \quad (\text{A.30})$$

Obviously, the left side may be regarded as a function of all x_t and M_t if the equilibrium output price p_T^* is substituted by $s_T(x_0, \dots, x_{T-1}, M_0, \dots, M_{T-1})$.

The equality in (A.30) allows us to set up the last but one exit condition in the following

way

$$\begin{aligned}
& \int_{\mathbb{R}} \pi(\varphi', p_{T-1}^*) dF(\varphi' | x_{T-2}) + \beta \int_{\varphi \geq x_{T-1}} \int_{\mathbb{R}} \pi(\varphi', p_T^*) dF(\varphi' | \varphi) dF(\varphi | x_{T-2}) \\
& + \beta \int_{\varphi < x_{T-1}} r_T dF(\varphi | x_{T-2}) - r_{T-1} = 0
\end{aligned} \tag{A.31}$$

By moving further backwards in time we can consecutively specify the other exit and entry conditions as well. The conditions (A.28) and (A.29) thus add up to a system of $2T$ equations with $\mathbf{x} = (x_0, \dots, x_{T-1})$ and $\mathbf{M} = (M_0, \dots, M_{T-1})$ being the only unknown variables. The roots of this system, which represent a dynamic equilibrium, cannot be found analytically, however. They have to be computed with numerical methods, e.g., the Newton method.

A.3 Farm gate milk prices

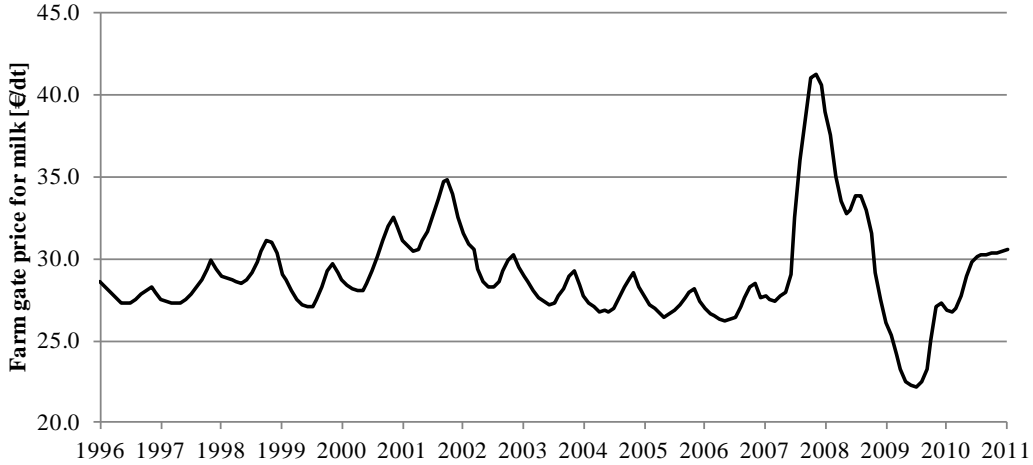


Figure 7: Farm gate price for milk in Germany

Source: Zentrale Markt- und Preisberichtsstelle (ZMP), Statistisches Bundesamt (2013)

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