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Compensating justice beats leaky buckets: an experimental investigation

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COMPENSATING JUSTICE BEATS LEAKY BUCKETS: AN EXPERIMENTAL INVESTIGATION

by Eva Camacho-Cuena, Tibor Neugebauer, and Christian Seidl

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COMPENSATING JUSTICE BEATS LEAKY BUCKETS: AN EXPERIMENTAL INVESTIGATION

by

Eva Camacho-Cuena[*], Tibor Neugebauer[**], and Christian Seidl[*]

Abstract

Leaky-bucket transactions can be regarded as income transfers allowing for transaction costs. In its most rudimentary form, leaky-bucket transactions trace out the maximum "leakage" of transaction costs before income inequality is exacerbated, or before a welfare loss is experienced. This notion suggests that part of the income transfer should reach the transferee in order to keep the degree of income inequality or social welfare intact.

However, in general, this conjecture is theoretically wrong. Rather there exists a unique benchmark such that it holds only for transfers among income recipients below the benchmark. When both are above the benchmark, the transferee has to be given more than the amount taken from the transferor, and when they are on opposite sides of the benchmark, both should experience an income loss. These three cases cover progressive transfers only. Three more cases apply to regressive transfers, and six more cases apply to income gains. Each of these twelve cases is covered by the present paper.

Yet experimental research for three families of renowned income inequality measures and their associated social welfare functions shows poor empirical evidence. Subjects' perceptions to maintain the degree of income inequality rather follow a simple precept: If someone gains income, the other person involved should be positively compensated, and if someone loses income, the other person involved should be negatively compensated. This expresses sort of *compensating justice* rather than restoration of the former degree of income inequality, or of the former level of social welfare. It requires different axioms of inequality measurement to comply with subjects' perceptions.

JEL Classification: D31, D63, C91.

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All common income inequality measures are required to satisfy the *transfer principle*. That is, if income is transferred from an income recipient to a poorer one without changing the order of income recipients (this is called a *progressive transfer*), then an income inequality measure is required to decrease because income becomes more equally distributed. The opposite holds if the transfer goes to a richer income recipient (this is called a *regressive transfer*).

Alternatively, income transfers affect social welfare: Keeping constant aggregate income and the number of income recipients, more equally distributed incomes are usually assumed to carry higher social welfare. In particular, progressive transfers increase social welfare.

When a transfer between two income recipients entails transaction costs, then the following question arises: Which are the maximum transaction costs such that the degree of income inequality or social welfare does remain the same in this society? Preliminary reasoning would suggest that transaction costs are only acceptable if they leave part of the transfer for the transferee.[1]

However, Seidl (2001) and Lambert and Lanza (2003) showed that this conjecture is theoretically wrong, at least for the degree of income inequality. Rather there exists a *unique benchmark* as a function of the income inequality measure and the income distribution applied, such that the above conjecture holds only when both parties to a progressive transfer have incomes *below* the benchmark. When both parties have incomes *above* the benchmark, then the transferee has even to receive a higher amount than the transfer taken from the transferor in order to maintain the degree of income inequality. When the parties lie on *opposite sides* of the benchmark, then also the "transferee" should suffer an income loss in order to maintain the degree of income inequality. This pattern is known as the leaky-bucket phenomenon.

Moreover, note that all income inequality measures are associated with particular families of social welfare functions (Blackorby and Donaldson, 1978). Preliminary reasoning may, thus, suggest that the theoretical results for income inequality measures do not necessarily carry over to social welfare. That is, it may be argued that a smaller, but positive, amount of the income transfer has to reach the transferee to maintain social welfare, irrespective of the benchmark. Again it can be shown theoretically that this conjecture does not necessarily hold.

An experimental test of the newly discovered theoretical relationships should encompass four categories: The income of an income recipient may either be *increased* or *decreased* by a certain amount, and the other income recipient, whose income should be adapted, may either be *richer* or *poorer* than the first one. For each of these four categories the respective incomes can lie above, below, or on opposite sides of the benchmark. This makes twelve cases to be analyzed.[2]

So far, experimental investigation of the leaky-bucket phenomenon has focused only on one of these twelve cases, viz. on the case of a progressive transfer for which the income of both, transferor and transferee, are below the benchmark, or when no benchmark exists (which is the case for some social welfare functions).

This restriction is revealed through the chosen experimental design of other research, which allowed only responses of income adjustments which were restricted to be positive and smaller than the amount of the income reduction of the richer party. Some experimenters, e.g. Amiel et al. (1999), asked their subjects to state the minimum amount that needs to be *given* to the transferee "to make the transfer worthwhile" without having stated how "worthwhile" should be understood.[3] Related work was done by Beckman and associates: Beckman et al. (2003b) used an experimental design based on leaky-bucket lotteries. Beckman et al. (2003a) investigated inefficiencies in income redistribution in a majoritarian democracy. They found "that voters are willing to support very large efficiency losses to transfer income if they do not have to pay and do not have any chance of obtaining the top positions. Support for redistribution, however, drops off markedly when voters are asked to contribute to the transfer or if they know their position before voting and they are at the top. Preferences for mandated redistribution thus are strongly conditioned by self interest and the opportunities which exist to occupy positions which receive the highest payoffs." (Beckman et al. 2003a, pp. 3–4.) In contrast to that, our experimental design avoids both risk and redistributive connotations. Subjects were rather solicited to focus on the *maintenance of the degree of income inequality*.

The experimental design used in this paper investigates all four basic cases for three income inequality measures, viz. Atkinson[4], generalized Gini[5], and entropy[6]. Although, in our experimental design, subjects were asked to adjust the other person's income such as to maintain the degree of income inequality within the society, we also investigated whether the results conform to social-welfare considerations as well.

The paper is arranged as follows: Section I gives an appraisal on the theory of leaky-bucket transactions, Section II presents the experiment, Section III contains the results, and Section IV concludes.

I Theory

In this section we describe the leaky-bucket theory. It is only presented in the form required by our experimental design. Consequently, we allow only for strictly positive incomes, which are different for different income recipients. The number of income recipients is assumed to be finite. The inequality measures and the social welfare functions are assumed to be continuously differentiable.

Thus, we consider only income distributions $y = \{y_i \mid i = 1, \ldots, n;\ 1 < n < \infty\}$ such that $0 < y_1 < y_2 < \ldots < y_n < \infty$. The set of such income distributions is denoted by Y. Income inequality measures are denoted by $I : \mathbb{R}^n_{++} \to \mathbb{R}_+$,[7] and social welfare functions are denoted by $W : \mathbb{R}^n_{++} \to \mathbb{R}$.

Income inequality measures may either process incomes only in increasing (or decreasing) order, or may process them in any order. The former ones are called *positional* income inequality measures, the latter ones are called *nonpositional* income inequality measures. When nonpositional income inequality measures satisfy symmetry, the ordering of an income distribution becomes immaterial.

Definition 1: $I(\cdot)$ is scale invariant if $I(y) = I(\lambda y)$ for all $y \in \mathbb{R}^n_{++},\ \lambda > 0$.

Lemma 2: If $I(\cdot)$ is scale invariant, then

$$\sum_{i=1}^{n} \frac{\partial I}{\partial y_i} y_i = 0.$$

Proof: Consider a scale variation $\lambda y,\ \lambda > 0$ of y. Then $I(\lambda y) \equiv I(y)$ for all $\lambda > 0$ implies

$$\frac{\partial I(\lambda y)}{\partial \lambda} = \sum_{i=1}^{n} \frac{\partial I(\lambda y)}{\partial \lambda y_i} y_i \equiv 0 \text{ for all } \lambda > 0. \text{ Hence, } \lim_{\lambda \to 1} \frac{\partial I(\lambda y)}{\partial \lambda} = \sum_{i=1}^{n} \frac{\partial I(y)}{\partial y_i} y_i = 0.$$

Q.E.D.

DEFINITION 3: $I(\cdot)$ is inequality averse if

$$\frac{\partial I}{\partial y_k} < \frac{\partial I}{\partial y_j} \text{ for } y_k < y_j.$$

DEFINITION 4: $W(\cdot)$ is inequality averse if

$$\frac{\partial W}{\partial y_k} > \frac{\partial W}{\partial y_j} \text{ for } y_k < y_j.$$

Let e_i denote an $n-$dimensional vector with a 1 on the $i-$th position and zeros everywhere else, and let τ be such that $0 < \tau < \min_{i,\ell \in \{1,\ldots,n\}} \mid y_i - y_\ell \mid$.

DEFINITION 5: $I(\cdot)$ satisfies the transfer principle if

$$I(y + \tau e_k - \tau e_j) < I(y) \text{ and } I(y - \tau e_k + \tau e_j) > I(y) \text{ for all } y_k < y_j.$$

DEFINITION 6: $I(\cdot)$ is Lorenz-consistent if $I(y) < I(y')$ when y Lorenz-dominates y'.

DEFINITION 7: $I(\cdot)$ is strictly S-convex if $I(yP) < I(y)$ for all bistochastic matrices P except permutation matrices. $W(\cdot)$ is strictly S-concave if $W(yP) > W(y)$ for all bistochastic matrices P except permutation matrices.

THEOREM 8: The following statements are equivalent:

(i) $I(\cdot)$ is inequality averse.

(ii) $I(\cdot)$ satisfies the transfer principle.

(iii) $I(\cdot)$ is strictly S-convex.

(iv) $I(\cdot)$ is Lorenz-consistent.

PROOF:

(i) $\Rightarrow$ (ii). Inequality aversion implies $\frac{\partial I}{\partial y_k} < \frac{\partial I}{\partial y_j}$ for all $y_k < y_j$. Hence, $\frac{\partial I}{\partial y_k} - \frac{\partial I}{\partial y_j} < 0$, which implies for $0 < \tau < \min_{i,\ell \in \{1,\ldots,n\}} \mid y_i - y_\ell \mid$:

$$0 > \left(\frac{\partial I}{\partial y_k} - \frac{\partial I}{\partial y_j} \right) \tau := \Delta I \text{ for all } y_k < y_j \, .$$

But this implies the transfer principle.

(ii) $\Rightarrow$ (iii). Define $\kappa := \frac{1}{\tau}(y_j - y_k)$, where $y_k < y_j$. Because of

$$0 < \tau < \min_{i,\ell \in \{1,\ldots,n\}} \mid y_i - y_\ell \mid$$

we have $\kappa > 1$. Then $(y + \tau e_k - \tau e_j)$ can equivalently be achieved by multiplying y by the n–dimensional bistochastic matrix

$$A := E - \frac{1}{\kappa} E^{jj}_{kk} + \frac{1}{\kappa} E^{jk}_{kj},$$

where E denotes the unit matrix, E^{jj}_{kk} denotes a matrix whose elements $\psi_{i\ell}$ are $\psi_{i\ell} = 1$ for $i = \ell = j$ and for $i = \ell = k$, and 0 otherwise, and E^{jk}_{kj} denotes a matrix whose elements $\varphi_{i\ell}$ are $\varphi_{i\ell} = 1$ for $i = j$, $\ell = k$ and for $i = k$, $\ell = j$, and 0 otherwise. By the transfer principle we have

$$I(y) > I(y + \tau e_k - \tau e_j) = I(yA),$$

which implies strict S-convexity of $I(\cdot)$, as there exists a bistochastic matrix for any sequence of progressive transfers.

(iii) $\Rightarrow$ (iv). Strict S-convexity of $I(\cdot)$ implies

$$I(y) > I(yP) = I(yP\Pi)$$

for all bistochastic matrices P and all permutation matrices Π. Let $(z_1, \ldots, z_n)$ denote a permutation of yP such that $z_1 \leq z_2 \leq \ldots \leq z_n$. Then

$$y_1 \leq \min_{\ell \in \{1,\ldots,n\}} \{\sum_{i=1}^{n} y_i p_{i\ell}\} := z_1, \text{ and } y_n \geq \max_{\ell \in \{1,\ldots,n\}} \{\sum_{i=1}^{n} y_i p_{i\ell}\} := z_n.$$

Because $\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} z_i$ and $y \neq z$ (as P is not a permutation matrix), there exists a unique k, such that $y_i \leq z_i$ for all $i \leq k$, and $y_k < z_k$, $y_{k+1} > z_{k+1}$. Hence, z Lorenz-dominates y.

(iv) $\Rightarrow$ (i). For $0 < \tau < \min_{i,\ell \in \{1,\ldots,n\}} \mid y_i - y_\ell \mid$ the income distribution $(y + \tau e_k - \tau e_j)$, $k < j$, Lorenz-dominates the income distribution y. For a Lorenz-consistent income inequality measure we have, thus, $I(y + \tau e_k - \tau e_j) < I(y)$, $k < j$. This implies $\frac{\partial I}{\partial y_k} - \frac{\partial I}{\partial y_j} < 0$ and, thus, $\frac{\partial I}{\partial y_k} < \frac{\partial I}{\partial y_j}$, which is inequality aversion.

Q.E.D.

THEOREM 9: For scale invariant and inequality averse income inequality measures there exists a benchmark y^*, $y_1 < y^* < y_n$ such that $\dfrac{\partial I}{\partial y_i} < 0$ for all $y_i < y^*$, and $\dfrac{\partial I}{\partial y_i} > 0$ for all $y_i > y^*$.

PROOF: By Lemma 2, scale invariance implies

$$\sum_{i=1}^{n} \frac{\partial I}{\partial y_i} y_i = 0.$$

Provided that not all items of the sum equal zero, this implies the existence of negative and positive components. As $y_i > 0,\ i = 1, \ldots, n$, this means the existence of negative and positive partial derivatives. As we have, by inequality aversion, $\dfrac{\partial I}{\partial y_k} < \dfrac{\partial I}{\partial y_j}$ for $y_k < y_j$, the occurrence of negative and positive partial derivatives implies $\dfrac{\partial I}{\partial y_1} < 0 < \dfrac{\partial I}{\partial y_n}$. Hence, the joint conditions imply the existence of a benchmark, y^*, such that $\dfrac{\partial I}{\partial y_i} < 0$ for all $y_i < y^*$ and $\dfrac{\partial I}{\partial y_i} > 0$ for all $y_i > y^*$.

Q.E.D.

DEFINITION 10: A general income inequality measure defined on Y, $I : \mathbb{R}^n_{++} \to \mathbb{R}_+$, is defined as

$$I(y) = \Psi\left[J(y)\right],\ \Psi' > 0,\ \text{where } J(y) = \frac{1}{n}\sum_{i=1}^{n} w(i) u\left(\frac{y_i}{\mu}\right),$$

and μ denotes mean income, the $w(i)$'s denote some weights, and $u(\cdot)$ denotes the utility of income normalized for mean income.

(i) For the nonpositional income inequality measures we have $w(i) \equiv 1$, and, after the appropriate substitution of $\Psi(\cdot)$ and $u(\cdot)$ (which are indicated after the respective expression), we have:

(a) $I(y) = \dfrac{1}{c(c-1)}\left[\dfrac{1}{n}\sum_{i=1}^{n}\left(\dfrac{y_i}{\mu}\right)^c - 1\right]$ for $c \neq 0, 1$; $\Psi(J) = \dfrac{J-1}{c(c-1)}$; $u(t) = t^c$;

$I(y) = \dfrac{1}{n}\sum_{i=1}^{n}\dfrac{y_i}{\mu}\ln\left(\dfrac{y_i}{\mu}\right)$ for $c = 1$; $\Psi(J) = J$; $u(t) = t\ln t$;

$I(y) = \dfrac{1}{n}\sum_{i=1}^{n}\ln\left(\dfrac{\mu}{y_i}\right)$ for $c = 0$; $\Psi(J) = J$; $u(t) = \ln\dfrac{1}{t}$;

for the entropy class of income inequality measures;

(b) $I(y) = 1 - \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\mu}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$ for $\theta \neq 1$; $\Psi(J) = 1 - J^{\frac{1}{1-\theta}}$; $u(t) = t^{1-\theta}$;

$I(y) = 1 - \left[\prod_{i=1}^{n}\left(\frac{y_i}{\mu}\right)\right]^{\frac{1}{n}}$ for $\theta = 1$; $\Psi(J) = 1 - \theta^J$; $u(t) = \ln t$;

for the Atkinson inequality measure.

(ii) For positional income inequality measures we have $\Psi(J) = J$, $u(t) = t$, and $w(i) = 1 + n\left[f\left(\frac{n-i}{n}\right) - f\left(\frac{n-i+1}{n}\right)\right]$, $f(0) = 0, f(1) = 1, f'(s) > 0\ \forall\ s \in (0,1)$ for the Yaari (1988) inequality index. For $f(s) = s^\nu,\ \nu > 1$, it becomes the extended Gini coefficient, for $\nu = 2$ it becomes the Gini coefficient.

THEOREM 11: We have

(i) for nonpositional income inequality measures

$$\frac{\partial I}{\partial y_k} \gtrless 0 \Leftrightarrow y_k \gtrless y^* = \mu \times (u')^{-1}\left[\frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{\mu}u'\left(\frac{y_i}{\mu}\right)\right].$$

In particular, we have

$y^* = \mu$ for the entropy income inequality index for $c = 0$, and for the Atkinson income inequality index for $\theta = 1$;

$y^* = \mu \exp\left[\frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{\mu}\ln\frac{y_i}{\mu}\right]$ for the entropy income inequality index for $c = 1$;

$y^* = \mu\left[1 + \frac{1}{n}\sum_{i=1}^{n}\left\{\left(\frac{y_i}{\mu}\right)^c - 1\right\}\right]^{\frac{1}{c-1}}$ for the entropy income inequality index for $c \neq 0, 1$;

$y^* = \mu\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\mu}\right)^{1-\theta}\right]^{\frac{1}{\theta}}$ for the Atkinson income inequality index for $\theta \neq 1$;

(ii) for positional income inequality measures:

$$\frac{\partial I}{\partial y_k} \gtrless 0 \Leftrightarrow w(k) \gtrless I(\cdot),$$

where $w(k)$ denotes the weight associated with y_k. In particular, we have

$$w(k) = I(\cdot) \times \frac{1 + n\left[f\left(\frac{n-k}{n}\right) - f\left(\frac{n-k+1}{n}\right)\right]}{\frac{1}{n}\sum_{i=1}^{n}\left\{1 + n\left[f\left(\frac{n-i}{n}\right) - f\left(\frac{n-i+1}{n}\right)\right]\frac{y_i}{\mu}\right\}}$$

for the Yaari income inequality index. For $f(s) = s^\nu,\ \nu > 1$, it becomes the extended Gini coefficient, for $\nu = 2$ it becomes the Gini coefficient.

PROOF: Lambert and Lanza (2003).

THEOREM 12 (LEAKY-BUCKET THEOREM): Let $I(\cdot)$ denote a differentiable, inequality averse, and scale invariant income inequality measure with y^* as its benchmark. Consider two income recipients, j and k, and assume that j's income is changed by δ, where $\mid \delta \mid < \min_{i,\ell \in \{1,\ldots,n\}} \mid y_i - y_\ell \mid$. Determine the change γ of k's income such that $\Delta I = 0$. Then:

$$\gamma \doteq -\frac{\partial I/\partial y_j}{\partial I/\partial y_k}\delta,$$

which means:

Benchmark	$\delta > 0$		$\delta < 0$	
	$y_j < y_k$	$y_j > y_k$	$y_j < y_k$	$y_j > y_k$
$y^* > y_j, y_k$	$\gamma < -\delta$	$-\delta < \gamma < 0$	$\gamma > -\delta$	$-\delta > \gamma > 0$
$y^* < y_j, y_k$	$-\delta < \gamma < 0$	$\gamma < -\delta$	$-\delta > \gamma > 0$	$\gamma > -\delta$
y^* between y_j and y_k	$\gamma > 0$	$\gamma > 0$	$\gamma < 0$	$\gamma < 0$

PROOF: By differentiability we have

$$\frac{\partial I}{\partial y_j}\delta + \frac{\partial I}{\partial y_k}\gamma \doteq 0, \text{ which implies } \gamma = -\frac{\partial I/\partial y_j}{\partial I/\partial y_k}\delta.$$

The cells in the above table result from Definition 3 and Theorem 8. For $\delta < 0$ the sign of γ is changed.

Q.E.D.

Income inequality measures have associated social welfare functions. This is immediate for the Atkinson inequality index which is derived from a parent social welfare function. Other income inequality measures have to be doctored to be transformed into social welfare functions. We propose to follow Blackorby and Donaldson's (1978, pp. 69–70) suggestion to first multiply an income inequality measure by (-1) to change its welfare implication from a decreasing to an increasing scale, then add 1 to normalize for the value of 1 for income equality, and finally multiply by the mean income μ to enter an efficiency component. Notice that this procedure does not give rise to unique social welfare functions, but defines only a representative of a whole family of social welfare functions compatible with the respective income inequality measure.[8]

DEFINITION 13: Income inequality measures have associated social welfare functions:

(i) Atkinson social welfare function:

$$W(y) = \begin{cases} \left[\frac{1}{n}\sum_{i=1}^{n} y_i^{1-\theta}\right]^{\frac{1}{1-\theta}}, & \text{for } \theta \neq 1,\ \theta > 0; \\ \frac{1}{n}\sum_{i=1}^{n} \ln y_i, & \text{for } \theta = 1. \end{cases}$$

(ii) Yaari and Gini social welfare functions:

$$W(y) = \sum_{i=1}^{n} \left[f\left(\frac{n-i+1}{n}\right) - f\left(\frac{n-i}{n}\right)\right] y_i,$$

$$f(0) = 0, f(1) = 1, f'(s) > 0, \forall\ s \in (0,1).$$

For $f(s) = s^{\nu},\ \nu > 1$, this social welfare function becomes the extended Gini social welfare function, for $\nu = 2$ it becomes the Gini social welfare function.

(iii) Entropy social welfare function

$$W(y) = \frac{\mu\left[1 + c\,(c-1) - \frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\mu}\right)^{c}\right]}{c\,(c-1)},\ c \neq 0, 1;$$

$$W(y) = \frac{1}{n \ln n}\left[n\mu \ln n\mu - \sum_{i=1}^{n} y_i \ln y_i\right],\ c = 1;$$

$$W(y) = \mu\left[1 - \frac{1}{n}\sum_{i=1}^{n} \ln \frac{\mu}{y_i}\right],\ c = 0.$$

One can immediately check that these three social welfare functions are inequality averse. The benchmarks, y^*, equivalent to the benchmarks observed for income inequality measures, would demand that $\frac{\partial W}{\partial y_i} < 0$ for $y^* < y_i \leq y_n$. Out of the collection of social welfare functions, described in Definition 13, $\frac{\partial W}{\partial y_i} < 0$ may occur only for the entropy social welfare function.

THEOREM 14: For the entropy social welfare function we have

$$\frac{\partial W}{\partial y_k} \gtrless 0 \ \Leftrightarrow y_k \lessgtr y^*,\ \text{where } y^* = \mu\left[\frac{1 + c(c-1)}{c} - \frac{1}{cn}(c-1)\sum_{i=1}^{n}\left(\frac{y_i}{\mu}\right)^{c}\right]^{\frac{1}{c-1}}.$$

PROOF: Compute $\frac{\partial W}{\partial y} \doteq 0$ for the entropy social welfare function and isolate the value y^* for which it holds.

Q.E.D.

This allows us to formulate a leaky-bucket theorem for social welfare functions.

THEOREM 15: (LEAKY-BUCKET THEOREM FOR SOCIAL WELFARE FUNCTIONS) Let $W(\cdot)$ denote a differentiable and inequality averse social welfare function with y^* as its benchmark (if it exists). Consider two income recipients, j and k, and assume that j's income is changed by δ, where $\mid \delta \mid < \min_{i,\ell \in \{1,\ldots,n\}} \mid y_i - y_\ell \mid$. Determine the change γ of k's income such that $\Delta W = 0$. Then we have

$$\gamma \doteq -\frac{\partial W/\partial y_j}{\partial W/\partial y_k}\delta\,,$$

which means:

Benchmark	$\delta > 0$		$\delta < 0$	
	$y_j < y_k$	$y_j > y_k$	$y_j < y_k$	$y_j > y_k$
$y^* > y_j, y_k$ or no benchmark	$\gamma < -\delta$	$-\delta < \gamma < 0$	$\gamma > -\delta$	$-\delta > \gamma > 0$
$y^* < y_j, y_k$	$-\delta < \gamma < 0$	$\gamma < -\delta$	$-\delta > \gamma > 0$	$\gamma > -\delta$
y^* between y_j and y_k	$\gamma > 0$	$\gamma > 0$	$\gamma < 0$	$\gamma < 0$

PROOF: By differentiability we have

$$\frac{\partial W}{\partial y_j}\delta + \frac{\partial W}{\partial y_k}\gamma \doteq 0, \text{ which implies } \gamma = -\frac{\partial W/\partial y_j}{\partial W/\partial y_k}\delta\,.$$

The cells in the above table result from Definition 4 and the possible existence of a benchmark. Q.E.D.

Recall that most leaky-bucket experiments do not only rely exclusively on the case that j's income exceeds k's income and on $\delta < 0$, but also restrict subjects' responses to $0 < \gamma < -\delta$, that is, the (poorer) transferee receives only part of the transfer which diminishes the (richer) transferor's income. This holds only for the social-welfare-function approach if there is either no benchmark, or if both income recipients are below the benchmark. With respect to the inequality-measure approach it holds only if both income

recipients are below the benchmark. However, this covers only a segment of the leaky-bucket universe.

Notice that the leaky-bucket theory is not at variance with the transfer principle. The transfer principle focuses on changes in the degree of income inequality, whereas the leaky-bucket theory focuses on transfer leakages which maintain the degree of income inequality.

II The Experiment

A The Experimental Design

The experiment was conducted at the KiEEL laboratory (Kiel Experimental Economics Laboratory) at the University of Kiel, Germany. Subjects were volunteers recruited from the students of different departments of this university. Before coming into the laboratory, subjects received a short training in handling income distributions. In this training, the concepts of income distributions, scale invariance, and the transfer principle were explained. Subjects were alerted that income distributions may be evaluated along different criteria: First, they may be judged according to the subjects' perceptions of being *more equally distributed*. Second, they may be judged according to warranting a greater GDP for the economy, irrespective of the distribution of the incomes. Third, they may be judged according to securing higher aggregate social welfare, where we stressed that social welfare had to be judged according to the values of the beholders. Fourth, they may be judged according to the subjects' preferences for income distributions if their income positions were determined under an ex-ante veil of ignorance.

Then the instructions of the experiment were explained to the subjects. We proposed an income distribution [500 €, 750 €, 1000 €, 1250 €, 1500 €, 1750 €, 2000 €], which represents monthly incomes of seven income recipients. Using Abbink and Sadrieh's (1995) Ratimage toolbox, these incomes were presented in the upper half of a computer screen as illustrated in Figure 1. The same picture was displayed on the lower half of the screen. Upon pressing any key, the income of an income recipient was increased or diminished by 100 € (standing proxy for our $|\delta|$ in Theorems 12 and 15). At the same time, the income of another income recipient was set equal to zero.

The upper half of the screen continued to present the original picture highlighting now the incomes which was and which should be changed, reminding the subjects of the original allocation.

The subject was asked to adapt the income set to zero in the lower half of the screen such that the *degree of income inequality within this society should be maintained*. Any move of the subject was shown on the screen, and the subject was asked either to confirm the answer or to try out some other moves. The subject was allowed to play round as long as he or she wanted to do so before making his or her move definitive.[9] In total, 84 combinations [7×6, each for $+100$€ and -100€] were presented to each subject in a random order.

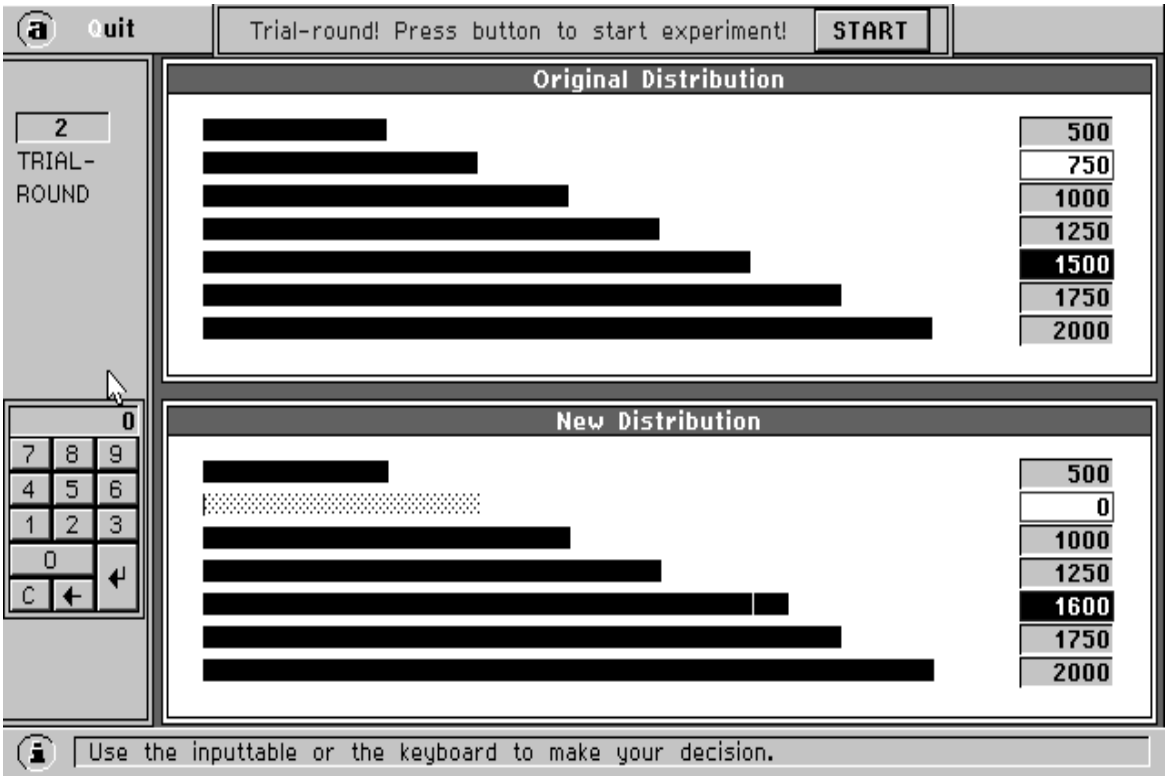


Figure 1: Screenshot

Subjects were also told that the change in an income recipient's income by 100 € was only caused by some chance event beyond the control of any party.[10] The subject was asked to state the other income recipient's income level which would restore the initial degree of income inequality within the society.[11]

This experiment required much attention and effort on the part of the subjects. To

induce subjects to participate in our experiment, we relied on their interest in the topic, since we did not know of an incentive-compatible payment régime that would not have biased subjects' responses. In total, 44 subjects participated in our experiment. The data of three subjects had to be eliminated because these subjects provided absurd responses.[12] Two responses of one subject, which were obviously misprints, were also eliminated.[13]

B The Numerical Benchmarks

Note from Theorems 11 and 14 that the benchmark is implicitly defined by setting the partial derivatives equal to zero. Now, the partial derivatives are functions of both the income distributions and the parameters of the respective income inequality measures applied. As the income distribution involved is given by our experimental design, the partial derivatives are functions of the parameters of the income inequality measures only.

When plotting these functions (their graphs are given in Figure 2 in the Appendix), we found that the function for the entropy inequality measure is S-shaped as a function of c, that the function for the Atkinson inequality measure is ogival-shaped (counter-S-shaped) as a function of θ,[14] and that the function for the extended Gini inequality measure is decreasingly convex-shaped as a function of ν. As concerns the entropy social welfare function, we observe something similar to a function with two hyperbolic branches for negative and positive values of c (the right branch is slightly increasing after having reached a minimum, which is not strictly hyperbolic), with an undefined interval for values of c somewhat between -2 and 0.

Inserting the seven incomes of our experimental design into the functions for the zero values of the partial derivatives gives us the parameter values for the critical benchmarks. They are summarized in Table 1.

Insert Table 1 about here

Table 1 shows us that only benchmarks between 750 € and 1500 € are associated with reasonable values of the parameters of the income inequality measures. All other benchmarks require excessive parameter values of inequality attitudes, which were never evi-

denced in empirical research. Therefore, we can restrict the benchmarks to lie in the open interval (€750, €1500).

III Results

Recall that we asked our subjects for the 84 income values of the y_k's which had to be adapted. These data allowed us to compute the γ's. They represent the data set of our analysis.

Summarizing all cases of Theorems 12 and 15, the theoretically correct sign of γ is determined by

$$\operatorname{sign}\gamma \doteq \operatorname{sign}\left(\left[|y_j - y_k| - \max\left\{|y_j - y^*|, |y_k - y^*|\right\}\right]\delta\right). \tag{1}$$

This means that, if y_j and y_k are on the *same* side of the benchmark, then $|y_j - y_k| < \max\left\{|y_j - y^*|, |y_k - y^*|\right\}$, and the sign of γ should be negative for $\delta > 0$ and positive for $\delta < 0$. If y_j and y_k are on *opposite* sides of the benchmark, then $|y_j - y_k| > \max\left\{|y_j - y^*|, |y_k - y^*|\right\}$ and the sign of γ should be positive for $\delta > 0$ and negative for $\delta < 0$.

Notice that we had no data on subjects' benchmarks. As our subjects did not know Theorems 12 and 15, they would not have understood queries for their benchmark. Thus, we were left to the message of Table 1, demonstrating that values of benchmarks outside the interval (750 €, 1500 €) are unlikely. This allowed us to assume for all benchmarks the condition $750\,€ < y^* < 1500\,€$. Given this assumption, we could partition our stimulus set into three subsets:

(a) The incomes involved lie certainly on the *same side* of the benchmark. This covers the stimuli $(y_i, y_k) \in \{$(500,750), (750,500), (1500,1750), (1750,1500), (1500,2000), (2000,1500), (1750,2000), (2000,1750)$\}$, both for $\delta > 0$ and $\delta < 0$, i.e., 16 stimuli.

(b) The incomes involved lie certainly on *opposite sides* of the benchmark. This covers the stimuli $(y_i, y_k) \in \{$(500,1500), (1500,500), (500,1750), (1750,500), (500,2000), (2000,500), (750,1500), (1500,750), (750,1750), (1750,750), (750,2000), (2000,750)$\}$, both for $\delta > 0$ and $\delta < 0$, i.e., 24 stimuli.

(c) The remaining 44 stimuli, on which *no a priori reasoning* on the values of the benchmarks applies.

Summary statistics of our results are provided in Tables 2 and 3.

Insert Table 2 about here

According to the leaky-bucket theory, for the subset (a) we should have observed negative γ's for $\delta > 0$ and positive γ's for $\delta < 0$. However, Table 2 shows us that the opposite holds for the mean value of the γ's. For the subset (b) we should have observed positive γ's for $\delta > 0$ and negative γ's for $\delta < 0$. Table 2 demonstrates that this is evidenced by the data for the mean value of the γ's, except for the case $\delta < 0$, $y_j > y_k$. The subset (c) covers the case for which we cannot rely on a priori reasoning about the benchmarks. Provided that subjects' benchmarks are symmetrically distributed around 1250 €, the leaky-bucket theory would demand a mean value of the γ's near zero. However, Table 2 exhibits positive mean values of the γ's for $\delta > 0$, and negative mean values of the γ's for $\delta < 0$. Whereas the mean values of γ are significantly different from zero for $\delta > 0$, the mean value of γ is significantly different from zero for $\delta < 0$ and $y_j < y_k$ only. For $\delta < 0$ and $y_j > y_k$ a Wilcoxon test shows that the mean value of γ is not significantly different from zero. With the exception of this latter case, the overall results confirm that positive γ's dominate for $\delta > 0$, and negative γ's for $\delta < 0$.

Insert Table 3 about here

In Table 3, we present the frequencies of positive, negative, and zero values of the γ's for the four basic cases for the subsets (a), (b), and (c). These results reinforce the behavioral pattern observed in Table 2. They show the preponderance of positive income compensations for income gains and negative income compensations for income losses. Broadly speaking, subjects' perceptions to maintain the degree of income inequality seem to follow a simple precept: If someone gains income, the other person involved should be positively compensated, and if someone loses income, the other person involved in the transfer should be negatively compensated. This expresses a sort of *compensating justice* rather than restoration of former degree of income inequality (or aggregate welfare), according to the leaky-bucket theory as explained in Theorems 12 and 15. Whenever

the leaky-bucket theory coincides with the compensating-justice hypothesis, then it is confirmed (e.g., for incomes lying on opposite sides of the benchmark), otherwise it is declined. Notice that neither the compensating-justice hypothesis, nor the leaky-bucket theory are at variance with the transfer principle, because they focus on the maintenance of the degree of income inequality, whereas the transfer principle focuses on changes in the degree of inequality.

Moreover, Tables 2 and 3 reveal a particular pattern of compensating justice:

(i) When an income recipient receives an extra income of €100 ($\delta > 0$), our subjects hold that the other income recipient should also receive an extra income ($\gamma > 0$) in order to maintain the degree of income inequality. However, when the other income recipient is poorer, he or she should receive considerably more extra income than when he or she is richer.

(ii) When an income recipient experiences an income loss of €100 ($\delta < 0$), our data show that, when the other income recipient is richer, he or she should also experience an income loss ($\gamma < 0$). However, when the other income recipient is poorer, our subjects hold that his or her income should stay put in order to maintain the degree of income inequality.

When screening the data for the various stimuli, these tendencies become more pronounced the poorer or richer the involved income recipients are.

More formally, let $\mid \delta \mid < \min_{i,\ell \in \{1,\ldots,n\}} \mid y_i - y_\ell \mid$, let γ_k and γ_i denote the γ's for income recipients k and i, respectively, and let $y_j \rightarrow y_j + \delta$. Then we have for $\delta > 0 : \gamma_k > \gamma_i > 0$ for $k < j < i$, and the γ_k's increase and the γ_i's decrease as j increases. For $\delta < 0$ we have: $\gamma_k > \gamma_i$, $\gamma_i < 0$ for $k < j < i$, and both the γ_k's and the γ_i's increase as j increases (for this case the γ_k's are not necessarily negative).

In order to capture these qualitative findings in quantitative terms, we estimated the following equation

$$\gamma \; = \; \beta + \alpha(y_j - y_k) + \varepsilon \tag{2}$$

using the panel-data estimation method with random effects for the four cases resulting from $\delta \gtrless 0$ and $y_j \lesseqgtr y_k$. The results are presented in Table 4.

Insert Table 4 about here

This table shows us that the income distance has a much higher impact on γ, when either the richer income recipient gains δ, or when the poorer income recipient loses δ. For instance, the coefficient estimates for Equation (2) show that, when an income recipient gains 100 €, a poorer income recipient is compensated with 45.78 € for the minimum distance of 250 €, and with 81.70 € for the maximum distance of 1500 €. When an income recipient loses 100 €, a richer income recipient's income should be reduced by 37.26 € for the minimum distance of 250 €, and by 121.52 € for the maximum distance of 1500 €.

For the remaining two cases, the tendency of compensating justice is much weaker. When an income recipient is given 100 €, a richer income recipient should receive a small positive compensation. Notice that the coefficient of the distance is small and nonsignificant. When an income recipient loses 100 €, a poorer income recipient should only lose income if his or her income is no more distant than 250 € (but note that the negative constant is nonsignificant).

These observations are confirmed by a χ^2 test. It shows that the distance between the two income recipients involved in a transfer has the strongest effect on γ when the poorer income recipient experiences a loss of €100, and the second strongest effect when the richer income recipient experiences a gain of €100.

Our qualitative results suggest that γ does not only depend on the *income distance*, but also on the *income position* of the income recipient whose income is changed by $\mid \delta \mid$. This led us to estimate a modified version of Equation (2) which includes dummies representing the income levels of the transferor:

$$(3) \qquad \gamma \;=\; \beta_0 + \alpha(y_j - y_k) + \beta_1\Delta_1 + \beta_2\Delta_2 + \beta_3\Delta_3 + \beta_5\Delta_5 + \beta_6\Delta_6 + \beta_7\Delta_7 + \varepsilon\,,$$

where $\Delta_i = 0$ for $i \neq j$ and $\Delta_i = 1$ for $i = j$. Equation (3) is calibrated for the median income $y_4 = 1250$ €, which implies that $\Delta_4 = 0$. We estimated again four equations corresponding to the four cases resulting from $\delta \lessgtr 0$ and $y_j \lessgtr y_k$. In Table 5 we present the results.

Insert Table 5 about here

We define the behavioral pattern as observed in Table 5 as *graded* compensating justice. It operates in this way:

- When an income recipient experiences an extra income of 100 € (first line of Table 5), a income recipient is first awarded a fixed amount of 19.68 €. This amount is diminished by 2.09% of the income difference between the two income recipients. The equalizing effect on the income distribution is higher, the poorer the income recipient who gets 100 € is. This obviously allows higher compensations to the richer income recipient. Thus, the values of the dummies decrease if better endowed income recipients receive 100 €. Notice that, although not all dummies are significant, their pattern conforms with the expectations of the compensating-justice hypothesis.

- When an income recipient experiences an extra income of 100 € (second line of Table 5), a poorer income recipient should even be higher compensated than the richer income recipient in the former case. He or she is awarded a fixed amount of 37.89 € plus 4.48% of the income difference between the two parties. Subjects seem, however, to perceive some overshooting due to the 4.48% of the income difference, and, hence, to correct that by another amount which decreases as the income position of the richer income recipient increases.

- When an income recipient experiences an income decrement of 100 € (third line of Table 5), then, according to the hypothesis of compensating justice, the income of a richer income recipient should also experience a loss. Table 5 shows that this income loss should be made up by a fixed amount of 18.22 € plus 8.19% of the income difference. Overshooting due to the 8.19% of the income difference is counteracted by another amount (represented by the dummies) which decreases as the income position of the poorer income recipient increases.

- When an income recipient experiences an income loss of 100 € (fourth line of Table 5), a poorer income recipient should experience an increase of income amounting to 4.25% of the income difference plus an amount which decreases as the income position of the richer income recipient increases. Calculations show that, with the exception of the lowest incomes among the poorer income recipients, this yields an income loss for the poorer income recipient, too.

Summarizing the results of our leaky–bucket experiment, we found that both leaky–bucket theorems (Theorems 12 and 15) lack empirical support. Rather compensating justice holds (Tables 2 and 3). Only when the leaky-bucket theory concurs with the compensating-justice hypothesis, it enjoys empirical support. This holds only for transfers between income recipients whose incomes lie on opposite sides of the benchmarks (i.e., the last lines of the tables in Theorems 12 and 15). As the social-welfare approach by and large lacks benchmarks for nondecreasing social welfare functions, it usually boils down to the first line of the table in Theorem 15. Hence, the social-welfare approach suffers from even less empirical support than the inequality-measure approach.

In face of the contradictions between the leaky-bucket theory and the compensating-justice hypothesis, income inequality measurement needs to be re-structured along special axioms if it should comply with subjects' perceptions. It is beyond the scope of this paper to provide a rigorous axiomatic system. All we can do now is to sketch two lines of reasoning which should be regarded—and which may turn out to be contradictory:

Sub-Translation Invariance for Increasing Incomes: If some incomes increase, other incomes should also increase to maintain the degree of inequality. Higher incomes should increase less than lower incomes. This means that for $\delta > 0$ and $y_j \to y_j + \delta$ we have $\gamma_k > \gamma_i > 0$ for $k < j < i$, and the y_k's increase and the y_i's decrease as j increases.

Super-Translation Invariance for Decreasing Incomes: If some incomes decrease, other incomes should also decrease to maintain the degree of inequality. Higher incomes should decrease more than lower incomes. This means that for $\delta < 0$ and $y_j \to y_j + \delta$ we have $\gamma_k > \gamma_i$, $\gamma_i < 0$, for $k < j < i$ and both the γ_k's and γ_i's increase as j increases.

IV Conclusion

During the last 110 years, admirable advances have been made in the fields of theoretical and empirical research of income inequality measurement, and the related field of concentration measurement.[15] Comprehensive information on this research can be gained from a great number of excellent surveys and textbooks.[16] Yet it is only a bit more than a decade since disillusion with the popular acceptance of central axioms of income inequality measurement began to undermine faith in the validity of inequality measurement. A number of questionnaire and experimental studies showed poor acceptance of

central distributional axioms such as scale invariance, the income equalizing effects of income translations, the population principle, Pareto-dominance, Lorenz-dominance, and the transfer principle.[17] Even for the simplest experimental designs in terms of numbers, the acceptance rates of these axioms hardly exceed some 40%. This seems to be caused by response-mode effects: if the axioms are presented in verbal form, agreement rises up to some 60%.[18] Subjects seem to have difficulties in transforming verbal convictions into numbers.

The theoretical analysis of leaky-bucket transactions, which can be seen as a generalization of the transfer principle with transaction costs, is of recent origin (Seidl (2001); Lambert and Lanza (2003)). It has opened up new avenues of analysis. Rather than tracing out the maximum leakage of transaction costs such that a transfer still "pays at the margin",[19] the theory has shown a plethora of possible results. In Theorems 12 and 15 we showed that leaky-bucket transactions encompass twelve cases each of which entails different results. Only one of them covers the traditional case of transfers which allows the transferee to receive a positive fraction of the transfer taken from a richer transferor.

Our experimental research showed that leaky-bucket theory is poorly evidenced by the data. Subjects rather follow some notion of compensating justice: If an income recipient loses income, the other income recipient involved should be negatively compensated, and if an income recipient gains income, the other income recipient involved should be positively compensated. Whenever the leaky-bucket theory coincides with the compensating-justice hypothesis, then it is confirmed (e.g., for incomes lying on opposite sides of the benchmark), otherwise it is declined.

Our central finding is that compensating justice asks for a higher compensation for the poorer income recipient (as compared to richer income recipients) in the case of income gains, and a lower loss (or even a small gain) for the poorer income recipient (as compared to richer income recipients) in the case of income losses. When screening the data for the various stimuli, these tendencies become more pronounced the poorer or richer the involved income recipients are.

The compensating-justice hypothesis demands a re-structuring of inequality measurement based on different axioms to comply with subjects' perceptions. This concerns two basic axioms, viz. sub-translation invariance for increasing incomes and super-translation invariance for decreasing incomes. It is beyond the scope of this paper to provide a rigor-

ous axiomatic system for a completely new theory of income inequality measurement.

References

Abbink, K. and Sadrieh, A. (1995). Ratimage, Research Assistance Toolbox for Computer-Aided Human Behavior Experiments. *Discussion Paper* B-325, University of Bonn.

Amato, V. (1968). *Metodologia Statistica Strutturale*, Volume 1. Cacucci, Bari.

Anand, S. (1982). *Inequality and Poverty in Malaysia*. Oxford University Press, Oxford.

Amiel, Y. and Cowell, F. A. (1992). Measurement of Income Inequality: Experimental Test by Questionnaire. *Journal of Public Economics*, 47:3–26.

Amiel, Y. and Cowell, F. A. (1994a). Income Inequality and Social Welfare. In Creedy, J., editor, *Taxation, Poverty and Income Distribution*, pages 193–219. Edward Elgar, Aldershot.

Amiel, Y. and Cowell, F. A. (1994b). Inequality Changes and Income Growth. In Eichhorn, W., editor, *Measurement of Welfare and Inequality*, pages 3–26. Springer, Berlin.

Amiel, Y. and Cowell, F. A. (1998). Distributional Orderings and the Transfer Principle: A Reexamination. *Research on Economic Inequality*, 8:195–215.

Amiel, Y. and Cowell, F. A. (1999a). Income Transformation and Income Inequality. In Slottje, D. J., editor, *Advances in Econometrics, Income Distribution and Scientific Methodology. Essays in Honor of Camilo Dagum*, pages 209–232. Physica, Heidelberg.

Amiel, Y. and Cowell, F. A. (1999b). *Thinking about Inequality: Personal Judgment and Income Distributions*. Cambridge University Press, Cambridge.

Amiel, Y. and Cowell, F. A. (2000). *Attitudes to Risk and Inequality: A New Twist on the Transfer Principle. STICERD Distributional Analysis Discussion Paper No. 56*. London School of Economics, London.

Amiel, Y., Creedy, J., and Hurn, S. (1999). Measuring Attitudes towards Inequality. *Scandinavian Journal of Economics*, 101:83–96.

Atkinson, A. B. (1970). On the Measurement of Inequality. *Journal of Economic Theory*, 2:244–263.

Ballano, C. and Ruiz-Castillo, J. (1993). Searching by Questionnaire for the Meaning of Income Inequality. *Revista Española de Economía*, 10:233–259.

Beckman, S. R., Cheng, D., Formby, J. P., and Smith, W. J. (1994). Preferences for Income Distributions and Redistributions: Evidence from Experiments with Real Income at Stake. Technical Report, University of Colorado, Denver, and University of Alabama.

Beckman, S. R., Formby, J. P., and Smith, J. W. (2003a). Efficiency, Equity and Democracy: Experimental Evidence on Okun's Leaky Bucket. Mimeo, University of Colorado at Denver. [revised version of a working paper by Beckman and Smith, 1994.]

Beckman, S. R., Formby, J. P., Smith, W. J., and Zheng, B. (2003b). Risk, Inequality Aversion and Biases Born of Social Position: Further Experimental Tests of the Leaky Bucket. Working Paper, University of Colorado, Denver, and University of Alabama.

Bernasconi, M. (2002). How Should Income be Divided? Questionnaire Evidence from the Theory of 'Impartial Preferences'. In Moyes, P., Seidl, C., and Shorrocks, A., editors, *Inequalities: Theory, Experiments and Applications*, pages 169–195. Springer-Verlag, Vienna–New York. Journal of Economics/Zeitschrift für Nationalökonomie, Supplement 9.

Blackorby, C. and Donaldson, D. (1978). Measures of Relative Equality and their Meaning in Terms of Social Welfare. *Journal of Economic Theory*, 18:59–80.

Bonferroni, C. E. (1930). *Elementi di Statistica Generale*. Seeber, Florence.

Bourguignon, F. (1979). Decomposable Income Inequality Measures. *Econometrica*, 47:901–920.

Chakravarty, S. R. (1988). Extended Gini Indexes of Inequality. *International Economic Review*, 29:147–156.

Chakravarty, S. R. (1990). *Ethical Social Index Numbers*. Springer-Verlag, Berlin.

Champernowne, D. G. (1952). The Graduation of Income Distributions. *Econometrica*, 20:591–615.

Champernowne, D. G. (1974). A Comparison of Measures of Inequality of Income Distribution. *The Economic Journal*, 84:787–816.

Champernowne, D. G. and Cowell, F. A. (1998). *Economic Inequality and Income Distribution*. Cambridge University Press, Cambridge.

Cowell, F. A. (1977). *Measuring Inequality*. Phillip Allan, Oxford. 2nd ed.: Prentice Hall/Harvester Wheatsheaf, London 1995.

Cowell, F. A. (1980). On the Structure of Additive Inequality Measures. *The Review of Economic Studies*, 47:521–531.

Cowell, F. A. (1985). 'A Fair Suck of the Source Bottle' – or What do you Mean by Inequality? *Economic Review*, 6:567–579.

Cowell, F. A. (2000). Measurement of Inequality. In Atkinson, A. B. and Bourguignon, F., editors, *Handbook of Income Distribution*, volume 1, pages 87–166. Elsevier Science, Amsterdam.

Dagum, C. (1990). On the Relationship Between Income Inequality Measures and Social Welfare Functions. *Journal of Econometrics*, 43:91–102.

Dagum, C. (1993). The Social Welfare Bases of Gini and other Income Inequality Measures. *Statistica*, 53:3–30.

Dalton, H. (1920). The Measurement of the Inequality of Incomes. *The Economic Journal*, 30:348–361.

David, H. A. (1968). Gini's Mean Difference Rediscovered. *Biometrika*, 55:573–575.

Donaldson, D. and Weymark, J. A. (1980). A Single-Parameter Generalization of the Gini Indices of Inequality. *Journal of Economic Theory*, 22:67–86.

Fishburn, P. C. and Willig, R. D. (1984). Transfer Principles in Income Distribution. *Journal of Public Economics*, 25:323–328.

Foster, J. E. (1985). Inequality Measurement. In Young, H. P., editor, *Fair Allocation, Proceedings of Symposia in Applied Mathematics*, Volume 33, pages 31–68. The American Mathematical Society, Providence RI.

Gini, C. (1912). Variabilitá e Mutabilita. Bologna. Reprinted in Pizetti, E. and Salvemini, T., editors, *Memorie di Metodologia Statistica*, (1955), pages 211–382. Liberia Eredi Virgilio Veschi, Rome.

Gini, C. (1914). Sulla misura della concentrazione e della variabilitá dei caratteri. *Atti del R. Instituto Veneto di SS. LL. AA.*, 73:1203–1248.

Harrison, E. and Seidl, C. (1994a). Acceptance of Distributional Axioms: Experimental Findings. In Eichhorn, W., editor, *Models and Measurement of Welfare and Inequality*, pages 67–99. Springer-Verlag, Berlin.

Harrison, E. and Seidl, C. (1994b). Perceptional Inequality and Preferential Judgments: An Empirical Examination of Distributional Judgements. *Public Choice*, 19:61–81.

Herfindahl, O. C. (1950). Concentration in the Steel Industry. PhD. dissertation, Columbia University.

Jenkins, S. (1991). The Measurement of Inequality. In Osberg, L., editor, *Economic Inequality and Poverty: International Perspectives*, pages 3–38. M. E. Sharp, Armank NY–London.

Kanbur, S. M. R. (1984). The Measurement and Decomposition of Inequality and Poverty. In Ploeg, F. Van der, editor, *Mathematical Methods in Economics*, pages 403–432. John Wiley, Chichester.

Kolm, S.-C. (1969). The Optimal Production of Social Justice. In Margolis, J. and Guitton, H., editors, *Public Economics*, pages 145–200. Macmillan, London.

Lambert, P. J. (1989). *The Distribution and Redistribution of Income.* Basil Blackwell, Cambridge MA and Oxford.

Lambert, P. J. and Lanza, G. (2003). The Effect on Inequality of Changing One or Two Incomes. working paper, Universities of York, Oregon, and Palermo.

Nygård F. and Sandström A. (1981). *Measuring Income Inequality.* Almqvist and Wicksell, Stockholm.

Ok, E. A. (1995). Fuzzy Measurement of Income Inequality: A Class of Fuzzy Inequality Measures. *Social Choice and Welfare*, 12:111–136.

Okun, A. M. (1975). Equality and Efficiency: The Big Trade-Off. Brookings Institution, Washington DC.

Pareto, V. (1895). La legge della domanda. *Giornale degli Economisti*, 10:59–68.

Piesch, W. (1975). *Statistische Konzentrationsmaße.* J.C.B. Mohr, Tübingen.

Seidl, C. (2001). Inequality Measurement and the Leaky–Bucket Paradox. *Economics Bulletin*, 4(6):1–7. http://www.economicsbulletin.com/2001volume/4-01D30001A.pdf.

Shorrocks, A. F. (1980). The Class of Additively Decomposable Inequality Measures. *Econometrica*, 48:613–625.

Shorrocks, A. F. (1983). Ranking Income Distributions. *Economica*, 50:3–17.

Shorrocks, A. F. (1984). Inequality Decomposition by Population Subgroups. *Econometrica*, 52:1369–1385.

Silber, J. (1999). *Handbook of Income Inequality Measurement*. Kluwer, Boston–Dodrecht–London.

Traub, S., Seidl, C., and Schmidt, U. (2003). Lorenz, Pareto, Pigou: Who Scores Best? Economics Working Paper No. 2003–04, University of Kiel.

Weymark, J. A. (1981). Generalized Gini Inequality Indices. *Mathematical Social Sciences*, 1:409–430.

Yaari, M. (1988). A Controversial Proposal Concerning Inequality Measurement. *Journal of Economic Theory*, 44:381–397.

Yitzhaki, S. (1983). On an Extension of the Gini Inequality Index. *International Economic Review*, 24:617–628.

Notes

[1]These beliefs were entertained, e.g., by Atkinson (1970, p. 5), Okun (1975, pp. 91–95), Jenkins (1991, pp. 28–29), and Amiel et al. (1999, pp. 87–89).—Notice the difference from the plain transfer principle: Whereas the plain transfer principle focuses on *changes* in the degree of income inequality, the transfer principle with transfers focuses on the *maintenance* of the degree of income inequality.

[2]Even the seminal theoretical paper of Lambert and Lanza (2003), which treats leaky buckets in terms of inequality measures exclusively, focuses on progressive transfers only, which covers just three out of twelve possible cases.

[3]It could refer to the minimum amount before a welfare loss is experienced, or before income inequality is exacerbated (See Lambert and Lanza 2003, p. 17).

[4]This income inequality measure is based on the concept of the *equally distributed equivalent income*, which was foreshadowed by Champernowne (1952), and first formulated by Kolm (1969). Atkinson (1970) re-established this concept and made it the centerpiece of his famous income inequality measure.

[5]The Gini coefficient was introduced by Gini (1912; 1914). It can be expressed in several different ways. For a concise survey see, e.g., Anand (1982, appendix). The Gini coefficient may, akin to the Atkinson income inequality measure, be extended to include an inequality-aversion parameter. Pioneering work was done by Donaldson and Weymark (1980), Weymark (1981), Yitzhaki (1983), and Chakravarty (1988).

[6]The entropy income inequality measures were developed by Bourguignon (1979), Cowell (1980), and Shorrocks (1980; 1984) to identify income inequality measures with are decomposable for different homogenous subgroups of income recipients. Decomposition occurs with respect to a within-group component (a weighted sum of the inequality indices of the subgroups), and a between-group component (the inequality measure evaluated by assigning every member of a subgroup the mean income of the respective subgroup). Entropy income inequality measures are, too, driven by an inequality-aversion parameter. They are generalizations of two non-parameterized income inequality measures originally proposed by Theil.

[7]It might sound strange to some readers that we do not confine the range of income inequality measures to the unit interval. Yet there are some recognized income inequality measures which do not satisfy this condition, most notably the entropy income inequality measure. When its inequality aversion parameter c converges to $+\infty$ or $-\infty$, then the entropy income inequality measure converges to $+\infty$.

[8]For further work on the relationship between income inequality measures and social welfare functions see Dagum (1990; 1993).

[9]Already before embarking on the phase of real data collection, subjects were invited to trial plays in order to become fully acquainted with the program. Only when they pressed a "start"-button did the actual experiment begin. Moreover, an experimenter was present all the time who could be addressed in case of questions or problems.

[10]Other experimenters, e.g., Amiel et al. (1999, pp. 94-95), stated explicitly in their instructions that some amount was *taken* from a person and ask for the minimum amount that should be *given* to some other person to make the transfer worthwhile. This addressed an act of redistribution and appealed to subjects' desire for transfers or feelings of social envy rather than to their perception of the degree of income inequality. We were only interested in the latter aspect.

[11]In the software they could enter any integer number between zero and 2,200.

[12]We conjecture that they were perhaps tempted to test the limits and confines of our software.

[13]This explains why the case numbers in the aggregate line of Table 3 sum to 861 for $\delta > 0$, $y_j < y_k$ and for $\delta < 0$, $y_j > y_k$, whereas they sum to 860 for $\delta > 0$, $y_j > y_k$ and for $\delta < 0$, $y_j < y_k$.

[14]This shape results from taking the real numbers as the domain of θ. Notice, however, that $\theta < 0$ indicates income inequality sympathy. Therefore, this part of the domain of θ is usually excluded from the analysis.

[15]Cf., e.g., Pareto (1895), Lorenz (1905), Gini (1912; 1914), Dalton (1920), Bonferroni (1930), Herfindahl (1950), Champernowne (1952; 1974), Amato (1968), David (1968), Kolm (1969), Atkinson (1970), Piesch (1975), Fishburn and Willig (1984), Ok (1995).

[16]Cf., e.g., Cowell (1977; 2000), Nygård and Sandström (1981), Kanbur (1984), Foster (1985), Lambert (1989), Chakravarty (1990), Jenkins (1991), Champernowne and Cowell (1998), Silber (1999).

[17]Cf. Amiel and Cowell (1992; 1994a,b; 1998; 1999a,b; 2000); Ballano and Ruiz-Castillo (1993); Harrison and Seidl (1994a,b); Bernasconi (2002); Traub et al. (2003).

[18]Our experiment was carried out in terms of numbers. It would not be viable to ask respective questions in verbal form. While the transfer principle can be communicated to any simpleton, the theory of leaky-bucket transactions has proved be a sealed book even to first-rate experts, as they did not allow the respondents to their questionnaire the appropriate domain of responses (Amiel et al. (1999)).

[19]See, e.g., Atkinson (1970, p. 5), Okun (1975, pp. 91–95), Jenkins (1991, pp. 28–29), and Amiel et al. (1999, pp. 87–89).

Measures	Income levels y_i						
	500	750	1000	1250	1500	1750	2000
Atkinson's θ	3.2×10^7	6.77	3.11	1	-1.68 [a]	-8.88 [a]	-3.3×10^6 [a]
Gini's ν [d]	[b]	18.12	7.27	3.54	1.72	[b]	[b]
Entropy inc. inequ. c	-3.26×10^6	-5.77	-2.11	0	2.68	9.88	3.2×10^6
Entropy swf. c	-3.27×10^6	-5.87	-3.82	-3.297	-3.08	-2.973	$^{+4.918}_{-2.915}$ [c]

[a] Domain of inequality sympathy.
[b] Not defined.
[c] Not uniquely defined.
[d] Rank positions converted into income levels.

Table 1: Zero Positions of Partial Derivatives Evaluated at $y_i,\ i = 1, \ldots, 7$.

Stimulus subsets	$\delta > 0$				$\delta < 0$			
	$y_j < y_k$		$y_j > y_k$		$y_j < y_k$		$y_j > y_k$	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
(a) Same side	18.64	97.54	45.66	86.23	-56.37	101.15	-2.52	84.42
(b) Opposite sides	39.75	121.53	72.63	84.80	-94.42	118.85	12.76	90.36
(c) Benchmark?	25.34	109.97	54.16	85.81	-52.53	96.10	2.37	87.89
Aggregate	28.19	111.36	57.81	86.07	-65.19	105.52	4.40	88.09

Table 2: Summary statistics for the values of γ.

Cases		β	α
$\delta > 0$	$y_j < y_k$	23.16033 (0.076)	−0.0074954 (0.330)
	$y_j > y_k$	38.59084 (0.000)	0.0287423 (0.000)
$\delta < 0$	$y_j < y_k$	−20.40951 (0.091)	0.0674103 (0.000)
	$y_j > y_k$	−9.889199 (0.336)	0.0214383 (0.000)

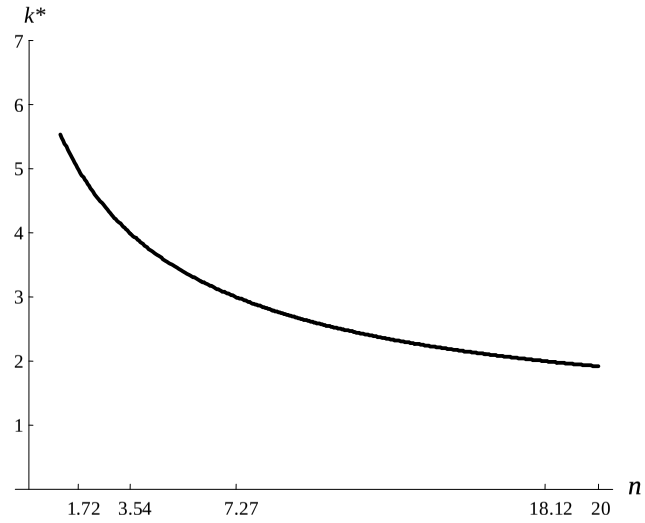
Table 4: Simple Compensating Justice (p-values in parentheses)

Stimulus subsets		$\delta > 0$						$\delta < 0$					
		$y_j < y_k$			$y_j > y_k$			$y_j < y_k$			$y_j > y_k$		
		pos.	neg.	zero	pos.	neg.	zero	pos.	neg.	zero	pos.	neg.	zero
(a) Same side	n	95	59	10	115	39	10	44	112	8	59	95	10
	%	57.92	35.97	6.09	70.12	23.78	6.09	26.83	68.29	4.88	35.98	57.93	6.09
(b) Opposite sides	n	148	77	21	205	27	14	45	192	8	101	125	20
	%	60.16	31.31	8.53	83.33	10.98	5.69	18.37	78.37	3.26	41.06	50.81	8.13
(c) Benchmark?	n	274	142	35	342	90	18	105	328	18	183	232	36
	%	60.75	31.49	7.76	76.0	20.0	4.0	23.28	72.73	3.99	40.58	51.44	7.99
Aggregate	n	517	278	66	662	156	42	194	632	34	343	452	66
	%	60.05	32.29	7.67	76.98	18.14	4.88	22.56	73.49	3.95	39.84	52.50	7.67

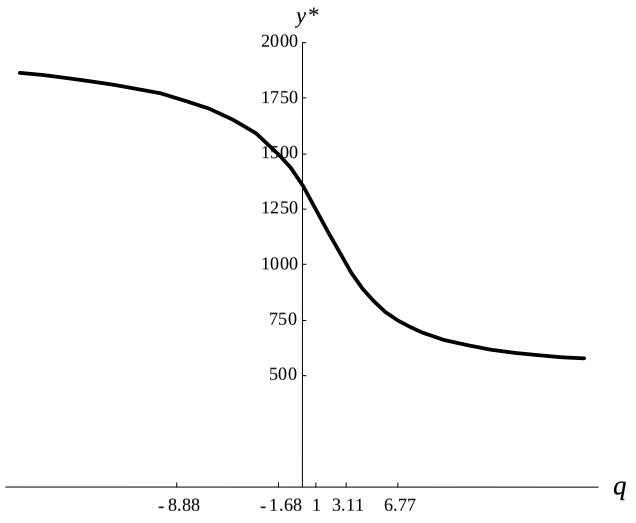
Table 3: Numbers and Percentages of the γ's

Cases		β_0	α	β_1 $y_j = 500$	β_2 $y_j = 750$	β_3 $y_j = 1000$	β_5 $y_j = 1500$	β_6 $y_j = 1750$	β_7 $y_j = 2000$
$\delta > 0$	$y_j < y_k$	19.67573	0.0208962	49.374013	31.18177	11.87218	-7.486005	-14.44789	–
		(0.174)	(0.016)	(0.000)	(0.001)	(0.228)	(0.524)	(0.339)	–
	$y_j > y_k$	37.89269	0.0448488	–	23.40732	8.545121	-7.500397	-20.3	-19.98902
		(0.001)	(0.000)	–	(0.050)	(0.361)	(0.340)	(0.008)	(0.009)
$\delta < 0$	$y_j < y_k$	-18.22181	0.0819304	10.89055	20.05978	11.47097	-.07494253	-52.58828	–
		(0.173)	(0.000)	(0.210)	(0.021)	(0.196)	(0.944)	(0.000)	–
	$y_j > y_k$	-0.3070848	0.0425491	–	11.96249	-2.600058	-24.44466	-35.76574	-37.82178
		(0.979)	(0.000)	–	(0.305)	(0.777)	(0.001)	(0.000)	(0.000)

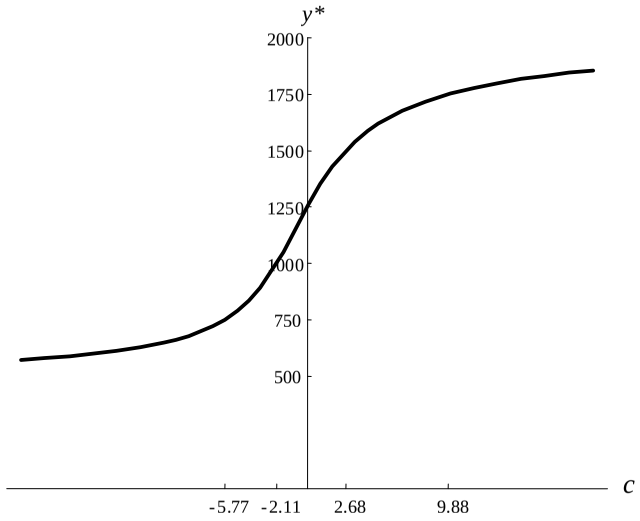
Table 5: Graded Compensating Justice (p-values in parentheses)



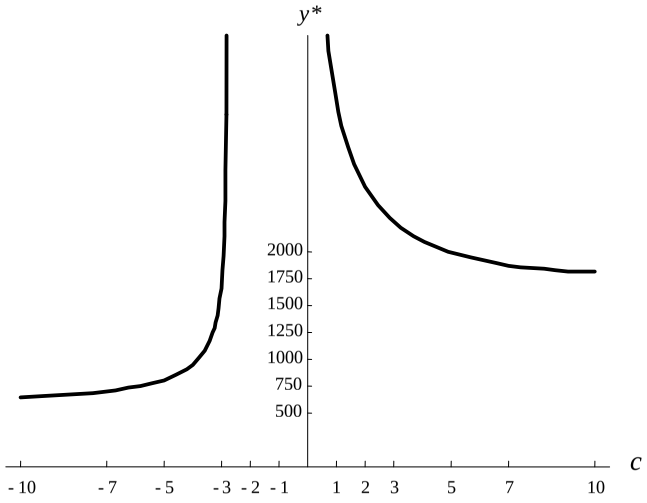
(a) Benchmarks of the Extended Gini Inequality Measure



(b) Benchmarks of the Atkinson Inequality Measure



(c) Benchmarks of the Entropy Inequality Measure



(d) Benchmarks of the Entropy Social Welfare Function

Figure 2: Benchmarks