

Lengnick, Matthias; Wohltmann, Hans-Werner

**Working Paper**

## Agent-based financial markets and New Keynesian macroeconomics: A synthesis

Economics Working Paper, No. 2011-09

**Provided in Cooperation with:**

Christian-Albrechts-University of Kiel, Department of Economics

*Suggested Citation:* Lengnick, Matthias; Wohltmann, Hans-Werner (2011) : Agent-based financial markets and New Keynesian macroeconomics: A synthesis, Economics Working Paper, No. 2011-09, Kiel University, Department of Economics, Kiel

This Version is available at:

<https://hdl.handle.net/10419/49990>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

C | A | U

Christian-Albrechts-Universität zu Kiel

Department of Economics

Economics Working Paper  
No 2011-09

# agent-based financial markets and new keynesian macroeconomics -a synthesis- -updated version-

by Matthias Lengnick and Hans-Werner Wohltmann



# Agent-Based Financial Markets and New Keynesian Macroeconomics – A Synthesis –

**Matthias Lengnick\***

*University of Kiel*

*University of Mannheim*

**Hans-Werner Wohltmann\*\***

*University of Kiel*

September 13, 2011

## Abstract

We combine a simple agent-based model of financial markets and a New Keynesian macroeconomic model with bounded rationality via two straightforward channels. The result is a macroeconomic model that allows for the endogenous development of business cycles and stock price bubbles. We show that market sentiments exert important influence on the macroeconomy. They introduce high volatility into impulse-response functions of macroeconomic variables and thus make the effect of a given shock hard to predict. We also analyze the impact of different financial transaction taxes (FTT, FAT, progressive FAT) and find that such taxes can be used to stabilize the economy and raise funds from the financial sector as a contribution to the costs produced by the recent crisis. Our results suggest that the FTT leads to higher tax revenues and better stabilization results than the FAT. However, the FTT might also create huge distortion if set too high, a threat which the FAT does not imply.

*JEL classification: E0, E62, G01, G18*

*Keywords: Agent-based modeling; stock market; New Keynesian macroeconomics; financial transaction tax; financial activities tax*

---

\* *E-mail:* matthias.lengnick@vwl.uni-mannheim.de, *Tel:* +49 621 181 1897, *Fax:* +49 621 181 1774

\*\* *E-mail:* wohltmann@economics.uni-kiel.de

Economists [...] have to do their best to incorporate the realities of finance into macroeconomics.

---

*Paul Krugman (2009)*

## 1 Introduction

The economies of almost every country have recently been hit by a turmoil in the financial markets. This so-called financial crisis has vividly demonstrated that developments in the financial markets can have major impacts on the real economy. Interdependencies between real and financial markets should therefore obviously be taken into account when doing macroeconomics. Natural questions to ask after the recent crisis are: To which extent does the formation and bursting of bubbles spill over into real markets? Can financial market regulation be used to reduce disturbances of the real economy? How can the financial sector be held to account for the enormous costs created by the recent crisis?

For about two decades now, a relatively new modeling approach has been applied to the analysis of financial and foreign exchange markets. This approach builds on the method of agent-based computational (ABC) simulation, it drops the assumptions of rational expectations, homogeneous individuals, perfect ex ante coordination and often also market equilibria, in favor of adaptive learning, simple interactions of heterogeneous agents, and emerging complex macroscopic phenomena.<sup>1</sup> The approach seems very promising thus far since, on the one hand, it is grounded in the results of survey studies<sup>2</sup> and laboratory experiments<sup>3</sup>, and on the other hand, the emerging macro-dynamics mimic the properties of real world data (such as martingale property of stock prices, fat tails of return distribution, volatility clustering and dependency in higher moments)<sup>4</sup> quite well, a success that traditional financial market models, building on equilibrium and rationality, do not provide.<sup>5</sup> A huge literature has already developed on this topic that – despite its success – is largely ignored by macroeconomists.

---

<sup>1</sup> For an introduction into ABC financial market modeling see, e.g., Samanidou et al. (2006), Hommes (2006) or LeBaron (2006). Outstanding examples of such models are Kirman (1993), Brock and Hommes (1998), and Lux and Marchesi (2000).

<sup>2</sup> Consult Frankel and Froot (1987), Ito (1990), Taylor and Allen (1992) and Lui and Mole (1998).

<sup>3</sup> Consult Caginalp et al. (2001), Sonnemans et al. (2004) and Hommes et al. (2005).

<sup>4</sup> A detailed description of these *stylized facts* can be found in Lux (2009).

<sup>5</sup> De Grauwe and Grimaldi (2006), for example, compare the performance of an agent-based model with popular models like that of Obstfeld and Rogoff in explaining the stylized facts of foreign exchange rates. They find that the former performs much better.

One strength of the ABC method is that it naturally allows for the endogenous emergence of bubbles. In such models, investors can typically choose from a set of different non-rational trading strategies. The decision which strategy to employ is reached by using an evolutionary approach: A continuous evaluation of those strategies according to past performance leads to changes in the size of the different investor groups. In phases that are dominated by technically operating investors, stock prices can deviate sharply from their underlying fundamental value. If market sentiments change and fundamentalists dominate, convergence towards the fundamental value sets in. Inspired by the spectacular failure of mainstream macroeconomics to provide an explanation of the current crisis and an agenda of how to deal with it, a number of authors are calling for the use of ABC models in macroeconomics.<sup>6</sup> According to them, the assumptions of equilibrium, perfect ex ante coordination, rational expectations and representative agents are very unrealistic and the reason that macroeconomists have become blind to crisis.

The emergence of asset price misalignments (i.e. bubbles) on the financial markets is often seen as having the most devastating impact on the real economy. Some macroeconomic models already allow for such misalignments. Bernanke and Gertler (1999), for example, augment the model of Bernanke et al. (1999) by imposing an exogenously given path for asset price misalignment. In their model, each bubble has a constant exogenous probability to burst, where "burst" simply means that asset prices immediately return to their fundamental value. More recently Milani (2008) and Castelnuovo and Nistico (2010) have integrated stock price misalignment into a New Keynesian DSGE model. Their aim is to provide insights into the dynamics of the stock price component that is driven by utility-optimizing, rational-expecting agents. Stock price dynamics in such models are a rational response of an ex ante perfectly coordinated economy in equilibrium. With the mentioned criticism in mind, it is hard to imagine stock price bubbles or financial crisis in such frameworks.

Kontonikas & Ioannidis [KI] (2005) and Kontonikas & Montagnoli [KM] (2006) use forward- and backward-looking New Keynesian macroeconomic (NKM) models with lagged stock wealth effects. Stock price dynamics in these models are not exogenously imposed and the crash of a bubble does not simply occur with a fixed probability. Instead they make use of an endogenous dynamic process that binds stock prices to two different forces: One of which leads to a return towards the fundamental

---

<sup>6</sup> See, e.g., Colander et al. (2008), Colander et al. (2009), Lux and Westerhoff (2009), Krugman (2009), Kirman (2010), Delli Gatti et al. (2010), and Dawid and Neugart (forthcoming). Examples of purely agent-based macro models (with no connection to NKM) are Gaffeo et al. (2008) or Deissenberg et al. (2008).

value, and the other – so-called momentum effect – relates stock prices to their own past development. While KI (2005) and KM (2006) are clearly inspired by the agent-based financial markets literature with its fundamentalist and chartist trading rules, none of the above models explicitly motivates the dynamics of stock price misalignment by boundedly rational investor behavior and none makes use of an evolutionary selection mechanism of trading strategies that is used in ABC type models.

In a recent paper Bask (2009) uses a New Keynesian dynamic stochastic general equilibrium (DSGE) framework with stock prices that are determined by the demand of two different types of investors: chartists and fundamentalists. While the model provides the major advantage that it justifies stock price movements by the behavior of these two types of investors, it does not allow for an endogenous evaluation of the different investment strategies. As in KI (2005) and KM (2006) the aspect of evolutionary learning, that is so important for ABC financial markets, is missing. Investors therefore keep employing the same investment rule and do not try to learn from past observations.

In this paper, we merge a simple ABC model of financial markets with the New Keynesian DSGE model. Such a comprehensive model allows for the simultaneous development of endogenous business cycles and stock price bubbles. Expectations in both submodels are formed by an evolutionary selection process. To the best of our knowledge, no such attempt has been made so far. Since we combine two separate subdisciplines of economics, and do not want to exclude readers who are not familiar with both of these areas, our approach focuses on simplicity. Nonetheless, our model leads to a number of interesting insights. We find that the transmission of shocks is dependent on the state of market sentiments<sup>7</sup> at the time of its occurrence. We also find that the negative impact that speculative behavior of financial market participants exerts on the macroeconomy, can be reduced by introducing a tax on financial transactions. We use our model to answer two questions that are currently on the international policy agenda: (1) Is a *Financial Transaction Tax* (FTT) or a *Financial Activities Tax* (FAT) better suited for regulating financial markets and generating tax income for the state? (2) Should the rate of a FAT be flat or progressive?

The model is developed in section 2. We demonstrate the working of our model by means of numerical simulation and impulse response analysis in section 3. In section 4 we analyze the

---

<sup>7</sup> By *market sentiments* we mean the state of agents opinions about economic variables that are the result of an evolutionary process and not of rational forecasting. The expression is taken from De Grauwe (2010a) where it is used as a synonym to *animal spirits*.

introduction of different kinds of taxes levied on financial transactions. Our results are checked for robustness in section 5. Section 6 concludes.

## 2 The Model

Our model consists of two parts, one describing the financial sector, and one the real sector of the economy. We use the ABC chartist-fundamentalist model proposed by Westerhoff (2008) to model the financial market. The real sector is described by the NKM framework augmented by a cost effect of stock prices. Since we allow for an *endogenous* development of business cycles and stock price bubbles, our model is an augmentation of NKM models that already include stock price bubbles, but impose their dynamics exogenously (section 1). It is also an augmentation of those models that integrate a stock market with different types of investors into macroeconomics, but do not employ endogenous learning (section 1).

An approach which is related to ours can be found in Proaño (2011). The author makes use of an ACE foreign exchange market in the context of an open economy macro model. While the general idea is similar to ours, our way of integrating the financial market and the real economy is very different. Our financial market, in contrast to Proaño (2011), first, contains noise and is therefore not completely deterministic. Second, it operates on a smaller time interval than the real economy. The paper at hand can thus be seen as a complementary approach to a similar research target.

The first problem one has to deal with is that the rules determining the dynamics of financial markets are likely to be very different from those of the real markets. Economic transactions in the former seem to take place much more frequently than in the latter.<sup>8</sup> For example, a large fraction of financial transactions (10%-60% according to market)<sup>9</sup> are accounted by algorithmic trading which is typically of an extremely short-term intra-daily nature. This implies that both can not be modeled on the same time scale.<sup>10</sup>

The two modeling methodologies employed throughout this paper are building on very different assumptions. In order to prohibit contradictions stemming from these different assumptions of

---

<sup>8</sup> Although this argument seems to be straightforward it is also backed empirically by Aoki and Yoshikawa (2007), who find that time series of real economic data do not share the power law distribution of financial markets which implies that the latter are characterized by higher economic activity.

<sup>9</sup> Consult Matheson (2011), p. 19.

<sup>10</sup> On the explicit modeling of high frequency New Keynesian models see Franke and Sacht (2010).

ABC and DSGE modeling, we do not simply integrate one into the other, but take the differences seriously. As a result, we must assume that real and financial markets are populated by different kinds of agents. We interpret those of the financial market to be institutional investors, who have the resources to participate in high frequency trading. Conversely, real market agents have neither detailed knowledge about financial markets, nor the possibility to participate in high frequency trading. Subsection 2.1 defines the financial sector of our economy, while 2.2 defines the real one. Subsection 2.3 brings the two sectors together.

## 2.1 Financial Market

We use the model proposed by Westerhoff (2008) to define the financial sector of our economy for two reasons: First, because of its straightforward assumptions and easy implementation, and second, because it has already been used for policy analysis (especially transaction taxes) so that its behavior in this respect is well known.<sup>11</sup> In this model, stock price adjustment is given by a price impact function:

$$s_{t+1} = s_t + a (W_t^C D_t^C + W_t^F D_t^F) + \epsilon_t^s \quad (1)$$

$D^C$  and  $D^F$  stand for the orders generated by chartists and fundamentalists, respectively.<sup>12</sup>  $W^C$  and  $W^F$  denote the fractions of agents using these strategies, and  $a$  is a positive reaction parameter. Eq. (1) can be interpreted as a market maker scenario, where prices are adjusted according to observed excess demand.<sup>13</sup> Since fundamentalist and chartist investment strategies do not account for all possible strategies that exist in real markets, a noise term  $\epsilon^s$  is added that is i.i.d. normally distributed with standard deviation  $\sigma^s$ . It can be interpreted as the influence of those other strategies.  $t$  denotes the time index which is interpreted as days. For the sake of simplicity, we make use of the standard assumption that the true (log) fundamental value of the stock price  $\bar{s}^f$  equals zero. Thus, the stock price  $s_t$  also equals the stock price misalignment.

---

<sup>11</sup>The approach is, for example, also used in Westerhoff and Dieci (2006) who model two financial markets and their interaction when introducing transaction taxes. Demary (2010) also analyzes the effects of introducing such taxes in a basic Westerhoff-model augmented by different time horizons of investors.

<sup>12</sup>Negative orders denote a supply of stock.

<sup>13</sup>There are also agent-based financial models that make use of Walrasian market clearing. See for example Brock and Hommes (1998).



Chartists expect that the direction of the recently observed price trend is going to continue:

$$\tilde{\mathbb{E}}_t^C [s_{t+1} - s_t] = k^C [s_t - s_{t-1}] \quad (2)$$

$k^C$  is a positive parameter that denotes the strength of trend extrapolation. The tilde on the expectations operator indicates that the expectation is not formed rationally. Fundamentalists, on the other hand, expect that  $k^F \cdot 100$  % of the actual perceived mispricing is corrected during the next period:

$$\tilde{\mathbb{E}}_t^F [s_{t+1} - s_t] = k^F [s_t^f - s_t] \quad (3)$$

$s_t^f$  is the perceived fundamental value that does not necessarily equal its true counterpart  $\bar{s}^f$ . The difference between  $s_t^f$  and  $\bar{s}^f$  is explained in detail in subsection 2.3. Assuming that the demand generated by each type of investors depends positively on the expected price development leads to:

$$D_t^i = \ell \mathbb{E}_t^i [s_{t+1} - s_t] + \epsilon_t^i \quad i = \{C, F\} \quad (4)$$

$\ell$  is a positive reaction parameter. Since (2) and (3) do not reflect the great amount of chartist and fundamentalist trading strategies that exist in real world markets, the noise term  $\epsilon_t^i$  is added. It is normally distributed with standard deviation  $\sigma^i$  and can be interpreted as the influence of all other forecasting strategies different from (2) and (3). The demand generated by chartist and fundamentalist trading rules is therefore given by:<sup>14</sup>

$$D_t^C = b (s_t - s_{t-1}) + \epsilon_t^C \quad b = \ell \cdot k^C \quad (5)$$

$$D_t^F = c (s_t^f - s_t) + \epsilon_t^F \quad c = \ell \cdot k^F \quad (6)$$

The fractions of agents using the two different investment strategies are not fixed over time. Instead, agents continuously evaluate the strategies they use according to past performance. The better a strategy performs relative to the other, the more likely it is that agents will employ it. It

---

<sup>14</sup>Westerhoff (2008) directly assumes eq. (5) and (6) and does not explicitly state the different types of expectation formations.

is assumed that the attractiveness of a particular strategy depends on its most recent performance  $(\exp\{s_t\} - \exp\{s_{t-1}\}) D_{t-2}^i$  as well as its past attractiveness  $A_{t-1}^i$ :<sup>15</sup>

$$A_t^i = (\exp\{s_t\} - \exp\{s_{t-1}\}) D_{t-2}^i + dA_{t-1}^i \quad i = \{C, F\} \quad (7)$$

The memory parameter  $0 \leq d \leq 1$  defines the strength with which agents discount past profits. The extreme cases  $d = 0$  and  $d = 1$  relate to scenarios where agents have zero and infinite memory. Note the timing of the model: Orders submitted in  $t - 2$  are executed in  $t - 1$ . Their profitability ultimately depends on the price realization in  $t$ . Agents may also withdraw from trading (strategy “0”). The attractiveness of this strategy  $A_t^0$  is normalized to zero ( $A_t^0 = 0$ ). The fraction of agents that employ strategy  $i$  is given by the well known *discrete choice* or *Gibbs probabilities*:<sup>16</sup>

$$W_t^i = \frac{\exp\{eA_t^i\}}{\exp\{eA_t^C\} + \exp\{eA_t^F\} + \exp\{eA_t^0\}} \quad i = \{C, F, 0\} \quad (8)$$

The more attractive a strategy, the higher the fraction of agents using it. Note that the probability of choosing one of the three strategies is bounded between zero and one. The positive parameter  $e$  measures the *intensity of choice*. The higher (lower)  $e$ , the greater (lesser) the fraction of agents that will employ the strategy with the highest attractiveness. This parameter is often called the *rationality parameter* in ABC financial market models.<sup>17</sup> The described mechanism can be interpreted as an evolutionary survival of the most profitable forecasting strategy.

The only difference between our financial market submodel and that of Westerhoff (2008) is that we distinguish between the true fundamental value  $\bar{s}^f$  and the trader’s perception of it,  $s_t^f$ . Both models are equivalent if  $s_t^f = \bar{s}^f$ .

<sup>15</sup> Recall that  $s_t$  is the logarithm of the stock price. In order to calculate nominal profits,  $s_t$  has to be delogarithmized.

<sup>16</sup> See, e.g., Manski and McFadden (1981) for a detailed explanation of discrete choice models.

<sup>17</sup> Consult Westerhoff and Dieci (2006), Hommes (2006) and Westerhoff (2008).

## 2.2 Real Markets

The partial model describing the real sector is given by a hybrid NKM model. New Keynesian models are widely used in macroeconomics because they typically allow for a good fit of real world data, and they are derived from individual optimization.

$$i_q = \delta_\pi \tilde{\mathbb{E}}_q [\pi_{q+1}] + \delta_x \tilde{\mathbb{E}}_q [x_{q+1}] + \epsilon_q^i \quad (9)$$

$$x_q = \chi \tilde{\mathbb{E}}_q [x_{q+1}] + (1 - \chi)x_{q-1} - \frac{1}{\sigma} \left( i_q - \tilde{\mathbb{E}}_q [\pi_{q+1}] \right) + \epsilon_q^x \quad (10)$$

$$\pi_q = \beta \left( \psi \tilde{\mathbb{E}}_q [\pi_{q+1}] + (1 - \psi)\pi_{q-1} \right) + \gamma x_q - \kappa s_q + \epsilon_q^\pi \quad (11)$$

The notation of the variables is as follows:  $i$  is the deviation of the nominal interest rate from its target,  $\pi$  the deviation of the inflation rate from its target,  $x$  the (log) output gap (i.e. its deviation from steady state), and  $s$  the deviation of the (log) nominal stock price from its true fundamental value  $\bar{s}^f$ . The subscript  $q = 1, \dots, Q$  denotes the time index. We keep the common interpretation of the time index in New Keynesian models and assume that it denotes quarters.  $\tilde{\mathbb{E}}_q [\cdot]$  is the expectations operator conditional on knowledge available in  $q$ , where the tilde indicates that they are formed non-rationally. The dynamic path of the stock price  $s$  is determined by the model developed in the previous subsection. The variables  $\epsilon_q^i$ ,  $\epsilon_q^x$ ,  $\epsilon_q^\pi$  are stochastic elements with zero mean.

Equation (9) is a standard monetary policy interest rule. The central bank reacts to expected deviations of inflation and output from its target. For now, we use equal expectations formation for the central bank and the market. In section 5.2 we will generalize the model by assuming that the central bank's expectations are different from those of the market. Equation (10) is referred to as the dynamic IS-curve that describes the demand side of the economy. It results from the Euler equation (which is the result of intertemporal utility maximization) and market clearing in the goods market. Equation (11) is a New Keynesian Phillips curve that represents the supply side. It can be derived under the assumptions of nominal price rigidity and monopolistic competition. Asset prices influence the economy through a *balance sheet channel* that works as follows: The willingness of banks to grant credits typically depends on the borrowers' financial position. For example, agents could use assets they hold as collateral when borrowing money. The more collateral a debtor has to offer, the more advantageous his credit contract will be. In this context, "advantageous" may

mean that either credits of larger size are offered or that credits of the same size could be obtained cheaper (lower interest payments). The first argument can be used to relate asset prices positively to aggregate demand, as for example done in Bernanke and Gertler (1999), Kontonikas and Ioannidis (2005), Kontonikas and Montagnoli (2006), or Bask (2009). We stress the second argument in this paper. Higher prices of assets owned by firms increase their creditworthiness, and allow them access to cheaper credits. Since most firms' production is largely financed through credits, asset prices are inversely related to firms marginal (real) costs of production. This argument allows the addition of the term  $-\kappa s_q$  to equation (11).<sup>18</sup> This verbal kind of micro foundation is sufficient for our purposes. The reader is referred to Bernanke and Gertler (1999) who discuss a balance sheet channel (and its microfoundation) in more detail.

Note that we defined  $s_q$  as the *nominal* stock price gap. The so-called *cost channel of monetary transmission* is commonly introduced into New Keynesian models by adding the nominal interest rate into the Phillips-curve (see for example Ravenna and Walsh (2006) or Lam (2010)). Analogously to this channel, we also decided to insert the nominal (and not the real) stock price gap into (11). Note also that our definition of the stock price gap is very different from that of Milani (2008) or Castelnuovo and Nistico (2010), who define it as the difference between the stock price under fully flexible and somewhat rigid market conditions. Both, of course, are the result of utility optimal paths under rational expectations. ABC financial market models could also be employed for the analysis of foreign exchange rates. Since a rise (fall) of foreign exchange rates would also raise (lower) production costs – via more expensive (cheaper) intermediate inputs – they would be included with the opposite sign (i.e.  $+\kappa s_q$ ). To avoid confusion, we want to point out again that we are modeling stock prices with the ABC submodel and not foreign exchange rates.

To derive eq. (10), it is commonly assumed that the household's only possibility of transferring wealth into future periods is by demanding bonds. Households therefore do not hold or trade stock. We keep this assumption in order to allow for analysis of the isolated impact of the speculation of financial market participants on stock prices. We further assume that firms hold an initial amount of stock but do not participate in stock trading. Consequently, they are only affected by the financial sector via the balance sheet channel, and not via speculative gains. The financial sector can not generate profits on the aggregate level by selling and reselling stock. If one agent wins from a

---

<sup>18</sup>Formally, the cost channel is typically introduced by adding interest costs ( $+\alpha \cdot i_q$ ) into the Phillips-Curve. If interest costs are negatively depending on solvency and thus stock prices, the term  $-\kappa \cdot s_q$  has to be added instead.

beneficial transaction, others must lose. The only possibility for the aggregate stock market to earn profits is by dividend payments from the real sector. Because their relative size is small when calculated for a daily basis, and because the Westerhoff-model does not explicitly take financial wealth into account, we do not model the stream of dividend payments from firms to financial investors. As a result of the above arguments and assumptions, financial streams between the real and financial sector do not exist.

The model can be rearranged as follows:

$$\begin{aligned}
\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{\sigma} & 1 & 0 \\ 0 & -\gamma & 1 \end{pmatrix}}_{\mathbf{A}} \begin{pmatrix} i_q \\ x_q \\ \pi_q \end{pmatrix} &= \underbrace{\begin{pmatrix} 0 & \delta_x & \delta_\pi \\ 0 & \chi & \frac{1}{\sigma} \\ 0 & 0 & \beta\psi \end{pmatrix}}_{\mathbf{B}} \begin{pmatrix} 0 \\ \tilde{\mathbb{E}}_q[x_{q+1}] \\ \tilde{\mathbb{E}}_q[\pi_{q+1}] \end{pmatrix} + \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1-\chi & 0 \\ 0 & 0 & \beta(1-\psi) \end{pmatrix}}_{\mathbf{C}} \begin{pmatrix} i_{q-1} \\ x_{q-1} \\ \pi_{q-1} \end{pmatrix} \\
&+ \begin{pmatrix} 0 \\ 0 \\ -\kappa \end{pmatrix} s_q + \begin{pmatrix} \epsilon_q^i \\ \epsilon_q^x \\ \epsilon_q^\pi \end{pmatrix} \tag{12}
\end{aligned}$$

The dynamics of the forward-looking variables  $i$ ,  $x$  and  $\pi$  depend on the future expectations  $\tilde{\mathbb{E}}_q[x_{q+1}]$ ,  $\tilde{\mathbb{E}}_q[\pi_{q+1}]$ , their own history  $(i_{q-1}, x_{q-1}, \pi_{q-1})$ , the current value of  $s_q$  and the realizations of the noise terms  $(\epsilon_q^i, \epsilon_q^x, \epsilon_q^\pi)$ .

The endogenous real sector variables  $i_q$ ,  $x_q$  and  $\pi_q$  can be calculated as follows:

$$\begin{pmatrix} i_q \\ x_q \\ \pi_q \end{pmatrix} = \mathbf{A}^{-1} \mathbf{B} \begin{pmatrix} 0 \\ \tilde{\mathbb{E}}_q[x_{q+1}] \\ \tilde{\mathbb{E}}_q[\pi_{q+1}] \end{pmatrix} + \mathbf{A}^{-1} \mathbf{C} \begin{pmatrix} i_{q-1} \\ x_{q-1} \\ \pi_{q-1} \end{pmatrix} + \mathbf{A}^{-1} \begin{pmatrix} 0 \\ 0 \\ -\kappa \end{pmatrix} s_q + \mathbf{A}^{-1} \begin{pmatrix} \epsilon_q^i \\ \epsilon_q^x \\ \epsilon_q^\pi \end{pmatrix} \tag{13}$$

Of course the parameters must be selected in a way that the system is stable. We take a closer look on the stability conditions of the system in the next subsection after the model has been stated completely.

Expectations in the real sector are also formed in a non-rational way. Following De Grauwe (2010a) and De Grauwe (2010b) we assume that a certain fraction  $\omega_q^{\text{opt}}$  of agents is optimistic about

the future development of output while another  $\omega_q^{\text{pes}}$  is pessimistic. Both groups form expectations according to:

$$\text{Optimists expectation: } \tilde{E}_q^{\text{opt}} [x_{q+1}] = g_t \quad (14)$$

$$\text{Pessimists expectation: } \tilde{E}_q^{\text{pes}} [x_{q+1}] = -g_t \quad \left( = -\tilde{E}_q^{\text{opt}} [x_{q+1}] \right) \quad (15)$$

The spread between the two expectations ( $2g_t$ ) is assumed to vary over time according to:

$$2g_t = \mu + \nu \cdot \text{Std} [x_t] \quad (16)$$

The parameters satisfy  $\mu, \nu \geq 0$  and  $\text{Std} [x_t]$  denotes the unconditional standard deviation of the output gap computed over a fixed window of past observations.<sup>19</sup> The economic rationale behind this implementation is that the agents beliefs diverge more when uncertainty surrounding the output gap is high. In the special case of  $\nu = 0$  the divergence of beliefs is constant over time. In line with the method provided in the previous subsection, we define the attractiveness of the different strategies as:

$$A_q^{\text{opt}} = - \left( x_{q-1} - \tilde{E}_{q-2}^{\text{opt}} [x_{q-1}] \right)^2 + \zeta A_{q-1}^{\text{opt}} \quad (17)$$

$$A_q^{\text{pes}} = - \left( x_{q-1} - \tilde{E}_{q-2}^{\text{pes}} [x_{q-1}] \right)^2 + \zeta A_{q-1}^{\text{pes}} \quad (18)$$

The attractiveness of forecasting strategies are therefore determined by past mean squared forecast errors (MSFEs) weighted with decaying weights. Applying discrete choice theory, the weights that determine the fractions of agents are given by:

$$\omega_q^{\text{opt}} = \frac{\exp\{\phi A_q^{\text{opt}}\}}{\exp\{\phi A_q^{\text{opt}}\} + \exp\{\phi A_q^{\text{pes}}\}} \quad (19)$$

$$\text{and } \omega_q^{\text{pes}} = \frac{\exp\{\phi A_q^{\text{pes}}\}}{\exp\{\phi A_q^{\text{opt}}\} + \exp\{\phi A_q^{\text{pes}}\}} \quad (20)$$

<sup>19</sup>In all numerical simulations we set  $\mu = 0.5$  and  $\nu = 2$ . The mentioned time windows is set to 20 periods.

The market's expectation of the output gap is given by the weighted average of the two different forecasting strategies.

$$\tilde{E}_q [x_{q+1}] = \omega_q^{\text{opt}} \tilde{E}_q^{\text{opt}} [x_{q+1}] + \omega_q^{\text{pes}} \tilde{E}_q^{\text{pes}} [x_{q+1}] \quad (21)$$

Expectations about the inflation rate in De Grauwe (2010a) and De Grauwe (2010b) are formed in a similar way. One type of agents (the *targeters*) believes in the inflation target that the central bank has announced, hence their expectations are given by:

$$\tilde{E}_q^{\text{tar}} [\pi_{q+1}] = \pi^* \quad (22)$$

Another group (the *extrapolators*) expect that the future inflation rate is given by the most recently observed one, i.e. they extrapolate past values into the future. Expectations of this group are given by:

$$\tilde{E}_q^{\text{ext}} [\pi_{q+1}] = \pi_{q-1} \quad (23)$$

The markets expectation  $\tilde{E}_q [\pi_{q+1}]$  is again determined as the weighted average of these two groups. Where the fractions of targeters and extrapolators ( $\omega^{\text{tar}}$  and  $\omega^{\text{ext}}$ ) are again determined by the same evolutionary approach used for expectations about the output gap.

Both expectations,  $\tilde{E}_q [x_{t+1}]$  and  $\tilde{E}_q [\pi_{t+1}]$ , are not unrational. The difference to conventional rational expectations (RE) is that no single agent is required to expect future dynamics rationally. It has been pointed out by a number of authors, that forming conventional RE would indeed be impossibly complicate.<sup>20</sup> It would require every agent to know how everybody else would react in every possible situation and to calculate the resulting mean time paths in advance. It is implausible that real world human beings are capable of solving such highly complex problems. In our model, agents choose from a set of forecasting rules that are so simple that real world human beings would be able to employ them. Using such simple rules is not unrational, it can be understood as the best way to deal with an overwhelmingly complex world. An evolutionary mechanism is used to permanently evaluate these strategies and sort out the poorly performing in favor of the better ones.

---

<sup>20</sup> Consult Ackerman (2002), Gaffeo et al. (2008), Fair (2009) and Kirman (2010).

Hence, instead of requiring rationality from the individuals (as conventional rational expectations do), it is the result of an evolutionary dynamic market process.

## 2.3 Bringing the Two Sectors Together

As already mentioned, the two parts of the model run on different time scales. The real markets operate quarterly while the financial market operates daily. We assume that one quarter consists of 64 trading days. Therefore, the financial sector performs 64 increments of the time index  $t$  within one increment of the real market's time index  $q$  (figure 1). Quarter  $q$  is defined to contain the days  $64(q - 1) + 1, \dots, 64q$ .

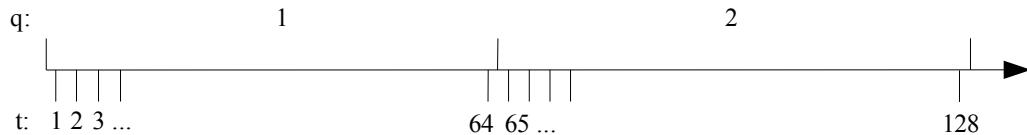


Figure 1: Time scale as indexed by days ( $t$ ) and quarters ( $q$ )

We assume that the relevant value of the quarterly stock price  $s_q$  that affects the real sector via eq. (13) is the average of the daily realizations of  $s_t$  of the corresponding quarter  $q$ . Thus  $s_q$  is given by:<sup>21</sup>

$$s_q = \frac{1}{64} \sum_{t=64(q-1)+1}^{64q} s_t \quad (24)$$

Using the definitions above, we calculate the recursive dynamics of the financial market for one quarter  $q$  (in days:  $t = (q - 1) \cdot 64 + 1, \dots, q \cdot 64$ ) with the agent-based model defined in section 2.1, and insert the mean of the resulting  $s_t$ 's into eq. (13) in order to get the impact on real sector variables.

Now that we have set up the real and financial markets we can define the difference between the true fundamental stock price ( $\bar{s}^f$ ) and the fundamentalist's perception of it ( $s_t^f$ ). The fundamental

<sup>21</sup>Eq. (24) assumes that the influence of daily stock prices on the real economy is equal for each day in the quarter. One could instead also introduce a discounting factor into (24) to raise the relative influence of the more recent days. We show in the online appendix that our results depend only marginally on the decision to use such a discounting factor.



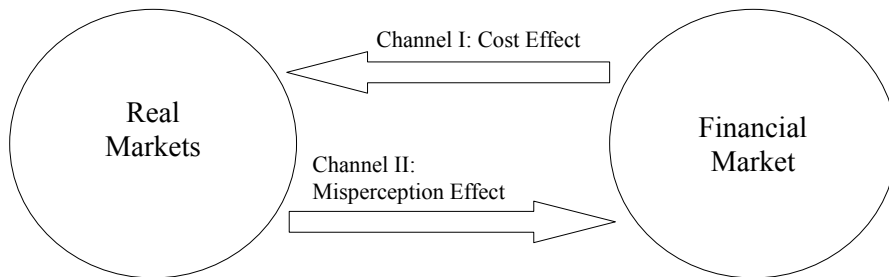


Figure 2: Channels between real and financial markets

value of any given stock is commonly understood to be the sum of all discounted future dividend payments  $d_{t+k}$ . In the most simple case it could be given by something similar to:<sup>22</sup>

$$s_t^f = \sum_{k=1}^{\infty} \rho^k \mathbf{E}_t [d_{t+k}] \quad (25)$$

Dividends are typically closely related to real economic conditions ( $x_q$  in our model). Therefore,  $s_t^f$  would depend on the expectation of  $x$  for all future days. We decided to model the perception of the fundamental value in a different way for two reasons: First, it has been empirically found that stock markets overreact to new information, i.e. stock prices show stronger reactions to new information than they should, given that agents behave rationally.<sup>23</sup> Second, it has been argued that in reality it is very difficult (if not impossible) to identify the *true* fundamental value of any stock.<sup>24</sup> Given these problems, it seems reasonable to assume that agents do not know the true value of  $\bar{s}^f$  or calculate it in a rational way (as in eq. (25)), but instead simply take the current development of the real economy as a proxy for it.

$$s_t^f = h \cdot x_q \quad q = \text{floor} \left( \frac{t-1}{64} \right), \quad h \geq 0 \quad (26)$$

The floor-function rounds a real number down to the next integer. Eq. (26) states that the fundamentalists' perception  $s_t^f$  is biased in the direction of the most recent real economic activity, i.e. if output is high (low) the fundamental stock price is perceived to lie above (below) its true counterpart. Note that ABC models of financial markets can typically not relate the fundamental value to the recent economic development, since the latter is not modeled endogenously. Most models do

<sup>22</sup> Consult Campbell et al. (1997) chapter 7 for the derivation of this equation and more general versions.

<sup>23</sup> De Bondt and Thaler (1985) were among the first to describe this phenomenon.

<sup>24</sup> For example Rudebusch (2005) or Bernanke and Gertler (1999) raise doubts of this kind.

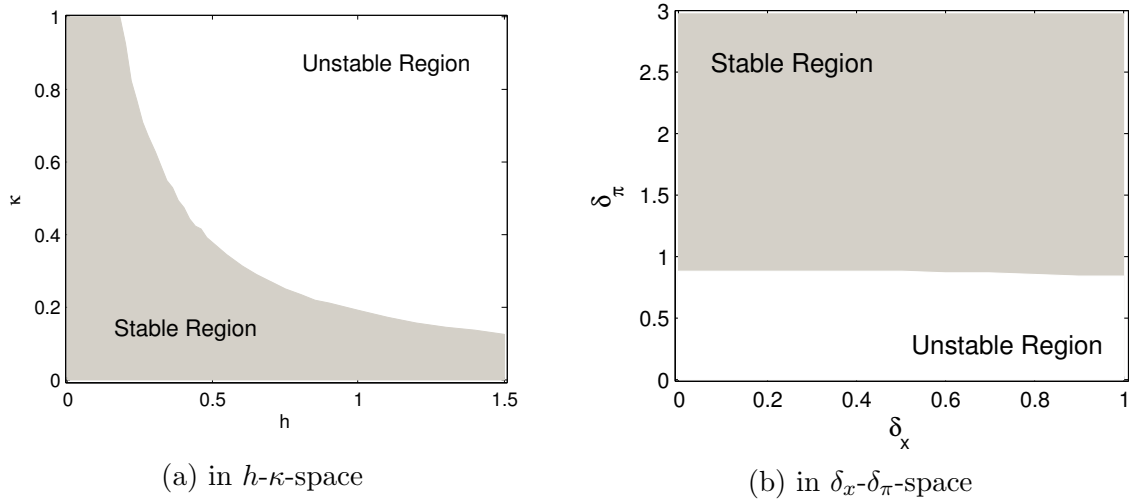


Figure 3: Stability Analysis<sup>26</sup>

not distinguish between  $s_t^f$  and  $\bar{s}^f$ , they set both equal to zero or assume them to follow a random walk.<sup>25</sup> Figure 2 illustrates the two channels that exist between the real and the financial market. Channel I (the cost channel) allows the financial market to influence the real sector and disappears if  $\kappa$  in eq. (11) is set equal to  $\kappa = 0$ . Channel II (the misperception of  $\bar{s}^f$  channel) allows for influence in the opposite direction, and disappears if  $h$  in eq. (25) is set equal to  $h = 0$ . If both of these cross-sectoral parameters are set equal to zero ( $\kappa = 0$  &  $h = 0$ ), both sectors (i.e. both submodels) operate independently of each other.

The two cross-sectoral channels feed on each other. If stock prices are high, Channel I exerts a positive influence on output: Solvency of firms rises which lowers their credit costs. Marginal costs and thus inflation fall. As a result, output rises, which in turn exerts a positive influence on stock prices through Channel II, and so on. To exclude explosive paths,  $\kappa$  has to be lower the higher  $h$  and vice versa. Figure 3(a) shows a numerical approximation of the stability region in  $h$ - $\kappa$ -space. It is known that the policy parameters  $\delta_\pi$  and  $\delta_x$  are crucial for the stability of the NKM model. Under standard specification, a sufficient condition for stability is  $\delta_\pi > 1$  and  $\delta_x \geq 0$ . To check whether our behavioral model possesses a similar property, we generate a numerical approximation of the stability region in  $\delta_x$ - $\delta_\pi$ -space (figure 3(b)). The system is stable for  $\delta_\pi > 1$  and  $\delta_x \geq 0$  and thus features the typical stability properties of NKM models.

<sup>25</sup> Again, Westerhoff (2008) is a good example to look at since both of these approaches are discussed there.

<sup>26</sup> The parameterization used for this numerical investigation is discussed in detail below.

### 3 Numerical Simulations

The analysis of our model is performed by means of numerical simulation. The calibration is given in Table 1. The parameter values for the financial sector are exactly the same as in Westerhoff (2008). The values of  $\sigma$ ,  $\gamma$  and  $\beta$  are so-called "deep parameters" (or functions of such) and common in New Keynesian models. For the policy parameters  $\delta_x$  and  $\delta_\pi$  we use the values that have originally been suggested by Taylor (1993). The hybridity parameters  $\chi$  and  $\psi$  are set to 0.8. These parameters have to be set larger than 0.5 in order to maintain the endogenous business cycles of the De Grauwe (2010a) model.<sup>27</sup> In order to set the cross-sectoral parameters, we assume that the real sector is much less influenced by the financial sector than the other way round.<sup>28</sup> Therefore we set  $h$  to be ten times larger than  $\kappa$ .

Table 1: Baseline Calibration of the Model

Financial sector	Real sector	Interaction
$a = 1$	$\sigma = 1$	$\kappa = 0.1$
$K^C = 0.04$	$\gamma = 0.17166$	$h = 1$
$K^F = 0.04$	$\beta = 0.99$	
$\ell = 1$	$\delta_x = 0.5$	
$d = 0.975$	$\delta_\pi = 1.5$	
$e = 300$	$\zeta = 0.5$	
$\sigma^s = 0.01$	$\phi = 10$	
$\sigma^C = 0.05$	$\chi = 0.8$	
$\sigma^F = 0.01$	$\psi = 0.8$	
	$\sigma_\epsilon = 0.15$	

The memory parameter for the financial sector  $d = 0.975$  (which is taken from Westerhoff (2008)) is much higher than that for the real sector  $\zeta = 0.5$  (taken from De Grauwe (2010a)). However, when comparing these two values, it has to be taken into account that  $d$  refers to the discounting of information that is only one day old. Bringing it to a quarterly basis we obtain  $d^{64} = 0.2$ . Therefore our calibration presumes that past information are less taken into account by financial market agents than by real sector agents. The *intensity of choice* parameters ( $e$  and  $\phi$ ) seem most problematic

<sup>27</sup> One can easily see that if (9) is inserted into (10), the two terms containing  $\tilde{E}_q[x_{q+1}]$  cancel out if  $\chi = 0.5$ . Whether agents are optimistic or pessimistic does not play any role for the determination of  $x_q$  any more. The self-fulfilling character of expectations (explained in detail below) will ultimately break down.

<sup>28</sup> We have assumed that the value of  $x$  is taken as information for the development of the real sector. If stock prices *overreact* to new information (De Bondt and Thaler (1985), Nam et al. (2001), Becker et al. (2007)), this implies a strong reaction of  $s$  to  $x$  and thus a high value of  $h$ .

since their impact on the results depends on the latent attractiveness values. We will therefore check the robustness of our obtained result to different parameterization of  $e$  and  $\phi$  in section 5.

### 3.1 Dynamics of one Simulation

To demonstrate the working of our model, we perform one “representative” run. The simulated time period consists of 100 quarters (or 6400 days). To eliminate the influence of arbitrary initial conditions each simulation is performed with a ”burn-in” phase of 20 quarters. Figure 4 shows the resulting dynamics for  $x_q$ ,  $\pi_q$ ,  $\omega^{\text{opt}}$ ,  $\omega^{\text{tar}}$ ,  $i_q$ ,  $\left(i_q - \tilde{E}_q[\pi_{q+1}]\right)$ ,  $s_t$ , and a variable called *market sentiments*. The latter represents the fraction of agents, employing the three trading strategies. Black denotes chartist trading ( $W^C$ ), gray fundamentalist trading ( $W^F$ ), and white no trading ( $W^0$ ). To generate the dynamics, a series of pseudo random numbers has to be drawn. Hence each realization of simulated data is a unique result of the underlying random seed. The horizontal time axes are quarterly scaled. In the diagrams containing daily data, quarters cover an interval of 64 data points.

The output gap is characterized by cyclical ups and downs. Hence the model generates an endogenous business cycle. Closely connected to the up and down phases of output is the fraction of optimists ( $\omega^{\text{opt}}$ ). This result obviously follows from the specification of our learning mechanism. In times of high output, the forecasting rule of optimists (14) performs much better than the pessimists rule (15). This results in a larger fraction of optimists and thus in a market expectation above the steady state of zero ( $\tilde{E}_q[x_{q+1}] > 0$ ). Since the resulting high output, again, favors optimist forecasting, the situation has a tendency to reproduce itself. Vice versa for phases dominated by pessimists. Hence, expectations about the output gap have a self-fulfilling nature and generate business cycles.

A similar pattern can be observed for expectation formation of the inflation rate. However, the self-fulfilling tendency is not as strong as for the output gap. If inflation largely deviates from the target (e.g. around  $q = 35$ ), the *targeter’s* expectation rule performs poorly relative to that of the *extrapolators*. As a result, the fraction of *targeters* ( $\omega^{\text{tar}}$ ) decreases and inflation can become very low. In contrast to the expectations of the output gap, the mechanism is not strong enough to generate a self-fulfilling, cyclical regime switching.

The stock market is also characterized by the emergence of different regimes. Most of the time,

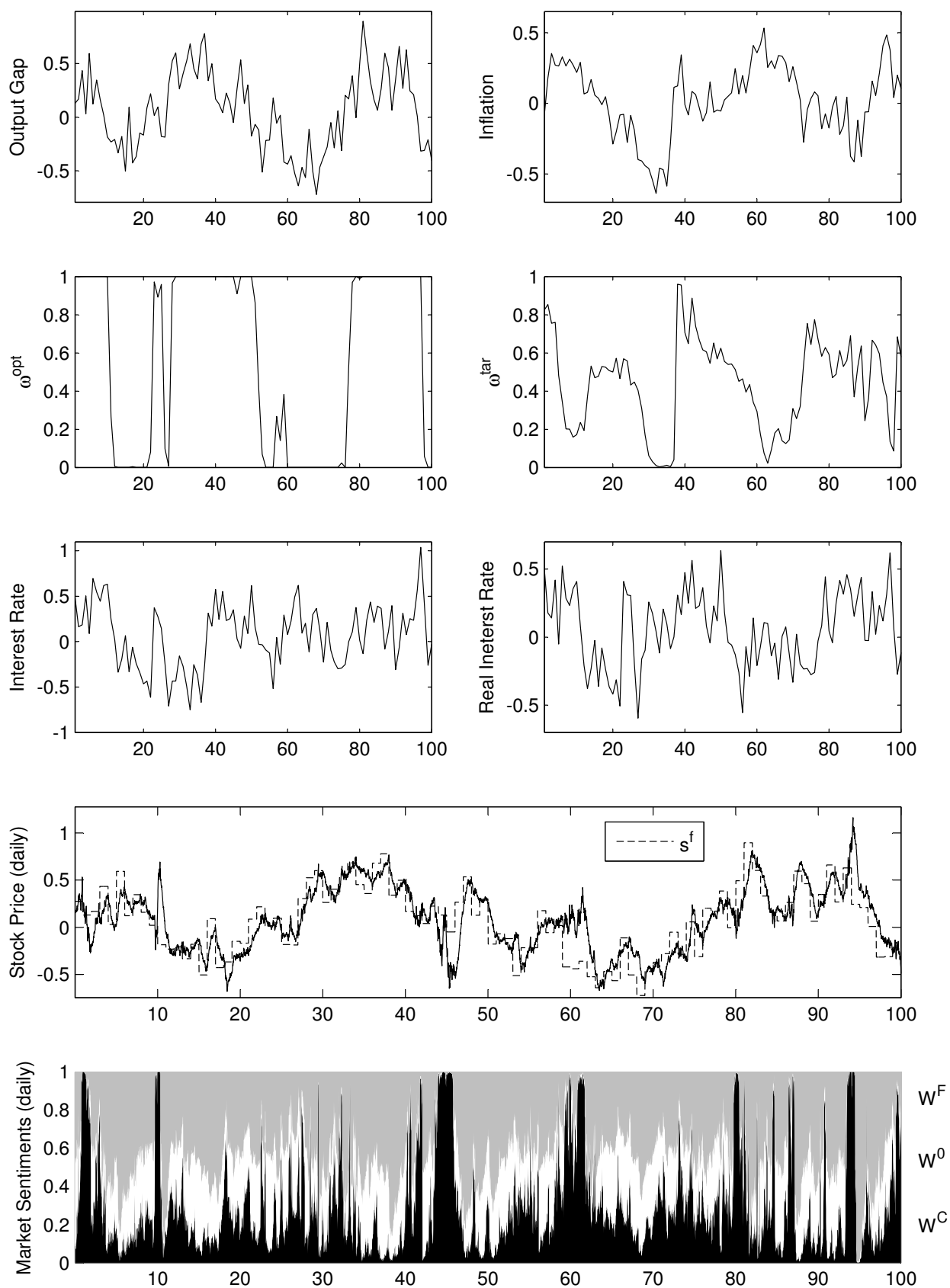


Figure 4: Model Output for a Time Period of  $Q = 40$

when a certain amount of fundamentalists is present in the market, the stock price follows its perceived fundamental value (which is given by the dashed line) closely. Deviations from  $s^f$  during such phases are transitory and small in size. The course of the stock price is thus largely influenced by the underlying real economy:  $s_t$  is high during the booms ( $q = 30, \dots, 40$  and  $q = 80, \dots, 95$ ) and low during recessions ( $q = 10, \dots, 20$  and  $q = 59, \dots, 70$ ). If the market sentiments change in favor of the chartists, stock prices become disconnected from the underlying fundamentals. Around  $q = 60$ , for example, chartists form the dominating majority and the stock price moves away from  $s^f$ . It continues to follow a slight upward trend, although the underlying value falls, i.e. a bubble builds up. In  $q = 62$  market sentiments turn around, fundamentalists, who judge  $s_t$  as extremely over-valued, become dominating and drive the price down again, i.e. the bubble bursts. The opposite case of chartists driving  $s_t$  down below the fundamental value can be found for example around  $q = 45$ .

The model generates endogenous waves of optimism and pessimism, inflation-targeting and inflation-extrapolation as well as chartism and fundamentalism. Each forecasting strategy is able to dominate the market from time to time, but the evolutionary learning assures that none dominates forever. The result is an endogenously occurring business cycle and endogenous stock price bubbles.

### 3.2 Impulse Response Analysis

In this subsection we analyze the effects of an exogenous shock to the real sector. In DSGE models, such questions are typically analyzed via impulse response functions that try to isolate the effects of an exogenous realization of the stochastic terms  $\epsilon_q^i$ ,  $\epsilon_q^x$  and  $\epsilon_q^\pi$ . We focus on the impact of an unanticipated, transitory cost shock without persistence of size  $\epsilon_5^{\pi,+} = 1$ . In order to allow for impulse response analysis in a way similar to that typically used in DSGE models, we perform the following experiment:

1. Generate the model dynamics for one particular random seed.
2. Generate the same dynamics with the same random seed (i.e. identical realizations of the pseudo random numbers), but with  $\epsilon_5^\pi$  increased by  $\epsilon_5^{\pi,+} = 1$ .
3. Calculate the differences between the trajectories of step 1 and 2 which gives the isolated impact of the cost shock. Note that the noise terms are identical in both runs. Differences are thus not a result of different random numbers, but solely due to the imposed shock.
4. Repeat steps 1-3 10,000 times.

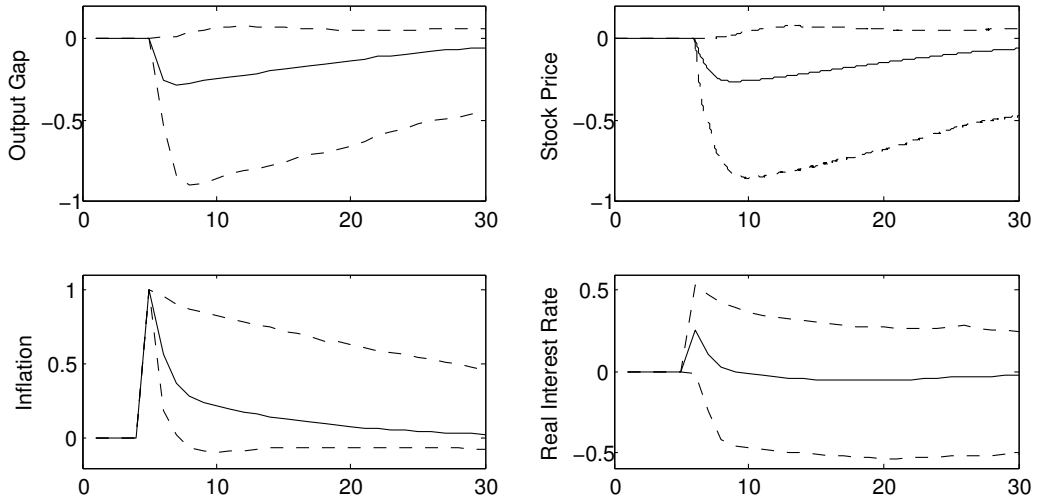


Figure 5: Mean response to a exogenous cost shock of size one. Dashed lines are 95 % quantiles.

Figure 5 shows the resulting responses to an exogenous shock of  $\epsilon_5^{\pi,+} = 1$  for our baseline calibration. The solid lines illustrate mean responses, while the dashed lines represent 95% quantiles. On average, the economy shows the typical stagflationary response to the cost shock. Inflation and the real interest rate rise, while output and the stock price fall. All impulse responses show high volatility. The quantiles for the output gap, for example, illustrate that the reaction of  $x_q$  can be located anywhere between (a) a strong negative reaction on impact that is accelerated during the subsequent two periods and followed by a hump-shaped path back towards trend and (b) no reaction on impact followed by a slightly positive path in the medium and long run. The other time series exhibit similarly volatile impulse responses. The only exception is (by construction of the shock) the reaction of inflation on impact.

To analyze the source of this high volatility, we generate the impulse response functions of  $\pi$  and  $s$  that result if either optimists or pessimists are dominating the market during the shock period. We define a situation in which  $\omega_q^{\text{opt}} \geq 0.75$  as dominated by optimists and a situation in which  $\omega_q^{\text{pes}} \geq 0.75$  as dominated by pessimists. The top row in figure 6 shows the effect on inflation and stock prices. The reaction of stock prices is stronger when optimists dominate. If pessimists form the majority, the amplitude is smaller for both time series. The economic logic underlying this phenomenon is the following. If the number of optimists is high, the economy is caught in a self-reproducing circle of high output and optimistic expectations. A huge contractionary shock can break this circle and thus turn the boom into a recession. If pessimists dominate, such an amplification mechanism can

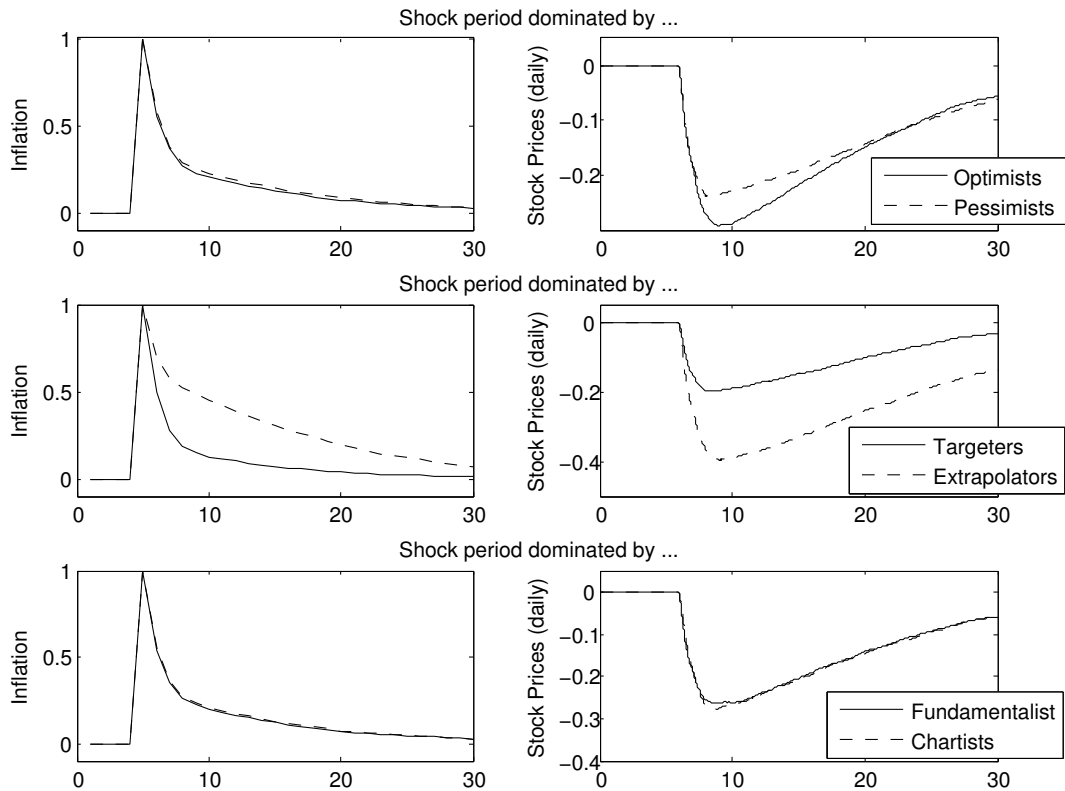


Figure 6: Mean response of output and stock price with initial conditions dominated by different groups of agents

not emerge and the fall in GDP is much smaller. The same contractionary shock therefore has a higher mean impact during a boom than during a recession.

Comparing impulse responses for those cases when targeters dominate with those dominated by extrapolators,<sup>29</sup> yields a similar picture (second row in figure 6). If the latter are dominating, the impact of the shock is stronger on impact and more persistent for both time series. The economic rationale is that extrapolators do not generate a mean-reversion. Since they expect the current state of inflation to persist, they create persistence in the system. Persistence, in turn, makes their own forecasting strategy more attractive and the number of extrapolators might increase further. Therefore, the higher persistence in this case is also a result of self-fulfilling expectations. If targeters dominate (i.e. the inflation target of the central bank is credible), the persistence of the shock is much lower. Strong believes in a stable system obviously lead to a dampening of shocks. Whether fundamentalists or chartists dominate the stock market does not play a major role in the transmission of cost shocks.

<sup>29</sup> Dominance has been defined analogously to the above case.



The high volatility that has been found in the impulse responses of figure 5 is therefore partly a result of the history dependence. Since the shock can have a different impact on average depending on the initial beliefs of agents, the uncertainty about the impact of the shock increases. De Grauwe (2010a) has also analyzed the origin of persistence in his behavioral NKM and finds that responses maintain persistent even if the hybrid character is turned off. This point is of interest, because persistence has been a matter of concern in NKM modeling. In its baseline notation (for  $\chi = 1$  and  $\psi = 1$  in (10)-(11)) those models do not produce persistent responses to non-persistent shocks. De Grauwe argues that the evolutionary learning algorithm produces *endogenous* persistence, while in standard NKM models persistence is introduced *exogenously* by assuming a hybrid form. Now that we have gained some understanding of the dynamics of the model, we can use it to analyze a prevailing question currently debated among policy makers.

## 4 Taxing Financial Transactions

The recent financial crisis has created enormous costs in all industrialized economies. First, it produced a huge decline in GDP and rise in unemployment. Second, the financial positions of states have been very negatively affected because of several necessary stabilization policies like capital injections, purchase of assets, fiscal stimuli, direct support and many more. On average, the advanced G20 countries suffered a rise in government debt by 40%.<sup>30</sup> Because of such rising debt, several countries directly stumbled from the financial- into the fiscal crisis. As a response to those devastating externalities of the financial sector, it should be asked, first, whether new regulatory policies are needed to stabilize this sector for the future and, second, whether it should provide a financial contribution to the recently generated costs. The traditional way of achieving both would be to levy a financial transaction tax (FTT) in the spirit of Tobin (1978). Such a tax would make short term trading less attractive, while having no significant influence on long term trading. Since high frequency, speculative trading is a socially wasteful business, it would be beneficial to curtail it by introducing a FTT. Long run oriented trading that is based on underlying fundamental values, is not affected.<sup>31</sup>

In approaching this question, the G20 leaders have recently asked the IMF to prepare a report

---

<sup>30</sup> Consult IMF (2010).

<sup>31</sup> A nice executive summary of arguments in favor and against a FTT can be found in Schulmeister et al. (2008).

on how the "financial sector could make a fair and substantial contribution" in bearing parts of the induced burden.<sup>32</sup> Summarizing the results of this report, the IMF argues that taxing financial transaction is generally a feasible policy instrument for achieving this goal and that "the FTT should not be dismissed on grounds of administrative practicality" (p. 19). However it also argues that the traditional FTT might not be the best instrument to "finance a resolution mechanism" and "focus on core sources of financial instability". Another slightly different type of tax – called financial activities tax (FAT) – might be better suited than the FTT.

While policy makers are currently intensively debating the introduction of such taxes,<sup>33</sup> one striking aspect of the debate is "that it is almost entirely unguided by the public finance literature on the topic – because there is hardly any"<sup>34</sup>. In this section, we use our model to analyze the advantages and disadvantages of both kinds of taxes with regard to their ability to stabilize markets and to raise fiscal income. To evaluate their effect on the variables of interest, we report the average fractions of fundamentalists  $\frac{1}{T} \sum_{t=1}^T W_t^F$  and chartists  $\frac{1}{T} \sum_{t=1}^T W_t^C$  resulting from a certain tax. We also report the average tax revenue per agent and day that is given as:

$$\text{Tax Revenue} = \frac{1}{T} \sum_{t=1}^T (W_t^F \cdot \text{Tax}_t^F + W_t^C \cdot \text{Tax}_t^C) \quad (27)$$

Where  $\text{Tax}^F$  is the tax paid by fundamentalists and  $\text{Tax}^C$  the tax paid by chartists. Additionally we define the following two measures:<sup>35</sup>

$$\text{vol}(s) = \frac{1}{T-1} \sum_{t=2}^T |s_{t-1} - s_t| \quad \text{dis}(s) = \frac{1}{T} \sum_{t=1}^T |s_t| \quad (28)$$

And for quarterly time series:

$$\text{vol}(z) = \frac{1}{Q-1} \sum_{q=2}^Q |z_{q-1} - z_q| \quad \text{dis}(z) = \frac{1}{Q} \sum_{q=1}^Q |z_q| \quad z = \{x, \pi\} \quad (29)$$

The measure  $\text{vol}(\cdot)$  denotes the volatility (i.e. rate of change) of a time series. Accordingly,  $\text{dis}(\cdot)$  measures its distortion (i.e. difference to fundamental steady state). We do not use the variance

<sup>32</sup>The mentioned report is IMF (2010).

<sup>33</sup>Besides the IMF and the G20, the European Commission and the European Parliament are currently examining whether a financial tax should be introduced. See for example EU (2010), EU (2011a) or EU (2011b).

<sup>34</sup>Quote taken from Keen (2011).

<sup>35</sup>Both measures closely follow Westerhoff (2008).

measure because it interprets volatility via the average squared distance from the mean. Our time series show long-lasting deviations from the mean (which we interpret as bubbles or distortion). When calculating the variance, one would not measure the volatility but rather the mean squared distortion. To avoid confusion we do not use the variance measure.

The introduction of a FTT has already often been analyzed in the ABC finance literature. Examples are Westerhoff (2003), Westerhoff (2008) or Demary (2008). These studies, however, limit their attention to the reduction of volatility and distortion of stock prices. Our study adds several aspects to this literature: (1) We do not restrict our analysis to the stabilization of financial markets but include real markets as well. (2) We contrast the classical FTT with the innovative, very recently proposed FAT. (3) We also answer a question that has become very prevailing during the recent fiscal crisis: how should a tax be designed in order to yield maximal tax revenues? There has also been a number of empirical studies that investigate the impact of FTTs.<sup>36</sup> These studies, however, focus on short-term volatility and neglect long-term mispricing. We take the latter into account, since it can lead to the built up and bursting of bubbles and therefore might have the most important impact on the real economy.

#### 4.1 Financial Transaction Tax

The basic characteristics of the FTT is that it is small in size but levied on a brought basis: the total value of transaction. To introduce it into our model, we assume that the tax has to be paid relative to the nominal value traded. Since complete investment consists of two transactions, the tax also has to be paid twice. Orders generated in  $D_{t-2}$  imply nominal transactions of  $D_{t-2} \cdot \exp\{s_{t-1}\}$  in  $t-1$  and  $D_{t-2} \cdot \exp\{s_t\}$  in  $t$ . The tax rate  $\tau$  is applied to the absolute nominal value of both transactions (i.e. buys and sells are equally taxed). Since tax payments directly reduce the profitability of an investment, eq. (7) changes to:<sup>37</sup>

$$A_t^i = (\exp\{s_t\} - \exp\{s_{t-1}\}) D_{t-2}^i - \tau (\exp\{s_t\} + \exp\{s_{t-1}\}) |D_{t-2}^i| + dA_{t-1}^i \quad (30)$$

The transaction tax is represented by  $\tau$  and  $|D_{t-2}^i|$  is the absolute value of  $D_{t-2}^i$ . We run the model for 500 quarters (32,000 days) with different values for  $\tau$  as well as 1000 different realizations of the

<sup>36</sup> See IMF (2010) p. 20 for a summary of those studies.

<sup>37</sup> Consult also Westerhoff (2008) for the introduction of an FTT into his model.

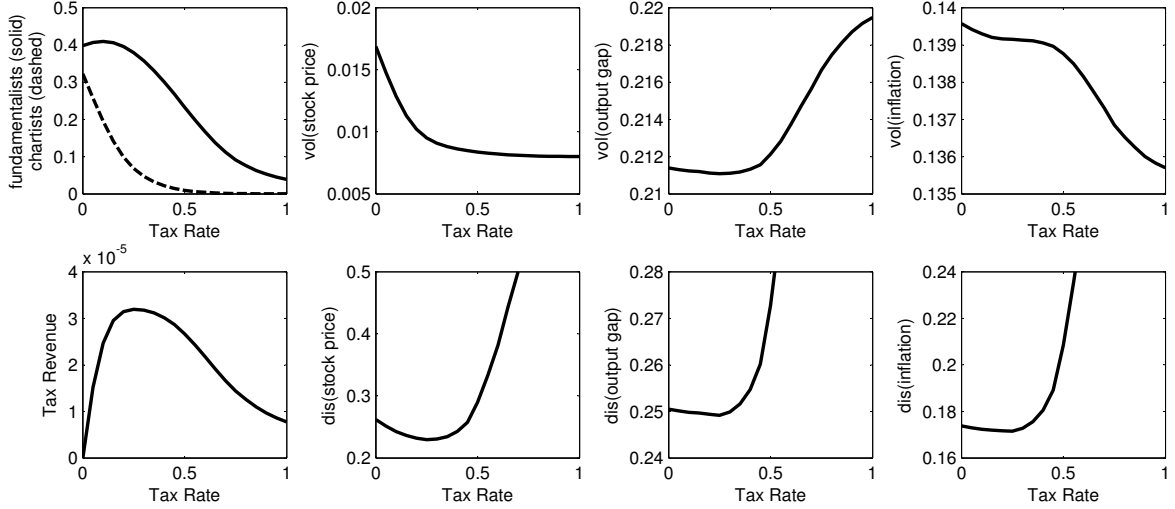


Figure 7: Impact of Financial Transaction Tax (FTT)

pseudo random number generator for each  $\tau$ . Figure 7 shows the average fraction of chartists and fundamentalists, the gained tax revenue as well as volatility and distortion of  $s$ ,  $x$  and  $\pi$  with respect to the imposed FTT.

Increasing the tax rate (starting from zero), leads to a sharp decline in the fraction of chartist traders which approximately equals zero for  $\tau \geq 0.6\%$ . At the same time it slightly increases the number of fundamentalist traders up to  $\tau \approx 0.1\%$  and decreases it gradually for higher tax rates. Tax revenue follows a typical Laffer curve: Increasing the tax rate up to  $\tau \approx 0.23\%$  leads to rising tax revenue. But increasing the tax rate further crowds too many agents out of the market and thus leads to a falling tax income.

Concerning stability, we evaluate the FTT by how well it is capable of reducing volatility and distortion of  $s$ ,  $x$  and  $\pi$ . With respect to  $\text{vol}(s)$  and  $\text{vol}(\pi)$ , the FTT has an exclusively positive influence on stability. Increasing the FTT leads to monotonically decreasing volatility of stock prices and inflation. The measures  $\text{vol}(x)$ ,  $\text{dis}(s)$ ,  $\text{dis}(x)$  and  $\text{dis}(\pi)$  recommend a different conclusion. All four follow a u-shaped pattern with minimum near  $\tau = 0.3\%$ . Therefore, with respect to these variables, the FTT has an ambiguous impact. It stabilizes the market for small tax rates. If it becomes too large ( $\tau > 0.3\%$ ) the market is destabilized and the values of  $\text{dis}(s)$ ,  $\text{dis}(x)$  and  $\text{dis}(\pi)$  become quickly very large.

## 4.2 Financial Activities Tax

A FAT, as proposed in the report of the IMF (2010) is "levied on the sum of profits and remuneration of financial institutions" (p. 21). Several detailed examples of how such a FAT could look like can be found in the report (p. 66-70). In our model, we have to use a more stylized version of course. Since, we do not consider labor costs in the financial sector, all gains from stock trading  $(\exp\{s_t\} - \exp\{s_{t-1}\}) D_{t-2}^i$  are profits. Thus we introduce a FAT into our model by taxing profits a constant rate of  $\tau$ . If we assume further that the FAT only applies if profits are positive, tax payments are given by:

$$\mathbf{1}_{\{\mathbb{R}^+\}} [(\exp\{s_t\} - \exp\{s_{t-1}\}) D_{t-2}^i] \cdot \tau \cdot (\exp\{s_t\} - \exp\{s_{t-1}\}) D_{t-2}^i$$

Where  $\mathbf{1}_{\{\mathbb{R}^+\}} [y]$  is the indicator function that becomes 1 if  $y \in \mathbb{R}^+$  and zero otherwise. Equation (7) changes to:

$$A_t^i = (\exp\{s_t\} - \exp\{s_{t-1}\}) D_{t-2}^i + dA_{t-1}^i \quad (31)$$

$$- \mathbf{1}_{\{\mathbb{R}^+\}} [(\exp\{s_t\} - \exp\{s_{t-1}\}) D_{t-2}^i] \cdot \tau \cdot (\exp\{s_t\} - \exp\{s_{t-1}\}) D_{t-2}^i \quad (32)$$

We perform the same experiment that we used to analyze the impact of the FTT. Results are illustrated in figure 8. The rate of a FAT has to be much higher than that of a FTT because its base is much smaller. In our analysis we account for tax rates between 0% and 50%. To allow for a better comparability of the results, scaling of the ordinates is carried over from figure 7.

The tax slowly decreases the number of chartist while keeping the number of fundamentalists almost constant. All other measures follow a monotonic path. Tax revenues rise while volatility and distortion of  $s$ ,  $x$  and  $\pi$  fall. The effect of the FAT is thus clearly positive. Increasing it leads to an improvement of all of our measures. An optimal value of  $\tau$  can not be identified. Therefore the question for the FAT's size is not a question of economic optimality, but mainly one of political feasibility.

For comparing both taxes with each other, we assume that the FTT is set in the range of optimal values at  $\tau = 0.28\%$ . This rate leads to low market volatility and distortion at a high revenue (section 4.1). For the FAT we assume different values between 5% and 50%.<sup>38</sup> The impact on all

<sup>38</sup>For illustration purpose, IMF (2010) [p. 22, 69] and EU (2010) [p. 6] assume a rate of 5%.

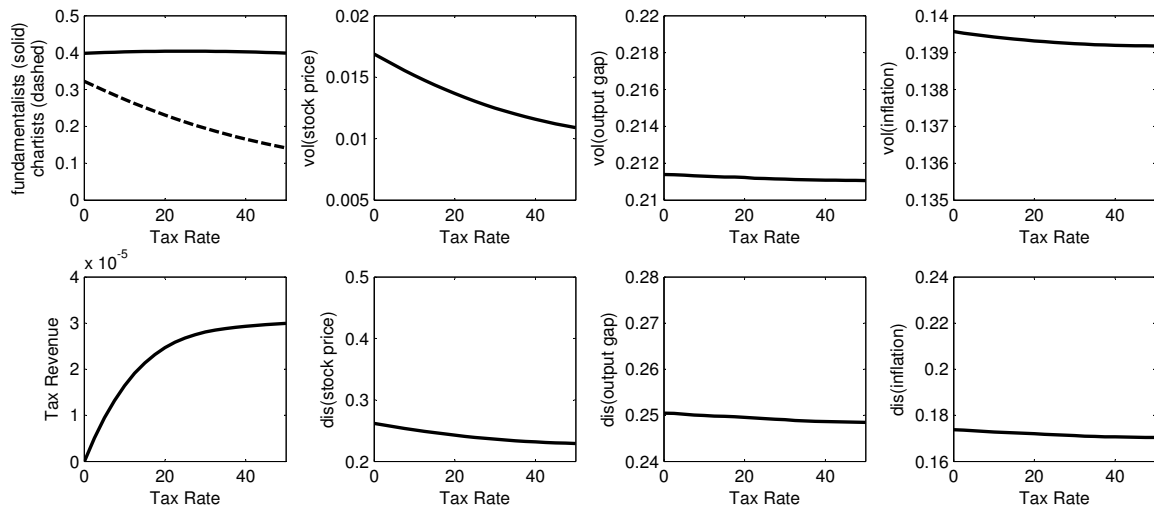


Figure 8: Impact of Financial Activities Tax (FAT)

measures of interest are contrasted in table 2. For all values, the FAT leads to a higher fraction of fundamentalists. All other measures are in favor of the FTT. The tax revenue is larger or at least equally good for the FTT while all undesirable characteristics ( $W^C$ ,  $dis(\cdot)$  and  $vol(\cdot)$ ) are smaller. Note however, that a large fraction of fundamentalists is not an end in itself, it is only a useful means to stabilize the market. Therefore, it is only a measure of second order compared to values of distortion and volatility. Since the FTT leads to better (or at least equally good) results in all other respect, we can conclude that the FTT strictly dominates the FAT.

A point in favor of the FAT is that there is no danger of setting the rate too high. If the optimal rate for the FTT is missed and a larger rate is set (0.8% for example), huge distortions might occur (figure 7). The FAT does not suffer from such a problem (figure 8) because of its monotonic impact.

The IMF also considers another variant of the FAT in its report and proposes: "*taxing high returns more heavily than low*" (p. 68). Adding such an element of progressivity, should discourage risk-taking. To test the scope of such a progressive FAT (called FAT3 in the IMF report), we perform the above experiment a third time. Instead of assuming a flat tax as in equation (31) we let the tax rate grow with profits. The first step in defining such a tax, is to identify a threshold value of profits. Profits above this threshold are defined '*excess profits*' and thus taxed higher. Let

Table 2: Comparison of FTT with FAT

	FTT	FAT				
	0.28%	5%	10%	20%	35%	50%
$W^C$	0.054	0.297	0.273	0.231	0.179	0.142
$W^F$	0.367	<b>0.400<sup>a</sup></b>	<b>0.402</b>	<b>0.404</b>	<b>0.403</b>	<b>0.399</b>
Revenue	$3.2 \cdot 10^{-5}$	$0.9 \cdot 10^{-5}$	$1.6 \cdot 10^{-5}$	$2.5 \cdot 10^{-5}$	$2.9 \cdot 10^{-5}$	$3.0 \cdot 10^{-5}$
dis( $s$ )	0.228	0.256	0.251	0.243	0.234	0.229
dis( $x$ )	0.249	0.250	0.250	0.250	0.249	0.249
dis( $\pi$ )	0.171	0.174	0.173	0.173	0.172	0.171
vol( $s$ )	0.009	0.016	0.015	0.014	0.012	0.011
vol( $x$ )	0.211	0.212	0.212	0.212	0.212	0.211
vol( $\pi$ )	0.139	0.140	0.139	0.139	0.139	0.139

<sup>a</sup> Bold numbers indicate that the FAT leads to better results than the FTT

$P_t = (\exp\{s_t\} - \exp\{s_{t-1}\}) D_{t-2}^i$  denote profits. We define the benchmark  $P^*$  to be the standard deviation of profits:

$$P^* := \text{std}(P_t) \quad (33)$$

A tax rate that is quadratically growing in  $P$  can be defined as:

$$\text{FAT rate} = \mathbf{1}_{\{\mathbb{R}^+\}} [P] \cdot \tau \cdot \left(\frac{P}{P^*}\right)^2 \quad (34)$$

Such a tax definition has the following nice properties. If profits equal  $P^*$  (the benchmark value), they are taxed by a rate of  $\tau$  (figure 9). If they are above  $P^*$  (excess profits) they are taxed by a higher rate that grows quadratically in  $P$ .<sup>39</sup> For profits below the threshold, the tax falls and smoothly approaches zero at  $P = 0$ . The tax rate is thus progressive and not subject to any steps. It also allows us to perform experiments similar to the ones above. By increasing (decreasing)  $\tau$ , we increase (decrease) the FAT for all (positive) values of  $P$  in the same direction while preserving the general shape.

Figure 10 compares the results for the progressive FAT (bold line) with the flat tax scenario of the

<sup>39</sup>For very high profits eq. (34) might result in tax rates above 100%. Since such rates are unrealistic, we restrict the tax to values  $\leq 100\%$ .

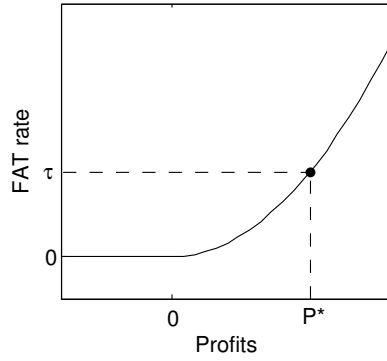


Figure 9: Rate of the progressive FAT

previous experiment (thin line). Comparing both FAT's leads to ambitious results. The progressive FAT gives rise to a better stabilization for rates up to  $\tau \approx 30\%$ : The number of fundamentalists is higher while the number of chartists is lower compared to the flat FAT. At the same time, all volatility and distortion measures are lower. Only for high tax rates of 30% or more the results turn around partly:  $\text{vol}(s)$ ,  $\text{dis}(s)$ ,  $\text{vol}(x)$ ,  $\text{dis}(x)$  and  $\text{dis}(\pi)$  are advantages under a flat tax. The revenue is larger under a progressive FAT between 0% and 11%. For tax rates above 11% the flat FAT yields more income. The nice property of monotonically decaying volatility and distortion that has already been found for the flat FAT is also found for the progressive one.

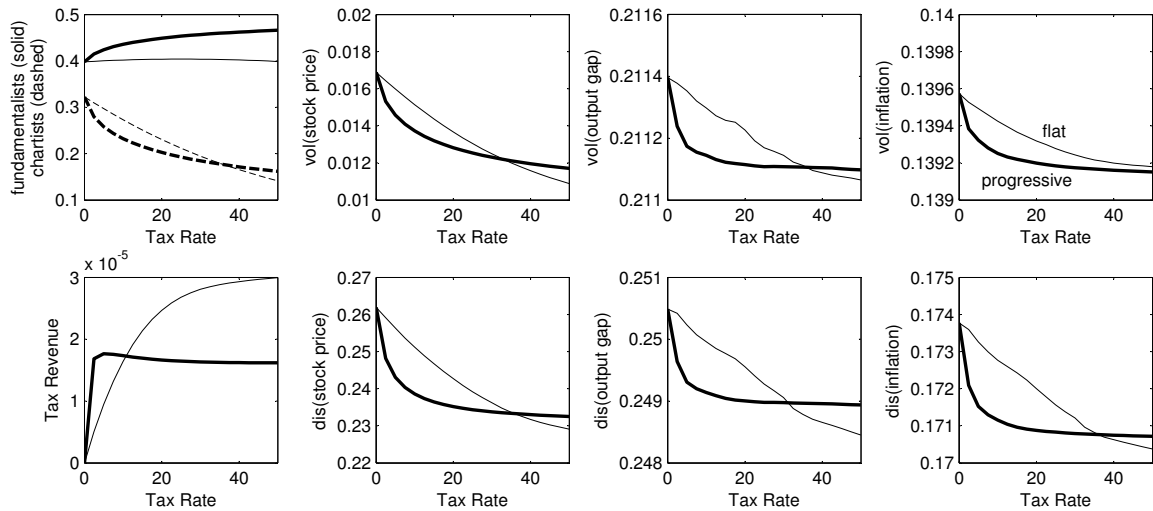


Figure 10: Impact of *progressive* Financial Activities Tax (FAT)

The reason for this result can easily be explained by taking a look at the distribution of profits. Figure 11 shows the distribution of fundamentalist's profits (solid line) and chartist's profits (dashed



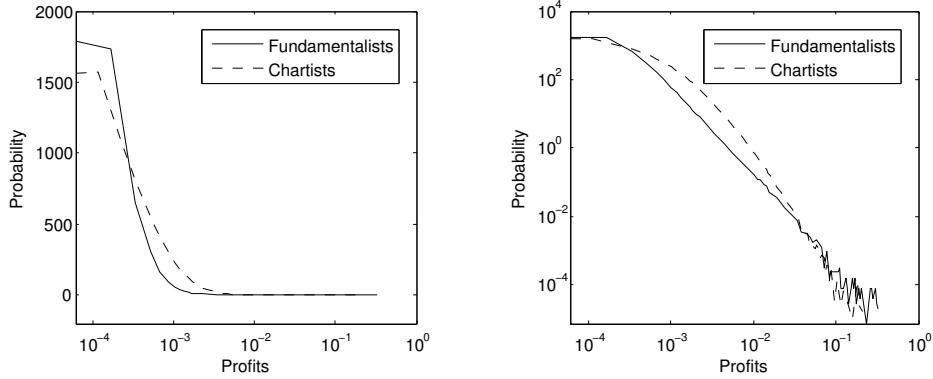


Figure 11: Distribution of Profits in a semi-log (left) and log-log scaled plot (right).

line), both, in a semi-log scaled plot and a log-log scaled plot. Both lines show the characteristic fat tails.<sup>40</sup> At the same time, the distribution of fundamentalist's profits has more density located at low values while chartists have more density located at higher values. In the tail (i.e. for very high profits) both distributions are approximately equal. Fundamentalists (who earn lower profits on average) are thus favored by the progressive FAT, while chartists (who earn higher profits on average) are disadvantaged. As a result, the fraction of fundamentalists increases while that of chartists decreases compared to a flat FAT (figure 10, upper left panel). A more stable economy is the result.

Finally, we compare the progressive FAT at different rates with the FTT of 0.28%. Results are given in table 3. For all values of the tax, the fraction of fundamentalists is higher than under the FTT while the values of  $\text{vol}(\pi)$  are equal throughout all simulations. If we leave the fractions of fundamentalists and chartists aside, the FTT again strictly dominates the FAT. We can therefore conclude that the classical FTT is better suited than the new FAT in order to achieve stabilization of markets and raise funds from the financial sector.

<sup>40</sup> Consult Lux (2009) on the empirical properties of financial data.

Table 3: Comparison of FTT with progressive FAT

	FTT	FAT (progressive)				
	0.28%	5%	10%	20%	35%	50%
$W^C$	0.054	0.258	0.233	0.203	0.178	0.162
$W^F$	0.367	<b>0.424<sup>a</sup></b>	<b>0.436</b>	<b>0.449</b>	<b>0.460</b>	<b>0.466</b>
Revenue	$3.2 \cdot 10^{-5}$	$1.8 \cdot 10^{-5}$	$1.7 \cdot 10^{-5}$	$1.7 \cdot 10^{-5}$	$1.6 \cdot 10^{-5}$	$1.6 \cdot 10^{-5}$
dis( $s$ )	0.228	0.243	0.239	0.235	0.234	0.233
dis( $x$ )	0.249	0.250	0.249	0.249	0.249	0.249
dis( $\pi$ )	0.171	0.172	0.172	0.172	0.172	0.171
vol( $s$ )	0.009	0.015	0.014	0.013	0.012	0.012
vol( $x$ )	0.211	0.212	0.212	0.212	0.212	0.212
vol( $\pi$ )	0.139	0.139	0.139	0.139	0.139	0.139

<sup>a</sup> Bold numbers indicate that the FAT leads to better results than the FTT

## 5 Robustness Checks

In this section, we check the robustness of our result with respect to some assumptions that we had to make throughout our analysis. Since our simulations suggests that the FTT is the best way of taxing the financial sector, we focus on the robustness of our derived optimal FTT.

### 5.1 Parameterization

As mentioned in section 3, calibration of the intensity of choice parameters  $\phi$  and  $e$  is probably the most problematic one. Our first robustness checks are therefore concerned with these parameters.

Table 4 and 5 show how some important results change with the variation of  $\phi$  and  $e$ .

Table 4: Robustness check of intensity of choice parameter  $\phi$ 

	$\phi = 5$	$\phi = 8$	$\phi = 10$	$\phi = 12$	$\phi = 15$
$\min_{\tau} \text{vol}(x)$	0.205	0.209	0.211	0.212	0.214
$\arg \min_{\tau} \text{vol}(x)$	0.30%	0.28%	0.28%	0.28%	0.25%
$\min_{\tau} \text{dis}(x)$	0.226	0.241	0.249	0.255	0.262
$\arg \min_{\tau} \text{dis}(x)$	0.30%	0.29%	0.28%	0.28%	0.27%
$\min_{\tau} \text{Revenue}$	$3.2 \cdot 10^{-5}$	$3.2 \cdot 10^{-5}$	$3.2 \cdot 10^{-5}$	$3.2 \cdot 10^{-5}$	$3.2 \cdot 10^{-5}$
$\arg \min_{\tau} \text{Revenue}$	0.26%	0.26%	0.26%	0.26%	0.26%

Our results are fairly robust against different values of the parameter  $\phi$  (table 4). The optimal tax rate as well as the induced optimal values of  $\text{vol}(x)$ ,  $\text{dis}(x)$  and Revenue change only slightly. The opposite holds for the parameter  $e$ . The optimal tax rate diverges strongly between 0.18% and 0.54% for different values of  $e$  (table 5). The induced Tax revenues are also subject to high uncertainty.

This robustness test suggests, that reliable estimations of the *intensity of choice* parameter for the financial sector are of major importance for reliable policy suggestions. We have shown in section 4.1 that a too high FTT can result in huge distortion of the real economy. The negative effects of a too low FTT will instead be much smaller. A good strategy might thus be to set the FTT significantly below the value that is optimal with respect to a given parameterization in order to deal with the uncertainty in an appropriately careful way.

Table 5: Robustness check of intensity of choice parameter  $e$

	$e = 100$	$e = 200$	$e = 300$	$e = 400$	$e = 500$
$\min_{\tau} \text{vol}(x)$	0.211	0.211	0.211	0.211	0.211
$\arg \min_{\tau} \text{vol}(x)$	0.54%	0.36%	0.28%	0.25%	0.18%
$\min_{\tau} \text{dis}(x)$	0.248	0.248	0.249	0.250	0.251
$\arg \min_{\tau} \text{dis}(x)$	0.80%	0.41%	0.28%	0.10%	0.12%
$\min_{\tau} \text{Revenue}$	$8.4 \cdot 10^{-5}$	$4.4 \cdot 10^{-5}$	$3.2 \cdot 10^{-5}$	$2.7 \cdot 10^{-5}$	$2.3 \cdot 10^{-5}$
$\arg \min_{\tau} \text{Revenue}$	0.81%	0.41%	0.26%	0.18%	0.16%

## 5.2 Taylor Rule

In this section we test the robustness against different specifications of the Taylor rule. We begin by introducing an interest smoothing parameter into (9). The current interest rate is thus not only concerned with reducing inflation and the output gap but also with producing a smooth path of  $i_q$  over time. Variation A of the policy rule is thus given by:

$$i_q = \lambda \left( \delta_{\pi} \tilde{\mathbb{E}}_q [\pi_{q+1}] + \delta_x \tilde{\mathbb{E}}_q [x_{q+1}] \right) + (1 - \lambda) i_{q-1} + \epsilon_q^i \quad (35)$$

As a second variation, we assume that the board of the central bank is composed of a set of heterogeneous agents who form expectations differently. Some are optimistic about future output and some are pessimistic. The expectations and fractions of optimistic and pessimistic central

bankers are the same as for the other agents. Thus, they are given by  $\tilde{E}_q^{\text{opt}} [x_{q+1}]$ ,  $\tilde{E}_q^{\text{pes}} [x_{q+1}]$ ,  $\omega_q^{\text{opt}}$  and  $\omega_q^{\text{pes}}$  as defined in section 2.2. Assume that the majority of central bankers can fully introduce their expectations into the policy rule. The expectation that enters the Taylor rule is then given by:

$$\hat{E}_q [x_{q+1}] = \begin{cases} \tilde{E}_q^{\text{opt}} [x_{q+1}] & \text{if } \omega_q^{\text{opt}} > \omega_q^{\text{pes}} \\ \tilde{E}_q^{\text{pes}} [x_{q+1}] & \text{otherwise} \end{cases} \quad (36)$$

In other words, the central bank's decision structure is such that it become fully optimistic if optimists dominate and fully pessimistic if pessimists dominate. Variation B of the Taylor rule is given by:

$$i_q = \delta_\pi \hat{E}_q [\pi_{q+1}] + \delta_x \hat{E}_q [x_{q+1}] + \epsilon_q^i \quad (37)$$

Where  $\hat{E}_q [\pi_{q+1}]$  is formed analogous to  $\hat{E}_q [x_{q+1}]$ .

Table 6: Robustness check of different versions of the Taylor rule

Taylor Rule	Baseline specification	Variation A ( $\lambda = 0.75$ )	Variation B
$\min_\tau \text{vol}(x)$	0.211	0.203	0.228
$\arg \min_\tau \text{vol}(x)$	0.28%	0.24%	0.20%
$\min_\tau \text{dis}(x)$	0.249	0.278	0.272
$\arg \min_\tau \text{dis}(x)$	0.28%	0.24%	0.29%
$\min_\tau \text{Revenue}$	$3.2 \cdot 10^{-5}$	$3.2 \cdot 10^{-5}$	$3.3 \cdot 10^{-5}$
$\arg \min_\tau \text{Revenue}$	0.26%	0.26%	0.27%

Table 6 compares our result for the different specifications of the Taylor rule. The parameter  $\lambda$  in Variation A is set to  $\lambda = 0.75$ .<sup>41</sup> The results show that the measures of stability and tax revenue are only subject to minor change. The optimal tax rate that minimizes  $\text{vol}(x)$  undergoes the largest change. It falls by 0.04% in case of variation A and 0.08% for variation B. This uncertainty again suggests that the rate of the FTT should be set to lower rates compared to those that are optimal given our baseline specification.

<sup>41</sup>The same analysis for a wider  $\lambda$ -range as well as the impulse response functions resulting from the variation of the Taylor Rules can be found in the online appendix.

## 6 Conclusion

We have developed a model that combines agent-based financial market theory with New Keynesian macroeconomics. The two employed submodels are simple representatives of their respective discipline. They are both subject to an evolutionary process of expectation formation that sorts out the poorly performing strategies in favor of the good ones. Interaction between the two models is brought about by two straightforward channels. Our comprehensive model is very stylized and not yet ready for econometric analysis. But even with this simplistic methodology, we are able to show that the behavioral structure of our model has a strong impact on the transmission of shocks. The market sentiments in the shock period, for example, can lead to a very different average transmissions.

We also used the model to analyze a question that is currently debated among policy makers. Namely, if the introduction of a tax on financial transactions can bring about positive developments for the overall economy. We find that such a tax could generally reduce volatility and distortion of the real and financial market variables, but that its size and type plays an important role. If the tax is of the FTT type, it is very efficient in bringing down volatility and raising tax revenue, but if set too high, the macroeconomy might also be subject to very strong distortion. The FAT is less able to stabilize the market and also generates less revenue for the state. But in contrast to the FTT, it does not create large distortions when set too high. We have shown that the optimal decision of making the FAT flat or progressive is depending on the tax rate. For values below 11% the progressive version is the best choice, while for rates above 40% the flat tax version is preferable. In between, the progressive tax leads to better stabilization while the flat tax generates more revenue.

Our model is stylized and simple to implement. Of course, it can also be used for numerous augmentations: (1) The effects of different cross-sectoral channels (e.g. Tobin's  $q$  or stock wealth effect) can be analyzed. (2) The rules that define the behavior of the financial market agents (like the time horizon of investors' strategies) can be changed. (3) Since the occurrence of bubbles implies large deviations from the fundamental steady state, one might also use a version of the NKM submodel that is not log-linearized. (4) Moreover, we do not take financial streams between the real and the financial sector explicitly into account. A simplification that might be relaxed in future research.

We see this paper as an early stage in a broader research agenda. The agent-based method offers enormous new possibilities for macroeconomics. Our research agenda is targeted at further

exploration of these possibilities. The paper at hand tries to bridge the gap between this newly emerging field<sup>42</sup> and mainstream macro. It uses new methods (like interaction and evolutionary learning), but also builds on traditional methods and assumption (like market equilibrium or utility maximization in the NKM part). Future research will focus on working out the agent-based part further.

## References

- Ackerman, F. (2002). Still Dead After All These Years: Interpreting the Failure of General Equilibrium Theory, *Journal of Economic Methodology* **9**(2): p. 119–139.
- Aoki, M. and Yoshikawa, H. (2007). *Reconstructing Macroeconomics: A Perspective from Statistical Physics and Combinatorial Stochastic Processes*, Cambridge University Press.
- Bask, M. (2009). Monetary Policy, Stock Price Misalignments and Macroeconomic Instability, *Hanken School of Economics, Working Papers* **540**.
- Becker, M., Friedmann, R. and Klößner, S. (2007). Intraday Overreaction of Stock Prices, *Saarland University, Working Papers*.
- Bernanke, B. and Gertler, M. (1999). Monetary Policy and Asset Price Volatility, *Economic Review* **4**: 17–51.
- Bernanke, B. S., Gertler, M. and Gilchrist, S. (1999). *The Financial Accelerator in a Quantitative Business Cycle Framework*, In: *Handbook of Macroeconomics, Volume I*, Edited by J. B. Taylor and M. Woodford, chapter 21, pp. 1341–1393.
- Brock, W. A. and Hommes, C. H. (1998). Heterogeneous Beliefs and Routes to Chaos in a Simple Asset Pricing Model, *Journal of Economic Dynamics and Control* **22**: 1235–1274.
- Caginalp, G., Porter, D. and Smith, V. (2001). Financial Bubbles: Excess Cash, Momentum and Incomplete Information, *Journal of Behavioral Finance* **2**: 80–99.
- Campbell, J. Y., Lo, A. W. and MacKinlay, A. C. (1997). *The Econometrics of Financial Markets*, Princeton University Press.
- Castelnuovo, E. and Nistico, S. (2010). Stock Market Conditions and Monetary Policy in a DSGE Model, *Journal of Economic Dynamics and Control* **34**: 1700–1731.

---

<sup>42</sup> See for example Gaffeo et al. (2008) or Dosi et al. (2008).

- Colander, D., Föllmer, H., Haas, A., Goldberg, M., Juselius, K., Kirman, A., Lux, T. and Sloth, B. (2009). The Financial Crisis and the Systemic Failure of the Economics Profession, *Critical Review* **21**(2-3): p. 249–267.
- Colander, D., Howitt, P., Kirman, A., Leijonhufvud, A. and Mehrling, P. (2008). Beyond DSGE Models: Toward an Empirically Based Macroeconomics, *American Economic Review* **98**: 236–240.
- Dawid, H. and Neugart, M. (forthcoming). Agent-Based Models for Economic Policy Design, *Eastern Economic Journal*.
- De Bondt, W. F. M. and Thaler, R. (1985). Does the Stock Market Overreact?, *The Journal of Finance* **40**: 793–805.
- De Grauwe, P. (2010a). Animal Spirits and Monetary Policy, *Economic Theory*.
- De Grauwe, P. (2010b). The scientific foundation of dynamic stochastic general equilibrium (dsge) models, *Public Choice* **144**: 413–443.
- De Grauwe, P. and Grimaldi, M. (2006). Exchange Rate Puzzles: A Tale of Switching Attractors, *European Economic Review* **50**: 1–33.
- Deissenberg, C., van der Hoog, S. and Dawid, H. (2008). EURACE: A Massively Parallel Agent-Based Model of the European Economy, *Applied Mathematics and Computation* **204**: 541–552.
- Delli Gatti, D., Gaffeo, E. and Gallegati, M. (2010). Complex Agent-Based Macroeconomics: A Manifesto for a New Paradigm, *Journal of Economic Interaction and Coordination*.
- Demary, M. (2008). Who Does a Currency Transaction Tax Harm More: Short-Term Speculators or Long-Term Investors?, *Jahrbücher für Nationalökonomie und Statistik (Journal of Economics and Statistics)* **228**: 228–250.
- Demary, M. (2010). Transaction Taxes and Traders with Heterogeneous Investment Horizons in an Agent-Based Financial Market Model, *Economics: The Open-Access, Open-Assessment E-Journal* **4**.
- Dosi, G., Fagiolo, G. and Roventini, A. (2008). The Microfoundations of Business Cycles: An Evolutionary, Multi-Agent Model, *Journal of Evolutionary Economics* **18**: 413–432.
- EU (2010). Financial Sector Taxation, *European Commission Services: Working Document SEC(2010)1166*.
- EU (2011a). Investing Today for Growth Tomorrow, *Press Releases* **IP/11/799**.

- EU (2011b). Multiannual Financial Framework (MFF): Questions and Answers, *Press Releases MEMO/11/468*.
- Fair, R. C. (2009). Has Macro Progressed?, *Cowles Foundation Discussion Papers* **1728**.
- Franke, R. and Sacht, S. (2010). Some Observations in the High-Frequency Versions of a Standard New-Keynesian Model, *Christian-Albrechts-Universität Kiel, Economic Working Papers* **1**.
- Frankel, J. A. and Froot, K. A. (1987). Using Survey Data to Test Standard Propositions Regarding Exchange Rate Expectations, *The American Economic Review* **77**: 133–153.
- Gaffeo, E., Delli Gatti, D., Desiderio, S. and Gallegati, M. (2008). Adaptive Microfoundations for Emergent Macroeconomics, *Eastern Economic Journal* **34**: 441–463.
- Hommes, C. H. (2006). *Heterogeneous Agent Models in Economics and Finance*, In: Handbook of Computational Economics, Volume 2, Edited by Leigh Tesfatsion and Kenneth L. Judd., chapter 23, pp. 1009–1186.
- Hommes, C., Sonnemans, J., Tuinstra, J. and van de Velden, H. (2005). Coordination of Expectations in Asset Pricing Experiments, *Review of Financial Studies* **18**: 955–980.
- IMF (2010). A Fair and Substantial Contribution by the Financial Sector, *Staff of the International Monetary Fund: Final Report for the G-20* .
- Ito, T. (1990). Foreign Exchange Rate Expectations: Micro Survey Data, *The American Economic Review* **80**: 434–449.
- Keen, M. (2011). Rethinking the Taxation of the Financial Sector, *CESifo Economic Studies* **57**.
- Kirman, A. (1993). Ants, Rationality and Recruitment, *The Quarterly Journal of Economics* **108**: 137–156.
- Kirman, A. (2010). The Economic Crisis is a Crisis for Economic Theory, *Paper presented at the CESifo Economic Studies Conference on “What’s Wrong with Modern Macroeconomics?”*.
- Kontonikas, A. and Ioannidis, C. (2005). Should Monetary Policy Respond to Asset Price Misalignments?, *Economic Modelling* **22**: 1105–1121.
- Kontonikas, A. and Montagnoli, A. (2006). Optimal Monetary Policy and Asset Price Misalignments, *Scottish Journal of Political Economy* **53**: 636–654.
- Krugman, P. (2009). How Did Economists Get it so Wrong?, *New York Times* **Sept 2**.
- Lam, J.-P. (2010). The Importance of Commitment in the New Keynesian Model, *The B.E. Journal of Macroeconomics* **10**.



- LeBaron, B. (2006). *Agent-Based Computational Finance*, In: Handbook of Computational Economics, Volume 2, Edited by Leigh Tesfatsion and Kenneth L. Judd., chapter 24, pp. 1187–1233.
- Lui, Y.-H. and Mole, D. (1998). The Use of Fundamental and Technical Analyses by Foreign Exchange Dealers: Hong Kong Evidence, *Journal of International Money and Finance* **17**: 535–545.
- Lux, T. (2009). *Stochastic Behavioral Asset-Pricing Models and the Stylized Facts*, In: Handbook of Financial Markets: Dynamics and Evolution. Edited by Thorsten Hens and Klaus R. Schenk-Hoppé, chapter 3, pp. 161–215.
- Lux, T. and Marchesi, M. (2000). Volatility Clustering in Financial Markets: A Micro-Simulation of Interactive Agents, *International Journal of Theoretical and Applied Finance* **3**: 675–702.
- Lux, T. and Westerhoff, F. (2009). Economics Crisis, *Nature Physics* **5**.
- Manski, C. and McFadden, D. (1981). *Structural Analysis of Discrete Data with Econometric Applications*, MIT Press, Cambridge.
- Matheson, T. (2011). Taxing Financial Transactions: Issues and Evidence, *IMF Working Paper* **54**.
- Milani, F. (2008). Learning About the Interdependence Between the Macroeconomy and the Stock Market, *University of California-Irvine, Department of Economics Working Papers* **070819**.
- Nam, K., Pyun, C. S. and Avard, S. L. (2001). Asymmetric Reverting Behavior of Short-Horizon Stock Returns: An Evidence of Stock Market Overreaction, *Journal of Banking & Finance* **25**: 807–824.
- Proaño, C. R. (2011). Exchange Rate Determination, Macroeconomic Dynamics and Stability under Heterogeneous Behavioral FX Expectations, *Journal of Economic Behavior & Organization* **77**: 177–188.
- Ravenna, F. and Walsh, C. E. (2006). Optimal Monetary Policy with the Cost Channel, *Journal of Monetary Economics* **53**: 199–216.
- Rudebusch, G. D. (2005). Monetary Policy and Asset Price Bubbles, *Federal Reserve Bank of San Francisco Economic Letter* **5**.
- Samanidou, E., Zschischang, E., Stauffer, D. and Lux, T. (2006). Microscopic Models for Financial Markets, *Christian-Albrechts-Universität Kiel, Economic Working Papers* **15**.
- Schulmeister, S., Schratzenstaller, M. and Picek, O. (2008). A General Financial Transaction Tax: Motives, Revenues, Feasibility and Effects, *Österreichisches Institut für Wirtschaftsforschung Working Paper*.

- Sonnemans, J., Hommes, C., Tuinstra, J. and van de Velden, H. (2004). The Instability of a Heterogeneous Cobweb Economy: A Strategy Experiment on Expectation Formation, *Journal of Economic Behavior and Organization* **54**: 453–481.
- Taylor, J. B. (1993). Discretion versus Policy Rules in Practice, *Carnegie-Rochester Conference Series on Public Policy* **39**: 195–214.
- Taylor, M. P. and Allen, H. (1992). The Use of Technical Analysis in the Foreign Exchange Market, *Journal of International Money and Finance* **11**: 304–314.
- Tobin, J. (1978). A Proposal for International Monetary Reform, *Eastern Economic Journal* **4**: 153–159.
- Westerhoff, F. (2003). Heterogeneous Traders and the Tobin Tax, *Journal of Evolutionary Economics* **13**: 53–70.
- Westerhoff, F. (2008). The Use of Agent-Based Financial Market Models to Test the Effectiveness of Regulatory Policies, *Jahrbücher für Nationalökonomie und Statistik (Journal of Economics and Statistics)* **228**: 195–227.
- Westerhoff, F. H. and Dieci, R. (2006). The Effectiveness of Keynes-Tobin Transaction Taxes When Heterogeneous Agents Can Trade in Different Markets: A Behavioral Finance Approach, *Journal of Economic Dynamics and Control* **30**: 293–322.

# Online Appendix

## Agent-Based Financial Markets and New Keynesian Macroeconomics – A Synthesis –

**Matthias Lengnick\***

*University of Kiel*

*University of Mannheim*

**Hans-Werner Wohltmann\*\***

*University of Kiel*

September 13, 2011

---

\* *E-mail:* [matthias.lengnick@vwl.uni-mannheim.de](mailto:matthias.lengnick@vwl.uni-mannheim.de)

\*\* *E-mail:* [wohltmann@economics.uni-kiel.de](mailto:wohltmann@economics.uni-kiel.de)

# 1 Robustness to Different Definitions of Weights

For integrating the real- and financial sector, we have assumed that all daily realizations of  $s_t$  enter the quarterly value  $s_q$  with equal weights. An alternative would be to assume geometrically decaying weights. Equation (24) becomes:

$$s_q = \sum_{t=64(q-1)+1}^{64q} \text{weight}_t s_t \quad (1)$$

$$\text{with: } \text{weight}_t = (1 - \rho) \cdot \rho^{64q-t} \quad 0 < \rho < 1 \quad (2)$$

High values for  $\rho$  lead to slowly decaying weights, low values result in quickly decaying weights. Geometric weights of this form add up to one for very long time periods. For a smaller period of 64 days they do not. We thus rescale our weights by multiplying with a constant that keeps the relative weight between different  $s_t$  constant but yields

$$\sum_{t=64(q-1)+1}^{64q} \text{weight}_t = 1 \quad (3)$$

Table 1: Robustness check of weighting assumption in equation (24)

Weights	equal	geometrically decaying			
		$\rho = 0.99$	$\rho = 0.9$	$\rho = 0.75$	$\rho = 0.5$
$\min_{\tau} \text{vol}(x)$	0.211	0.211	0.211	0.211	0.211
$\arg \min_{\tau} \text{vol}(x)$	0.28%	0.28%	0.28%	0.28%	0.28%
$\min_{\tau} \text{dis}(x)$	0.249	0.249	0.250	0.251	0.251
$\arg \min_{\tau} \text{dis}(x)$	0.28%	0.28%	0.28%	0.28%	0.28%
$\min_{\tau} \text{Revenue}$	$3.2 \cdot 10^{-5}$	$3.2 \cdot 10^{-5}$	$3.2 \cdot 10^{-5}$	$3.2 \cdot 10^{-5}$	$3.2 \cdot 10^{-5}$
$\arg \min_{\tau} \text{Revenue}$	0.26%	0.26%	0.26%	0.26%	0.26%

Table 1 shows that our results are very robust against the assumption of different weights in (24). The minimal distortion of the output gap changes marginally. All other results stay constant.

## 1.1 Taylor Rule

Variation A of the Taylor rule has been defined as:

$$i_q = \lambda \left( \delta_\pi \hat{E}_q [\pi_{q+1}] + \delta_x \hat{E}_q [x_{q+1}] \right) + (1 - \lambda) i_{q-1} + \epsilon_q^i \quad (4)$$

Table 2 shows the results of a robustness check for different values of  $\lambda$ . Moving away from our baseline calibration ( $\lambda = 1.0$ ) monotonically increases distortion of  $x$  in the optimum. This result obviously follows from the fact that the central bank is less concerned with stabilizing output and more with smoothing the interest rate. The volatility of  $x$ , instead, falls until  $\lambda = 0.75$  and rises afterwards. A moderate weight on interest smoothing therefore reduces volatility of output while a high weight leads to an increase. The reason for this result is the following. The Taylor rule depends on expectations about the future values of  $x$  and  $\pi$ . Since these are formed in a non-rational way, they are a potential source of volatility. Reducing the weight of these expectations therefore leads to a decline of  $\text{vol}(x)$ . If  $\lambda$  becomes too small, the influence of the stabilizing role of monetary policy declines by so much that  $\text{vol}(x)$  rises again.

Table 2: Robustness check of the interest smoothing parameter  $\lambda$

	$\lambda = 1.0$	$\lambda = 0.9$	$\lambda = 0.75$	$\lambda = 0.5$	$\lambda = 0.3$
$\min_\tau \text{vol}(x)$	0.211	0.206	0.203	0.212	0.235
$\arg \min_\tau \text{vol}(x)$	0.28%	0.27%	0.24%	0.22%	0.14%
$\min_\tau \text{dis}(x)$	0.249	0.257	0.278	0.355	0.543
$\arg \min_\tau \text{dis}(x)$	0.28%	0.27%	0.24%	0.11%	0.08%
$\min_\tau \text{Revenue}$	$3.2 \cdot 10^{-5}$	$3.2 \cdot 10^{-5}$	$3.2 \cdot 10^{-5}$	$3.3 \cdot 10^{-5}$	$3.5 \cdot 10^{-5}$
$\arg \min_\tau \text{Revenue}$	0.26%	0.26%	0.26%	0.25%	0.18%

Concerning the tax rates, we find again that an optimal value might lie significantly below the values that have been found to be optimal in our baseline calibration. Figures 1 and 2 show the impulse responses under our alternative Taylor rules.

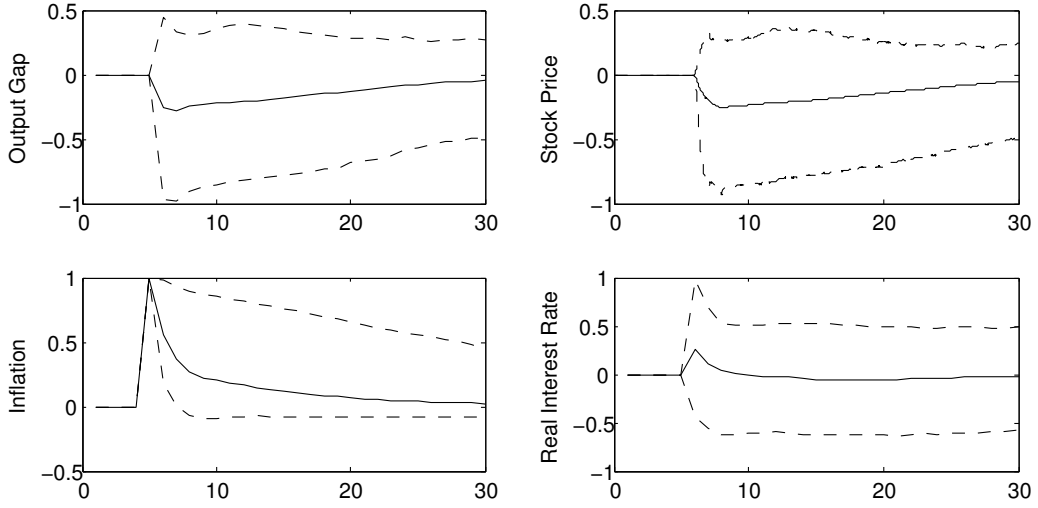


Figure 1: Cost shock for  $i_q = \delta_\pi \hat{E}_q [\pi_{q+1}] + \delta_x \hat{E}_q [x_{q+1}] + \epsilon_q^i$

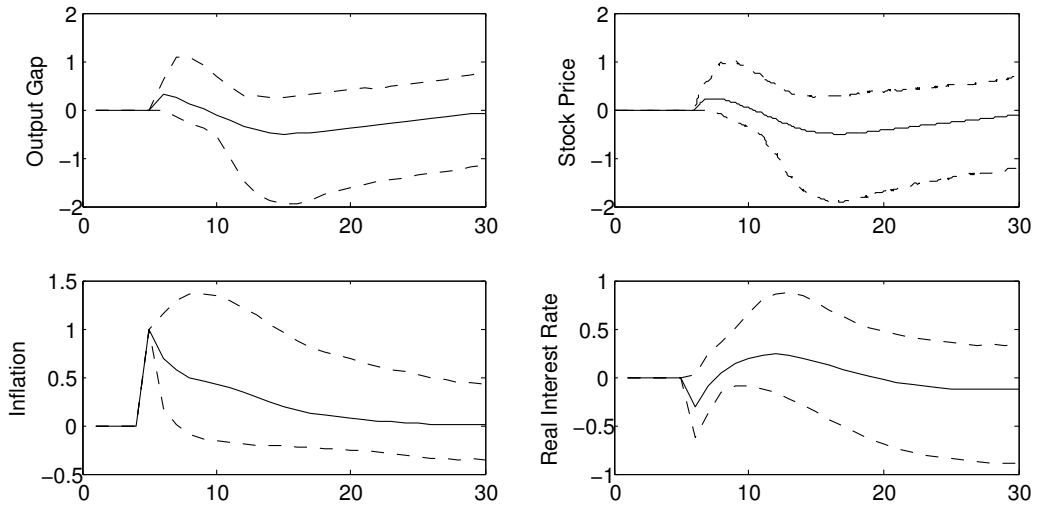


Figure 2: Cost shock for  $i_q = \lambda \left( \delta_\pi \hat{E}_q [\pi_{q+1}] + \delta_x \hat{E}_q [x_{q+1}] \right) + (1 - \lambda) i_{q-1} + \epsilon_q^i$  with  $\lambda = 0.3$