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Banks, Markets, and Financial Stability

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Banks, Markets, and Financial Stability

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September 30, 2012

Abstract

In a theoretical model of the Diamond-Dybvig style, in which deposit-taking banks and financial markets coexist, bank behavior is analyzed taking into account a positive ex-ante probability of a future financial crisis. We focus on the role of the interaction of market liquidity and banks funding liquidity in the propagation of shocks in the financial system. Our findings suggest that in particular bank-dominated financial systems are prone to contagious bank runs due to asset price deteriorations as a consequence of fire sales of assets in financial markets. Nevertheless, banks only prefer holding liquidity buffers to weather future crises if the ex-ante crisis probability exceeds a certain threshold. Moreover, central bank interventions are shown to have de-stabilizing effects because they reduce banks incentives to hold liquidity buffers. This, in turn, may be interpreted as a justification for prudential regulation in terms of minimum liquidity buffer requirements.

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1 Introduction

Liquidity arises endogenously from the behavior of actors in the financial system. This is, in particular, relevant for banks which increasingly engaged in financial market transactions to complement and facilitate their traditional lending and deposit-taking business over the past ten to fifteen years. From their point of view the endogeneity of liquidity creates liquidity risk which may quickly translate into solvency risk. This appears to be one of the major lessons learned from the financial crisis of the years 2007 to 2009. Consequently, top-ranking politicians as well as banking supervisors called for a fundamental revision of the regulation of financial institutions. The Basel III framework represents an important milestone in this regard. Taking into account the lessons learned from the financial crisis, especially internationally standardized liquidity requirements have been added to the regulatory framework.

The difficulty in revising the regulatory framework, however, became clear bit by bit when banks, supervisors, and researchers went into the details of the crisis events. They found that, from the perspective of financial institutions, liquidity may be best understood as a complex concept comprising several dimensions. In particular, banks' traditional funding liquidity – ie their ability to meet payment obligations when they fall due – was observed to affect and to be affected by liquidity of financial markets – ie the ability to sell any amount of a certain asset in a financial market at any time without significant negative effect on market prices. Moreover, this interdependency may have played a crucial role in the propagation of liquidity shocks among banks. Even banks that have been basically sound faced funding strains due to illiquid asset markets even endangering the solvency of these banks in some cases. The relevant academic literature, however, was found to be incomplete in explaining these interrelationships and failed in answering a number of questions which came up in the process of revising banking regulation.¹

The present paper analyzes the role of the interaction of liquidity risk in asset markets and funding liquidity risk of banks for the propagation of liquidity shocks among banks and the translation of liquidity risk into solvency risk. The major contribution to the fast growing literature addressing the

¹See also the excellent review of Tirole (2010) on what is known and what is not known about liquidity, illiquidity, and associated phenomena just like market freezes, fire sales, contagion.

before-mentioned lessons learned from the 2007-2009 crisis is to consider a strictly positive probability for a future financial crisis in a theoretical model based on Fecht (2004).² Banks as well as households make investment decisions taking into account a strictly positive ex-ante probability for a future run on one of the banks in the financial system under consideration. As a result, optimal decisions may be expected to differ from the standard results of the relevant literature which usually assumes that future crises are not anticipated in the decision-making process.³

Although the assumption is not completely new to the literature⁴, we are – to our best knowledge – the first who consider a positive ex-ante crisis probability in a setting where banks are interconnected via asset markets which also may be directly used by households. Market liquidity, then, arises from the joint effect of household behavior and bank behavior. The model of the present paper, therefore, allows for an detailed analysis off the behavior of banks and households in the face of future financial crises, the propagation of liquidity shock in the financial system due to the interaction of market liquidity and funding liquidity, and the impact on the solvency of basically sound banks.

We find that banks may prefer to hold a liquidity buffer to forearm against future financial crises. When the ex-ante crisis probability is beyond a certain threshold value, banks find it beneficial to hold a liquidity buffer. As long as the ex-ante crisis probability is below this threshold, banks prefer not to hold a liquidity buffer. The threshold value of the ex-ante crisis probability arises endogenously from the fundamentals in our financial system – in particular the return of the long-term investment project and the probability that the households in one region are hit by a liquidity shock – and interacts with the structure of the financial system. This structure may be either bank-dominated or market-oriented and also arises endogenously in the model. Moreover, we observe that if banks decide not to hold liquidity

²Historical data show that severe financial crises occur approximately once in 25 years (cf. BCBS, 2010a, p. 3 and Annex 1).

³For example the papers of Allen and Gale (2000), Allen and Gale (2004a), Fecht (2004), and Caballero and Simsek (2011) analyze bank behavior and the propagation of shocks via financial markets. However, all the papers share the common assumption, that ex ante the probability of a future financial crisis is zero. As a result, decisions to be made in these papers do not take future financial crises into account.

⁴Cooper and Ross (1998) analyze the effect of a strictly positive ex-ante crisis probability in the basic Diamond-Dybvig model.

buffers, a liquidity shock that hits one of the banks leads to contagion of the other bank and the financial market serves as a propagation mechanism. Unfortunately, the before mentioned interaction of fundamentals and financial system structure makes it difficult to derive additional general results on bank behavior and contagion in our model. Therefore, we run simulations to characterize complex interactions between the ex-ante crisis probability, the structure of financial markets, and the fundamentals of the economy to determine whether a single bank prefers to take precautionary measures – ie holds a liquidity buffer – and whether financial markets act as a mechanism for the propagation of shocks in the financial system.

From a policy perspective our results shed some light on the role of mark-to-market accounting in the propagation of shocks in the financial system, on the value of liquidity risk stress tests in banks and the value of liquidity standards as those recently formulated by the BCBS. However, accounting standards, banking regulation and monetary policy actions of central banks are not explicitly taken into account and are, hence, left for future research.

The remainder of the paper is organized as follows: section 2 presents related literature. Section 3 sets up the model which is analyzed in sections 4 through 7. Section 8 concludes.

2 Relationship to the literature

The model of the present paper builds on the seminal papers of Diamond and Dybvig (1983), Jacklin (1987), and Diamond (1997). That is, we consider households that are exposed to liquidity risk and banks that are able to provide liquidity insurance to households. Just like in Allen and Gale (2004b) the financial system of our paper comprises financial intermediaries and financial markets as well. Financial markets allow for trading – and hence liquidating – claims on long-term investment projects before maturity. However, financial markets in our model are more general as they may be used by banks as well as households to exchange liquid funds for claims on illiquid (long-term) investment projects. As a result, we are able to include the aspect of market participation into the analysis of market liquidity.⁵

Based on arguments that were brought forward by Diamond and Rajan (2001) and Diamond and Rajan (2005) the present model extends this general

⁵Huang and Wang (2008) analyze the effect of market participation on market liquidity and asset price formation in financial markets in more detail.

setting in several ways which were first described in Fecht (2004): we consider two identical (geographical) regions with one monopolistic bank in each. The financial market interconnects both regions. Moreover, households can be either of two types. Sophisticated households are – just like banks – able to extract high returns from directly investing into long-term projects. In contrast, naive households are only able to extract a minimum return from holding claims on long-term projects directly. Naive households get efficient access to long-term investment projects only via banks since the latter are able to credibly commit to completely pass on the returns from their portfolio of liquid funds and claims on long-term investment projects to households.

The main extension of the present model compared to the literature, however, is to consider a commonly known ex-ante probability of a financial crisis – in the sense of a run on one of the banks in the financial system – in the future. In this way we are able to analyze banks’ risk management decisions – ie the allocation of deposits on liquid assets (reserves) and illiquid assets – in more detail. While we are not the first to address this issue⁶, in contrast to earlier papers our setting allows for considering the role of the interaction of funding and market liquidity as well as conclusions regarding financial system stability.

Our paper is, moreover, quite closely related to recent papers of Freixas et al. (2010) and Carletti and Leonello (2011). In particular Freixas et al. (2010) address an objective similar to ours also assuming a non-zero crisis probability. In contrast to our approach, however, they consider direct links between banks via interbank market exposures. In their model the interbank market redistributes liquidity in the financial system. The aggregate amount of liquidity, however, is fixed. In contrast, our model considers an asset market (instead of interbank lending) that allows for an early liquidation of claims on long-term assets. The market is generally accessible, ie also households may enter and demand or supply claims on long-term investment projects. As a consequence, the asset market provides liquidity to market participants who supply claims on long-term assets. And the aggregate amount of liquidity in the market is endogenously determined by the initial decisions of banks and households to invest their funds into short-term or long-term assets.

Just like in our paper asset market liquidity in Carletti and Leonello

⁶See, eg, Cooper and Ross (1998), Holmstrom and Tirole (2000), Bougheas and Ruis-Porras (2005), and Garleanu and Pedersen (2007).

(2011) arises endogenously from banks' initial decisions to invest funds into liquid short-term or illiquid long-term assets. In contrast to our model the asset market in Carletti and Leonello (2011) is a pure interbank market, ie households do not directly have access to the market. Moreover, Carletti and Leonello (2011) do not consider a strictly positive ex-ante probability of a future financial crisis. Instead they focus on the question whether the strength of credit market competition between banks affects bank behavior and, in turn, financial stability. The aspect of competition between banks is, however, beyond the scope of our paper.

In addition to the previously mentioned papers our paper is also related to a recent strand of the literature addressing observations made and lessons learned in the 2007 - 2009 financial crisis: Brunnermeier and Pedersen (2009) and Acharya and Viswanathan (2011) find that liquidity risk, in general, arises endogenously in the financial system and that there exists a strong interaction between funding liquidity and market liquidity. With banks increasingly using operations in financial markets to fund payment obligations when they fall due, a squeeze in market liquidity may immediately reduce a bank's funding liquidity. Moreover, Brunnermeier and Pedersen (2009) show that this interaction may cause liquidity spirals: reduced funding liquidity may negatively affect market liquidity because the more banks are in need of funds the lower supply relative to demand in financial markets. This further reduces market liquidity and creates an additional negative effect on banks' funding liquidity, and so on. Acharya and Viswanathan (2011) provide an agency theoretic explanation for the interaction between market and funding liquidity. They argue that financial firms have incentives to build up high leverage in good times, which forces strong de-leveraging in case of an adverse shock. As a result market and funding liquidity dry up.

While liquidity spirals cannot be directly observed in our model and we do not consider agency-theoretic elements in our setting, we also find funding liquidity of banks and market liquidity of banks' assets to arise endogenously from banks decisions with respect to investment and asset market participation. Brunnermeier and Pedersen (2009), however, focus on the interaction of margin requirements and model-based market pricing to explain endogenous formation of market participants' funding liquidity, assets' market liquidity and liquidity spirals. They do not consider initial investment decisions of banks and households which may be affected by a commonly known ex-ante probability of a future financial crisis.

Our findings also fit into a very topical strand of the recent literature

which addresses the impact of mark-to-market valuation of financial intermediaries' assets on financial stability. For instance, recent papers of Cifuentes et al. (2005), Allen and Carletti (2008), and Adrian and Shin (2010) analyze whether mark-to-market valuation may trigger liquidity-driven prices in asset markets and whether this creates a feedback-loop to market behavior of financial intermediaries. In this context our model may be interpreted as considering an extreme form of mark-to-market valuation. That is, claims on long-term investment projects are permanently marked to market and the market value of these claims acts as a benchmark for households to decide whether they run a bank or not. Moreover, we show that this market-value driven decision may propagate to other banks as the decision to run a bank increases pressure on that bank to sell long-term assets to meet payment obligations which reduces market liquidity and market prices.

Our paper is also related to another aspect which has been already addressed in the literature – liquidity hoarding. A large part of this literature finds liquidity hoarding to be a result of informational asymmetries in financial markets.⁷ But recent papers of Acharya and Merrouche (2009) and Gale and Yorulmazer (2011) address a different motivation for liquidity hoarding. Gale and Yorulmazer (2011) argue there may be a so-called precautionary motive for banks to hoard liquid assets. They explain that banks that could also use financial markets to fund future payment obligations may prefer holding a certain buffer of liquidity because doing so insures these banks against funding liquidity risk due to adverse market situations in the future. The results of our model are basically in line with the findings of Gale and Yorulmazer (2011). However, while Gale and Yorulmazer (2011) focus on analyzing banks' liquidity management in a general equilibrium setting, our paper addresses aspects of financial system stability. Our results suggest that ex-ante liquidity hoarding not only serves the precautionary motivation of a single bank, it may moreover generate positive external effects and hence

⁷For instance Heider et al. (2009) analyze in a theoretical model how the risk of banks' long-term assets may cause the liquidity in unsecured interbank money markets to evaporate. The model builds on counterparty risk as the key friction which is amplified by asymmetric information. Heider et al. (2009) argue that the level and the distribution of counterparty risk crucially affects the functioning of unsecured interbank money markets. In particular when counterparty risk is highly dispersed, interbank markets may break down since liquidity rich banks, then, prefer to hoard liquidity. In effect the market price of liquidity starts to increase until borrowing in the interbank market is too expensive for riskier banks.

stabilizes the financial system. That is, holding a liquidity buffer reduces pressure on banks to sell claims on long-term assets in secondary markets in order to meet payment obligations when there appears a shock or a financial crisis in the future. Empirical evidence for precautionary liquidity hoarding of banks after the onset of the financial crisis 2007-2009 is found in Acharya and Merrouche (2009).

3 Assumptions

Consider a Diamond-Dybvig-style economy with one good, three-dates ($t = 0, 1, 2$), two identical regions $\{I; II\}$, and a continuum of ex ante identical households of measure 1. A non-random proportion π of households will prefer to consume early – at time $t = 1$ – and the complementary proportion $1 - \pi$ will prefer to consume late – at time $t = 2$. Each household is endowed with one unit of goods and has preferences over consumption c_t at date $t = 0$ given by

$$U(c_1, c_2) = \begin{cases} u_1(c_1) & \text{with probability } \pi \\ u_2(c_2) & \text{with probability } (1 - \pi) \end{cases} \quad (1)$$

The uncertainty about the preferred consumption date resolves at $t = 1$. This means that every household learns at $t = 1$ whether it is patient (prefers consuming at $t = 2$) or impatient (prefers consuming at $t = 1$). However, the individual realization is private information of the respective household and not publicly observable. There is no aggregate uncertainty regarding the share of patient and impatient households. Therefore, from the law of large numbers the portion of impatient and patient households in the economy as a whole is given by π and $1 - \pi$, respectively. For simplicity, there is no discounting and we assume risk neutral households, i.e. linear utility functions.

$$U(c_1, c_2) = \begin{cases} x_1 c_1 & \text{with probability } \pi \\ c_2 & \text{with probability } (1 - \pi) \end{cases} \quad (2)$$

$$x_1 > R \quad (3)$$

In the economy there are two different production technologies available. The first is a pure storage technology that yields a zero net interest and enables households to transfer units between any two dates. The second production technology is owned by a continuum of entrepreneurs, who do

not have any initial endowment, but offer a long-run investment project to households. Investments, however, can be only made in $t = 0$ to get payed some outcome in $t = 2$. At $t = 1$ the entrepreneurs decide whether they spend their entire effort and generate a return of $R > 1$ at $t = 2$ for every unit invested at $t = 0$ or whether they shirk. Entrepreneurs have an incentive to reduce their effort since doing so increases their private benefit. Shirking, however, reduces the return of the long-run project to $\epsilon = 0$. If the project is prematurely liquidated, it yields also a return of $\epsilon = 0$. Table 1 summarizes the investment options.

	t=0	t=1	t=2
Storage	-1	+1	
		-1	+1
Production			
behave	-1	0	R
shirk	-1	0	0
liquidate	-1	0	0

Table 1: Investment Options

In order to invest in the long-run project, investors can use a centralized financial market. In $t = 0$ households use the primary market to invest in the long-run project by buying financial claims from an entrepreneur. Since funds are assumed to be scarce, competition between entrepreneurs will lead to a promised repayment of R in $t = 2$. Depending on their consumption needs, households may be inclined to trade the claims on the long-run investment project against consumption goods with other agents in a secondary market in $t = 1$. At $t = 2$ the entrepreneurs pay out the actual return of the project to the final claim holder.

Moreover, households are assumed to be either of two types. A fraction $(1 - i)$ of households is sophisticated (henceforth Type-A). They are able to learn everything about the project and to monitor entrepreneurs. Therefore, these households have the ability to force the entrepreneurs to spend their entire effort for the long-term project and realize a return of R on the financial claims. This is the case because these households will replace misbehaving entrepreneurs without incurring any profit loss. The complementary fraction i of households is of the naive type (henceforth Type-B). They are not

able to monitor the entrepreneurs and achieve a return of ϵ since, then, the entrepreneurs have an incentive to shirk.

Besides the direct investment strategy consumers can decide to deposit their funds with a bank. A bank is a financial institution that offers deposit contracts against households' initial endowments. The proceeds from deposit contracts are then used to build up a portfolio containing the investments in the storage technology and claims on the long-term production technology. In each region of the economy is one bank which offers identical deposit contracts. The uniform contract arises due to competition between banks and the contestability of the banking market. Like Type-A households, banks are able to monitor the effort level of entrepreneurs accurately and achieve a return of R on financial claims. In contrast to sophisticated households, banks are able to credibly commit to pass on the entire return to naive households. The argument was brought forward by Diamond and Rajan (2001) who argued that the attempt to renegotiate the deposit contract would lead in a run on the bank due to sequential service property of deposit contracts (first-come, first-served). Thus, only banks have the ability to provide naive households with efficient access to the long-run investment opportunity.

The innovation that distinguishes our model from that of Fecht (2004) is that we also consider two states of the world. The probability of a financial crisis, i.e. a bank run due to a coordination failure of depositors, is assumed to be higher than zero in our model and anticipated by market participants who will adjust their expectations accordingly. Let $m \in M \equiv \{0; 1\}$, where

$$m = \begin{cases} 1 & \text{with probability } \theta & \text{crisis state} \\ 0 & \text{with probability } (1 - \theta) & \text{normal times state} \end{cases} \quad (4)$$

and $\theta \in [0, 1]$ is the probability of a coordination failure state $m = 1$. We assume that m is observable but not verifiable and thus contracts cannot be written contingent on the realization of m . Since we have two banks of which one is subject to a run at a time. The probability of a specific bank to be subject to a run is $\theta/2$.

4 No Crisis

For a start assume that $\theta = 0$. Since the consumer type and the consumption preferences are private information, banks are not able to provide contracts contingent on the realization of these characteristics. Thus banks can offer

only one deposit contract. If banks offer a deposit contract that provides depositors with some option for consumption smoothing, i.e. $d_2/d_1 < R$, then the optimal deposit contract banks can offer solves the optimization problem (P1).

$$(P1) \left\{ \begin{array}{ll} \max_{l,k} & E[U] = \pi i x_1 d_1 + (1 - \pi) i d_2 \\ & \frac{R}{p_n} d_1 \geq d_2 \quad (IC_A) \\ & d_1 \leq d_2 \quad (IC_B) \\ \text{s.t.} & d_1 \leq \frac{l + p_n k}{1 - (1 - \pi)i} \quad (BC_1) \\ & d_2 \leq \frac{1 - l - k}{(1 - \pi)i} \cdot R \quad (BC_2) \\ & \pi x_1 d_1 + (1 - \pi) \frac{R}{p_n} d_1 \quad (PC) \\ & > \max \left\{ \pi x_1 + (1 - \pi) \frac{R}{p_n}, \pi x_1 p_n + (1 - \pi) R \right\} \end{array} \right.$$

Given that the deposit contract provides some insurance against liquidity risks, also sophisticated households might find it optimal to invest in bank deposits in $t = 0$. But in contrast to naive households, sophisticated depositors can withdraw and reinvest in assets in the financial market if they turn out to be patient. While patient naive depositors will have incentives to hold on to their deposits as long as (IC_B) holds, patient sophisticated households will rather withdraw their deposits to reinvest in financial markets, if (IC_A) holds. Given that they plan to withdraw and reinvest if they turn out to be patient, sophisticated households have an ex-ante incentive to invest in deposits rather than hold a portfolio of liquidity (storage technology) and assets (claims against entrepreneurs) and reallocate the portfolio in $t = 1$ according to their consumption preferences if (PC) holds.

Given that only naive patient households keep their deposits until $t = 2$, the bank must dispose of sufficient liquidity in $t = 1$ to refinance the repayment d_1 to all but the patient naive households. Thus the initial liquidity l holding plus the revenues from selling assets in the financial market $p_n k$ must suffice to repay d_1 to the fraction $[1 - (1 - \pi)i]$ of households. Returns on the long term asset holdings must suffice to refinance the repayment to patient depositors. Consequently, we have the two budget constraints (BC_1) and (BC_2) .

Because the banking market is assumed to be contestable, banks will offer a deposit contract that maximizes naive households expected utility. Given that $d_2/d_1 < R$ and that sophisticated households withdraw irrespective of

whether they are patient or impatient, the deposit contract involves a cross-subsidization from naive to sophisticated households. Therefore, if a bank does not maximize the expected utility of naive households given this cross-subsidization a competitor could always offer a deposit contract preferable to the naive households leaving the incumbent bank with only the sophisticated households.

Since we assume that banks act as price takers in the financial market, it is easy to see from (BC_1) that for $p_n > 1$ banks will only invest in assets and try to refinance short-term repayments solely with the revenue from asset sales. But this would mean that no liquidity is held in the economy. Thus banks actually could not exchange their asset against liquidity and this cannot be an equilibrium. For $p_n < 1$ banks would only hold liquidity. Patient sophisticated depositors receiving liquidity when withdrawing their deposits will not find any supply of assets in the market. Thus only for $p_n = 1$ banks are indifferent and will sell assets in the financial market while at the same time also investing some of their portfolio in liquidity. Taking this equilibrium asset price into account it is easy to see that both (IC_A) and (PC) hold for any deposit contract with $d_1 > 1$ that provides some liquidity insurance, i.e. $d_2/d_1 < R$.

Assuming a symmetric equilibrium in which all banks hold the same amount of assets in their trading book we can derive from the no-arbitrage condition $p_n = 1$ the market clearing condition:

$$k = d_1(1 - \pi)(1 - i) \quad (MC_n)$$

Given the no-arbitrage condition we can simplify the budget constraints to:

$$1 \geq (1 - (1 - \pi)i) d_1 + (1 - \pi)id_2/R \quad (BC)$$

Consequently, as long as the costs of increasing the short-term repayments in terms of forgone long-term repayment are lower than the marginal rate of substitution between short and long-term repayment for naive households the bank will choose the maximum incentive compatible short-term repayment: Whenever the budget constraint is flatter than the indifference curve the bank will choose d_1 such that (IC_B) holds with equality, i.e.

$$x_1 > \frac{1 - (1 - \pi)i}{\pi i} \cdot R$$

and we have $d_1 = d_2$. Reinserting in the budget constraint allows us to derive

$$d^* = d_1 = d_2 = \frac{R}{R - (1 - \pi)i(R - 1)} \quad (5)$$

Thus he have the following proposition:

Proposition 1 (Bank-dominated financial system) *If the fraction of naive households i is higher than the threshold level \hat{i} with*

$$\hat{i} = \frac{R}{\pi x_1 + (1 - \pi)R}$$

banks offer the same short and long-term repayment d^ on deposits. While this contract allows naive depositors not only to benefit from the long-term productive investment it also provides them with a maximum liquidity insurance. While also sophisticated households initially deposit their funds with the bank they withdraw their deposits irrespective whether or not they are patient or impatient. Patient sophisticated households reinvest the proceeds in the financial market in $t = 1$ buying assets from the banks at the arbitrage free price $p_n = 1$.*

If, however, the fraction of naive households is small such that

$$i < \frac{R}{\pi x_1 + (1 - \pi)R}$$

the cross subsidization of sophisticated households becomes too costly for naive depositors. In this case banks offer a deposit contract

$$\{d_1; d_2\} = \{1; R\}$$

which is only attractive for naive households. While this contract allows naive households to benefit from the productive investment, it does not provide any insurance against liquidity risks. Banks (and sophisticated households) are again indifferent between holding assets or liquidity at the arbitrage free price $p_n = 1$. Thus sophisticated households do not fare better investing directly than holding deposits initially. Thus sophisticated households invest directly in liquidity and assets and reallocate their portfolio according to their consumption preference shock. In this case banks do only hold assets to refinance the repayment to patient naive depositors. They do not hold assets to sell them in the financial market.

Proposition 2 (Market-oriented financial system) *If the fraction of naive households i is smaller than the threshold level \hat{i} . Banks only provide efficient access for naive households to the long term investment opportunity. Banks are not selling assets in the financial market. Both naive as well as sophisticated households retain considerable liquidity risk.*

Now consider in this benchmark case the effects of a run on one bank. In a bank-dominated financial system with $i > \hat{i}$ the bank affected by the run will not only sell k assets. This bank is forced to fire sale in addition its $(1 - l - k)$ assets. Thus per-capita repayment to depositors is then given by

$$d_c = l + p_c(1 - l) \quad (6)$$

Given that patient sophisticated depositors will also in the run use the repayment to reinvest in the financial market in the event of a crisis the market clearing condition would be

$$(d_c + d_1)(1 - \pi)(1 - i) = p_c[k + (1 - l)]$$

assuming that the other bank remained solvent and could still repay d_1 to its patient sophisticated depositors and sell only k in the financial market. Because $d_c < d_1$, the liquidity that patient sophisticated depositors receive from the failing bank and that they use to demand assets in the financial market falls short of the liquidity that they would provide to the asset market if their bank would be solvent. At the same time $k < (1 - l)$. Consequently, due to the fire sales of the failing banks, asset supply increases while at the same time asset demand is being reduced. Thus in the zero probability event of a crisis the asset price drops to $p_c < 1$ in a bank dominated financial system. But from (MC_n) immediately follows that for $p_c < 1$ the other bank does not receive sufficient liquidity from assets sales out of its trading book. Since all of the remaining assets are needed to refinance the repayment to patient naive households, the bank can not sell additional assets to increase the liquidity inflow. Thus an asset price drop due to one bank's fire sales cannot be sustained by the other bank and will always lead to contagion in a bank-dominated financial system.

In a market oriented financial system with $i \leq \hat{i}$ banks do not rely on liquidity inflow from the financial market. If a run hits one bank and forces it into fire sales any detrimental effect on asset prices of these fire sales will not destabilize the other bank.

Proposition 3 (Stability) *If the run on one bank is a zero probability event, this run with subsequent fire sales of assets will lead to an asset price deterioration. In a bank-dominated financial system the asset deterioration is unsustainable for the other bank and will inevitably lead to contagion. In a market oriented financial system the asset price drop does not affect other banks.*

Finally, consider the constrained efficient solution in this setting. The social planner that can shut down financial markets but cannot observe the type of an individual household solves the following problem.

$$(P^{sp}) \left\{ \begin{array}{ll} \max_{l,k} & E[U] = \pi x_1 c_1 + (1 - \pi) c_2 \\ & c_1 \leq c_2 \quad (IC) \\ \text{s.t.} & \pi c_1 + (1 - \pi) \frac{c_2}{R} \leq 1 \quad (BC) \end{array} \right.$$

He maximizes overall expected utility of naive and sophisticated households. Taking into account only the budget constraint and the incentives constant ensuring that patient households do not withdraw early. For $x_1 > R$ both (IC) and (BC) are binding and the constrained efficient consumption allocation is given by

$$\{c_1^{sp}; c_2^{sp}\} = \left\{ \frac{R}{R - (1 - \pi)(R - 1)}; \frac{R}{R - (1 - \pi)(R - 1)} \right\}$$

Thus the level of risk sharing provided by banks in a bank-dominated financial system ($i > \hat{i}$) is optimal: $d_2/d_1 = c_2/c_1 = 1$. However, the consumption level of patient and impatient naive households is lower than in the optimal allocation, because of the information rent extracted by patient sophisticated investors. The difference between the optimal consumption level and the level achieved by naive households in a bank-dominated financial system increases in the share of patient sophisticated households. Sophisticated investors bear considerable liquidity risk. Their consumption level is d_1 when patient and Rd_1 when impatient. Only if no investors can invest in financial market ($i = 0$) the allocation achieved by a bank-dominated financial system is optimal.

Sophisticated investors cannot extract an information rent in a market-oriented financial systems ($i \leq \hat{i}$). However, in such a system neither markets nor banks provide the optimal liquidity insurance. The interest rate from $t = 1$ to $t = 2$ is the same in financial markets as well as in bank deposits.

Compared to the constraint efficient allocation banks underinvest in liquidity in both market and banks-dominated financial systems.

Proposition 4 (Efficiency) *For $i < 1$ neither the allocation in a market-oriented ($i \leq \hat{i}$) nor in a bank-dominated financial system ($i > \hat{i}$) is contained efficient. A market-oriented system provides inefficient liquidity insurance. In a bank-dominated financial system naive households achieve optimal liquidity insurance but pay an information rent to patient sophisticated investors. The larger this information rent, i.e. the larger the share of patient sophisticated investors $(1 - \pi)(1 - i)$, the less efficient the allocation in a bank-dominated financial system.*

A simple regulatory solution to implement efficiency often proposed in similar settings is a regulatory minimum reserve holding. Farhi et al. (2009), for instance, show that a minimum reserve requirement can implement an efficient solution in a model, in which depositors can engage in hidden side trades in $t = 1$. If banks are required to hold a minimum share c_1^{sp} of the deposits received in $t = 0$ in liquidity the amount of goods available in $t = 1$ and $t = 2$ are determined and rate of exchange of $t = 2$ -consumption goods against $t = 1$ -consumption goods is fixed at c_2^{sp}/c_1^{sp} .

However, in our setting depositors not only have the possibility to trade in a $t = 1$ financial market. In our model investors can only decide about how much to invest in liquidity, the long-term investment and in bank deposits in $t = 0$. As a consequence, liquidity regulation cannot implement the constraint efficient allocation in our framework.

To see this consider a liquidity regulation that requires banks to hold a share of c_1^{sp} of each unit of deposits received in $t = 0$ in the storage technology.

++++++problem needs to be completed and solved++++++

$$(P^{reg}) \left\{ \begin{array}{ll} \max_{l,k} & E[U] = \pi i x_1 d_1 + (1 - \pi) i d_2 \\ & \frac{R}{p_n} d_1 \geq d_2 \quad (IC_A) \\ & d_1 \leq d_2 \quad (IC_B) \\ & d_1 \leq \frac{l + p_n k}{1 - (1 - \pi)^i} \quad (BC_1) \\ \text{s.t.} & d_2 \leq \frac{1 - l - k}{(1 - \pi)^i} \cdot R \quad (BC_2) \\ & \pi x_1 d_1 + (1 - \pi) \frac{R}{p_n} d_1 \quad (PC) \\ & > \max \left\{ \pi x_1 + (1 - \pi) \frac{R}{p_n}, \pi x_1 p_n + (1 - \pi) R \right\} \end{array} \right.$$

5 Infrequent crisis

Consider now the run on one bank as an event that occurs with a small but positive probability. In a bank-dominated financial system a run on one bank and the resulting fire sales will always induce a liquidity shortage at the other bank unless banks hold liquidity buffers. However, as long as the run on one bank occurs with a sufficiently low probability the expected costs of holding liquidity buffers to avoid contagion, i.e. the reduced repayment on deposits in a no-crisis state, overcompensate the expected benefits from being able to sustain the asset price drop following fire sales of the other bank. Thus for a sufficiently low θ the possibility of a liquidity crisis will only affect asset prices. In normal times the asset price includes a liquidity risk premium in order to compensate banks for the liquidity risk they incur, i.e. for the expected costs of contagion through financial markets.

Consequently, if a run on one bank is sufficiently unlikely ($\theta \leq \bar{\theta}$) and banks can still provide some consumption smoothing for naive households $R > d_2 > d_1 > 1$ (i.e. the fraction of naive households is again sufficiently high $i \geq \bar{i}$) banks offer a deposit contract that solves the optimization problem (P2).

$$(P2) \left\{ \begin{array}{l} \max_{l,k} \quad E[U] = (1 - \theta) [\pi i x_1 d_1 + (1 - \pi) i d_2] + \theta [\pi i x_1 d_c + (1 - \pi) i d_c] \\ \frac{R}{p_n} d_1 \geq d_2 \quad (IC_A) \\ d_1 \leq d_2 \quad (IC_B) \\ d_1 \leq \frac{l + p_n k}{1 - (1 - \pi) i} \quad (BC_1) \\ d_2 \leq \frac{(1 - l - k)}{(1 - \pi) i} \cdot R \quad (BC_2) \\ \text{s.t.} \quad d_c \leq (1 - l) p_c + l \quad (BC_c) \\ (1 - \theta) \left[(1 - \pi) \frac{R}{p_n} d_1 + \pi x_1 d_1 \right] + \theta \left[(1 - \pi) \frac{R}{p_c} d_c + \pi x_1 d_c \right] \quad (PC) \\ > \max \left\{ (1 - \pi) \left[\theta \frac{R}{p_c} + (1 - \theta) \frac{R}{p_n} \right] + \pi x_1, \right. \\ \left. \pi x_1 [\theta p_c + (1 - \theta) p_n] + (1 - \pi) R \right\} \end{array} \right.$$

When designing the optimal deposit contract, banks must also take into account the amount that they can repay in a crisis if such an event has a positive probability. Due to their financial market activity in a bank-dominated financial system not only the bank that directly suffers from a run is forced to liquidate its entire portfolio in the market. Also the other bank will be liquidated because of a liquidity shortage given that it does not

preserve a liquidity buffer. Consequently, in the depositors' expected utility function that banks maximize we only need to consider the banks' repayment d_1 and d_2 to patient and impatient depositors when there is no crisis and the per capita liquidation return d_c that both banks can distribute in the crisis.

While the incentive compatibility constraints for naive and sophisticated households ((IC_A) and (IC_B)) and the budget constraints for early and late repayment ((BC_1) and (BC_2)) remain unchanged, we have also to take into account the budget constraint (BC_c) for the crisis situation. This constraint simply states that the repayment per capita after the liquidation equals at most the entire liquidation proceeds whereby all assets are sold off in the financial market at the crisis price p_c .

Finally, in contrast to the no-crisis case, the participation constraint of sophisticated depositors (PC) must now take into account that prices in the asset market and the repayment of banks vary depending on the different states that can occur. Thus it is only preferable for sophisticated depositors to initially invest in deposits if the expected payoff, that they can realize by withdrawing and consuming if impatient or reinvesting in financial markets if patient, is larger than the payoff they realize by investing either only in liquidity or assets and trade in the financial market in $t = 1$ according to their realized consumption preferences.

In addition to the optimal deposit contract solving $(P2)$ the equilibrium with a bank-dominated financial system and an infrequent crisis is characterized by the market clearing condition for the asset market in the good and in the bad state. In the no crisis state the market value of the bank's trading portfolio must be equal to the withdrawals of patient sophisticated households who reinvest in financial markets.

$$p_n k = d_1(1 - \pi)(1 - i) \quad (MC_n)$$

In the crisis situation the market value of the entire asset holding, i.e. the trading book plus the banking book, must be equal the cash received by the patient sophisticated households from the liquidation of their respective bank.

$$p_c(1 - l) = d_c(1 - \pi)(1 - i) \quad (MC_c)$$

Because of the higher marginal utility of impatient depositors, depositors' expected utility in the no-crisis state is optimized with a maximum repayment on deposits in the short-run for $p_n \geq 1$. Taking the incentive constraints

of patient naive households into account, maximum expected utility for the non-crisis period is achieved with $d_1 = d_2$. In the crisis state depositors' utility is maximized with a maximum l for $p_c \leq 1$. Increasing the liquidity holdings beyond the amount required to implement $d_1 = d_2$ is costly in the no-crisis state, because holding such a liquidity buffer would imply that the repayment to patient depositors in the no-crisis state is inefficiently refinanced with proceeds from the storage technology rather than the long-term investment technology. Consequently, it is efficient for the bank not to increase its liquidity holdings beyond what is needed to implement the optimal repayments in the no-crisis state, if the marginal disutility from holding a liquidity buffer in the no-crisis state is not smaller than the benefits in the crisis period:

$$(1 - \theta) i [\pi x_1 + (1 - \pi)] \frac{(R - 1)}{(1 - \pi) i + (1 - (1 - \pi) i) R} \geq \theta i [\pi x_1 + (1 - \pi) i] (1 - p_c)$$

Thus, as long as the crisis probability is lower than a threshold $\tilde{\theta}$ with⁸

$$\tilde{\theta} = \frac{(R - 1)}{[R - (1 - \pi) i (R - 1)] (1 - p_c) + (R - 1)}$$

banks will not hold excess liquidity and will choose a portfolio to maximize depositors' expected utility in the no-crisis state.

Taking as given that prices p_n and p_c adjust such that banks are indifferent between holding liquidity or investing in assets banks hold in equilibrium exactly enough assets in their trading book such that (MC_n) holds. The withdrawals of all impatient depositors must be financed with liquidity holdings and the repayment to patient naive households who only withdraw their depositors in $t = 2$ will be financed out of the banking book, i.e. assets held until maturity. Since (IC_b) is the binding constraint it will hold with equality. Thus given $\theta \leq \tilde{\theta}$ the optimal repayment in no-crisis situations is given by the general budget constraint:

$$(1 - i) (1 - \pi) d^* / p_n + \pi d^* + i (1 - \pi) d^* / R = 1$$

⁸Note that this implies that $p_c \geq 1 - \frac{(1-\theta)}{\theta} \frac{(R-1)}{[R-(1-\pi)i(R-1)]}$. If p_c drops below this threshold, banks would find it beneficial to only invest in liquidity. However, if banks only invest in liquidity, a bank dominated financial system does not emerge and banks are redundant.

Consequently, the optimal deposit contract is given by:

$$d^{**} = d_1 = d_2 = \frac{p_n R}{(1 - \pi) [(1 - i) R + i p_n] + \pi p_n R} \quad (7)$$

and banks' liquidity holding is

$$l^{**} = \frac{\pi p_n R}{(1 - \pi) [(1 - i) R + i p_n] + \pi p_n R} \quad (8)$$

Obviously, both banks' liquidity holdings as well as their repayments in no-crisis times increase in the asset price in no-crisis states.

Inserting (8) and (BC_c) form $(P2)$ in the market clearing condition for the crisis period (MC_c) gives the following cash-in-the-market equilibrium condition for the asset price in the crisis period:⁹

$$p_c = \frac{\pi p_n R}{[(1 - i) R + i p_n]} \cdot \frac{(1 - i)}{(1 - (1 - \pi)(1 - i))} \quad (\text{CMP})$$

which implies that the asset price in the crisis state increases in the price in normal times:

$$\frac{\partial p_c}{\partial p_n} = \frac{\pi}{(1 - (1 - \pi)(1 - i))} \cdot \left(\frac{R(1 - i)}{[(1 - i) R + i p_n]} \right)^2 > 0 \quad (9)$$

The intuition for this is that the larger the price in the no-crisis state the larger is the general repayment that banks can afford in no-crisis times. To fund the higher repayment for impatient households banks hold somewhat more liquidity. In the crisis state a larger liquidity holding reduces the amount of assets thrown on the market and reduces asset price deterioration during the banking crisis.¹⁰

$$d_c^{**} = \frac{\pi p_n R}{(1 - \pi) [(1 - i) R + i p_n] + \pi p_n R} \cdot \frac{1}{1 - (1 - \pi)(1 - i)} \quad (10)$$

The condition that we did not consider so far, but that is required to close the model is the no-arbitrage condition. At the equilibrium prices in the crisis and no-crisis states banks must be indifferent between investing in asset or holding liquidity ex ante.

⁹See the appendix for details.

¹⁰See appendix for details.

If the bank would only hold liquidity it could repay all early withdrawing depositors, impatient and patient sophisticated ones, with liquidity and use liquidity to buy assets at the no-crisis price p_n to finance the repayments to impatient naive depositors. Following that strategy the bank could pay depositors in $t = 1$ and $t = 2$:¹¹

$$d = \frac{R}{(1 - (1 - \pi) i)R + (1 - \pi) i p_n}$$

Since a bank holding only liquidity could repay $d_c = 1$ in the crisis the expected utility a bank following that strategy could provide to naive households is given by

$$(1 - \theta) [\pi x_1 + (1 - \pi)] i \frac{R}{(1 - (1 - \pi) i)R + (1 - \pi) i p_n} + \theta [\pi x_1 + (1 - \pi)] i$$

A bank that only invests in assets and sells some of them off in $t = 1$ to finance the short-term repayments would be able to pay

$$d = \frac{R}{(1 - (1 - \pi) i) \frac{R}{p_n} + (1 - \pi) i}$$

Given that during a crisis the bank would have to sell off all its assets at the equilibrium price p_c , expected utility of naive households depositing at a bank that only invests in asset amounts to:

$$(1 - \theta) [\pi x_1 + (1 - \pi)] i \frac{R}{(1 - (1 - \pi) i) \frac{R}{p_n} + (1 - \pi) i} + \theta [\pi x_1 + (1 - \pi)] i p_c$$

Thus banks will be indifferent between holding liquidity and investing in assets given the following no-arbitrage condition

$$p_c = 1 - \frac{(1 - \theta)}{\theta} \left(\frac{(p_n - 1)R}{(1 - (1 - \pi) i)R + (1 - \pi) i p_n} \right) \quad (\text{NAC})$$

Following the no-arbitrage condition the equilibrium asset price in crisis states is a decreasing function of the asset price in no-crisis states:

$$\frac{\partial p_c}{\partial p_n} = - \frac{(1 - \theta)}{\theta} \frac{(1 - (1 - \pi) i)R^2 + (1 - \pi) i R}{[(1 - (1 - \pi) i)R + (1 - \pi) i p_n]^2} < 0 \quad (11)$$

¹¹See the appendix for details.

Intuitively, a higher asset price in normal times makes asset holdings more attractive. In order to ensure that banks are indifferent, the price in crisis periods must be lower.

Using CMP and NAC finally allows us to determine the equilibrium asset price in no-crisis and in crisis states. From (9) it is easy to see that according to CMP p_c is a monotonically increasing concave function in $p_n \forall p_n \in \mathbb{R}_{\leq 0}$, while (11) indicates that p_c according to NAC is a monotonically decreasing convex function of $p_n \forall p_n \in \mathbb{R}_{\leq 0}$. Consequently, there exists only one equilibrium combination of asset prices.

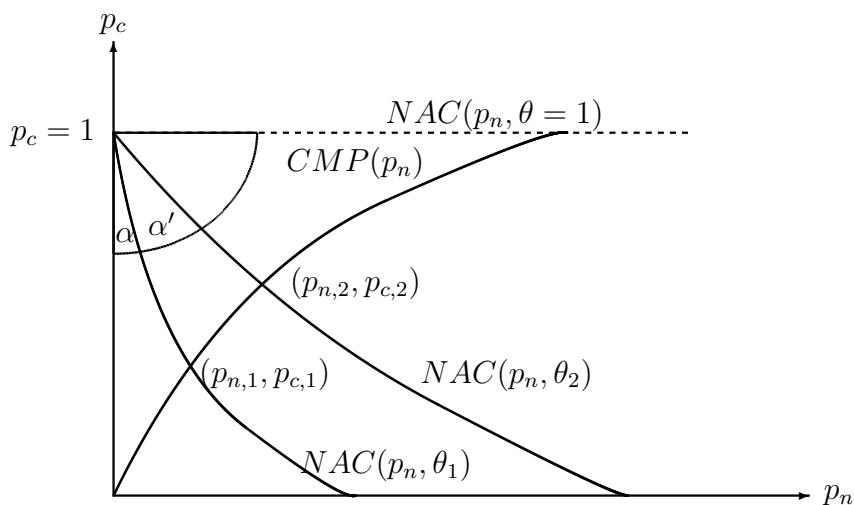


Figure 1: Impact of a Change in θ on Equilibrium Prices

More importantly, note that the CMP is no function of θ but an increasing function of p_n , thus a change in θ moves equilibrium prices along CMP (see Figure 1). Additionally, NAC is a decreasing function of p_n but the slope of the tangent increases with θ . Since the NAC passes independently of the parameter setting through $p_c = p_n = 1$, an increase in θ , from θ_1 to θ_2 leads to a raise of the angle α moving low asset prices $(p_{n,1}, p_{c,1})$ to a higher price level $(p_{n,2}, p_{c,2})$. Thus as depicted in figure 1 an increase in the crisis probability (θ) leads to soaring prices in both the crisis as well as the no-crisis state. This intuitively gives us the following proposition:

Proposition 5 For $\theta \leq \min\{\bar{\theta}, \tilde{\theta}\}$ there is only one equilibrium related to one combination of p_n and p_c . The larger θ in this interval the larger are both p_n and p_c .

Proof 1 Given that i, π and θ are $\in (0, 1)$ and $R \geq 1$ a formal proof requires the three following insights:

(a) Note that $CMP(p_n)$ is strictly increasing in $p_n \forall p_n \in \mathbb{R}_{\leq 0}$ and is no function of θ implying if the intersection point increases in p_n , the intersection value $CMP(p_n)$, i.e. p_c , must also increase.

(b) Furthermore, $NAC(p_n, \theta)$, for a fixed value θ , is a monotonically decreasing function of p_n , leading to $CMP(p_n)$ and $NAC(p_n, \theta)$ intersect only once $\forall p_n \in \mathbb{R}_{\leq 0}$. Now consider θ_1 and θ_2 . Let $p_{n,1}$ be the value such that $CMP(p_{n,1}) = NAC(p_{n,1}, \theta_1)$ and suppose that $NAC(p_{n,1}, \theta_2) > NAC(p_{n,1}, \theta_1)$, then necessarily the intersection point $p_{n,2}$ which solves $CMP(p_{n,2}) = NAC(p_{n,2}, \theta_2)$ must satisfy $p_{n,2} > p_{n,1}$.

Due to monotonicity, for every $0 \leq y \leq p_{n,1}$ we have $CMP(y) \leq CMP(p_{n,1})$. Similarly we also have $NAC(y, \theta_2) \geq NAC(p_{n,1}, \theta_2)$. So if $NAC(p_{n,1}, \theta_2) > CMP(p_{n,1})$, it is impossible that $NAC(y, \theta_2) = CMP(y)$. Since the two curves must intersect, they have to intersect somewhere $y > p_{n,1}$.

(c) Finally, we need to show that if (p_n, θ) solves $NAC(p_n, \theta) = CMP(\theta)$, for any $\theta' > \theta$, we have that $NAC(p_n, \theta') > NAC(p_n, \theta)$. Note that $NAC(p_n, \theta)$ is an increasing function of θ ($\partial NAC(p_n, \theta) / \partial \theta$) for all $p_n > 1$. Thus the proof is accomplish by showing that the intersection cannot be at $p_n < 1$. Using monotonicity again it suffices two show that $NAC(1, \theta) > CPM(1)$. One immediately sees that $NAC(1, \theta) = 1$ and $CPM(1) < 1$ which completes the proof. ■

Thus in a bank-dominated financial system a higher probability of a run on one bank with the associated fire sales and contagion of the other bank leads to overall higher asset prices. It increases the overall value of liquidity which in turn induces banks to hold more liquidity. Thus more can be paid for assets both in the crisis as well as in the no-crisis case.

From (7) and (10) is is easy to see that a higher asset price in the no crisis state induces increases both the repayment investors receive. The intuition is that with a higher asset price the information rent that patient sophisticated investors can extract is lower. Therefore, the repayment that banks can provide to depositors is higher. Banks have invest fewer resources ex ante

in assets that are only held to sell that to patient sophisticated investors in the market. Thus the liquidity insurance provided by the banking sector in the no-crisis case becomes more effects and approaches the constant efficient allocation.

Thus the threat of a crisis with the fostered incentives to withhold liquidity improves the efficiency of the deposit contract and the allocation achieved if banks are stable.

6 Central Bank Interventions

In a liquidity crisis the central bank (CB) may improve the welfare by acting as market maker (MMOR) or lender (LOLR) of last resort. While a MMOR tries to prevent the asset price from dropping by injecting liquidity in the financial market, a LOLR socializes the failing bank. A CB can act as MMOR in two different ways. They either accept a wide range of securities as collateral in repos or they purchase and sale the assets directly. In our model the MMOR intervenes directly by buying assets at secondary market, thereby stabilizing the asset price. A LOLR, on the other hand, takes the the failing bank over, by socializing its liabilities and assets.

In the following two subsection we analyse the implication of a credible commitment of a CB to act as MMOR or LOLR on the deposit contract banks offer and subsequently we judge both interventions in terms of welfare improvement.

6.1 The Market Maker of Last Resort

Imagine an economy in which bank runs happen infrequently, but these runs are characterized by contagion, since no bank holds liquidity in excess. A central bank can avoid contagion by increasing the nominal amount of money, i.e. printing money, and then demanding assets at secondary market. This will decrease the real deposit repayment to early consuming households, but also stops the propagation of financial shocks. Making contagion impossible, implies that the asset price is state invariant, that is $p_n = p_c$. The only price which is in line with the arbitrage free condition NAC and is state invariant is

$$p_n = p_c = 1.$$

Thus the CB has to increase the liquidity in the secondary market such that the cash in the market price in the crisis state equals 1. As usual the bank will maximize the expected utility of naive households, taking into account that 3 different states can evolve. 1) no bank is affected by a bank run which happens with probability $(1-\theta)$. In this state the bank repays the contractual amount d_1 to impatient households and d_2 to patient households. 2) the other bank is affected by a bank run. In this state the CB will inject money in the secondary market by increasing the nominal amount of money, in order to stop the asset price from dropping. This implies that the sound bank is able to honour the deposit contract agreement in nominal terms and thus the bank is not affected by contagion. The early withdrawing households, however, face a real reduction of the contract repayment. Consequently, the real consumption of impatient households is reduced by the factor $(1-\tau)$. 3) the bank itself is affected by a run and will be liquidated. In that state the households will suffer in two ways. First, all customers of the liquidated bank only receive the liquidation value of the assets and secondly all impatient households face a real reduction of their consumption basket.

In both crisis states the CB purchases assets at the secondary market, holds them until $t = 2$ and finally subsidizes patient households with the return on the long-run asset. In order not to create an incentive for patient households to run the bank, only households, in the region without a bank run, are subsidized. The redistribution of funds to only one region is a pure technical assumption which does not affect the optimal contracts and the expected utility of households. We denote the amount of assets, the CB purchased at the secondary market Ψ . The optimization problem is summarized in (P6). In order to simplify notation we already inserted the arbitrage free market

price $p = 1$.

$$(P6) \left\{ \begin{array}{l} \max_{l,k} \quad E[U] = (1 - \theta) [\pi i x_1 d_1 + (1 - \pi) i d_2] + \\ \quad \frac{\theta}{2} [\pi i x_1 (1 - \tau) d_1 + (1 - \pi) i d_2] + \\ \quad \frac{\theta}{2} [\pi i x_1 (1 - \tau) d_c + (1 - \pi) i d_c] + \\ \quad \frac{\theta}{2} i \Psi R \\ \quad d_1 \leq d_2 \quad (IC_B) \\ \quad d_1 \leq \frac{l+k}{1-(1-\pi)i} \quad (BC_1) \\ \quad d_2 \leq \frac{(1-l-k)}{(1-\pi)i} \cdot R \quad (BC_2) \\ \text{s.t.} \quad d_c = l + p c (1 - l) = 1 \\ \quad (1 - \theta) [(1 - \pi) R d_1 + \pi x_1 d_1] + \\ \quad \frac{\theta}{2} [(1 - \pi) R d_1 + \pi (1 - \tau) x_1 d_1] + \frac{\theta}{2} [(1 - \pi) R d_c + (1 - \tau) \pi x_1 d_c] \\ \quad > (1 - \pi) R + \pi x_1 [(1 - \theta) + \theta (1 - \tau)] \quad (PC) \end{array} \right.$$

By increasing the nominal amount of money, the CB implicitly taxes all impatient customers, leading to tax revenues, i.e. liquidity injection, amounting to

$$\Psi = \pi \tau [d_c + d_1]$$

Combing (BC_1) and (BC_2) leads to

$$1 \geq (1 - (1 - \pi)i) d_1 + (1 - \pi) i d_2 / R \quad (12)$$

As long as the utility function is steeper than the budget constraint we have $d_1 = d_2$. Reinserting in the budget constraint leads to the deposit contract repayment

$$d^* = d_1 = d_2 = \frac{R}{R - (1 - \pi)i(R - 1)} \quad (13)$$

Not surprisingly the MMOR intervention leads to a deposit contract equal to the no crisis situation, but comparing the deposit contract repayment with that of the infrequent crisis case without CB actions, shows that the contractual repayment is higher without interventions in the normal periods. The intuition is, if central banks do not intervene $p_n > 1$ and banks return from selling assets at secondary market is higher which enables the banks to offer a higher short-run deposit repayment ex ante. Therefore, a MMOR reduces asset price bubbles, arising from a illiquidity premium, which adversely affects the deposit repayment in calm periods.

The tax rate, which ensures $p_c = 1$, can be derived from the market clearing

condition. The CB demands assets with the entire tax revenues, but also patient sophisticated households of both banks will buy assets at the secondary market, while the liquidated bank sells $(1 - l)$ and the sound bank $d^*(1 - \pi)(1 - i)$ assets.

$$\Psi + (1 + d^*)(1 - \tau)(1 - \pi)(1 - i) = (1 - l) + d^*(1 - \pi)(1 - i) \quad (14)$$

Inserting $l = \pi d^*$ and $\Psi = \pi\tau[1 + d^*]$ leads to

$$\tau_{MMOR}^* = \frac{1 - \pi d^* - (1 - \pi)(1 - i)}{(1 + d^*)\pi} \quad (15)$$

6.2 Lender of Last Resort

A LOLR does not stabilize the security prices directly, but indirectly by purchasing the entire balance sheet of the struggling bank. Thus, the bank facing a bank run does not firesale assets at the secondary market, the CB steps in and meets the banks obligations. The CB finances the liquidity shortage by increasing the nominal amount of money, which again implicitly leads to a taxation of early consuming households. Since acting as a LOLR averts firesales and therefore no price distortion take place, implying a state invariant asset price. The only state independent, arbitrage free asset price is

$$p_n = p_c = 1.$$

If the CB acts as LOLR, the optimization problem (P7) leads to the optimal deposit contract. Banks take into account that either one bank is affected by a bank run, which happens with probability θ or no bank is affected, happening with the converse probability. If banks end up in a liquidity crisis, the CB creates liquidity, by increasing the nominal amount of money, being an implicit tax for early consuming households and nationalizes the failing bank. The CB then holds the assets of the struggling bank until $t = 2$ and subsidizes patient households which did not cause a bank run. The total tax revenue is denoted by Ψ . In the no-crisis state banks repay the contractual amount d_1 to early consumers and d_2 to patient households. We

again inserted the arbitrage free market prices in order to simplify notation.

$$(P6) \left\{ \begin{array}{l} \max_{l,k} \quad E[U] = (1 - \theta) [\pi i x_1 d_1 + (1 - \pi) i d_2] + \\ \quad \theta [\pi i x_1 (1 - \tau) d_1 + (1 - \pi) i d_2] + \\ \quad \frac{\theta}{2} i \Psi R \\ \quad d_1 \leq d_2 \quad (IC_B) \\ \quad d_1 \leq \frac{l+k}{1-(1-\pi)i} \quad (BC_1) \\ \quad d_2 \leq \frac{(1-l-k)}{(1-\pi)i} \cdot R \quad (BC_2) \\ \text{s.t.} \quad d_c \leq 1 \quad (BC_c) \\ \quad (1 - \theta) [(1 - \pi) R d_1 + \pi x_1 d_1] + \quad (PC) \\ \quad \theta [(1 - \pi) R d_1 + \pi x_1 (1 - \tau) d_1] + \\ \quad > (1 - \pi) R + \pi x_1 [(1 - \theta) + \theta(1 - \tau)] \end{array} \right.$$

In the crisis state the CB takes the struggling bank over. That is, the CB fulfils all obligations of the bank and obtains in exchange all the assets which are intended to be held until maturity. Since the bank holds liquidity for impatient households and already plans to sell $d_1(1 - \pi)(1 - i)$ assets to patient, sophisticated households at the secondary market, only $d_2(1 - \pi)i/R$ assets is planned to be held until maturity. Therefore, the CB creates liquidity amounting to

$$\Psi = \frac{d_2(1 - \pi)i}{R} = 2d_1\pi\tau. \quad (16)$$

Putting (BC_1) and (BC_2) together and using $d_1 = d_2$ for a utility function being steeper than the budget constraint, leads to the optimal deposit contract repayment

$$d^* = d_1 = d_2 = \frac{R}{R - (1 - \pi)i(R - 1)}. \quad (17)$$

This is exactly the same contract as a bank can offer if the CB acts as MMOR. Reinserting the optimal deposit contract, $l = \pi d^*$ and $k = d^*(1 - \pi)(1 - i)$ in 16, leads to the tax rate

$$\tau_{LOLR}^* = \frac{1 - \pi d^* - d(1 - \pi)(1 - i)}{2d^*\pi}$$

Obviously, the tax rate which is imposed on early consuming households is lower when CB acts as LOLR in comparison with MMOR, $\tau_{LOLR}^* < \tau_{MMOR}^*$. The underlying reason is that in LOLR case no bank is liquidate and thus the taxable income is higher. A lower tax also implies a higher expected utility and thus the LOLR dominates the MMOR.

6.3 CB Intervention in an Infrequent Crisis Environment

Judging the profitability of the a MMOR or LOLR means analysing the CB interventions in terms of welfare improvement. We already noticed that a LOLR is always superior to a MMOR, but it is not clear if an intervention leads to an increase in expected utility. Obviously, the interventions avoid the propagation of financial shocks. On the other hand, banks are able to provide a higher deposit repayment in the no-crisis state, if no CB intervenes due to a higher asset price. Secondly, a higher short-term deposit repayment implies that banks hold more liquidity and thus banks are better prepared for a crisis. In order to shed more light on that issue we plot the excess utility of households depending on the CB action in Figure 2. Excess utility is defined as the utility of a deposit contract when CB intervene minus the utility of a deposit contract without policy actions.

The graphs on the left hand side of Figure 2 show a comparative static analysis of the crisis probability θ , whereas the graphs on the right hand side analyse the impact of a change of naive households. We used the parameter setting $\pi = 0.5$, $i = 0.6$, $R = 1.5$ and $x_1 = 3$ for the graphs on the right side and $\pi = 0.5$, $\theta = 0.1$, $R = 1.5$ and $x_1 = 3$ for those on the left hand side.

The first row of graphes shows the total excess utility of all households. Interestingly, it severely depends on the fundamentals of the economy if a MMOR is able to increase welfare. Especially when the crisis probability is high, a MMOR destroys utility. When CB do not intervene, a high crisis probability implies a relative high asset price in the no-crisis state, which leads to a high short-term deposit repayment and higher liquidity holdings. In the crisis state the liquidity helps to buffer the asset price from dropping too much. A MMOR kills the incentive to hold more liquidity by fixing the asset price to 1. Also quiet interesting is that the naive households utility increases with a MMOR while the utility of sophisticated households decreases. The underlying mechanics are that sophisticated households normally benefit from a decreased asset price in the crisis state. If a central bank acts as MMOR, the price is stabilized in the crisis state and consequently the patient sophisticated households do not benefit to the same extend. Finally, when the crisis probability goes to zero, a MMOR, a LOLR as well as no CB intervention leads to identical utility levels.

The figures on the left hand side tell a similar story. It can be seen that the fraction of naive households is critical for MMOR in order to be welfare

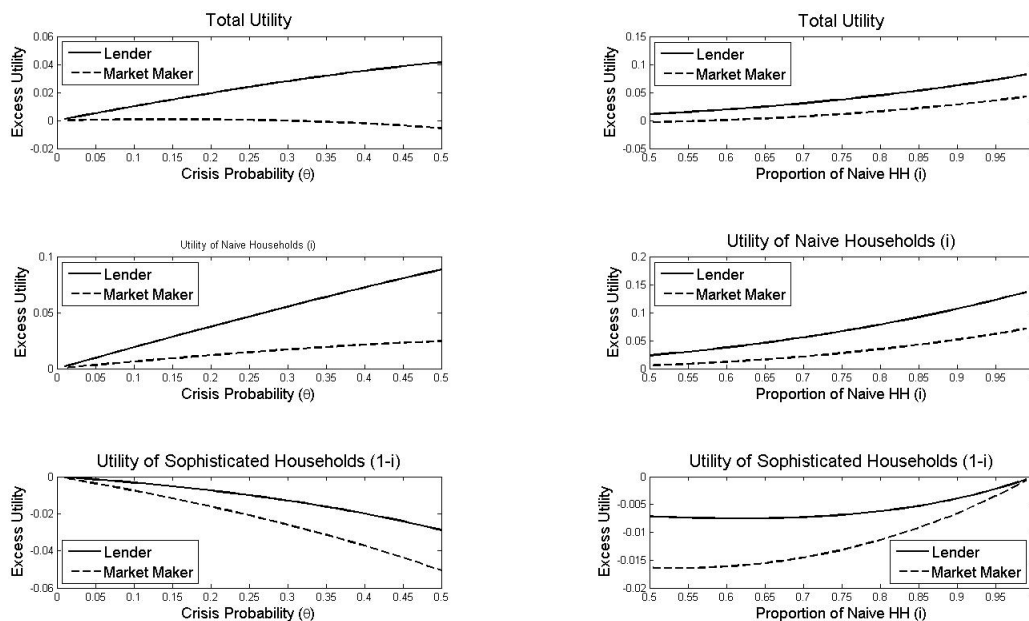


Figure 2: CB Interventions in an Infrequent Crisis Environment

improving. Having a higher proportion of naive households in the economy implies a severe price reduction in the crisis state without policy actions. Consequently a MMOR outperforms the "not acting strategy" when there are many naive households in the economy. Even more interesting is the performance of a LOLR. It not only permanently outperforms the MMOR but also the "not acting strategy".

7 Conclusion

This paper extends the model of Fecht (2004) in a way that allows for an analysis of the role of the interaction of funding liquidity and market liquidity for the propagation of liquidity shocks in a financial system and the translation of liquidity risk into solvency risk of banks. The main model innovation is to consider a commonly known ex-ante probability of a future financial

crisis which implies that banks and households anticipate future crisis events at the time when they decide on investing funds in liquid short-term assets (reserves) and illiquid long-term assets.

We find a general incentive for banks to hold liquidity buffers. They insure banks against funding liquidity risk due to adverse market situations in the future. Whether banks will effectively hold a liquidity buffer in the equilibrium, however, depends on a threshold ex-ante probability for a future crisis that is endogenously determined by the interplay of the economic fundamentals of the model (eg return of long-term investment projects, probability of liquidity shocks on households) and the endogenously arising structure of the financial system (bank-dominated vs. market oriented). Observing an ex-ante crisis probability below the endogenous threshold probability makes banks not to hold liquidity buffers. In this case a liquidity shock on one of the banks in our model financial system leads to contagion of the other banks with the financial markets serving as propagation mechanism. When banks, however, prefer to hold liquidity buffers there is no contagion. We further characterize banks' decisions to hold liquidity buffers by means of simulating the complex interaction between the ex-ante crisis probability, the structure of the financial system, and the fundamentals of the economy.

Our results shed some light on the effects of mark-to-market accounting and the potential value of liquidity risk stress tests and regulatory liquidity requirements – although not explicitly considered in the model.

The model of our paper implicitly assumes that long-term assets are valued at (liquidity-driven) market prices. This valuation is taken into account when households decide on running a bank or not. When they decide to run, the affected bank starts to sell long-term assets in the financial market which reduces market liquidity and asset prices. This, in turn, creates a problem for other banks in the financial system. Their long-term assets are also evaluated at market prices. Decreasing market prices, then, reduce asset values which may induce households also to run other banks. In other words, our results implicitly show that mark-to-market accounting fosters the propagation of shocks in the model financial system via financial markets.

Moreover, this mechanism helps to understand why liquidity risk stress tests may be valuable as a part of a bank's liquidity risk measurement and management. Liquidity risk stress testing is a tool that detects vulnerabilities in a bank's liquidity status in a crisis situation. In addition liquidity stress tests help to quantify the size of a potential crisis impact and adequate countermeasures as well. In our model vulnerabilities arise from the mark-to-

market evaluation of banks' long-term assets which are used to fund higher repayments to households in case of a run. Based on stress test results, a bank might be better able to decide on the adequate amount of liquid assets (reserves). This, in turn, makes a bank less prone to a bank run and reduces pressure on banks to sell long-term assets in financial markets. As a consequence, market prices become less volatile and contagion becomes less likely. In other words, our results suggest that liquidity risk management based on liquidity stress tests improves the soundness of a certain bank and generates positive external effects for the financial system as a whole.

However, in our model banks' decisions to hold a liquidity buffer trade off positive effects from a more stable financial system against other effects arising from the fundamentals (return of long-term investment projects, share of households active in asset markets, etc.). Against this background liquidity regulation, eg in the form of the internationally harmonized liquidity standards that have been recently published by the Basel Committee on Banking Supervision (BCBS), may be desirable. These standards aim at ensuring that banks hold sufficient amounts of liquid funds to meet payment obligations not only in the short run (30 days) but in particular also in the long run (one year)(cf. BCBS, 2010b). Reliable conclusions of the effective value of liquidity risk stress tests and liquidity regulation, however, requires to extent our model in this regard which is left for future analysis.

Appendix A

Equilibrium conditions given infrequent crisis

Deriving the crisis price: Inserting 8 and (BC_c) from $(P2)$ in the market clearing condition for the crisis period (MC_c) yields:

$$\begin{aligned}
 p_c(1-l) &= [(1-l)p_c + l](1-\pi)(1-i) \\
 p_c(1-l)(1-(1-\pi)(1-i)) &= l(1-\pi)(1-i) \\
 p_c &= \frac{l}{(1-l)} \cdot \frac{(1-\pi)(1-i)}{(1-(1-\pi)(1-i))} \tag{18}
 \end{aligned}$$

$$\frac{l}{(1-l)} = \frac{\frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R}}{\frac{(1-\pi)[(1-i)R + ip_n] + \pi p_n R - \pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R}} = \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n]}$$

Inserting in ?? gives

$$\begin{aligned}
 p_c &= \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n]} \cdot \frac{(1-\pi)(1-i)}{(1-(1-\pi)(1-i))} \\
 p_c &= \frac{\pi p_n R}{[(1-i)R + ip_n]} \cdot \frac{(1-i)}{(1-(1-\pi)(1-i))}
 \end{aligned}$$

Deriving the per capita repayment in crisis Inserting (CMP) and (8) in (BC_c) from $(P2)$

$$\begin{aligned}
 d_c &= \left(1 - \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R}\right) \frac{\pi p_n R}{[(1-i)R + ip_n]} \\
 &\quad \cdot \frac{(1-i)}{(1-(1-\pi)(1-i))} + \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R} \\
 d_c &= \frac{(1-\pi)[(1-i)R + ip_n]}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R} \cdot \frac{\pi p_n R}{[(1-i)R + ip_n]} \cdot \frac{(1-i)}{(1-(1-\pi)(1-i))} + \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R} \\
 d_c &= \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R} \cdot \frac{(1-\pi)(1-i)}{(1-(1-\pi)(1-i))} + \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R} \\
 d_c &= \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R} \cdot \left(1 + \frac{(1-\pi)(1-i)}{(1-(1-\pi)(1-i))}\right) \\
 d_c &= \frac{\pi p_n R}{(1-\pi)[(1-i)R + ip_n] + \pi p_n R} \cdot \frac{1}{1-(1-\pi)(1-i)}
 \end{aligned}$$

Deriving the no-arbitrage condition If the bank would only hold liquidity it could repay all early withdrawing depositors, impatient and patient sophisticated ones, with liquidity and use liquidity to buy assets at the no-crisis price p_n to refinance the repayments to impatient naive depositors. As a consequence, when only holding liquidity a bank would face the budget constraint:

$$\begin{aligned}\pi d + (1 - \pi)(1 - i)d + (1 - \pi)i\frac{p_n}{R}d &= 1 \\ \left[\pi + (1 - \pi)(1 - i) + (1 - \pi)i\frac{p_n}{R} \right] d &= 1\end{aligned}$$

Following that strategy the bank could pay depositors in $t = 1$ and $t = 2$

$$d = \frac{1}{\pi + (1 - \pi)(1 - i) + (1 - \pi)i\frac{p_n}{R}}$$

Since holding only liquidity permits the bank to pay $d_c = 1$ in the crisis period, expected utility that a bank could provide to naive households would be

$$(1 - \theta)[\pi x_1 + (1 - \pi)]i\frac{1}{\pi + (1 - \pi)(1 - i) + (1 - \pi)i\frac{p_n}{R}} + \theta[\pi x_1 + (1 - \pi)]i$$

A bank that only invests in assets and sells some of them off in $t = 1$ to refinance the short-term repayments would be able to repay

$$d = \frac{1}{\frac{\pi}{p_n} + \frac{(1 - \pi)(1 - i)}{p_n} + (1 - \pi)i\frac{1}{R}}$$

Given that during a crisis the bank would have to sell off all its assets at the equilibrium price p_c expected utility of naive households depositing at a bank that only invests in asset amounts to:

$$(1 - \theta)[\pi x_1 + (1 - \pi)]i\frac{1}{\frac{\pi}{p_n} + \frac{(1 - \pi)(1 - i)}{p_n} + (1 - \pi)i\frac{1}{R}} + \theta[\pi x_1 + (1 - \pi)]ip_c$$

Thus banks will be indifferent between holding liquidity and investing in assets if

$$(1 - \theta)\frac{1}{\frac{\pi}{p_n} + \frac{(1 - \pi)(1 - i)}{p_n} + (1 - \pi)i\frac{1}{R}} + \theta p_c = (1 - \theta)\frac{1}{\pi + (1 - \pi)(1 - i) + (1 - \pi)i\frac{p_n}{R}} + \theta$$

$$\begin{aligned}
p_c &= \frac{(1-\theta)}{\theta} \left(\frac{1}{\pi + (1-\pi)(1-i) + (1-\pi)i\frac{p_n}{R}} - \frac{1}{\frac{\pi}{p_n} + \frac{(1-\pi)(1-i)}{p_n} + (1-\pi)i\frac{1}{R}} \right) + 1 \\
p_c &= \frac{(1-\theta)}{\theta} \left(\frac{1}{\pi + (1-\pi)(1-i) + (1-\pi)i\frac{p_n}{R}} - \frac{p_n}{\pi + (1-\pi)(1-i) + (1-\pi)i\frac{p_n}{R}} \right) + 1 \\
p_c &= 1 - \frac{(1-\theta)}{\theta} \left(\frac{p_n - 1}{\pi + (1-\pi)(1-i) + (1-\pi)i\frac{p_n}{R}} \right)
\end{aligned}$$

Appendix B

Frequent crisis

Assume now that the crisis probability is high. Thus each bank anticipates that it is likely that either itself or the bank in the other region is subject to a run. With a high probability of a crisis on either bank it might be worthwhile for banks to hold excess reserves.

Given our assumption that a run on a bank is characterized by *all* patient depositors withdrawing, remaining liquid during a run of its own depositors would require a bank to invest all funds in reserves. However, in this case the bank would be redundant.

However, if the probability of depressed asset prices is high because of fire sales of the bank in the other region, banks might want to hold excess liquidity to repay withdrawing patient sophisticated depositors rather than refinancing these repayments with the proceeds from asset sales. Given that banks hold liquidity in excess of what they need in normal times to refinance the early withdrawals, they use this extra liquidity in normal times to increase the $t = 2$ repayment. As long as $p_n < R$ they would do so using this excess liquidity to buy assets. Thus, only $p_n = R$ can be an equilibrium price in normal times given that banks hold excess liquidity. Moreover, at $p_n = R$ sophisticated patient households do not benefit from pretending being impatient and withdrawing the funds from the bank. Since both strategies – holding the funds with the bank or withdrawing the funds and investing the proceeds directly – yield the same return, sophisticated households are indifferent and we assume that they will keep the funds with the bank.¹² Having already determined the price in normal times and the arbitrage free condition (NAC), p_c follows from reinserting p_n in NAC,

$$p_c = 1 - \frac{1 - \theta}{\theta} \cdot (R - 1)$$

First, p_c does only depend on the return of the long-run technology and the crisis probability, but neither on the amount of people having an efficient access to the secondary market, nor on the deposit contract repayment. Secondly, if the crisis probability converges to $\theta \rightarrow 1$, $p_c = 1$ as we already

¹²The different strategies (keeping the funds with the bank or withdrawing and investing the proceeds in assets) have the same effect on banks budget constraint given $p_n = R$. Therefore both are inline with optimization problem (P4).

discussed in the permanent crisis case. Finally, the banks have to hold enough liquidity in excess such that the arbitrage free price equals the cash in the market price.

If a bank contains the destabilizing effect of asset price deterioration with excess reserves, the optimal deposit contract banks offer solves the optimization problem (P4) where we already replaced $p_n = R$.

$$(P4) \left\{ \begin{array}{l} \max_{l,k} \quad E[U] = (1 - \theta)[\pi i x_1 d_1 + (1 - \pi) i d_2^n] + \\ \quad \frac{\theta}{2} [\pi i x_1 d_1 + (1 - \pi) i d_2^c] + \frac{\theta}{2} [\pi i x_1 d^c + (1 - \pi) i d^c] \\ \quad \max \left\{ 1, \frac{R}{p_c} \right\} d_1 \geq d_2^c \quad (IC_A^c) \\ \quad d_1 \leq d_2^n \quad (IC_B^n) \\ \quad d_1 \leq d_2^c \quad (IC_B^c) \\ \quad d_1 \leq \frac{l}{\pi} \quad (BC_1^n) \\ \quad d_2^n \leq \frac{(1-l-l_e)R+l_e}{1-\pi} \quad (BC_2^n) \\ \quad d_1 \leq \frac{l+l_e+k p_c}{1-(1-\pi)i} \quad (BC_1^c) \\ \text{s.t.} \quad d_2^c \leq \frac{(1-l-l_e-k)R}{(1-\pi)i} \quad (BC_2^c) \\ \quad d_c \leq (1-l-l_e)p_c + l + l_e \quad (BC_c) \\ \quad (1-\pi) \left[\frac{\theta}{2} \frac{R}{p_c} d_2^c + \frac{\theta}{2} \frac{R}{p_c} d^c + (1-\theta) d_2^n \right] + \\ \quad + \pi x_1 \left[\frac{\theta}{2} d_1 + \frac{\theta}{2} d^c + (1-\theta) d_1 \right] \quad (PC) \\ \quad > \max \left\{ (1-\pi) \left[\theta \frac{R}{p_c} + (1-\theta) \right] + \pi x_1, \right. \\ \quad \left. (1-\pi)R + \pi x_1 [\theta p_c + (1-\theta)R] \right\} \end{array} \right.$$

Banks maximize again naive households' expected utility taking now into account that the bank might be in three different states of the world: 1) no bank is affected by a run, 2) the bank in the other region is affected by a run and asset prices are depressed due to the other bank's fire sales, and 3) the bank itself being affected by a run.

If the bank is itself affected by the run, which happens with probability $\theta/2$, it is liquidated. In this case it repays d^c , which is the per capita liquidation proceeds determined by (BC_c) , in $t = 1$ to patient and impatient depositors. If the bank in the other region is experiencing a run, sells off assets and depresses asset prices, the considered bank uses its liquidity holdings $l + l_e$, i.e. the regular liquidity holdings for repayment of impatient households plus excess reserves held to refinance the withdrawals during phases of depressed asset prices, together with the proceeds from asset sales $p_c k$ to

repay d_1^c to impatient depositors and patient sophisticated ones (see (BC_1^c)). The remaining asset holdings are used to refinance the repayments d_2^c to patient naive households who only withdraw deposits in $t = 2$ (see (BC_2^c)).

If neither bank is affected by a run, which happens with probability $(1 - \theta)$, the bank repays the contractual amount d_1 to impatient and d_1^n to patient depositors.

Assuming that the utility function is steeper than the budget constraint, such that a bank dominated financial system evolves, either $d_1 = d_2^n$ or $d_1 = d_2^c$ leads to the optimal deposit contract. If $d_2^n \geq d_2^c$ it follows that $d_1 = d_2^c$ is the optimality condition and vice versa. Assuming that $d_1 = d_2^c$ is binding and taking (BC_1^c) and (BC_2^c) together, banks end up with the following budget constraint:

$$d \leq (1 - l - l_e - k)R + l_e + l + kp_c \quad (BC^c)$$

However, if the fundamentals of the economy are such that $d_1 = d_2^n$ leads to the optimal deposit contract, the banks budget constraint can be derived by combining (BC_1^n) and (BC_2^n) .

$$d \leq (1 - l_e - l)R + l_e + l \quad (BC^n)$$

Setting $BC^n \leq BC^c$ leads to

$$0 \leq -k(R - p_c). \quad (19)$$

Thus, the optimal contract is given by $d_1 = d_2^n$ if $BC^n \leq BC^c$ which holds for $k > 0$. Vice versa, if k is negative, meaning the banks hold enough excess liquidity in order to demand assets at secondary market in the crisis state when the other bank is affected by a bank run, the optimal contract is given by $d_1 = d_2^c$. This will happen if there are only few patient sophisticated households in the economy. In this case only a small fraction of households have access to the financial market, implying huge price deterioration and making it especially interesting for banks to hold excess liquidity. In other words, in this economy banks speculate for a crisis and hope for "cheap" assets at the secondary market in the crisis state.

First we will now derive the result for the case $d_1 = d_2^n$ which is denoted by *I* and subsequently we focus on the second case $d_1 = d_2^c$ which is highlighted by *II*. Assuming $d_1 = d_2^n$, the two budget constraints (BC_1^n) and (BC_2^n) can be summarized to

$$\pi d^I R + (1 - \pi)d^I = (1 - l_e^I)R + l_e^I.$$

Since the excess liquidity holdings have to suffice such that the outpayment to patient sophisticated can be financed without being affected by contagion, we have

$$d^I(1 - \pi)(1 - i) = l_e^I + k^I p_c. \quad (20)$$

The excess liquidity cushion plus the cash inflow from selling assets to sophisticated households have to suffice in order to repay the contractual amount to patient sophisticated households. Reinserting l_e^I in the budget constraint and solving for d^I gives the deposit contract repayment as function of k :

$$d^I(k^I) = \frac{(R + k^I p_c)(R - 1)}{R - (R - 1)(1 - \pi)i} \quad (21)$$

It is particularly interesting that the deposit repayment is increasing with the amount of assets being sold to sophisticated households in a financial crisis. If $k = 0$, the bank does not sell any assets at the secondary market and the demand of assets is only satisfied with assets being sold from the bank experiencing a bank run. In this case the contractual repayment equals the deposit contract repayment in the "no crisis case" where θ is set to 0, otherwise banks offer a higher deposit contract repayment when bank runs happen frequently. This is of course in line with the observations we made in the "infrequent crisis case", where banks were able to offer a higher short-term repayment, due to a higher asset price in normal times.

The remaining unknown, i.e. the amount of assets being sold to patient sophisticated households (k) in a crisis, can be derived from the market clearing condition.

$$p_c[1 - l^I(k^I) - l_e^I(k^I) + k^I] = [d_c^I(k^I) + d^I(k^I)](1 - \pi)(1 - i)$$

All patient households will demand assets. Those, who withdraw from the sound bank, receive d , while those from the struggling bank only gain d_c . Assets are supplied by the struggling bank which is liquidated and sells $(1 - l - l_e)$ assets. The sound bank will also sell k assets to sophisticated households. Obviously, the deposit repayment to early consuming households is satisfied with pure liquidity, thus $l = \pi d^I(k)$ and the market clearing condition simplifies to:

$$p_c[1 - \pi d^I(k^I) - l_e^I(k^I) + k^I] = [d_c^I(k^I) + d^I(k^I)](1 - \pi)(1 - i)$$

These are linear, but quite complex functions in k^I . We forgo deriving the explicit solution here, but we will show the solution graphical later on and

proceed now by developing the optimal contract for an economy in which banks speculate for a crisis by holding huge amounts of liquidity reserves, i.e. $d_1 = d_2^c$, for a sufficiently steep utility function. Inserting $d_1 = d_2^c$ in (BC_1^c) and (BC_2^c) , respectively yields

$$[1 - (1 - \pi)i]d^c = l + l_e^c + kp_c \quad (22)$$

$$[(1 - \pi)i] \frac{d^c}{R} = (1 - l - l_e^c - k) \quad (23)$$

Taking both budget constraints together gives

$$1 - k(1 - p_c) = d^c \left[1 - \frac{1}{R} \right] \quad ((BC))$$

Solving the budget constraint for d allows us to derive the optimal deposit contract.

$$d^{II}(k^{II}) = \frac{[1 - k^{II}(1 - p_c^{II})]R}{R - (R - 1)(1 - \pi)i} \quad (24)$$

Self evidently, both solutions, d^I and d^{II} , are identical, given $k^I = k^{II} = 0$. Since $k^{II} \leq 0$, i.e. the sound bank plans to buy assets at the secondary market in a crisis, and $p_c^{II} \leq 1$ in equilibrium, the deposit repayment increases with the amount of assets the sound bank buys at the secondary market. Assuming a symmetric portfolio holding of banks in both regions, we know that the consumption of sophisticated as well as naive impatient households must be refinanced by storing: $d = l/\pi$. This also means that the withdrawals of patient sophisticated households in times of depressed asset prices are refinanced with excess liquidity and proceeds of asset sales:

$$(1 - \pi)(1 - i)d^{II}(k^{II}) = l_e^{II}(k) + p_c^{II}k^{II}$$

The remaining unknown to be determined is k^{II} , by solving the secondary market equilibrium k^{II} ,

$$p_c[1 - \pi d^{II}(k^{II}) - l_e^{II}(k^{II}) + k^{II}] = [d_c^{II}(k^{II}) + d^{II}(k^{II})](1 - \pi)(1 - i).$$

Finally, Figure 3 shows how different fundamentals, that is the crisis probability θ and the fraction of naive households i , lead to the emergence different financial systems. If the fraction of naive households is low, the cross subsidization of sophisticated households becomes too costly for naive

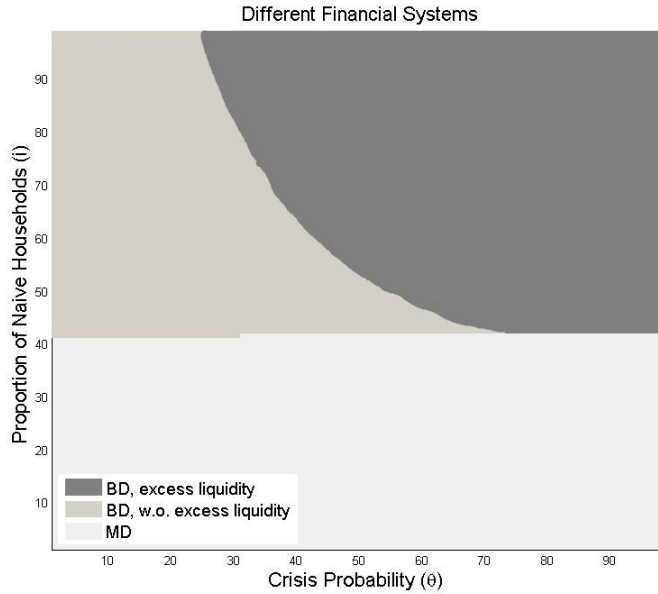


Figure 3: Emergence of Different Financial Systems

households and a market oriented financial system will evolve. On the other hand, if the fraction of naive households is high and the crisis probability is low, banks have no incentive to hold liquidity in excess and a particular fragile bank dominated financial system evolves. Holding no liquidity buffers, leads to a propagation of financial shocks and to a collapse of the entire financial system in the crisis state. Finally, if there are many naive households in the economy and the crisis probability is high, a bank oriented financial system evolves where banks hold excess liquidity and thus no financial contagion takes place.

7.1 CB Intervention in a Frequent Crisis Environment

In a frequent crisis economy banks do have an incentive to prepare for an adverse event by holding excess liquidity. It is therefore not obvious, if CB interventions dominate the self-insurance. Figure 4 depicts the excess utility of the different CB interventions. Excess utility is again defined as the difference between the utility of a deposit contract with policy intervention and without intervention. It can be clearly seen that in most of the cases

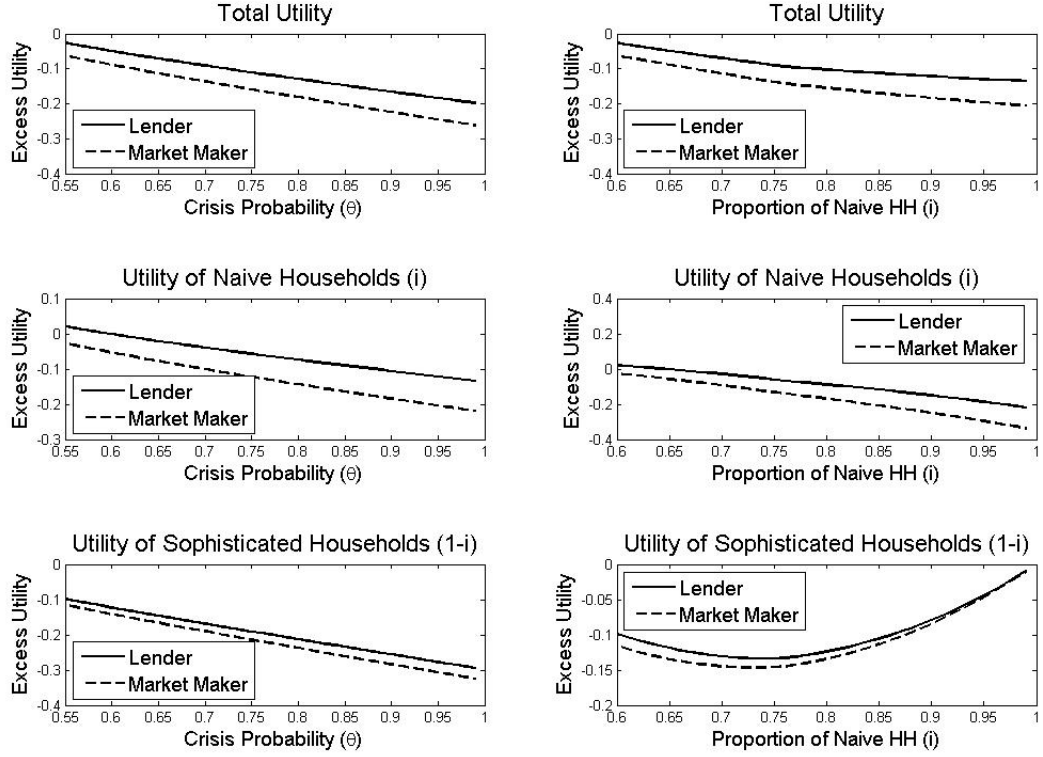


Figure 4: CB Interventions in an Infrequent Crisis Environment

the self insurance dominates any CB interventions. However, if the expected excess utility of naive households is bigger than the utility without CB interventions, the bank will offer a contract which counts on the CB intervention, even though the total excess utility is negative. Due to the contestability of the banking sector, banks only take the utility of naive households into account. Thus, a credible commitment of a CB to intervene can adversely affect the self-insurance of banks, causing systemic risk and destroying expected utility.

To summarize, when a bank faces a run the optimal policy response is acting as a lender of last resort. This is especially beneficial when banks do not protect it self against the propagation of financial shocks. On the other hand,

self-insurance is most of the times optimal in a frequent crisis economy. At the discontinuity point $\hat{\theta}$, where banks start holding liquidity in excess, CB interventions increase the expected utility of naive households, but decrease the total expected utility. Consequently, banks start relying on the CB and do not insure against a liquidity crisis, causing a change from a sound financial system to a fragile.

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