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Working Paper — Digitized Version

Sustainability in closed and open economies

Kiel Working Paper, No. 741

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Suggested Citation: Klepper, Gernot; Stähler, Frank (1996) : Sustainability in closed and open economies, Kiel Working Paper, No. 741, Kiel Institute of World Economics (IfW), Kiel

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Kiel Working Papers

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SUSTAINABILITY IN CLOSED AND OPEN ECONOMIES

by

Gernot Klepper and Frank Staehler

June 1996



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ISSN 0342 - 0787

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SUSTAINABILITY IN CLOSED AND OPEN ECONOMIES

by

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Key words. International trade, factor mobility, sustainability, international environmental problems.

JEL classification: F 20, Q 20.

Abstract. *This paper discusses unilateral sustainability policies for tradable resources in closed and open economies. The effects of sustainability policies are modelled in an intertemporal, competitive framework by applying different sustainability rules which are introduced unilaterally in the domestic country. The paper shows that no sustainability rule will lead to a slower rate of extraction of the resource. Instead, resource extraction is increased in both countries. It is also shown that the foreign country may well gain in terms of consumption and real income from such unilateral sustainability policies but not in terms of sustainability.*

1. Introduction

The impact of environmental policies on international trade has recently received considerable attention. Several papers have dealt with this impact either in a Heckscher-Ohlin framework or a strategic trade theory setting (for an overview, see Ulph, 1994). In a Heckscher-Ohlin world where environmental resources are an intersectorally mobile but internationally immobile factors, strict environmental policies induce a country to specialize on resource-extensively produced goods. This structural change is the natural consequence of moving from relative resource abundance to relative resource scarcity. If environmental policies substitute for trade policies and markets are oligopolistic, the impact of environmental policies is unclear. Whether a country has an incentive to cut or to exceed the socially optimal regulation level in order to shift rents homewards depends on the parameters of the model and the type of competition. Both approaches assume that environmental resources are internationally immobile.

In some cases, however, environmental deterioration depends on the excessive use of resources which are mobile across countries. The most outstanding example are fossil fuels which are rarely used in the country of production. Mobility of factors is a feature in international trade modelling which has been rarely dealt with in a general equilibrium framework (e.g. Svensson, 1984, Ethier, Svensson, 1986). One reason is that factor trade and goods trade are in many cases perfect substitutes, and trade theory has mainly focused on the structural changes associated with trade. When structural changes may be caused by both features, there is no need to take recourse to factor mobility when the conventional Heckscher-Ohlin model produces the same result. From an environmental policy perspective, however, the impact on trade flows itself deserves attention. If a resource is mobile, strict domestic environmental policies may increase the resource imports from other countries. If the increase of resource imports overcompensates the decrease of the domestic resource extraction, total resource use is increased. This paper shows that this case cannot be ruled out such that a unilateral environmental policy may imply an increased resource use. As we focus on the trade aspect, we employ a simple model of one tradable good, one tradable factor, one non-tradable factor and two countries.

There are two strands of literature which set the stage for the sustainability rules to be discussed by this paper. On the one hand, the question was raised which investment rule could guarantee a constant, strictly positive consumption level when production is based on the use of capital and non-renewable resources. The so-called Hartwick Rule (Hartwick, 1977, 1978) set up an investment calculus which ensures constant consumption in a closed economy. Other papers (Asheim, 1986, Hartwick, 1994) have considered necessary changes of this rule when resource trade has to be taken into ac-

count. As these papers focus on the sustainability of a certain consumption level, they deal only implicitly with the impact on resource extraction and resource trade.

On the other hand, rules have been developed by taking resource considerations of intergenerational justice. The principle of intergenerational equity would require each generation to use only so much of a resource such that it will leave constant the stock which it has inherited. In other words, the resource extraction must not exceed its regeneration. This concept is called "strong sustainability" (Pearce, Atkinson 1993) since it sets a physical constraint for every single resource.

However, for nonrenewable resources strong sustainability is not a feasible and meaningful approach because it would require not to extract any of the resource as the example of fossil fuels illustrates. In addition, there is no reason why different resources should not be substitutable, i.e. why the decrease in one stock should not be compensated by an increase in another resource stock. Therefore, the concept of "weak sustainability" was introduced (Pearce, Atkinson 1993). Weak sustainability as an intergenerational equity criterion takes into account the limited but possible substitutability of resources and requires not a constant stock of each resource but of the value of the total resource stock. In this case, resources are defined in a wide sense including natural, environmental, and man-made capital, including human-capital. We will refer to this rule as the rule of constant wealth.

The papers which deal with sustainability aspects in closed and in open economies have not discussed the increase of resource extraction as a reaction on capital accumulation as explicit as it is done here. As these papers and the rules they develop for investment originate from quite different objectives, we do not try to evaluate their appropriateness. Instead, we take the above mentioned two rules as they stand, and we discuss their impact on resource extraction, on resource trade when economies are open, on production, and on consumption.

The paper is organized as follows. Section 2 gives the model and the model assumptions. Section 3 discusses the two sustainability rules in a closed economy. Section 4 discusses four sustainability rules in an open economy and compares the results with the previous section. In section 5 a numerical example illustrates the possible consumption and income paths. Section 6 concludes the paper.

2. The Model

The model is a simple version of the classical growth model with exhaustible natural resources by Stiglitz (1974) which is extended to incorporate different sustainability rules and international trade. The adoption and extension of this model is due to three reasons. Firstly, in order to derive at some conclusions, the fairly general assumption of linear-homogeneous production functions cannot be made. Linear-homogenous production functions make no assumption about the behavior of the income factor shares in the course of time, and thereby imply ambiguity at a very early stage of the model which has not addressed trade issues yet. As we are strongly interested in trade effects, we keep the tradition of the Stiglitz-paper and assume constant income factor shares.

Secondly, modelling intertemporal behavior under competitive market conditions entails several complications unless the existence of future markets for all goods and factors is assumed. These complications arise because under perfect competition it is assumed that all resource owners base their resource extraction policy on a set of given current and future resource and capital prices in order to maximize the present value of their assets. If such future markets do not exist, an assumption is needed explaining which price resource owners expect in the next period. Since we are interested in long-run effects and indeed in intergenerational issues, we find that the lack of future markets is a more realistic assumption, and therefore this paper assumes that resource owners expect resource prices to rise with the current price of capital, i.e. the marginal productivity of capital.¹ This means that the agents use the Hotelling Rule for predicting resource price changes. If we assume the existence of all future markets, we would basically discuss a static problem without any case for policy intervention because it is well known since Arrow and Delreau that the existence of all relevant markets guarantees efficiency. As future markets do not exist, we also assume that trade is balanced at each point in time (for a paper adopting an intertemporal budget constraint, see van Geldrop, Withagen, 1993).

Thirdly, we are interested in a scenario in which doomsday is avoidable in principle because any intergenerational rule makes only sense if every generation has a chance to survive. Solow (1974) has shown that any production based on linear-homogenous functions must be of the Cobb-Douglas type in order to allow to realize strictly positive consumption levels in the future.

We assume two countries, a home country and a foreign country, which both produce one good Y by the use of two factors, resources R and a composite factor K . All terms

¹ When future markets do not exist, the resource extraction policy of a competitive economy is indeterminate and may imply too slow or too quick exploitation. See Stiglitz (1974).

referring to the foreign country will be denoted by a star. Resource extraction will be denoted by E and resource use in production will be denoted by R . Obviously, $R = E$ holds in a closed economy. Resources are extracted from a given exhaustible and non-renewable stock which is denoted by S . The composite factor K will sometimes be referred to as capital and is, for simplicity, assumed not to depreciate. Good Y is the numeraire in this model. Production is based on perfectly competitive market structures. According to the usual Heckscher-Ohlin assumptions, capital is internationally immobile. Resources, however, are assumed to be internationally mobile if economies are open. Output of good Y which is mobile in the case of open economies is produced with a Cobb-Douglas production function

$$(1) \quad Y(t) = K(t)^\alpha R(t)^{1-\alpha}.$$

The resource stock decreases through extraction such that

$$(2) \quad E(t) \geq 0, \quad -\dot{E}(t) = \dot{S}(t)$$

holds. The dot denotes the first derivative with respect to time. Profit maximization under perfect competition requires factor prices to equal the value of their marginal product

$$(3a) \quad \pi_R(t) = (1 - \alpha) \frac{Y(t)}{R(t)},$$

$$(3b) \quad \pi_K(t) = \alpha \frac{Y(t)}{K(t)} < 1.$$

π denotes the factor prices. In order to restrict the analysis to realistic cases, we assume that the price of capital does not exceed unity. The output/capital ratio is defined by

$$(4) \quad \beta(t) = \frac{Y(t)}{K(t)}.$$

Resource owners will extract resources such that the growth rate of the resource prices is equal to the interest rate at each point in time, i.e.

$$(5) \quad \frac{\dot{\pi}_R(t)}{\pi_R(t)} = \pi_K(t) = \alpha \frac{Y(t)}{K(t)} = \alpha \beta(t).$$

(5) mirrors the Hotelling Rule for resource extraction².

From the production technology (1), profit maximization (3a,b), and the Hotelling Rule (5), a number of identities for the growth rates of Y and R can be derived. The growth rate of output Y is

$$(6) \quad g_Y = \frac{\dot{Y}(t)}{Y(t)} = \alpha g_K + (1 - \alpha) g_R$$

where g_K denotes the growth rate of the capital stock, and g_R denotes the growth rate of the resource input in production.

Differentiating (3a) w.r.t. time and using (5) yields

$$(7) \quad g_Y - g_R = \alpha \beta(t).$$

In the case of an open economy, it is assumed that there is no intertemporal trade, i.e. excess demand for goods and resources and the balance of payments should be zero at any moment in time. The conditions for international market clearing and the balance of payments are given through

$$(8a) \quad \forall t: \quad Y(t) - C(t) - \dot{K}(t) + Y^*(t) - C^*(t) - \dot{K}^*(t) = 0,$$

$$(8b) \quad \forall t: \quad R(t) + R^*(t) = E(t) + E^*(t),$$

$$(8c) \quad \forall t: \quad \pi_R(t) [E(t) - R(t)] + Y(t) - C(t) - \dot{K}(t) = 0.$$

C denotes consumption in the respective country.

The Hotelling Rule describes the extraction path for a closed economy or for the world supply in open economies. We, therefore, need to make a specific assumption about the local resource extraction decisions. We assume that both economies are equally endowed with resources at the time of introducing sustainability policies. If the two economies possess the same resource stock at time t_0 , then the growth rates of extraction are also equal since they depend on the growth rate of the resource price.

² (5) is the Stiglitz-assumption of short-run perfect foresight. It should be clear that any policy which varies π_K does only imply a revised expected change of π_R under these assumptions.

Before turning to specific sustainability rules, the growth of an unregulated economy is modelled. In the case of - so to speak - laissez-faire, we adopt the standard assumption that every country saves a constant fraction σ of production and invests it in the capital stock K such that:

$$\forall t: \quad \dot{K}(t) = \sigma Y(t) \Leftrightarrow g_K^L = g_K = \sigma \beta(t),$$

$$\sigma < 1 - \alpha.$$

The superscript L denotes the laissez-faire case for both countries. In order to simplify the presentation, we will only differentiate the growth rates. The qualification for σ will ensure that domestic investment is too low compared to a sustainable development at least in the beginning.

If both countries invest the same rate and are equally endowed with capital and resources at the beginning, both economies remain identical, prices remain the same, and trade will never take place even if both economies are open. The growth rate of output will be determined by (6) and (7) such that

$$(9) \quad g_Y^L = g_Y^L = -(1 - \alpha)\beta(t) + \sigma\beta(t)$$

holds. Similarly, resource extraction and resource use will fall according to

$$(10) \quad g_R^L = g_R^L = -[1 - \sigma]\beta(t).$$

Since $g_p = g_Y - g_K$,

$$(11) \quad g_p^L = g_p^L = -(1 - \alpha)\beta(t)$$

indicates that the output/capital ratio is falling as well.

We would like to stress again that adopting the Stiglitz-model with short-run perfect foresight means that all resource owners choose an extraction path the price of which follows the Hotelling's Rule. Since future capital prices are unknown to resource owners, they adjust to policy variations through changes in their extraction path. Sections 3 and 4 will compare the bilateral laissez-faire policy with a unilateral sustainability policy of the domestic country. This policy considers the voluntary saving and adds public saving until the respective investment rule is met.

3. Sustainability in Closed Economies

For the closed economy, we consider two rules:

$$(a) \quad \forall t: \quad \dot{K}(t) = \pi_R(t) R(t),$$

$$(b) \quad \forall t: \quad \pi_K(t) \dot{K}(t) = \pi_R(t) R(t).$$

These sustainability rules require that the domestic country compensates its resource use or its resource extractions by investment. If the saving in the laissez-faire case does not suffice to meet the respective rule (which is necessary to face a sustainability problem), governmental action will ensure that the sum of private and public spending fulfils the respective sustainability rule. Rule (a) represents the well-known Hartwick Rule (Hartwick, 1977, 1978) which requires to invest the resource rent in physical capital. Rule (b) requires to compensate the resources extracted or used through an increase of the present value of the capital stock by the same amount by which the value of the resource stock has fallen.

In a closed economy, there are no repercussion of trade on the resource use of the domestic country. According to (a) and (b), the growth rates of capital are

$$(12a) \quad g_K^a = (1 - \alpha)\beta(t),$$

$$(12b) \quad g_K^b = \frac{1 - \alpha}{\alpha},$$

respectively. The superscript denotes which rule is under consideration in the determination of a growth rate. (12a) shows that a marginal saving rate α which falls short of $(1 - \alpha)$ leads to a too low degree of investment. As we have assumed that the price of capital falls short of unity, $\alpha\beta$ falls short of unity (see (5)) and the growth rate of (12b) exceeds the growth rate of (12a). From (6) and (7), one may now compute the growth rates of the domestic resource use:

$$(13a) \quad g_R^a = -\alpha\beta(t),$$

$$(13b) \quad g_R^b = \frac{1 - \alpha}{\alpha} - \beta(t).$$

From (12) and (13) together with (6) we derive the output growth rates:

$$(14a) \quad g_Y^* = 0,$$

$$(14b) \quad g_Y^b = \frac{1-\alpha}{\alpha} [1 - \alpha\beta(t)].$$

Rule (a) guarantees a constant output level. According to (3) and (4), $\alpha\beta(t) = \pi_K(t)$. Hence, the output growth rate of rule (b) is positive for empirically relevant cases because rule (b) leads to higher investment levels for all interest rates which fall short of unity.

In a closed economy which employs production technology (1), consumption and the change of consumption are given by

$$(15) \quad C(t) = Y(t) - K(t) \Rightarrow \begin{aligned} \dot{C}^*(t) &= \alpha \dot{Y}(t) \\ \dot{C}^b(t) &= \dot{Y}(t) - \frac{1-\alpha}{\alpha} \dot{K}(t) \end{aligned}$$

(14) and (15) determine the change of consumption in time of rules (a) and (b)

$$(16a) \quad \dot{C}^*(t) = 0$$

$$(16b) \quad \begin{aligned} \dot{C}^b(t) &= \frac{1-\alpha}{\alpha} \left[(1 - \alpha\beta(t)) \dot{Y}(t) - \frac{1-\alpha}{\alpha} \dot{K}(t) \right] \\ &= -[1-\alpha] \beta(t) \dot{Y}(t) \left[\beta(t)^2 - \frac{1}{\alpha} \beta(t) - \frac{1-\alpha}{\alpha} \right] \end{aligned}$$

(16a) is the well-known result of the Hartwick Rule which guarantees constant consumption in closed economies. The sign of (16b) is undetermined in general and depends on $\beta(t)$. Define

$$(17) \quad \mu(\beta) := \beta^2 - \frac{1}{\alpha} \beta + \frac{1-\alpha}{\alpha^2} = 0 \quad \text{with}$$

$$\frac{d\mu}{d\beta} = 2\beta - \frac{1}{\alpha}, \quad \frac{d\mu}{d\beta} = 0 \Rightarrow \beta = \frac{1}{2\alpha}, \quad \frac{d^2\mu}{d\beta^2} = 2,$$

$$\beta = \frac{1}{2\alpha} \pm \frac{1}{\alpha} \sqrt{\alpha - \frac{3}{4}}.$$

If $\mu(\beta)$ is positive (negative), \dot{C} is negative (positive). (16) shows that a negative change is guaranteed if α falls short of 0.75. For $\alpha = 0.75$, a β which exceeds (falls short of) $1/2\alpha = 2/3$, gives a positive (negative) change in consumption. Lower and higher α 's require higher β 's.

Before going into more detail, we consider the feasibility of sustainability policies and the behavior of β . Only such investment rules are feasible for which investment is smaller than total production, i.e. $\dot{K} \leq Y$. The feasibility condition can be written as

$$(18) \quad g_K \leq \frac{Y(t)}{K(t)} = \beta(t).$$

This condition is always met for the Hartwick Rule (a) but it is not automatically fulfilled under rule (b). Since under rule (b) $g_K^b = 1 - \alpha/\alpha$, we need $\beta > 1 - \alpha/\alpha$. Given this restriction on feasibility, (13b) shows that any feasible sustainability policy which applies rule (b) is only possible when the growth rate of resource use is negative.

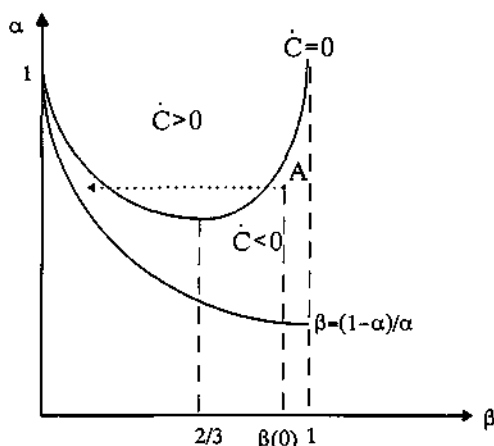
Additionally, we find from (12a), (12b), (14a) and (14b) that the growth rates of β coincide for both rules and are equal to the growth rate of laissez-faire policies (see (11)):

$$(19) \quad g_\beta^a = g_\beta^b = -(1 - \alpha)\beta(t).$$

Under short-run perfect foresight of resource owners, $\beta(0)$ is identical for all cases under consideration (see footnote 2). If β has the same starting-point and the same growth rates under all policies under consideration, $\beta(t)$ is the same for all cases and does not depend on the policy variant. Then, comparing (10) with both (13a) and (13b) reveals that resource use falls slower if either of these rules is adopted. This means that adopting one of the sustainability rules increases resource use compared to laissez-faire investment. The reason is that increasing the capital stock implies a lower marginal productivity of capital. A lower marginal productivity of capital makes resource extraction more profitable compared to resource conservation. Hence, investment policies for a sustainable resource use accelerate resource extraction compared to no sustainability policies. This means that both sustainability rules compensate for resource extraction, but that achieving weak sustainability as it was defined does not mean that fewer resources are extracted. To the contrary, both rules require higher extraction for achieving intergenerational equity.

Since the output/capital ratio is declining over time, we are also able to reconsider the change of consumption under rule (b). Figure 1 depicts $\mu(\beta)$ and the feasibility constraint in the α - β -space. Figure 1 illustrates that under rule (b) consumption may rise or decline depending on the capital productivity and the production function. Suppose that the economy starts with a $\beta > 2/3$ and a sufficiently high α , e.g. point A. Then consumption will be falling. Since β is also falling over time the economy will move along the dotted arrow, hence consumption will increase after the line $\dot{C} = 0$ has been crossed. Later consumption will fall again and eventually the economy will reach a capital productivity where sustainability will be infeasible.

Figure 1: Consumption change and feasibility constraint for rule (b)



4. Sustainability in Open Economies

We now turn to the analysis of a sustainability policy in an open economy. Starting point are two identical economies, the home and the foreign country. Without any policy intervention these countries will grow or shrink at the same rate, there will be no trade, thus they remain identical. The imposition of an investment rule for achieving one of the sustainability objectives will distort this symmetry and will induce trade in the resource and in the commodity.

Under free trade, factor prices equalize such that - given the assumption about the production technology - the factor intensities will equalize as well for all t :

$$(20) \quad \frac{R(t)}{K(t)} = \frac{R^*(t)}{K^*(t)}$$

Differentiation of (20) with respect to time leads to

$$(21) \quad g_R - g_{R^*} = g_K - g_{K^*}$$

g_K represents the growth rate of the capital stock under the sustainability rule and g_{K^*} that of the foreign country without policy intervention. According to equation (21), the difference in the growth rate of capital accumulation has the opposite sign of the difference of the growth rates of resource extraction. In other words, the larger the capital accumulation under the sustainability rule relative to the growth rate of capital accumulation in the foreign country, the faster will resource use fall in the foreign country relative to the home country.

From the Cobb-Douglas production functions and (21) one can also deduct that the difference in the growth rate of investment is equal to the difference in the growth rate of output.

$$(22) \quad g_R - g_{R^*} = g_K - g_{K^*} = g_Y - g_{Y^*}$$

Since we have assumed integrated resource markets, resource owners in both countries follow the same extraction rules, namely the Hotelling-Rule. Consequently, extractions and hence the growth rates of extraction are identical, i.e. $g_E = g_{E^*}$. Differentiating the market clearing condition for the world market for the resource (8b) and using (21) allows us to determine the difference in the growth rates of resource extraction and resource use in production:

$$(23) \quad g_E - g_R = (g_{K^*} - g_K) \left[1 - \frac{R(t)}{E(t) + E^*(t)} \right]$$

If the investment is identical, then the difference between resource extraction and resource use will not change either. If - as it is assumed under a sustainability policy - the home country invests more than the foreign country, then the growth rate in extraction is smaller than that in resource use. In other words, the home country becomes an importer of the resource and an exporter of the output. This happens irrespectively of which particular sustainability rule is applied.

The development of the two economies can be illustrated by computing the growth rates of the different variables. For the foreign country, investment is still determined

by the savings rate α . But since output and income are not identical in an open economy, the investment is not equal to that in a closed economy. Total income can be written as³

$$I^* = Y^* + \pi_R(E^* - R^*)$$

Given the Cobb-Douglas technology, income can also be written as

$$(24) \quad I^* = Y^* \left[1 + (1-\alpha) \frac{E^* - R^*}{R^*} \right] = Y^* [1 - (1-\alpha)m^*] = Y^* \xi^*$$

where m^* denotes the import quota of resources for the foreign country and $\xi^* = 1 + (1-\alpha)m^*$. (24) is equivalent for the home country if all stars are removed and the domestic import quota is used.

The investment rule of the foreign country is therefore

$$(25) \quad K^* = \alpha I^* = \alpha Y^* \xi^*$$

and the corresponding growth rate g_K is

$$(26) \quad g_{K^*} = \alpha \xi^* \beta$$

From (6) and (7), the growth rate of output and of resource use can be computed as

$$(27) \quad g_{Y^*} = (1-\alpha)\beta \left[\frac{\sigma}{1-\alpha} \xi^* - 1 \right]$$

$$(28) \quad g_{R^*} = -(1-\sigma\xi^*)\beta.$$

By assumption, the foreign country does not pursue any sustainability policy. If the domestic country also does not intervene in the capital accumulation both countries will remain identical and trade will not take place. Hence ξ and ξ^* will in that case be equal to one such that equations (23) to (28) also describe the non-intervention case for both countries if $\xi = \xi^* = 1$.

³ All variables are functions of time.

Sustainability Rules for an Open Economy

The two rules of sustainability which have been discussed in the section on the closed economy need to be differentiated further in the case of an open economy. Whereas in a closed economy there is no difference in resources extracted, E , and resources used, R , in an open economy they will differ. As a consequence one needs to define whether the sustainability rule refers to the domestic resource stock or to the overall resource stock. In the first case investment needs only to compensate for domestic resource extraction, in the second it needs to compensate for domestic resource use.

The Hartwick Rule concentrating on resource use would then amount to

$$(a) \quad \dot{K}(t) = \pi_R(t)R(t) \quad \forall t.$$

The corresponding rule for constant wealth of the economy would be

$$(b) \quad \pi_K(t)\dot{K}(t) = \pi_R(t)R(t) \quad \forall t.$$

Both rules in terms of extraction amount to

$$(c) \quad \dot{K}(t) = \pi_R(t)E(t) \quad \forall t$$

$$(d) \quad \pi_K\dot{K}(t) = \pi_R(t)E(t) \quad \forall t.$$

Sustainable Growth

Given the identity of the differences in the growth rates of the relevant variables in equation (22), it is now straightforward to compute the development of the domestic and the foreign country. Translating the sustainability rules (a) to (d) into growth rates of capital yields

$$(29 a) \quad g_K^a = (1 - \alpha)\beta(t),$$

$$(29 b) \quad g_K^b = \frac{1 - \alpha}{\alpha},$$

$$(29 \text{ c}) \quad g_K^c = (1-\alpha)\beta(t) \frac{E(t)}{R(t)},$$

$$(29 \text{ d}) \quad g_K^d = \frac{1-\alpha}{\alpha} \frac{E(t)}{R(t)}.$$

Since the domestic country is a resource importer, the extraction-based rules (c) and (d) require a slower increase in the capital stock than the use-based rules (a) and (b).

The difference in the growth rate of capital, resource extraction, and output is given by

$$(30 \text{ a}) \quad g_K^a - g_{K^*} = (1-\alpha)\beta(t) \left(1 - \frac{\sigma}{1-\alpha} \xi^*(t) \right)$$

$$(30 \text{ b}) \quad g_K^b - g_{K^*} = (1-\alpha)\beta(t) \left(\frac{1}{\alpha\beta(t)} - \frac{\sigma}{1-\alpha} \xi^*(t) \right)$$

$$(30 \text{ c}) \quad g_K^c - g_{K^*} = (1-\alpha)\beta(t) \left(\frac{E(t)}{R(t)} - \frac{\sigma}{1-\alpha} \xi^*(t) \right)$$

$$(30 \text{ d}) \quad g_K^d - g_{K^*} = (1-\alpha)\beta(t) \left(\frac{1}{\alpha\beta(t)} \frac{E(t)}{R(t)} - \frac{\sigma}{1-\alpha} \xi^*(t) \right)$$

By assumption, the following relationships hold: $\alpha > 0$ and $1-\alpha > \sigma$. Also $\xi^*(t) = 1$ if there is not trade and $\beta(t)$ is falling since $g_R < 0$. For a complete characterization of (30 a-d) we need to determine the development of $\xi^*(t)$. As mentioned, at the beginning of the imposition of the sustainability rule $\xi^*(t) = \xi(t) = 1$. Hence, at least in the initial period the growth rates of capital and output are larger at home than abroad and the home country is importing the resource and exporting the produced commodity Y.

The examination of ξ and ξ^* also gives further insight into the development of the import quota for the resource. Differentiating ξ and ξ^* w.r.t. time yields

$$(31) \quad \dot{\xi}(t) = \frac{1}{2} (1-\alpha) \frac{R'(t)}{R(t)} (g_{K^*} - g_K)$$

$$(32) \quad \dot{\xi}^*(t) = \frac{1}{2} (1-\alpha) \frac{R(t)}{R'(t)} (g_K - g_{K^*})$$

Since $g_k > g_{k^*}$ - at least in the beginning - ξ is falling over time and ξ^* is increasing. It follows directly that the import quota of the home country is increasing and that of the foreign country, the resource exporter, is decreasing.

The appendix proves that the growth rate of the domestic capital stock cannot fall below that of the foreign country, regardless which sustainability rule is chosen in the domestic country. This has important implications since it also means that the growth rate of imports of resources in the home country is always positive (see equation 23).

Resource use falls in both countries, yet it falls faster in the foreign country than in the home country which pursues the sustainability rule (equation 21). From (22) it is also obvious that output grows faster in the home country than in the foreign country. And finally, the import quota of the domestic country is always increasing.

$$(33) \quad g_m = -\frac{1}{2}(g_k - g_{k^*})\frac{1}{\bar{m}}$$

and that of the foreign country is always decreasing

$$(34) \quad g_{m^*} = -\frac{1}{2}(g_k - g_{k^*})\frac{1}{\bar{m}^*}$$

Although the import quota of the home (foreign) country always increases (decreases) it stays within certain limits. These limits, \bar{m} and \bar{m}^* , can be determined for the different sustainability rules as:

$$(35 \text{ a}) \quad m^a < \bar{m}^a = \frac{1 - \alpha - \sigma}{2(1 - \alpha) - (1 + \alpha)\sigma} \quad m^{*a} > \bar{m}^{*a} = -\frac{1 - \alpha - \sigma}{(1 - \alpha)\sigma}$$

$$(35 \text{ b}) \quad m^b < \bar{m}^b = \frac{1 - \alpha - \alpha\beta\sigma}{2(1 - \alpha) - (1 + \alpha)\alpha\beta\sigma} \quad m^{*b} > \bar{m}^{*b} = -\frac{1 - \alpha - \alpha\beta\sigma}{(1 - \alpha)\alpha\beta\sigma}$$

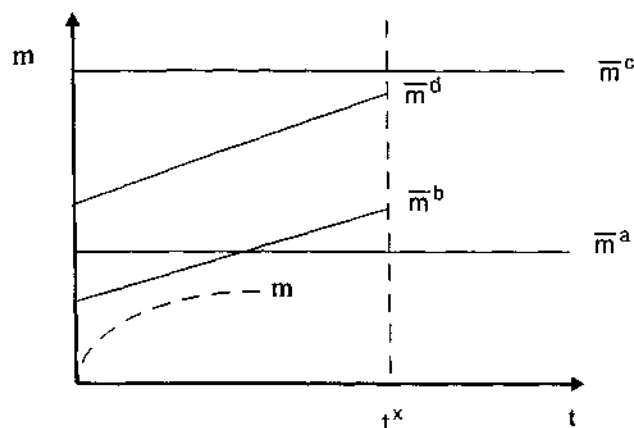
$$(35 \text{ c}) \quad m^c < \bar{m}^c = \frac{1 - \alpha - \sigma}{1 - \alpha - (1 + \alpha)\sigma} \quad m^{*c} > \bar{m}^{*c} = -\frac{1 - \alpha - \sigma}{(1 - \alpha)(1 - \sigma)}$$

$$(35 \text{ d}) \quad m^d < \bar{m}^d = -\frac{1 - \alpha - \alpha\beta\sigma}{1 - \alpha - (1 + \alpha)\alpha\beta\sigma} \quad m^{*d} > \bar{m}^{*d} = -\frac{1 - \alpha - \alpha\beta\sigma}{(1 - \alpha)(1 - \alpha\beta\sigma)}$$

The limits to which the import quota of the home country converges under the different sustainability rules are illustrated in Figure 2. Under the Hartwick rule with respect

to resource use (a), the limit is the horizontal line \bar{m}^a intersecting the vertical axis at the limit given in (35 a)⁴.

Figure 2 - Import quotas for the home country



If the rule for a constant value of the capital and resource stocks is used (b) the limit for the home country's import share is an increasing function over time because the capital productivity β is falling over time. The import share \bar{m}^b will also reach a limit at that point in time t^* in which the feasibility constraint, $\alpha\beta > 1 - \alpha$, is violated. The reason for such a limit originates from the constant growth rate of investment which eventually violates the feasibility constraint of non-negative consumption since output is falling over time. This is illustrated in Figure 2 by the maximum import share \bar{m}^b which intersects the largest feasible limit $\bar{m}^{bf} = (1 - \alpha)(2 - \alpha(1 + \alpha))^{-1}$ at time t^* . Beyond the time t^* , the sustainability rule cannot be followed any more. Similarly, we get for rule (d) $\bar{m}^{to} = (1 - \alpha)(1 - (1 + \alpha)\alpha)^{-1}$.

Under the extraction-based rules the same patterns emerges. The extraction-based Hartwick rule (c) leads to import shares approaching a constant which lies above that of rule (a) for reasonable parameter values of α and σ . The constant overall capital stock required by rule (d) has an increasing import share such that the feasibility constraint on the investment rule will be binding like in the corresponding rule (b).

⁴ For half way realistic values of $\alpha = 0,75$ and $\sigma = 0,15$ one can compute the limit as $\bar{m}_a = 0,42$. The corresponding export quota of the foreign country, i.e. exports divided by resource extraction, is 0.30.

The development of output, income, etc. of the home country and the foreign country can now be described, although not in detail for all variables. The most straightforward results can be computed for rule (a).

As in the case of a closed economy the output in the home country does not change, i.e. $g_y = 0$. Income, however, falls since $g_i = g_y + g_k$ with $g_k < 0$. Resource imports are increasing as is the import quota. However, the import quota converges towards the limit given in (35 a). In the closed economy, constant output also implies constant consumption (equation 15). This is not the case in the open economy. The growth rate in consumption can be derived from differentiating the identity $C = \xi Y - K$ under the investment rule (a):

$$(36) \quad g_c = \frac{\xi Y}{C} g_i - g_v.$$

Since $g_y = 0$ and $g_k < 0$, consumption will fall.

The results for the foreign country are not so obvious. It is a net exporter of the resource and its output is falling (see equation (27) and the appendix). It is unclear whether income rises since $g_r = g_y + g_k$ with $g_k < 0$ and $g_k > 0$. Consequently, the consumption path cannot be predicted as well.

If the Hartwick rule is applied to the rents from resource extraction instead of those from resource use, then the output in the home country will no longer remain constant. Under rule (c) investment will not be high enough to sustain a constant output. The growth rate of output is then $g_y^c = -(1-\alpha)\beta m$ where both β and m are positive and declining. The sign of the growth rate of income is negative but that of consumption cannot be determined. By substituting the sustainability rule (c) into the consumption identity one can compute

$$(37) \quad \dot{C}^c \geq 0 \quad \text{if} \quad \frac{C}{Y} m^* + (1-\alpha) \left(\frac{E}{R} - \frac{\sigma}{1-\alpha} \xi^* \right) \geq 0.$$

As ξ^* approaches its limit, the last bracket will become zero such that eventually \dot{C}^c has a negative sign but one cannot determine whether it might have had a positive sign for lower ξ^* . The reason for this result stems from the fact that the investment under rule (c) is smaller than under rule (a). Hence consumption may be increasing for a while although output and income is falling.

The rules of constant wealth produce conditions for a positive change in consumption for which an intuitive interpretation can not be given. One may, however, discuss the impact on production. The growth rates of production amount to

$$(38b) \quad g_Y^b = \frac{1-\alpha}{\alpha} - [1-\alpha]\beta(t) \quad \text{and}$$

$$(38d) \quad g_Y^d = \frac{1-\alpha}{\alpha} \frac{E(t)}{R(t)} - [1-\alpha]\beta(t).$$

Because the price of capital $\alpha\beta$ was assumed to fall short of unity, (38b) shows that production is increased under rule (b). Under rule (d), production is increasing at the beginning of sustainability policies, but may decline with an increasing import quota.

We are now able to compare the growth paths of the two economies with and without sustainability rules. Again, determining the growth rates of β for the different policies shows that

$$(39) \quad g_\beta^a = g_\beta^b = g_\beta^c = g_\beta^d = -(1-\alpha)\beta(t)$$

holds for all cases. As $\beta(0)$ is identical for all cases, identical growth rates imply identical $\beta(t)$ for all cases under consideration.

The comparison of growth paths under sustainability policies with those under laissez-faire is straight forward. First, we observe that g_R rises by switching from laissez-faire to either sustainability rule because $\xi' > 1$ whereas it was equal to one in the laissez-faire case:

$$(40) \quad g_R \geq g_R^L.$$

According to (22),

$$(41) \quad g_R = g_K - g_{K^*} + g_{R^*} > g_{R^*}$$

holds because g_K does never fall short of g_{K^*} . (41) shows that both growth rates of the resource use exceed the corresponding laissez-faire growth rates. From (23), we know that

$$(42) \quad g_E = [g_K - g_{K^*}] \left[1 - \frac{R(t)}{2E(t)} \right] + g_R < g_R,$$

$$g_{E^*} = [g_{K^*} - g_K] \left[1 - \frac{R^*(t)}{2E(t)} \right] + g_{R^*} > g_{R^*},$$

is valid. (42) demonstrates that the growth rate of extractions falls short of (exceeds) the growth rate of the resource use in the domestic (foreign) country. From (41) and (42), we derive

$$(43) \quad g_E = [g_K - g_{K^*}] \frac{R(t)}{2E(t)} + g_{R^*} > g_R > g_R^t = g_E^t.$$

(43) together with (42) shows that unilateral policies will lead to a *further resource extraction in both countries* because both resource markets are now integrated. The foreign country will export its increasing extraction such that the domestic increase in extraction does not cover the increase in domestic resource use but imports are increased as well.

If one compares a sustainability policy in a country without resource trade, we derive from (6) and (7) that

$$(44) \quad g_R = g_K - \beta(t)$$

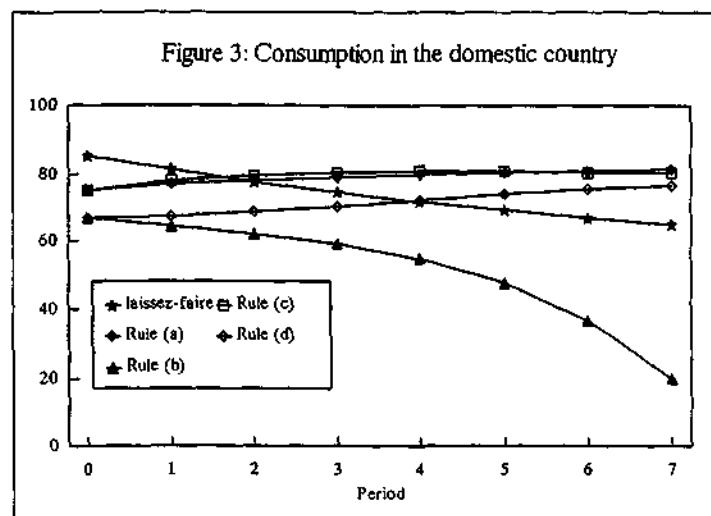
holds. Inserting (26) and (28) into (41) gives the same result. Hence, we may conclude that domestic resource use is not changed under rules (a) and (b) when resource trade is allowed. Because rules (c) and (d) imply lower investment in the course of time, they imply a lower resource use. As a country engaging in resource trade is always a resource importer, we find that the growth rate of extraction does always fall short from the one of a closed economy. From this comparison, we conclude that an economy with a unilateral sustainability policy can slow down its resource extraction further if it engages in resource trade, i.e. it can "import sustainability".

5. A Numerical Illustration

In order to shed more light on the behavior of consumption and income, we compute the different sustainability rules in a numerical model in discrete time covering eight periods. We use the following parameters and starting values: $\alpha = 0.75$, $\sigma = 0.15$,

$K(0) = K^*(0) = R(0) = R^*(0) = 100$.⁵ Discrete time modelling implies that the resource price in period $t+1$ equals the resource price in period t times one plus the capital price in period t . Hence, discrete time modelling leads to a slightly higher increase of the resource price than continuous time modelling because the decline of the capital price is delayed by one period.

From this simulation, we find that production increases in the domestic country and decreases in the foreign country. Furthermore, we find quite different paths for consumption in the home country and in the foreign country which are depicted in Figures 3 and 4.



⁵ The simulation is performed using the GAMS/MINOS software developed by the World Bank. The input files are available upon request.

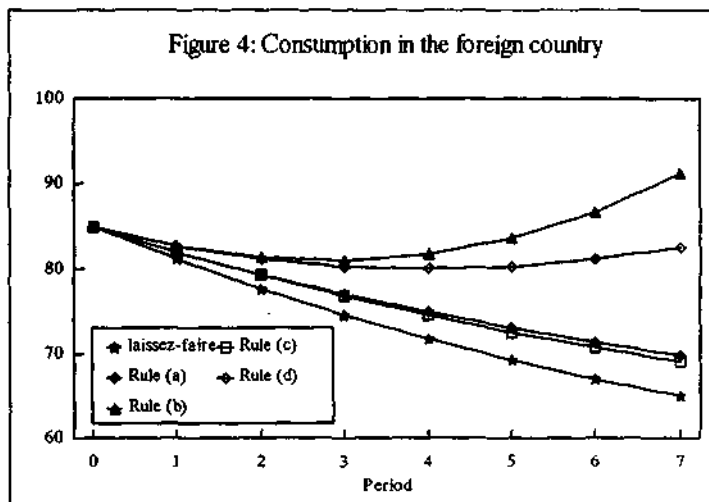
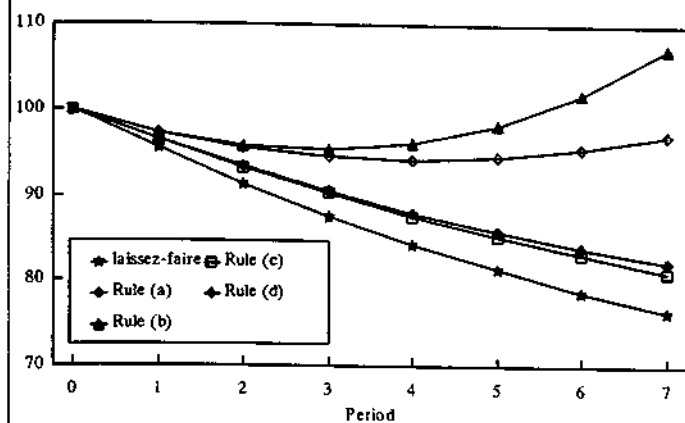


Figure 3 shows that consumption in the home country strongly declines under rule (b). Rule (b) does never reach the consumption level of the laissez-faire economy, but all other rules eventually will. Both Hartwick Rules [(a) and (c)] and the extraction-based rule (d) lead to an increasing consumption path. Hence, investment is not too large and resource use is not too small to increase consumption in later periods compared to laissez-faire.

Figure 4 shows that - compared to laissez-faire - the foreign country will be able to consume more under all sustainability rules. Rules (a) and (c) imply decreasing consumption in the foreign country whereas rules (b) and (d) result in a u-shaped consumption path. Hence, the rules of constant wealth make the resource-exporting country sufficiently rich to eventually surpass their initial consumption level. This consumption is made possible by the increasing demand of the home country for resource imports which is paid for with consumption goods. This effect is also visible in figure 5 which shows that the income in the foreign country follows the same paths as consumption. Since rule (b) implies the highest investment level in the domestic country, both consumption and income of the foreign country will surpass their initial values. The foreign country gains from the increase in the capital stock of the home country and the subsequent scarcity of resources.

Figure 5: Income in the foreign country



6. Concluding Remarks

The analysis of sustainability policies on resource extraction in a dynamic context first yields the result that compensating investments in the capital stock do not slow down the resource extraction path. This result originates from the dynamic behavior of the resource extraction decision which depends according to the Hotelling Rule on the interest rate, i.e. the marginal product of capital. But since investment lowers the marginal capital productivity, the path of resource prices increases over time at a lower speed than with lower investment. This is due to the fact that this model has no technical progress which increases the productivity of capital. Allowing trade in resources, the country following the unilateral sustainability rule can import sustainability, thus preserving its resource stock better than under autarky. However, even with trade it cannot achieve a lower resource extraction path than without any compensating investment rule at all.

There are three escape routes which may reconcile sustainability policies with modest resource extractions. Firstly, one may take the strong sustainability concept which does not allow for substitution between man-made and natural capital. This concept, however, is obviously not apt to deal with problems of non-renewable resources. Secondly, one may define weak sustainability as not adding the value of resource extraction to the value of physical stocks but to another stock the marginal productivity of which does not determine resource extraction. For example, if the value of extracted or used resources is invested into human capital which in turn might influence technical progress, the marginal productivity of physical capital may be rising instead of falling as in one case. As the marginal productivity of physical capital determines the resource price change, resource extraction and resource use are likely to be more modest. Thirdly, one may find that sustainability must be reached at by direct resource regulation. Taxing the resource use may decrease resource extraction at a point in time but not the extraction path. It is, therefore, necessary that taxation follows a dynamic scheme. If resource use is taxed at a constant rate, Welsch and Stähler (1990) have demonstrated that such a tax merely shifts resource exploitation from the present to the future.

From this discussion, we draw two policy conclusions. Firstly, we find that trade in resources may lead to an import of sustainability at the expense of unsustainability in other countries. Unless the more unsustainable use of resources abroad raises no problem, trade in resources stresses the importance of international policy coordination. Secondly, we find that the concept of weak sustainability should be redefined in order to avoid investment into physical capital. It is not only useless but even harmful just to accumulate physical capital as resource extraction in all countries and resource

imports are likely to be increased. In this sense, increasing physical capital leads to a faster exploitation of resources. The concept of weak sustainability should therefore focus more on the productivity enhancing part of investment decisions.

Acknowledgements

This paper is part of a research project on "The Measurement and Achievement of Sustainable Development". We gratefully acknowledge financial support by the European Union. It was presented at the Annual Meeting of the Ausschluß für Umwelt- und Ressourcenökonomie of the Verein für Socialpolitik in Dresden. We are grateful to the participants for helpful comments. The usual disclaimer applies.

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Appendix

This appendix will prove that ξ^* will never reach a level which makes the growth rate of the foreign capital stock exceed the growth rate of the domestic capital stock. It does so in two steps:

- Firstly, it proves that the foreign stock's growth rate can never strictly exceed the domestic stock's growth rate. This result can be shown easily by considering the behavior of the growth rates in the case of equalized growth rates. However, it does not rule out that both growth rates coincide.
- Secondly, the appendix proves that the foreign stock's growth rate does always strictly fall short of the domestic stock's growth rate. This result will be arrived at by contradiction. We will assume that both growth rates coincide and will show that the required slopes contradict the actual slopes at this point.

In general, the change of ξ^* is given by

$$(A1) \quad \dot{\xi}^*(t) = \frac{1-\alpha}{2} \frac{R(t)}{R'(t)} [g_K(t) - g_K^*(t)].$$

As we know that $\xi^*(0) = 1$, ξ^* will increase over time. If the growth rates of the capital stocks were identical, ξ^* would reach a critical level which will be denoted by $\bar{\xi}_x^*$ where $x \in \{a, b, c, d\}$ denotes the sustainability rule under consideration. $\bar{\xi}_x^*$ would be zero due to (A1). In that case, the growth rates of resource use and capital accumulation remained identical, and the growth rate of the import quota were zero. Thus, (A1) proves that the growth rate of the foreign capital stock cannot strictly exceed the growth rate of the domestic capital stock because it cannot overtake. This result shows that the foreign country's capital stock will always be significantly lower than the domestic country's capital stock.

(A1), however, does not rule out that both growth rates coincide. The critical $\bar{\xi}_x^*$'s for which $g_K = g_K^*$ for the four rules under consideration are given in (A2).

$$(A2) \quad \bar{\xi}^{**}(t) = \frac{1-\alpha}{\sigma} = \text{const.}, \quad \bar{\xi}^{**}(0) = \frac{1-\alpha}{\sigma} > 1,$$

$$\bar{\xi}^{*b}(t) = \frac{1-\alpha}{\alpha\sigma\beta(t)}, \quad \bar{\xi}^{*b}(0) = \frac{1-\alpha}{\alpha\sigma\beta(0)} > \frac{1}{\sigma} > 1 \text{ because } \beta(\cdot) > \frac{1-\alpha}{\alpha},$$

$$\bar{\xi}^{*c}(t) = \frac{1-\alpha}{\sigma} \frac{E(t)}{R'(t)}, \quad \bar{\xi}^{*c}(0) = \frac{1-\alpha}{\sigma} \frac{E(0)}{R'(0)} = \frac{1-\alpha}{\sigma} > 1,$$

$$\bar{\xi}^{*d}(t) = \frac{1-\alpha}{\alpha\sigma\beta(t)} \frac{E(t)}{R'(t)}, \quad \bar{\xi}^{*d}(0) = \frac{1-\alpha}{\alpha\sigma\beta(0)} \frac{E(0)}{R'(0)} = \frac{1-\alpha}{\alpha\sigma\beta(0)} > \frac{1}{\sigma} > 1,$$

$$\text{because } \beta(\cdot) > \frac{1-\alpha}{\alpha}.$$

(A2) shows that the critical values exceed the actual values at the beginning of sustainability policies which are equal to one. Note that (A2) uses the feasibility constraint (18) for rules (b) and (d). Differentiation of the critical values with respect to time gives

$$(A3) \quad \dot{\bar{\xi}}^{*a}(t) = 0,$$

$$\dot{\bar{\xi}}^{*b}(t) = \frac{(1-\alpha)^2}{\alpha\sigma} = \text{const.} > 0,$$

$$\dot{\bar{\xi}}^{*c}(t) = -\bar{\xi}^{*c}(t) \left[g_K^c(t) - g_{K^*}(t) \right] \left[1 - \frac{R(t)}{2E(t)} \right] \leq 0,$$

$$\dot{\bar{\xi}}^{*d}(t) = \bar{\xi}^{*d}(t) \left\{ (1-\alpha)\beta(t) - \left[g_K^d(t) - g_{K^*}(t) \right] \left[1 - \frac{R(t)}{2E(t)} \right] \right\}.$$

Due to (A1), we may restrict the analysis on cases for which $g_K \geq g_{K^*}$ holds. (A3) shows that the critical ξ^* increases linearly under rule (b) and does not increase under rule (c). For rule (d), the sign is undetermined for $g_K^d > g_{K^*}$ and the shape of the curve is not known unless the relationship between β and the import quota is further speci-

fied. We do not even know the change at the beginning of sustainability policies because

$$\dot{\xi}^{*d}(0) = (1-\alpha)\beta(0) - \frac{1}{2} \left[\frac{1-\alpha}{\alpha} - \sigma\beta(0) \right]$$

may be negative or positive. If $\beta(0)$ reaches the upper limit $1/\alpha$, $\dot{\xi}^{*d}(0)$ is clearly positive, if $\beta(0)$ reaches the lower limit $(1-\alpha)/\alpha$ (such that any sustainability policy is feasible only in the beginning),

$$\dot{\xi}^{*d}(0) = \frac{1-\alpha}{\alpha} [1 - 2\alpha + \sigma]$$

and likely to be negative for a high α and a low σ .

Suppose that $g_k(T) = g_k^*(T)$ holds for a certain $T < \infty$. At that time, the actual ξ^* is assumed to reach the critical $\bar{\xi}^*$. When the actual ξ^* reaches the critical $\bar{\xi}^*$ in finite time, and ξ^* starts below $\bar{\xi}^*$, the implication is that the slope of ξ^* must exceed the slope of $\bar{\xi}^*$ strictly at that point. This requirement gives a contradiction because

$$(A4) \quad \dot{\xi}^{*s}(T) = 0,$$

$$\dot{\xi}^{*b}(T) = \frac{(1-\alpha)^2}{\alpha\sigma} > 0,$$

$$\dot{\xi}^{*c}(T) = 0,$$

$$\dot{\xi}^{*d}(T) = \bar{\xi}^{*d}(T)(1-\alpha)\beta(T) > 0.$$

whereas $\dot{\xi}^{**}(T) = 0$ by (A1). (A4) shows that ξ^* will never reach $\bar{\xi}^{**}$ for all cases because the slope of every $\bar{\xi}^{**}$ does not fall short of the slope of ξ^* at T . Hence, both growth rates can never coincide in finite time.