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#### Abstract

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# State Capacity and Public Debt: A political economy analysis * 

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#### Abstract

High public debt combined with low capacities of the state to raise taxes and to support markets can put even developed countries into turmoil. However, the existing political economy literature of state capacity, pioneered by Besley and Persson (2009), does not investigate the interaction of these capacities with public debt. This paper studies the incentives behind raising debt and building state capacity in an integrated analytical framework. We examine the impact of political stability, cohesiveness of institutions, and income fluctuations on the political outcome, while allowing for sovereign default. We investigate when public debt and state capacity investments move in the same or opposite directions in response to exogenous parameter changes. This allows us to show when a state will simultaneously accumulate high public debt and invest only little in its capacities to raise taxes and to support markets, leading to a positive probability of sovereign default.


JEL codes: D72, E62, H11, H20, H60.
Keywords: fiscal capacity, legal capacity, public debt, sovereign default

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## 1 Introduction

The combination of high public debt and low capacities of the state to raise taxes and to support markets can upset even developed economies. A recent example is the case of Greece. Its rather high shadow economy ${ }^{1}$ points to a low level of fiscal capacity, the institutional infrastructure necessary to collect and enforce taxes. Furthermore, low property rights protection ${ }^{2}$ indicates a low level of legal capacity, the legal infrastructure necessary to provide a secure investment climate. State capacity, the combination of legal and fiscal capacity, is a crucial determinant of a state's financial strength. A country with low state capacity that has at the same time a tendency to accumulate high public debt might run into severe problems. The European debt crisis has exemplified this in an inglorious way. In light of this, it is important to understand the mechanisms underlying the combined evolution of state capacity and public debt.

However, the political economy literature of public debt ${ }^{3}$ usually takes the institutional infrastructure necessary to raise taxes as given and does not consider this fiscal capacity as an investment object of the government. The legal infrastructure necessary for the proper functioning of a market economy is also not modeled as an endogenous political choice in this literature. In contrast, the recent political economy literature of state capacity, pioneered by Besley and Persson (2009), endogenizes fiscal and legal capacity as investment objects of the state. However, this literature does not include public debt. Analyzing the combined evolution of state capacity and public debt necessitates an integrated analytical framework. Otherwise, important aspects of the interaction between these dynamic variables will remain unexplored. We provide such an integrated model, and we uncover interactions between state capacity and public debt that cannot be understood by studying the two issues separately.

We generalize the baseline model of state capacity investment in Besley and Persson $(2010,2011)$ to include public debt, fluctuating incomes, and the possibility of default. In a dynamic framework, an incumbent policymaker cannot be sure to remain in power in the future. It wants to benefit its own clientele, and decides about investments in the future fiscal and legal capacities. The incumbent can additionally spend on a commoninterest public good or redistribute money towards its own clientele. The "cohesiveness" of institutions determines to what degree this redistribution is possible. Following Besley

[^1]and Persson, we say a country has low cohesiveness, when it is very easy to do clientele politics to the benefit of the own group. Importantly, the incumbent can also raise debt now. Debt is restricted by future state capacity, because the latter determines repayment capacity in the second period. The income level attainable for a given legal support as well as the value of public goods fluctuate over time, introducing a business cycle component into the model. The implied possibility of ability-to-pay default gives us a tractable way to study the effects of increasing costs of debt financing.

We derive two main sets of results. First, in a simple basic model without fluctuating incomes and without default, we show that the possibility to raise debt can create an additional incentive to invest in state capacity. The intuition is that debt allows to draw future tax resources to the present. This circumvents the problem of a use of future public funds that is not in line with the current incumbent's objective. Specifically, high political instability and low cohesiveness make the first period incumbent afraid of giving additional state capacity to the future government. By high political instability, this government is likely to be from the opposed group, and by low cohesiveness, it can use the higher taxing power to heavily redistribute away from the period- 1 incumbent group. In the model without debt (c.f. Besley and Persson, 2010, 2011), the only possibility to protect against such an adverse use of future public funds is to lower investments in fiscal and legal capacity. We call this the low-investment-mechanism. However, if debt can be used to bring future public funds at the disposal of the first period incumbent, then this incumbent can decide about their use. Given that there are profitable uses of tax resources in the first period, the incumbent now has higher incentives to invest in state capacities in order to increase the amount of public funds at its disposal. We call this the debt-mechanism. The strength of this mechanism depends on how easy it can be used. For the basic model without default, there are no restrictions on using this mechanism. Therefore, it can completely cancel out the original low-investment-mechanism.

However, our second set of results shows how the debt-mechanism can be weakened, thereby allowing the original low-investment-mechanism to partly resurface. Specifically, with fluctuating incomes, the cost of raising additional debt depends on the possibility of default. When debt is raised to the point where default becomes possible, it becomes increasingly expensive to use the debt channel to draw future public funds to the present. In particular, for investments in fiscal capacity, a part of the implied future public funds can then only be drawn to the present at high costs. To the extent that it is very costly to draw newly created future public funds to the present, the low-investment-mechanism resurfaces. Specifically, it resurfaces the stronger, the higher are the income fluctuations. For high income fluctuations, we therefore get results close to the original no-debt model.

In particular, a country with low cohesiveness and high political instability will invest only little in state capacities. Furthermore, this 'weak state' situation is now worsened by a built-up of high debt, leading to a positive probability of default.

From these results, we can draw the following policy implications. Besides political instability, cohesiveness is identified as an important driving force behind the combined evolution of state capacity and public debt. Cohesiveness enters as an exogenous parameter in our model, but our comparative static results show which implications follow from changing it. In our model, a country with high cohesiveness will be close to the social planner optimum. Increasing cohesiveness in the real world necessitates deep reforms that go at the core of the functioning of the state. Examples of such reforms include implementing a functioning system of checks and balances, establishing an independent press that names and shames clientele politics, creating provisions in the constitution that prevent clientele politics, or strengthening the constitutional court in its power to enforce such provisions. Comprehensive reforms in these directions should prevent a country from running into a situation of high debt and low state capacity.

The rest of this paper is organized as follows. Section 2 discusses the relation of this paper to the existing literature. Section 3 sets out a basic model of state capacity and public debt which does not yet include fluctuating incomes or default. We extend the setup of Besley and Persson $(2010,2011)$ by introducing the possibility to raise debt to the government's policy set. Comparing the results to the model without debt, we find that the possibility to raise debt can create a novel incentive to invest in state capacity. In Section 4, we introduce exogenous income fluctuations into the model to allow for sovereign default. We investigate when public debt and state capacity investments move in the same or opposite directions in response to exogenous parameter changes. In the latter case, countries can run into the situation with low state capacity and high public debt. Section 5 generalizes our model by introducing quasi-linear utility functions. This allows us to establish the robustness of the previous results and to gain a deeper understanding of the underlying mechanisms. Section 6 concludes and discusses topics for future research.

## 2 Relation to the Literature

Analyzing the political incentives behind investing in state capacity and raising public debt, we bring together the two strands of the political economy literature that have analyzed these concepts in isolation. The concept of state capacity was brought back to the minds of economists by Timothy Besley and Torsten Persson in a series of recent papers (Besley
and Persson, 2009, 2010). ${ }^{4}$ These were condensed into their book Pillars of Prosperity (Besley and Persson, 2011). All of these models include two aspects of state capacity, legal and fiscal capacity, in a tractable political economy model with two periods.

Our model builds on the workhorse model in Besley and Persson (2011). This model has been extended in several directions. Besley and Persson (2009), for instance, provide a micro foundation for the growth enhancing effect of legal support by explicitly modeling a credit market whose effectiveness depends on the level of legal support. Besley et al. (2013) drop legal capacity and extend the remaining fiscal capacity model to comprise multiple periods and to include decreasing marginal benefits of public good spending. They show that the main results from the two-period model generalize to this setup.

The main novel feature of our paper is that we consider in one model the interaction of the strategic use of debt and the decision of an incumbent government to invest in its future powers to raise taxes and to grant legal support. With regard to the state capacity literature, we find that the possibility to raise debt can provide an additional mechanism to incentivize state capacity investments that cannot be seen in a model without debt. However, debt might be used to tie down the additional investments for uses that are not in accordance with the social planner's objective. However, we also derive conditions under which the link between debt and state capacity investments is weak. In this case, the weak state situation of low investments in state capacity, identified by Besley and Persson, is worsened by an additional buildup of high debt.

While the state capacity literature has not included the debt channel at all, the debt literature usually includes taxes. Nevertheless, this latter literature takes fiscal capacity as given. Implicitly, institutional capacity to raise taxes is often assumed to be maximal in this context. In particular, when labor taxes are considered, the upper bound on taxation is given by the tax rate maximizing the resulting Laffer curve. Therefore, fiscal capacity is not included as an endogenous dynamic variable in this literature.

The strand of the debt literature closest to our setup is the literature on strategic debt initiated by Persson and Svensson (1989), Alesina and Tabellini (1990) and Tabellini and Alesina (1990). ${ }^{5}$ Persson and Svensson (1989) and Alesina and Tabellini (1990) consider debt in the setup of distortionary labor taxes. The cost of raising debt therefore involves

[^2]higher tax distortions in the future. Both papers show that in political competition between parties with differing objectives, too much debt is raised compared to a normative benchmark. This happens because a currently ruling government cannot be sure to remain in power in the future and therefore uses debt to bind its successor's hands. Since our model has non-distortionary taxation, it is more closely related to the one of Tabellini and Alesina (1990). They examine a two-period model with non-distortionary taxation and a group of heterogeneous individuals with different preferences over two public goods. Again, the social planner would run a balanced budget, but in the political equilibrium the uncertainty regarding the future median voter leads to a positive debt level to bind the hands of the future median voter.

A similar trade-off as in these models also arises in our setup. The spending purpose on which groups have differing preferences is now redistributive transfers. Also similarly, our model can produce the incentive to over-accumulate debt compared to a social planner. The difference is that in Alesina and Tabellini (1990) the over-accumulation of debt is resolved as soon as an incumbent government can be sure to remain in power. In this case of their model, there is no reason for a government to distort the inter-temporal spending profile through debt. The intra-temporal spending decision might still differ from a social planner, but this decision is unrelated to debt.

In our case, even with full political stability, we might get over-accumulation of debt because the uncertainty about the value of the pubic good remains. This can give rise to a situation where differing preferences of the Utilitarian social planner and a political government on the form of spending imply a distortion of the inter-temporal spending decision. In our setup, redistributive transfers correspond to clientele politics which are not benefitial from a social planner's point of view. A social planner will therefore only spend money on public goods. This implies that the social planner will not raise debt when the future value of public goods is expected to be higher than in the present. At the same time, a political government might still accumulate debt in order to finance redistribution towards its own clientele in the present. In general, the difference to the normative benchmark in our setup is always due to distortions of the spending decision and does not involve an overly distorted tax profile. Since we do not include distortionary taxation, this latter kind of distortion cannot arise in our model.

In addition to the literatures about state capacity and public debt, the extended version of the model in section 4 is related to a third strand of literature, the literature about sovereign default. The literature of borrowing with default goes back to the seminal study of Eaton and Gersovitz (1981). Arellano (2008) extends their approach and applies it to sovereign debt default, especially in the context of developing countries. Both studies use
an infinite horizon model in which the borrower can choose to default. The incentives to not default are given by an embargo on future borrowing, an additional penalty, or direct output costs. Due to the two period setting, our model does not include the embargo on future borrowing. It is therefore somewhat in the spirit of Alesina and Tabellini (1989), who also rely an a two period model of sovereign debt default. Furthermore, we only model ability-to-pay default and not willingness-to-pay default. The reason is that the modeling of default is used here mainly to include increasing costs of debt financing in a tractable way. A more involved modeling of the default decision is left to future research.

## 3 Basic Model Setup

To focus ideas and establish a benchmark for our further analysis, we first extend the simple workhorse model of state capacity investment in Besley and Persson (2011) to include public debt. The following presentation of the basic model setup is therefore mainly a condensed presentation of the model set out in Besley and Persson (2011) with the necessary modifications for the inclusion of public debt. The full model with fluctuating incomes and the possibility of default will be presented in the next section.

The model has two periods $s=1,2$ and considers a country consisting of two equally sized groups of individuals. Total population size is normalized to 1 . One of the groups holds governmental power in the first period. Individuals that are a member of the incumbent group in a given period are superscripted by the letter $I$, whereas members of the opposition group are superscripted by the letter $O$. With exogenous probability $\gamma$, power is transferred to the other group after the first period. Higher $\gamma$ thus captures higher political instability from the point of view of the first period incumbent group.

In period $s$, an individual of group $J \in\{I, O\}$ has an income of $\omega\left(p_{s}^{J}\right)$, where $\omega(\cdot)$ is an increasing and concave function of legal support $p_{s}^{J}$ granted to group $J$. More broadly, one can think of $p_{s}^{J}$ as any kind of market supporting policies that increase the private income of individuals of group $J$. Following Besley and Persson (2009, 2010), we interpret $p_{s}^{J}$ as legal enforcement which is conducive to a more efficient functioning of capital markets. ${ }^{6}$

The utility of an individual of group $J \in\{I, O\}$ in period $s$ is linear in private consumption $c_{s}^{J}$ and public good consumption $g_{s}:{ }^{7}$

$$
\begin{equation*}
u_{s}^{J}=\alpha_{s} g_{s}+c_{s}^{J} \tag{1}
\end{equation*}
$$

where $\alpha_{s}$ parametrizes the marginal value of public good consumption relative to that of

[^3]private consumption. Income available for private consumption is determined by the nondistortionary tax rate $t_{s}$ on income $w$ and the per-capita transfers $r_{s}^{I}, r_{s}^{O}$ awarded by the government to the different groups. Therefore, individual utility in period $s$ becomes:
\[

$$
\begin{equation*}
u_{s}^{J}=\alpha_{s} g_{s}+\left(1-t_{s}\right) \omega\left(p_{s}^{J}\right)+r_{s}^{J} \tag{2}
\end{equation*}
$$

\]

Future utility is discounted with the discount factor $\delta \in(0,1)$.
The value of public goods fluctuates over time. In a developing country setup, a period with a high value of public good spending can be interpreted as a situation with a high threat of an external war. For a developed country, it is harder to find a perfect real world match for this assumption. Nevertheless, we can think of certain rescue actions in times of an economic crisis whose benefit to the overall economy exceeds possible additional private benefits by far. The stabilization of the economy can then be interpreted as a quasi-public good whose value is high in crisis times. One example would be the nationalization of a system-relevant bank.

To model this fluctuation in the simplest possible way, the value of public goods is drawn each period from a two-point distribution: $\alpha_{s} \in\left\{\alpha_{H}, \alpha_{L}\right\}$, with $\alpha_{H}>2>\alpha_{L}>1$, and $\operatorname{Prob}\left[\alpha_{s}=\alpha_{H}\right]=\phi$. As will become clear in the subsequent analysis, the high value $\alpha_{H}$ is chosen such that public good spending in this state of the world will be preferable to transfer spending. In a situation with $\alpha_{L}$, this is not necessarily the case. Since public goods benefit everybody the same, the desired size of fiscal infrastructure will depend on the probability $\phi$ of ending up in a situation where the state definitely spends on commoninterest public goods.

The crucial feature of the model in Besley and Persson (2011) is that it includes two aspects of state capacity, fiscal capacity $\tau_{s}$ and legal capacity $\pi_{s}$. Existing fiscal capacity $\tau_{s}$ puts an upper bound on the tax rate that can be raised from income in period $s$ : $t_{s} \leq \tau_{s}$. In this simple model, $\left(1-\tau_{s}\right)$ can be interpreted as the fraction of income that an individual can earn in an informal sector. To increase second period fiscal capacity $\tau_{2}$, the period1 government can invest in the built-up of $\left[\tau_{2}-\tau_{1}\right.$ ] additional units of fiscal capacity. ${ }^{8}$ For the sake of parsimony, we assume zero depreciation of the stocks of state capacity, in contrast to Besley and Persson (2011). We require $\left[\tau_{2}-\tau_{1}\right] \geq 0$, so disinvestment is not allowed. There is an increasing and convex cost $F\left(\tau_{2}-\tau_{1}\right)$ of carrying out the investment,

[^4]with $F(0)=F_{\tau}(0)=0$. Here, as in the following, subscripts on functions denote partial derivatives, and the last equation can be interpreted as the first marginal investment having negligible costs.

Legal capacity $\pi_{s}$ puts an upper bound on the legal support to both groups: $p_{s}^{J} \leq \pi_{s}$ for $J \in\{I, O\}$. The idea is that existing legal infrastructure restricts the level of legal support a government can grant to any group. The government in period 1 can invest in the future legal capacity that becomes available in period 2 via an increasing and convex cost function $L\left(\pi_{2}-\pi_{1}\right)$ with $L(0)=L_{\pi}(0)=0$. As for fiscal capacity, we require $\left[\pi_{2}-\pi_{1}\right] \geq 0$, so disinvestment is not allowed.

Our main innovation in this benchmark section is to introduce the possibility to raise debt. Specifically, the country is assumed to start with a stock of debt equal to zero, $b_{0}=0$. The period-1 incumbent government can now issue one-period risk-free bonds $b_{1} \geq 0$ on an international bond market. These bonds have to be repaid in the second period. The interest rate which has to be paid on bonds is given by $\rho=1 / \delta-1$, where $\delta$ is the discount factor of the individuals. Since the bonds are supposed to be risk-free, the maximal amount of bonds is determined by the requirement that fiscal and legal capacity in the second period must be high enough to repay the bonds: $b_{1} \leq \frac{\tau_{2} \omega\left(\pi_{2}\right)}{1+\rho}$. The righthand side of this inequality is the discounted maximal tax revenue that can be raised in the second period. Following Besley and Persson (2011) we still assume that the citizens themselves cannot save or borrow. For one, due to the linearity of the utility function, this assumption does not alter the results. Secondly, at least for the developing world, there is evidence for private agents' lack of access to credit markets (see Claessens, 2006).

The incumbent group government is assumed to maximize its own group's utility subject to a usual budget constraint and a constraint imposed by the country's "cohesiveness" of institutions. The budget constraint requires that government revenues are enough to finance all government expenditures: ${ }^{9}$

$$
\begin{equation*}
\sum_{J \in\{I, O\}} \frac{t_{s} \omega\left(p_{s}^{J}\right)}{2}+b_{s} \geq g_{s}+m_{s}+n_{s}+\frac{r_{s}^{I}+r_{s}^{O}}{2}+(1+\rho) b_{s-1} \tag{3}
\end{equation*}
$$

where $m_{s}$ and $n_{s}$ represent the investment costs in fiscal and legal capacity, which only occur in period 1, and hence are given by

$$
m_{s}= \begin{cases}F\left(\tau_{2}-\tau_{1}\right) & \text { if } s=1  \tag{4}\\ 0 & \text { if } s=2\end{cases}
$$

[^5]and
\[

n_{s}= $$
\begin{cases}L\left(\pi_{2}-\pi_{1}\right) & \text { if } s=1  \tag{5}\\ 0 & \text { if } s=2\end{cases}
$$
\]

Since the groups have equal size, $\frac{r_{s}^{I}+r_{s}^{O}}{2}$ is the average per-capita transfer that the government pays out.

The institutional constraint requires that for each unit of transfers awarded by the government to its own group it must transfer at least $\sigma \in[0,1]$ units to the other group. Besley and Persson (2011) introduce the parameter $\theta=\frac{\sigma}{1+\sigma} \in\left[0, \frac{1}{2}\right]$ to describe the "cohesiveness" of institutions. $\theta=\frac{1}{2}$ refers to completely cohesive institutions which make sure that the opposition is treated in exactly the same way as the incumbent group. For $\theta<\frac{1}{2}$, clientele politics are possible that lead to a redistribution of money towards the incumbent group. Given that the incumbent government respects the institutional setting as just another constraint, but ultimately is only concerned about its own group's utility, it will set transfers to the opposition as small as possible:

$$
\begin{equation*}
r_{s}^{O}=\sigma r_{s}^{I}=\frac{\theta}{1-\theta} r_{s}^{I} \tag{6}
\end{equation*}
$$

In the following, we therefore assume the government is choosing only transfers $r_{s}^{I}$ to its own group, while implicitly setting $r_{s}^{O}$ according to (6).

Plugging (6) into the budget constraint (3), we arrive at a modified budget constraint that already includes the constitutional constraint:

$$
\begin{equation*}
\sum_{J \in\{I, O\}} \frac{t_{s} \omega\left(p_{s}^{J}\right)}{2}+b_{s} \geq g_{s}+m_{s}+n_{s}+\frac{r_{s}^{I}}{2(1-\theta)}+(1+\rho) b_{s-1}, \tag{7}
\end{equation*}
$$

with $b_{0}=b_{2}=0$.
The timing of the whole two-period model is now as follows:

1. The initial stock of fiscal capacity is $\tau_{1}$ and group $I_{1}$ is in power. Nature draws the public good value $\alpha_{1}$.
2. The government from the currently incumbent group $I_{1}$ chooses the set of period1 policies $\left\{t_{1}, g_{1}, r_{1}^{I}, b_{1}, p_{1}^{I}, p_{1}^{O}\right\}$ and by its investment decision chooses the period-2 stocks of fiscal capacity $\tau_{2}$ and legal capacity $\pi_{2}$.
3. $I_{1}$ remains in power with probability $1-\gamma$, and nature draws $\alpha_{2}$
4. The government from the future incumbent group $I_{2}$ chooses period-2 policies $\left\{t_{2}, g_{2}, r_{2}^{I}, p_{2}^{I}, p_{2}^{O}\right\}$ while honoring the debt commitments.

The applied solution concept is subgame perfect equilibrium.

### 3.1 Analysis of the basic model

Debt and state capacity investments generate a dynamic link across periods. However, given the linear utility function, we can derive the optimal policy decision between public good spending and transfer spending for any period taking as given the levels of state capacity, state capacity investments and debt. Furthermore, the non-distortionary nature of taxes makes the level of taxes in a given period depend only on the level of fiscal capacity in that period. In a second step, the optimal debt level will be determined using the optimal policy functions on public good and transfer spending and still taking state capacity and state capacity investments as given. ${ }^{10}$ Having derived the optimal policy decisions on spending and debt for different levels of state capacity investments, the optimal level of these investments can then be determined in a last third step.

## Intra-temporal policies

Turning to the first step, legal protection will be set maximally for both groups: $p_{s}^{I}=$ $p_{s}^{O}=\pi_{s}$. This is because, first, the incumbent group gains from an increase of the own income. Second, it also benefits from an increase of the other group's income, because the resulting higher tax revenues can be used for additional public good or transfer spending.

Taxes will be used up to the full fiscal capacity: $t_{s}=\tau_{s}$. The reason is the following: The marginal benefit of public spending is always at least as high as the opportunity cost of lost private consumption, since $\max \left\{\alpha_{s}, 2(1-\theta)\right\} \omega\left(\pi_{s}\right) \geq 2(1-\theta) \omega\left(\pi_{s}\right) \geq \omega\left(\pi_{s}\right)$.

Compared to the model without debt, the introduction of debt does not change the trade-off between public goods and transfers. This trade-off depends solely on the constant marginal benefits of these two forms of spending. The only effect is on the level of spending. Specifically, the residual revenues have to be adjusted for the net inflow of money after issuing new debt and repaying old debt, $b_{s}-(1+\rho) b_{s-1}$. The optimal policy function for public good spending becomes

$$
G\left(\alpha_{s}, \tau_{s}, \pi_{s}, m_{s}, n_{s}, b_{s}, b_{s-1}\right)= \begin{cases}\tau_{s} \omega\left(\pi_{s}\right)-m_{s}-n_{s}+b_{s}-(1+\rho) b_{s-1} & \text { if } \alpha_{s} \geq 2(1-\theta)  \tag{8}\\ 0 & \text { otherwise }\end{cases}
$$

That is, public goods are provided at the maximal level, if the gross marginal value of public good spending, $\alpha_{s}$, exceeds the gross marginal value of transfers for the incumbent group, $2(1-\theta)$. At the same time, transfers and therefore redistribution towards the

[^6]incumbent group are zero. If the ordering between the marginal values is the opposite way, we only get redistributive transfers and no public goods.

Using the new budget constraint (7) with $t_{s}$ set to $\tau_{s}$ and $p_{s}^{J}$ set to $\pi_{s}$, the indirect payoff function for group $J \in\{I, O\}$ in period $s$ becomes:
$W\left(\alpha_{s}, \tau_{s}, \pi_{s}, m_{s}, n_{s} b_{s-1}, b_{s}, \beta^{J}\right)=\alpha_{s} G+\left(1-\tau_{s}\right) \omega\left(\pi_{s}\right)+\beta^{J}\left[\tau_{s} \omega\left(\pi_{s}\right)-G-m_{s}-n_{s}+b_{s}-(1+\rho) b_{s-1}\right]$
where $\beta^{I}=2(1-\theta)$ and $\beta^{O}=2 \theta$ can be interpreted respectively as the gross marginal value of transfer spending for the incumbent (I) and for the opposition (O). Note that we have suppressed the arguments of the $G$ function. Furthermore, $\beta^{J}\left[\tau_{s} \omega\left(\pi_{s}\right)-G-m_{s}-\right.$ $\left.n_{s}+b_{s}-(1+\rho) b_{s-1}\right] \geq 0$ are the transfers to group $J$.

The "value functions" capturing the within-period utility in the second period for a group that is the incumbent (I) or the opposition (O) become:

$$
\begin{align*}
& U^{I}\left(\tau_{2}, \pi_{2}, b_{1}\right)  \tag{10}\\
& =\phi W\left[\alpha_{H}, \tau_{2}, \pi_{2}, 0,0, b_{1}, 0,2(1-\theta)\right]+(1-\phi) W\left[\alpha_{L}, \tau_{2}, \pi_{2}, 0,0, b_{1}, 0,2(1-\theta)\right] \\
& =\phi\left[\alpha_{H}\left(\tau_{2} \omega\left(\pi_{2}\right)-(1+\rho) b_{1}\right)+\left(1-\tau_{2}\right) \omega\left(\pi_{2}\right)\right]+(1-\phi) W\left[\alpha_{L}, \tau_{2}, \pi_{2}, 0,0, b_{1}, 0,2(1-\theta)\right] \\
& U^{O}\left(\tau_{2}, \pi_{2}, b_{1}\right)  \tag{11}\\
& =\phi W\left[\alpha_{H}, \tau_{2}, \pi_{2}, 0,0, b_{1}, 0,2 \theta\right]+(1-\phi) W\left[\alpha_{L}, \tau_{2}, \pi_{2}, 0,0, b_{1}, 0,2 \theta\right] \\
& =\phi\left[\alpha_{H}\left(\tau_{2} \omega\left(\pi_{2}\right)-(1+\rho) b_{1}\right)+\left(1-\tau_{2}\right) \omega\left(\pi_{2}\right)\right]+(1-\phi) W\left[\alpha_{L}, \tau_{2}, \pi_{2}, 0,0, b_{1}, 0,2 \theta\right]
\end{align*}
$$

Note that, when the value of the public good is high, whatever is left after repaying debt, $\left(\tau_{2} \omega\left(\pi_{2}\right)-(1+\rho) b_{1}\right)$, will be spent on the public good. Finally, the total expected utility of the period-1 incumbent group, as seen from the first period, is:

$$
\begin{equation*}
W\left(\alpha_{1}, \tau_{1}, \pi_{1}, F\left(\tau_{2}-\tau_{1}\right), L\left(\pi_{2}-\pi_{1}\right), 0, b_{1}, 2(1-\theta)\right)+\delta\left([1-\gamma] U^{I}\left(\tau_{2}, \pi_{2}, b_{1}\right)+\gamma U^{O}\left(\tau_{2}, \pi_{2}, b_{1}\right)\right) \tag{12}
\end{equation*}
$$

## Inter-temporal policies

Having solved for the optimal intra-temporal policies, we now turn to the inter-temporal policies $b_{1}, \tau_{2}$ and $\pi_{2}$. To make the following analysis easier, we define $\lambda_{1}$, the gross marginal benefit of public funds in period 1 , and $E\left(\lambda_{2}\right)$, the expected gross marginal benefit of public funds in period 2. We have

$$
\lambda_{1} \equiv \max \left\{\alpha_{1}, 2(1-\theta)\right\}
$$

and

$$
\begin{equation*}
E\left(\lambda_{2}\right) \equiv \phi \alpha_{H}+(1-\phi) \lambda_{2}^{L} \tag{13}
\end{equation*}
$$

with

$$
\lambda_{2}^{L}= \begin{cases}\alpha_{L} & \text { if } \alpha_{L} \geq 2(1-\theta)  \tag{14}\\ (1-\gamma) 2(1-\theta)+\gamma 2 \theta & \text { otherwise }\end{cases}
$$

since $E\left(\lambda_{2}\right)$ depends on the use of public funds in the future, which is uncertain.
With this notation, the inter-temporal maximization problem of the incumbent group in period $\mathrm{s}=1$ becomes:

$$
\begin{align*}
\max _{\tau_{2}, \pi_{2}, b_{1}} E V^{I_{1}}\left(\tau_{2}, \pi_{2}, b_{1}\right) & -\lambda_{1}\left(F\left(\tau_{2}-\tau_{1}\right)+L\left(\pi_{2}-\pi_{1}\right)-b_{1}\right)  \tag{15}\\
\text { s.t. } \tau_{2} & \geq \tau_{1}, \\
\pi_{2} & \geq \pi_{1}, \\
b_{1} & \leq \frac{\tau_{2} \omega\left(\pi_{2}\right)}{1+\rho}, \\
\lambda_{1} & \equiv \max \left\{\alpha_{1}, 2(1-\theta)\right\}
\end{align*}
$$

with:

$$
\begin{equation*}
E V^{I_{1}}\left(\tau_{2}, \pi_{2}, b_{1}\right)=\delta\left((1-\gamma) U^{I}\left(\tau_{2}, \pi_{2}, b_{1}\right)+\gamma U^{0}\left(\tau_{2}, \pi_{2}, b_{1}\right)\right) \tag{16}
\end{equation*}
$$

## Choice of debt

The three dynamic variables fiscal capacity, legal capacity and debt are interlinked by the following fact. The amount of debt which can be raised is limited by the amount of future fiscal and legal capacity. The latter two determine the money the state can raise to repay debt. Furthermore, the investment and debt decisions determine the amount of residual revenue a government has at its disposal for financing public good spending or transfers after all other expenditures are covered. We now analyze the choice of debt taking the levels of fiscal and legal capacity investments as given.

The simple linear model has the advantage that, in each period, the use of residual government revenues either on public goods or transfers is exactly determined. Furthermore, the marginal benefit of that residual use is constant in either case. This marginal benefit is what we referred to as the gross marginal benefit of public funds and denoted by $\lambda_{s}$.

Debt allows to make future public funds available in the present. Therefore, the optimal debt level can be found by a simple comparison of the gross marginal benefit of public funds in the two periods. Specifically, if $\lambda_{1}$, the gross marginal benefit in the first period,
is higher than $E\left(\lambda_{2}\right)$, the expected gross marginal benefit in the second period, then it is optimal to raise the maximal debt that is allowed by future state capacity: $b_{1}=\frac{\tau_{2} \omega\left(\pi_{2}\right)}{1+\rho}$. If $E\left(\lambda_{2}\right)>\lambda_{1}$, then no debt is raised: $b_{1}=0 .{ }^{11,12}$

Summarizing the above analysis in a policy function for the debt level chosen in period 1, we have:

$$
B\left(\alpha_{1}, \tau_{2}, \pi_{2}\right)= \begin{cases}\frac{\tau_{2} \omega\left(\pi_{2}\right)}{1+\rho} & \text { if } \lambda_{1}>E\left(\lambda_{2}\right)  \tag{17}\\ 0 & \text { otherwise }\end{cases}
$$

## Choice of fiscal and legal capacity

With this, we have arrived at the third step of the analysis, the decision about fiscal and legal capacity investment. We substitute the policy function (17) for $b_{1}$ in (12) and maximize the resulting function with respect to future fiscal capacity $\tau_{2}$ and legal capacity $\pi_{2}$, subject to the constraints that fiscal and legal capacity investments can't be negative, $\left[\tau_{2}-\tau_{1}\right] \geq 0$ and $\left[\pi_{2}-\pi_{1}\right] \geq 0$, and transfers must also be weakly positive. From this, we get the following "Euler equations"

$$
\begin{align*}
& \delta\left([1-\gamma] \frac{d U^{I}\left[\tau_{2}, \pi_{2}, B\left(\alpha_{1}, \tau_{2}, \pi_{2}\right)\right]}{d \tau_{2}}+\gamma \frac{d U^{O}\left[\tau_{2}, \pi_{2}, B\left(\alpha_{1}, \tau_{2}, \pi_{2}\right)\right]}{d \tau_{2}}\right) \\
& +W_{b_{1}}\left[\alpha_{1}, \tau_{1}, \pi_{1}, m_{1}, n_{1}, B\left(\alpha_{1}, \tau_{2}, \pi_{2}\right), 0,2(1-\theta)\right] \omega\left(\pi_{2}\right)  \tag{18}\\
& \leq-W_{m}\left[\alpha_{1}, \tau_{1}, \pi_{1}, m_{1}, n_{1}, B\left(\alpha_{1}, \tau_{2}, \pi_{2}\right), 0,2(1-\theta)\right] F_{\tau}\left(\tau_{2}-\tau_{1}\right)
\end{align*}
$$

$$
\text { c. s. } \tau_{2}-\tau_{1} \geq 0
$$

and

$$
\begin{align*}
& \delta\left([1-\gamma] \frac{d U^{I}\left[\tau_{2}, \pi_{2}, B\left(\alpha_{1}, \tau_{2}, \pi_{2}\right)\right]}{d \pi_{2}}+\gamma \frac{d U^{O}\left[\tau_{2}, \pi_{2}, B\left(\alpha_{1}, \tau_{2}, \pi_{2}\right)\right]}{d \pi_{2}}\right) \\
& +W_{b_{1}}\left[\alpha_{1}, \tau_{1}, \pi_{1}, m_{1}, n_{1}, B\left(\alpha_{1}, \tau_{2}, \pi_{2}\right), 0,2(1-\theta)\right] \tau_{2} \omega^{\prime}\left(\pi_{2}\right)  \tag{19}\\
& \leq-W_{n}\left[\alpha_{1}, \tau_{1}, \pi_{1}, m_{1}, n_{1}, B\left(\alpha_{1}, \tau_{2}, \pi_{2}\right), 0,2(1-\theta)\right] L_{\pi}\left(\pi_{2}-\pi_{1}\right) \\
& \text { c. s. } \pi_{2}-\pi_{1} \geq 0,
\end{align*}
$$

where $\frac{d U^{I}}{d \tau_{2}}, \frac{d U^{O}}{d \tau_{2}}, \frac{d U^{I}}{d \pi_{2}}$ and $\frac{d U^{O}}{d \pi_{2}}$ are now total derivatives. The trade-off is between the marginal benefit of future fiscal or legal capacity (left-hand side) against the marginal cost

[^7]of financing that fiscal or legal capacity (right-hand side). Also analogously to the model without debt:
\[

$$
\begin{aligned}
\lambda_{1} & \equiv-W_{m}\left[\alpha_{1}, \tau_{1}, \pi_{1}, m_{1}, n_{1}, B\left(\alpha_{1}, \tau_{2}, \pi_{2}\right), 0,2(1-\theta)\right] \\
& =-W_{n}\left[\alpha_{1}, \tau_{1}, \pi_{1}, m_{1}, n_{1}, B\left(\alpha_{1}, \tau_{2}, \pi_{2}\right), 0,2(1-\theta)\right]=\max \left\{\alpha_{1}, 2(1-\theta)\right\}
\end{aligned}
$$
\]

The opportunity cost of using government revenues for financing investments is the gross marginal benefit of period-1 public funds. ${ }^{13}$ It depends on the form of residual spending (public goods or transfers) in period 1.

The crucial difference to a model without debt are the left-hand sides of (18) and (19). The left-hand side of (18) describes the marginal benefit of additional future fiscal capacity, as seen from the first period. If no debt is raised, it is $\delta \omega\left(\pi_{2}\right)\left[E\left(\lambda_{2}\right)-1\right] . E\left(\lambda_{2}\right)$ is the expected gross marginal benefit of future public funds and is given in (13) and (14). However, when debt is raised, it is raised maximally and uses up all public funds in the second period. Therefore, the gross marginal value of public funds is then determined by the use of debt. Since debt is used to finance first-period expenditures on public goods or transfers, the gross marginal benefit of future public funds becomes $\lambda_{1}$. The point is that debt allows to make future public funds available in the present. Therefore, the benefit of future public funds is then given by the present benefit of residual spending. According to this discussion, the optimality condition (18) can be rewritten as:

$$
\begin{align*}
& \delta \omega\left(\pi_{2}\right)\left[\max \left\{\lambda_{1}, E\left(\lambda_{2}\right)\right\}-1\right] \leq \lambda_{1} F_{\tau}\left(\tau_{2}-\tau_{1}\right)  \tag{20}\\
& \text { c. s. } \tau_{2}-\tau_{1} \geq 0
\end{align*}
$$

Given the assumption $F_{\mathcal{\tau}}(0)=0$, a necessary and sufficient condition for positive investments in fiscal capacity is now $\max \left\{\lambda_{1}, E\left(\lambda_{2}\right)\right\}>1$.

The left-hand side of (19) describes the marginal benefit of additional future legal capacity, as seen from the first period. Following the same reasoning as for fiscal capacity, the optimality condition (19) can be rewritten as:

$$
\begin{align*}
& \delta \omega^{\prime}\left(\pi_{2}\right)\left[1+\tau_{2}\left[\max \left\{\lambda_{1}, E\left(\lambda_{2}\right)\right\}-1\right]\right] \leq \lambda_{1} L_{\pi}\left(\pi_{2}-\pi_{1}\right)  \tag{21}\\
& \text { c. s. } \pi_{2}-\pi_{1} \geq 0
\end{align*}
$$

Given the assumption $L_{\pi}(0)=0$, there is always positive investment in legal capacity.
Considering the left-hand sides of equation (20) and equation (21), we notice that an investment in one of the two state capacities increases the marginal return of the other.

[^8]This is because we have $\max \left\{\lambda_{1}, E\left(\lambda_{2}\right)\right\}>1$. So, fiscal and legal capacity are complements. Note that the analogous condition in Besley and Persson (2011), $E\left(\lambda_{2}\right)>1$, was not guaranteed to always hold. In contrast, the introduction of debt into the basic model implies that complementarity between the two forms of state capacity investment will always hold.

As in Besley and Persson (2011) this complementarity is not only an interesting fact, but also allows us to apply results on monotone comparative statics. By Theorem 5 and 6 of Milgrom and Shannon (1994), any factor that increases the left-hand sides of equation (20) and equation (21) leads to an increase of both fiscal and legal capacity investments. ${ }^{14}$ This reasoning is used to establish the comparative statics stated in the propositions in the following two subsections.

For all of the subsequent analysis, we make the following assumptions.

## Assumptions

$$
\begin{align*}
& \delta \omega\left(\pi_{2}\right)\left[\alpha_{H}-1\right]<\alpha_{L} F_{\tau}\left(\overline{\tau_{2}}-\tau_{1}\right)  \tag{22}\\
& \delta \omega^{\prime}\left(\pi_{2}\right)\left[1+\tau_{2}\left[\alpha_{H}-1\right]\right]<\alpha_{L} L_{\pi}\left(\overline{\pi_{2}}-\pi_{1}\right) \tag{23}
\end{align*}
$$

for some $\overline{\tau_{2}}, \overline{\pi_{2}}$, so that $L\left(\overline{\pi_{2}}-\pi_{1}\right)+F\left(\overline{\tau_{2}}-\tau_{1}\right)=\tau_{1} \omega\left(\pi_{1}\right)$. So, $\overline{\tau_{2}}, \overline{\pi_{2}}$ are levels of future state capacity which can be financed if the current tax revenue is only and fully used for that purpose. These assumptions mean that the curvature of the cost functions $F()$ and $L()$ is high enough for the marginal cost of increasing fiscal and legal capacity to surpass the marginal benefit at an interior level of investment. That is, we don't allow the marginal benefit of investment to still surpass the marginal cost at the point where all possible tax revenues are only used for investments in fiscal and legal capacity. ${ }^{15}$ These technical assumptions are only necessary in the linear model. As we will see, they can be dispensed with in the quasi-linear setup of section 5 .

In the following, we first analyze the normative benchmark of a social planner.

### 3.2 The social planner's solution

The problem of a Utilitarian social planner is equivalent to the version of the model where full cohesiveness $(\theta=1 / 2)$ restricts the incumbent group in both periods to provide the

[^9]same transfers to both groups. Then, since $\alpha_{L}>2(1-\theta)=1$, the social planner always uses all residual money to provide public goods. For the basic model with debt, the results about debt and state capacity investments of a social planner are summarized in the following proposition: ${ }^{16}$

Proposition 3.1. Suppose that the decisions about debt and state capacity investments are made by a Pigouvian planner with Utilitarian preferences. Then:

1. If $\alpha_{1}=\alpha_{L}$ :
(a) No debt is raised.
(b) No transfers are paid.
(c) There are positive investments in fiscal and legal capacity.
(d) Higher $\phi$ increases investment in fiscal and legal capacity.
2. If $\alpha_{1}=\alpha_{H}$ :
(a) Debt is raised maximally: $b_{1}=\frac{\tau_{2} \omega\left(\pi_{2}\right)}{1+\rho}$.
(b) No transfers are paid.
(c) There are positive investments in fiscal and legal capacity and the investments are higher than when no debt can be raised.

For a social planner $\lambda_{1}=\alpha_{1}$. That is, the gross marginal value of public funds in the first period corresponds to the value of public goods in the first period. Moreover, for the social planner the gross marginal value of public funds in the second period is $E\left(\lambda_{2}\right)=\phi \alpha_{H}+(1-\phi) \alpha_{L}>1$.

For the first part, note that $\lambda_{1}=\alpha_{L}<E\left(\lambda_{2}\right)$ implies that no debt will be raised. In a model without debt, the results of the first part are valid for both, $\alpha_{1}=\alpha_{L}$ and $\alpha_{1}=\alpha_{H}$ as stated in Proposition 2.1 in Besley and Persson (2011).

In the model with debt, if $\alpha_{1}=\alpha_{H}$, we have $\lambda_{1}=\alpha_{H}>E\left(\lambda_{2}\right)$ and debt will be raised maximally in order to make future public funds available in the present. Therefore, the net marginal benefit of future public funds is greater than it was without debt, which raises incentives to invest in fiscal capacity. Basically, debt allows the social planner to use the tax system of the future to finance a highly-valued public good today. Given a high need for public funds today versus a lower need tomorrow this increases incentives to invest in fiscal capacity for the purpose of increasing spending today. By complementarity, investments in legal capacity increase as well.

We now turn to the analysis of the political equilibrium.

[^10]
### 3.3 Three types of states revisited

The outcome of the political game will depend on the interplay of the parameters governing how cohesive political institutions are ( $\theta$ ) and how stable the political system is $(\gamma)$ with the public good parameters $\left(\phi, \alpha_{H}, \alpha_{L}\right)$ summarizing the main features of the economic environment. In Besley and Persson (2011), the following condition ensures that political institutions are sufficiently cohesive to make the political outcome coincide with the outcome under a social planner:

Cohesiveness: $\quad \alpha_{L} \geq 2(1-\theta)$
This condition will hold if the parameter governing the cohesiveness of political institutions, $\theta$, is close enough to $1 / 2$, the value it takes for a social planner. Recall that $\theta=1 / 2$ ensures that both groups have to be treated equally and therefore captures perfectly cohesive political institutions.

If the cohesiveness condition fails, but the stability of the political system is high enough, Besley and Persson (2011) get a state that still has positive investments in state capacity. The corresponding stability condition is:

Stability: $\quad \phi \alpha_{H}+(1-\phi)[(1-\gamma) 2(1-\theta)+\gamma 2 \theta]>1$
This condition will hold when the probability of staying in political power, $1-\gamma$, is big enough. That is, from the point of view of the period-1 incumbent government, the political system is stable in the sense of not endangering its power. However, the condition goes further. In fact, it refers to stability in the sense of not endangering the interests of the period-1 government. This can also be ensured by the economic environment. For instance, if a high value of public good spending is expected with certainty $(\phi \rightarrow 1)$, the stability condition will also hold. The interest of the period-1 government in future public good spending is then respected no matter who is in power in the future. In order to compare our results to the ones in Besley and Persson (2011), we consider the same three types of states that they derive and investigate if these types still arise after the introduction of debt.

## Common-Interest State

In the case where the cohesiveness condition holds, we get the following result:
Proposition 3.2. If Cohesiveness holds, then the outcome is the same as under a social planner (see Proposition 3.1).

This result is analogous to the model without debt (Proposition 2.2 in Besley and Persson
(2011)). The reason is that high cohesiveness makes redistribution unattractive compared to public good spending even when the latter has a low value. Therefore, by choice, each government will provide only public goods thereby behaving exactly like a social planner. Additionally to the model without debt, the shifting of public resources over time also follows the structure of the public good values and again coincides with the social planner behavior. In line with Besley and Persson (2011), we call this state the common interest state.

## Redistributive State

Assume that Cohesiveness fails, but Stability holds. We get the following results:
Proposition 3.3. If Cohesiveness fails and Stability holds, then:

1. If $\alpha_{1}=\alpha_{L}$ and $2(1-\theta)<\phi \alpha_{H}+(1-\phi)[(1-\gamma) 2(1-\theta)+\gamma 2 \theta]$ :
(a) No debt is raised.
(b) Residual revenues in period 1 are used to finance transfers.
(c) There are positive investments in fiscal and legal capacity.
(d) Higher $\phi$ increases investments in fiscal and legal capacity.
(e) A lower value of $\gamma$ unambiguously raises investments, whereas an increase in $\theta$ raises investments if $\gamma>1 / 2$.
2. If $\alpha_{1}=\alpha_{H}$ or if $\alpha_{1}=\alpha_{L}$ and $2(1-\theta)>\phi \alpha_{H}+(1-\phi)[(1-\gamma) 2(1-\theta)+\gamma 2 \theta]$ :
(a) Debt is raised maximally: $b_{1}=\frac{\tau_{2} \omega\left(\pi_{2}\right)}{1+\rho}$.
(b) There are positive investments in fiscal and legal capacity and the investments are higher than when no debt can be raised.
(c) If $\alpha_{1}=\alpha_{H}$, the levels of fiscal and legal capacity investments are the same as those chosen by a social planner in the same situation (see Proposition 3.1 Part 2).
(d) If $\alpha_{1}=\alpha_{L}$, residual revenues in period 1 are used to finance transfers.
(e) Political instability $\gamma$ does not have an influence on the investment decisions. If $\alpha_{1}=\alpha_{H}$, cohesiveness $\theta$ does not have an influence either.

In an environment with a low public good value in the first period, the incumbent government prefers to spend on redistributive transfers in this first period. However, when the condition in part 1 of the proposition holds, the expected value of future public spending is still higher than the value of this first-period transfer spending. The implied preference
for future spending leads to the result that no debt is raised, because this would mean taking resources from the future. Therefore, the remaining results in the first part of the proposition are analogous to the model without debt.

When one of the conditions in part 2 of the proposition holds, there is a preference for the present. Specifically, the incumbent group in period 1 can no longer be sure that spending in the second period will be in its interest in expectation. However, with debt, the period-1 government now has the possibility to bring future public funds at its disposal. Thereby, it can decide about the spending purposes which these future public funds will be used for. This will actually allow the incumbent group to solve the problem of future redistribution against itself and hence raises incentives for investing in fiscal and legal capacity.

Importantly, these bigger incentives can be driven by the desire to finance redistributive transfers in the present, which is not in the spirit of a Utilitarian social planner. Therefore, we do not only get spending on the 'wrong' issues, we can even get the incentive to finance more of this 'wrong' spending through the issuance of debt. In this case of the basic model, the additional debt-induced incentive to invest in fiscal and legal capacity therefore creates a bigger deviation of the political outcome from the social planner optimum.

## ‘Weak' State

The last possibility arises when both the cohesiveness and the stability condition fail. Besley and Persson (2011) call such a state a weak state. In their model without debt, such a state has no incentive to invest in fiscal capacity (Proposition 2.4 in Besley and Persson (2011)).

As we have already seen, the introduction of debt can raise incentives to invest because what drives these incentives is now the use that period- 2 public funds can be put to in the first period. This strongly suggests that the weak state situation, which is based on the fear of future public funds being used against the own group, will no longer arise in this basic model with debt. The next proposition confirms this hypothesis:

Proposition 3.4. In the basic model without the possibility of default, if Cohesiveness and Stability fail, then:

1. There is positive investment in fiscal capacity which is higher than the zero investment in the case without debt. Moreover, there is positive investment in legal capacity which is higher than in the case without debt.
2. If $\alpha_{1}=\alpha_{H}$, the levels of investment in fiscal and legal capacity are the same as those chosen by a social planner in the same situation (see Proposition 3.1 Part 2).
3. Debt is raised maximally: $b_{1}=\frac{\tau_{2} \omega\left(\pi_{2}\right)}{1+\rho}$.
4. If $\alpha_{1}=\alpha_{L}$, residual revenues are used to finance transfers.

So, the weak state situation as in Besley and Persson (2011) does no longer arise when debt is allowed since the possibility to raise debt creates incentives for investing in state capacity. We even get the social planner's investment level if $\alpha_{1}=\alpha_{H}$. For the case $\alpha_{1}=\alpha_{L}$, we also get positive investments in state capacity, potentially even higher than the social planner's investments. However, from the perspective of a social planner, this case is now even worse than in the model without debt since all future tax revenue is now drawn to the present and used for transfers directed to the incumbents clientele.

The arguments for all these results and their interpretations are the same as for Part 2 of Proposition 3.3, which described a redistributive state.

It is important to note, however, that this result of the weak state situation no longer arising depends on the highly stylized setup of the basic model. As soon as we introduce the possibility of default, the costs of raising high debt enter the analysis. Doing so in the extended setup of the next section, we can reestablish the possibility of a weak state.

## 4 Full Model

Despite providing a very good starting point for further analysis, the basic model needs some modification to accommodate the introduction of debt in a more realistic manner. Until now, the interest rate of government bonds is constant and so it is independent of the level of debt, future income and tax revenues are perfectly predictable, government default is impossible and the model has a bang-bang solution, i.e. either maximal debt or no debt at all. In the following section we extend the model by allowing income $\omega$ to be subject to shocks. As we will see below, this leads to varying interest rates, the possibility of government default and inner solutions for public debt.

In this section, we allow the income $\omega_{s}\left(\pi_{s}\right)$ of the economy in period $s$ to fluctuate so that tax revenues $\tau_{s} \omega_{s}\left(\pi_{s}\right)$ are also uncertain. The following can thus be interpreted as a parsimonious way of including an exogenous business cycle component into the model. Specifically, we assume $\bar{\omega}\left(\pi_{s}\right)>\underline{\omega}\left(\pi_{s}\right), \bar{\omega}^{\prime}\left(\pi_{s}\right)=\underline{\omega}^{\prime}\left(\pi_{s}\right) \equiv \omega^{\prime}\left(\pi_{s}\right)^{17}$ and $\operatorname{Prob}\left(\omega_{s}\left(\pi_{s}\right)=\bar{\omega}\left(\pi_{s}\right)\right)=$ $\psi$.

[^11]In such a setting, the interest rate is endogenously determined by $R(b)=\rho+r(b) . r(b)$ is the risk premium, which is nonzero if $b$ exceeds a certain threshold $\bar{b}$, characterized below. So far, the interest rate $\rho=1 / \delta-1$ was pinned down by the discount factor $\delta$ of the consumers, and therefore it was independent of the debt level $b$. However, in reality the interest rate a country has to pay in order to issue government bonds depends on the level of public debt. The higher interest rate captures the risk premium due to a higher probability of default.

The analysis now requires a more precise description of the timing in the game. Let the timing be as described in section 3. However, since the investment decision regarding fiscal capacity might influence the solvency of the state and therefore the credit terms, one has to be careful regarding the timing of issuing debt and investing in fiscal capacity. Let us divide stage 2 of the timing from section 3 into two stages 2 a and 2b. Assume that in stage $2 a$ the government makes the decision regarding the investment in fiscal capacity and in stage 2 b all other decisions including debt. ${ }^{18}$ In such a setting, the investors that buy the government bonds condition their expectations regarding the future solvency of the state on the future levels of fiscal and legal capacity, $\tau_{2}$ and $\pi_{2}$.

Besides lending money to the model country, we assume that the international investors have the possibility to invest in riskless bonds which just compensate them for their time preference. These riskless bonds therefore have an interest rate of $\rho=1 / \delta-1$. Since investors are assumed to be risk neutral, the risk premium has to be just high enough to make them indifferent between lending money to the model country and investing in the risk-free asset.

The threshold $\bar{b}$ is defined such that for $b \leq \bar{b}$, bonds plus interest are fully payed back even for the low income realization $\underline{\omega}\left(\pi_{2}\right)$. In this case, there is no risk that needs compensation, so $R(b)=\rho$. The threshold $\bar{b}$ is given by:

$$
\begin{equation*}
\bar{b}\left(\tau_{2}, \pi_{2}\right)=\frac{\tau_{2} \underline{\omega}\left(\pi_{2}\right)}{1+\rho} \tag{24}
\end{equation*}
$$

For $b>\bar{b}$, debt will be payed back fully in case of high income $\bar{\omega}\left(\pi_{2}\right)$ but partially else. The function for the risk premium, $r(b)$, that makes investors indifferent between lending to the country and investing in the risk-free asset is defined by

$$
\begin{equation*}
(1+\rho) b_{1}=\psi \cdot \underbrace{\left(1+\rho+r\left(b_{1}\right)\right) b_{1}}_{\text {repayment in case of } \bar{\omega}\left(\pi_{2}\right)}+(1-\psi) \cdot \underbrace{\left(\tau_{2} \underline{\omega}\left(\pi_{2}\right)\right)}_{\text {repayment in case of } \underline{\omega}\left(\pi_{2}\right)} \tag{25}
\end{equation*}
$$

Rearranging terms leads to the expression $r\left(b_{1}\right)=\frac{1-\psi}{\psi}\left(1+\rho-\frac{\tau_{2} \omega\left(\pi_{2}\right)}{b_{1}}\right)$.

[^12]It is clear that there has to be a maximum level of debt $\overline{\bar{b}}$. This is defined by the maximum debt that can be fully payed back including interest in the case of high income $\bar{\omega}\left(\pi_{2}\right)$. This level is given by $\overline{\bar{b}}=\frac{\tau_{2} \bar{\omega}\left(\pi_{2}\right)}{1+\rho+r(\overline{\bar{b}})}$. Solving for $\overline{\bar{b}}$, this leads to:

$$
\begin{equation*}
\overline{\bar{b}}\left(\tau_{2}, \pi_{2}\right)=\frac{\tau_{2}\left(\psi \bar{\omega}\left(\pi_{2}\right)+(1-\psi) \underline{\omega}\left(\pi_{2}\right)\right)}{1+\rho} \tag{26}
\end{equation*}
$$

When debt is not completely payed back, which means that there is sovereign debt default, the country incurs a penalty $P$. For reasons of tractability, the penalty is assumed to reduce the after-tax income. It would certainly be more realistic to have the penalty reduce gross income as in many models of sovereign debt default (e.g. Alesina and Tabellini (1989), Arellano (2008), Eaton and Gersovitz (1981)). However, the above assumption is made to avoid technical complications. The penalty can be interpreted, for instance, as credit restrictions on retailers which make it more expensive to supply imported goods. The ensuing reduction in the purchasing power of income is captured in our simple model by the direct reduction of after-tax income through the penalty.

We assume that the penalty has the following form (where $\Delta$ is the amount not repaid):

$$
P= \begin{cases}0 & \text { if } \quad \Delta=0 \quad(\text { no default })  \tag{27}\\ P(\Delta) & \text { if } \Delta=\left(1+R\left(b_{1}\right)\right) b_{1}-\left(\tau_{2} \underline{\omega}\left(\pi_{2}\right)\right) \quad \text { and } \omega_{2}\left(\pi_{2}\right)=\underline{\omega}\left(\pi_{2}\right) \\ P_{\max } & \text { else. }\end{cases}
$$

This means, as long as the government shows good will, in the sense that it repays as much debt as it can, the penalty depends on the amount of debt that is not repaid. $P(\Delta)$ is assumed to be increasing and convex for $\Delta \in\left[0,(1+R(\overline{\bar{b}})) \overline{\bar{b}}-\left(\tau_{2} \underline{\omega}\left(\pi_{2}\right)\right)\right]=$ $\left[0, \tau_{2}\left(\bar{\omega}\left(\pi_{2}\right)-\underline{\omega}\left(\pi_{2}\right)\right)\right]$ with $P(0)=0$. If the country repays less than possible and defaults purposely, we assume the punishment to be maximal, $P_{\max }$. We assume $P_{\max }$ to be high enough to prevent the government from defaulting purposely. That is, we only consider ability-to-pay default and not willingness-to-pay default. This allows us to model rising costs of debt financing without having to burden the analysis with a more involved modeling of the default decision.

Concerning the intra-temporal policies, with the same reasoning as in the model without debt, fiscal and legal capacities are always fully employed. As for the policy function for public good spending, it also looks analogous to before:

$$
G\left(\alpha_{s}, \tau_{s}, \pi_{s}, m_{s}, n_{s}, b_{s}, b_{s-1}, \hat{\omega}\right)= \begin{cases}\tau_{s} \hat{\omega}\left(\pi_{s}\right)-m_{s}-n_{s}+b_{s}  \tag{28}\\ -\min \left\{\left(1+R\left(b_{s-1}\right)\right) b_{s-1}, \tau_{s} \hat{\omega}\left(\pi_{s}\right)\right\} & \text { if } \alpha_{s} \geq 2(1-\theta) \\ 0 & \text { otherwise }\end{cases}
$$

where $\hat{\omega}$ can now be the high or low income realization $\bar{\omega}$ or $\underline{\omega}$. For simplifying the notation, the following contains the "expected" policy function $G=\psi G(\bar{\omega})+(1-\psi) G(\underline{\omega})$, where $\psi$ is the probability for the high income realization $\bar{\omega}$.

The inter-temporal maximization problem of the incumbent group in period $\mathrm{s}=1$ becomes:

$$
\begin{align*}
\max _{\tau_{2}, \pi_{2}, b_{1}} E V^{I_{1}}\left(\tau_{2}, \pi_{2}, b_{1}\right) & -\lambda_{1}\left(F\left(\tau_{2}-\tau_{1}\right)+L\left(\pi_{2}-\pi_{1}\right)-b_{1}\right)  \tag{29}\\
\text { s.t. } \tau_{2} & \geq \tau_{1}, \\
\pi_{2} & \geq \pi_{1}, \\
b_{1} & \leq \overline{\bar{b}}\left(\tau_{2}\right), \\
\lambda_{1} & \equiv \max \left\{\alpha_{1}, 2(1-\theta)\right\}
\end{align*}
$$

with

$$
\begin{align*}
E V^{I_{1}}\left(\tau_{2}, \pi_{2}, b_{1}\right) & =\delta\left((1-\gamma) U^{I}\left(\tau_{2}, \pi_{2}, b_{1}\right)+\gamma U^{0}\left(\tau_{2}, \pi_{2}, b_{1}\right)\right)  \tag{30}\\
& =\delta\left(( 1 - \gamma ) \left(\phi E W\left(\alpha_{H}, \tau_{2}, \pi_{2}, 0, b_{1}, 0,2(1-\theta)\right)\right.\right. \\
& \left.+(1-\phi) E W\left(\alpha_{L}, \tau_{2}, \pi_{2}, 0, b_{1}, 0,2(1-\theta)\right)\right) \\
& \left.+\gamma\left(\phi E W\left(\alpha_{H}, \tau_{2}, \pi_{2}, 0, b_{1}, 0,2 \theta\right)+(1-\phi) E W\left(\alpha_{L}, \tau_{2}, \pi_{2}, 0, b_{1}, 0,2 \theta\right)\right)\right)
\end{align*}
$$

and

$$
\begin{aligned}
& E W\left(\alpha_{2}, \tau_{2}, \pi_{2}, m_{2}=0, n_{2}=0, b_{1}, b_{2}=0, \beta^{J}\right)= \\
& \alpha_{2} G+\left(1-\tau_{2}\right)\left(\psi \bar{\omega}\left(\pi_{2}\right)+(1-\psi) \underline{\omega}\left(\pi_{2}\right)\right)-(1-\psi) P+\beta^{J}\left[\tau_{2}\left(\psi \bar{\omega}\left(\pi_{2}\right)+(1-\psi) \underline{\omega}\left(\pi_{2}\right)\right)\right. \\
& \left.-G-\psi\left(1+R\left(b_{1}\right)\right) b_{1}-(1-\psi) \min \left\{\left(1+R\left(b_{1}\right)\right) b_{1}, \tau_{2} \underline{\omega}\left(\pi_{2}\right)\right\}\right],
\end{aligned}
$$

which is the indirect payoff function for group $J \in I, O$ in period $s$. This function is now an expected value itself, because future income $\omega_{2}\left(\pi_{2}\right)$ is uncertain. Note that for the analysis here, we assume that the fluctuations in income and in the valuation of public goods are independent. In the quasi-linear setup of section 5 , we will consider the other extreme, a perfect correlation between the two in the sense that public good spending has a higher value in times with low income. The most realistic modeling probably lies at some intermediate level of correlation, but the extreme cases allow us to keep the analysis tractable.

Plugging (31) into (30), leads to:

$$
\begin{align*}
E V^{I_{1}}\left(\tau_{2}, \pi_{2}, b_{1}\right)= & \delta\left[\left(\psi \bar{\omega}\left(\pi_{2}\right)+(1-\psi) \underline{\omega}\left(\pi_{2}\right)\right)\left(1-\tau_{2}\right)-(1-\psi) P\right.  \tag{32}\\
& +\underbrace{\left[\phi \alpha_{H}+(1-\phi) \lambda_{2}^{L}\right]}_{\equiv E\left(\lambda_{2}\right)} \cdot\left[\tau_{2}\left(\psi \bar{\omega}\left(\pi_{2}\right)+(1-\psi) \underline{\omega}\left(\pi_{2}\right)\right)\right. \\
& \left.\left.-\psi\left(1+R\left(b_{1}\right)\right) b_{1}-(1-\psi) \min \left\{\left(1+R\left(b_{1}\right)\right) b_{1}, \tau_{2} \underline{\omega}\left(\pi_{2}\right)\right\}\right]\right]
\end{align*}
$$

with $\lambda_{2}^{L}$ defined by (14).

### 4.1 Solution of the Model

Regarding the solution of this optimization problem, several cases can arise:

- case a) : $b_{1}=0^{19}$
- case b) : $b_{1} \in(0, \bar{b})$
- case c) : $b_{1}=\bar{b}$
- case d) : $b_{1} \in(\bar{b}, \overline{\bar{b}})$
- case e) : $b_{1}=\overline{\bar{b}}$

When do these cases emerge? The first order conditions with respect to $b_{1}$ in case b) and d) lead us to the relevant conditions. In case b), the first order condition with respect to $b_{1}$ gives us $\lambda_{1}=E\left(\lambda_{2}\right)$ and in case d) $\lambda_{1}=E\left(\lambda_{2}\right)+\frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}$. It follows immediately that if $E\left(\lambda_{2}\right)<\lambda_{1}<E\left(\lambda_{2}\right)+\left.\frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\bar{b}}$, we are in case c), if $\lambda_{1}$ exceeds $E\left(\lambda_{2}\right)+$ $\left.\frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\overline{\bar{b}}}$, we are in case e) and if $\lambda_{1}$ is smaller than $E\left(\lambda_{2}\right)$, we are in case a).

Since the function $R\left(b_{1}\right)$ depends on the case and includes $\tau_{2}$ and $\pi_{2}$, also the first order conditions that determine $\tau_{2}$ and $\pi_{2}$ vary over the cases. The following table summarizes the expressions: ${ }^{20}$

| Case | Condition |
| :--- | :--- |
| a) | $\lambda_{1}<E\left(\lambda_{2}\right)$ |
| b) | $\lambda_{1}=E\left(\lambda_{2}\right)$ |
| c) | $E\left(\lambda_{2}\right)<\lambda_{1}<E\left(\lambda_{2}\right)+\left.\frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}\right\|_{b=\bar{b}}$ |
| d) | $\lambda_{1}=E\left(\lambda_{2}\right)+\frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}$ |
| e) | $\lambda_{1}>E\left(\lambda_{2}\right)+\left.\frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}\right\|_{b=\overline{\bar{b}}}$ |


| Case | FOC for $\tau_{2}$ |
| :--- | :--- |
| a) | $\delta\left\{\psi \bar{\omega}\left(\pi_{2}\right)+(1-\psi) \underline{\omega}\left(\pi_{2}\right)\right\}\left(E\left(\lambda_{2}\right)-1\right)=\lambda_{1} \frac{\partial F\left(\tau_{2}-\tau_{1}\right)}{\partial \tau_{2}}$ |
| $\mathrm{~b})$ | $\left.\delta\left\{\psi \bar{\omega}\left(\pi_{2}\right)+(1-\psi) \underline{\omega}\left(\pi_{2}\right)\right)\right\}\left(E\left(\lambda_{2}\right)-1\right)=\lambda_{1} \frac{\partial F\left(\tau_{2}-\tau_{1}\right)}{\partial \tau_{2}}$ |
| c) | $\delta\left\{\psi\left[\bar{\omega}\left(\pi_{2}\right)-\underline{\omega}\left(\pi_{2}\right)\right]\left(E\left(\lambda_{2}\right)-1\right)+\underline{\omega}\left(\pi_{2}\right)\left(\lambda_{1}-1\right)\right\} \leq \lambda_{1} \frac{\partial F\left(\tau_{2}-\tau_{1}\right)}{\partial \tau_{2}}$ |
| $\mathrm{~d})$ | $\delta\left\{\psi\left[\bar{\omega}\left(\pi_{2}\right)-\underline{\omega}\left(\pi_{2}\right)\right]\left(E\left(\lambda_{2}\right)-1\right)+\underline{\omega}\left(\pi_{2}\right)\left(\lambda_{1}-1\right)\right\} \leq \lambda_{1} \frac{\partial F\left(\tau_{2}-\tau_{1}\right)}{\partial \tau_{2}}$ |
| $\mathrm{e})$ | $\delta\left\{\left(\psi \bar{\omega}\left(\pi_{2}\right)+(1-\psi) \underline{\omega}\left(\pi_{2}\right)\right)\left(\lambda_{1}-1\right)-\left.(1-\psi) \frac{\partial P(\Delta)}{\partial \Delta}\right\|_{b=\overline{\bar{b}}}\left(\bar{\omega}\left(\pi_{2}\right)-\underline{\omega}\left(\pi_{2}\right)\right)\right\} \leq \lambda_{1} \frac{\partial F\left(\tau_{2}-\tau_{1}\right)}{\partial \tau_{2}}$ |

[^13]| Case | FOC for $\pi_{2}$ |
| :--- | :--- |
| a) | $\delta \omega^{\prime}\left(\pi_{2}\right)\left[1+\tau_{2}\left(E\left(\lambda_{2}\right)-1\right)\right]=\lambda_{1} \frac{\partial L\left(\pi_{2}-\pi_{1}\right)}{\partial \pi_{2}}$ |
| b) | $\delta \omega^{\prime}\left(\pi_{2}\right)\left[1+\tau_{2}\left(E\left(\lambda_{2}\right)-1\right)\right]=\lambda_{1} \frac{\partial L\left(\pi_{2}-\pi_{1}\right)}{\partial \pi_{2}}$ |
| c) | $\delta \omega^{\prime}\left(\pi_{2}\right)\left[1+\tau_{2}\left(\lambda_{1}-1\right)\right]=\lambda_{1} \frac{\partial L\left(\pi_{2}-\pi_{1}\right)}{\partial \pi_{2}}$ |
| d) | $\delta \omega^{\prime}\left(\pi_{2}\right)\left[1+\tau_{2}\left(\lambda_{1}-1\right)\right]=\lambda_{1} \frac{\partial L\left(\pi_{2}-\pi_{1}\right)}{\partial \pi_{2}}$ |
| e) | $\delta \omega^{\prime}\left(\pi_{2}\right)\left[1+\tau_{2}\left(\lambda_{1}-1\right)\right]=\lambda_{1} \frac{\partial L\left(\pi_{2}-\pi_{1}\right)}{\partial \pi_{2}}$ |

In the following analysis, we consider an economy that has a low value of public good spending in the first period, $\alpha_{1}=\alpha_{L}$. In this environment, a social planner will not want to raise debt. In particular, the social planner's solution is clearly case a), since the social planner has $\lambda_{1}=\alpha_{1}<(1-\phi) \alpha_{L}+\phi \alpha_{H}=E\left(\lambda_{2}\right) .{ }^{21}$

We want to see whether for such an environment, governments with a preference for the own group could exhibit a bias towards the present and hence towards excessive debt. Such a bias would make the political equilibrium differ from the social planner's solution in a significant way.

Furthermore, for the following analysis, we define 'free future revenues' as the discounted expected future tax revenues minus debt, $\delta E\left(\tau_{2} \omega_{2}\left(\pi_{2}\right)\right)-b_{1}$. Free future revenues refer to the expected future tax revenues that are not bound by debt and therefore measure the 'free' resources of the future government. We define this measure in order to measure debt in relation to a state's fiscal power, which is more informative than the absolute debt level itself.

So what is the political equilibrium? Let's distinguish between countries with high and low cohesiveness $\theta$. Assume first that cohesiveness is sufficiently high, in the sense that the cohesiveness condition of section 3 holds: $\alpha_{L}>2(1-\theta)$. Then the political equilibrium is case a), since we have $\lambda_{1}=\alpha_{1}$ and $E\left(\lambda_{2}\right)=(1-\phi) \alpha_{L}+\phi \alpha_{H}$, as it was the case for the social planner. That is, high enough cohesiveness will make the political equilibrium coincide with the social planner outcome. The respective comparative statics are summarized in part 1 of the following proposition.

Now, consider countries with low cohesiveness, in the sense that $\theta$ is sufficiently below $1 / 2$. These countries can end up in each of the cases a)-e), depending on the parameters $\gamma, \phi$ and $\alpha_{H}$ summarized in $E\left(\lambda_{2}\right) .{ }^{22}$ Parts 2-4 of the following proposition summarize the corresponding results:

[^14]Proposition 4.1. Suppose an economy in the model with sovereign default starts in the first period with $\alpha_{1}=\alpha_{L}$. Moreover, suppose that the constraint $\tau_{2} \geq \tau_{1}$ does not bind. ${ }^{23}$ Then:

1. In the social planner's solution as well as in the political equilibrium, if $\alpha_{L} \geq 2(1-\theta)$
(a) No debt is raised [case a) holds].
(b) No transfers are paid.
(c) There are positive investments in fiscal and legal capacity.
(d) Higher $\phi$ increases investment in fiscal and legal capacity.
(e) Higher $\psi$ increases investment in fiscal and legal capacity.
(f) Neither $\gamma$ nor $\theta$ have an influence on the investment decisions.
(g) Free future revenues are increasing in $\phi$ and $\psi$.

In the following, consider the political equilibrium for $\alpha_{L}<2(1-\theta)$. That is, we have $\lambda_{1}=2(1-\theta)$ and $E\left(\lambda_{2}\right)=\phi \alpha_{H}+(1-\phi)[(1-\gamma) 2(1-\theta)+\gamma 2 \theta]$.
2. If $\lambda_{1}<E\left(\lambda_{2}\right)$
(a) No debt is raised [case a) holds].
(b) There are positive investments in fiscal and legal capacity.
(c) Higher $\phi$ increases investment in fiscal and legal capacity.
(d) Higher $\psi$ increases investment in fiscal and legal capacity.
(e) A lower value of $\gamma$ unambiguously raises investments, whereas an increase in $\theta$ raises investments if $\gamma$ is above 1/2.
(f) Free future revenues are increasing in $\theta$ (if $\gamma>1 / 2$ ), $\phi, \psi$ and $\gamma$.
3. If $E\left(\lambda_{2}\right)<\lambda_{1}<E\left(\lambda_{2}\right)+\left.\frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\overline{\bar{b}}}$
(a) If $E\left(\lambda_{2}\right)<\lambda_{1}<E\left(\lambda_{2}\right)+\left.\frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\bar{b}}$, then $b_{1}=\bar{b}$ is the optimal debt level. That is case c) holds.
i. Higher $\phi$ as well as lower $\gamma$ lead to an increase of both $b_{1}$ and free future revenues.
ii. Higher $\psi$ leads to an increase of both $b_{1}$ and free future revenues.

[^15](b) If $\lambda_{1}=E\left(\lambda_{2}\right)+\left.\frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\tilde{b}}$, with $\bar{b}<\tilde{b}<\overline{\bar{b}}$, then $b_{1}=\tilde{b}$ is the optimal debt level (case d) holds), so there is sovereign default in the second period with probability $1-\psi$.
i. Higher $\psi$ leads to an increase of $b_{1}$.
ii. Higher $\phi$ as well as lower $\gamma$ lead to an increase of free future revenues.
(c) Residual revenues are used to finance transfers.
(d) There are positive investments in fiscal and legal capacity.
(e) Higher $\phi$ as well as lower $\gamma$ increase investments in fiscal and legal capacity.
(f) Higher $\psi$ increases investment in fiscal and legal capacity.
4. If $\lambda_{1}>E\left(\lambda_{2}\right)+\left.\frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\overline{\bar{b}}}$
(a) Debt is raised maximally, $b_{1}=\overline{\bar{b}}$ (case e) holds).
(b) Neither $\phi$ nor $\gamma$ have an influence on the debt level, but it is increasing in $\psi$.
(c) Free future revenues are 0 and therefore they are constant with respect to $\phi, \gamma$, $\psi$ and $\theta$.
(d) Residual revenues are used to finance transfers.
(e) There are positive investments in fiscal and legal capacity.
(f) Neither $\phi$ nor $\gamma$ have an influence on the investment decision.
(g) Higher $\psi$ increases investment in fiscal and legal capacity.

As already mentioned, part 1 of the proposition establishes the social planner solution, which coincides with the political equilibrium under the cohesiveness condition, $\alpha_{L} \geq$ $2(1-\theta)$. To understand the comparative static results, we can look at the respective first order conditions for fiscal and legal capacity in case a). Recall that the right-hand side of these conditions gives the marginal benefit of higher investments. Since no debt is raised and future tax resources are left in the future, investment incentives are driven by the expected value of future public funds, $E\left(\lambda_{2}\right)=\phi \alpha_{H}+(1-\phi) \alpha_{L}$. This explains the comparative static results (d) and (e). For instance, the benefit of future fiscal capacity depends on the expected income base to which it can be applied, $\left\{\psi \bar{\omega}\left(\pi_{2}\right)+(1-\psi) \underline{\omega}\left(\pi_{2}\right)\right\}$. Since this is increasing in $\psi$, the probability of a high income realization, investment incentives increase in $\psi$.

Parts 2-4 of Proposition 4.1 illustrate the outcome of the political equilibrium when the cohesiveness condition fails. That is, cohesiveness $\theta$ is low enough such that $\alpha_{L}<$ $2(1-\theta)$. The proposition contains the comparative static results that can be shown using the techniques of monotone comparative statics. It turns out that it is hard to get


Figure 1
unambiguous comparative static results with respect to cohesiveness $\theta$, as soon as debt is used. Therefore, we concentrate, for the following illustration, on parameter changes which do not alter cohesiveness $\theta$. This implies that we keep the value of present public funds $\lambda_{1}=2(1-\theta)$ constant. Most of the remaining parameters enter $E\left(\lambda_{2}\right)$, the expected value of future public funds. Therefore, we can illustrate most comparative static results by considering the reactions to a change in $E\left(\lambda_{2}\right)$, keeping $\lambda_{1}$ constant.

For a fixed and sufficiently low value of cohesiveness, Figure 1 illustrates the relation
between $E\left(\lambda_{2}\right)$, future fiscal capacity $\tau_{2}$, future legal capacity $\pi_{2}$, debt $b_{1}$ and free future revenues. ${ }^{24}$

The letters below the graphs refer to the different cases we identified above. Given an expected value of future public funds, $E\left(\lambda_{2}\right)$, higher than the present value $\lambda_{1}$, we start on the right of the figure in case a). The corresponding results are summarized in Part 2 of Proposition 4.1. No debt is raised and the comparative static results correspond to a model without debt. In particular, all parameter changes that decrease $E\left(\lambda_{2}\right)$ will decrease investments in fiscal and legal capacity. One important possibility to lower $E\left(\lambda_{2}\right)$ is an increase in political instability.

Continuing to decrease $E\left(\lambda_{2}\right)$, at some point, we reach the knive-edge case b) with $E\left(\lambda_{2}\right)=\lambda_{1} \cdot{ }^{25}$ The debt level in this case is indeterminate in the range $[0, \bar{b}]$.

By lowering $E\left(\lambda_{2}\right)$ further, we move to case c). To understand the comparative static results for case c), it is helpful to consider the respective first order condition for fiscal capacity $\tau_{2}$. Note that $\underline{\omega}\left(\pi_{2}\right)$ is the "low income part" of the expected future tax base. This part will be available for sure in the future. In contrast, $\psi\left[\bar{\omega}\left(\pi_{2}\right)-\underline{\omega}\left(\pi_{2}\right)\right]$ describes the additional expected value of the (income) tax base that will be available if the high income realizes. We call this the "high income part".

In case c), it is optimal to exactly raise a debt level of $\bar{b}\left(\tau_{2}, \pi_{2}\right)=\frac{\tau_{2} \omega\left(\pi_{2}\right)}{1+\rho}$. Because of the ensuing penalty, it would be too expensive to raise more debt. This means that the low income part of the future tax base is fully drawn to the present. For this part, the marginal benefit of making more of it available to the state through fiscal capacity investments is thus proportional to ( $\lambda_{1}-1$ ). For case c), $E\left(\lambda_{2}\right)<\lambda_{1}$, so future public funds are more valuable when they can be used in the present through debt. Therefore, investment incentives are higher than in a world without debt through the influence of the low income part. ${ }^{26}$

In contrast, the high income part of expected future tax resources is not drawn to the present. For this part of the future tax base, the marginal benefit of making more of it available through fiscal capacity investments is thus proportional to $\left(E\left(\lambda_{2}\right)-1\right)$. As far as this part is concerned, we therefore have the same effects as in a model without debt. For instance, given low cohesiveness, increasing political instability makes it more likely that the current government's group gets screwed over in the future by a rival government. This

[^16]decreases $E\left(\lambda_{2}\right)$ and, through the influence of the high income part, decreases incentives to invest in fiscal capacity. By complementarity between the two forms of state capacity investment, we also get less investments in legal capacity $\pi_{2} .{ }^{27}$ Lower levels of fiscal and legal capacity decrease $\bar{b}\left(\tau_{2}, \pi_{2}\right)=\frac{\tau_{2} \omega\left(\pi_{2}\right)}{1+\rho}$. The latter is just the present value of the low income part of future public funds. Since exactly this part is drawn to the present in case c), the debt level decreases when investments in state capacity decrease. ${ }^{28}$ This is illustrated in the third panel of Figure 1. For case c), we thus have debt and state capacity moving in the same direction in response to exogenous parameter changes.

Continuing to decrease the value of future public funds, $E\left(\lambda_{2}\right)$, we enter case d). In this case, it is optimal to incur some penalty. The optimality condition for debt implies that the optimal debt level lies a certain amount $d b$ above $\bar{b}$. Since optimal debt $b_{1}$ is still lower than the maximal debt level $\overline{\bar{b}}$, there is still some proportion of expected future tax resources which is left in the future. In particular, for the marginal investment in fiscal capacity, the high income part of future tax resources is not drawn to the present. Therefore, as in case c), the marginal benefit of additional fiscal capacity contains a term proportional to ( $E\left(\lambda_{2}\right)-1$ ). Lowering $E\left(\lambda_{2}\right)$ will thus again decrease incentives to invest in fiscal and legal capacity. Since this decreases $\bar{b}\left(\tau_{2}, \pi_{2}\right)=\frac{\tau_{2} \omega\left(\pi_{2}\right)}{1+\rho}$, we have one force that pulls debt down. Specifically, for a fixed level $d b$ of debt which is raised above $\bar{b}$, the total amount of debt, $\bar{b}+d b$, will go down. The low income part of future public funds decreases and less debt is needed to draw it to the present. This effect was the only effect at work in case c) and it was responsible for debt and state capacity moving in the same direction. We call this effect the low income effect.

In case d), we get a second effect. Specifically, from the first order condition for debt in case d) we can see the following. If $E\left(\lambda_{2}\right)$ decreases, $\Delta$ will adjust upwards such that the first order condition holds again with equality. That is, the amount $d b$ of debt which is raised above $\bar{b}$ will increase and trigger a higher penalty. Given, for instance, an increasing level of political instability, the danger of getting screwed over by a rival government in the future gets bigger. Therefore, it becomes optimal to raise more debt and avoid the rival redistribution for a higher proportion of expected future tax resources. In particular, for more units of fiscal capacity, the high income part is now also drawn to the present. ${ }^{29}$ The higher penalty implied by this is justified because the alternative of leaving money to the

[^17]future also gets more expensive. We call this the high income effect.
The overall effect on the level of debt is therefore ambiguous. However, if the high income effect dominates the low income effect, we get an increase in debt as indicated by the dashed line in the third panel of Figure 1. In region d), debt and state capacities can therefore start to move in opposite directions in response to exogenous parameter changes. ${ }^{30}$

Decreasing the value of future public funds, $E\left(\lambda_{2}\right)$, further, we finally enter case e). In this case, debt is raised maximally, so all future public funds are drawn to the present. However, this comes at the price of a high penalty. As the first order condition for fiscal capacity investment in case e) shows, investment incentives for the high income part are now driven by a combination of $\left(\lambda_{1}-1\right)$, the net marginal benefit of public funds used in the present, and the marginal penalty at the maximal debt level. It turns out that we enter case e) when $E\left(\lambda_{2}\right)$ takes the value $\lambda_{1}-\left.\frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\overline{\bar{b}}}$. Because of the penalty, investment incentives are therefore as low as if they were driven by this very low value of $E\left(\lambda_{2}\right)$. If $E\left(\lambda_{2}\right)$ drops further, investment incentives stay constant at a the level implied by $\lambda_{1}-\left.\frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\bar{b}}$.
Even in case e), through the influence of the low income part, we still get higher investment incentives than in a world without debt under realistic restrictions on the income levels. Nevertheless, case e) can reestablish a weak state situation similar to Besley and Persson (2011), where we get very low investments in fiscal and legal capacity. ${ }^{31}$ These low investments arise from a low value of future public funds, $E\left(\lambda_{2}\right)$. Given the possibility to raise debt, $E\left(\lambda_{2}\right)$ influences investment incentives only through the high income part. Therefore, state capacity investments will be the lower, the higher the influence of the high income part, in the sense that the difference between the income levels, $\bar{\omega}-\underline{\omega}$, is bigger. Furthermore, the situation of low investments in state capacity is now worsened by a high debt level. Debt is bad here because in the political equilibrium, $\lambda_{1}=2(1-\theta)>\alpha_{L}$ holds. Therefore, in case e), debt is used to fully tie down future public funds for clientele politics today. A social planner, on the other hand, would not draw future public funds to the present but would use them instead for public good expenditures in the future.

Note that the weak state situation in the preceding analysis arises for similar parameter values as in a model without debt. In particular, recall that the above analysis was done for countries with low enough cohesiveness. Furthermore, a low value of $E\left(\lambda_{2}\right)$ leading to case e) can be driven by high political instability $\gamma$. Low cohesiveness and high political

[^18]instability constitute the parameter constellation that defines a weak state in the basic model without debt (Besley and Persson, 2011). ${ }^{32}$ The mechanism at work has now already been highlighted several times: When the incumbent government is afraid that future public funds will be used against its interest, it will not invest in the additional creation of these public funds. We call this the low-investment-mechanism.

However, the preceding analysis also cautions that the low-investment-mechanism is only partly at work in the default setup. Specifically, we have just argued that the size of the difference between low and high income realizations plays a crucial role. Note in particular that, as this difference goes to zero, we move back to the debt model without default from section 3. For this model, investment incentives were driven by the present value of public funds as soon as the future value dropped low enough. Therefore, the low-investmentmechanism was completely broken by the mechanism of using debt to bring future public funds at the disposal of the present. We call the latter the debt-mechanism.

The above analysis presents a first step in analyzing the relative strength of these two mechanisms. Specifically, we have highlighted how the two effects interact in a specific framework. The following conclusions and implications drawn from the above analysis depend on this framework. Therefore, we do not want to stretch them too far.

As can be seen in Figure 1, for countries that differ only in $E\left(\lambda_{2}\right)$, our model setup leads to a negative correlation of debt and state capacity. Moreover, 'crisis countries' with very high public debt and low fiscal and legal capacity can arise. Greece could be an example of such a country. In our model, these are countries with low cohesiveness $\theta$ and a low value of future public funds, $E\left(\lambda_{2}\right)$. The latter could be due, for instance, to high political instability $\gamma$.

Since $E\left(\lambda_{2}\right)<\lambda_{1}$ defines cases c)-e), only for sufficiently low cohesiveness we can ever end up in the high debt - low state capacity situation of case e). Therefore, the lower is cohesiveness, the more likely (i.e. for a larger range of parameter values) a country will end up in such a situation.

From this analysis, we can already draw some policy implications. A crucial factor that keeps a country from running into a debt trap, that is a situation with very high debt and low state capacity, seems to be a sufficient level of cohesiveness. High cohesiveness entails provisions in a country's constitution and other institutional features which prohibit clientele politics by the political group in power. In a politically unstable political environment uncohesive countries, where it is easy to benefit your preferred clientele, will end up in the problematic situation of a debt trap. As the following quotation from Lyrintzis (2011)

[^19]shows, Greece seems to have suffered from exactly this kind of uncohesiveness:
"Patronage and clientele networks have marked Greek politics since the creation of the modern Greek state [...] The alternation of the two major parties in power led to political polarization and after each governmental change to massive allocation of favours to the party's clientele."

To avoid to get back in such a situation, a reform of cohesiveness seems to be beneficial. This necessitates deep reforms that go at the core of the functioning of the state. Examples of such reforms include implementing a functioning system of checks and balances, establishing an independent press that names and shames clientele politics, establishing provisions in the constitution that prevent clientele politics, or strengthening the constitutional court in its power to enforce such provisions.

## 5 Quasi-linear Model

The extension to include ability-to-pay default added an important feature to the analysis. For a rising debt level, the risk of default makes it more and more expensive to use debt for drawing future public funds to the present. This can partly break the incentivizing effect which the possibility to raise debt can have on state capacity investments. Nevertheless, even with this extension, the linear modeling remains rather extreme in some of its predictions. For instance, it does not seem realistic that public good spending would go down to zero in some future instances. To see if the predictions from the linear model carry over to a more realistic preference specification, we modify the model from the last section to allow for quasi-linear preferences.

Specifically, utility of an individual of group $J$ in period $s$ changes from (2) to:

$$
\begin{equation*}
u_{s}^{J}=\alpha_{s} V\left(g_{s}\right)+\left(1-t_{s}\right) \omega\left(p_{s}^{J}\right)+r_{s}^{J} \tag{33}
\end{equation*}
$$

$V()$ is increasing and concave and is assumed to fulfill the Inada conditions. The latter will make sure that there is preference for some positive public good spending in each period. Within a period, a government will now provide public goods until the marginal benefit of doing so equals the marginal benefit of transfer spending:

$$
\begin{equation*}
V_{g}(\hat{g})=2(1-\theta) \tag{34}
\end{equation*}
$$

If there is more money than $\hat{g}$ to spend, then this rest of the money will be spent on redistributive transfers.

Even this seemingly little modification of changing the public good sub-utility leads to quite some complications in the analytical solution of the model. In the analysis below,
we therefore focus on a special case of the quasi-linear model environment. Furthermore, to reduce the number of cases to consider, we make the following assumption. The high income $\bar{\omega}\left(\pi_{s}\right)$ realizes exactly then when public good spending has a low value $\alpha_{L}$. This can be justified by interpreting certain measures to avert an economic crisis as public good spending. Such a measure could be the nationalization of a system relevant bank in trouble. In a boom with high income, such public spending is not necessary and therefore public good spending can be seen as having a lower general benefit there. The complementary case that corresponds to an economic crisis has $\underline{\omega}\left(\pi_{s}\right)$ and $\alpha_{H}$. The probability to end up in a boom time with a lower value $\alpha_{L}$ of public good spending is given by:

$$
\begin{equation*}
\operatorname{Prob}\left(\bar{\omega}, \alpha_{L}\right)=(1-\phi) \tag{35}
\end{equation*}
$$

The probability of a crisis with a high public good value $\alpha_{H}$ is given by:

$$
\begin{equation*}
\operatorname{Prob}\left(\underline{\omega}, \alpha_{H}\right)=\phi \tag{36}
\end{equation*}
$$

Everything else stays the same compared to the linear model with default, except for the specification of the penalty. We now assume that the penalty has the following form (where $\Delta$ is the amount not repaid):

$$
P= \begin{cases}0 & \text { if } \Delta=0 \quad(\text { no default })  \tag{37}\\ P(\Delta) & \text { if } \Delta=\left(1+R\left(b_{1}\right)\right) b_{1}-\left(\tau_{2} \underline{\omega}\left(\pi_{2}\right)-\bar{g}\right) \quad \text { and }\left(\alpha_{2}, \omega_{2}\left(\pi_{2}\right)\right)=\left(\alpha_{H}, \underline{\omega}\left(\pi_{2}\right)\right) \\ P_{\max } & \text { else. }\end{cases}
$$

In particular, there is now an amount $\bar{g}$ which is left to the country even in case of default, without triggering the maximal penalty $P_{\max }$. This acknowledges the fact that some basic public good spending is necessary each period in the sense of being highly valuable. Then even the maximal penalty $P_{\max }$ that the international market can inflict will not be enough to keep the country from defaulting, if full repayment is required. Therefore, as long as the government repays $\tau_{2} \underline{\omega}-\bar{g}$, the international market acknowledges that the country does all that it can. When the government shows this kind of good will, the penalty depends on the amount of debt that is not repaid in the same way as before.

If the country repays less, that is the government defaults purposely, we assume the punishment to take its maximal value, $P_{\max }$. Given $P_{\max }$, the amount $\bar{g}$ of public good spending that is 'left' to the country is set such that the government does not want to default purposely. In particular, even with the maximally inflictable penalty, a country would prefer to default, if honoring the debt commitments to the demanded degree, $\tau_{2} \underline{\omega}\left(\pi_{2}\right)-\bar{g}$, would leave it with less welfare. To keep the analysis simple, we assume that $\bar{g}$ is such that
the country would not want to default purposely even with the maximal debt $\overline{\bar{b}}$. For lower equilibrium levels of debt, $\bar{g}$ could be lower, but we neglect this subtlety for now. More specifically, $\bar{g}$ is implicitly defined by:

$$
\begin{aligned}
& \alpha_{H} V(\bar{g})+\left(1-\tau_{2}\right) \underline{\omega}\left(\pi_{2}\right)-P(\overbrace{\tau_{2}\left(\bar{\omega}\left(\pi_{2}\right)-\underline{\omega}\left(\pi_{2}\right)\right)}^{\Delta(\overline{\bar{b}})})= \\
& \quad \alpha_{H} V\left(\min \left\{\tau_{2} \underline{\omega}\left(\pi_{2}\right), \hat{g}\right\}\right)+\left(1-\tau_{2}\right) \underline{\omega}\left(\pi_{2}\right)+2(1-\theta) \max \left\{0, \tau_{2} \underline{\omega}\left(\pi_{2}\right)-\hat{g}\right\}-P_{\max },
\end{aligned}
$$

with $\hat{g}$ defined through $\alpha_{H} V_{g}(\hat{g})=2(1-\theta)$.
The left-hand side of this equation is the utility in the case of maximally honoring the debt commitments, whereas the right-hand side captures the utility of default.

Define $\Gamma \equiv(1-\gamma) 2(1-\theta)+\gamma 2 \theta$. We make the following Assumptions:

$$
\begin{align*}
\phi \alpha_{H} V_{g+}(\bar{g})+(1-\phi) \Gamma & <\left.\frac{\phi}{1-\phi} \frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\bar{b}}+\Gamma  \tag{38}\\
\alpha_{H} V_{g-}(\bar{g}) & >\left.\frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\overline{\bar{b}}} \tag{39}
\end{align*}
$$

Here $V_{g+}$ denotes the derivative from the right and $V_{g-}$ the derivative from the left. ${ }^{33}$ Note also that $\frac{\partial P(\Delta)}{\partial \Delta}$ is evaluated at $\bar{b}=\frac{\tau_{2} \omega\left(\pi_{2}\right)-\bar{g}}{1+\rho}$ in (38) and at $\overline{\bar{b}}=\frac{\tau_{2}\left[(1-\phi) \bar{\omega}\left(\pi_{2}\right)+\phi \underline{\omega}\left(\pi_{2}\right)\right]-\bar{g}}{1+\rho}$ in (39). Assumption (39) therefore says that public good spending below $\bar{g}$ is very valuable. Specifically, it is so valuable that even in the face of penalties the country will not reduce these penalties by lowering its public good consumption below $\bar{g}$, the level it is accorded by the international market in the case of default. Economically, this public good consumption can be interpreted as public expenditures for crucial projects. Examples include maintenance of basic medical care, security, infrastructure and schools. A reduction of public good spending below $\bar{g}$ would go at the core of these projects. Assumption (38) implies that the marginal penalty at $\bar{b}$ is high enough to get unique solutions from the optimization problem, that is either $b_{1}^{*} \leq \bar{b}$ or $b_{1}^{*}>\bar{b}$.

## Intra-temporal policies

Considering the intra-temporal maximization problem solved by the government, we now have to distinguish two scenarios.

Scenario 1: $\tau_{s} \omega_{s}\left(\pi_{s}\right)-m_{s}-n_{s}+b_{s}-\left(1+R\left(b_{s-1}\right)\right) b_{s-1}>\bar{g}$

That is, the residual public resources after taking care of the investment expenditures $\left(m_{s}, n_{s}\right)$ and debt obligations are larger than the amount of public good spending which is

[^20]'left' to the country in any case. In this scenario, the intra-temporal maximization problem of the incumbent group is analogous to the linear case:
$\max _{\left\{g_{s}\right\}} u_{s}^{I}=\alpha_{s} V\left(g_{s}\right)+\left(1-\tau_{s}\right) \omega_{s}\left(\pi_{s}\right)+2(1-\theta)\left[\tau_{s} \omega_{s}\left(\pi_{s}\right)-m_{s}-n_{s}+b_{s}-\left(1+R\left(b_{s-1}\right)\right) b_{s-1}-g_{s}\right]$

Given a public good provision of $g_{s}$ by the incumbent government, the utility of the opposition group will be:

$$
\begin{equation*}
u_{s}^{O}=\alpha_{s} V\left(g_{s}\right)+\left(1-\tau_{s}\right) \omega_{s}\left(\pi_{s}\right)+2 \theta\left[\tau_{s} \omega_{s}\left(\pi_{s}\right)-m_{s}-n_{s}+b_{s}-\left(1+R\left(b_{s-1}\right)\right) b_{s-1}-g_{s}\right] \tag{41}
\end{equation*}
$$

In the following, denote the residual resources left after investment expenditures and debt obligations as:

$$
T \equiv \tau_{s} \omega_{s}\left(\pi_{s}\right)-m_{s}-n_{s}+b_{s}-\left(1+R\left(b_{s-1}\right)\right) b_{s-1}
$$

The policy function for public good provision then becomes:

$$
G\left(\alpha_{s}, \tau_{s}, \pi_{s}, m_{s}, n_{s}, b_{s-1}\right)= \begin{cases}T & \text { if } V^{\prime}(T) \geq 2(1-\theta)  \tag{42}\\ \hat{g} & \text { otherwise }\end{cases}
$$

Plugging (42) into (40) and (41), yields the following 'indirect' payoff function for group $J \in\{I, O\}$ in period $s$ :

$$
\begin{align*}
W\left(\alpha_{s}, \tau_{s}, \pi_{s}, m_{s}, n_{s}, b_{s-1}, b_{s}, \beta^{J}\right. & )=\alpha_{s} V(G)+\left(1-\tau_{s}\right) \omega_{s}\left(\pi_{s}\right)  \tag{43}\\
+ & \beta^{J}\left[\tau_{s} \omega_{s}\left(\pi_{s}\right)-m_{s}-n_{s}+b_{s}-\left(1+R\left(b_{s-1}\right)\right) b_{s-1}-G\right]
\end{align*}
$$

where $\beta^{I}=2(1-\theta)$ and $\beta^{O}=2 \theta$.

Scenario 2: $\tau_{s} \omega_{s}\left(\pi_{s}\right)-m_{s}-n_{s}+b_{s}-\left(1+R\left(b_{s-1}\right)\right) b_{s-1} \leq \bar{g}$
In this case, the resources available after honoring the debt commitments and covering the investment expenditures would be less than $\bar{g}$, the spending on public goods which is allowed without triggering the maximal penalty.

By assumption (39) and the convexity of the penalty function, we have $\alpha_{H} V_{g-}(\bar{g})>$ $\left.\frac{\partial P(\Delta)}{\partial \Delta}\right|_{b}$ for all $b \in[\bar{b}, \bar{b}]$. This implies that for scenario 2 and with $\alpha_{s}=\alpha_{H}$, the government will provide the public good level $\bar{g}$ and incur the resulting penalty from not being able to fully honor its debt commitments. With $\alpha_{s}=\alpha_{L}$ we would need to make additional assumptions. However, it turns out that for the situation we want to focus on below, the government will never end up in scenario 2 for $\alpha_{s}=\alpha_{L}$.

The 'indirect' payoff function for group $J \in\{I, O\}$ in period $s$ becomes:

$$
\begin{align*}
W\left(\alpha_{s}, \tau_{s}, \pi_{s}, m_{s}, n_{s}, b_{s-1}, b_{s}, \beta^{J}\right)= & \alpha_{s} V(\bar{g})+\left(1-\tau_{s}\right) \omega_{s}\left(\pi_{s}\right)  \tag{44}\\
& -P\left[\left(1+R\left(b_{s-1}\right)\right) b_{s-1}-\left(\tau_{s} \omega_{s}\left(\pi_{s}\right)-\bar{g}\right)\right]
\end{align*}
$$

## Inter-temporal policies

The inter-temporal maximization problem of the incumbent group of period $s=1$ is:

$$
\begin{align*}
\max _{\left\{\tau_{2}, \pi_{2}, b_{1}\right\}} & W\left(\alpha_{1}, \tau_{1}, \pi_{1}, m_{1}, n_{1}, b_{0}, b_{1}, 2(1-\theta)\right)+\delta E V^{I_{1}}\left(\tau_{2}, \pi_{2}, b_{1}\right)  \tag{45}\\
\text { s.t. } \tau_{2} & \geq \tau_{1}, \\
\pi_{2} & \geq \pi_{1}, \\
b_{1} & \leq \overline{\bar{b}}\left(\tau_{2}, \pi_{2}\right),
\end{align*}
$$

In the following, replace $W\left(\alpha_{s}, \tau_{s}, \pi_{s}, m_{s}, n_{s}, b_{s-1}, b_{s}, \beta^{J}\right)$ by $W\left(\alpha_{s}, \beta^{J}\right)$. Then we get:

$$
\begin{align*}
E V^{I_{1}}\left(\tau_{2}, \pi_{2}, b_{1}\right)= & (1-\gamma)\left[\phi W\left(\alpha_{H}, 2(1-\theta)\right)+(1-\phi) W\left(\alpha_{L}, 2(1-\theta)\right)\right]  \tag{46}\\
& +\gamma\left[\phi W\left(\alpha_{H}, 2 \theta\right)+(1-\phi) W\left(\alpha_{L}, 2 \theta\right)\right]
\end{align*}
$$

In the appendix, we derive comparative static results for this setup, summarized in the following proposition.

Proposition 5.1. Suppose an economy in the quasi-linear model with sovereign default starts in the first period with $\alpha_{1}=\alpha_{L}$. Moreover, suppose that there are enough resources to provide transfers and that the constraint $\tau_{2} \geq \tau_{1}$ does not bind. ${ }^{34}$ Then:

1. If $\alpha_{2}=\alpha_{L}\left(\alpha_{2}=\alpha_{H}\right)$, transfers (no transfers) will be provided in the second period.
2. There are positive investments in fiscal and legal capacity.
3. Higher $\phi$ increases(decreases) investments in fiscal and legal capacity if $\Gamma<1(\Gamma>1)$.
4. Lower $\gamma$ increases investments in fiscal and legal capacity.
5. Lower $\gamma$ leads to an increase of free future revenues.
6. Higher $\phi$ leads to an increase of free future revenues if $\Gamma<1$.
7. Higher $\phi$ leads to a decrease of debt if $\Gamma>1$.

[^21]8. The higher $\theta$, the less likely it is that a country will end up in a situation with sovereign default.

Under some further technical assumptions spelled out in the appendix, we get:
9. The higher $\gamma$, the more likely it is that debt and state capacity investments move in opposite rather than the same direction in response to a change in $\gamma$.

Regarding part 1 to part 8, the analogy to the findings from the linear case is clear. ${ }^{35}$ Part 9 is worth an explanation. Recall that in the linear case, we had that debt and state capacity investments moved in the same direction for $b_{1}^{*}<\bar{b}$ but possibly moved in opposite directions for $b_{1}^{*}>\bar{b}$ in response to exogenous parameter changes. This was driven by the fact that when debt was raised above $\bar{b}$, the ensuing penalty payments made it more and more expensive to further increase debt. On the other hand, for $b_{1}^{*}<\bar{b}$ raising debt was essentially 'costless'. In the quasi-linear setup, we do not have this cutoff at $\bar{b}$. As it turns out, the qualitative comparative statics for debt on first sight look analogous for $b_{1}^{*}<\bar{b}$ and $b_{1}^{*}>\bar{b}$. The reason is the following. By the concave subutility $V(g)$ in the quasi-linear case, we get that raising more debt is now also costly for $b_{1}^{*}<\bar{b}$. Specifically, this cost now corresponds to the lost utility from less public good consumption. Due to the concavity of $V()$, this 'cost' of debt is also increasing in the debt level. This is because more debt means less public good consumption that can be financed in the second period.

In order to have debt and state capacity investments change from moving in the same to moving in opposite directions, we actually need ambiguity in the way that debt and state capacity investments react to exogenous parameter changes. This ambiguity shows up clearly for the comparative statics with respect to political instability $\gamma$. In particular, both state capacities $\tau_{2}^{*}$ and $\pi_{2}^{*}$ decrease with a higher value of political instability. The reaction of debt $b_{1}^{*}$, however, is ambiguous. In the appendix, we show that the marginal effect of an increase of $\gamma$ on both state capacities $\tau_{2}^{*}$ and $\pi_{2}^{*}$ becomes stronger for higher $\gamma$. We then argue that for higher $\gamma$, it becomes more likely that debt and state capacity investments move in opposite rather than the same direction in response to a change in $\gamma$. Note that this result points in the same direction as our findings in the linear case. Admittedly, we do no longer have a clear cutoff at $\bar{b}$. However, whether an economy raises debt above or below $\bar{b}$ in the linear case was also driven by $\gamma$. Therefore, in the linear case we also had the conclusion that for high $\gamma$ (and therefore low $E\left(\lambda_{2}\right)$ ), it is likely for debt

[^22]and state capacity investments to move in opposite directions in response to exogenous parameter changes.

## 6 Conclusion

This paper presented an integrated analytical framework for analyzing the interaction between public debt and state capacity, the power of a state to raise taxes and to provide market supporting policies. We showed that the possibility to raise debt can provide a novel incentive to invest in state capacity, because debt allows to bring future state capacity at the disposal of the current government. Insofar as debt can be used to protect the current government from an adverse use of future public funds, it is no longer necessary to use low investments in state capacity as a protection device.

However, we also showed how this novel mechanism can be weakened in a world with income fluctuations and the possibility of default. When high costs of raising debt make it very expensive to draw all relevant future public funds to the present, the mechanism of lowering investments resurfaces. Specifically, this mechanism is more prominent for high income fluctuations, because these increase the proportion of public funds that can only be drawn to the present at high costs. For such an environment, we get results that are closest to the original no-debt model by Besley and Persson (2011). In particular, an unstable political environment combined with insufficient institutional provisions to prevent clientele politics can then be conducive to a situation of low state capacity. Furthermore, this weak state situation is now worsened by a high built up of debt, increasing the probability of sovereign default.

The results of our model are in principle testable. In cross-country data, we predict to observe a negative correlation between public debt and measures of state capacity for countries with uncohesive institutions. For instance, countries with high political instability are expected to show high public debt and low fiscal and legal capacity. There is anecdotal evidence that this might be the case for countries such as Greece.

Our model leaves room for several generalizations which should be investigated by future research. First, to qualify the model results in light of the tax smoothing literature, it would be interesting to allow for distortionary taxation. Second, the modeling of default could be extended to a full-fledged model of willingness-to-pay default. This would certainly allow to uncover additional interesting channels that shape the interaction between debt and state capacity. Third, in light of this, it could make sense to extend the model to more than two periods.

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## Appendix

## Proof of proposition 3.1:

Part 1 a) follows from (17) and part b) follows from (8). Part c) follows from the FOC for fiscal and legal capacity (i.e. (20) and (21)). Part 1 d): We first want to show that fiscal and legal capacity are both monotone nondecreasing in $E\left(\lambda_{2}\right)$. Since $\phi$ enters the model only through $E\left(\lambda_{2}\right)$ and $E\left(\lambda_{2}\right)$ is increasing in $\phi$, this suffices. Let us rewrite equation (15):

$$
\begin{equation*}
f\left(b_{1}, \tau_{2}, \pi_{2}\right)=E V^{I_{1}}\left(\tau_{2}, \pi_{2}, b_{1}\right)-\lambda_{1}\left(F\left(\tau_{2}-\tau_{1}\right)+L\left(\pi_{2}-\pi_{1}\right)-b_{1}\right) \tag{47}
\end{equation*}
$$

As stated in Part 1 a), $b_{1}=0$ is the debt level that maximizes (47). Define $g\left(\tau_{2}, \pi_{2}\right) \equiv$ $f\left(0, \tau_{2}, \pi_{2}\right)$. Following Corollary 3 of Milgrom and Shannon (1994), it remains to show that $g\left(\tau_{2}, \pi_{2}\right)$ is quasisupermodular in $\left(\tau_{2}, \pi_{2}\right)$ and satisfies the single crossing property in $\left(\tau_{2}, \pi_{2}, E\left(\lambda_{2}\right)\right)$. We have:

$$
\begin{align*}
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2} \partial \tau_{2}}=\delta \omega^{\prime}\left(\pi_{2}\right)\left(E\left(\lambda_{2}\right)-1\right)>0  \tag{48}\\
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2} \partial E\left(\lambda_{2}\right)}=\delta \omega^{\prime}\left(\pi_{2}\right) \tau_{2}>0  \tag{49}\\
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \tau_{2} \partial E\left(\lambda_{2}\right)}=\delta \omega\left(\pi_{2}\right)>0 \tag{50}
\end{align*}
$$

By Theorem 6 of Milgrom and Shannon (1994), $g\left(\tau_{2}, \pi_{2}\right)$ has increasing differences in $\left(\tau_{2}, \pi_{2}, E\left(\lambda_{2}\right)\right)$ and is supermodular in $\left(\tau_{2}, \pi_{2}\right)$. It follows that $g\left(\tau_{2}, \pi_{2}\right)$ satisfies the single crossing property in $\left(\tau_{2}, \pi_{2}, E\left(\lambda_{2}\right)\right.$ ) and is quasisupermodular in $\left(\tau_{2}, \pi_{2}\right)$. So, fiscal and legal capacity are both monotone nondecreasing in $E\left(\lambda_{2}\right)$.

It remains to show that fiscal and legal capacity are both strictly increasing in $E\left(\lambda_{2}\right)$. Since we have shown that both are monotone nondecreasing, there are three other potential possibilities: fiscal capacity is strictly increasing while legal capacity remains constant, legal capacity is strictly increasing while fiscal capacity remains constant, both are constant. However, all of them lead to a contradiction regarding the FOC (20) and (21), so they can be ruled out.

Part 2 a) follows from (17) and b) follows from (8). Part c): The positive investments follow from the FOC for fiscal and legal capacity (i.e. (20) and (21)). The comparison to the model without debt is due to the fact that when no debt can be raised, $\max \left\{\lambda_{1}, E\left(\lambda_{2}\right)\right\}$ has to be replaced by $E\left(\lambda_{2}\right)$ which is smaller (the comparison can be shown rigorously using monotone comparative statics similar to the proof of part 1 d ) of proposition 3.1).

## Proof of proposition 3.2:

If the cohesiveness condition holds, $\lambda_{1}=\alpha_{1}$ and $E\left(\lambda_{2}\right)=\phi \alpha_{H}+(1-\phi) \alpha_{L}$. These are the same terms as for a social planner. Therefore, the results of proposition 3.1 hold.

## Proof of proposition 3.3:

Part 1 a) follows from (17) and part b) follows from (8). Part c) follows from the FOC for fiscal and legal capacity (i.e. (20) and (21)). For part 1 d) and e) we apply again monotone comparative statics as in part 1 d ) of proposition 3.1. As stated in part a), $b_{1}=\frac{\tau_{2} \omega\left(\pi_{2}\right)}{1+\rho}$ is the debt level that maximizes (47). Define $g\left(\tau_{2}, \pi_{2}\right) \equiv f\left(\frac{\tau_{2} \omega\left(\pi_{2}\right)}{1+\rho}, \tau_{2}, \pi_{2}\right)$. We have:

$$
\begin{equation*}
\frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2} \partial \tau_{2}}=\delta \omega^{\prime}\left(\pi_{2}\right)\left(E\left(\lambda_{2}\right)-1\right)>0 \tag{51}
\end{equation*}
$$

since the stability condition holds.

$$
\begin{align*}
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2} \partial E\left(\lambda_{2}\right)}=\delta \omega^{\prime}\left(\pi_{2}\right) \tau_{2}>0  \tag{52}\\
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \tau_{2} \partial E\left(\lambda_{2}\right)}=\delta \omega\left(\pi_{2}\right)>0  \tag{53}\\
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2} \partial \theta}=\delta \omega^{\prime}\left(\pi_{2}\right) \tau_{2}(1-\phi) 2(2 \gamma-1)+2 \frac{\partial L(\cdot)}{\partial \pi_{2}}>0 \quad \text { for } \gamma>1 / 2  \tag{54}\\
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \tau_{2} \partial \theta}=\delta \omega\left(\pi_{2}\right)(1-\phi) 2(2 \gamma-1)+2 \frac{\partial F(\cdot)}{\partial \tau_{2}}>0 \quad \text { for } \gamma>1 / 2 \tag{55}
\end{align*}
$$

Following the same reasoning as in the proof of part 1 d ) of proposition 3.1, we are done.
Part 2 a) follows from (17). Part 2 b): The positive investments follow from the FOC for fiscal and legal capacity (i.e. (20) and (21)). The comparison to the model without debt is due to the fact that when no debt can be raised, $\max \left\{\lambda_{1}, E\left(\lambda_{2}\right)\right\}$ has to be replaced by $E\left(\lambda_{2}\right)$ which is smaller (the comparison can be shown rigorously using monotone comparative statics similar to the proof of part 1 d ) of proposition 3.1). Part 2 c ) and e) follow from the FOC for fiscal and legal capacity (i.e. (20) and (21)) and the definitions of $\lambda_{1}$ and $E\left(\lambda_{2}\right)$. Part 2 d ) follows from (8).

## Proof of proposition 3.4:

Part 1: The positive investments follow from the FOC for fiscal and legal capacity (i.e. (20) and (21)). The comparison to the model without debt is due to the fact that when no debt can be raised, $\max \left\{\lambda_{1}, E\left(\lambda_{2}\right)\right\}$ has to be replaced by $E\left(\lambda_{2}\right)$ which is smaller (the comparison can be shown rigorously using monotone comparative statics similar to the proof of part 1 d ) of proposition 3.1). Part 2 follows from the FOC for fiscal and legal
capacity (i.e. (20) and (21)) and the definitions of $\lambda_{1}$ and $E\left(\lambda_{2}\right)$. Part 3 follows from (17) and part 4 follows from (8).

## Proof of proposition 4.1:

Part 1 a)-f) of proposition 4.1 directly follows from proposition 3.1 and proposition 3.2, except for the comparative statics w.r.t. $\psi$. The comparative statics for $\psi$ (and for $\phi$ ) are shown in the following. We first want to show that fiscal and legal capacity are both monotone nondecreasing in $\psi$ and $E\left(\lambda_{2}\right)$. Since $\phi$ enters the model only through $E\left(\lambda_{2}\right)$ and $E\left(\lambda_{2}\right)$ is increasing in $\phi$, this suffices.

Let us rewrite equation (29):

$$
\begin{equation*}
f\left(b_{1}, \tau_{2}, \pi_{2}\right)=E V^{I_{1}}\left(\tau_{2}, \pi_{2}, b_{1}\right)-\lambda_{1}\left(F\left(\tau_{2}-\tau_{1}\right)+L\left(\pi_{2}-\pi_{1}\right)-b_{1}\right) \tag{56}
\end{equation*}
$$

Since we are in case a), $b_{1}=0$ is the debt level that maximizes (56). Define $g\left(\tau_{2}, \pi_{2}\right) \equiv$ $f\left(0, \tau_{2}, \pi_{2}\right)$. Following Corollary 3 of Milgrom and Shannon (1994), it remains to show that $g\left(\tau_{2}, \pi_{2}\right)$ is quasisupermodular in $\left(\tau_{2}, \pi_{2}\right)$ and satisfies the single crossing property in $\left(\tau_{2}, \pi_{2}, E\left(\lambda_{2}\right), \psi\right)$. We have:

$$
\begin{align*}
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2} \partial \tau_{2}}=\delta \omega^{\prime}\left(\pi_{2}\right)\left(E\left(\lambda_{2}\right)-1\right)>0  \tag{57}\\
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2} \partial E\left(\lambda_{2}\right)}=\delta \omega^{\prime}\left(\pi_{2}\right) \tau_{2}>0  \tag{58}\\
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2} \partial \psi}=0  \tag{59}\\
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \tau_{2} \partial E\left(\lambda_{2}\right)}=\delta\left(\psi \bar{\omega}\left(\pi_{2}\right)+(1-\psi) \underline{\omega}\left(\pi_{2}\right)\right)>0  \tag{60}\\
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \tau_{2} \partial \psi}=\delta\left(\bar{\omega}\left(\pi_{2}\right)-\underline{\omega}\left(\pi_{2}\right)\right)\left(E\left(\lambda_{2}\right)-1\right)>0 \tag{61}
\end{align*}
$$

By Theorem 6 of Milgrom and Shannon (1994), $g\left(\tau_{2}, \pi_{2}\right)$ has increasing differences in $\left(\tau_{2}, \pi_{2}, E\left(\lambda_{2}\right), \psi\right)$ and is supermodular in $\left(\tau_{2}, \pi_{2}\right)$. It follows that $g\left(\tau_{2}, \pi_{2}\right)$ satisfies the single crossing property in $\left(\tau_{2}, \pi_{2}, E\left(\lambda_{2}\right), \psi\right)$ and is quasisupermodular in $\left(\tau_{2}, \pi_{2}\right)$. So, fiscal and legal capacity are both monotone nondecreasing in $E\left(\lambda_{2}\right)$ and $\psi$.

It remains to show that fiscal and legal capacity are both strictly increasing in $E\left(\lambda_{2}\right)$ and $\psi$. Since we have shown that both are monotone nondecreasing, there are three other possibilities which we have to check: fiscal capacity is strictly increasing while legal capacity remains constant, legal capacity is strictly increasing while fiscal capacity remains constant, both are constant. All of these can be shown to lead to a contradiction as demonstrated
for the following case. Consider an increase in $E\left(\lambda_{2}\right)$. Assume fiscal capacity is strictly increasing while legal capacity remains constant. From (57) and (58) it follows that the LHS of the FOC for $\pi$ is increasing, so the RHS has to increase as well. Since $L()$ is convex, $\pi$ has to increase. This is a contradiction. In an analogous way, all other cases can be handled. The same can be done for an increase in $\psi$. In total, we conclude that fiscal and legal capacity must be strictly increasing in $E\left(\lambda_{2}\right)$ as well as $\psi$.

Part 1 g ) follows from the definition of free future revenues and the comparative statics for $\tau_{2}$ and $\pi_{2}$.

Part 2 a)-e) of proposition 4.1 directly follows from proposition 3.3, except for the comparative statics w.r.t. $\psi$. The comparative statics for $\psi$ (and for $\phi, \gamma$ and $\theta$ ) are shown in the following. We want to show that fiscal and legal capacity are both strictly increasing in $\psi, \theta$ (for $\gamma>1 / 2$ ) and $E\left(\lambda_{2}\right)$. Since $\phi$ and $\gamma$ enter the model only through $E\left(\lambda_{2}\right)$ and $E\left(\lambda_{2}\right)$ is increasing in $\phi$ and decreasing in $\gamma$, this suffices. For $\psi$ and $E\left(\lambda_{2}\right)$, the proof is exactly as in Part 1. It remains to show that fiscal and legal capacity are both monotone nondecreasing in $\theta$ (for $\gamma>1 / 2$ ).

Since we are case a), $b_{1}=0$ is the debt level that maximizes (56). Define $g\left(\tau_{2}, \pi_{2}\right) \equiv$ $f\left(0, \tau_{2}, \pi_{2}\right)$. We have:

$$
\begin{gather*}
\frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2} \partial \tau_{2}}=\delta \omega^{\prime}\left(\pi_{2}\right)\left(E\left(\lambda_{2}\right)-1\right)>0  \tag{62}\\
\frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2} \partial \theta}=\delta \omega^{\prime}\left(\pi_{2}\right) \tau_{2} 2(1-\phi)(2 \gamma-1)+2 \frac{\partial L\left(\pi_{2}-\pi_{1}\right)}{\partial \pi_{2}}>0 \quad \text { if } \gamma>\frac{1}{2}  \tag{63}\\
\frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \tau_{2} \partial \theta}=\delta\left(\psi \bar{\omega}\left(\pi_{2}\right)+(1-\psi) \underline{\omega}\left(\pi_{2}\right)\right) 2(1-\phi)(2 \gamma-1)+2 \frac{\partial F\left(\tau_{2}-\tau_{1}\right)}{\partial \tau_{2}}>0 \quad \text { if } \gamma>\frac{1}{2} \tag{64}
\end{gather*}
$$

Following the same reasoning as in the proof of part 1 , we are done.
Part 2 f) follows from the definition of free future revenues and the comparative statics for $\tau_{2}$ and $\pi_{2}$.

## Part 3:

Part 3 a): $b_{1}=\bar{b}$ follows from the FOC for debt.
The comparative statics for $b_{1}$ follow from the definition of $b_{1}=\bar{b}$ and the comparative statics for $\tau_{2}$ and $\pi_{2}$ in part 3 e$)-\mathrm{g}$ ). Note that the formula for free future revenues becomes $\delta \tau_{2} \psi(\bar{v}-\underline{v})$. This term is increasing in $\tau_{2}$. The comparative statics for free future revenues w.r.t $\phi$ and $\gamma$ follow immediately from the comparative statics for $\tau_{2}$ stated in part 3 e) and g). The comparative statics w.r.t $\psi$ follow from $\frac{\partial \delta \tau_{2} \psi(\bar{v}-\underline{v})}{\partial \psi}=\delta \tau_{2}(\bar{v}-\underline{v})+\delta \psi(\bar{v}-\underline{v}) \frac{\partial \tau}{\partial \psi}$ and the comparative statics for $\tau_{2}$ stated in part 3 f ).

Part 3 b): $\bar{b}<b_{1}<\overline{\bar{b}}$ follows from the FOC for debt.
$i$ : If $\psi$ increases, the FOC for debt says that $\Delta$ has to increase. Since $\tau_{2}$ and $\pi_{2}$ increase as stated by part 3 f ), $b_{1}$ has to increase. $i i$ : Since $\phi$ and $\gamma$ enter the model only through $E\left(\lambda_{2}\right)$ and $E\left(\lambda_{2}\right)$ is increasing in $\phi$ and decreasing in $\gamma$, we have to show that free future revenues $(F R)$ are increasing in $E\left(\lambda_{2}\right)$.

We have $\frac{\partial F R}{\partial E\left(\lambda_{2}\right)}=\frac{\partial \delta \tau_{2}\left(\psi \bar{\omega}\left(\pi_{2}\right)+(1-\psi) \omega\left(\pi_{2}\right)\right)}{\partial E\left(\lambda_{2}\right)}-\frac{\partial b_{1}}{\partial E\left(\lambda_{2}\right)}$ and $\frac{\partial \delta \tau_{2}\left(\psi \bar{\omega}\left(\pi_{2}\right)+(1-\psi) \omega\left(\pi_{2}\right)\right)}{\partial E\left(\lambda_{2}\right)}=\delta\left(\psi \bar{\omega}\left(\pi_{2}\right)+\right.$ $\left.(1-\psi) \underline{\omega}\left(\pi_{2}\right)\right) \frac{\partial \tau_{2}}{\partial E\left(\lambda_{2}\right)}+\delta \tau_{2} w^{\prime}\left(\pi_{2}\right) \frac{\partial \pi_{2}}{\partial E\left(\lambda_{2}\right)}$. We cannot solve for $b_{1}$ explicitly, still we can say something about $\frac{\partial b_{1}}{\partial E\left(\lambda_{2}\right)}$. Taking the total differential w.r.t. $E\left(\lambda_{2}\right)$ of the FOC for $b_{1}$ we obtain $0=1+\frac{(1-\psi)}{\psi} \frac{\partial^{2} P(\Delta)}{\partial \Delta^{2}} \frac{\partial \Delta}{\partial E\left(\lambda_{2}\right)}$.
Since $P(\Delta)$ is convex, $\frac{\partial \Delta}{\partial E\left(\lambda_{2}\right)}<0$. Since we are in case d), $\Delta=(1+R(b)) b-\left(\tau_{2} \underline{\omega}\left(\pi_{2}\right)\right)=$ $\frac{1}{\psi}\left((1+\rho) b_{1}-\tau_{2} \underline{\omega}\left(\pi_{2}\right)\right)$ so $\frac{\partial \Delta}{\partial E\left(\lambda_{2}\right)}=\frac{1}{\psi}\left((1+\rho) \frac{\partial b_{1}}{\partial E\left(\lambda_{2}\right)}-\underline{\omega}\left(\pi_{2}\right) \frac{\partial \tau_{2}}{\partial E\left(\lambda_{2}\right)}-\tau_{2} \underline{\omega}^{\prime}\left(\pi_{2}\right) \frac{\partial \pi_{2}}{\partial E\left(\lambda_{2}\right)}\right)$.
This leads to $\frac{\partial b_{1}}{\partial E\left(\lambda_{2}\right)}<\delta\left(\underline{\omega}\left(\pi_{2}\right) \frac{\partial \tau_{2}}{\partial E\left(\lambda_{2}\right)}+\tau_{2} \underline{\omega}^{\prime}\left(\pi_{2}\right) \frac{\partial \pi_{2}}{\partial E\left(\lambda_{2}\right)}\right)=\delta\left(\underline{\omega}\left(\pi_{2}\right) \frac{\partial \tau_{2}}{\partial E\left(\lambda_{2}\right)}+\tau_{2} w^{\prime}\left(\pi_{2}\right) \frac{\partial \pi_{2}}{\partial E\left(\lambda_{2}\right)}\right)$.
Therefore, $\frac{\partial \delta \tau_{2}\left(\psi \bar{\omega}\left(\pi_{2}\right)+(1-\psi) \omega\left(\pi_{2}\right)\right)}{\partial E\left(\lambda_{2}\right)}>\frac{\partial b_{1}}{\partial E\left(\lambda_{2}\right)}$. So, free future revenues are increasing in $E\left(\lambda_{2}\right)$.

Part 3 c ) is due to $\alpha_{L}<2(1-\theta)$. Part 3 d ) follows from the FOCs for $\tau_{2}$ and $\pi_{2}$ in case c) and d). For Part 3 e), f) and g) we apply again monotone comparative statics.

We want to show that fiscal and legal capacity are both strictly increasing in $\psi$ and $E\left(\lambda_{2}\right)$ (since $\phi$ and $\gamma$ enter the model only through $E\left(\lambda_{2}\right)$ and $E\left(\lambda_{2}\right)$ is increasing in $\phi$ and decreasing in $\gamma$, this suffices). We proceed in two stages, first we consider an economy in case c) (i.e. $E\left(\lambda_{2}\right)<\lambda_{1}<E\left(\lambda_{2}\right)+\left.\frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\bar{b}}$ ) and than in case d) (i.e. $\left.\lambda_{1}=E\left(\lambda_{2}\right)+\frac{(1-\psi)}{\psi} \frac{\partial P(\Delta)}{\partial \Delta}\right)$.

In case c ), $b_{1}=\bar{b}$ is the debt level that maximizes (56). Define $g\left(\tau_{2}, \pi_{2}\right) \equiv f\left(\bar{b}, \tau_{2}, \pi_{2}\right)$. We have:

$$
\begin{align*}
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2} \partial \tau_{2}}=\delta\left(\omega^{\prime}\left(\pi_{2}\right)\left(E\left(\lambda_{2}\right)-1\right)+\omega^{\prime}\left(\pi_{2}\right)\left(\lambda_{1}-E\left(\lambda_{2}\right)\right)\right)=\delta\left(\omega^{\prime}\left(\pi_{2}\right)\left(\lambda_{1}-1\right)\right)>0  \tag{65}\\
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2} \partial E\left(\lambda_{2}\right)}=\delta \psi\left(\omega^{\prime}\left(\pi_{2}\right) \tau_{2}-\omega^{\prime}\left(\pi_{2}\right) \tau_{2}\right)=0  \tag{66}\\
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \tau_{2} \partial E\left(\lambda_{2}\right)}=\delta\left(\left(\psi \bar{\omega}\left(\pi_{2}\right)+(1-\psi) \underline{\omega}\left(\pi_{2}\right)\right)-\underline{\omega}\left(\pi_{2}\right)\right)>0  \tag{67}\\
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2} \partial \psi}=0  \tag{68}\\
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \tau_{2} \partial \psi}=\delta\left(\bar{\omega}\left(\pi_{2}\right)-\underline{\omega}\left(\pi_{2}\right)\right)\left(E\left(\lambda_{2}\right)-1\right)>0 \tag{69}
\end{align*}
$$

Following the same reasoning as in the proof of part 1, we are done. Note that we have made use of the assumption $\bar{\omega}^{\prime}\left(\pi_{s}\right)=\underline{\omega}^{\prime}\left(\pi_{s}\right) \equiv \omega^{\prime}\left(\pi_{s}\right)$.

In case d), the debt level $b^{*} \in(\bar{b}, \overline{\bar{b}})$ that maximizes (56) is implicitly defined by the FOC of $b_{1}$. Define $g\left(\tau_{2}, \pi_{2}\right) \equiv f\left(b^{*}, \tau_{2}, \pi_{2}\right)$. By the Envelope Theorem and plugging in the first order condition for debt (81), we obtain:

$$
\begin{equation*}
\frac{\partial g\left(\tau_{2}, \pi_{2}\right)}{\partial \tau_{2}}=\delta\left(\left(\psi \bar{\omega}\left(\pi_{2}\right)+(1-\psi) \underline{\omega}\left(\pi_{2}\right)\right)\left(E\left(\lambda_{2}\right)-1\right)+\underline{\omega}\left(\pi_{2}\right)\left(\lambda_{1}-E\left(\lambda_{2}\right)\right)\right)-\lambda_{1} \frac{\partial F\left(\tau_{2}-\tau_{1}\right)}{\partial \tau_{2}} \tag{70}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2}}=\delta\left(\omega^{\prime}\left(\pi_{2}\right)\left[1+\tau_{2}\left(E\left(\lambda_{2}\right)-1\right)\right]+\tau_{2} \omega^{\prime}\left(\pi_{2}\right)\left(\lambda_{1}-E\left(\lambda_{2}\right)\right)\right)-\lambda_{1} \frac{\partial L\left(\pi_{2}-\pi_{1}\right)}{\partial \pi_{2}} \tag{71}
\end{equation*}
$$

These are the same derivatives as in case c), which therefore will lead to the same crossderivatives. Following the same reasoning as in case c), we are done.

Part 4: a) follows from the FOC for debt, b) from the definition of $\overline{\bar{b}}$, c) from the definition of $\overline{\bar{b}}$ and free future revenues. d) is due to $\alpha_{L}<2(1-\theta)$. e) and f) follow from the FOCs for $\tau_{2}$ and $\pi_{2}$ in case d) and e). For g) we apply again monotone comparative statics.

Since we are in case e), $b_{1}=\overline{\bar{b}}$ is the debt level that maximizes (56). Define $g\left(\tau_{2}, \pi_{2}\right) \equiv$ $f\left(\overline{\bar{b}}, \tau_{2}, \pi_{2}\right)$. We have:

$$
\begin{align*}
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2} \partial \tau_{2}}=\delta \omega^{\prime}\left(\pi_{2}\right)\left(\lambda_{1}-1\right)>0  \tag{72}\\
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2} \partial \psi}=0  \tag{73}\\
& \frac{\partial^{2} g\left(\tau_{2}, \pi_{2}\right)}{\partial \tau_{2} \partial \psi}=\delta\left(\bar{\omega}\left(\pi_{2}\right)-\underline{\omega}\left(\pi_{2}\right)\right)\left(\lambda_{1}-1\right)+\left.\psi \frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\overline{\bar{b}}}\left(\bar{\omega}\left(\pi_{2}\right)-\underline{\omega}\left(\pi_{2}\right)\right)>0 \tag{74}
\end{align*}
$$

Following the same reasoning as in the proof of part 1, we are done.
Q.E.D.

## Proof of proposition 5.1:

In order to derive the relevant FOC, we have to consider two cases, $b_{1}^{*}<\bar{b}$ and $b_{1}^{*}>\bar{b}$.
First, suppose that given the optimal $\tau_{2}^{*}$ and $\pi_{2}^{*}$, the optimal debt fulfills $b_{1}^{*}<\bar{b}$. For this constellation, we will never end up in scenario 2 in the second period.

Therefore, using (42) and (43) in (45), we arrive at the following first order condition for debt $b_{1}$ :
$\max \left\{2(1-\theta), V_{g}\left[\tau_{1} \omega_{1}\left(\pi_{1}\right)-F\left(\tau_{2}^{*}-\tau_{1}\right)-L\left(\pi_{2}^{*}-\pi_{1}\right)+b_{1}^{*}\right]\right\}=-\delta \frac{\partial E V^{I_{1}}\left(\tau_{2}^{*}, \pi_{2}^{*}, b_{1}^{*}\right)}{\partial b_{1}}$

Since we assume that, at the solution $\left(\tau_{2}^{*}, \pi_{2}^{*}, b_{1}^{*}\right)$, it is optimal in the first period to provide transfers, the first term in the above equation evaluates to $2(1-\theta)$. We show below that the only possible configuration such that the first order condition for debt holds is the following: In the second period, for the boom situation $\left(\alpha_{L}, \bar{\omega}\left(\pi_{2}\right)\right)$ we get transfers, while for the crisis situation $\left(\alpha_{H}, \underline{\omega}\left(\pi_{2}\right)\right)$ no transfers are provided. ${ }^{36}$ The resulting first order condition for debt is:

$$
\begin{equation*}
2(1-\theta)=\phi \underbrace{\alpha_{H} V_{g}\left[\tau_{2}^{*} \underline{\omega}\left(\pi_{2}\right)-(1+\rho) b_{1}^{*}\right]}_{>2(1-\theta)}+(1-\phi)[\underbrace{(1-\gamma) 2(1-\theta)+\gamma 2 \theta}_{\equiv \Gamma<2(1-\theta)}] \tag{76}
\end{equation*}
$$

Note that this implies $\tau_{2}^{*} \underline{\omega}\left(\pi_{2}\right)-(1+\rho) b_{1}^{*}<\widehat{g}$.
Let us denote the objective function to be maximized (see equation (45)) as:

$$
\begin{equation*}
f\left(b_{1}, \tau_{2}, \pi_{2}\right)=W\left(\alpha_{1}, \tau_{1}, \pi_{1}, m_{1}, n_{1}, b_{0}, b_{1}, 2(1-\theta)\right)+\delta E V^{I_{1}}\left(\tau_{2}, \pi_{2}, b_{1}\right) \tag{77}
\end{equation*}
$$

The debt level $b_{1}^{*}$ that maximizes (77) is implicitly defined by the FOC for debt (76). Define $g\left(\tau_{2}, \pi_{2}\right) \equiv f\left(b_{1}^{*}, \tau_{2}, \pi_{2}\right)$. By the Envelope Theorem and plugging in the first order condition for debt (76), we arrive at the following first order conditions for future state capacities: ${ }^{37}$

$$
\begin{align*}
& \frac{\partial g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \tau_{2}}=\delta\{(1-\phi)(\bar{\omega}-\underline{\omega})(\Gamma-1)+\underline{\omega}[2(1-\theta)-1]\}-2(1-\theta) \frac{\partial F\left(\tau_{2}^{*}-\tau_{1}\right)}{\partial \tau_{2}}=0  \tag{78}\\
& \frac{\partial g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \pi_{2}}= \delta\left\{(1-\phi)\left[\bar{\omega}^{\prime}\left(\pi_{2}\right)-\underline{\omega}^{\prime}\left(\pi_{2}\right)\right]\left[1-(1-\Gamma) \tau_{2}\right]\right.  \tag{79}\\
&\left.+[2(1-\theta)-1] \tau_{2} \underline{\omega}^{\prime}\left(\pi_{2}\right)+\underline{\omega}^{\prime}\left(\pi_{2}\right)\right\}-2(1-\theta) \frac{\partial L\left(\pi_{2}^{*}-\pi_{1}\right)}{\partial \pi_{2}}=0
\end{align*}
$$

Now, suppose that given the optimal $\tau_{2}^{*}$ and $\pi_{2}^{*}$, the optimal debt fulfills $b_{1}^{*}>\bar{b}$. For this constellation, we will end up in scenario 2 in the second period if $\alpha_{2}=\alpha_{H}$. Therefore, in $E V^{I_{1}}$, i.e in equation (46), the terms $W\left(\alpha_{H}, \ldots\right)$ are now given by (44) while the terms $W\left(\alpha_{L}, \ldots\right)$ are still given by (43). The first order condition for debt $b_{1}$ is:
$\max \left\{2(1-\theta), V_{g}\left[\tau_{1} \omega_{1}\left(\pi_{1}\right)-F\left(\tau_{2}^{*}-\tau_{1}\right)-L\left(\pi_{2}^{*}-\pi_{1}\right)+b_{1}^{*}\right]\right\}=-\delta \frac{\partial E V^{I_{1}}\left(\tau_{2}^{*}, \pi_{2}^{*}, b_{1}^{*}\right)}{\partial b_{1}}$
It has the same structure as before, but $E V^{I_{1}}$ differs in the just described way. In particular, since we assume that at the solution $\left(\tau_{2}^{*}, \pi_{2}^{*}, b_{1}^{*}\right)$, it is optimal in the first period to provide transfers, the first term in the above equation evaluates to $2(1-\theta)$. Analogously

[^23]to the case $b_{1}^{*}<\bar{b}$, we show below that the only possible configuration such that the first order condition for debt holds is the following: In the second period, for the situation $\left(\alpha_{L}, \bar{\omega}\right)$ we get transfers, while for $\left(\alpha_{H}, \underline{\omega}\right)$ no transfers are provided. The resulting first order condition for debt is:
\[

$$
\begin{equation*}
\frac{\phi}{1-\phi} \frac{\partial P(\Delta)}{\partial \Delta}=\underbrace{2(1-\theta)-[\underbrace{(1-\gamma) 2(1-\theta)+\gamma 2 \theta}_{\equiv \Gamma<2(1-\theta)}]}_{\equiv N} \tag{81}
\end{equation*}
$$

\]

As before, write equation (45) as:

$$
\begin{equation*}
f\left(b_{1}, \tau_{2}, \pi_{2}\right)=W\left(\alpha_{1}, \tau_{1}, \pi_{1}, m_{1}, n_{1}, b_{0}, b_{1}, 2(1-\theta)\right)+\delta E V^{I_{1}}\left(\tau_{2}, \pi_{2}, b_{1}\right) \tag{82}
\end{equation*}
$$

The debt level $b_{1}^{*}$ that maximizes (82) is implicitly defined by (81), the FOC with respect to debt. Define $g\left(\tau_{2}, \pi_{2}\right) \equiv f\left(b_{1}^{*}, \tau_{2}, \pi_{2}\right)$. By the Envelope Function Theorem and plugging in the first order condition for debt (81), we obtain:

$$
\begin{align*}
\frac{\partial g\left(\tau_{2}, \pi_{2}\right)}{\partial \tau_{2}}= & \delta\{(1-\phi)(\bar{\omega}-\underline{\omega})(\Gamma-1)+\underline{\omega}[2(1-\theta)-1]\}-2(1-\theta) \frac{\partial F\left(\tau_{2}-\tau_{1}\right)}{\partial \tau_{2}}  \tag{83}\\
\frac{\partial g\left(\tau_{2}, \pi_{2}\right)}{\partial \pi_{2}}= & \delta\left\{(1-\phi)\left[\bar{\omega}^{\prime}\left(\pi_{2}\right)-\underline{\omega}^{\prime}\left(\pi_{2}\right)\right]\left[1-(1-\Gamma) \tau_{2}\right]\right.  \tag{84}\\
& \left.+[2(1-\theta)-1] \tau_{2} \underline{\omega}^{\prime}\left(\pi_{2}\right)+\underline{\omega}^{\prime}\left(\pi_{2}\right)\right\}-2(1-\theta) \frac{\partial L\left(\pi_{2}-\pi_{1}\right)}{\partial \pi_{2}}
\end{align*}
$$

Note that (83) and (84) are exactly the same as (78) and (79), the respective conditions in the case $b_{1}^{*}<\bar{b}$. So, the FOCs for state capacity investments are the same for $b_{1}^{*}<\bar{b}$ and $b_{1}^{*}>\bar{b}$.

Having derived these relevant FOCs, we now turn to the proof:
Part 1: We have to show that the alternatives that there are no transfers at all and that there are transfers in both situations $\alpha_{2}=\alpha_{L}$ and $\alpha_{2}=\alpha_{H}$ cannot be an equilibrium. First, assume that there are no transfers in the second period. So, the marginal benefit of public funds in the future (determined by the marginal value of the public good) is larger than $2(1-\theta)$ for sure. Since there are transfers in period 1 , the marginal benefit of public funds in the present seen from the perspective of the incumbent government is $2(1-\theta)$. This cannot be an equilibrium. Second, assume that there are transfers in the second period for $\alpha_{2}=\alpha_{L}$ and $\alpha_{2}=\alpha_{H}$. Due to political instability, the marginal benefit of public funds in the future seen from the perspective of the incumbent government is $(1-\gamma) 2(1-\theta)+\gamma 2 \theta<2(1-\theta)$. So, from the perspective of the incumbent government, the marginal benefit of public funds in the future would be smaller than in the present, which cannot be an equilibrium.

Part 2 follows from the FOC for fiscal and legal capacity and the assumption that $\tau_{2} \geq \tau_{1}$ does not bind.

Part 3: We have

$$
\begin{align*}
& \frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \tau_{2} \partial \pi_{2}}=\delta\left\{\underline{\omega}^{\prime}\left(\pi_{2}^{*}\right)[2(1-\theta)-1]\right\}>0  \tag{85}\\
& \frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \tau_{2} \partial \phi}=-\delta\left(\bar{\omega}\left(\pi_{2}^{*}\right)-\underline{\omega}\left(\pi_{2}^{*}\right)\right)(\Gamma-1)\left\{\begin{array}{l}
>0 \text { if } \Gamma<1 \\
<0 \text { if } \Gamma>1
\end{array}\right.  \tag{86}\\
& \frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \pi_{2} \partial \phi}=0 \tag{87}
\end{align*}
$$

Following the same steps as in the proof of Part 1 of proposition 4.1, we are done.
Part 4: We have:

$$
\begin{align*}
& \frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \tau_{2} \partial \gamma}=2 \delta\{(1-\phi)(\bar{\omega}-\underline{\omega})[2 \theta-1]\}<0  \tag{88}\\
& \frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \pi_{2} \partial \gamma}=0 \tag{89}
\end{align*}
$$

and $\frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \tau_{2} \partial \pi_{2}}$ as above. Following the same steps as in the proof of Part 1 of proposition 4.1, we are done.

Part 5: From the FOCs for debt for both cases $b_{1}^{*}>\bar{b}$ and $b_{1}^{*}<\bar{b}$ it follows that $\tau_{2} \underline{\omega}\left(\pi_{2}\right)-(1+\rho) b_{1}$ is decreasing in $\gamma$ (note that $\Delta=(1+R(b)) b-\left(\tau_{2} \underline{\omega}\left(\pi_{2}\right)-\bar{g}\right)=$ $\left.\frac{1}{1-\phi}\left((1+\rho) b_{1}-\left(\tau_{2} \underline{\omega}\left(\pi_{2}\right)-\bar{g}\right)\right)\right)$, so $\delta \tau_{2}\left(\omega\left(\pi_{2}\right)+\underline{v}\right)-b_{1}$ is decreasing in $\gamma$. Free future revenues are given by $F R=\delta E\left(\tau_{2} \omega\left(\pi_{2}\right)\right)-b_{1}=\delta \tau_{2}\left(\phi \underline{\omega}\left(\pi_{2}\right)+(1-\phi) \bar{\omega}\left(\pi_{2}\right)\right)-b_{1}=$ $\delta \tau_{2}\left(\omega\left(\pi_{2}\right)+\underline{v}\right)-b_{1}+\delta \tau_{2}(1-\phi)(\bar{v}-\underline{v})$. Since $\tau_{2}$ is also decreasing in $\gamma$, we are done.

Part 6 and 7: From the FOCs for debt for both cases $b_{1}^{*}>\bar{b}$ and $b_{1}^{*}<\bar{b}$ it follows that $\tau_{2} \underline{\omega}\left(\pi_{2}\right)-(1+\rho) b_{1}$ is increasing in $\phi$. Part 7 follows form the comparative statics for $\tau_{2}$ and $\pi_{2}$. Part 6 follows from the same reasoning as in the proof of part 5 (w.r.t. $\phi$ instead of $\gamma$ ).

Part 8: We want to show that for $\theta \rightarrow 1 / 2, b_{1}^{*}<\bar{b}$ is the case. For the FOCs for debt, we had (76) for $b_{1}^{*}<\bar{b}$ :

$$
\begin{equation*}
2(1-\theta)=\underbrace{\phi \underbrace{\alpha_{H} V_{g}\left[\tau_{2}^{*} \underline{\omega}\left(\pi_{2}\right)-(1+\rho) b_{1}^{*}\right]}_{>2(1-\theta)}+(1-\phi)[\underbrace{(1-\gamma) 2(1-\theta)+\gamma 2 \theta}_{\equiv \Gamma<2(1-\theta)}]}_{R H S_{1}} \tag{90}
\end{equation*}
$$

and (81) for $b_{1}^{*}>\bar{b}$ :

$$
\begin{equation*}
2(1-\theta)=\underbrace{\frac{\phi}{1-\phi} \frac{\partial P(\Delta)}{\partial \Delta}-[\underbrace{(1-\gamma) 2(1-\theta)+\gamma 2 \theta}_{\equiv \Gamma<2(1-\theta)}]}_{R H S_{2}} \tag{91}
\end{equation*}
$$

Since $2(1-\theta) \stackrel{\theta \rightarrow 1 / 2}{\rightarrow} 1$ and $R H S_{2} \xrightarrow{\theta \rightarrow 1 / 2} x \geq\left.\frac{\phi}{1-\phi} \frac{\partial P(\Delta)}{\partial \Delta}\right|_{b=\bar{b}}+1>1$, we have that $2(1-\theta)<R H S_{2}$ for $\theta \rightarrow 1 / 2$. So we cannot be in case $b_{1}^{*}>\bar{b}$ and therefore have to be in case $b_{1}^{*}<\bar{b}$. ${ }^{38}$

Part 9: From the FOCs for debt for both cases $b_{1}^{*}>\bar{b}$ and $b_{1}^{*}<\bar{b}$ it follows that $\tau_{2} \underline{\omega}\left(\pi_{2}\right)-(1+\rho) b_{1}$ is decreasing in $\gamma$.

As we know, $\tau_{2}^{*}$ and $\pi_{2}^{*}$ decrease with an increase in $\gamma$, thereby moving down $\tau_{2} \underline{\omega}\left(\pi_{2}\right)-(1+$ $\rho) b_{1}$. This implies that $\tau_{2} \underline{\omega}\left(\pi_{2}\right)-(1+\rho) b_{1}$ is adjusted in the right direction, which makes it unclear if $b_{1}^{*}$ also has to decrease or should increase in order to make $\tau_{2} \underline{\omega}\left(\pi_{2}\right)-(1+\rho) b_{1}$ move down by the required amount. However, the stronger $\tau_{2}^{*}$ and $\pi_{2}^{*}$ decrease, the more likely that the induced decrease in $\tau_{2} \underline{\omega}\left(\pi_{2}\right)-(1+\rho) b_{1}$ is already enough. Even more, if they decrease a lot, they could induce a decrease in $\tau_{2} \underline{\omega}\left(\pi_{2}\right)-(1+\rho) b_{1}$ that is too high and $b_{1}^{*}$ would then have to decrease.

Therefore, in the following, we investigate if the optimal values for the state capacities react stronger to an increase in $\gamma$ for higher $\gamma$. If this is the case, then we get debt and state capacity investments to move in opposite directions in response to a change in $\gamma$ for high $\gamma$.

By the implicit function theorem, we get:

$$
\begin{align*}
& \frac{\partial \tau_{2}^{*}}{\partial \gamma}=\frac{1}{\operatorname{det}(J)}(\underbrace{-\frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \pi_{2}^{2}}}_{>0} \cdot \underbrace{\frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \tau_{2} \partial \gamma}}_{<0}+\underbrace{\frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \tau_{2} \partial \pi_{2}}}_{>0} \cdot \underbrace{\frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \pi_{2} \partial \gamma}}_{=0})<0  \tag{92}\\
& \frac{\partial \pi_{2}^{*}}{\partial \gamma}=\frac{1}{\operatorname{det}(J)}(\underbrace{-\frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \tau_{2}^{2}}}_{>0} \cdot \underbrace{\frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \pi_{2} \partial \gamma}}_{=0}+\underbrace{\frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \tau_{2} \partial \pi_{2}}}_{>0} \cdot \underbrace{\frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \tau_{2} \partial \gamma}}_{<0})<0 \tag{93}
\end{align*}
$$

where $\operatorname{det}(J)$ denotes the determinant of the Hessian of $g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)$. By assuming enough curvature in the cost functions $\left(F_{\tau \tau}, L_{\pi \pi}\right.$ big enough), we can ensure that $\operatorname{det}(J)>0$ and that the function $g\left(\tau_{2}, \pi_{2}\right)$ is strictly concave.

Preferably, we would like to derive $\frac{\partial^{2} \tau_{2}^{*}}{\partial \gamma^{2}}$ and $\frac{\partial^{2} \pi_{2}^{*}}{\partial \gamma^{2}}$. However, the resulting formulas are intractable without additional assumptions. Therefore, we opt here for the approach of considering first the individual terms in the equations and then determining if their inter-

[^24]action is likely to move $\frac{\partial \tau_{2}^{*}}{\partial \gamma}$ and $\frac{\partial \pi_{2}^{*}}{\partial \gamma}$ up. In the following, we keep all other parameters except for $\gamma$ constant.

1. $\frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \tau_{2}^{2}}=-2(1-\theta) F_{\tau \tau}\left(\tau_{2}^{*}-\tau_{1}\right)$

Assuming that $\frac{\partial F_{\tau \tau}()}{\partial \tau}$ is constant, this term does not change with $\gamma$.
2. $\frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \pi_{2}^{2}}=\delta \omega^{\prime \prime}\left(\pi_{2}^{*}\right)\left\{1+\tau_{2}^{*}(2(1-\theta)-1)\right\}-2(1-\theta) L_{\pi \pi}\left(\pi_{2}^{*}-\pi_{1}\right)$

Assume that $\frac{\partial L_{\pi \pi}()}{\partial \pi}$ and $\frac{\partial \omega^{\prime \prime}()}{\partial \pi}$ are constant. ${ }^{39}$ Then we get:

$$
\frac{\frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \pi_{2}^{2}}}{\partial \gamma}=\delta\{\underbrace{\omega^{\prime \prime}}_{<0} \underbrace{[2(1-\theta)-1]}_{>0} \underbrace{\frac{\partial \tau_{2}^{*}}{\partial \gamma}}_{<0}\}>0
$$

3. $\frac{\partial^{2} g\left(\tau_{2}^{*},,_{2}^{*}\right)}{\partial \tau_{2} \partial \pi_{2}} \stackrel{\bar{\omega}^{\prime} \equiv \underline{\omega}^{\prime}}{=} \delta\left\{\underline{\omega}^{\prime}\left(\pi_{2}^{*}\right)[2(1-\theta)-1]\right\}$

This expression increases with $\gamma$, since $\pi_{2}^{*}$ is decreasing with $\gamma$ and $\omega()$ is concave.
4. $\operatorname{det}(J)=\underbrace{\left|\frac{\partial^{2} g}{\partial \pi_{2}^{2}}\right|}_{\downarrow} \left\lvert\, \cdot \underbrace{\left|\frac{\partial^{2} g}{\partial \tau_{2}^{2}}\right|}_{\text {constant }}+\underbrace{\left(-\left[\frac{\partial^{2} g}{\partial \tau_{2} \partial \pi_{2}}\right]^{2}\right)}_{\downarrow}\right.$

The directional indicators refer to the reaction of the respective term to an increase in $\gamma$. As can be seen, $\operatorname{det}(J)$ moves down in reaction to an increase in $\gamma$.

With all these results, we can now go back to (92) and (93):

$$
\frac{\partial \pi_{2}^{*}}{\partial \gamma}=\underbrace{\frac{1}{\operatorname{det}(J)}}_{\uparrow}(\underbrace{\frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \tau_{2} \partial \pi_{2}}}_{\uparrow} \cdot \underbrace{\frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \tau_{2} \partial \gamma}}_{\text {constant }})
$$

Thus, the reaction of legal capacity $\pi_{2}^{*}$ to an increase in $\gamma$ unambiguously gets stronger under the assumptions used in the preceding derivations. This is because $\frac{\partial^{2} g\left(\tau_{2}^{*}, \pi_{2}^{*}\right)}{\partial \tau_{2} \partial \gamma}<0$ and therefore the increase in the first two (positive) factors makes the whole (negative) expression $\frac{\partial \pi_{2}^{*}}{\partial \gamma}$ more negative. Stated differently $\frac{\partial \pi_{2}^{*}}{\partial \gamma}$ goes up in absolute value.

The reaction of fiscal capacity $\tau_{2}^{*}$ to an increase in $\gamma$ also unambiguously gets stronger,

[^25]as can be seen from the following:
\[

$$
\begin{aligned}
\frac{\partial\left(\frac{\partial \tau_{2}^{*}}{\partial \gamma}\right)}{\partial \gamma}= & -\frac{1}{\left[\left|\frac{\partial^{2} g}{\partial \pi_{2}^{2}}\right|\left|\frac{\partial^{2} g}{\partial \tau_{2}^{2}}\right|-\left(\frac{\partial^{2} g}{\partial \tau_{2} \partial \pi_{2}}\right)^{2}\right]^{2}} \\
& \underbrace{(22 \underbrace{\left|\frac{\partial^{2} g}{\partial \pi_{2}^{2}}\right|}_{>0} \underbrace{\left|\frac{\partial^{2} g}{\partial \tau_{2} \partial \gamma}\right|}_{>0} \underbrace{\left\lvert\, \frac{\partial^{2} g}{\partial \tau_{2} \partial \pi_{2}}\right.}_{>0} \underbrace{\frac{\partial\left(\frac{\partial^{2} g}{\partial \tau_{2} \partial \pi_{2}}\right)}{\partial \gamma}}_{>0}+\underbrace{\left[-\frac{\partial\left|\frac{\partial^{2} g}{\partial \pi_{2}^{2}}\right|}{\partial \gamma}\right]}_{>0} \underbrace{\left|\frac{\partial^{2} g}{\partial \tau_{2} \partial \gamma}\right|}_{>0} \underbrace{\left.\left(\frac{\partial^{2} g}{\partial \tau_{2} \partial \pi_{2}}\right)^{2}\right)}_{>0}<0}_{<0} \underbrace{>0}_{>0})
\end{aligned}
$$
\]

Again, the whole (negative) expression $\frac{\partial \tau_{2}^{*}}{\partial \gamma}$ gets more negative or stated differently $\frac{\partial \tau_{2}^{*}}{\partial \gamma}$ goes up in absolute value. The reaction is stronger.

Therefore, we get indeed that for rising political instability $\gamma$, it is more likely that debt and state capacity investments move in opposite directions in response to a change in $\gamma$.
Q.E.D.

Proposition 6.1. Suppose an economy in the quasi-linear model with sovereign default starts in the first period with $\alpha_{1}=\alpha_{L}$. Moreover, suppose that there are enough resources to provide transfers and that the constraint $\tau_{2} \geq \tau_{1}$ does bind. Then:

1. If $\alpha_{2}=\alpha_{L}\left(\alpha_{2}=\alpha_{H}\right)$, transfers (no transfers) will be provided in the second period.
2. There is positive investment in legal capacity and zero investment in fiscal capacity.
3. Investment in legal capacity is constant w.r.t. $\phi$.
4. Investment in legal capacity is constant w.r.t. $\gamma$.
5. Lower $\gamma$ leads to an increase of free future revenues.
6. Lower $\gamma$ leads to a decrease of debt.
7. Higher $\phi$ leads to an increase of free future revenues.
8. Higher $\phi$ leads to a decrease of debt.
9. The higher $\theta$, the less likely it is that a country will end up in a situation with sovereign default.

## Proof of proposition 6.1:

Part 1: This proof is equivalent to the proof of part 1 of proposition 5.1. Part 2, 3 and 4 follow from the FOC for legal capacity and the assumption that $\tau_{2} \geq \tau_{1}$ binds. Part 5
and 7 follow from the definition of free future revenues and the comparative statics for $b_{1}$, $\pi_{2}$ and the assumption that $\tau_{2} \geq \tau_{1}$ binds. Part 6 and 8 follow from the FOC for debt, the comparative statics for $\pi_{2}$ and the assumption that $\tau_{2} \geq \tau_{1}$ binds. Part 9: This proof is equivalent to the proof of part 8 of proposition 5.1.
Q.E.D.


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[^1]:    ${ }^{1}$ Buehn and Schneider (2012) estimate that the average size of the Greek shadow economy between 1999 and 2007 amounted to 27.5 percent of official GDP.
    ${ }^{2}$ The International Property Rights Index 2013 ranks Greece on the 62 th place behind China and India, which is the lowest rank of a western European country (see Property Rights Alliance, 2013).
    ${ }^{3}$ A review of the recent contributions to this literature can be found in Battaglini (2011). For a survey of the earlier contributions see Alesina and Perotti (1995).

[^2]:    ${ }^{4}$ Early studies concerning state capacity are the ones of Cukierman et al. (1992) regarding fiscal capacity and Svensson (1998) regarding legal capacity. In recent years, Acemoglu (2005) and Acemoglu et al. (2011) made further contributions to the literature on state building
    ${ }^{5}$ Recent models in the field of political economy of public debt with rich dynamic frameworks can be found in Battaglini and Coate (2008), Yared (2010), Song et al. (2012). For a political economy model of debt which endogenizes political turnover but shares the two-period setup with our model see Lizzeri (1999).

[^3]:    ${ }^{6}$ Note however, that we do not model these capital markets explicitly. For a microfoundation of the above reduced form modeling, see Besley and Persson (2009).
    ${ }^{7}$ Note that the whole analysis is in per-capita terms.

[^4]:    ${ }^{8}$ There will be a technological maximum $\bar{\tau}<1$ above which fiscal capacity cannot be expanded $\left(\tau_{s} \leq \bar{\tau}\right)$. Here, this would be determined by the fact that some small black market jobs just cannot be detected. Besley et al. (2013) interpret $\bar{\tau}$ as the highest technologically feasible tax rate (while $\tau_{s}$ is the highest institutionally feasible tax rate) and argue that in a richer model with distortionary taxation, $\bar{\tau}$ could be the peak of the Laffer curve. However, in the following, we focus on a situation where the optimal level $\tau_{2}$ will not hit this upper bound.

[^5]:    ${ }^{9}$ Recall that total population is normalized to 1.

[^6]:    ${ }^{10}$ The reason that the debt decision can be analyzed before the state capacity decisions has to do with the constancy of the marginal value of spending in each period.

[^7]:    ${ }^{11}$ The implicit assumption behind $b_{1} \geq 0$ is that the government cannot invest in assets on the bond market.
    ${ }^{12}$ Furthermore, this result requires Assumptions (22) and (23) which will be introduced after having derived the optimal state capacity investments. We need these additional technical assumptions here because for $E\left(\lambda_{2}\right)>\lambda_{1}$, we could otherwise get that all first-period tax revenue is used for investments in future state capacity. With low enough costs of investment, it could then be beneficial to use debt for bringing future tax revenues to the present and finance even more future state capacity.

[^8]:    ${ }^{13}$ This result depends again on Assumptions 22 and 23. These technical assumptions exclude the case where it is optimal (and through debt possible) that the marginal money to finance future fiscal capacity actually comes from the future. Cf. the previous footnote.

[^9]:    ${ }^{14}$ For a more detailed formal treatment, see the proofs in the Appendix.
    ${ }^{15}$ Note that the left-hand sides of (22) and (23) give the absolute maximum for the marginal benefits of fiscal and legal capacity investment. The right-hand sides give the absolute minimum for the marginal costs.

[^10]:    ${ }^{16}$ The proofs of this and of all subsequent propositions are collected in the Appendix.

[^11]:    ${ }^{17}$ Note that this is equivalent to assuming $\omega_{s}\left(\pi_{s}\right)$ to have the following form: $\bar{\omega}\left(\pi_{s}\right)=w\left(\pi_{s}\right)+\bar{v}$ and $\underline{\omega}\left(\pi_{s}\right)=w\left(\pi_{s}\right)+\underline{v}$. This means we assume the income shock (e.g. due to business cycles) to be additive and not depending on the level of legal capacity. Therefore, increasing legal capacity leads to a higher expected income (a positive growth trend), around which the actual income fluctuates with an amplitude that is constant with respect to $\pi$.

[^12]:    ${ }^{18}$ We can also think of these actions as taking place simultaneously under the constraint that the bundle of debt, interest rate and state capacity investments is so that investors are indifferent.

[^13]:    ${ }^{19}$ This is because we assume $b_{0}=0$ and that governments cannot accumulate assets. If we allowed for assets, governments would use their revenue to buy bonds in this case. If we additionally allowed $b_{0}>0$, revenues would be used to reduce debt (and possibly to buy bonds).
    ${ }^{20}$ The FOCs for $\tau_{2}$ and $\pi_{2}$ in case d) have been rearranged by using the FOC for $b_{1}$. The $\leq$ in the FOC for $\tau_{2}$ in case c ), d) and e) are due to the constraint $\tau_{2} \geq \tau_{1}$ which might bind in these cases (Note that $\pi_{2} \geq \pi_{1}$ does never bind).

[^14]:    ${ }^{21}$ If we had $\alpha_{1}=\alpha_{H}$, we would have $\lambda_{1}>E\left(\lambda_{2}\right)$ for both the social planner and for a government with own group bias. Both would have a preference for the present and an incentive to raise debt, and we would end up in one of the cases c), d) or e).
    ${ }^{22}$ For the sake of completeness, further channels of influence are $\psi, \bar{\omega}(\cdot)-\underline{\omega}(\cdot)$ and $P(\cdot)$, which also enter $E\left(\lambda_{2}\right)$.

[^15]:    ${ }^{23}$ It turns out that this is not a very restrictive assumption. It would only be violated if the high income realization $\bar{\omega}(\cdot)$ lied unrealistically high above the low income realization $\underline{\omega}(\cdot)$. A sufficient condition for this assumption is that $\bar{\omega}(\cdot) \leq 2 \underline{\omega}(\cdot)$.

[^16]:    ${ }^{24}$ The continuity at the border between two cases can be seen from the first order conditions. Free future revenues jump in case b) since debt jumps from $\bar{b}$ to 0 . Depending on the functional forms of $F()$, $L()$ and $\omega()$, the upward/downward sloping lines of the diagram are not necessarily linear.
    ${ }^{25}$ This case is not included in the proposition because there are no comparative static results to derive.
    ${ }^{26}$ Note that the proposition does not contain results about this level comparison to a world without debt.

[^17]:    ${ }^{27}$ These comparative static results are captured in part 3 (e) of Proposition 4.1. Note that $E\left(\lambda_{2}\right)$ does not enter the first order condition for legal capacity due to our assumption $\bar{\omega}^{\prime}\left(\pi_{2}\right)=\underline{\omega}^{\prime}\left(\pi_{2}\right)$. This implies that the high income part is not altered by investments in legal capacity.
    ${ }^{28}$ Part 3 (a) i. of the proposition.
    ${ }^{29}$ Note, however, that as long as we are in case d), the high income part is not drawn to the present for the marginal investment unit of fiscal capacity. That is why investment incentives are still driven partly by $E\left(\lambda_{2}\right)$.

[^18]:    ${ }^{30}$ Note that the absolute level of debt could also decrease with a decreasing value $E\left(\lambda_{2}\right)$ of future public funds in case d). However, decreasing $E\left(\lambda_{2}\right)$ in case d) will always lead to a decrease in the level of free future revenues, that is of expected future public funds minus debt.
    ${ }^{31}$ Note that in Besley and Persson (2011), a weak state did not invest at all in fiscal capacity.

[^19]:    ${ }^{32}$ To be more precise, recall from the discussion of the stability condition in section 3.3 that the probability $\phi$ for a high value of public good spending also has to be low enough

[^20]:    ${ }^{33}$ This allows for the theoretical possibility of a kink at $\bar{g}$.

[^21]:    ${ }^{34}$ A sufficient but not necessary condition that $\tau_{2} \geq \tau_{1}$ never binds is $\Gamma>1$. The results when $\tau_{2} \geq \tau_{1}$ binds are established in proposition 6.1 in the appendix.

[^22]:    ${ }^{35}$ However, the comparative statics with respect to $\phi$ now depend on $\Gamma$. The reason for this difference compared to the comparative statics in the model of section 4 is that a change in $\phi$ captures now also a change in $\psi$.

[^23]:    ${ }^{36}$ There is the possibility that the first order condition for debt will never hold with equality. It can be that the objective function with respect to debt is not differentiable but has a kink/peak. We exclude this case for now. It will be discussed in more detail later.
    ${ }^{37}$ Since these conditions will determine optimal values, we now move the ${ }^{*}$-notation to denote the values that maximize $g\left(\tau_{2}, \pi_{2}\right)$.

[^24]:    ${ }^{38}$ This can also be directly shown: By assumption (38), we have that for each constellation of parameters and optimal policies, $R H S_{1}<R H S_{2}$. Moreover, for every $\theta=1 / 2-\epsilon$ with $\epsilon \rightarrow 1 / 2$ there exist optimal policies such that $(90)$ holds with equality since $\Gamma \xrightarrow{\theta \rightarrow 1 / 2} 1$ and $\alpha_{H} V_{g}\left[\tau_{2}^{*} \underline{\omega}\left(\pi_{2}\right)-(1+\rho) b_{1}^{*}\right] \in(2(1-$ $\left.\theta), \alpha_{H} V_{g}(\bar{g})\right]$.

[^25]:    ${ }^{39}$ Note that $\underline{\omega}^{\prime \prime}\left(\pi_{2}^{*}\right)=\bar{\omega}^{\prime \prime}\left(\pi_{2}^{*}\right)=\omega^{\prime \prime}\left(\pi_{2}^{*}\right)$ follows from $\underline{\omega}^{\prime}\left(\pi_{2}^{*}\right)=\bar{\omega}^{\prime}\left(\pi_{2}^{*}\right)=\omega^{\prime}\left(\pi_{2}^{*}\right) . \omega^{\prime \prime}\left(\pi_{2}^{*}\right)<0$ follows from the concavity of $\omega()$.

