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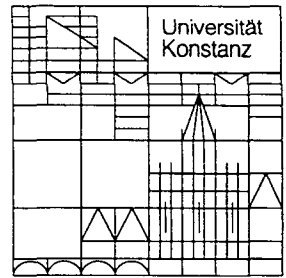
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Sonderforschungsbereich 178
„Internationalisierung der Wirtschaft“

Diskussionsbeiträge



Juristische
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Fakultät für Wirtschafts-
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Itzhak Zilcha

Multiple Currencies and Cross Hedging

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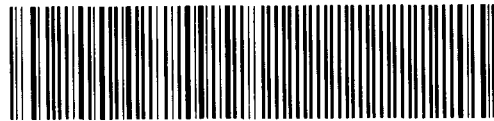
Multiple Currencies and Cross Hedging

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Multiple Currencies and Cross Hedging

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Abstract

The paper derives optimal cross hedging and production rules for an exporting firm which faces multiple exchange rate risks. We study the impact of currency cross hedging upon the firm's export production for two countries. We demonstrate that when the forward market for cross hedging is unbiased there is a full hedge. However, the profits remain stochastic. The cross hedge reduces uncertainty about the producer's income except that part which is unhedgeable. Furthermore we show that introducing an unbiased forward market for a cross-currency hedging will not affect the firm's total production level, even though it will increase the export to one of the two countries. This is in contrast to the usual impact which unbiased forward market has upon the risk-averse firm's production.

Key words: exports, cross hedging, forward markets

JEL Classification: F21; F31

Multiple currencies and cross hedging

UDO BROLL and ITZHAK ZILCHA

The paper derives optimal cross hedging and production rules for an exporting firm which faces multiple exchange rate risks. We study the impact of currency cross hedging upon the firm's export production for two countries. We demonstrate that when the forward market for cross hedging is unbiased there is a full hedge. However, the profits remain stochastic. The cross hedge reduces uncertainty about the producer's income except that part which is unhedgeable. Furthermore we show that introducing an unbiased forward market for a cross-currency hedging will not affect the firm's total production level, even though it will increase the export to one of the two countries. This is in contrast to the usual impact which unbiased forward market has upon the risk-averse firm's production.

1. Introduction

Exchange rates of the major industrial countries have been substantially volatile in the last decade. As a result, exchange rate uncertainties have become of increasing concern to international firms (e.g., Giavazzi and Giovannini (1989), Shapiro (1991), Buckley (1992), Fry (1992), Grobar (1993)). To respond to these uncertainties, a risk-averse firm can adjust its international trade and think about hedging opportunities. Thus we witness an intensified use of risk-sharing markets such as currency forwards as well as currency options. However, not every currency is traded in a future or forward market (see Buckley (1992), Fry (1992)). Therefore the exporting firm uses forward contracts with other underlying assets whose spot prices are correlated with the foreign exchange. Such hedging is called cross hedging (see Anderson and Danthine (1981), Powers and Castellino (1991)).

Recently there have been contributions to the theory of international firms under stochastic exchange rates in which the effects and the role of hedging

markets have been analyzed (see Kawai (1981), Benninga, Eldor, and Zilcha (1985)). It has been shown in the literature that an international firm facing exchange rate risk can eliminate, at least partly, exchange rate risk by using direct currency forward market, or another financial asset which is perfectly correlated to the exchange rate (Kawai and Zilcha (1986), Ware and Winter (1988)). In the absence of such markets, the firm can reduce its income risk by engaging in hedging activity of other currencies correlated to the exchange rate (Broll and Wahl (1995), Broll, Wahl, and Zilcha (1995)).

In the following we show that a multi-country exporting firm can benefit from hedging exchange rate risks even when the hedging tool is imperfect. In our study we present a model of a risk-averse two-country exporting firm when perfect hedging instruments (i.e., a forward market for the currency in which export is invoiced) are not available. Instead the firm can cross hedge the exchange rate risks by using the forward markets of a third country's currency. Hence cross hedging is a method applied to manage foreign currency risk when there are no direct forward markets in a currency.

Cross hedging is important because it expands the opportunity set of hedging alternatives. Our study provides some insight into the economic impact of cross hedges and their effectiveness. First we demonstrate that when the forward market for cross hedging is unbiased there is a full hedge. However, profits remain stochastic. The cross hedge reduces uncertainty about the producer's income except that part which is unhedgeable due to the missing forward market. Furthermore we show that introducing an unbiased forward market for a cross-currency hedging will not affect the firm's total production level, even though it will increase the export to one of the two countries. This is in contrast to the usual impact which unbiased forward market has upon the risk-averse competitive firm's production. One way to explain this result is that cross hedging does not remove the exchange risks in the profits of the firm, since the 'full hedging theorem' does not hold. On the other hand, such a market eliminates part of the risk by making the

profit dependent on one exchange rate only and this will affect the exports. The optimal production in the case where only a cross exchange rate hedge is possible depends on the risk aversion of the firm. It can be shown that an increase in the volatility of the cross exchange rate results in lower production and exports if absolute risk aversion is decreasing and convex function. Increasing volatility of the cross exchange rate increases the production level of the firm when the absolute risk aversion is non-decreasing and concave.

The organization of the paper is as follows. Section 2 introduces our model of an exporting firm with multiple random currencies and a cross hedge. In Sections 3 and 4 we examine the impact of cross hedging of exchange rate risk on exports to two countries. It is shown that in the case of currency cross hedging there is a ‘full hedge’ if the forward market for the cross exchange rate risk is unbiased; however, such unbiased market does not eliminate currency risk altogether. The effect of cross exchange rate volatility on production decisions is also discussed. Furthermore, we show that the availability of one currency forward market leads to the separation property for the firm’s output. Section 5 concludes the paper.

2. Multi-country exporting firm

Consider a competitive risk-averse firm which exports all output to two different countries 1 and 2. The production process gives rise to a cost function (denominated in domestic currency) $C(x)$, where $x = x_1 + x_2$ is the total quantity to be exported. We assume that $C(x)$ is a strictly convex, increasing and differentiable function. The firm faces random exchange rates of countries 1 and 2, \tilde{e}_1 and \tilde{e}_2 correspondingly. Denote the price of this commodity in the currency of country i by P_i ($i = 1, 2$). We assume that the firm is interested in profits denominated in its domestic currency and that it has to make the following decisions before the exchange rates are revealed: How much to produce, how to allocate production between the foreign separated

commodity markets and how to reduce the multiple exchange rate risks.

In our study the firm cannot hedge directly its multiple currency risks in currency forward markets. However, there is a forward market for a *cross* exchange rate \tilde{s} whose random spot price is correlated to the exchange rates \tilde{e}_1 and \tilde{e}_2 (see the diagram below: Fig. 1). More specifically we shall take $\tilde{e}_2 = \tilde{s}\tilde{e}_1$ with probability 1. Thus \tilde{s} is the spot exchange rate between these two currencies. The existence of a cross-currency forward market for these two currencies allows to use some forward rate s_f (at date $t = 0$) for the spot exchange \tilde{s} . If such a cross-currency forward market exists the firm can sell (or buy) forward the currency of country 2, z_2 , at a competitive forward price s_f , and receive (or pay) $s_f z_2$ units of currency of country 1 next period. Such a forward commitment increases (or reduces) the firm's profits (denominated in domestic currency) by $z_2 s_f \tilde{e}_1$ but reduces it by $z_2 \tilde{e}_2$. Thus the net local currency receipts from such cross hedging is $z_2 (s_f - \tilde{s}) \tilde{e}_1$. We shall assume that all transactions take place after production is completed (say, date $t = 1$) and hence the realization of the exchange rate is known.

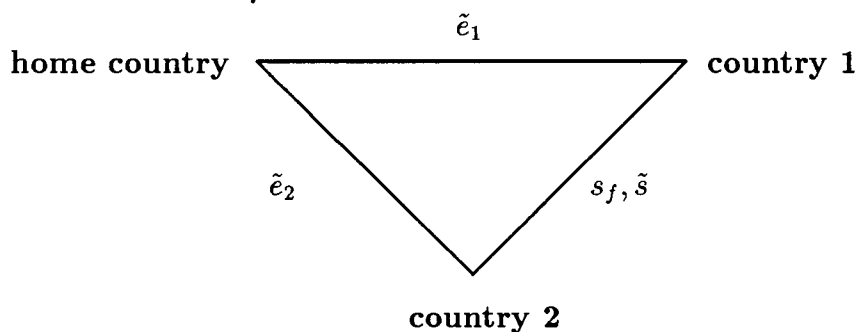


Fig. 1

The optimal choice of exports x_1 and x_2 to country 1 and 2 respectively and the optimal cross-currency forward commitment z_2 are determined by

the firm's maximization of expected utility of profits $\tilde{\Pi}$, denominated in local currency. The firm's von Neumann-Morgenstern utility function for profits is denoted by U and $U' > 0, U'' < 0$. We shall assume that the random variables \tilde{e}_1 and \tilde{s} are stochastically independent.

The firm's optimization problem is

$$\max_{x_1, x_2, z_2} EU(\tilde{\Pi}),$$

where profits are defined as follows

$$\tilde{\Pi} = \tilde{e}_1 P_1 x_1 + \tilde{e}_2 P_2 x_2 - C(x_1 + x_2) + z_2(s_f - \tilde{s})\tilde{e}_1.$$

Since the maximand is strictly concave in x_1, x_2, z_2 necessary and sufficient conditions for a maximum $EU(\tilde{\Pi})$, assuming that x_1^* and x_2^* are positive at the optimum, are:

$$E(\tilde{e}_1 P_1 - C'(x^*))U'(\tilde{\Pi}^*) = 0, \quad (1)$$

$$E(\tilde{e}_2 P_2 - C'(x^*))U'(\tilde{\Pi}^*) = 0, \quad (2)$$

$$E(s_f - \tilde{s})\tilde{e}_1 U'(\tilde{\Pi}^*) = 0, \quad (3)$$

where U' is the marginal utility, E the mathematical expectations operator with respect to the joint c.d.f. $G(e_1, e_2)$, and $x^* = x_1^* + x_2^*$. We shall denote by \star the optimum for this problem. In the following we use (1)-(3) to explore the effects of multiple random currencies and cross hedging on the firm's export production.

From the first-order conditions (1)-(3) (obtained under the assumption that the firm exports to both countries) we find the following non-arbitrage condition in the commodity market.

Corollary 1. The non-arbitrage condition between the two commodity markets must hold: $s_f P_2 = P_1$.

The *proof* follows directly from equations (1)-(3). □

3. Optimal cross hedge

Let us consider now the optimal cross hedge of the exporting firm. First we claim that when the forward market for cross hedging is unbiased and \tilde{s} is stochastically independent of \tilde{e}_1 there is a full hedge. However, profits remain stochastic. From equation (3) we have:

$$(s_f - E\tilde{s})E\tilde{e}_1U'(\tilde{\Pi}^*) - Cov(\tilde{s}, \tilde{e}_1U'(\tilde{\Pi}^*)) = 0, \quad (4)$$

where profits are

$$\tilde{\Pi}^* = \tilde{e}_1(P_1x_1^* + z_2^*s_f) + (P_2x_2^* - z_2^*)\tilde{e}_2 - C(x^*).$$

Now we claim:

Proposition 1. If a cross currency forward market exists, then:

- (a) *The firm fully hedges, i.e., $P_2x_2^* = z_2^*$, if $s_f = E\tilde{s}$.*
- (b) *The firm underhedges, i.e., $P_2x_2^* > z_2^*$, if $s_f < E\tilde{s}$.*
- (c) *The firm overhedges, i.e., $P_2x_2^* < z_2^*$, if $s_f > E\tilde{s}$.*

Proof. By the unbiasedness assumption, equation (4) can be rewritten as

$$Cov(\tilde{s}, \tilde{e}_1U'[\tilde{e}_1(P_1x_1^* + z_2^*s_f) + (P_2x_2^* - z_2^*)\tilde{e}_2 - C(x^*)]) = 0. \quad (5)$$

In order for the covariance to be zero, by the independence assumption we must have a full hedge, i.e. $P_2x_2^* = z_2^*$. Now suppose that the forward market incorporates a risk premium, so that $s_f < E\tilde{s}$. Then the first term on the left-hand side of equation (4) is negative (this follows since $E\tilde{e}_1U'(\tilde{\Pi}) > 0$) and therefore the covariance term must, at the optimum, be negative. In order for the covariance to be negative, the value of $U'(\tilde{\Pi}^*)$ must decline when \tilde{s} rises, which means that $\tilde{\Pi}^*$ must rise when \tilde{s} rises. With the independence assumption this can happen only if $P_2x_2^* > z_2^*$. The proof for the case where $s_f > E\tilde{s}$ is similar. \square

Contrary to the ‘usual’ full-hedging theorem (Kawai and Zilcha (1986)) in our case ‘full hedge’ does not imply nonrandom profits. Here, although the forward market is unbiased some exchange rate risk remains. The cross hedge reduces uncertainty about the producer’s income except that part which is unhedgeable due to the missing forward market where domestic currency can be purchased (or sold) using country 1 currency.

4. Exports and cross hedging

The availability of hedging opportunities affects production decision for exports. However, as we demonstrate later the allocation of the output between the two markets is not determined uniquely by equations (1) and (2). Only the total output $x = x_1 + x_2$ is determined.

Denote by $\bar{e}_1 = E\tilde{e}_1$ and $\bar{s} = E\tilde{s}$. Using Corollary 1 the optimality conditions for the exports can be rewritten as follows:

$$(\bar{e}_1 P_1 - C'(x^*))EU'(\tilde{\Pi}^*) + P_1 Cov(\tilde{e}_1, U'(\tilde{\Pi}^*)) = 0, \quad (6)$$

$$(\bar{s} P_2 \bar{e}_1 - C'(x^*))EU'(\tilde{\Pi}^*) + P_2 Cov(\tilde{s}\tilde{e}_1, U'(\tilde{\Pi}^*)) = 0. \quad (7)$$

Now we claim:

Proposition 2. Assume that unbiased cross-currency forward market exists, then: (a) Only the total output $x^ = x_1^* + x_2^*$ is uniquely determined. Export allocation x_1^*, x_2^* is indeterminate. (b) The volatility of \tilde{s} does not affect the total output level x^* .*

Proof. (a) Using Proposition 1 and the definition of $\tilde{\Pi}^*$ we find that

$$\tilde{\Pi}^* = \tilde{e}_1 P_1 (x_1^* + x_2^*) - C(x_1^* + x_2^*). \quad (8)$$

In this case equation (3) becomes:

$$E[(s_f - \tilde{s})\tilde{e}_1 U'(\tilde{e}_1 P_1 (x_1^* + x_2^*) - C(x_1^* + x_2^*))] = 0. \quad (9)$$

Which trivially holds for any $x^* = x_1^* + x_2^*$ since \tilde{s} and \tilde{e}_1 are independent and $s_f = E\tilde{s}$. The optimal x^* will be uniquely determined from equation (2) (note that equations (1) and (2) are identical in this case). Clearly only x^* can be determined hence the allocation to x_1^* and x_2^* is indeterminate. To prove part (b) it is obvious from equation (1) (or equation (2)) that the expression in (8) results in $U'(\tilde{\Pi}^*)$ being independent of \tilde{s} hence only $s_f = E\tilde{s}$ affects the optimum level of output x^* . \square

4.1 Export production without forward markets

Consider the case where the exporting firm has no access to a forward market for a cross exchange rate. Denote by \bar{x}_1 and \bar{x}_2 the optimal exports which are determined by maximizing $EU(\bar{\Pi})$ where profits are

$$\bar{\Pi} = \tilde{e}_1 P_1 \bar{x}_1 + \tilde{e}_2 P_2 \bar{x}_2 - C(\bar{x}_1 + \bar{x}_2).$$

The first order conditions for optimality of export production $\bar{x} = \bar{x}_1 + \bar{x}_2$ are:

$$E(\tilde{e}_1 P_1 - C'(\bar{x}))U'(\bar{\Pi}) = 0, \quad (10)$$

$$E(\tilde{s}\tilde{e}_1 P_2 - C'(\bar{x}))U'(\bar{\Pi}) \leq 0. \quad (11)$$

If $\bar{x}_2 > 0$ condition (11) holds as equality.

Let us show now that introducing unbiased forward market for a cross-currency hedging will not affect the firm's total production level, even though it will increase the export to country 2. This is in contrast to the usual impact which unbiased forward market has upon the risk-averse competitive firm's production: output level increases.

Proposition 3. Assume that initially no forward market for cross exchange hedging exists. Introducing unbiased market for cross hedging will not change the total output, i.e., $x^ = \bar{x}$; however, the export to country 2 will increase.*

Proof. Assume first that without cross currency forward market the firm exports to country 2, namely, that $\bar{x}_2 > 0$. From condition (11) we have equality to zero of the left hand side in (11). Thus from (10) and (11) we derive,

$$E[(\tilde{s}P_2 - P_1)\tilde{e}_1U'(\bar{\Pi})] = 0. \quad (12)$$

Since $\bar{x}_2 > 0$ we know that $Cov(\tilde{s}, \tilde{e}_1U'(\bar{\Pi})) < 0$ hence from (12) we find that,

$$(\bar{s}P_2 - P_1)E\tilde{e}_1U'(\bar{\Pi}) + P_2Cov(\tilde{s}, \tilde{e}_1U'(\bar{\Pi})) = 0.$$

Which implies $\bar{s}P_2 - P_1 > 0$ or $s_fP_2 - P_1 > 0$, which is impossible when no arbitrage is allowed; thus $\bar{x}_2 = 0$.

When an unbiased currency cross forward market exists the total output x^* is determined, using Proposition 2, by

$$E(\tilde{s}\tilde{e}_1P_2 - C'(x^*))U'(\tilde{e}_1P_1x^* - C(x^*)) = 0. \quad (13)$$

Which can be rewritten as (\tilde{s} and \tilde{e}_1 are independent random variables)

$$E[(s_fP_2\tilde{e}_1 - C'(x^*))U'(\tilde{e}_1P_1x^* - C(x^*))] = 0.$$

But $\bar{x} = \bar{x}_1$ is the solution to the equation:

$$E[(\tilde{e}_1P_1 - C'(\bar{x}))U'(\tilde{e}_1P_1\bar{x} - C(\bar{x}))] = 0. \quad (14)$$

Since $EU[\tilde{e}_1P_1x - C(x)]$ is a strictly concave function of x , hence equation (13), or (14), has a unique solution $x^* = \bar{x}$. Clearly, in the presence of cross hedging x_2^* can be positive. \square

One way to explain the result in Proposition 3 is that the unbiased forward market for cross hedging in this case does not remove the exchange risks in the profits of the firm, since the ‘full hedging theorem’ does not hold. On the other hand, such a market eliminates ‘part’ of the risk by making the profit dependent on \tilde{e}_1 only (note that \tilde{e}_2 is ‘riskier’ than \tilde{e}_1), and this should have some affect on the output for exports.

4.2 Higher cross exchange rate volatility

In this section we would like to study the effects of higher exchange rate volatility in the cross exchange rate on the firm's export production. The optimal production in the case where only a cross exchange rate hedge is possible depends on the risk aversion of the firm (the separation property does not hold).

Let \tilde{e}_1 assume values in $[\alpha, \beta]$ where $0 < \alpha < \beta < \infty$ and denote by $\mu(\tilde{e}_1)$ its distribution function. We make the following assumption about the utility function needed for Proposition 4. Define $R_a(X) = -U''(X)/U'(X)$ the absolute measure of risk aversion.

Assumption (A.1) $R_a(X)$ is decreasing and convex function.

Assumption (A.1) holds for many widely used families of utility functions such as

$$U(X) = \frac{X^{1-\gamma}}{1-\gamma}, \quad \gamma > 0, \quad U(X) = \ln X \quad \text{for } \gamma = 1.$$

Proposition 4. Consider the exporting firm described above and assume that (A1) holds, and that unbiased currency forward market exists. Increasing volatility of the cross exchange rate \tilde{s} results in lower production (and exports).

Proof. Let us prove this Proposition by comparing production levels for the case $\tilde{s} = E\tilde{s} = \bar{s}$ and the case of random \tilde{s} (with positive variance). Denote by x_1^*, x_2^* the optimal exports for the random case \tilde{s} , assuming that the currency forward market is unbiased, i.e., $s_f = E\tilde{s}$. Thus x_1^*, x_2^*, z^* are the solution to equations (1)-(3). Let

$$\tilde{\Pi}^*(e_1) = e_1 P_1 x_1^* + \tilde{s} e_1 P_2 x_2^* + z^* e_1 (s_f - \tilde{s}) - C(x_1^* + x_2^*).$$

Before we continue let us prove the following Lemma, which is required for our proof.

LEMMA. Let \tilde{A}, \tilde{X} be random variables which are independent. Define

$$\xi(\tilde{A}) = \frac{E_X U'(\tilde{A} + b\tilde{X})}{U'(\tilde{A} + b\bar{X})},$$

where $\bar{X} = E_X \tilde{X}$ is the expectation with respect to the distribution function of \tilde{X} . Then $\xi(A)$ is decreasing in A if assumption (A.1) holds.

Proof of the Lemma. Differentiating the function $\xi(A)$, we obtain

$$\xi'(A) = \frac{E_X U''(A + b\tilde{X})}{U'(A + b\bar{X})} + \left[\frac{-U''(A + b\bar{X})}{U'(A + b\bar{X})} \right] \frac{E_X U'(A + b\tilde{X})}{U'(A + b\bar{X})}.$$

Now let us write,

$$\begin{aligned} \xi'(A)U'(A + b\bar{X}) &= E_X \left[\frac{U''(A + b\tilde{X})}{U'(A + b\tilde{X})} U'(A + b\tilde{X}) \right] \\ &\quad + R_a(A + b\bar{X})E_X U'(A + b\tilde{X}) = \\ &= \left[R_a(A + b\bar{X}) - E_X R_a(A + b\tilde{X}) \right] E_X U'(A + b\tilde{X}) \\ &\quad - Cov(R_a(A + b\tilde{X}), U'(A + b\tilde{X})) E_X U'(A + b\tilde{X}). \end{aligned}$$

But this covariance is positive since both functions are monotone decreasing in \tilde{X} . Therefore,

$$\begin{aligned} \xi'(A)U'(A + b\bar{X}) &< \left[R_a(A + b\bar{X}) - E_X R_a(A + b\tilde{X}) \right] E_X U'(A + b\tilde{X}) < \\ &\quad \left[R_a(A + b\bar{X}) - R_a(A + b\bar{X}) \right] E_X U'(A + b\tilde{X}) = 0. \end{aligned}$$

Due to the convexity of the absolute risk aversion, assumption (A1). Thus $\xi'(A) < 0$, which proves the Lemma.

Define now,

$$m(\tilde{e}_1) = \frac{E_s U'[\tilde{\Pi}^*(\tilde{e}_1)]}{U'[\tilde{e}_1 P_1 x_1^* + \bar{s}\tilde{e}_1 P_2 x_2^* - C(x^*)]},$$

where $x^* = x_1^* + x_2^*$. Let $\bar{\lambda}$, $\alpha < \bar{\lambda} < \beta$, be defined by $\bar{\lambda}P_1 - C'(x^*) = 0$. Thus for $e_1 < \bar{\lambda}$ we have $e_1P_2 - C'(x^*) < 0$ and for $e_1 > \bar{\lambda}$, $e_1P_1 - C'(x^*) > 0$. By the above Lemma $m(e_1)$ is decreasing in e_1 . Therefore,

$$m(e_1) \leq m(e'_1) \quad \text{for any } e_1 \in (\bar{\lambda}, \beta], \quad e'_1 \in [\alpha, \bar{\lambda}] \quad (15)$$

Consider equation (1) and let us take expectation with respect to the distribution of \tilde{s} on both sides, we obtain,

$$\int_{\alpha}^{\beta} [e_1P_1 - C'(x^*)]E_s U'(\tilde{\Pi}^*(e_1))d\mu(e_1) = 0 \quad (16)$$

where $\mu(e_1)$ is the c.d.f. of \tilde{e}_1 . Using the definition of $m(e_1)$ equation (16) can be written as follows:

$$\int_{\alpha}^{\beta} [e_1P_1 - C'(x^*)]m(e_1)U'(e_1P_1x_1^* + \bar{s}e_1P_2x_2^* - C(x^*)) = 0.$$

Which can be written as,

$$\int_{\bar{\lambda}}^{\beta} [e_1P_1 - C'(x^*)]m(e_1)U'(\hat{\Pi})d\mu = \int_{\alpha}^{\bar{\lambda}} [C'(x^*) - e_1P_1]m(e_1)U'(\hat{\Pi})d\mu. \quad (17)$$

But with (15) we derive from (17),

$$\int_{\bar{\lambda}}^{\beta} [e_1P_1 - C'(x^*)]U'(\hat{\Pi})d\mu > \int_{\alpha}^{\bar{\lambda}} [C'(x^*) - e_1P_1]U'(\hat{\Pi})d\mu. \quad (18)$$

Thus we obtained that

$$E[\tilde{e}_1P_1 - C'(x^*)]U'(e_1P_1(x_1^* + x_2^*) - C(x^*)) > 0. \quad (19)$$

Since $e_1P_1 = \bar{s}e_1P_2$. But the optimal output when \bar{s} is fixed, denoted by \bar{x} , satisfies:

$$E[e_1P_1 - C'(\bar{x})]U'(e_1P_1\bar{x} - C(\bar{x})) = 0. \quad (20)$$

But the function $EU[e_1P_1x - C(x)]$ is strictly concave in x , therefore from (19) and (20) we proved that $x^* < \bar{x}$. \square

Corollary 2. If $R_a(X)$ is non-decreasing and concave then the result in Proposition 4 is reversed, i.e., increasing volatility of \tilde{s} increases the production level of the firm.

Note that the assumptions in Corollary 2 hold for quadratic utility function.

Corollary 3. When absolute risk aversion is constant then $\xi(A) = \text{constant}$ and, hence, increasing volatility of \tilde{s} does not change the production level.

The proofs of Corollary 2 and Corollary 3 follow from that of Proposition 4.

4.3 Exports when one currency has forward market

So far we have considered the case where only cross hedging exists. Now we analyze the question what will happen in only unbiased forward market for one currency exists, for example, for the exchange rate \tilde{e}_1 with a given forward exchange rate e_f^1 . The optimal choice of exports x_1 and x_2 and optimal currency forward commitment z_1 are determined by the firm's maximization of expected utility of profits $\tilde{\Pi}$ (denominated in domestic currency),

$$\tilde{\Pi} = \tilde{e}_1 P_1 x_1 + \tilde{e}_2 P_2 x_2 - C(x_1 + x_2) + z_1(e_f^1 - \tilde{e}_1).$$

Necessary and sufficient conditions for optimum, where $\hat{x} = \hat{x}_1 + \hat{x}_2$, are

$$E(\tilde{e}_1 P_1 - C'(\hat{x}))U'(\hat{\Pi}) = 0, \quad (21)$$

$$E(\tilde{e}_2 P_2 - C'(\hat{x}))U'(\hat{\Pi}) = 0, \quad (22)$$

$$E(e_f^1 - \tilde{e}_1)U'(\hat{\Pi}) = 0. \quad (23)$$

First, since we want to study the case where the international firm exports to both countries we find out

Corollary 4. The firm exports to country 2 (for which no currency forward market exists) if the expected marginal revenue is higher than the marginal revenue guaranteed in country 1, i.e., $e_f^1 P_1 < E\tilde{e}_2 P_2$.

Proof of the Corollary 4. Assume that $E\tilde{e}_2P_2 > e_f^1P_1$ and $x_2^* = 0$. In this case condition (22) is an inequality. Hence, we find out

$$E[(\tilde{e}_2P_2 - e_f^1P_1)U'(\tilde{\Pi})] < 0. \quad (24)$$

However, the covariance $Cov(\tilde{e}_2P_2, U'(\tilde{\Pi})) = 0$, since Π is independent of \tilde{e}_2 if $\hat{x}_2 = 0$. Thus condition (24) cannot hold. \square

Now let us show that separation property is restored if only one currency forward market exists. We shall make the following assumption first.

Assumption (A.2) The firm exports to both countries, i.e., $E\tilde{e}_2P_2 > e_f^1P_1$.

Now we claim:

Proposition 5. Assume the firm has access to one unbiased currency forward market (let us say for country 1), then: (a) total output \hat{x} of the firm is independent of the distribution function of the exchange rates and on the firm's attitude towards risk, (b) total production is higher in this case than in the case where only cross hedging exists, i.e., $x^* < \hat{x}$.

Proof. The proof of (a) is given by the following condition. From equation (21) and (23) we obtain

$$C'(\hat{x}) = e_f^1P_1. \quad (25)$$

(b) From equation (1) and (25) we obtain $C'(x^*) < E\tilde{e}_1P_1 = e_f^1P_1$. Therefore $x^* < \hat{x}$. \square

Condition (25) is a generalization of the separation property to our model (see, for example, Benninga, Eldor and Zilcha (1985)). The existence of one currency forward market is sufficient for the separation theorem (see equation (25)), which shows that the total production is independent of the distribution and the utility function.

5. Concluding remarks

Given the great volatility of foreign exchange rates firms engaged in international operations have been highly interested in developing ways to protect themselves from exchange rate risk. Currency forwards and options are used by international firms for insurance against price and exchange rate risk. However there are only a limited number of currencies which are traded actively in forward markets or for which forward contracts are typically available.

In our study the exporting firm hedges multi-currency risk by engaging in a forward contract for a cross hedging. We show that the affect of cross hedging on production and exports differs from that of a direct hedging markets. Cross hedging is a method used by international firms to manage foreign currency risk when there are no currency forward markets which are applicable directly. Cross hedging is important because it expands the opportunity set of hedging alternatives. Our study provides some insight into the selection of cross hedges and their effectiveness.

We demonstrate that existence of one currency forward market is sufficient for the separation theorem, which implies that the total production of the firm is indepenent of the distribution and the utility function.

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