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## The equilibrium risk premium on the forward market for foreign currency

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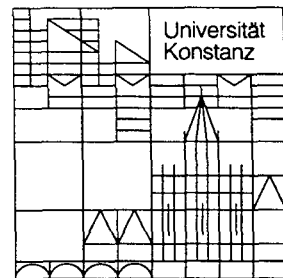
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Udo Broll  
Bernhard Eckwert

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# The Equilibrium Risk Premium on the Forward Market for Foreign Currency

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## Abstract

This paper constructs an intertemporal model of the spot and forward markets for foreign exchange and shows that in equilibrium the forward market is unbiased, i.e., the forward rate is equal to the expected spot rate which will prevail in the market next period. This holds true as long as the monetary authorities do not exogenously intervene in the foreign exchange market. Our analysis suggests that nominal exchange rate variability can affect the real sector of the economy only if active intervention policies are carried out on the spot exchange market.

*JEL: F31, F33*

*Key words: exchange rate risk, currency forward markets, risk premium*

## **The Equilibrium Risk Premium on the Forward Market for Foreign Currency**

UDO BROLL and BERNHARD ECKWERT

This paper constructs an intertemporal model of the spot and forward markets for foreign exchange and shows that in equilibrium the forward market is unbiased, i.e., the forward rate is equal to the expected spot rate which will prevail in the market next period. This holds true as long as the monetary authorities do not exogenously intervene in the foreign exchange market. Our analysis suggests that nominal exchange rate variability can affect the real sector of the economy only if active intervention policies are carried out on the spot exchange market.

Central to the dispute on the pros and cons of flexible exchange rates is the popular conjecture that exchange rate variability increases uncertainty in international transactions and, hence, has a negative impact on the volume of international trade (Cushman, 1983, 1986; Steinherr and Perée, 1989; Broll, Wahl and Zilcha, 1995; Broll and Eckwert, 1996). Most work in this vein uses a partial equilibrium approach focusing on exporters or importers concerned with revenues in home currency. In the absence of well developed forward markets there is some unequivocal evidence of a negative impact of exchange rate risk on export and import volumes.

If exchange rate risk can be hedged in a forward market then the export/import level of an international firm only depends on the forward rate (Ethier, 1973; Baron, 1976; Kawai, 1981). Thus, under proper optimization, the probability distribution of the exchange rate will affect the real economy only to the extent that it affects the forward rate. If investors were risk-neutral, then the forward market would be unbiased, i.e. the market should set the forward rate equal to the expected future spot rate. In this case, greater volatility (mean preserving spread) of the foreign exchange rate will have no effects on the allocation of resources, and therefore on foreign trade.

Yet, if the participants in the foreign exchange market are risk-averse, then the forward market might be biased through the presence of a risk premium. Moreover, the risk premium could vary over time and thereby affect international trade.

While extensive research has been conducted on the relationship between the forward rate and the spot rate that occurs on the maturity date of the forward contract, the evidence is still inconclusive: most research is unable to demonstrate empirically a significant bias on the forward exchange market (Hodrick, 1987; Goodhard, 1988; Macklem, 1991; Beetsma, 1995; Malliaropulos, 1995). This is the topic taken up in this paper. From an equilibrium point of view, we argue that exchange rate risk is non-systematic as long as the monetary authorities refrain from intervening in the spot market for foreign exchange: since importers and exporters are on opposite sides of the market and hence are affected differently by fluctuations of the exchange rate, there is scope for diversification of the exchange rate risk. Under the assumption that the foreign exchange market is competitive, there will be no risk premium on the forward market.

In this theoretical context, exchange rate volatility cannot spill over into the real sector of the economy unless the central bank exogenously intervenes in the foreign exchange market. Thus, our analysis suggests that (costly) real sector adjustments may be due to active intervention policies rather than exchange rate variability. The study also demonstrates that the widely used partial equilibrium approach which analyzes the forward market in isolation can be quite misleading, because the premium specified there might not be compatible with full equilibrium, i.e., with equilibrium on both the forward market and the spot market for foreign exchange.

The plan of the paper is as follows. Section I lays out an intertemporal model of an open economy and derives the optimal behavior of the market participants. In Section II we study the implications for the forward rate of equilibrium on the spot and forward exchange markets. A summary and some conclusions are provided in Section III.

## I. A simple equilibrium model of the markets for foreign exchange

Consider a two-period model with three types of economic agents: importers, exporters, and the central bank. At date 0, the forward market for foreign exchange opens and the contracts on the import and export markets are concluded. We assume that invoicing occurs in the foreign currency and that all balances are settled at date 1 (one-period trade credit). In order to close open positions for foreign currency the agents can trade on a spot foreign exchange market at time 1. The (random) one period ahead spot foreign exchange rate is  $\tilde{e}$ .

### I.A. Importing firms

Following Viaene and de Vries, 1992 we model the import activity in our economy by means of the optimizing behavior of a trade house: the importers buy commodities internationally at a fixed price of one unit of foreign currency and retail them locally. On the local market they face the inverse demand function  $P(Y)$ ,  $P'(Y) < 0$ , where  $Y$  is the quantity demanded at price  $P$ . Thus, if calculated in terms of domestic currency, the importers sell the commodities at a deterministic price,  $P$ , while they buy them at stochastic unit costs equal to  $\tilde{e}$ . The importers may hedge against the involved exchange rate risk by contracting on the forward foreign currency market. Denote by  $L$  the amount of foreign currency sold forward against the forward rate  $e_f$ . Then the profit function of the import sector reads:

$$(1) \quad \tilde{\Pi}_I = P(Y)Y - \tilde{e}Y + (e_f - \tilde{e})L.$$

The importer evaluates his random domestic profits according to a risk-averse von Neumann-Morgenstern utility function  $U_I : \mathbb{R}_+ \rightarrow \mathbb{R}$ ,  $U' > 0$ ,  $U'' < 0$ .

The necessary and sufficient conditions for the importer's decision problem

$$\max_{Y,L} EU_I(\tilde{\Pi}_I),$$

are

$$\begin{aligned} EU'_I[P'(Y)Y + P(Y) - \tilde{e}] &= 0, \\ EU'_I(e_f - \tilde{e}) &= 0. \end{aligned}$$



These two equations can be rewritten as<sup>1</sup>

$$(2) \quad P(Y)(\varepsilon[P, Y] + 1) = e_f,$$

$$(3) \quad EU'_I(e_f - \tilde{e}) = 0.$$

Observe that the equation system (2)-(3) exhibits a recursive structure which implies that the import decision can be separated from the hedging decision. In particular, according to equation (2), the import decision  $Y$  depends only on the forward rate  $e_f$ , while the optimal hedge  $L$  is also a function of the importer's preferences and of the distribution of the spot exchange rate  $\tilde{e}$ . This is a consequence of the well-known separation theorem (Ethier, 1973; Danthine, 1978; Holthausen, 1979).

### I.B. Exporting firms

The export sector produces a quantity  $X$  of an export good at the domestic costs  $C(X)$ , and sells on competitive world markets at a price of unity. The cost function is strictly increasing and convex, i.e.  $C'(X) > 0$ ,  $C''(X) > 0$ . If  $K$  denotes the amount of foreign currency sold forward, the profit function of the export sector takes the form

$$(4) \quad \tilde{\Pi}_E = \tilde{e}X - C(X) + (e_f - \tilde{e})K.$$

Again we have assumed that a one-period trade credit will be extended which implies that the export revenues depend on the random spot exchange rate at date 1.

The exporter's decision problem

$$\max_{X, K} EU_E(\tilde{\Pi}_E),$$

yields the first order conditions

$$EU'_E[\tilde{e} - C'(X)] = 0,$$

$$EU'_E(e_f - \tilde{e}) = 0.$$

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<sup>1</sup>The elasticity  $\varepsilon$  is defined in the usual way by  $\varepsilon[P, Y] := \frac{\partial P}{\partial Y} \frac{Y}{P}$ .

These equations can be rewritten as

$$(5) \quad C'(X) = e_f,$$

$$(6) \quad EU'_E(e_f - \tilde{e}) = 0.$$

The optimal export volume, which can directly be read off equation (5), depends positively on the forward rate. Equation (6) determines the hedging position of the export sector. The term  $EU'_E e_f$  represents the utility loss of paying the forward price for one unit of foreign currency. At the optimum this marginal loss matches the expected marginal utility gains,  $EU'_E \tilde{e}$ , provided by the future value of the foreign currency unit on the spot exchange market.

If the forward market is unbiased, i.e.  $e_f = E\tilde{e}$  holds, then equations (3) and (6) take the form

$$(7) \quad \text{Cov}[U'_\rho(\Pi_\rho), \tilde{e}] = 0, \quad \rho = I, E,$$

from which  $L = -Y$  and  $K = X$  follow immediately. Thus, the speculative positions of importers and exporters vanish if and only if the risk premium on the forward market is zero.

### *I.C. Central bank*

The third actor in our model is the central bank. Having in mind a floating exchange rate regime, we assume that the central bank does not intervene on the spot market through foreign currency swaps. Instead it sells forward an amount  $F$  of foreign currency at date 0 which will be determined endogenously in equilibrium.

## **II. Equilibrium on the forward and spot exchange markets**

In this section we study the implications for the forward rate of equilibrium on the spot and forward exchange markets. The forward market at date 0 clears if  $L + K + F = 0$  holds. At the final date 1, no private actor wishes to hold foreign currency.

Therefore, at this date the excess supply of foreign exchange on the spot market equals  $(X - K) - (L + Y)$ , where  $X - K$  and  $-(L + Y)$  are, respectively, the amounts of foreign currency supplied by the export sector and by the import sector. Thus, using the equilibrium condition for the forward market and defining the trade balance as the difference between exports and imports,  $T := X - Y$ , the clearing condition for the spot market can be stated as  $T + F = 0$ . This leads us to

**Definition 1 (Equilibrium)** . *An equilibrium for foreign exchange consists of a forward rate  $e_f$  and a forward exchange market intervention by the central bank,  $F$ , such that the forward foreign exchange market and the spot foreign exchange market clear:*

$$(8) \quad L + K + F = 0, \quad (\text{forward market})$$

$$(9) \quad T + F = 0. \quad (\text{spot market})$$

The next theorem contains the main result of this paper. It claims that in the equilibrium defined in Definition 1 the risk premium on the forward market for foreign exchange will be zero.

**Theorem 1** *In equilibrium the forward market is unbiased, i.e.  $e_f = E\tilde{e}$  holds.*

**Proof:** Let  $\bar{e} := E\tilde{e}$ , denote by  $R := \bar{e} - e_f$  the premium on the forward market and define  $g : \mathbb{R}_{++} \rightarrow \mathbb{R}$ ,  $g(Y) := P(Y)(\varepsilon[P, Y] + 1)$ . We show that  $R = 0$  and  $F = g^{-1}(\bar{e}) - C'^{-1}(\bar{e})$  constitutes an equilibrium for the economy.

(i) Equations (2) and (5) allow us to state the trade balance as

$$(10) \quad T := X - Y = C'^{-1}(e_f) - g^{-1}(e_f).$$

Inserting (10) and the above given equilibrium values for  $F$  and  $e_f$  into (9), one easily verifies that the spot exchange market clears.

(ii) For  $R = 0$  equations (3) and (6) take the form derived in (7):

$$(11) \quad \text{Cov}[U'_I(\tilde{\Pi}_I), \tilde{e}] = 0,$$

$$(12) \quad \text{Cov}[U'_E(\tilde{\Pi}_E), \tilde{e}] = 0.$$

From the definition of  $\tilde{\Pi}_I$  and  $\tilde{\Pi}_E$ , (11) and (12) hold if and only if  $L = -Y$  and  $K = X$ . Thus,  $L + K = X - Y = T$ , from which (8) follows immediately if we take (9) into account. ■

According to Theorem 1 partial equilibrium models which analyze the forward market in isolation can be quite misleading, because the premium specified there might not be compatible with equilibrium on the spot market. In fact, as long as the central bank refrains from intervening on the spot exchange market, any (partial equilibrium) specification of the forward market which involves a non-zero risk premium violates a basic consistency condition in full equilibrium.

The central economic mechanism underlying the result in Theorem 1 is quite simple: since the agents are symmetrically informed and risk-averse they all speculate in the same direction, i.e. they all take positive (negative) speculative positions on the forward market if the risk premium is positive (negative). Speculation is absent only if the risk premium vanishes.<sup>2</sup> However, no private agent wants to hold foreign currency beyond date 1. Thus all traders use the spot market to close their open speculative positions. By assumption, the central bank does not intervene on the spot exchange market, so in equilibrium the speculative positions of the agents must cancel out. This in turn implies a zero risk premium, because importers and exporters never take opposite speculative positions on the forward market.

In the terminology of modern finance theory, Theorem 1 says that exchange rate fluctuations do not constitute a systematic risk factor for an open economy. Risks resulting from movements of the exchange rate can be fully diversified, because any appreciation of the home country's currency is linked to a depreciation of the foreign country's currency. Exchange rate losses of the import sector are therefore offset by exchange rate gains of the export sector and vice versa, so that the exchange rate risks cancel out in the aggregate if the markets for foreign exchange clear.

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<sup>2</sup>The speculative parts of the hedges of the import sector and of the export sector are  $L + Y$  and  $K - X$ , respectively. If the risk premium on the forward market is positive (negative), i.e.  $E\tilde{e} - e_f \stackrel{(<)}{>} 0$ , then eqs. (3) and (6) imply  $L + Y \stackrel{(>)}{<} 0$ ,  $K - X \stackrel{(>)}{<} 0$ .

In view of the above discussion the finding that in full equilibrium the forward market for foreign exchange cannot be biased is suggestive in nature, because it seems to be based on a robust economic mechanism which is likely to remain valid even in a more general theoretical framework. E.g., the time horizon of the model is not of critical importance for this mechanism. As long as the time horizon is finite there exists some final date  $T$  where all agents wish to exchange their holdings of foreign currency for domestic currency. Thus at time  $T-1$  the forward market will be unbiased according to Theorem 1. In that case, however, a risk-averse agent has no incentive to hold foreign currency in his portfolio. Thus, the situation on the spot market at time  $T-1$  is the same as at time  $T$ . Therefore, by simple backward induction we find that the risk premium on the forward market is zero at all dates. Of course, the backward induction argument cannot be applied to models with infinite time horizons. Even so, it is quite obvious that in all stationary equilibria of an infinite horizon economy (and, possibly, in some equilibria which converge to a steady state) the result in Theorem 1 will survive in some modified form.

We could easily introduce into our model other agents (e.g. consumers or speculators) who do not take part in import or export activities but nevertheless have access to the foreign exchange markets. As long as these agents are risk-averse they would not affect the result in Theorem 1, because they have no incentive to speculate if the forward market is unbiased. Thus, in equilibrium the demand and supply on the foreign exchange markets depends only on the behavior of the three sectors that have explicitly been taken into account.

If the central bank does not intervene on the spot exchange market and hence the forward market is unbiased, then the risk allocation in the economy is optimal in the sense that all risks will be perfectly hedged at no costs. This follows immediately from equation (7) in combination with the definition of the profit functions in (1) and (4). Central bank interventions on the spot market, in turn, would leave the forward market biased in equilibrium which implies non-zero speculative positions of the private market participants. Thus the equilibrium profits of the import sector and of the export sector would both be random. Since the distribution of the economy-

wide risk factor, the stochastic exchange rate, has not changed, we may conclude that central bank interventions on the spot exchange market prevent full risk diversification and therefore result in a suboptimal allocation of exchange rate risks.

### III. Conclusions

This paper studies the implications of equilibrium on the spot and forward markets for foreign exchange. It is shown that the widely used partial equilibrium approach which links the volume of international trade to the exogenously specified risk premium on the forward market misses the point, because in full equilibrium this risk premium must vanish. While our study provides a theoretical basis for the strong empirical evidence that forward foreign exchange markets are approximately unbiased, there is still a question as to whether risk premia on these markets, if they are small but different from zero, need necessarily be understood as a disequilibrium phenomenon. This is possibly not so, because the analysis in this work is subject to a number of limitations which might have contributed to the sharp characterization of the forward market in Theorem 1.

First, most spot markets for foreign exchange are not perfectly free from outside interventions. In fact, since the breakdown of the Bretton Woods System there have been countless occasions on which monetary authorities have officially intervened in the foreign exchange market. Even if the central banks agree that the time path of the exchange rate should be determined by market forces, it might nevertheless be in their interest to smooth out the daily erratic movements of the exchange rate without affecting its time trend. In a generalized version of our model such interventions would lead to a risk premium on the forward market, which fluctuates randomly around zero.

Second, in our model the random spot exchange rate constitutes the only source of uncertainty which affects the agents' decision problems. In economies with multiple risk factors (endowment risks, dividend risks, policy risks etc.) agents might be willing to hold long positions or short positions on the forward exchange market even if the risk premium is zero. E.g., agents who face some endowment risk in the future could construct a partial hedge at no costs by selling forward foreign currency, if the spot

exchange rate is positively correlated to the endowment risk. In such a generalized framework the interactions of the spot exchange rate with other risk factors could generate a positive or negative risk premium on the forward market in full equilibrium. Even so, the risk premium is likely to be small in absolute value, since the effects on the forward market of those interactions may offset each other to a large extent.

Third, the analysis in this paper assumes away all informational asymmetries across agents. Obviously, expectations about the distribution of the future spot exchange rate may affect the forward rate. If these expectations depend on private information which is asymmetrically distributed in the economy, then part of this information will be revealed by the equilibrium prices. The process of information transmittal may affect the forward rate through its impact on the agents' expectations.

Finally, our simplified approach proceeds on the assumption that importers and exporters evaluate their profits in domestic currency. In equilibrium, the speculative positions of both sectors vanish, which implies that the *domestic* forward market for foreign exchange is unbiased (i.e.,  $e_f = E\tilde{e}$  holds). However, if e.g. the importers were to evaluate their profits in foreign currency then they would refrain from speculating only if the *foreign* forward market was unbiased (i.e.,  $1/e_f = E1/\tilde{e}$  holds). By Jensen's inequality, it is impossible that both forward markets are simultaneously unbiased. Thus, in equilibrium the two sectors must take opposite speculative positions on the forward market which implies  $\frac{1}{E1/\tilde{e}} < e_f < E\tilde{e}$ , i.e., the risk premium on the forward market is positive but small (less than  $E\tilde{e} - \frac{1}{E1/\tilde{e}}$ ).

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