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Optimal intergenerational redistribution and strategic incentives with two countries and endogenous fertility: Theory and application to the European Union

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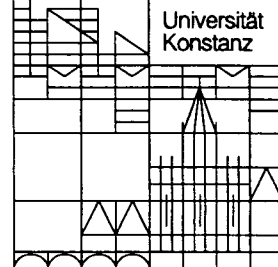
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Diskussionsbeiträge

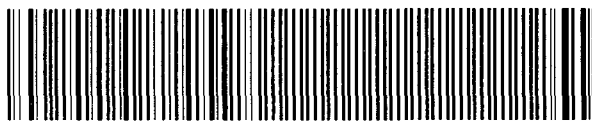
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Martin Kolmar

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# Optimal Intergenerational Redistribution and Strategic Incentives with Two Countries and Endogenous Fertility: Theory and Application to the European Union\*

Martin Kolmar<sup>†</sup>

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## Abstract

Intergenerational redistribution is one of the major aims of national welfare states that have to prove their reliability in a world of internationally integrated goods and factor markets. In this paper, we analyse the conditions for the existence of steady-state equilibria and the (Pareto-) optimal structure of national PAYG systems in a two-country model with endogenous fertility. Secondly we demonstrate that there are strategic incentives for national authorities to deviate from the optimal pension scheme even without labor mobility, which is the common source of inefficiencies in models of interregional social competition. They have two sources of strategic behavior in the model: One is an interest externality, the other one is a growth externality. This second externality shows that these incentives exist even with an exogenous interest rate. Policy implications for the European Union are discussed.

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# 1 Introduction

The creation of internationally integrated goods and factor markets puts pressure on the national welfare systems. This pressure implies two things: First, existing inefficiencies are becoming increasingly visible and costly in a world of integrated markets; therefore, reforms following the integration of markets may be efficiency-enhancing. This effect is strengthened if individuals can “vote by feet” and choose a region with a supply of social security that fits well with their preferences. Second, it is getting increasingly difficult to achieve national social-policy targets. The market pressure may lead to an increasing competition for mobile factors and the resulting equilibrium may be inefficient.

The theoretical literature dealing with the question of efficient national social security in a world of internationally integrated markets mainly focusses attention to the systems that redistribute within a given generation, like unemployment insurance (Verbon (1990), Wildasin (1991, 1994)) and social assistance (Brown and Oates (1987)). In addition, most of the papers analyse the consequences of labor mobility on the functioning of the welfare state.<sup>1</sup> The main insight of this literature is that decentralized policy making can be expected to be inefficient because: a) Mobile factors do not allocate according to their highest productivities because different contribution rates drive different wedges between marginal productivities and net incomes. b) The level of social security in every region is inefficiently low because the effectiveness of every unit of transfer income is reduced due to the immigration of net beneficiaries and the emigration of net payers.

The papers by Homburg and Richter (1993) and Breyer and Kolmar (1995) extend the question to intergenerational redistribution in systems like public-pension systems, long-term-care and health-insurance.<sup>2</sup> They show that there are idiosyncratic problems for systems that depend on a “generational contract” in which contributions and benefits are calculated according to the res-

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<sup>1</sup>For a recent survey of the literature see Breyer and Kolmar (1996).

<sup>2</sup>Health insurance redistributes intergenerationally even if premium payments are actuarially fair from a life-cycle perspective because the risk of getting sick is (more or less) positively correlated with the age of the individual.

idence principle<sup>3</sup>: The young generation has a legal way to terminate the contract by migrating to a different country. The exit-option of future generations creates an incentive to emigrate from regions with high- and immigrate to countries with low contribution rates. This induces an inefficient allocation of labor on the labor markets. Additionally, the rate of return of a public pension system becomes highly unpredictable.

One of the major critiques of the literature is the assumption of labor mobility. Especially in the European Union, it is argued<sup>4</sup>, mobility costs (like language barriers and cultural background) are prohibitively high. Thus, it is argued, the results are interesting from a theoretical point of view, but without empirical relevance.

Without adding another argument to this discussion, we develop a model without labor mobility and demonstrate that the responsibility of regional authorities for public pensions still creates strategic incentives for setting policy-parameters that make the resulting equilibrium inefficient.<sup>5</sup>

In order to do so, we merge two strands of the literature. The first strand is the literature on interregional spillovers in open economies (see for example Wildasin (1991) and Persson (1985)). The second strand is the literature on endogenous fertility initiated by Becker (1960). Following this approach, a number of articles on public-pension systems have been published over the last years.<sup>6</sup>

The merging of these strands of the literature causes several interesting insights into the existence of steady-state equilibria, the efficiency of decen-

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<sup>3</sup>The residence principle is applied to calculate contributions and old-age pension benefits for migrating workers in the European Union.

<sup>4</sup>For example by Raffelhueschen (1996).

<sup>5</sup>One relatively easy way to avoid inefficiencies due to migration that is in accordance with the principle of subsidiarity has been advocated by Sinn (1990). He argues that a change to an origin principle in social policy removes incentives to migrate that are motivated by social arbitrage. Without labor-mobility, the residence and origin principles coincide. Thus, changing systems cannot be the end of the story in our model.

<sup>6</sup>For example Bental (1989), Cigno (1986), Eckstein and Wolpin (1985), Felderer (1992), Lapan and Enders (1990), Nishimura and Zhang (1992), Peters (1995), Raut (1991), Razin and Sadka (1995), Samuelson (1975), Schweizer (1995), Straub and Wenig (1984), Wigniolle (1995), Wildasin (1990) and the references therein.

tralized equilibria, and therefore the role of government interventions. They are derived throughout the paper: In section 2 we introduce the notation. In section 3 we analyse conditions for the existence of steady-state equilibria. In section 4 we derive optimality conditions for a one-period equilibrium. We compare these conditions with a decentralized equilibrium in section 5 and derive optimal intervention schemes. Section 6 analyses the strategic incentives for a regional policy-maker. Section 7 concludes with a brief discussion of the policy implications for the European Union.

## 2 The Model

We analyse an economy with two regions, 1 and 2, in which policymakers autonomously determine the contribution rate of a pay-as-you-go public-pension system (PAYG).

### 2.1 Firms

Every region produces a homogeneous output by the use of labor and capital. We assume perfectly integrated goods and capital markets and totally separate labor markets between the two regions. The production technology is linear homogeneous and identical between the two regions.<sup>7</sup> The technology is represented by a production function  $X_i^t = F(K_i^t, L_i^t)$ ;  $t = 1, 2, \dots, \infty$  is the period of production and  $i = 1, 2$  is a regional index. The market structure is perfectly competitive in both regions, the individual firm behaves as a price-taker. Capital is completely used up after the period of production.<sup>8</sup>

We can rewrite the production function in per-worker terms due to its linear homogeneity:

$$X_i^t = F(K_i^t, L_i^t)$$

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<sup>7</sup>We assume that technological knowledge is part of the capital used in production and that there is no region-specific human capital that creates differences in productivities.

<sup>8</sup>This assumption is necessary in models of endogenous population growth in order to avoid problems of interpretation if the population growth is too low to absorb the capital stock of the old generation.

$$\begin{aligned}
&= L_i^t F(K_i^t/L_i^t, 1) \\
&= L_i^t f(k_i^t), \quad k_i^t = (K_i^t/L_i^t)
\end{aligned}$$

$f$  fulfills the Inada conditions. From profit-maximisation of the firms we get the usual factor-price frontier:

$$1 + r_i^t = f'(k_i^t) \quad (1)$$

$$w_i^t = f(k_i^t) - k_i^t f'(k_i^t) \quad (2)$$

From the assumption of a completely integrated capital-market it follows that  $r_1^{t+1} = r_2^{t+1} = r^{t+1}$ , together with the assumption of identical technologies we can conclude that  $k_1^t = k_2^t$ , and therefore  $w_1^t = w_2^t = w^t$ .

## 2.2 Households

At every point of time  $t$  there are  $N_i^{t-1}$ ,  $N_i^t$ ,  $N_i^{t+1}$  identical (overlapping) old, working and young individuals (households) in region  $i = 1, 2$ . Every individual lives for exactly three periods, childhood, working-life and retirement.

In the first period of life, the individual has no economic decision to make. He consumes an exogenous amount  $x_i^t$  of the homogeneous output, that is paid by his parent.<sup>9</sup>

In the second period of life, every individual works one unit of time and receives a wage  $w_i^t$ . Furthermore, he has to decide on consumption  $c_{2i}^t$ , the number of children  $1 + n_i^{t+1}$  he wants to bring up, and savings on capital markets  $s_{2i}^t$ . Every individual has to pay a fraction  $\tau_i$  of his gross income ( $w_i^t$ ) as contribution to a pay-as-you-go financed public pension system. Accordingly, his net income is  $(1 - \tau_i)w_i^t$ .

During his third period of life, an individual consumes ( $c_{3i}^{t+1}$ ) which equals his savings  $(1 + r_i^{t+1})s_{2i}^t$  plus the benefits from the PAYG-system  $(1 + n_i^{t+1})\tau_i w_i^{t+1}$ . We assume a constant contribution rate.

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<sup>9</sup>Following the literature on endogenous fertility, we assume that every individual can procreate by himself.



Every individual has a twice-continuously differentiable, strictly quasi-concave utility function that is increasing in working-life and retirement consumption and the number of own children. For convenience we assume identical utility functions in both regions.<sup>10</sup> Thus, there are two motives to have children in this model: First, parents derive utility from having children (consumption motive). Second, a PAYG is a channel to transfer income from the second to the third period of life (investment motive).

With these specifications we get the following optimisation problem for a representative individual, written as a Lagrange-problem ( $\lambda_i$  and  $\mu_i$  are the associated Lagrange-parameters.):

$$\begin{aligned}
\mathcal{L}_i &= U_i(c_{2i}^t, c_{3i}^{t+1}, (1 + n_i^{t+1})) & (3) \\
&+ \lambda_i[(1 - \tau_i)w_i^t - c_{2i}^t - (1 + n_i^{t+1})x_i^t - s_{2i}^t] \\
&+ \mu_i[(1 + r^{t+1})s_{2i}^t + \tau_i w_i^{t+1}(1 + n_i^{t+1}) - c_{3i}^{t+1}] \\
&n_i \in [-1, \bar{n}]
\end{aligned}$$

We restrict attention to interior solutions of the above problem.<sup>11</sup> We get the following first-order conditions for a maximum ( $U_{y_i}$  is the partial derivative of  $U$  with respect to  $y$  in region  $i$ ):

$$U_{2i} - \lambda_i = 0 \quad (4)$$

$$U_{3i} - \mu_i = 0 \quad (5)$$

$$U_{n_i} - \lambda_i x_i^t + \mu_i \tau_i w_i^{t+1} = 0 \quad (6)$$

$$-\lambda_i + \mu_i(1 + r^{t+1}) = 0 \quad (7)$$

After some elementary transformations it follows for region  $i$ :

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<sup>10</sup>This assumption has no influence on the qualitative nature of the results. It will be marked throughout the text if results change quantitatively. We will nevertheless use a region index at the utility function in order to simplify the discussion.

<sup>11</sup>The reader interested in a detailed analysis of interior and corner solutions is referred to Kolmar (1997).

$$GRS_2^n(i) := \frac{U_{ni}}{U_{2i}} = x_i^t - \frac{\tau_i w_i^{t+1}}{1+r^{t+1}} \quad (8)$$

$$GRS_2^n(i) := \frac{U_{3i}}{U_{2i}} = \frac{1}{1+r^{t+1}} \quad (9)$$

The marginal rate of substitution between an additional (marginal) child and consumption is equal to the net costs of raising this child. The net costs are equal to the direct costs  $x_i^t$  minus future pension payments by this child,  $\frac{\tau_i w_i^{t+1}}{1+r^{t+1}}$ . This expression may be either positive or negative depending on the amount of intergenerational redistribution. If the ‘net price’ of children is below a certain bound (e.g. is negative), maximum fertility would be realized and the maximisation problem would generate a corner solution. The marginal rate of substitution between working-life and retirement consumption must be equal to the usual discount rate given by the capital-market interest factor.

### 3 The Existence of Steady-State Equilibria

We have an interregional goods and capital market and regional labor markets in this model. For the labor markets to clear, it must be that

$$L_i^t = N_i^t, \quad i = 1, 2 \quad (10)$$

Due to the functional form of the production function, this equality is fulfilled if there is an equilibrium in both other markets. From “Walras law” we know that if one of the remaining markets is in equilibrium, the other one is too. Thus we can restrict attention to the capital market. This market is in equilibrium if total savings in one period are equal to the total stock of capital in the next period:<sup>12</sup>

$$(K_1^{t+1} - N_1^t s_{21}^t) + (K_2^{t+1} - N_2^t s_{22}^t) = 0 \quad (11)$$

A steady-state equilibrium is a time-path of population growth rates, savings, consumption and capital stocks that is compatible with a) individual

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<sup>12</sup>See Persson (1985) for further details on Walras law in OLG-economies.

utility maximisation, b) profit maximisation of the firms c) the equilibrium conditions for every period and d) constant growth rates for all variables such that consumption and utility remains constant over time. We get for  $i = 1, 2$ :

$$\begin{aligned}
c_{2i}^t &= c_{2i}^{t+1} = c_{2i} \\
c_{3i}^t &= c_{3i}^{t+1} = c_{3i} \\
s_{2i}^t &= s_{2i}^{t+1} = s_{2i} \\
n_i^t &= n_i^{t+1} = n_i \\
k_i^t &= k_i^{t+1} = k \\
1 + r &= f'(k) \\
w_1 &= w_2 = f(k) - kf'(k) \\
(K_1^{t+1} - N_1^t s_{21}) + (K_2^{t+1} - N_2^t s_{22}) &= 0 \\
GRS_2^n(i) &= x_i^t - \frac{\tau_i w}{1+r} \\
GRS_2^3(i) &= \frac{1}{1+r}
\end{aligned}$$

Next, we will analyse the conditions under which a steady-state equilibrium exists. In order to do so we will first prove three elementary Lemmata:

**Lemma 1:** *There are either equal population-growth rates in both regions or foreign debt is equal to zero in a steady-state equilibrium.*

**Proof:** The equilibrium condition for the capital market is in period  $t$ :

$$\begin{aligned}
(K_1^{t+1} - N_1^t s_{21}^t) + (K_2^{t+1} - N_2^t s_{22}^t) &= 0 \\
\Leftrightarrow \frac{N_1^t}{N_2^t} ((1 + n_1^{t+1})k_1^{t+1} - s_1^t) + ((1 + n_2^{t+1})k_2^{t+1} - s_2^t) &= 0
\end{aligned}$$

By the use of the conditions for a steady-state equilibrium, this equality can be transformed ( $N_i^0$  is the population in region  $i$  in the first period of the steady-state):

$$\Psi = \left( \frac{1 + n_1}{1 + n_2} \right)^t \frac{N_1^0}{N_2^0} ((1 + n_1)k_1 - s_1) + ((1 + n_2)k_2 - s_2) = 0 \quad (12)$$

In general, population-growth rates differ between regions. We therefore have to analyse the long-run behavior of  $\left(\frac{1+n_1}{1+n_2}\right)^t \frac{N_1^0}{N_2^0}$ . We get:

$$\lim_{t \rightarrow \infty} \left(\frac{1+n_1}{1+n_2}\right)^t \frac{N_1^0}{N_2^0} = \begin{cases} \infty & : n_1 > n_2 \\ 0 & : n_1 < n_2 \\ (N_1^0/N_2^0) & : n_1 = n_2 \end{cases} \quad (13)$$

Thus, in equilibrium we get  $\lim_{t \rightarrow \infty}(\Psi) = \infty \neq 0$  for  $n_1 > n_2$  and  $(1+n_1)k_1 - s_1 \neq 0$ ; alternatively we get  $\lim_{t \rightarrow \infty} \Psi = (1+n_2)k_2 - s_2$  for  $n_1 < n_2$ . If the population growth rates differ, there can only be an equilibrium in the capital market if foreign debt is equal to zero. On the other hand, for equal population-growth rates we get  $\frac{N_1^0}{N_2^0}((1+n_1)k_1 - s_1) + ((1+n_2)k_2 - s_2) = 0$ , this may be compatible with a positive stock of foreign debt. **q.e.d.**

The intuition for the result is straightforward: If both regions differ in their steady-state population-growth rates, every amount of foreign debt (positive or negative) of the faster growing region would grow without bounds. This would contradict the equilibrium in the capital market.

**Lemma 2:** *With identical regional technologies, a steady-state exists only if the ratio of population growth factors equals the ratio of per-capita savings.*

**Proof:** As was demonstrated above, with identical interregional technologies and an integrated capital market, the per-worker capital stock has to be equal in both regions,  $r^{t+1} = f'(k_1^{t+1}) = f'(k_2^{t+1}) \Leftrightarrow k_1^{t+1} = k_2^{t+1} = k^{t+1}$ . If we insert this condition into the equilibrium condition and evaluate for a steady-state, we get:

$$\left(\frac{1+n_1}{1+n_2}\right)^t \frac{N_1^0}{N_2^0} ((1+n_1)k - s_1) + ((1+n_2)k - s_2) = 0 \quad (14)$$

This yields for  $k$ :

$$k = \frac{\left(\frac{1+n_1}{1+n_2}\right)^t \frac{N_1^0}{N_2^0} s_1 + s_2}{\left(\frac{1+n_1}{1+n_2}\right)^t \frac{N_1^0}{N_2^0} (1+n_1) + (1+n_2)} \quad (15)$$

$k$  has to stay constant in a steady-state. The easiest way to check whether  $k$  is in fact independent of  $t$ , is to differentiate (15) with respect to  $t$  and to set this derivative equal to zero:

$$\begin{aligned} \frac{\partial k}{\partial t} &= t \left( \frac{1+n_1}{1+n_2} \right)^{t-1} \frac{N_1^0}{N_2^0} \frac{s_1(1+n_2) - s_2(1+n_1)}{\left( \left( \frac{1+n_1}{1+n_2} \right)^t \frac{N_1^0}{N_2^0} (1+n_1) + (1+n_2) \right)^2} = 0 \\ &\Leftrightarrow s_1(1+n_2) - s_2(1+n_1) = 0 \end{aligned} \quad (16)$$

In other words, it must be that  $s_1/s_2 = (1+n_1)/(1+n_2)$ . **q.e.d.**

**Lemma 3:** *With a foreign debt of zero, identical technologies and a perfectly integrated capital market, a region with a higher population-growth rate needs higher per-capita savings.*

**Proof:** Due to the identical technologies it must be that  $k_1^{t+1} = k_2^{t+1}$ . With a foreign debt of zero we have  $(1+n_i^{t+1})k_i^{t+1} - s_i^t = 0$ , thus

$$k_i^{t+1} = \frac{s_i^t}{1+n_i^{t+1}} \quad (17)$$

It follows

$$\frac{s_1^t}{1+n_1^{t+1}} = \frac{s_2^t}{1+n_2^{t+1}} \quad (18)$$

which proves Lemma 3. **q.e.d.**

Lemmata 2 and 3 also have a straightforward interpretation: With different population-growth rates, foreign debt must be zero from Lemma 1. Due to the integrated capital-market, the per-worker capital stock must be equal in both regions. With zero foreign debt, per-capita savings must be higher in the faster growing region in order to finance the per-worker capital stock. Hence, the ratio of savings and population growth must be equal.

In order to check the requirements for the existence of a steady-state equilibrium, we have to evaluate the first-order conditions of the individual maximisation problem evaluated in a steady-state:

$$\begin{aligned}
\frac{U_n(1)}{U_2} &= \left[ x_i^t - \frac{\tau_1 w}{1+r} \right] \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \left[ x_i^t - \frac{\tau_2 w}{1+r} \right] = \frac{U_n(2)}{U_2} \\
&\Leftrightarrow \tau^2 \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \tau^1 \\
\frac{U_3(1)}{U_2} &= \frac{1}{1+r} = \frac{U_3(2)}{U_2} \\
w &= f(k) - k f'(k) \\
1+r &= f'(k)
\end{aligned}$$

**Case 1:** Assume that  $\tau_1 = \tau_2$ . In this case, the marginal rates of substitution between the regions are equal. In addition, the budget constraints are the same for individuals in both regions due to factor-price equalisation. Thus, we get a steady-state equilibrium where individuals in both regions act in exactly the same way,  $c_{21} = c_{22}$ ,  $c_{31} = c_{32}$ ,  $n_1 = n_2$ .<sup>13</sup>

**Case 2:** Assume that  $\tau_1 \neq \tau_2$ . In this case, the marginal rates of substitution between consumption and children differ between the regions. Furthermore, due to the equalisation of factor prices, gross incomes are equal. In general we can expect that population growth rates differ except for very special preference structures. In this case, it follows from Lemma 1 that foreign debt must be equal to zero. Thus, from Lemma 3 we have that the faster growing region needs higher per-capita savings. On the other hand, *ceteris paribus*, with an existing PAYG a higher number of children leads to a higher transfer of income to the next period. Thus, there can be no substitution between consumption in both periods (the marginal rate of substitution between working life and retirement consumption is equal in both regions) and children; retirement consumption and children must be complements. This condition is rather restrictive and is violated by usual preference relations. Lemma 4 gives an example that makes the intuition precise:

**Lemma 4:** *If households have Cobb-Douglas utility functions  $U =$*

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<sup>13</sup>For the case of different regional utility functions we would get a coordination of the form  $\tau_1 = \alpha \tau_2$ .

$c_2^\alpha c_3^\beta (1+n)^\gamma$ ,  $\alpha + \beta + \gamma = 1$ , there exists no steady-state equilibrium with different PAYG contribution rates.

**Proof:** Let  $\tau_1 \neq \tau_2$ . Due to factor-price equalisation, the Marshallian demand functions can be calculated for the same wage- and interest rates but for different contribution rates. They are:

$$\begin{aligned} s_{2i} &= \beta(1 - \tau_i)w \\ (1 + n_i) &= \gamma \frac{(1 - \tau_i)w}{x_i^t - \frac{\tau_i w}{1+r}} \end{aligned}$$

From Lemma 1 and 3 it must be that  $s_1/s_2 = (1+n_1)/(1+n_2)$ . If we insert the demand functions into this condition we get  $\tau_1 = \tau_2$ , a contradiction to the assumption. **q.e.d.**

Figure 1 gives a graphical representation of the argument.  $\mathcal{C}$  measures aggregate consumption  $c_2 + \frac{1}{1+r}c_3$ .

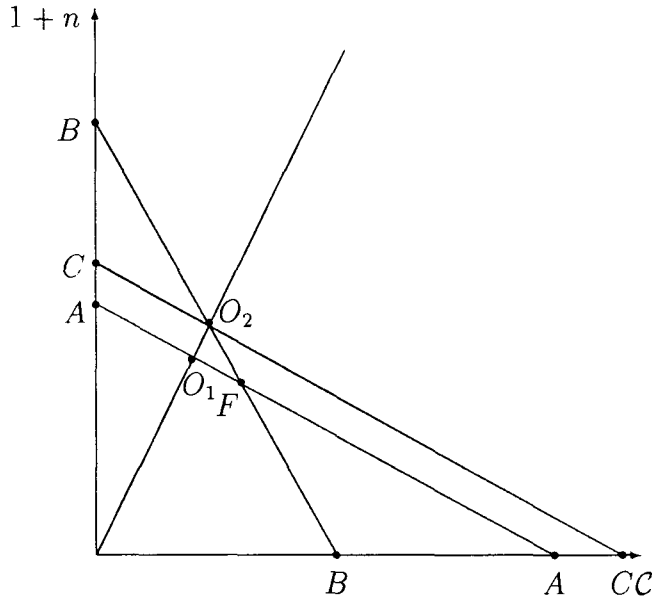


Figure 1: The non-existence of steady-state equilibria with Cobb-Douglas preferences

The income-consumption path of a Cobb-Douglas utility function is a straight line through the origin. Equation (18) also defines a straight line

through the origin in  $((1+n), c_3)$ -space. Due to factor-price equalisation, region  $j$ 's budget line can be constructed by the rotation of region  $i$ 's budget line around a straight line in  $(c_2, c_3, (1+n))$ -space (lines  $AA$  and  $BB$  and point  $F$  in figure 1). Thus, they have different slopes for different contribution rates. If you rotate the budget line, for the existence of a steady-state equilibrium the utility maximising choices must be along a straight line through the origin (points  $O_1$  and  $O_2$  in the figure). On the other hand, the utility maximising choices are a straight line for different parallel budget lines in the case of Cobb-Douglas preferences (lines  $AA$  and  $CC$  in the figure). Thus,  $O_2$  cannot be simultaneously optimal for  $BB$  and  $CC$ .

**Result 1:** *a) With harmonised PAYG-systems there exists a steady-state equilibrium. b) With non-harmonized PAYG-systems there exists no steady-state equilibrium except for special utility functions.<sup>14</sup>*

## 4 Intertemporal and Interregional Optimality

Let  $\alpha_i^t = N_i^t/N^t$  be the fraction of the total period- $t$  population living in region  $i$ . The per-capita<sup>15</sup> resource constraint of both regions is in  $t$ :

$$\sum_{i=1}^2 \alpha_i^t \left( f_i(k_i^t) - c_{2i}^t - \frac{c_{3i}^t}{(1+n_i^t)} - (1+n_i^{t+1})[x_i^t + k_i^{t+1}] \right) = 0 \quad (19)$$

A central planner maximises the utility of a representative working individual in  $t$  in region 1 subject to the following constraints: The utility of a representative working individual in  $t$  in 2 must not fall below a minimum utility level (Lagrange multiplier  $\zeta_2$ ), the utility levels of the retired individuals must not fall below a minimum utility level (Lagrange multipliers  $\mu_i$ ), the resource constraints in  $t$  and  $t+1$  (Lagrange multipliers  $\lambda$ ). Additionally,

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<sup>14</sup>One example for a special utility function is the generalized perfect-complements function  $u = \min[u_n(1+n), u_c(c_2, c_3)]$ . For this function, the elasticity of substitution between consumption and children is zero.

<sup>15</sup>In the following, the term "per capita" is used for quantities per worker. If  $\kappa_i^{t+1}$  is the capital stock per pensioner in period  $t+1$ , it follows that  $(1+n_i^{t+1})k_i^{t+1} = \kappa_i^{t+1}$ .



the working generation in  $t$  has to take care of the next (unborn) generation. This is done by choosing a minimum capital stock for the future generation in order to make sure that the production possibilities of this generation are large enough (Lagrange multipliers  $\theta_i$ ).<sup>16</sup>

$$\begin{aligned}
\mathcal{L} &= u_1^t(c_{21}^t, c_{31}^{t+1}, 1 + n_1^{t+1}) \\
&+ \zeta_2 \left( u_2^t(c_{22}^t, c_{32}^{t+1}, 1 + n_2^{t+1}) - \bar{u}_2^t \right) \\
&+ \lambda^t \sum_{i=1}^2 \alpha_i^t \left( f_i(k_i^t) - c_{2i}^t - \frac{c_{3i}^t}{(1 + n_i^t)} - (1 + n_i^{t+1})[x_i^t + k_i^{t+1}] \right) \\
&+ \lambda^{t+1} \sum_{i=1}^2 \alpha_i^{t+1} \left( f_i(k_i^{t+1}) - c_{2i}^{t+1} - \frac{c_{3i}^{t+1}}{(1 + n_i^{t+1})} - (1 + n_i^{t+2})[x_i^t + k_i^{t+2}] \right) \\
&+ \sum_{i=1}^2 \mu_i \left( u_i^{t-1}(c_{2i}^{t-1}, c_{3i}^t, 1 + n_i^t) - \bar{u}_i^{t-1} \right) \\
&+ \sum_{i=1}^2 \theta_i \left( k_i^{t+2} - \bar{k}_i^{t+1} \right) \tag{20}
\end{aligned}$$

This problem must be solved for  $X = \{c_{2i}^t, c_{3i}^t, c_{3i}^{t+1}, 1 + n_i^{t+1}, k_i^{t+1}\}_{i=1,2}$ .<sup>17</sup> The calculation of the first-order conditions of the above problem is straightforward<sup>18</sup>:

$$\zeta_i \frac{\partial u_i^t}{\partial c_{2i}^t} = \lambda^t \alpha_i^t \quad i = 1, 2 \tag{21}$$

$$\zeta_i \frac{\partial u_i^t}{\partial c_{3i}^{t+1}} = \lambda^{t+1} \frac{\alpha_i^{t+1}}{(1 + n_i^{t+1})} \quad i = 1, 2 \tag{22}$$

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<sup>16</sup>For a justification of this criterion and the general structure of normative intergenerational criteria see Kolmar and Stolte (1996).

<sup>17</sup>The derivatives of  $\alpha$  with respect to the population growth rates are

$$D\alpha^{t+1} = (\alpha_1^{t+1} \alpha_2^{t+1}) \begin{bmatrix} \frac{1}{1+n_1^{t+1}} & -\frac{1}{1+n_2^{t+1}} \\ -\frac{1}{1+n_2^{t+1}} & \frac{1}{1+n_1^{t+1}} \end{bmatrix}$$

<sup>18</sup>We introduce the additional Lagrange parameter  $\zeta_1 = 1$  as weight for the working individual in region 1.

$$\zeta_i \frac{\partial u_i^t}{\partial (1 + n_i^{t+1})} = \lambda^t \alpha_i^t (x_i^t + k_i^{t+1}) - \lambda^{t+1} \alpha_i^{t+1} \frac{c_{3i}^{t+1}}{(1 + n_i^{t+1})^2} \quad i = 1, 2 \quad (23)$$

$$\lambda^t \alpha_i^t (1 + n_i^{t+1}) = \lambda^{t+1} \alpha_i^{t+1} f'_i(k_i^{t+1}) \quad i = 1, 2 \quad (24)$$

$$\mu_i \frac{\partial u_i^{t-1}}{\partial c_{3i}^{t+1}} \leq \frac{\lambda^t}{(1 + n_i^{t+1})} \quad i = 1, 2 \quad (25)$$

As a preliminary result we can conclude from (24), using  $\frac{\alpha_i^t}{\alpha_i^{t+1}} = \frac{(1+n^{t+1})}{(1+n_i^{t+1})}$ , that:

$$(1 + n^{t+1}) = \frac{\lambda^{t+1}}{\lambda^t} f'_i(k_i^{t+1}) \quad i = 1, 2 \quad (26)$$

This condition defines a link between the optimal population growth and the optimal productivity.  $\lambda_t$  is the marginal (indirect) utility of an increase in productivity. In a steady-state equilibrium ( $\lambda^{t+1} = \lambda^t$ ), all variables grow according to the so-called *golden rule*,  $(1 + n^{t+1}) = f'_i(k_i^{t+1})$ . In a two-country world, the rule implies that the *regional* marginal productivity of per-capita capital has to be equal to the *average* population growth factor. This condition gives a first intuition into the strategic aspects of decentralized policy-making: For the sake of simplicity let  $\lambda^{t+1} = \lambda^t$  and assume that  $(1 + n_i^{t+1}) > (1 + n_j^{t+1})$ . In this case we get:

$$\begin{aligned} (1 + n_i^{t+1}) &> (1 + n^{t+1}) = f'_i(k_i^{t+1}) \\ (1 + n_j^{t+1}) &< (1 + n^{t+1}) = f'_j(k_j^{t+1}) \end{aligned}$$

An optimal policy requires that one region has a marginal productivity below the the regional population growth rate. Thus, the virtual interest rate of a regional PAYG exceeds the capital-market interest rate. This is the so-called Aaron-condition<sup>19</sup> for a steady-state welfare-improving increase in intergenerational redistribution. Therefore, with regional pension systems, the Aaron

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<sup>19</sup>See Aaron (1966).

condition predicts that an increase in the contribution rate could increase utility in this region.

The solution of the optimisation problem boils down to the following marginal rates of substitution (*MRS*):

1. Intertemporal Optimality:

$$MRS_3^2(i) = f'_i(k_i^{t+1}) \quad i = 1, 2 \quad (27)$$

$$MRS_2^n(i) = x_i^t + k_i^{t+1} - \frac{1}{f'_i(k_i^{t+1})} \frac{c_{3i}^{t+1}}{(1 + n_i^{t+1})} \quad i = 1, 2 \quad (28)$$

Equation (27) needs no further explanation. The first two terms in (28) measure the costs of an additional child holding the capital-labor ratio constant. The third term measures the discounted return of an additional child. Therefore, the optimal “price” of an additional child is equal to its direct costs  $x_i^t$  if and only if the retirement consumption of a given generation is entirely financed by the capital accumulation of this generation,  $\kappa_i^{t+1} = c_{3i}^{t+1} / f'_i(k_i^{t+1})$ . The condition is intuitive: Without any intergenerational transfer, any future consumption must be financed by savings via capital. If the optimal retirement consumption - for example due to a low rate of time preference - requires too much capital accumulation, this overaccumulation of capital can be avoided by intergenerational transfers from young to old individuals. In this case, the sum of the second and third term is negative. The opposite argument holds for the case of underaccumulation of capital. The only difference is that in the first period, the old generation will never have an incentive to give a transfer to the working generation.

2. Interregional Optimality

$$\frac{u_{13}^t}{u_{23}^t} = \frac{u_{12}^t}{u_{22}^t} = \frac{\zeta_2 N_1^t}{\zeta_1 N_2^t} \quad (29)$$

$$f'_1(k_1^{t+1}) = f'_2(k_2^{t+1}) \quad (30)$$

$$\frac{u_{1n}^t}{u_{2n}^t} = \frac{\zeta_2}{\zeta_1} \frac{N_1^t}{N_2^t} \frac{(x_1^t + k_1^{t+1}) - \frac{1}{f_1'(k_1^{t+1})} \frac{c_{31}^{t+1}}{(1 + n_1^{t+1})}}{(x_2^t + k_2^{t+1}) - \frac{1}{f_2'(k_2^{t+1})} \frac{c_{32}^{t+1}}{(1 + n_2^{t+1})}} \quad (31)$$

The weight  $\frac{\zeta_2}{\zeta_1} \frac{N_1^t}{N_2^t}$  measures the “importance” of region 2 for the central planner. Variations in  $\bar{u}_2^t$  change the value of this weight. The intertemporal and interregional optimality conditions are not independent of each other. On the contrary, it follows:

**Lemma 5:** *If both countries have an optimal intertemporal policy, and (30) is fulfilled, (29) and (31) are also fulfilled.*

**proof:** Follows immediately.

## 5 Interregional One-Period Equilibrium

From individual utility- and profit maximisation we get the following conditions for a one-period equilibrium in region  $i$  in period  $t$ :

1. Intertemporal conditions for an equilibrium:

$$MRS_2^n(i) = x_i^t - \frac{\tau_i(f_i(k_i^{t+1}) - k_i^{t+1} f_i'(k_i^{t+1}))}{f_i'(k_i^{t+1})} \quad i = 1, 2 \quad (32)$$

$$MRS_2^3(i) = \frac{1}{f_i'(k_i^{t+1})} \quad i = 1, 2 \quad (33)$$

$$(1 + r^{t+1}) = f_i'(k_i^{t+1}) \quad i = 1, 2 \quad (34)$$

2. Interregional conditions for an equilibrium:

$$f_1'(k_1^{t+1}) = (1 + r^{t+1}) = f_2'(k_2^{t+1}) \quad (35)$$

$$\frac{u_{13}^t}{u_{23}^t} = \frac{u_{12}^t}{u_{22}^t} \quad (36)$$

$$\frac{u_{1n}^t}{u_{2n}^t} = \frac{u_{12}^t}{u_{22}^t} \frac{x_1^t - \frac{\tau_1(f_1(k_1^{t+1}) - k_1^{t+1} f_1'(k_1^{t+1}))}{f_1'(k_1^{t+1})}}{x_2^t - \frac{\tau_2(f_2(k_2^{t+1}) - k_2^{t+1} f_2'(k_2^{t+1}))}{f_2'(k_2^{t+1})}} \quad (37)$$

The first observation is that there is no original interregional source of inefficiency in the model. If every region chooses its contribution rate  $\tau_i$  in a way to fulfill its intertemporal optimality conditions, the resulting equilibrium will be interregionally efficient:

**Lemma 6** *Every equilibrium that is intertemporally efficient for both regions is also interregionally efficient.*

**Proof:** (36) is a special case of (29). (27) and (33), as well as (30) and (35) coincide. Thus, only (28) and (32) may differ. If you choose  $\tau_i$  in a way to fulfill (28) = (32), it follows that (31) and (37) coincide. **q.e.d.**

The second observation is that a one-period equilibrium may be intertemporally inefficient. Equation (37) need not coincide with optimality condition (31). Thus, the economy may end up with an inefficient rate of population growth. This inefficiency is a result of the fact that a single individual neglects the effect of his fertility decision on the capital-market interest rate.<sup>20</sup>

As a special case we can analyse a Walrasian equilibrium (a fully funded system,  $\tau_i = 0$ ). A comparison of equilibrium- and optimality conditions shows that a Walrasian equilibrium is only efficient if

$$k_i^{t+1} = \frac{1}{f_i'(k_i^{t+1})} \frac{c_{3i}^{t+1}}{(1 + n_i^{t+1})} \quad (38)$$

is fulfilled in both regions.

**Result 2:** *A Walrasian equilibrium is efficient if and only if at the optimum, retirement consumption is completely financed through savings on capital markets.*

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<sup>20</sup>See Peters (1995) for a detailed discussion.

The result shows that the globalisation of markets is not sufficient to improve this kind of inefficiency: Even if an excess supply of capital in region  $i$  is completely absorbed by an excess demand of capital in region  $j$ , the resulting equilibrium would not be efficient because the relative price of children deviates from the optimal shadow price.

If there exists an optimal contribution rate, it must solve the following condition:

$$\frac{\tau^i(f_i(k_i^{t+1}) - k_i^{t+1}f'_i(k_i^{t+1}))}{f'_i(k_i^{t+1})} = -k_i^{t+1} + \frac{1}{f'_i(k_i^{t+1})} \frac{c_{3i}^{t+1}}{(1+n_i^{t+1})} \quad (39)$$

In order to simplify notation we will use prices in the following:

$$\frac{\tau^i w_i^{t+1}}{(1+r^{t+1})} = -k_i^{t+1} + \frac{1}{(1+r^{t+1})} \frac{c_{3i}^{t+1}}{(1+n_i^{t+1})} \quad (40)$$

If we solve the above equation we end up with the following optimal contribution rate for a PAYG in region  $i$ :

$$\tau_i^o = \frac{c_{3i}^{t+1} - (1+r^{t+1})(1+n_i^{t+1})k_i^{t+1}}{(1+n_i^{t+1})w_i^{t+1}} \quad (41)$$

The contribution rate has two objectives: It redistributes income between generations and it distorts market prices in order to internalize the interest-rate externality. Due to the proportional structure of the tax it has to be equal to the difference between the optimal retirement consumption and the optimal capital accumulation divided by the optimal wage income.

In order to get a better understanding of the structure of regional PAYGs in open economies we will first analyse an autarkic equilibrium. For such an equilibrium we have  $k_i^{t+1} = \frac{s_i^t}{(1+n_i^{t+1})}$  for  $i = 1, 2$ . The optimal contribution rate simplifies to:

$$\tau_i^o = \frac{c_{3i}^{t+1} - (1+r^{t+1})s_i^t}{(1+n_i^{t+1})w_i^{t+1}} \quad (42)$$

This equation corresponds to the budget constraint of an old individual, evaluated at the optimum. It is the percentage excess of consumption over savings. A remark on the enforcement of the optimal Pigou-tax is in order: In case

of a positive tax - the decentralized capital stock would be too high - we can expect the implementation of a PAYG. The old generation and every successor is able to increase its utility by the implementation of a PAYG-pension scheme. Especially the old generation gets a “free lunch” from the working generation at the period of implementation. In case of underaccumulation we would get the opposite result: If the economy was in a steady-state with a working PAYG-pension system, every generation would be better off. But there is a problem in the first period of implementation: The old generation would have to pay contributions without getting any return. This makes the introduction of a negative Pigou tax unlikely.

In the general case of a nonautarkic equilibrium, let 1 (2) be the capital importing region (capital exporting region). Thus,  $k_1^{t+1} > \frac{s_1^t}{(1+n_1^{t+1})}$  and  $k_2^{t+1} < \frac{s_2^t}{(1+n_2^{t+1})}$ . This implies for the optimal contribution rate:

$$\tau_1^o = \frac{c_{31}^{t+1} - (1+r^{t+1})(1+n_1^{t+1})k_1^{t+1}}{(1+n_1^{t+1})w_1^{t+1}} < \frac{c_{31}^{t+1} - (1+r^{t+1})s_1^t}{(1+n_1^{t+1})w_1^{t+1}} \quad (43)$$

$$\tau_2^o = \frac{c_{32}^{t+1} - (1+r^{t+1})(1+n_2^{t+1})k_2^{t+1}}{(1+n_2^{t+1})w_2^{t+1}} > \frac{c_{32}^{t+1} - (1+r^{t+1})s_2^t}{(1+n_2^{t+1})w_2^{t+1}} \quad (44)$$

The capital importing region has a contribution rate that is smaller than the one that would follow in autarky and vice versa. For the purpose of intuition we look at the following example: Let both regions be identical except for their preference for old age consumption. We assume that region 2 has a c.p. higher preference for old age consumption, which means that, for every  $\tau$ , region 2 has a higher level of savings and a lower interest rate in an autarkic solution. Furthermore, both regions would be over-capitalized without intergenerational transfers. Opening up capital markets would induce movements of capital from region 2 to region 1. This leads to a lower interest rate in 1 and a higher interest rate in 2. This effect induces a distortion of the effective price for children. In order to balance this distortion, region 1 has to decrease and region 2 has to increase the contribution rate, because 2 is to a larger extent “responsible” for the distortion.

## 6 Strategic Aspects of Decentralized Pension Systems

As a next step we will ask whether there are regional incentives to set contribution rates strategically. The conclusion from the last section would be that regional policies have to be coordinated, but can remain within the authority of local governments. This conclusion is only correct if one can show that decentralized authorities do in fact have the right incentives to set tax rates according to (43) and (44).

In order to answer this question it is necessary to specify the optimal regional strategy. We assume Nash-behavior of both regional governments in the determination of contribution rates. We can think of two different regional policies: 1. A change in the contribution rate only affects the benefits of the currently working generation. 2. A change in the contribution rate affects the benefits of the generation currently working as well as the benefits of the currently retired generation. We will analyse the second case in the following.

Without loss of generality we will analyse the decision problem of a representative working individual in region 1 in period  $t$ . He maximises his indirect utility function  $V_1^t(\tau_1, \tau_2)$  by the choice of  $\tau_1$  subject to the capital-market equilibrium constraint. Using the envelope theorem and  $(\partial w_1^{t+1})/(\partial \tau_1) = (\partial w_1^{t+1})/(\partial r^{t+1}) * (\partial r^{t+1})/(\partial \tau_1) = -k_1^{t+1}(\partial r^{t+1})/(\partial \tau_1)$ , we get:

$$\frac{\partial V_1^t}{\partial \tau_1} = \mu_1 \left( w_1^{t+1}(1 + n_1^{t+1}) - w_1^t(1 + r^{t+1}) + (s_{21}^t - (1 + n_1^{t+1})\tau_1 k_1^{t+1}) \frac{\partial r^{t+1}}{\partial \tau_1} \right) \leq 0 \quad (45)$$

$$\frac{\partial V_2^t}{\partial \tau_2} = \mu_2 \left( w_2^{t+1}(1 + n_2^{t+1}) - w_2^t(1 + r^{t+1}) + (s_{22}^t - (1 + n_2^{t+1})\tau_2 k_2^{t+1}) \frac{\partial r^{t+1}}{\partial \tau_2} \right) \leq 0 \quad (46)$$

The (endogenous) effect of the interest rate can be determined by totally differentiating the condition for capital-market equilibrium:

$$\Psi = \alpha_1^{t+1} \left( k_1^{t+1} - \frac{s_1^t}{1 + n_1^{t+1}} \right) + \alpha_2^{t+1} \left( k_2^{t+1} - \frac{s_2^t}{1 + n_2^{t+1}} \right) = 0 \quad (47)$$

The first two terms in (45) and (46) are the regional Aaron conditions for the optimal contribution rate of a PAYG. The third term measures the influence



of the change of the interest rate. It consists of the change in interest payments for savings,  $s_{21}^t$ , and the change in pension payments due to the change of the wages,  $-\tau_1(1 + n_1^{t+1})k_1^{t+1}$ . The sign of this expression is ambiguous in general.

In order to determine strategic incentives for the regional determination of contributions it is easier to construct a reference model of the determination of optimal contribution. This is done by the simultaneous determination of both contribution rates and the maximisation of region 1's indirect utility subject to a minimum-utility constraint for region 2. Again, this maximisation has to take the capital-market equilibrium-condition into account. The optimisation problem reads as follows:

$$\mathcal{W}(\tau_1, \tau_2) = V_1^t(\tau_1, \tau_2) + \xi(V_2^t(\tau_1, \tau_2) - \bar{V}_2^t) \quad (48)$$

We first have to prove a simple result on the equivalence of the direct and the indirect optimisation problem:

**Lemma 7:**  $(\tau_1^o, \tau_2^o) \in \operatorname{argmax} \mathcal{W}(\tau_1, \tau_2)$  for  $\bar{V}_2^t = \bar{U}_2^t$ .

**Proof:** Let  $X(\tau_1, \tau_2)$  be the vector of Marshallian reaction functions of the decentralized optimisation problem and  $X^*$  be the optimal allocation for a given  $\bar{U}_2^t$ . It follows that  $X(\tau_1^o, \tau_2^o) = X^*$ . Thus,  $U_i^t(X^*) = U_i^t(X(\tau_1^o, \tau_2^o)) := V(\tau_1^o, \tau_2^o)$ . **q.e.d.**

Using the Envelope-theorem we get the following first-order conditions of the above problem:

$$\begin{aligned} \frac{\partial \mathcal{W}}{\partial \tau_1} = & \left[ \mu_1 \left( w_{21}^{t+1}(1 + n_1^{t+1}) - w_{21}^t(1 + r^{t+1}) \right) \right. \\ & \left. + \left( \mu_1(s_{21}^t - (1 + n_1^{t+1})\tau_1 k_1^{t+1}) + \xi \mu_2(s_{22}^t - (1 + n_2^{t+1})\tau_2 k_2^{t+1}) \right) \frac{\partial r^{t+1}}{\partial \tau_1} \right] \quad (49) \end{aligned}$$

$$\begin{aligned} \frac{\partial \mathcal{W}}{\partial \tau_2} = & \left[ \xi \mu_2 \left( w_{22}^{t+1}(1 + n_2^{t+1}) - w_{22}^t(1 + r^{t+1}) \right) \right. \\ & \left. + \left( \mu_1(s_{21}^t - (1 + n_1^{t+1})\tau_1 k_1^{t+1}) + \xi \mu_2(s_{22}^t - (1 + n_2^{t+1})\tau_2 k_2^{t+1}) \right) \frac{\partial r^{t+1}}{\partial \tau_2} \right] \quad (50) \end{aligned}$$

The equilibrium condition is again

$$\Psi = \alpha_1^{t+1} \left( k_1^{t+1} - \frac{s_1^t}{1 + n_1^{t+1}} \right) + \alpha_2^{t+1} \left( k_2^{t+1} - \frac{s_2^t}{1 + n_2^{t+1}} \right) = 0$$

A comparison of these conditions with the conditions for a decentralized Nash-equilibrium shows that the decentralized determination of PAYG-contribution rates is in general not optimal. This specifies the following Lemma:

**Lemma 8:** *Necessary conditions for the optimality of the decentralized Nash equilibrium are alternatively: i) The contribution rate is compatible with the following condition:  $s_{2i}^t = \tau_i(1 + n_i^{t+1})k_i^{t+1}$   $i = 1, 2$  and  $\xi = 1$ . ii) The interest rate does not react to changes in the contribution rates ( $\frac{\partial(1+r^{t+1})}{\partial\tau_i} = 0$ ) and  $\xi = 1$ .*

In order to analyse condition i), we distinguish two cases:

1. autarkic equilibrium:

In an autarkic equilibrium it must be that  $s_{2i}^t = (1 + n_i^{t+1})k_i^{t+1}$  for both regions. Thus, it must be that  $\tau_i = \frac{s_{2i}^t}{(1+n_i^{t+1})k_i^{t+1}} = 1$ . From the individual budget constraint and  $c_{2i}^t \geq 0$ ,  $(1 + n_i^{t+1}) \geq 0$  and  $c_{2i}^t + x_i^t(1 + n_i^{t+1}) = -s_{2i}^t = -(1 + n_i^{t+1})k_i^{t+1}$  it can be deduced that per-capita savings, and thus the per-capita stock of capital have to be smaller or equal to zero. The only allocation that is compatible with this restriction is  $(0, 0, 0)$ . This cannot be optimal.

2. nonautarkic equilibrium:

Let region 1 be the capital-importing region. Thus,  $s_{21}^t < (1 + n_1^{t+1})k_1^{t+1}$  and  $s_{22}^t > (1 + n_2^{t+1})k_2^{t+1}$ . It follows that:

$$\tau_1 = \frac{s_{21}^t}{(1 + n_1^{t+1})k_1^{t+1}} < 1$$

$$\tau_2 = \frac{s_{22}^t}{(1 + n_2^{t+1})k_2^{t+1}} > 1$$

If we exclude a contribution rate higher than 100 %, this case cannot exist.

ii) We do not characterize the technological constraints necessary to get a fixed interest rate. We rather continue to analyse the strategic structure of decentralized policy-making for this case:

## 6.1 Strategic incentives with an exogenous interest rate

This section is devoted to the simplest case of an exogenous interest rate. The first order conditions for the optimal contribution rates simplify substantially:

$$\frac{\partial V_1^t}{\partial \tau_1} = \mu_1 \left( w_1^{t+1}(1 + n_1^{t+1}) - w_{21}^t(1 + r^{t+1}) \right) := \mu_1 A_1 \quad (51)$$

$$\frac{\partial V_2^t}{\partial \tau_2} = \mu_2 \left( w_2^{t+1}(1 + n_2^{t+1}) - w_2^t(1 + r^{t+1}) \right) := \mu_2 A_2 \quad (52)$$

$$\Psi = \alpha_1^{t+1} \left( k_1^{t+1} - \frac{s_1^t}{1 + n_1^{t+1}} \right) + \alpha_2^{t+1} \left( k_2^{t+1} - \frac{s_2^t}{1 + n_2^{t+1}} \right) = 0$$

$A_i$  is the abbreviation of the regional Aaron-index. From the structure of the matrix of regional second-order conditions we can show that every interior extremum must be a minimum:

$$H = \begin{bmatrix} \mathcal{W}_{\tau_1 \tau_1} & \mathcal{W}_{\tau_1 \tau_2} \\ \mathcal{W}_{\tau_2 \tau_1} & \mathcal{W}_{\tau_2 \tau_2} \end{bmatrix} = \begin{bmatrix} \mu_1 w_1^{t+1} \frac{\partial(1+n_1^{t+1})}{\partial \tau_1} & 0 \\ 0 & \mu_2 w_2^{t+1} \frac{\partial(1+n_2^{t+1})}{\partial \tau_2} \end{bmatrix} \quad (53)$$

We will evaluate  $H$  for the extrema of  $V_i$ . According to (51) and (52), they are reached if  $A_i = 0$ . The first principal minor  $\mathcal{W}_{\tau_1 \tau_1} = \mu_1 w_1^{t+1} \frac{\partial(1+n_1^{t+1})}{\partial \tau_1}$  is weakly positive: The sign of this condition depends on the derivative of  $1 + n_i$  with respect to  $\tau_i$ . If we look at the intertemporal budget constraint of a household working in  $t$ , we can define the effective price  $P_i$  of a (marginal) child and the effective income  $M_i$ :

$$(1 + r^{t+1})c_{2i}^t + c_{3i}^{t+1} + \underbrace{((1 + r^{t+1})x_i^t - \tau_i w_i^{t+1})}_{=P_i} (1 + n_i^{t+1}) = \underbrace{(1 + r^{t+1})(1 - \tau_i)w_i^t}_{=M_i}$$

Using the Slutsky decomposition, we get:<sup>21</sup>

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<sup>21</sup>The general form of the Slutsky equation is for  $q_i$  being an element of  $\{c_2^t, c_3^{t+1}, n^{t+1}\}$ :

$$\frac{\partial q_i}{\partial \tau} = \frac{\partial q_i}{\partial M} \left( \frac{\partial M}{\partial \tau} - \sum_{j=1}^3 q_j \frac{\partial p_j}{\partial \tau} \right) + \sum_{j=1}^3 \left[ \frac{\partial q_i}{\partial p_j} \right]_{\bar{u}} \frac{\partial p_j}{\partial \tau}$$

$$\begin{aligned}
\frac{\partial(1+n_i^{t+1})}{\partial\tau_i} &= \frac{\partial(1+n_i^{t+1})}{\partial P_i} \frac{\partial P_i}{\partial\tau_i} + \frac{\partial(1+n_i^{t+1})}{\partial M_i} \frac{\partial M_i}{\partial\tau_i} \\
&= -w_i^{t+1} \left[ \frac{\partial(1+n_i^{t+1})}{\partial P_i} \right]_{\bar{u}} \\
&\quad + \underbrace{\left( (1+n_i^{t+1})w_i^{t+1} - w_i^t(1+r^{t+1}) \right)}_{=0} \frac{\partial(1+n_i^{t+1})}{\partial M_i}.
\end{aligned}$$

Because the effect of substitution is always non-positive, the first principal minor is weakly positive. By the same argument, the determinant of  $H$  is also (weakly) positive,  $\mathcal{W}_{\tau_1\tau_1}\mathcal{W}_{\tau_2\tau_2} = \mu_1\mu_2w_{21}^{t+1}w_{22}^{t+1}\frac{\partial(1+n_1^{t+1})}{\partial\tau_1}\frac{\partial(1+n_2^{t+1})}{\partial\tau_2} \geq 0$ . Thus, the matrix of second-order effects of  $\mathcal{W}$  is positive (semi-) definite.

**Lemma 9:** *With an exogenous interest rate, every interior extremum must be a minimum.*

From the individual budget constraint it can be concluded that  $\tau = 1$  must be a minimum of the problem, because the demand for all goods must be 0. Thus,  $V_i(1, 1) = U_i(0, 0, 0)$ . It follows that  $\tau_i = 0$  must be the maximum of  $V_i$ . Two consequences follow from the argument:

**Result 3:** *With an exogenous interest rate, the optimal contribution rates for a PAYG are zero in both regions.*

The result is intuitive: The contribution rate should distort the effective price of children in order to internalize the interest-rate externality. With an exogenous interest rate there is no externality, and therefore no need to correct prices. The optimal stock of capital can be determined such that the optimal old-age consumption is completely financed out of private savings. But what about dynamic efficiency? One of the major advantages of a PAYG-pension

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This expression simplifies to:

$$\frac{\partial q_i}{\partial\tau} = \frac{n-r}{1+r} \frac{\partial q_i}{\partial M} - \frac{w_2}{1+r} \left[ \frac{\partial q_i}{\partial P} \right]_{\bar{u}}$$

system in models with exogenous fertility is that it may avoid dynamic inefficiencies and thus Pareto-dominates the Walrasian equilibrium. Result 3 states that this function of PAYG-pension systems disappears in models with endogenous fertility. The proof of the next result shows why:

**Result 4:** *With endogenous fertility and an exogenous interest rate, any Walrasian equilibrium is dynamically efficient.*

**Proof:** Due to the fact that  $\tau_i = 1$  is a minimum, and every other interior extremum must also be a minimum and due to the continuity of all functions, it follows that  $V_i$  is (weakly) monotonous in  $\tau_i$ . This implies  $w_i^{t+1}(1 + n_i^{t+1}) - w_i^t(1 + r^{t+1}) \leq 0$ . In a steady-state equilibrium with constant per-capita income it follows furthermore, that  $w_i^{t+1} = w_i^t$ , and thus  $(1 + r^{t+1}) \geq (1 + n_i^{t+1})$ . **q.e.d.**

Next, we will answer the question whether the optimal contribution rates  $\tau_1 = 0, \tau_2 = 0$  will be realized in a Nash-equilibrium.

Due to the (weak) convexity of  $V$  we can restrict attention to the corner solutions  $\tau_i = 0$  und  $\tau_i = 1$  as candidates for an equilibrium.

Assume region  $j$  chooses  $\tau_j = 1$ , what is a best answer of region  $i$ ? As for the derivation of the optimum,  $\tau_i = 1$  must be a minimum because all demands must be 0. Thus,  $\tau_i = 0$  is a best answer of  $i$ . Assume region  $j$  chooses  $\tau_j = 0$ . In this case,  $\tau_i = 1$  is a best answer, if savings in region  $i$  are smaller or equal to the savings in region  $j$  and the possible revenue from arbitrage in  $i$  (the Aaron-condition) for  $\tau_i = 1$  exceeds the costs of arbitrage (the costs of the additional children). This sufficient condition follows from the comparison of individual budget constraints in region  $i$  for a contribution rate of zero and one:

$$\tau_i = 1 \Leftrightarrow w_i^{t+1}(1 + n_{i,\tau_i=1}^{t+1}) - w_i^t(1 + r^{t+1}) > (n_{i,\tau_i=1}^{t+1} - n_{i,\tau_i=0})x_i^t(1 + r^{t+1}) \quad (54)$$

The savers in  $j$  are indifferent between investments in capital or in foreign pensions because, due to arbitrage, both assets have to pay the same amount of interest. This behavior is compatible with an equilibrium on capital markets, as the supply of savings in  $j$  exceeds the demand of savings in  $i$ . We summarize with the following result:

**Result 5:** a) *With an exogenous interest rate, there exists a Nash-equilibrium  $\tau_i = 1, \tau_j = 0$ , if i) the supply of savings in region  $j$  meets the demand in  $i$  and ii) the Aaron-index in  $i$  exceeds the cost-differential of additional children. b) If the Aaron-condition in  $i$  does not exceed the cost-differential of additional children,  $\tau_i = 0, \tau_j = 0$  is the only Nash-equilibrium of the game.*

**Result 6:** *If the Aaron-condition in one region exceeds the cost-differential for additional children in that region, and the capital market equilibrium exists, there exist strategic incentives for this region to deviate from the optimal PAYG-structure.*

## 6.2 Decentralized equilibrium with an endogenous interest rate

With an endogenous interest rate, this variable is an additional source of strategic interaction. We assume in the following that there exists an interior Nash-equilibrium as well as an interior interregional optimum. We look at a tuple  $(\tau_1^N, \tau_2^N)$  for which (45) and (46) are fulfilled. We analyse the condition for the first country:

$$\begin{aligned} \frac{\partial V_1^t}{\partial \tau_1} &= \mu_1 \left( w_1^{t+1}(1 + n_1^{t+1}) - w_1^t(1 + r^{t+1}) + (s_{21}^t - (1 + n_1^{t+1})\tau_1 k_1^{t+1}) \frac{\partial r^{t+1}}{\partial \tau_1} \right) \leq 0 \\ &\alpha_1^{t+1} \left( k_1^{t+1} - \frac{s_1^t}{1 + n_1^{t+1}} \right) + \alpha_2^{t+1} \left( k_2^{t+1} - \frac{s_2^t}{1 + n_2^{t+1}} \right) = 0 \end{aligned}$$

Starting at such a Nash equilibrium, we calculate the necessary deviations of  $(\tau_1, \tau_2)$  in order to get closer to the regional optimum. Inserting  $(\tau_1^N, \tau_2^N)$  into (45), we get:

$$\frac{\partial \mathcal{W}}{\partial \tau_1}(\tau_1^N, \tau_2^N) = \xi \mu_2 (s_{22}^t - (1 + n_2^{t+1})\tau_2 k_2^{t+1}) \frac{\partial r^{t+1}}{\partial \tau_1}$$

The following four cases have to be distinguished:

1. Region 2 may be a) a capital-exporting  $(s_{22}^t - (1 + n_2^{t+1})k_2^{t+1} > 0)$  or b) a capital importing  $(s_{22}^t - (1 + n_2^{t+1})\tau_2 k_2^{t+1} < 0)$  region.

2. The interest rate may be a) increasing ( $\frac{\partial r^{t+1}}{\partial \tau_1} > 0$ ) or b) decreasing ( $\frac{\partial r^{t+1}}{\partial \tau_1} < 0$ ) in  $\tau_1$ .

We first look at a capital-exporting region 2. From  $s_{22}^t - (1 + n_2^{t+1})k_2^{t+1} > 0$  it follows  $s_{22}^t - (1 + n_2^{t+1})\tau_2 k_2^{t+1} > 0$  and thus  $\frac{\partial W}{\partial \tau_1} > 0 \Leftrightarrow \frac{\partial(1+r^{t+1})}{\partial \tau_1} > 0$  and vice-versa. The Nash-equilibrium tax-rate  $\tau_1^N$  will deviate from the optimum tax rate in a way that the capital-importing region 1 reaches a rate of interest that is below the optimum. This implies an increase (decrease) of the tax rate if and only if the interest rate is negatively (positively) related to the tax rate.  $\tau_i$  should work as a Pigou-tax to internalize the interest-rate externality. With decentralized policy-making, a capital-importing region may reduce future foreign claims by lowering the interest rate. Thus, from a regional perspective, there exists a trade-off between the internalisation of the fertility caused externality and the reduction of future foreign debt.

**Result 7:** *A capital-importing region with a positive (negative) relation between the PAYG-contribution rate and the interest rate has an incentive to choose a lower (higher) contribution rate than optimal.*

## 7 Consequences for Public Pension Systems within the European Union

The discussion of the consequences of the theoretical findings will proceed in two steps. First, the relevance of the inefficiencies for the process of European integration will be discussed. Second, we will discuss some policy implications that are in accordance with the principle of subsidiarity.

Article B of the Treaty of Maastricht (ToM) defines targets for the union that must be used to derive normative criteria according to which institutional and political structures can be judged. One of these targets is the balanced and persistent growth of all the member states the union. This target cannot be reduced to the common criterion of (Pareto-) efficiency. It must therefore be analysed as a complementary and independent goal. Balanced and persistent growth therefore have to be interpreted as the requirement to set policy

variables in accordance with the existence of a steady-state equilibrium. If this view is accepted, there is a strong argument for the coordination, harmonisation or even consolidation of contribution rates of the public pension systems.

On the other hand, decentralized policy making tends to induce incentives at the local level for setting inefficient contribution rates. Information asymmetries aside, this problem could be solved if the national pension systems are converted towards a European pension system. This solution is unsatisfactory for several reasons. First, information asymmetries cannot be neglected without justification. Individuals may differ in their degrees of time-preference. This information is necessary for the calculation of optimal Pigou-taxes. It is much easier to gather this information for local governments than for a centralized European pension fund. Information asymmetries are also important for political-economic reasons. A centralized system tends to be more inefficient and more open for interest group influence than a decentralized system. Thus, following the principle of subsidiarity, we have to look for the minimum coordination requirements compatible with efficiency.

The existing regulations 1408/71, 574/72 and 1274/92 in the field of public pension coordination are exclusively concerned with the calculation of benefits for migrating workers. Thus, so far there is no regulation for the kinds of inefficiencies worked out in this paper. What are the sources of the inefficiencies?

#### **Level externality:**

For an exogenous interest rate, it is the unilateral possibility to finance part of the national pensions via foreign debt. In the ToM, certain criteria are defined for the evolution of public debt. These criteria exclude the *hidden* debt present in all PAYG-financed social security systems. The possibility to avoid higher taxation or cuts in the benefits of social security increases the pressure to implement new “intergenerational contracts”<sup>22</sup> Thus, this hidden public debt should be included in the stabilisation criteria for national governments. Alternatively, an upper bound could restrict the level of foreign debt a country may have, a possibility that is clearly in conflict with the spirit of European integration.

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<sup>22</sup>In Germany, for example, a new PAYG-system, the compulsory long term care insurance, has been introduced in 1994.



## Interest-rate externality

With an endogenous interest rate it is a-priori unclear whether debt limits can avoid strategic incentives. The incentives for a net borrower and a positive relation between contribution rate and interest rate point exactly into the opposite direction. Such a country tends to reduce the level of PAYG-financed systems, thus reduces the implicit stock of public debt. This incentive is nevertheless strategic. The convergence criteria in the ToM concerning long-term interest rates does not seem to be adequate to solve the problem: Even with a common currency there are incentives for the countries to have some influence on the interest rate. Thus, a specific mechanism is needed that allows for the internalisation of this externality.

As for all externalities, changes of PAYG-contribution rates can be accompanied by transfer payments between the member states. These transfer payments must take the form of expected-externality payments as in d'Aspremont and Grard-Varet (1979) or Laffont and Maskin (1977) where each country receives or has to pay the net-expected externality on the other member states caused by its policy.<sup>23</sup> This system of incentive payments is compatible with the existing institutions in the EU. For example the European Regional and Social Funds could administer these payments if the objectives for contributions and transfers can be modified in a way as to be compatible with the incentive-mechanism. Thus, such a mechanism is the weakest form of central intervention compatible with efficiency.

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<sup>23</sup>We do not want to go into the details of the restrictive assumptions needed for the existence of such a mechanism.

## 8 References

- Aaron, H.J. (1966): "*The Social Insurance Paradox*". Canadian Journal of Economics and Political Science 32: 371-374.
- Becker, G. (1960) : "*An Economic Analysis of Fertility*" in: Easterlin, R.A. (ed.): Demographic and Economic Change. Princeton: Princeton University Press.
- Bental, B. (1989): "*The Old Age Security Hypothesis and Optimal Population Growth*". Journal of Population Economics 1, S. 285-301.
- Breyer, F. and M. Kolmar (1995): *Does the Common Labor Market Imply the Need for a European Public Pension Scheme?*, Discussion Paper 244, Sonderforschungsbereich 178, Universitaet Konstanz.
- Breyer, F. and M. Kolmar (1996): "*Social Policy in a Common Market: Labor Market, Social Arbitrage, and Public Finance*" in : R. Holzmann (Ed.), Maastricht: Monetary Constitution without a Fiscal Constitution?, 129-152. Nomos Verlag, Baden-Baden.
- Cigno, A. (1986): "*Fertility and the Tax-Benefit System: A Reconsideration of the Theory of Family Taxation*". The Economic Journal 96, 1035-1051.
- d'Aspremont, C. and L.A. Grard-Varet (1979): "*Incentives and Incomplete Information*". Journal of Public Economics 11, 25-45.
- Eckstein, Z. and K.I. Wolpin (1985): "*Endogenous fertility and Optimal Population Size*". Journal of Public Economics 27: 93-106.
- Felderer, B. (1992): "*Does a Public Pension Sceme Reduce Savings Rates and Birth Rates?*". Journal of Institutional and Theoretical Economics 148: 314-325.
- Homburg, S. and W. Richter (1993): "*Harmonizing Public Debt and Pension Schemes in the European Community*". Journal of Economics, Supplementum 7: 51-63.

- Kolmar, M. (1997): *“Intergenerational Redistribution in a Small open Economy with Endogenous Fertility”*. Forthcoming, Journal of Population Economics.
- Kolmar, M. and K. Stolte (1996): *“The Illusion of Intergenerational Preference Aggregation: Limits of Individualistic Population Ethics”*. Discussion Paper, University of Konstanz.
- Laffont, J.J. and E. Maskin (1977): *“A differential Approach to Expected Utility Maximizing Mechanisms”*, in: J.J. Laffont (Ed.): *Aggregation and Revelation of Preferences*, North-Holland, 289-308.
- Lapan, H.E. and W. Enders (1990): *“Endogenous Fertility, Ricardian Equivalence, and Debt Management Policy”*. Journal of Public Economics 41, 227-248.
- Nishimura, K. and J. Zhang (1992): *“Pay-As-You-Go Public Pensions with Endogenous Fertility”*. Journal of Public Economics 48, 239-258.
- Persson, T. (1985): *“Deficits and Intergenerational Welfare in Open Economies”*. Journal of International Economics 19, 67-84.
- Peters, W. (1995): *“Public Pensions, Family Allowances and Endogenous Demographic Change”*. Journal of Population Economics 8, 161-183.
- Raffelhueschen B. (1996): *Comment on the Paper: Social Policy in a Common Market: Labor Market, Social Arbitrage, and Public Finance*, in: R. Holzmann (Ed.), Maastricht: *Monetary Constitution Without a Fiscal Constitution?*, 159-161. Nomos Verlag, Baden-Baden.
- Raut, L.K. (1991): *“Capital Accumulation, Income Distribution and Endogenous Fertility in an Overlapping Generations General Equilibrium Model”*. Journal of Development Economics 34, 123-150.
- Razin, A., Sadka, E. (1995): *“Population Economics”*. MIT-Press, Cambridge, Massachusetts.
- Samuelson, P.A. (1975): *“The Optimum Growth Rate for Population ”*. International Economic Review 16, 531-538.

- Schweizer, U. (1995): "*Endogenous Fertility and the Henry George Theorem*".  
Forthcoming, Journal of Public Economics.
- Straub, M. and Wenig, A. (1984): "*Human Fertility and the Distribution of Wealth*". In: Steinmann, G. (ed.): Economic Consequences of Population Change in Industrialized Countries. Springer Verlag, Heidelberg, New York.
- Verbon, H.A.A. (1990): "*Social Insurance and the Free Internal Market*."  
European Journal of Political Economy 5: 487-500.
- Wigniolle, B. (1995): "*Fertility, Intergenerational Transfers and Economic Development*". MAD Université de Paris I, discussion paper.
- Wildasin, D.E. (1990): "*Non-Neutrality of Debt with Endogenous Fertility*".  
Oxford Economic Papers, 414-428.
- Wildasin, D.E. (1991): "*Income Redistribution in a Common Labor Market*".  
American Economic Review: 757-774.
- Wildasin, D.E. (1994): "*Income Redistribution and Migration*" , Canadian Journal of Economics 27, 637-656.