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A game theoretical analysis of household migration decisions in a static and deterministic world

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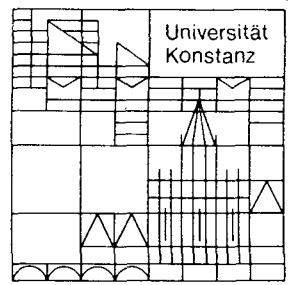
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Sonderforschungsbereich 178
„Internationalisierung der Wirtschaft“

Diskussionsbeiträge



Juristische
Fakultät

Fakultät für Wirtschafts-
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of Household Migration Decisions
in a Static and Deterministic World**

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A GAME THEORETICAL ANALYSIS
OF HOUSEHOLD MIGRATION DECISIONS
IN A STATIC AND DETERMINISTIC WORLD

Siegfried Berninghaus
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Serie II - Nr. 29



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O. INTRODUCTION

This paper contributes to the decision-theoretical foundation of migration behavior. Following an old tradition in microeconomic theory, past economic studies in migration theory did not distinguish between personal and family decisions. One important exception is the analysis of Mincer (1978).

The main conclusion in Mincer's paper are: There exists an influence of family ties on migration, and such ties result in negative personal externalities that are usually internalized by the family and thus tend to discourage migration. "Tied persons" in the family are "those whose gains from migration are (in absolute value) dominated by gains (or losses) of the spouse", (Mincer (1978), ; p. 753). Moreover such ties tend to reduce the employment and earnings of those wives who do migrate and to increase the employment and earnings of their husbands. Increased labor force participation rates of women cause an increase in migration ties, which results both in less migration and more marital instability. Mincer's results were supported by his own empirical findings, and by those of Graves and Linneman (1979) and Sandell (1977) as well.

Unfortunately Mincer's paper is lacking of a satisfactory theoretical base. He utilizes the well-known (deterministic) "Human Capital Model" of migration (see Sjaastad (1963), Cebula (1979)), modified by a "divorce constraint" which has been introduced by G. Becker into Human Capital models of marriage. According to our viewpoint there are two points missing in Mincer's sketchy theoretical analysis:

- Typically imperfect information about the destination region and thus costs of acquiring information (in a very

broad sense) play a crucial role in the migration decision: (this point has been emphasized recently by several authors e.g. David (1972), McCall/McCall (1984), Meyer (1984); see for this point also Berninghaus/Seifert-Vogt (1987)).

- A migration decision in a household consisting of more than one person who is authorized to decide is typically the solution of a conflict situation, at least in those cases where the net gains of migration for these persons are not all positive. (According to Mincer this is the real interesting aspect in the analysis of the influence of family ties on migration. But Mincer does not use the convenient analytical tool for analysing such conflict situations, namely game theoretical methods.)

In this paper we will make a first step towards a game theoretical foundation of the household migration decision. In this paper we omit the imperfect information aspect - for analytical convenience. An analysis taking regard of both of the above mentioned aspects will be deleted to a subsequent paper.

More precisely we will analyze the following decision problem: Consider a couple living in region h . Suppose the husband gets a job offer from a foreign country g . Then the husband has to decide whether to stay in h or to leave h for working in g . In case of migration the wife has to decide whether to join her husband for g or to stay in h . As it has been noticed before we will suppose complete information about all relevant economic variables in g and h .

Our goal then is to determine the (game theoretical) solution of this two-person decision problem, and to study the effects of varying exogenous parameters on this solution. Therefore we need a careful analysis of the

household's consumption decisions in the different situations: In the dissolution case we must specify how the income is divided between the two persons; given the respective income fractions each person will choose then a consumption plan which maximizes his individual utility subject to the relevant budget restriction. In the two marital cases the household is assumed to decide for a common consumption plan after some bargaining process; (our model follows the approaches of Manser and Brown (1980) and McElroy and Horney (1981)). Concerning this issue we met an unexpected difficulty which to overcome in a satisfactory way we were unfortunately not able, namely to obtain definite comparative static results for the (Nash-) bargaining solution outcomes. (Only in a very special symmetric case we will show that this solution yields increasing (rsp. decreasing) utilities for increasing income (rsp. prices).)

We claim that our contribution yields a sound theoretical foundation of the above mentioned works on family migration decisions, above all of the analysis of Mincer. Thus our analysis has the following advantages:

- The crucial role of the dissolution threat can be made explicit in the game theoretical framework, and it can be shown, under which conditions such a threat stabilizes or destabilizes the household.
- A microeconomic foundation of the causes of family ties is given by distinguishing personal (or private) goods and household (or - within the household - public) goods, and allowing externalities in the individual utility functions.
- The influence of the intra-household income relation (which is an indicator of the labor force participation of the woman under some circumstances) on migration behavior and marital stability is made precise.
- Whereas all articles on family migration known to us

describe interesting empirical facts, completed by some more or less sketchy theoretical considerations (in this respect Mincer's work is no exception in our view) our model and results make it possible to deduce hypotheses of empirical content.

The rest of this paper is organized as follows: In section A we present a general model - the A-model - of the household migration decision and characterize the game theoretical solution in its most general form. In section B a more restrictive form of the A-model - the B-model - is introduced to obtain more detailed results concerning the effects of altering the basic economic parameters (price and income proportions) on the household migration decision. In section C we specialize the B-model by introducing specific utility functions of the Cobb-Douglas type: this enables us to deduce hypotheses about the household migration decision which would be empirically testable. A concluding section summarizes the main results of the paper, stresses its limitations and gives hints for further research in the field. - Some often used mathematical results, and above all, many of the sometimes tedious proofs of statements in the text are collected in a mathematical appendix at the end of the paper.

A. A GENERAL MODEL OF HOUSEHOLD MIGRATION DECISIONS

A.1 The Migration Game

In this section we will specify the rules and outcomes of the general model of the migration game. First of all we need a specification of the economic environment in region g and h , and a specification of the rules according to which the consumption decisions of the household in these economies is taken to satisfy the needs and wishes of its

members. Finally the individual behaviour in the various socio-economic contexts must be specified.

- (A1) a) The household consists of two persons $i=1,2$ who can decide about consumption and migration.
 b) The migration decision is supposed to be the first stage M of a three-stage game M^* .
 c) The second and third stage of M^* models the consumption decision of the household in each of the possible outcomes of the migration decision.

At first we will specify now the rules of the first stage game M .

(A2) The migration decision game M is given in extensive form by the following game tree

Figure 1

The tree of game M

The interpretation of this game tree is the following. First person 1 (husband) must decide between

- $g_1 =$ 1 stays in g
 $h_1 =$ 1 migrates to h

Only in the case of 1's choice h_1 , person 2 (wife) has to decide between:

- $g_2 =$ 2 stays separate in g
 $h_2 =$ 2 migrates together with 1 to h

The endpoints of the game tree indicate the outcomes resulting from the respective decisions.

Before we are able to specify the consumption decision

games at the endpoints of the first stage of M^* , we must specify the economic environments in region g and h .

(A3) a) In each economy there are three types of commodities

Type 0: The commodities which can be consumed by persons of the household together: these are analytically treated as public goods within the household. There are no such public goods.

Type i : ($i=1,2$): These commodities can only be consumed by person i separately; they are analytically treated as private goods for person i . There are n_i such private goods.

The respective commodity bundles are denoted by $x_0 = (x_{01}, \dots, x_{0n_0})$ and $x_i = (x_{i1}, \dots, x_{in_i})$; furthermore we write $x = (x_0, x_1, x_2)$.

b) The consumption space X_j for each type $j=0,1,2$ of commodity vectors is the nonnegative orthant of \mathbb{R}^{n_j} .

c) The economies in g and h differ with respect to the commodity price vectors p_j^r ($j=0,1,2$; $r=g,h$) and the money incomes y_i^r ($i=1,2$; $r=g,h$). These are for both persons exogenously given by

$$p_j^r > 0 \quad (j=0,1,2; r=g,h)$$

$$y_i^r > 0 \quad (i=1,2; r=g,h)$$

Remark: The "public goods" mentioned above are typically interpreted to be energy, housing, car, radio, TV, newspapers, books etc. and - if there are children in the household - all private goods and services for the children, whereas the "private goods" typically are clothing, food etc. The nonnegativity of X_j and the assumption of strict positivity of commodity prices and money incomes seems to be not very restrictive. Concerning the money incomes it should be remarked here that we

suppose inelastic labor supply in both regions. Consequently differences in money incomes are generated exclusively by wage differences.

The disposable income of the household in r now is defined as

$$y^r := y_1^r + y_2^r$$

If a dissolution of a household is at debate the economic consequences will play a crucial role. We must distinguish three cases of dissolution:

Case r : The household dissolves at region r , because 1 and 2 cannot reach any agreement how to spend the disposable income y^r such that each of the two's needs and wishes would be satisfied as good as possible; ($r=g,h$).

Case 0: The household dissolves as result of the migration decision (h_1, g_2) .

Next we have to specify the economic consequences of the dissolution of the household.

(A4): By law and/or institution there are given proportions τ_g , τ_h , and τ_0 such that in "case k " ($k=g,h,0$) for the household with disposable income y^k person 1 resp. 2 gets the income

$${}^k y_1 = \tau_k y^k \text{ resp. } {}^k y_2 = (1-\tau_k) y^k$$

with $0 \leq \tau_k \leq 1$

Next we specify the needs and wishes of the household's persons.

(A5) a) Each person i has a preference relation on the space of all "lotteries" on

$$X = X_1 \times X_2 \times X_3$$

which can be represented by a von Neumann and Morgenstern utility function u_i

b) For the restriction of u_i on X (which we will denote by u_i again) the following holds

- u_i is continuous on X
- u_i is increasing in all components of (x_0, x_1)
- u_i is strictly concave
- $u_i(0) = 0$

Remark: The first part of (A5) is needed for the application of traditional game theoretical reasoning to the game M^* (In this context a "lottery" is a discrete probability measure on X with only finitely many points of non-zero probability). The second part is in accordance with traditional household theory.

Now we are prepared to specify the second and eventually arising third stage of the game M^* .

(A6) a) Both persons are informed about the outcome of the first stage game M .

b) - At the node (h_1, g_2) each person i must choose - independently of the other - a consumption bundle $({}^i x_0, x_1)$ from his "dissolution" budget set.

$$B_i(p^r, {}^0 y_1) =: \{(x_0, x_1) \in X_0 \times X_1 : p_0^r x_0 + p_1^r x_1 \leq {}^0 y_1\}$$

- h for $i=1$

with $r = |$

L g for $i=2$

- The outcome is then the consumption bundle $({}^1 x_0, x_1, 0)$ for 1. resp. $({}^2 x_0, 0, x_2)$ for 2.

c) At the node r ($r = (g_1, g_2)$ or $r = (h_1, h_2)$) the two

persons must bargain for a common consumption vector in X according to the following rules:

- Both persons propose independently of each other a consumption vector 1x resp. 2x within the common budget set

$$B(p^r, y^r) := \{x \in X : p_0^r x_0 + p_1^r x_1 + p_2^r x_2 \leq y^r\}$$

- The outcome is then

$$r \quad {}^1x = {}^2x =: x, \text{ if the two proposals coincide}$$

|

L case r , otherwise

d) If in the second stage of M^* the case r ($r=g, h$) occurs, there is a third stage of M^* . This consists of:

- both persons are informed, that "case r " has occurred;
- each person i must choose (independently of the other) a consumption bundle from the dissolution budget set

$$B(p^r, {}^r y_1) = \{(x_0, x_1) \in X_0 \times X_1 :$$

$$p_i^r x_0 + p_i^r x_1 \leq {}^r y_1\}$$

- the choices $({}^1x_0, x_1)$ resp. $({}^2x_0, x_2)$ lead to the outcomes $({}^1x_0, x_1, 0)$ resp. $({}^2x_0, 0, x_2)$ for 1 resp. 2.

The specification of the bargaining situation under c) may seem rather restrictive at a first glance, since it supposes that the two persons must agree - if any - already in the first bargaining round. It would seem more realistic to model some sort of a sequential bargaining process with a sequence of offers and replies made over time in the course of negotiations. In a series of recent papers on bargaining - starting with a paper of Rubinstein, (1982) - the sequential bargaining approach is discussed exhaustively. This sequential approach must be contrasted

with the so-called axiomatic approach to bargaining, initiated by Nash, (1950); (for a survey of axiomatic models see Roth, (1979) and Peters, (1986)).

In the axiomatic approach a two-person bargaining game is usually defined as a pair (S, d) with a compact subset S of the nonnegative orthant in \mathbb{R}^2 and an element $d \in S$.¹ Here S represents the set of feasible utility payoffs to the players, and d the vector of utility payoffs corresponding to the disagreement outcome of the bargaining situation. A possible interpretation of such a bargaining game is: each player must propose - independently of the other - an element s^1 resp, s^2 of S yielding the outcome $s := s^1 = s^2$, if the two proposals coincide or to the outcome d otherwise.

We may apply this definition in our context by setting

$$S^r := u(B(p^r, y^r)) \quad , \quad (r=g, h);$$

i.e. the image under $u = (u_1, u_2)$ of the common budget set, and specifying d^r as the vector of utility payoffs of the certain outcome of the third stage subgame occurring in case r .

Now we refer to Sutton (1986) and Binmore et al. (1986), who have pointed out, that the sequential approach to bargaining and the axiomatic approach are "complementary" in the sense that the model of a bargaining game in the latter approach can be viewed as the 'normal form' of the model of a sequential bargaining game. Thus - loosely speaking - the relation between the two approaches is an analogous one as that between an extensive form game and its induced normal form.

We consider this relation as a justification to use here the nonsequential approach.

¹ Usually the convexity of S is still supposed. We may omit this, for reasons explained in the remark preceding Theorem 2 in the Appendix.

At the present stage of our formal argument we cannot give a full specification of what would be the disagreement point d . - Intuitively we could expect that this should be the utility vector corresponding to the solution of the third-stage subgame specified under d). But for this specification to be meaningful we still must introduce two important assumptions of game theoretical content.

(A7) Each person is completely informed about

- a) the decision structure as modelled by the game M^*
- b) the utility functions u_1 and u_2
- c) the price vectors p^r
- d) the incomes y_1^r and y_2^r
- e) the institutional parameters τ_k .

This assumption makes M^* a game of complete information. In our opinion above all the points c) and d) here are at issue. Especially regarding y_2^h it seems very hard, since it practically supposes that the wife also could receive a job offer from region h , but this will typically not be the case for the situation we try to model here where there is a decisive assymetry between the two partners.

Finally we must specify the behaviour of the persons in the modelled decisions process.

(A8) Each person i behaves according to

- EUH : the Expected Utility Hypothesis
- SP : the Concept of Subgame Perfectness
- N : the Nash Bargaining Theory
- MR : the Hypothesis of Mutual Rationality

EUH asserts that person i , confronted with the choice of an element of a subset of the set of all lotteries will choose such a lottery which maximizes his utility u_i in this subset. The concept SP^2 requires that the solution of M^*

² It was introduced by Selten (1965).

must prescribe for each person a strategy in M^* which, if restricted to the various subgames of M^* , obeys the same standards of optimality as the original strategy in M^* . More specifically this implies a backward solution procedure for M^* :

The Nash Bargaining Theory N requires that the solution of a bargaining game (S,d) obeys a list of axioms which are (compare e.g. Roth (1979)):

Individual Rationality, (strong) Pareto Optimality, Invariance with respect to positive affin-linear Transformations of the utility functions, Independence of Irrelevant Alternatives, and Symmetry.

Finally the Hypothesis MR asserts that each player of a game - i.e. in our case: each person i of the household - knows for certain that the other players - i.e. here: the partner of i in the household - behaves according to the postulated rationality axioms. This hypothesis is a crucial one for every noncooperative solution concept; it relates the player of a game to each other. It means some tacit agreement between the players, but one which is enforced only by the players self-interest to solve the conflict modelled in the game; it needs no external institutional rule to enforce that agreement.

A.2 A General Characterization of the Household Migration Decision

In this subsection we will give a first characterization of the Household Migration Decision in the framework of the general model as given by the assumptions (A1), ..., (A8). First we define:

Definition 1: The Household Migration Decision (= HMD), as predicted by the Household Migration Decision Game M^* , specified by (A1), ..., (A8), is the solution of the truncated game M_r of M^* .

Figure 2a:
The truncated game M_r

The game M_r differs from the first stage M of M^* in so far as for the endpoints (g) , (h_1, g_2) resp. (h) the payoff vectors s^g , d^0 and s^h are inserted. These are obtained - using SP - by applying the backward solution procedure, namely

1. The solution $(^i x_0^r, x_1^r)$ of the third stage games occurring in case r - see (A6)d) - and the solutions $(^i x_0^k, x_1^k)$ of the second stage game specified in (A6)b) are determined for $i = 1, 2$. According to EUH these solutions are those commodity bundles in the budget sets $B_1(p^r, {}^r y_1)$ resp. $B_1(p^r, {}^0 y_1)$ which maximize u_1 in the respective budget set. By continuity and strict concavity of u_1 these solutions $(^i x_0^k, x_1^k)$, ($k = 0, g, h$, $i = 1, 2$) are uniquely determined as the respective budget set are compact and convex. For the respective solution payoffs we introduce the notation

$$d^k := (d_1^k, d_2^k) \quad (k=0, g, h)$$

$$d_1^k := \begin{cases} \top V_1(p^r, {}^r y_1) & \text{for } r = g, h, i = 1, 2 \\ \mid V_1(p^h, {}^0 y_1) & \text{for } k=0, i=1 \\ \perp V_2(p^g, {}^0 y_2) & \text{for } k = 0, i = 2 \end{cases}$$

where

$$V_1(p^k, {}^k y_1) = u_1(^1 x_0^k, x_1^k, 0) \text{ and}$$

$$V_2(p^k, {}^k y_1) = u_2(^2 x_0^k, 0, x_2^k).$$

2. The outcomes d^r ($r=g, h$) yield the complete specification of the bargaining games (S^r, d^r) of the second stage (specified in (A6)c)). According to N we obtain then the Nash solution

$$s^r := (s_1^r, s_2^r)$$

with $s_1^r := U_1(p^r, y^r) := u_1(x(p^r, y^r))$

where $x(p^r, y^r)$ is the solution of the following optimization problem:³

$$\max N(x; p^r, y^r) \text{ s.t. } x \in B(p^r, y^r)$$

where the Nash function $N(.; p^r, y^r) : X \rightarrow R$ is given by

$$N(x; p^r, y^r) := (u_1(x) - V_1(p^r, y_1^r)) (u_2(x) - V_2(p^r, y_2^r))$$

(It was shown first by Nash that $x(p^r, y^r)$ is the only solution fulfilling the axioms mentioned at the end of A.1 in connection with N of (A8), and that exists for any bargaining game (S, d) .)

3. According to SP the solution of M^* is then the solution of the truncated game M_r of Figure 2:

Thus for the purpose of characterizing the HMD the second and eventually third stages of the game M^* are only instruments for determining the possible outcomes of any Household Migration Decision.

Obviously the game M^* and a fortiori the truncated game M_r depend on p^r and y_1^r ($r=g, h, i=1, 2$). It will be convenient to make this dependence obvious at least in the solution of M_r .

Definition 2: The solution function

$$L : P \times Y \rightarrow \{(g_1, g_2), (h_1, g_2), (h_1, h_2)\} \text{ with}$$

$$P := \{p = (p^g, p^h); p^r \in R^n, p^r > 0, r=g, h\}^4$$

$$Y := \{y = (y_1^g, y_2^g, y_1^h, y_2^h); y_1^r > 0, r=g, h, i=1, 2\}$$

is a mapping, which assigns to each feasible price-income-combination (p, y) the HMD $L(p, y)$ of the truncated game M_r , if the underlying prices and incomes are given by (p, y) .

³ Strict concavity of the u_i guarantees the uniqueness of the Nash solution.

⁴ $n = n_0 + n_1 + n_2$

Now we are prepared to formulate the first result concerning the HMD.

Proposition 1: Given the assumptions (A1), ..., (A8) the solution function L is well-defined, i.e. the HMD exists for all $(p, y) \in P \times Y$. Furthermore the following three inequalities are decisive for the form of L :

$$(1) \quad U_2(p^h, y^h) > U_2(p^g, y^g)$$

$$(2) \quad U_1(p^g, y^g) > U_1(p^h, y^h)$$

$$(3) \quad U_1(p^g, y^g) > U_1(p^h, y^h);$$

more precisely:⁵

$$L(p, y) = \begin{cases} (g^1, g^2) & \text{iff (1) and (2) or (1)' and (3)} \\ (h^1, g^2) & \text{iff (1)' and (3)' hold} \\ (h^1, h^2) & \text{iff (1) and (2)' hold.} \end{cases}$$

Proof: The proof of the existence of the HMD will be constructive by establishing the asserted shape of the solution function H . Applying SP we must first solve the subgame M_{T^2} , given by the tree.

Figure 2b
The tree of the
subgame M_{T^2}

By EUH the solution of M_{T^2} is equal to h_2 resp. g_2 iff (1) resp. (1)' hold.

⁵ We assume here and in the following that only strict inequalities will hold, because equalities between utility payoffs are highly improbable. By $(.)'$ of a relation $(.)$ we denote the strict reverse of $(.)$, i.e. "not $(.)$ and equality excluded".

Let us first consider the case (1)': Then applying SP and EUH we can conclude that 1 will choose g_1 resp. h_1 iff (3) resp. (3)' holds. Thus in the case considered

$$L(p,y) = \begin{cases} \top (g_1, g_2) \text{ iff } (3) \\ \perp (h_1, g_2) \text{ iff } (3)' \end{cases}$$

Second we consider the case (1): Then again by SP and EUH we can conclude that 1 will choose g_1 resp. h_1 iff (2) resp. (2)' holds. Thus in this case we obtain:

$$L(p,y) = \begin{cases} \top (g_1, g_2) \text{ iff } (2) \\ \perp (h_1, h_2) \text{ iff } (2)' \end{cases}$$

By puzzling together these results, we get

$$L(p,y) = \begin{cases} \top (g_1, g_2) \text{ iff } (1)' \text{ and } (3) \text{ or } (3) (1) \text{ and } (2) \\ \perp (h_1, g_2) \text{ iff } (1)' \text{ and } (3)' \\ \perp (h_1, h_2) \text{ iff } (1) \text{ and } (2)' \end{cases}$$

i.e. the asserted conditions are necessary and sufficient for the form of $L(p,y)$.

It seems to be instructive, to reconsider the intuitive content of the proof: Obviously there would be no migration of the household, if region g would be more attractive to person 1 than h , i.e. if (2) would hold. On the other side, if (2)' holds, person 2 could deter the household to migrate provided she could credibly threat to dissolve the household (i.e. if (1)' holds) and this threat would be a decisive argument for 1, (i.e. simultaneously (3) holds). In all other cases, person 1 would migrate to h , and dissolution of the household or common migration to h would depend on person 2's preferences between (h_1, g_2) and (h_1, h_2) , i.e. on the validity of (1).

Unfortunately without further specification of the model we cannot obtain more decisive results concerning the solution function L which characterizes the HMD. Therefore we will try to get some more detailed results by specializing our general model in the next section.

B.1 THE HOUSEHOLD MIGRATION DECISION IN A MORE RESTRICTIVE MODEL.

In this section we will study the influence of variations of the economic variables (p,y) on the HMD in more detail, i.e. we will try some kind of comparative static. For this we will first specialize the model of section A in some respects.

B.1 The special model of a Household Migration Decision Game

To clarify the relations between the special model used in this section B and the more general discussed in section A we will label the constitutive assumptions by (B.).

The assumptions $(B1,B2,B4,B6,B7,B8)$ are the same as $(A1,A2,A4,A6,A7,A8)$.

(B3) : (A3) and additionally

There is some positive real α such that

$$p^h = \alpha p^g$$

Thus we suppose now, that all commodity prices in h differ from all commodity prices in g by the same proportion. By this assumption we can reduce the numbers of the parameters

of the model and concentrate on the relative income changes. In other words: the intended use of the model of this section is the analysis of the impact of real income differences on the HMD.

(B5) : (A5) and additionally

a) u_1 is twice continuously differentiable and unbounded above.

b) The optimal solutions of

$$- \max_{(k=0,g,h)} u_1(x_0^k, x_1^k, 0) \text{ s.t. } p_0^k x_0^k + p_1^k x_1^k \leq y_1^k$$

$$- \max_{(k=0,g,h)} u_2(x_0^k, 0, x_2^k) \text{ s.t. } p_0^k x_0^k + p_2^k x_2^k \leq y_2^k$$

$$- \max N(x, p^r, y^r) \text{ s.t. } p^r x \leq y^r \quad (r=g,h)$$

belong to the interior of $X_0 \times X_1$ resp. $X_0 \times X_2$

resp. X , for all $(p, y) \in P \times Y$.

c) The Nash solution utilities behave "normally" with respect to income, i.e. for all $(p, y) \in P \times Y$, $r=g,h$:

- $U_1(p^r, y^r)$ is continuously differentiable w.r. to income y

- $D_y U_1(p^r, y^r) > 0^6$

- $U_1(p^r, y^r) \rightarrow 0 \quad (y^r \rightarrow \infty)$ and $U_1(p^r, y^r) \rightarrow \infty \quad (y^r \rightarrow \infty)$

Parts a) and b) of (B5) are regularity assumptions to facilitate the application of usual calculus. In our view they are not really restrictive.

The character of c) is entirely different: Unfortunately we are not able to find plausible and enough general sufficient conditions which would guarantee c). The problem with the optimization problem

⁶ For a differentiable function $f(\alpha_1, \dots, \alpha_m)$ we denote by $D_{\alpha_k} f(\alpha_1, \dots, \alpha_m)$ the partial derivative of f w.r. to α_k at $(\alpha_1, \dots, \alpha_m)$.

$$\max N(x; p^r, y^r) \text{ s.t. } p^r x \leq y^r$$

is the objective function $N(\cdot; p^r, y^r)$ which depends on the parameters p^r and y^r , via the disagreement outcomes. This twofold dependence of the Nash solution and thus of the respective utilities $U_1(p^r, y^r)$ seems to make it impossible to derive definitive conclusions about the comparative static behaviour of U_1 on a level of generality comparable with that usually assumed in the microeconomic theory of consumption and demand. (This problem has also been observed by Elroy/Horney, (1981).)

In this paper, it suffices to assume "normal" behaviour of the Nash solution w.r. to the income component. (In the appendix it will be shown that c) and additionally normal behaviour w.r. to prices is satisfied, if we assume complete symmetry between the persons of the household and exclude external effects, i.e. if we assume

$$(*) \quad \begin{cases} \top & u_1 = u_2 \\ & D_{x_2} u_1(x_0, x_1, x_2) = D_{x_1} u_2(x_0, x_1, x_2) = 0 \\ \perp & p_1 = p_2 \end{cases}$$

Finally we need a technical assumption which will be shown to be fulfilled for an important class of examples in the Appendix.

(B9) : The quotient

$$D_y V_2(p^g, {}^0 y^2) / (D_y U_2(p^h, y^h))$$

is equal to a constant d for all $(p, y) \in P \times Y$ such that:

- $p^h = \alpha p^g$ with some $\alpha > 0$, (compare B3),
- $U_2(p^h, y^h) = V_2(p^g, {}^0 y^2)$

The quotient in (B9) measures the relation between the marginal utilities of person 2 with respect to income changes for the (individual) utility maximizing outcome, if the household dissolves by separate migration of 1 to

region h , and the Nash solution outcome, if both migrate to h . Now if the difference of commodity prices in g and h is proportional for all commodities, and if the respective utilities are equal (i.e. the constellation of (p,y) is such that neither (1) nor (1)' holds), then this quotient does not depend on (p,y) .

Remark: Using (5) we can conclude that the quotient d of (B9) must be positive.

B.2 Analysis of the HMD in the special model

For given $(p,y) \in P \times Y$ we introduce the new parameters

$$\begin{aligned} b_i &:= y_i / y_1^g \quad (i=1,2) , \quad c := y_2^g / y_1^g , \\ m &:= y_1^g , \quad q := p^g \end{aligned}$$

implying:

$$\begin{aligned} (4) \quad & \begin{cases} y_1^g = (1+c)m , & y_1^h = (b_1 + cb_2)m \\ y_1^g = \tau_g (1+c)m , & y_2^g = (1-\tau_g)(1+c)m \\ y_1^g = \tau_g (b_1 + cb_2)m , & y_2^h = (1-\tau_g)(b_1 + cb_2)m \\ y_1^g = \tau^0 (b_1 + c)m , & y_2^g = (1-\tau^0)(b_1 + c)m \end{cases} \end{aligned}$$

Furthermore we define $z := (q, \alpha, c, m)$.

Our goal then is to analyze the following:

PROBLEM 1: For given z determine the ranges $G(z)$ resp. $D(z)$ resp. $H(z)$ in the (b_1, b_2) -plane, i.e. in the non-negative orthant of R^2 where the HMD will be (g_1, g_2) resp. (h_1, g_2) resp. (h_1, h_2) .

PROBLEM 2: What can be said about the dependence of these ranges $G(z)$ resp. $D(z)$ resp. $H(z)$ on z ?

Therefore we will analyze at first the "boarder lines" of the inequalities (1), (2) and (3).

Define $Z := \{z \in R^{n+3} : z_j > 0 \ (j=1, \dots, n+3)\}$

$$B_1 := \{b_i \in R , b_i \geq 0\} \quad (i=1,2)$$

$$B := B_1 \times B_2 .$$

and the functions $F_1, F_2 : B \rightarrow R$, $F_3 : B_1 \rightarrow R$
with

$$F_1(b, z) := U_2(\alpha q, (b_1 + cb_2)m) - V_2(q, (1 - \tau_0)(b_1 + c)m)$$

$$F_2(b, z) := U_1(q, (1+c)m) - U_1(\alpha q, (b_1 + cb_2)m)$$

$$F_3(b_1, z) := U_1(q, (1+c)m) - V_1(\alpha q, \tau_0(b_1 + c)m)$$

Therefore we may rewrite the inequalities (1), (2), (3):

$$(1) \quad F_1(b, z) > 0$$

$$(2) \quad F_2(b, z) > 0$$

$$(3) \quad F_3(b, z) > 0$$

and the boarder lines of these inequalities are for given z
 εZ defined as:

$$B_k(z) := \{b \varepsilon B : F_k(b, z) = 0\} \quad (k=1, 2)$$

$$B_3(z) := \{b \varepsilon : F_3(b_1, z) = 0\}$$

Our main result concerning PROBLEM 1 will be prepared by
some lemmata the proof of which are given in the Appendix.

Lemma 1: The functions F_k may be written as:

$$F_1(b, z) = U_2(q, b_1 + cb_2)m/\alpha - V_2(q, (1 - \tau_0)(b_1 + c)m)$$

$$F_2(b, z) = U_1(q, (1+c)m) - U_1(q, (b_1 + cb_2)m/\alpha)$$

$$F_3(b_1, z) = U_1(q, (1+c)m) - V_1(q, \tau_0(b_1 + c)m/\alpha)$$

Lemma 2: Define $Z_3 := \{z \varepsilon Z : B_3(z) \text{ is nonempty}\}$. Then

- a) For $z \varepsilon Z_3$ the set $B_3(z)$ is a straight line in B parallel to the b_2 -axis, and for b_1 above (rsp. below) $B_3(z)$ the inequality (3)' (rsp. (3)) holds.
- b) For $z \varepsilon Z_3$ the inequality (3)' holds for all $b_1 \varepsilon B_1$
- c) $\tau_0 \neq 0$ implies: Z_3 is non-empty.

Lemma 3: For all $z \varepsilon Z$:

- a) $B_2(z)$ is a straight line in B intersecting the interior B' of B and given by the equation

$$b_1 + cb_2 = \alpha(1+c)$$
- b) For $b \varepsilon B$ such that $b_1 + cb_2 > \alpha(1+c)$ (rsp. $< \alpha(1+c)$) the inequality (2)' (rsp. 2) holds.

Lemma 4: For all $z \in Z$:

- a) $B_1(z)$ is a straight line in B intersecting the interior B' of B and given by the equation

$$\sigma(z) \cdot b_1 - b_2 = \beta(z)$$

where

$$\sigma(z) := (d\alpha(1-\tau_0)-1)/c \quad \text{with } d \text{ from (B9)}$$

and

$$\beta(z) \text{ is implicitly given by } F_1(0, -\beta(z), z) = 0 \\ \text{(implying } \beta(z) \leq 0)$$

- b) For $b \in B$ such that $\sigma(z) \cdot b_1 - b_2 > \beta(z)$ (rsp. $< \beta(z)$) the inequality (1)' (rsp. (1)) holds.

Next we will introduce some other useful notations:

$$B_k(z; >) := \{b \in B : F_k(b, z) > 0\}, \quad (k = 1, 2, 3).$$

$$b_1(z) := \text{the intercept of } B_3(z) \text{ with the } b_1\text{-axis.}$$

$$B_{1k}(z) := \text{proj}_1 B_k(z) = \{b_1 \in B_1 : \text{there is some } b_2 \in B_2 \\ \text{s.t. } (b_1, b_2) \in B_k(z)\} \quad (k = 1, 2)$$

$$f_k(\cdot; z) : B_{1k}(z) \rightarrow B_2 \text{ defined by}$$

$$f_1'(b_1; z) := \sigma(z) \cdot b_1 - \beta(z)$$

$$f_2'(b_1; z) := (\alpha(1+c) - b_1)/c$$

(i.e. $f_k'(\cdot; z)$ is a functional representation of $B_k(z)$);

and finally:

$$f_k(\cdot; z) : B_1 \rightarrow B_2, \text{ defined by}$$

$$= f_k'(b_1, z), \quad b_1 \in B_{1k}(z)$$

$$f_k(b_1, z) := \begin{cases} & \\ \perp 0 & , \text{ otherwise} \end{cases}$$

Lemma 5: For all $z \in Z$ and b_1 in the interior of $B_{11}(z) \cap B_{12}(z)$ the relation

$$D_{b_1} f_1'(b_1; z) \geq D_{b_1} f_2'(b_1; z)$$

holds with strict inequality for $\tau_0 < 1$.

Lemma 6: We get the following equivalences:

- a) $b_1 < b_1(z) \Leftrightarrow (3) \text{ holds} \Leftrightarrow b \in B_3(z; >)$ for all $b_2 \in B_2$
 b) $b_2 > f_1(b_1; z) \Leftrightarrow (1) \text{ holds} \Leftrightarrow b \in B_1(z; >)$
 c) $b_2 < f_2(b_1; z) \Leftrightarrow (2) \text{ holds} \Leftrightarrow b \in B_2(z; >)$

Lemma 7: For all $z \in Z$ the relation

$$-\beta(z) \cdot (1-\tau_h) < \alpha(1-\tau_0)$$

holds.

Lemma 8: If for $z = (q, \alpha, c, m)$ the relation

$$\alpha \cdot \tau_g \geq c \cdot \tau_0 / (1+c)$$

holds, then $z \in Z_3$.

Lemma 9: Increasing α leads for fixed q, m, c to:

- a) an increase of $b_1(z)$; especially $\alpha \rightarrow \infty$ implies $b_1(z) \rightarrow \infty$
- b) a parallel shift to above of $B_2(z)$
- c) an increase of $\sigma(z)$ and a decrease of $\beta(z)$.

Now we are prepared to formulate a Proposition which yields an answer to Problem 1. For this it is convenient to denote the solution function $L(p, q)$ by $L(b, z)$.

Proposition 2: Let (B_1, \dots, B_9) hold. Then the HMD-solution function L is given by:

$$L(b, z) = \begin{cases} (g_1, g_2) & \text{iff (5) or (6) and (7),} \\ (h_1, g_2) & \text{iff (6) and (7)',} \\ (h_1, h_2) & \text{iff (8),} \end{cases}$$

hold where

$$(5) \quad f_1(b_1, z) < b_2 < f_2(b_1, z)$$

$$(6) \quad b_2 < f_1(b_1, z)$$

$$(7) \quad b_1 < b_1(z)$$

$$(8) \quad b_2 < e(b_1, z) := \max(f_1(b_1, z), f_2(b_1, z)).$$

Proof : It must be shown that:

(i) : (5) is equivalent to (1) and (2)

(ii) : (6) is equivalent to (1)'

(iii) : (7) is equivalent to (3)

(iv) : (8) is equivalent to (1) and (2)';

because then the assertion follows immediately from Proposition 1. Now the equivalence (i) is a

consequence of Lemma (6), (b) and (c); the equivalence (ii) follows from Lemma (6), (b), (iii) from Lemma 6, (a), and finally (7) again from Lemma 6 (b) and (c).

In the following figures we give graphical representations of the different ranges of income-differential combinations $b=(b_1, b_2)$ with $b_1=y_1^h/y_1^g$ for different $z=(q, c, \alpha, m)$ with $c=y_2^g/y_1^g$, $q=p^g$ and $p^h=\alpha q$. At this we denote

$$\begin{aligned} \text{[diagonal lines]} &:= G(z) = \{b \in B : L(b, z) = (g_1, g_2)\} \\ \text{[horizontal lines]} &:= D(z) = \{b \in B : L(b, z) = (h_1, g_2)\} \\ \text{[vertical lines]} &:= H(z) = \{b \in B : L(b, z) = (h_1, h_2)\} \end{aligned}$$

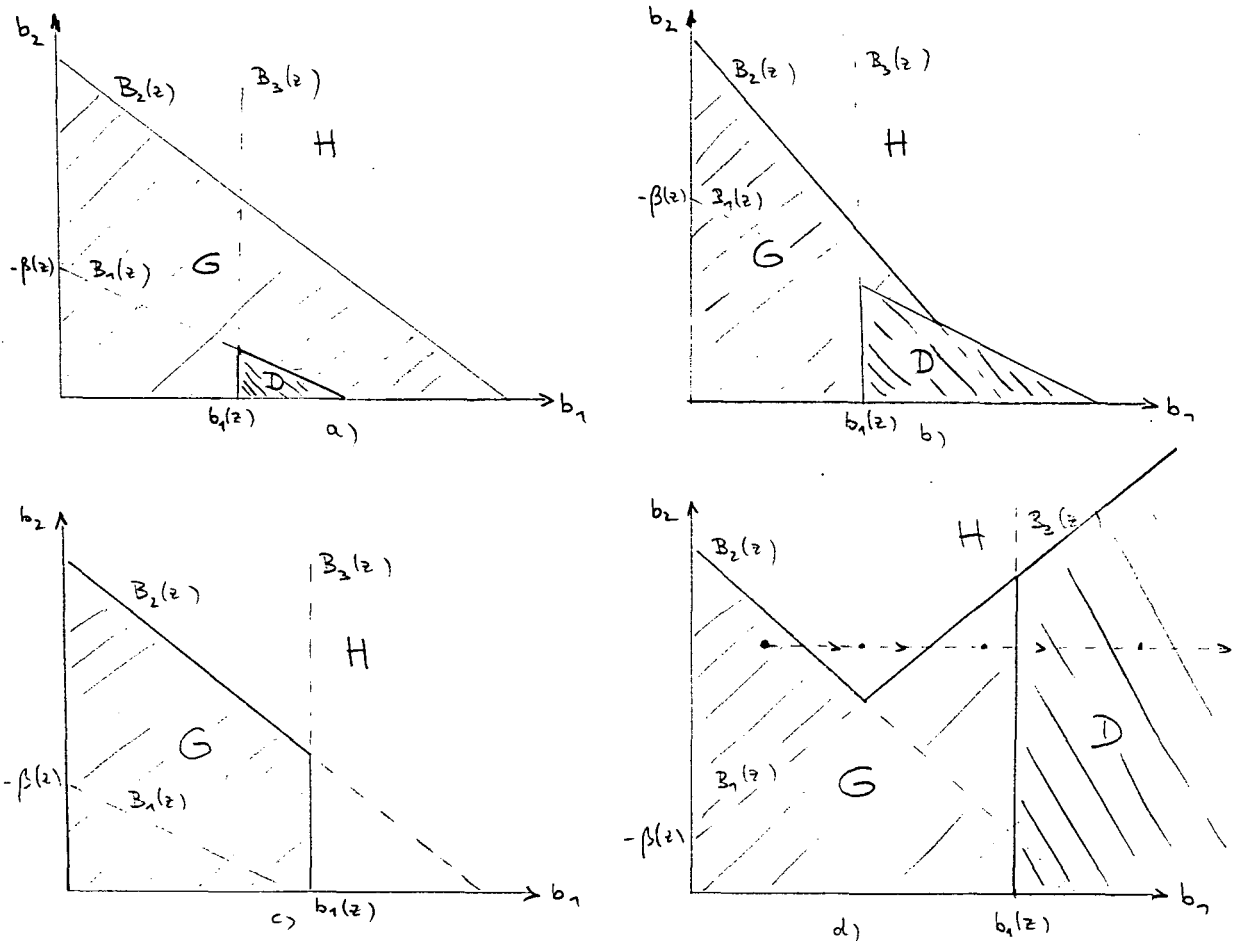


Figure 3: The HMD-ranges of income proportions b_1 and b_2 for normal z ; i.e. such z where (10) and/or (11) holds.

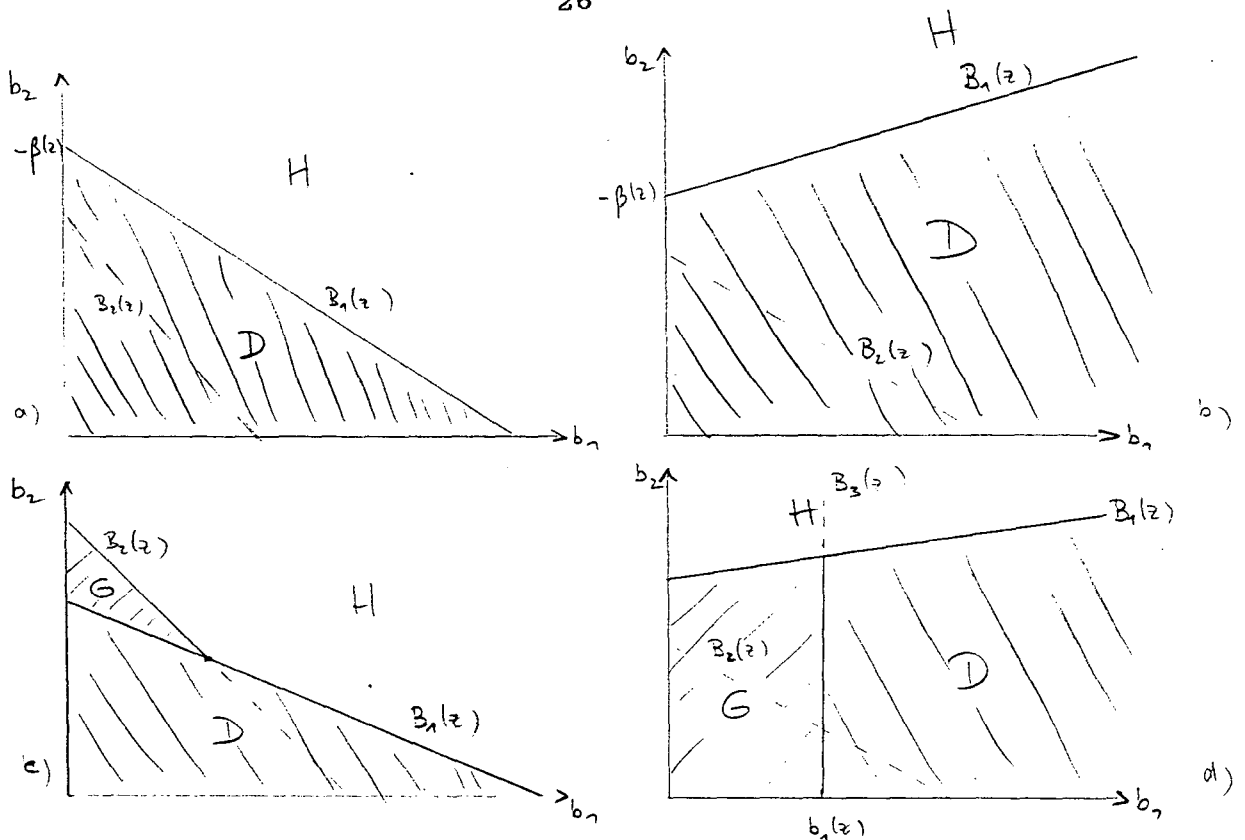


Figure 4: The HMD-ranges of income proportions b_1 and b_2 for unnormal z , i.e. $z \notin Z_3$ (a), b), c) and/or z fulfilling (9).

From Proposition 2 we may conclude some important results; (compare the figures 3 and 4):

Corollary 1: $G(z)$ is empty iff $z \in Z_3$ and the relation

$$(9) \quad \alpha(1+c) \leq -c\beta(z)$$

holds.

Proof: 1) If $z \in Z_3$, then (3)' resp. (7)' always hold, according to Lemma 2. Furthermore (9) together with Lemma 5 implies that $f_1(b_1, z) > f_2(b_1, z)$ for all b_1 , i.e. that (5) cannot hold. Thus $G(z)$ must be empty.

2) Now assume $G(z)$ empty. Since according to Lemma 4 there is a neighbourhood of the origin in \mathbb{R}^2 such that for all b in the intersection of this neighbourhood and B the relation (6) holds, we can conclude from Lemma 2, that z cannot be an element of Z_3 , because otherwise (6) and (7) would hold. - Since furthermore the relation $\alpha(1+c) > -c\beta(z)$ implies $f_2(b_1, z) > f_1(b_1, z)$ for some b_1 , it follows from (4)' that the

opposite, namely $\alpha(1+c) \leq -c\beta(z)$ is necessary for $G(z)$ to be empty.

Illustrations of Corollary 1 are given in Figures 4a) and b)

Corollary 2: Each of the following relations is sufficient for $G(z) \neq \emptyset$:

$$(10) \tau_0 \geq \tau_h, \text{ or}$$

$$(11) \tau_g \geq \tau_0 \text{ and } \alpha \geq 1$$

Proof: (10) implies, that $1-\tau_0 \leq 1-\tau_h$. Therefore it follows from Lemma 7, that $-\beta(z) < \alpha$, which implies that the relation (9) of Corollary 1 cannot hold, thus $G(z)$ is non-empty.

(11) implies that $(c(1+c) \cdot (\tau_0/\tau_g)) < 1$. Therefore $\alpha \geq 1$ and Lemma 8 imply. $z \in Z_{3,1}$; thus again by Corollary 1 $G(z)$ must be non-empty.

The most interesting aspect of these results seems to be, that equality of the dissolution parameters $\tau_0 = \tau_h$ implies the non-emptiness of $G(z)$, i.e. implies that there are always some income proportions b_1 and b_2 thus that the household will stay together in g .

Corollary 3: a) The set $G(z)$ is always bounded.

b) If $z \in Z_3$, then all b near zero belong to $G(z)$

c) If z does not belong to Z_3 and (9) holds, then to each b_1 near 0 there exists a b_2 such that $b = (b_1, b_2) \in G(z)$.

Proof: a) If $G(z)$ is empty, it is bounded by definition. Thus assume $G(z)$ non-empty. Now $G(z)$ is the union of $B_1(z; >) \cap B_2(z; >)$ and $B_1(z; <) \cap B_3(z; >)$. $B_2(z; >)$ is bounded, because $D_{b_1} f_2(b_1, z) < 0$, and similiary $B_1(z; <)$ is bounded because $D_{b_1} f_1(b_1, z) < 0$. Thus $G(z)$ is bounded.

b) If $z \in Z_3$, then $\emptyset \neq A := B_1(z; <) \cap B_3(z; >)$ is a subset of $G(z)$ and A is contained in some bounded

neighbourhood of the origin.

c) If z does not belong to Z_3 , but (8) holds, then $\Phi \neq \emptyset$ and $G(z) = B_1(z; \gamma) \cap B_2(z; \gamma)$. Then we distinguish two cases: α) there is some b_1' such that $f_1'(b_1', z) = f_2'(b_1', z)$; (i.e. $B_1(z)$ and $B_2(z)$ intersect); β): not α). In case α) for each $b_1 < b_1'$ we get the relation $f_1(b_1, z) < f_2(b_1, z)$, i.e. there is some b_2 such that (5) holds, resp. such that $(b_1, b_2) \in G(z)$. In the case β) for all $b_1 < b_{12}(z)$, where $b_{12}(z)$ is defined by $f_2'(b_{12}(z); z) = 0$, the relation $f_1(b_1, z) < f_2(b_1, z)$ holds, i.e. there is some b_2 such that $(b_1, b_2) \in G(z)$.

This corollary asserts economically that for small (b_1, b_2) we could expect that the household stays together in region g . The only exception is the case where z does not belong to Z . Then there exists for small b_1 some other small b_2 where the household will dissolve because of migration of the husband. Such b would belong to $B_1(z; \gamma)$ i.e. for such b the wife would threaten with dissolution, but because of (3)' the husband cannot be deterred to migrate by this threat. According to our model in this case 2's decision is caused by the small b_1 and b_2 whereas 1's decision is essentially caused by the small $\alpha < (c/1+c)(\tau_0/\tau_g)$ where the critical bound for α is increasing in c and τ_0 and decreasing in τ_g ; (compare Figure 4c) for this exceptional case).

Obviously there are cases where $G(z)$ is non-convex. That these cases are not nearly so atypical follows from:

Corollary 4: $G(z)$ is convex, iff (9) or (9)' and (12) or

(9)' and (13) hold, where

$$(12): \{b_1(z)\} = B_1(z) \cap B_2(z)$$

$$(13): B_1(z) \cap B_2(z) = \emptyset \text{ and } b_1(z) \geq \beta(z)/\sigma(z)$$

Remark: Since (10) implies (9)', we can conclude: if the dissolution parameters τ_0 and τ_h are equal, there remains only the case of condition (13) as a relevant case of convex $G(z)$, illustrated in Figure 3c); (but see section C).

Proof: 1) Let (9) hold: then $f_1(0; z) > f_2(0; z)$, implying that $G(z) = B_1(z; <) \cap B_3(z; >)$, which is convex. Let (9)' and (12) hold: then $G(z) = \{b \in B: b_1 \leq b_1(z), b_2 < f_2(b_1; z)\}$, which is convex.

If (9)' and (13) hold, then $G(z) = B_2(z; >) \cap B_3(z; >)$, which is convex.

2) We distinguish the following cases: α) $f_1(b_1, z) \geq f_2(b_1, z)$ for all b_1 . β) $B_1(z)$ and $B_2(z)$ intersect, \top) $f_2(b_1, z) \geq f_1(b_1, z)$. Case α) is equivalent to (9)', and this is equivalent to the convexity of $G(z)$. Assume now β) and let $b_1'(z)$ be given by $f_1'(b_1'(z), z) = f_2'(b_1'(z), z)$. If $b_1'(z) < b_1(z)$, then $G(z) = \{b \in B: b_1 \leq b_1'(z), b_2 \leq f_2(b_1, z)\} \cup \{b \in B: b_1'(z) < b_1 \leq b_1(z), b_2 \leq f_1(b_1, z)\}$, which is convex. If $b_1'(z) > b_1(z)$, then for ε small enough the points $(b_1(z) - \varepsilon, f_1(b_1(z), z) - \varepsilon)$ and $(b_1'(z) - \varepsilon, f_1(b_1'(z), z) + \varepsilon)$ belong to $G(z)$ whereas the straight line connecting these two points intersects $D(z)$, i.e. $G(z)$ is non-convex. Thus in case β) only $b_1'(z) = b_1(z)$ is compatible with $G(z)$ convex. Finally assume \top). Then $B_1(z) \cap B_2(z) = \emptyset$ and $G(z) = \{b \in B: b_1 \leq b_1(z), b_2 \leq f_2(b_1, z)\} \cup \{b \in B: b_1 > b_1(z), f_1(b_1, z) < b_2 < f_2(b_1, z)\}$. If then the second set in this union is non-empty (i.e. if $b_1(z) < \beta(z)/\sigma(z)$) then we could again construct a line connecting a point in the first with a point in the second set, which would intersect $D(z)$, a contradiction to the convexity of $G(z)$.

The most striking example of non-convex $G(z)$ is that illustrated in Figure 3d): Starting from income proportions (b_1, b_2) where the household stays together in g , we could

alone by an increase of b_1 , say to b_1+t_1 attain some income proportions (b_1+t_1, b_1) where the household migrates together to h , and by another increase $b_1+t_1+t_2$ we would again attain some income proportions $(b_1+t_1+t_2, b_1)$ where the household stays in region g , whereas we then could find some t_3 such that for all $t \geq t_3$ the income proportions $(b_1+t_1+t_2+t, b_2)$ would cause a dissolution of the household. How this seemingly surprising result can be explained within the model: Obviously the starting point (b_1, b_2) is one where the household stays in g , because of 1's preferences for g against h , which dominates in this case 2's preferences for h against dissolution. Now increasing b_1 to b_1+t_1 leads to a situation where this domination does not longer hold since now 1 prefers h to g . But increasing b_1+t_1 to $b_1+t_1+t_2$ leads to situation where now 2 prefers dissolution to h ; (an explanation of this follows later); but at this $b_1+t_1+t_2$ the dissolution threat deters person 1 from migration, such that they stay both in g . Only if the income proportion for 1 increases further, the dissolution of the household will occur, if not b_2 increases also above some level.

What is the most surprising aspect of the above consideration is: Given $b \in H(z)$ there are $b' \geq b$ and $b'' \geq b$ such that $b' \in G(z)$ and $b'' \in D(z)$. The reason for this is the positive slope of $f_1(\cdot, z)$. Therefore next we will analyse this slope in more detail.

At this consider the relations

$$(14) \quad d \cdot \alpha \cdot (1 - \tau_0) < 1$$

$$(15) \quad d \cdot \alpha \cdot (1 - \tau_0) = 1$$

$$(16) \quad d \cdot \alpha \cdot (1 - \tau_0) > 1$$

Obviously we get for the slope of $f_1(\cdot; z)$

$$D_{b_1} f_1(\cdot; z) \begin{cases} < 0, & \text{iff (14)} \\ = 0, & \text{iff (15)} \\ > 0, & \text{iff (16)} \end{cases}$$

Fixing d and τ_0 , where d depends essentially on the shape of the utility function u_2 , we can state that the slope of $f_1(\cdot; z)$ is negative (rsp. positive) for small (rsp. large) values of α , i.e. roughly speaking for a lower (rsp. higher) price level in h compared with that in g .

Corollary 5: The non-positivity of the slope of $f_1(\cdot; z)$ (i.e. (14) or (15)) is equivalent to the following condition:

$$b \in H(z), b' \geq b \text{ imply: } b' \in H(z).$$

Proof: 1) Let (14) or (15) hold and $b_1' > b_1, b_2' \geq b_2$, then $b_2' > f_2(b_1'; z)$ and $b_2' > f_1(b_1', z)$; therefore $b' \in H(z)$, according to relation (8) of Proposition 2. If $b_1' = b_1$ and $b_2' > b_2$, then (8) is again fulfilled, i.e. $b' \in H(z)$ again.

2) Assume now that neither (14) nor (15) holds which is equivalent to assuming (16). If we then choose $b_1' = b_1 + m$ with a natural number m large enough, and $b_2' = b_2$, we can show that $b_2' < f_1(b_1'; z)$, i.e. that $b' \in H(z)$.

The shape of the dissolution set $D(z)$ is also closely connected with the relations (14) to (16); more precisely:

Corollary 6: Given $z \in Z$; then $D(z)$ is bounded iff (14) holds.

Proof: (14) is equivalent to $\sigma(z) < 0$ rsp. $D_{b_1} f_1(\cdot; z) < 0$, and this in turn is equivalent to the boundedness of the set $B_1(z; <)$. Since $D(z) = B_1(z; <) \cap B_3(z; <)$, (14) is thus sufficient for $D(z)$ bounded, (eventually empty). Assume next, that (14) does not hold, i.e. that $\sigma(z) \geq 0$, and chose a $b \in D(z)$, characterized according to Proposition 2 by $b_1 > b_{13}(z)$ and $b_2 <$

$f_1(b_1, z)$. Then obviously $b_m := (b_1 + m, b_2)$ fulfills the relations (6) and (7)' too, for all natural m , i.e. $D(z)$ is not bounded.

There are cases, where the model predicts that for no income proportions the household will dissolve, i.e. where $D(z)$ will be empty:

Corollary 7: The dissolution set $D(z)$ is empty iff (14)

and

$$(17) \quad b_1(z) \geq \beta(z)/\sigma(z)$$

hold; (compare Corollary 4 and Figure 3c)

Proof: 1) Let (14) and (17) be given, then obviously for all $b \in B_1(z; <)$ we get $b \in B_3(z; >)$; i.e. $D(z)$ is empty.

2) If (14) does not hold, then we can always find a $b \in B$ fulfilling (6) and (7)', i.e. in this case $D(z)$ cannot be empty. Similarly, if (17) does not hold, there exists b_1 such that $b_1(z) < b_1$ and $f_1(b_1; z) > 0$, i.e. we can find a b with $0 < b_2 < f_1(b_1; z)$. Then (b_1, b_2) is an element of $D(z)$.

The relation (17) says: the income proportion y_1^h/y_1^g where the husband would be indifferent between staying in g and migrating alone to h is at least as great as the income proportion y_1^h/y_1^g where the wife would be indifferent between migrating together to h and staying alone in g , (if(!) she would get no income in h ; i.e. $b_2=0$). This condition together with not too large prices in h compared with that in g implies that, independent of the actual income proportions b_1 and b_2 , the dissolution will never be the HMD.

Finally we mention: ...

Corollary 8: a) For $z \in Z$: $H(z)$ is non-empty, convex and unbounded.

b) For all $z \in Z$: $D(z)$ non-empty implies: $D(z)$ convex.

Proof: a) The convexity of $H(z)$ follows, because of the convexity of the boarder-line $e(\cdot; z)$. Since $D_{b_1} f_1(\cdot; z)$ is always finite, and $D_{b_1} f_2(\cdot; z)$ is always negative, we can conclude from relation (8), that there is always some $b \in H(z)$, i.e. $H(z)$ is non-empty. Furthermore, if (14) or (15) holds, then (b_1+m, b_2+m) belongs to $H(z)$ for all natural m , implying that $H(z)$ is unbounded in these cases. If (16) holds, i.e. $\sigma(z) > 0$, then $b_2 + \sigma(z) \cdot (m+1) > f_1(b_1+m; z) > f_2(b_1+m; z)$, i.e. $(b_1+m, b_2 + \sigma(z)(m+1))$ belongs to $H(z)$ for all natural m , implying that $H(z)$ is unbounded in this case too.

b) Obvious.

Next it would be interesting to study the effects of varying z on the HMD, i.e. to solve Problem 2. Unfortunately this cannot be done without further assumptions, except for the case of a separate variation of α . Since in our opinion the assumptions we would need for the general variation of the other components of z would be qualitatively more restrictive than that of this section B we consider here only the variation of α , deleting the other variations to the example we will give in the next section C.

Proposition 3: Let (B_1, \dots, B_9) be given. Then an increase of α would make it "more probable" that the household stays together in region g ; i.e. more precisely:

a) Given b and $z = (q, \alpha, c, m)$ such that $b \in D(z)$, then there is an $\alpha' > \alpha$ such that $b \in G(z')$ for $z' = (q, \alpha', c, m)$.



b) Given b and $z = (q, \alpha, c, m)$ such that $b \in H(z)$, then there is an $\alpha' > \alpha$ such that $b \in G(z')$ for $z' = (q, \alpha', c, m)$.

Proof : a) $b \in D(z)$ is equivalent to $b_2 < \sigma(z)b_1 - \beta(z) = f_1(b_1; z)$ and $b_1 > b_1(z)$, according to Proposition 2. Increasing α to $\alpha' > \alpha$ leads to: $b_1(z') > b_1(z)$ and $f_1(b_1; z) < f_1(b_1; z')$, by Lemma 9. Therefore - again by Lemma 9 - we can choose α' large enough such that $b_1(z') > b_1$. Then $b \in G(z')$, because for (b, z') the relations (6) and (7) hold.

b) $b \in H(z)$ is equivalent to $b_2 > f_1(b_1; z)$ and $b_2 > f_2(b_1; z)$, according to Proposition 2. By Lemma 9 it follows that increasing α to $\alpha' > \alpha$ leads to: $f_2(b_1; z) > f_2(b_1; z')$ and $f_1(b_1; z') > f_1(b_1; z)$. This implies that there is some $\alpha' > \alpha$ such that either $f_1(b_1; z') < b_2 < f_2(b_1; z')$ or $b_2 < f_1(b_1; z')$, i.e. such that either (5) or (6) holds. If (5) holds, $b \in G(z')$. If (6) holds, then $b \in D(z')$, and we can apply the argument of a) to show the existence of some $\alpha'' > \alpha'$ such that $b \in G(z'')$.

The economic content of Proposition 3 seems highly plausible.

C: AN EXAMPLE: THE HMD IN THE SYMMETRIC COBB-DOUGLAS-MODEL

In order to illustrate the results of section B and - more urgently - to give a complete solution for PROBLEM 2 too, we consider in this section a special example of the restricted model of section B.

C.1 The Specification of the symmetric Cobb-Douglas-Model

The assumptions of this section are labeled by (C.) in a way to make it comparable with these of section A resp. B. The assumptions (C1,C2,C6,C7,C8) are the same as (A1,A2,A6,A7,A8)

(C3): (B3) and additionally

$$a) n_0 = n_1 = n_2 = 1 \quad \nabla$$

$$b) p_1^r = p_2^r \quad (r=g,h)$$

(C4): The dissolution parameters τ_g and τ_h of (A4) satisfy

$$\tau_g = \tau_h = 1/2$$

(C5): (A5)a) and additionally

$$a) u_1(x_0, x_1, x_2) = x_0^{a_0} x_1^{a_1}, \quad u_2(x_0, x_1, x_2) = x_0^{a_0} x_2^{a_2} \quad \text{with } 0 < a_0, a_1, a_2 < 1$$

$$b) a_1 = a_2$$

$$c) a_0 + a_1 = a_0 + a_2 < 1$$

For short we name the model of this section given by the assumption (C1,...,C8) the C-model, in contrast to the A- resp. B-model of sections A resp. B.

The C-model is introduced for two reasons: The main incentive is to prove the "normal" characterizations of the Nash solution by the application of its symmetry property and to give a complete solution to PROBLEM 2. Therefore we model the household's persons with identical tastes, equal income division in the dissolution case in g and h, and equal private good prices. This makes it necessary to exclude external effects between the private good preferences (see(C5)a)); for if we wanted to have symmetry with externalities we could only assume $u_1(x_0, x_1, x_2) = u_2(x_0, x_1, x_2) = x_0^{a_0} x_1^{a_2} x_2^{a_2}$; but this would have the unreasonable consequence, that the private good of 1 is for

1 equally important as for 2, and vice versa. It should be pointed out, that the utility functions u_i of (C5) are only concave but not more strictly concave, in spite of (C5)c). In the A- resp. B-model we used the strict concavity of the u_i to guarantee the uniqueness of the Nash solution. But it will turn out that this uniqueness is also given in the C-model; (see Lemma 10).

The assumption (C3)b) of equal positive good prices could be defended by the argument, that commodity bundles x_1 and x_2 will not be too different, if the respective preferences are identical (as assumed by (C5)b)).

Finally the assumption (C3)a) and the special functional Cobb-Douglas form of the utility functions serves for the second purpose of this C-model: to give an analytical illustration of the implications of the general model. Furthermore by assuming special elasticities a_0 and a_1 we can derive predictions from the model which are empirically testable - at least in principle. Observe that the relation between the elasticities a_0 and a_1 has effects on the strength of the family ties.

C.2 Analysis of the HMD in the symmetric Cobb-Douglas-Model

In the example we can give explicit formulas for the functions $U := U_1 = U_2$ resp. $V := V_1 = V_2$

Lemma 10: For all prices $p_0, p_1 > 0$ and incomes $y > 0$ the following formulas hold:

$$(18) \quad V(p_0, p_1, y) = (y/a)^a (a_0/p_0)^{a_0} (a_1/p_1)^{a_1}$$

$$(19) \quad U(p_0, p_1, y) = (y/a)^a (a_0/p_0)^{a_0} (a_1/(2p_1))^{a_1}$$

with $a := a_0 + a_1$

The proof is given in the Appendix.

Obviously the Nash solution utilities are normal in the sense of (B5), and even more:

Lemma 11: a) In the C-model the assumptions (B5) and (B9) are fulfilled, with

$$d = 2^{a_1/a}$$

b) $D_{p_j} U(p_0, p_1, y) < 0$ for all $p_0, p_1 > 0, y > 0, j = 0, 1$.

The proof of b) is obvious by (19), whereas that of a) may be found in the Appendix again.

In section B we could give no explicit formulas for $\beta(z)$ and $\sigma(z)$ the parameters of the straight line $B_1(z)$ - see Lemma 4 -, and for $b_1(z)$, the intercept of $B_3(z)$ with b_1 -axis - see Lemma 2. In the C-model we obtain

Lemma 12: Let d be given by Lemma 11; then:

a) the parameters of the straight lines $B_1(z)$ and $B_3(z)$ are given by:

$$(20) \beta(z) = -(1-\tau_0)\alpha \cdot d$$

$\tau_0 (1-\tau_0)$

$$(21) \sigma(z) = (d \cdot \alpha \cdot (1-\tau_0) - 1) / c$$

$$(22) b_1(z) = \alpha \cdot (1+c) / (d \cdot \tau_0) - c$$

b) if $B_1(z)$ and $B_2(z)$ intersect in B , then the b_1 -value of the intersection point is given by:

$$(23) b_1'(z) = (1+c) / (d \cdot (1-\tau_0)) - c$$

(see the Appendix for the proof).

These formulas in connection with the results of section B enable us to determine the possible HMD-solution-ranges in the (b_1, b_2) -plane quantitatively for fixed numerical values of $z = (q, \alpha, c, m)$. It should be remarked:

Proposition 4: In the C-model the solution function $L(b, z)$ does not depend on q and m :

Proof: Obvious.

Let us choose for examples:

$$a) a_0 = a_1 = 0.5, \tau_0 = 0.5, c = 1, \alpha = 1$$

$$b) a_0 = a_1 = 0.5, \tau_0 = 0.5, c = 1, \alpha = 1.8$$

$$c) a_0 = a_1 = 0.5, \tau_0 = 0.5, c = 0.5, \alpha = 1$$

We get then (approximately)

- a) $d = 1.4$, $\beta(z) = -0.7$, $\sigma(z) = -0.3$, $b_1(z) = 1.8$, $b_1'(z) = 1.8$
 b) $d = 1.4$, $\beta(z) = -1.3$, $\sigma(z) = 0.3$, $b_1(z) = 4.1$, $b_1'(z) = 1.8$
 c) $d = 1.4$, $\beta(z) = -0.7$, $\sigma(z) = -0.6$, $b_1(z) = 1.6$, $b_1'(z) = 1.6$

This leads to the following figures:

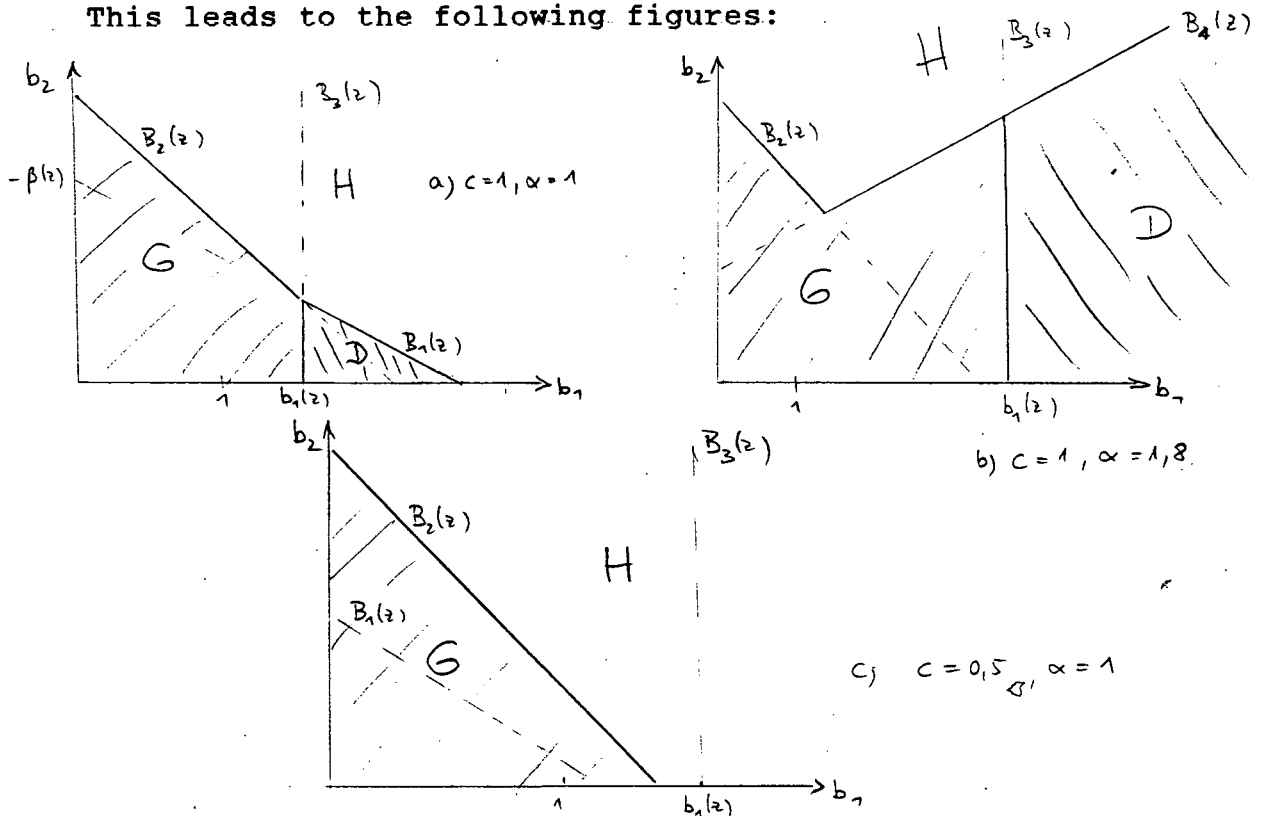


Figure 5: The HMD-ranges of income proportions b_1 and b_2 for specific numerical values of c and α .

Remark: It is a general result, that for $\tau_0 = 1/2$ and $\alpha = 1$ the intersection of $B_1(z)$ and $B_2(z)$, if it exists in B_1 , lies on $B_3(z)$, as in figure 5a) (see (22) and (23)). i.e. in this case the condition (12) for $G(z)$ convex in Corollary 4 is fulfilled.

Next we are interested in the effects of varying the proportion $c = y_2^g / y_1^g$ on the solution function $L(b, z)$. For this we need a further result (which will be proved in the Appendix too):

Lemma 13: For fixed α increasing (rsp. decreasing) c implies:

- a) $B_1(z)$ becomes more flat (rsp. more steep) with fixed intercept on the b_2 -axis. (For $c \rightarrow \infty$ it approaches the parallel line $b_2 = -\beta(z)$ and for $c \rightarrow 0$ it approaches the b_2 -axis).
- b) $B_2(z)$ becomes more flat (rsp. more steep) with fixed point $b^* = (\alpha, \alpha)$. (For $c \rightarrow \infty$ it approaches the parallel line $b_2 = \alpha$, and for $c \rightarrow 0$ it approaches the b_2 -axis).
- c) $b_1(z)$ increases (rsp. decreases) iff $\alpha > d\tau_0$ and decreases (rsp. increases) iff $\alpha < d\tau_0$.
($b_1(z)$ tends to ∞ for $c \rightarrow \infty$ and to $\alpha/(d\tau_0)$ for $c \rightarrow 0$ if $\alpha > d\tau_0$, and to 0 for $c \rightarrow \alpha/(d\tau_0 - \alpha)$ and to $\alpha/(d\tau_0)$ for $c \rightarrow 0$ if $\alpha < d\tau_0$).

Part c) of Lemma 13 suggests now to distinguish the cases $\alpha > d\tau_0$ and $\alpha < d\tau_0$. Since furthermore the sign of the slope of f_1 , i.e. the inequality $\sigma(z) \geq (\leq) 0$ (compare (14), (15) rsp. (16) in section B), will play a decisive role for the direction of the effects of an increased c , we will analyze separately the following four cases:

$$(I): \quad \alpha < \min(\alpha_1, \alpha_2)$$

$$(II): \quad \alpha_1 < \alpha < \alpha_2$$

$$(III): \quad \alpha_2 < \alpha < \alpha_1$$

$$(IV): \quad \max(\alpha_1, \alpha_2) < \alpha$$

$$\text{where } \alpha_1 := d \cdot (1 - \tau_0) - 1, \quad \alpha_2 := d\tau_0.$$

Notice, that for $a_0 = a_1 = 0.5$ and $\tau_0 = 0.5$, as it was supposed in the numerical example underlying Figure 5, $\alpha_2 < \alpha_1$, which implies that there case (II) cannot occur.

Proposition 5: Let the C-model be given and assume $z = (q, \alpha, c, m) \in Z_3$ such that (I) holds. Then:

- a) If $b \in G(z)$ and $b_2 < -\beta(z)$, there is a $c' > c$ such that $b \in D(z')$ for $z' = (q, \alpha, c', m)$
- b) If $b \in D(z)$ and $b_1 < \alpha$, there is a $c' < c$ such that $b \in G(z')$ for $z' = (q, \alpha, c', m)$.

Proof: a) $b \in G(z)$ implies $f_1(b_1; z) < b_2 < f_2(b_1; z)$ or $b_2 < f_1(b_1; z)$ and $b_1 < b_1(z)$. In the second case there is some $c' > c$ such that $b_2 < f_1(b_1; z) < f_1(b_1; z')$ and $b_1(z') < b_1 < b_1(z)$, according to Lemma 13a) and b). Thus $b \in D(z')$. In the first case there is again a $c' > c$ such that $b_2 < f_1(b_1; z')$; if then $b_1 > b_1(z')$, $b \in D(z')$; otherwise we can find another $c' > c$, greater than the first, such $b_1 > b_1(z')$ and a fortiori $b_2 < f_1(b_1; z')$, i.e. $b \in D(z')$ too.

b) $b \in D(z)$ implies $b_2 < f_1(b_1; z)$ and $b_1 > b_1(z)$. There is some $0 < c' < c$ such that $b_1 < b_1(z')$, and since $b_1 < \alpha$ this implies that c' may be chosen small enough so that $b_2 < f_2(b_1; z')$, whence it follows that $b \in G(z')$.

It can easily be seen by examples that without the additional restrictions on b , namely $b_2 < -\beta(z)$ resp. $b_1 < \alpha$, the conclusions need not hold.

Proposition 6: Let the C-model be given and assume $z = (q, \alpha, c, m) \in Z_3$ such that (II) holds. Then:

- a) Like Proposition 5, a).
- b) If $b \in D(z)$, $b_1 < \alpha/\alpha_2$, there is a $c' < c$ such that $b \in G(z')$ for $z' = (q, \alpha, c', m)$.

Proof: a) follows immediately from the proof of Proposition 5, a), since there the presupposition $\alpha < \alpha_1$ was not needed.

b) $b \in D(z)$ implies $b_2 < f_1(b_1; z)$ and $b_1 > b_1(z)$. Because of $b_1 < \alpha/\alpha_2$ there is some $0 < c' < c$ such that $b_1 < b_1(z')$, according to Lemma 13c) and $b_2 < f_1(b_1; z) < f_1(b_1; z')$, by Lemma 13a), thus $b \in G(z')$.

Without the additional assumption $b_1 < \alpha/\alpha_2$ in b), it could happen, that $b_1(z') \leq b_1$ for all c' .

Proposition 7: Let the C-model be given and assume $z = (q, \alpha, c, m)$ such that (III) holds; (notice that $\alpha > \alpha_2$ implies $z \in Z_3$). Then:

- a) If $b \in H(z)$ and $b_2 < -\beta(z)$, there is a $c' > c$ such that $b \in G(z')$ for $z' = (q, \alpha, c', m)$.
- b) If $b \in G(z)$ and $b_1 > \alpha$, there is a $c' < c$ such that $b \in H(z')$ for $z' = (q, \alpha, c', m)$.
- c) If $b \in D(z)$, there is a $c' > c$ such that $b \in G(z')$ for $z' = (q, \alpha, c', m)$.
- d) If $b \in G(z)$ and $b_1 > \alpha/\alpha_2$, there is a $c' < c$ such that $b \in D(z')$ for $z' = (q, \alpha, c', m)$.

Proof: a) $b \in H(z)$ implies $b_2 > f_1(b_1; z)$ and $b_2 > f_2(b_2; z)$. Since $\sigma(z) < 0$ and $b_2 < -\beta(z)$, there is some $c' > c$ such that $b_2 < f_1(b_1; z')$ - according to Lemma 13a) - and this c' may be chosen large enough such that $b_1 < b_1(z')$ - according to Lemma 13c). Thus $b \in G(z')$.

b) $b \in G(z)$ implies $f_1(b_1; z) < b_2 < f_2(b_1; z)$ or $b_1 < b_1(z)$ and $b_2 < f_1(b_1; z)$. In the first case there is a $c' < c$ such that $b_2 > f_2(b_1; z')$, because of $b_1 > \alpha$, and $b_2 > f_1(b_1; z')$, implying $b \in H(z')$. In the second case there is again a $c' > c$ such that $f_1(b_1; z') < b_2$ and $f_2(b_1; z') < b_2$, implying $b \in H(z')$.

c) $b \in D(z)$ implies $b_1 > b_1(z)$ and $b_2 < f_1(b_1; z)$. Then according to Lemma 13a) and c) there is some $c' > c$ such that $b_1 < b_1(z')$ and $b_2 < f_1(b_1; z')$, i.e. $b \in G(z')$.

d) Again it is easily seen that there is some $c' < c$ such that $b_1 > b_1(z')$ and $b_2 < f_1(b_1; z')$, i.e. $b \in D(z')$.

Again there can easily be constructed examples such that none of the above conclusions would hold without the additional restrictions in the respective "if"-parts. - Finally we get:

Proposition 8: Let the C-model be given and assume $z = (q, \alpha, c, m)$ such that (IV) holds (implying $z \in Z_3$ again). Then:

- a) If $b \in G(z)$ and $b_2 > \alpha$, there is a $c' > c$ such that $b \in H(z')$ for $z' = (q, \alpha, c', m)$.
- b) If $b \in H(z)$ and $b_1 < \min(\alpha, \alpha/\alpha_2)$, there is a $c' < c$ such that $b \in G(z')$ for $z' = (q, \alpha, c', m)$.
- c) If $b \in D(z)$ and $b_2 > \max(\alpha, -\beta(z))$, there is $c' > c$ such that $b \in H(z')$.
- d) If $b \in H(z)$ and $\alpha/\alpha_2 < b_1$, there is a $c' < c$ such that $b \in D(z')$ for $z' = (q, \alpha, c', m)$.

Proof: a) $b \in G(z)$ implies either $f_1(b_1; z) < b_2 < f_2(b_1; z)$ or $b_1 < b_1(z)$ and $b_2 < f_1(b_1; z)$. In the first case there is a $c' > c$ such that $b_2 > f_2(b_1; z')$ and $b_2 > f_1(b_1; z')$ (since $b_2 > \alpha$ and $\sigma(z) > 0$; according to Lemma 13b) and a)). Thus in this case $b \in H(z')$. The second case can be reduced to the first since there exists some $c' > c$ such that $b_2 > f_1(b_1; z')$, and for this c' we get either $b_2 > f_2(b_1; z')$, (implying $b \in H(z')$), or $b_2 < f_2(b_1; z')$ such that the situation of the first case appears.

b) $b_2 > f_2(b_1; z)$, $b_2 > f_1(b_1; z)$ and $b_1 < \alpha$ imply that there is a $c' < c$ such that either $f_1(b_1; z') < b_2 < f_2(b_1; z')$ (i.e. $b \in G(z')$) or $b_2 < f_1(b_1; z')$; now because of $b_1 < \alpha/\alpha_2$ for all c' : $b_1 < b_1(z')$, yielding in the second case $b \in G(z')$ too.

c) $b_1 > b_1(z)$, $\max(\alpha, -\beta(z)) < b_2 < f_1(b_1; z)$ imply that there is a $c' > c$ such that $b_2 < \max(f_1(b_1; z'), \alpha)$, whence it follows: $b \in H(z')$.

d) $b_1 > \alpha/\alpha_2$ and $b_2 > f_1(b_1; z)$ imply that there is $c' < c$ such that $b_1 > b_1(z')$ and $b_2 < f_1(b_1; z)$, i.e. $b \in H(z')$.

For the conclusions of Proposition 8 too examples could be given which would show that the respective restrictions in the "if"-parts were necessary.

Summarizing the Propositions 5 to 8 the following can be said roughly:

- There are two decisive points on which the direction of the effect of increasing resp. decreasing the proportion c between the wives and the husbands income in g depends: namely whether for the price ratio α between h and g $\alpha < \alpha_1$ and for $\alpha < \alpha_2$ holds. Here $\alpha < \alpha_2$ or $\alpha > \alpha_2$ determines whether the boarder line $B_3(z)$ of inequality (3) is decreasing or increasing in c , i.e. whether it becomes less or more probable that the husband prefers "staying together in g " or "migrating alone to h ". The inequality $\alpha < \alpha_1$ or $\alpha > \alpha_1$ determines the sign of the slope of the boarder line $B_1(z)$ of inequality (1): for $\alpha < \alpha_1$ this sign is negative, which implies that for increasing c it becomes more probable that the wife prefers "staying alone in g " to "migrating together to h ", (if the income proportion b_2 does not increase a certain limit, namely $-\beta(z)$); for $\alpha > \alpha_1$ the sign of $B_1(z)$'s slope is positive, which implies that for increasing c it becomes more probable that the wife prefers "migrating together to h " to "staying alone in g ".
- Then it can be shown (compare Prop. 5 and 6) that if $\alpha < \alpha_2$ by increasing (resp. decreasing) c the dissolution case becomes more resp. less probable compared with the case of staying together in g . I.e. if the prices in h are low enough compared with those in g , the dissolution threat of the wife cannot prevent the husband to migrate, and this is the more like the greater the wife's proportion in the households common income. - It should be pointed out, that this results depend crucially on restrictions for the income proportions, like $b_2 < -\beta(z) = \alpha/\alpha_1$ and/or $b_1 < \alpha$ resp. α/α_2 , and that for these restrictions necessarily b_1 and b_2 must be less or not essentially greater than 1.
- In contrast (see Prop. 7): If $\alpha_2 < \alpha < \alpha_1$, then the opposite relation between the dissolution and the staying together in g case appears. Furthermore in this case by

increasing (rsp. decreasing) c "staying together in g " becomes more (rsp. less) probable as "migrating together to h " - provided the wife's income proportion b_2 is not greater than α/α_1 again (rsp. the husbands income proportion b_1 not smaller than α).

- Finally (see Prop. 8): If $\alpha > \max(\alpha_1, \alpha_2)$, i.e. if the price ratio is sufficiently large, increasing (rsp. decreasing) c leads to higher (rsp. lower) probability for "migrating together to h " compared with the probability either of "staying together in g " or of "dissolving of the household" - provided the wife's income proportion b_2 is greater than α or even α/α_1 (rsp. the husbands income proportion is either less than α and α/α_2 or greater than α/α_2).

Thus even in the special C-model the effects of varying the proportion c are by no means unambiguous. Our analysis suggests to be careful when discussing the impacts of a policy, which is designed to improve the economic emancipation of the wife by an increase of her individual income, and by this way to strengthen the family ties such that the household would prefer staying in g .

D. CONCLUDING REMARKS

As pointed out in the introduction the model and its analysis in this paper should be seen as a first step towards a decision theoretical explanation of observed international migration behavior of households and the role family ties.

Now let us first briefly review this under the aspect how the model could easily be modified without changing its theoretical kernel, in order to enlarge its empirical content.

In the A-model we were able to construct the solution of the household migration decision problem, using very simple game theoretical arguments. To get some insights into the dependence of this solution on the relevant dates of the economies in g and h the more restricted B-model was designed. Two crucial assumptions were introduced at this:

A restriction on the variety of the underlying utility functions, above all to guarantee some normal comparative static results of the Nash solution outcomes, and a restriction on the variability of the prices: only over-all proportional changes between prices in g and in h were allowed. In our view to dispense with the normality assumption would be an interesting theoretical challenge. But since the normality of the Nash solution is met for symmetric household partners (according to Theorem 3 in the Appendix), there is some empirical evidence supporting this assumption. On the other side it should be remarked that it must be expected that more serious kinds of asymmetry are additional causes for family ties under some circumstances alone, because for the allowance of external effects between the persons private consumption plans the utility functions cannot be modelled symmetrically; (we suggest to study this in more detail by some numerical experiments). The restriction $p^h = \alpha p^g$ simplifies the analytical task for solving PROBLEM 1 and 2 considerably; compare Lemma 1), but it excludes the possibility to study a further determinant for a (household) migration decision of some empirical importance: that in one region some prices may be higher (rsp. lower) and other prices may be lower (rsp. higher) than in the other region. Concerning family migrations essentially differing price ratios between the public goods and the private goods must be expected to play some strong causes for family ties. Utilizing the facilities of computer machineries it will be possible to relax this proportionality restriction and to gain some useful

insights in the effects of differing weights in the price proportions between g and h on the household migration decision.

The highly special C-model finally was introduced to demonstrate how the model could be used to deduce predictions about household migrations which could be tested by econometric methods. Furthermore in this model one primary cause of family ties, namely the relation between the elasticities of public and private goods in the utility function can be studied explicitly. (The quotient d is increasing in a_1 for fixed a_0 , and decreasing in a_0 for fixed a_1 ; compare Lemma 11). - Again by the aid of computer machineries it would be possible to choose other examples of C-models (other utility functions) to cover a wider range of empirical applications.

Finally let us discuss the in our view most serious limitations of the model, and give some hints for further research: The model is of the Arrow-Debreu type, more specifically it is static and deterministic. Thus two important facts of observed (international) migration behavior, especially of guest-workers, cannot be captured by the model:

1. Often households dissolve by migration, typically of the husband, to another destination, but with the expectations that the dissolution is only for some years, and that the migrant will return into the family after this time - an expectation which is fulfilled in a lot of cases. During the dissolution period the migrant tries to remit a fraction of his income gained in h , partly for the expenditures of the rest of his family in g , partly for saving assets for the time after his return. The dissolution case 0 in our model can be only a rough approximation to this fact, first because it must be expected that the utility of the migrant then depends on that part of the remitted income which can really be

saved and second because the remittance rate typically will be the result of an economic decision of the migrant, i.e. it ought to be determined endogenously by an utility maximizing approach. - In this respect we refer to a paper of Djajić (1986), where the author designs a dynamic model which would be able to incorporate some of the above aspects. While Djajić's approach is individual utility maximizing without explicit regard of family migration, we suppose that it could be broadened to a dynamic family migration model.

2. If one of the persons in the household, e.g. the husband gets a job offer from another region h , neither he nor the spouse will be perfectly informed about the consumption possibilities, the prices and the spouse's income opportunities in h . The best what can be expected is that the household can reduce uncertainty about this by some process of information gathering. Thus the following scenario could be imagined: The husband decides to accept the offer and migrates alone to h , but first for one or two periods of time, utilizes this time for the acquisition of relevant information for him and the wife. Equipped with some information (typically imperfect yet always) the present final household migration decision takes place after this information gathering period.

At best, our model could capture this final decision stage. But it would be by far more satisfactory to design a model in which the first decision of the above scenario could be the result of some rational reflections about the possible opportunities. - For this we should have a stochastic dynamic model, the construction and analysis of which will be the task of another paper.

APPENDIX

Here first some general results concerning individual and household demand theory are given, and then in the second part the Lemmata 1, ..., 13 of the main text are proven.

1. Individual and Household Demand Theory

Let X_0, X_1, X_2 denote the non-negative orthant of some $\mathbb{R}^{n_0}, \mathbb{R}^{n_1}, \mathbb{R}^{n_2}$, and set $X := X_0 \times X_1 \times X_2$. Furthermore assume utility functions $u_i: X \rightarrow \mathbb{R}^+$ with:

- (U1) u_i is increasing in all components of (x_0, x_1)
 $\varepsilon X_0 \times X_1$
- (U2) u_i is strictly concave
- (U3) u_i is twice continuously differentiable
- (U4) $u_i(0) = 0$

Finally define for each $z \varepsilon Z := \{(p, m) \varepsilon X \times \mathbb{R} : p > 0 \text{ and } m > 0\}$ the budget sets:

$$B(z) := \{x \varepsilon X : px \leq m\}$$

$$B_i(z) := \{(x_0, x_1, 0) \varepsilon X : p_0 x_0 + p_1 x_1 \leq m_i\}$$

where

$$m_i = \begin{cases} \tau m, & i = 1 \\ (1-\tau)m, & i = 2 \end{cases}$$

$$m_i = \begin{cases} \tau m, & i = 1 \\ (1-\tau)m, & i = 2 \end{cases}$$

$$L (1-\tau)m, i = 2$$

for $0 \leq \tau \leq 1, (i = 1, 2)$

Then it is possible to define the indirect utility functions

$$V_i: Z \rightarrow \mathbb{R}, V_i(z) := \max \{u_i(x) : x \varepsilon B_i(z)\}.$$

Theorem 1: Given (U_1, U_2, U_3) . Then

- 1) V_i is well-defined for all $z \in Z$
- 2) For all $z \in Z_i := \{z \in Z : V_i(z) \neq 0\}$ V_i is continuously differentiable.
- 3) For the partial derivatives of V_i we get:

$$D_t V_i(z) = \begin{cases} r > 0, & \text{for } t = m \\ = 0, & \text{for } t = p_{jk}, k = 1, \dots, n_j, j = \\ & 1, 2, j \neq i \\ L < 0, & \text{for } t = p_{ik}, k = 1, \dots, n_i \end{cases}$$

- 4) If u_i is unbounded, then V_i is unbounded.

Proof: 1) follows immediately from (U2) and (U3). - Since the budget sets $B_i(z)$ are independent of p_j ($j \neq i$), we get $D_t V_i(z) = 0$ for all $t = p_{jk}, k = 1, \dots, n_j$ ($j \neq i$). Now consider the restricted utility function $u'_i : X_0 \times X_i \rightarrow R$ given by $u'_i(x_0, x_i) := u_i(x_0, x_i, 0)$. Then $V_i(z) = \max \{u'_i(x_0, x_i) : p_0 x_0 + p_i x_i \leq m\}$. Therefore V_i can be interpreted as an usual indirect utility function, implying by (U1), (U2), (U3) that 2), 3) and 4) are well-known results of (individual) demand theory; (see e.g. Katzner (1970), section 3).

Next we consider the optimization problem which yields the Nash solution for the bargaining game $(u(B(z)), V(z))$, with $u = (u_1, u_2)$, $V = (V_1, V_2)$. At this the Nash product function $N: X \times Z \rightarrow R$ with

$$N(x, z) := (u_1(x) - V_1(z))(u_2(x) - V_2(z))$$

must be maximized subject to $x \in B(z)$.

The solution $x(z)$ can be characterized as the solution of the following equation system for $x(z) \in B(z)$ and $\lambda(z) \in R$, using the Lagrangean method.

$$\begin{array}{l}
 \Gamma \quad D_{x_j} u_1(x(z)) (u_2(x(z)) - V_2(z)) + D_{x_j} u_2(x(z)) (u_1(x(z)) \\
 (N) \quad | \quad -V_1(z)) = \lambda(z) p_j, \\
 | \quad \quad \quad j=1, \dots, n_0, n_0+1, \dots, n_0+n_1, n_0+n_1+1, \dots, n_0+n_1+n_2) \\
 \perp \quad p(x(z)) = m.
 \end{array}$$

Typically this equation system cannot be solved explicitly because of its complex non-linearity, for any reasonable utility functions.

But let us use now these equations for the analysis of the Nash solution $U : Z \rightarrow \mathbb{R}_+^2$, $U = (U_1, U_2)$ with

$$U_1(z) := u_1(x(z)).$$

Remark: $U_1(z)$ is well-defined because of strict concavity of u_1 and u_2 . - It should be stressed at this occasion, that we did not follow the strict theory of Nash bargaining, in so far as for this, we should have assumed that $S(z)$ is the image under u of the set of all lotteries on $B(z)$, which would guarantee that $S(z)$ is convex - an usual requirement in Nash's theory. But since strict concavity of the u_i implies that the Nash solution is a degenerated lottery, representable by an $x(z) \in B(z)$ - the main reason for this is that the Pareto frontiers of the old $S(z)$ and the new $S(z)$ coincide, and that the Nash solution must lie on this frontier - we considered it permissible to relax the convexity assumption.

Theorem 2: Assuming that the solution $x(z)$ of (1) and (2) is continuously differentiable w. r. to z we get:

$$\begin{array}{l}
 D_m U_1(z) (U_2(z) - V_2(z)) + D_m U_2(z) (U_1(z) - V_1(z)) > 0 \\
 D_{p_j} U_1(z) (U_2(z) - V_2(z)) + D_{p_j} U_2(z) (U_1(z) - V_1(z)) < 0 \\
 \text{for all } j = 1, \dots, n := n_0 + n_1 + n_2
 \end{array}$$

Proof: By the Envelope-Theorem (see e.g. Varian (1985), A.13) we get for $M(z) := N(x(z), z)$ with $t = m$ or $= p_j$ and $\varepsilon_j = 1$ for $t = m$ resp. $\varepsilon_j = -x_j$ for $t = p_j$:

$$\begin{aligned} D_t M(z) &= D_t N(x(z), z) + \lambda(z) \varepsilon_j = \\ &= (D_{x_j} u_1(x(z)) p_j / \varepsilon_j - D_t V_1(z)) (U_2(z) - V_2(z)) \\ &+ (D_{x_j} u_2(x(z)) p_j / \varepsilon_j - D_t V_2(z)) (U_1(z) - V_1(z)). \end{aligned}$$

By differentiating $M(z)$ directly we get on the other side:

$$\begin{aligned} D_t M(z) &= (D_t U_1(z) - D_t V_1(z)) (U_2(z) - V_2(z)) \\ &+ (D_t U_2(z) - D_t V_2(z)) (U_1(z) - V_1(z)). \end{aligned}$$

Combining these two equations yields the asserted inequalities, since $D_{x_j} u_1(x(z)) p_j / \varepsilon_j$ is positive resp. negative for $\varepsilon_j = 1$ resp. $\varepsilon_j = -x_j$, and $U_1(z) > V_1(z)$ by the Individual-Rationality-Property of the Nash solution.

Thus generally we cannot state that the Nash solution utilities behave normally. But let us consider now an important special case, the symmetric case. This is given by the following properties:

$$\begin{aligned} \Gamma \quad z \in Z' &:= \{z \in Z : z = (p, m), p_1 = p_2\} \\ (S) \quad &| \\ &\perp \text{ For all } x \in X : u_1(x_0, x_1, x_2) = u_2(x_0, x_2, x_1) \end{aligned}$$

Theorem 3: Assuming that the solution function $x(z)$ of (N) is continuously differentiable w.r. to $z \in Z'$, and let (S) hold, then:

$$\begin{aligned} D_m U_1(z) &> 0 \quad (i = 1, 2) \\ D_{p_j} U_1(z) &< 0 \quad (i = 1, 2) \end{aligned}$$

Proof: For $z \in Z'$ we get $V_1(z) = V_2(z)$ and, according to (S): $(s_1, s_2) \in u(B(z))$ iff $(s_2, s_1) \in u(B(z))$

Therefore we get a symmetric bargaining game, and the Symmetry-Property of the Nash solution yields: $U_1(z) = U_2(z)$, and of course $D_t U_1(z) = D_t U_2(z)$. Thus the asserted inequalities are obvious consequences of Theorem 2.

2) Here we present the proofs of the Lemmata 1, ..., 13 in the main text:

Proof of Lemma 1: The proof consists of six steps:

1) For all $\alpha > 0$, $q > 0$, $m^7 > 0$, $i = 1, 2$:

$$V_1(\alpha q, m) = V_1(q, m/\alpha).$$

This follows because of

$$\begin{aligned} V_1(\alpha q, m) &= \max \{u_1(x_0, x_1, 0) : \alpha q_0 x_0 + \alpha q_1 x_1 \leq m\} \\ &= \max \{u_1(x_0, x_1, 0) : q_0 x_0 + q_1 x_1 \leq m/\alpha\} = \\ &V_1(q, m/\alpha), \end{aligned}$$

and analogously for V_2 .

2) From 1) we get the validity of the formula for F_3 .

3) For all $\alpha > 0$, $q > 0$, $m > 0$, $x \in X$:

$$N(x; \alpha q, m) = N(x; q, m/\alpha)$$

This is established by using 1) in the formula for the Nash product:

4) Obviously for all $\alpha > 0$, $q > 0$, $m > 0$ for the common budget sets holds:

$$B(\alpha q, m) = B(q, m/\alpha)$$

5) Finally we obtain for all $\alpha > 0$, $q > 0$, $m > 0$:

$$U_1(\alpha q, m) = U_1(q, m/\alpha)$$

for $U_1(\alpha q, m) = u_1(x(\alpha q, m))$ where $x(\alpha q, m)$ is the solution of

$$\max N(x; \alpha q, m) \quad \text{s.t. } x \in B(\alpha q, m).$$

Because of 3) and 4) this optimization problem is equivalent to

$$\max N(x; q, m/\alpha) \quad \text{s.t. } x \in B(q, m/\alpha)$$

⁷ This m must not coincide with the m in the main text.

with solution $x(q, m/\alpha)$ which must be equal to $x(\alpha q, m)$.

Therefore $u_1(x(\alpha q, m)) = u_1(x(q, m/\alpha)) = U_1(q, m/\alpha)$.

- 6) From 5) the asserted formulas for F_1 and F_2 follow.

Proof of Lemma 2:

- a) Since $V_1(q, \cdot)$ is strictly increasing, for $z \in Z_3$ the equation $F_3(b_1, z) = 0$ possesses exactly one solution $b_1(z)$, and since this solution is independent of b_2 , $B_3(z) = \{(b_1, b_2) : b_1 = b_1(z)\}$ is a straight line in B parallel to the b_2 -axis. For $b_1 > b_1(z)$ it follows by $D_y V_1 > 0$ that $F_3(b_1, z) < 0$, i.e. (9) holds, and analogously for $b_1 < b_1(z)$ the inequality (9) follows.
- b) Let us assume to the contrary that there is some b_1 such that $F_3(b_1, z) > 0$. Since by Theorem 1 V_1 is increasing in b_1 and unbounded we may choose $b'_1 > b_1$ large enough such that $F_3(b'_1, z) < 0$. By continuity then there must exist some $b_1(z)$ such that $F_3(b_1(z), z) = 0$. But this would imply $z \in Z_3$, in contradiction to the assumption that z does not belong to Z_3 .
- c) Choosing $z := (q, \alpha, c, m)$ with $\alpha = \tau_0/\tau_g$ and q, c, m at pleasure it follows that $F_3(b_1, z) = U_1(q, (1+c)m) - V_1(q, (b_1+c)m\tau_g) > 0$ for $b_1 = 1$, because of Individual Rationality of the Nash solution. Applying the same argument as under b) we may find some $b_1(z)$ such that $F_3(b_1(z), z) = 0$, i.e. $z \in Z_3$.

Proof of Lemma 3:

- a) Using the formula for F_2 in Lemma 1 we get because of $D_y U_1 \neq 0$ (according to (B5)) the equivalence $F_2(b, z) = 0$ iff $b_1 + cb_2 = \alpha(1+c)$, as asserted.
- b) For $b_1 >$ (rsp. $<$) $\alpha(1+c) - cb_2$ the assumption $D_y U_1 > 0$ implies $F_2(b_1, b_2, z) <$ (rsp. $>$) 0 , i.e. implies (2)' (rsp. (2)).

Proof of Lemma 4:

a) Because of $D_y U_2 \neq 0$ for each q the inverse function $v_q : U_2(q, Y) \rightarrow Y$ of $U_2(q, \cdot)$ exists. Therefore the function $f_1 : B_1 \times Z \rightarrow R$ given by

$$f_1(b_1, z) := v_q(V_2(q, (1-\tau_0)(b_1+c)m))\alpha/cm - b_1/c$$

is well defined. For its partial derivative w.r. to b_1 we get

$$\begin{aligned} D_{b_1} f_1(b_1, z) &= D_{v_q}(V_2(\dots)) D_{b_1} V_2(\dots) \alpha/cm - 1/c \\ &= (D_y U_2(\alpha q, y^h))^{-1} D_y V_2(q, {}^0 y_2) (1-\tau_0) \alpha/c - 1/c \end{aligned}$$

implying by (B_9) : $D_{b_1} f_1(b_1; z) = \sigma(z)$, which is independent of b_1 .

Therefore the graph of $f_1(\cdot; z)$ is a straight line in B . Now it is obvious that this graph of $f_1(\cdot; z)$ is equal to $B_1(z)$.

$$\text{Since } f_1(b_1, z) = \sigma(z)b_1 + f_1(0; z)$$

$$\text{and } f_1(0; z) = v_q(V_2(q, (1-\tau_0)cm))\alpha/cm = -\beta(z)$$

the assertion follows.

b) If for $b \in B$: $\sigma(z)b_1 - \beta(z) > b_2$ (rsp. $< b_2$), then $D_y U_2 > 0$ implies $F_1(b, z) < 0$ (rsp. > 0), i.e. the inequality (1)' (rsp. (1)).

Proof of Lemma 5:

The assertion follows immediately from

$$D_{b_1} f'_2(b_1, z) = -1/c, \quad D_{b_1} f'_1(b, z) = \sigma(z) = (d\alpha(1-\tau_0)-1)/c$$

and $d\alpha(1-\tau_0) \geq 0$ with strict inequality for $\tau_0 < 1$.

Proof of Lemma 6:

Obvious, by inserting the above definitions and using the results of Lemmata 2,3 and 4.

Proof of Lemma 7:

$-\beta(z)$ is defined as the solution of

$$U_2(q, -c\beta(z)m/\alpha) = V_2(q, (1-\tau_0)cm).$$

This implies because of Individual Rationality of the Nash solution: $V_2(q, (1-\tau_0)cm) > V_2(q, (1-\tau_h)(-c\beta(z))m/\alpha)$, and this the asserted inequality, because $V_2(q_1)$ is increasing.

Proof of Lemma 8:

Assume z does not belong to Z_3 , i.e. $U_1(q, (1+c)m) < V_1(q, \tau_0(b_1+c)m/\alpha)$. Then again Individual Rationality of the Nash solution implies that for all $b_1 \geq 0$: $V_1(q, \tau_0(b_1+c)m/\alpha) > V_1(q, \tau_g(1+c)m)$ resp. because of $V_1(q, \cdot)$ increasing: $\tau_0(b_1+c)m/\alpha > \tau_g(1+c)\alpha$, contradicting the presupposed inequality.

Proof of Lemma 9:

- a) Increasing α implies a decrease of $V_1(q, \tau_0(b_1+c)m/\alpha)$ and thus an increase of $F_3(b_1, z)$. Since $F_3(\cdot, z)$ is decreasing in b_1 , increasing α leads to an increase of $b_1(z)$, with $F_3(b_1(z), z) = 0$, and $\alpha \rightarrow \infty$ implies $b_1(z) \rightarrow \infty$.
- b) follows immediately by the definition of $B_2(z)$.
- c) Obviously $\sigma(z)$ is increasing in α . - Using the proof of Lemma 4 we know that $-\beta(z) = f_1(0, z) = k\alpha/cm$, where k is the positive value $v_q(q, (1-\tau_0)cm)$, whence it follows, that $\beta(z)$ is decreasing in α .

Proof of Lemma 10:

ad (18): $V(p_0, p_1, y) = u_1(x'_0, x'_1)$ where x'_0, x'_1 are the solutions of

$$D_{x_0} u_1(x'_0, x'_1) p_1 = D_{x_1} u_1(x'_0, x'_1) p_0$$

$$p_0 x'_0 + p_1 x'_1 = y$$

Solving these equations for u_1 given by (C5) we obtain: $x'_j = (a_j/a)(y/p_j)$, ($j = 0, 1$). Inserting this in u_1 yields (18).

ad (19): Specializing the equation system (N) for the Nash solution in the symmetric case (S) with $n_0 = n_1 = n_2 = 1$ we obtain (since $U = U_1 = U_2$ and $V = V_1 = V_2$):

$$(i) \quad (D_{x_0} u_1(x(z)) + D_{x_0} u_2(x(z))) (U(z) - V(z)) = \lambda(z) p_0$$

$$(ii) \quad (D_{x_1} u_1(x(z)) + D_{x_1} u_2(x(z))) (U(z) - V(z)) = \lambda(z) p_1$$

$$(iii) \quad (D_{x_2} u_1(x(z)) + D_{x_2} u_2(x(z))) (U(z) - V(z)) = \lambda(z) p_1$$

$$(iv) \quad p_0 x_0(z) + p_1 (x_1(z) + x_2(z)) = y$$

In the Cobb-Douglas-case of (C5) these equations yield:
 $x_1(z) = x_2(z)$ (from (ii) and (iii)),

$$2a_0 p_1 x_0^{a_0-1} x_1^{a_1} = a_1 p_0 x_0^{a_0} x_1^{a_1-1}$$

(from (ii), (iii) and (i))

and $p_0 x_0(z) + 2p_1 x_1(z) = y$

This gives: $x_0(z) = (a_0/a)(y/p_0)$ and $x_1(z) = (a_1/a)(y/2p_1)$, implying (19) by $U(p_0, p_1, y) = u_1(x_0(z), x_1(z))$.

Proof of Lemma 11a):

(B5)a)b) and c) are obvious. It remains to show that (B9) holds:

$$U_2(p^h, y^h) = V_2(p^g, {}^0y^2) \text{ is equivalent to}$$

$$U(\alpha q_0, \alpha q_1, (b_1 + cb_2)m) = V(q_0, q_1, (1-\tau_0)(b_1+c)m),$$

and this is equivalent to (by Lemma 10):

$$\begin{aligned} & ((b_1 + cb_2)m/\alpha)^a (a_0/q_0)^{a_0} (a_1/2q_1)^{a_1} \\ & = ((1-\tau_0)(b_1+c)m/a)^a (a_0/q_0)^{a_0} (a_1/q_1)^{a_1} \end{aligned}$$

rsp.

$$(b_1 + cb_2) = 2^{a_1/a} \alpha (1-\tau_0) (b_1 + c).$$

This implies

$$\begin{aligned} & D_y V_2(p^g, {}^0y^2) / D_y U_2(p^h, y^h) = \\ & = 2^{a_1} ((1-\tau_0)(b_1+c)m/a)^{a-1} / ((b_1 + cb_2)m/\alpha)^{a-1} = d \end{aligned}$$

Proof of Lemma 12:

(21) is clear.

ad (20): $\beta(z)$ is defined by the equation

$$F_1(0, -\beta(z), z) = 0 \text{ rsp.}$$

$$(-c\beta(z)m/\alpha)^a - ((1-\tau_0)cm/a)^a 2^{a_1} = 0$$

implying (20).

ad (22): $b_1(z)$ is the solution of $F_1(b_1(z), z) = 0$ rsp.

$$((1+c)m/a)^a = ((b_1(z)+c)m\tau_0/\alpha)^a 2^{a_1},$$

implying (22).

ad (23): Equating $f'_1(b_1, z) = f'_2(b_1, z)$ yields $b'_1(z)$. Thus we must solve

$$\sigma(z)b_1 - b_2 = \beta(z)$$

$$b_1 + cb_2 = \alpha(1+c)$$

for $b_1 = b'_1(z)$. Obviously this yields (23).

Proof of Lemma 13:

- a) Since $\beta(z)$ is independent of c (see (20)), and $|\sigma(z)|$ is decreasing in c , and $\sigma(z) \rightarrow 0$ (rsp. $\rightarrow \pm\infty$) for $c \rightarrow \infty$ (rsp. $\rightarrow 0$), the assertion follows.
- b) Obviously $b_1 = \alpha$, $b_2 = \alpha$ lies in $B_2(z)$ for all c . And since $b_2 = f'_2(b_1, z) = -(b_1/c) + \alpha(1+c)/c$ is the explicit representation of $B_2(z)$, the asserted properties of $B_2(z)$ follow immediately.
- c) Using (22) we see
- $$D_c b_1(z) = \alpha/(d\tau_0) - c \langle \rangle 0 \text{ iff } \alpha \langle \rangle d\tau_0,$$
- therefore the first part of the assertion is established. Again by (22) it follows that: for $\alpha > d\tau_0$ and $c \rightarrow \infty$: $b_1(z) \rightarrow \infty$; for $\alpha > d\tau_0$ and $c \rightarrow 0$: $b_1(z) \rightarrow \alpha/(d\tau_0)$ from above; for $\alpha < d\tau_0$ and $c \rightarrow \alpha/(d\tau_0 - \alpha)$: $b_1(z) \rightarrow 0$; and for $\alpha < d\tau_0$ and $c \rightarrow 0$: $b_1(z) \rightarrow \alpha/(d\tau_0)$ from below.

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