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## Enhanced reliability and the demand impact of jointly normal distributed signals

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**Wirtschaftswissenschaftliche Fakultät  
der Eberhard-Karls-Universität Tübingen**

*Enhanced reliability and the demand impact  
of jointly normal distributed signals*

Thomas Braun



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Enhanced reliability and the demand impact  
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# ENHANCED RELIABILITY AND THE DEMAND IMPACT OF JOINTLY NORMAL DISTRIBUTED SIGNALS

## 1. THE PROBLEM

The objective of this paper is to contribute to a better understanding of rational expectations equilibria. These equilibria emerge from demand decisions of investors who try to extract information about future market prices from current ones. Therefore from an economist's perspective it seems crucial to quantify the impact of information on investors' demand decisions in a way that readily allows for economic interpretation. Unfortunately matrix algebra although widely used in normal distribution theory has its shortfalls with respect to this aim. That is why this paper provides a different approach.

## 2. THE METHOD

Models of capital market equilibrium with risk averse investors having rational expectations are usually based on the so called *hybrid model*. According to this model prices  $P$  and signals  $S$  driving the economy are jointly normally distributed and investors have an exponential utility function.

If there are only two investment opportunities, one riskless asset earning an interest rate  $r$  and one risky asset with current market price  $P_0$ , an investor's optimal demand  $x$  for the risky asset is

$$x = \frac{t}{\text{var}(P_1 | I)} (E(P_1 | I) - (1+r) P_0) , \quad (1a)$$

where  $t$  relates to his risk tolerance and  $I = \{S_1 \dots S_n\}$  to the information available at the time of decision making.

Although being concise this representation does not allow to trace the demand impact of the signals which determines the existence and quality of not necessarily unique equilibria. Setting

$$RP := E(P_1) - (1+r) P_0 \text{ and } \hat{S}_j := S_j - E(S_j)$$

and rearranging (1a) to

$$x = \alpha RP + \sum_{j=1}^n \beta_j \hat{S}_j \quad (1b)$$

clearly appears more promising in this regard.

Usually the derivation of equation (1b) heavily draws on matrix algebra implying what is often referred to as tedious but straightforward calculations challenging readers' patience and imagination. This may be avoided by defining conditional covariances, variances and expectations of jointly normally distributed variables as recursions. Proper orthogonalisation of the original random variables [2] leads to the following expressions

$$\begin{aligned} & \text{cov}(P_1, S_n | S_{n-1} \dots S_1) \\ &= \text{cov}(P_1, S_n | S_{n-2} \dots S_1) - \frac{\text{cov}(P_1, S_{n-1} | S_{n-2} \dots S_1) \text{cov}(S_n, S_{n-1} | S_{n-2} \dots S_1)}{\text{var}(S_{n-1} | S_{n-2} \dots S_1)} \end{aligned}$$

$$\left[ \begin{array}{l} \text{implying} \\ \text{var}(P_1 | S_{n-1} \dots S_1) = \text{var}(P_1 | S_{n-2} \dots S_1) - \frac{\text{cov}(P_1, S_{n-1} | S_{n-2} \dots S_1)^2}{\text{var}(S_{n-1} | S_{n-2} \dots S_1)} \end{array} \right]$$

and

$$E(P_1 | S_n \dots S_1) = E(P_1 | S_{n-1} \dots S_1) + \frac{\text{cov}(P_1, S_n | S_{n-1} \dots S_1)}{\text{var}(S_n | S_{n-1} \dots S_1)} (S_n - E(S_n | S_{n-1} \dots S_1))$$

For

$$k_{S_n \rightarrow P_1} := \frac{\text{cov}(P_1, S_n | S_{n-1} \dots S_1)}{\text{var}(S_n | S_{n-1} \dots S_1)}$$

$$k_{S_i \rightarrow P_1} := \frac{\text{cov}(P_1, S_i | S_{i-1} \dots S_1)}{\text{var}(S_i | S_{i-1} \dots S_1)} - \sum_{j=1}^{n-i} \frac{\text{cov}(S_{i+j}, S_i | S_{i-1} \dots S_1)}{\text{var}(S_i | S_{i-1} \dots S_1)} k_{S_{i+j} \rightarrow P_1} \quad (i = 2 \dots n-1)$$

$$k_{S_1 \rightarrow P_1} := \frac{\text{cov}(P_1, S_1)}{\text{var}(S_1)} - \sum_{j=1}^{n-1} \frac{\text{cov}(S_{1+j}, S_1)}{\text{var}(S_1)} k_{S_{1+j} \rightarrow P_1}$$

the latter rearranges to

$$E(P_1 | S_n \dots S_1) = E(P_1) + \sum_{j=1}^n k_{S_j \rightarrow P_1} \hat{S}_j$$

Assuming risk tolerance normalized to  $t = 1$  for expositional purpose the above expressions yield

$$\alpha := \frac{1}{\text{var}(P_1 | I)} \quad \text{and} \quad \beta_j := \frac{k_{S_j \rightarrow P_1}}{\text{var}(P_1 | I)}$$

In the remaining part of this paper the relative ease and clearness of proceeding as outlined above shall be demonstrated for the two most prominent set-ups to analyse the use of private information in rational expectations models with heterogenously informed investors.

### 3. TWO EXAMPLES

In both cases the information set is restricted to only one signal and the current market price, i. e.

$$I = \{S, P_0\}$$

In the first set-up [1, 4, 5] an individual investor receives the signal

$$S = P_1 + \omega$$

which resembles the future market price plus noise. It is assumed, however, that the individual forecasting errors cancel out in the aggregate demand decisions of the theoretically infinitely large group of informed investors so that the the current market price

$$P_0 = a P_1 + b X + c$$

reflects the actual future market price besides some other source of uncertainty  $X$  uncorrelated with  $P_1$ . Now proceeding as outlined above and making proper use of

$$\text{cov}(\omega, P_1) = \text{cov}(\omega, P_0) = 0 \Leftrightarrow \text{cov}(P_1, S | P_0) = \text{cov}(P_1, P_1 + \omega | P_0) = \text{var}(P_1 | P_0)$$

simple manipulations lead to

$$\beta_S = \frac{1}{\text{var}(P_1 | S, P_0)} - \frac{1}{\text{var}(P_1 | P_0)} = \frac{1}{\text{var}(P_1 | S)} - \frac{1}{\text{var}(P_1)} = \frac{1}{\text{var}(\omega)} \quad (2a)$$

and

$$\beta_{P_0} = \frac{\text{cov}(P_0, P_1)}{\text{var}(P_0)} = \frac{1}{a} \left( \frac{1}{\text{var}(P_1 | P_0)} - \frac{1}{\text{var}(P_1)} \right) \quad (3a)$$

Equation (2a) reveals that the demand impact of signal  $S$  can be exclusively attributed to and measured in terms of the enhanced reliability it is providing without any interference with the current market price. Equation (3a) shows that the demand impact of the current market price is as well totally unaffected by the knowledge of  $S$ . It is rather the same as in the absence of any private information. This is explained by the fact that the information revealed by the current market price to the informed results from the knowledge that the current market price is not influenced by

individual forecasting errors. A fact that is, however, also well understood by investors without private information, thus explaining that their response is c. p. the same.

The second set-up [3] is based on

$$P_1 = S + \omega \text{ and } P_0 = dS - eX + f$$

presupposing that all privately informed investors are subject to the same forecasting error thereby excluding that individual errors cancel out in the aggregate. It is obvious that this assumption renders the current market price a worthless piece of information to those who already know the forecast as the current market price cannot tell them anything about the future market price beyond what is already revealed by the forecast. Again, economic reasoning is readily confirmed by proceeding as outlined above: Regarding

$$\text{cov}(P_1, S | P_0) = \text{cov}(S + \omega, S | P_0) = \text{var}(S | P_0) \text{ and } \text{cov}(P_1, P_0 | S) = 0$$

yields

$$\beta_S = \frac{\text{cov}(P_1, S | P_0)}{\text{var}(P_1 | S, P_0)} = \frac{\text{var}(S | P_0)}{\text{var}(P_1 | S, P_0)} = \frac{1}{\text{var}(P_1 | S, P_0)} = \frac{1}{\text{var}(P_1 | S)} = \frac{1}{\text{var}(\omega)} \quad (2b)$$

and

$$\beta_{P_0} = \frac{\text{cov}(P_0, P_1)}{\text{var}(P_0)} \left( \frac{1}{\text{var}(P_1 | S, P_0)} - \beta_S \right) = 0 \quad \text{q.e.d.} \quad (3b)$$

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