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Euro- US Real Exchange Rate Dynamics: How Far Can We Push General Equilibrium Models?*

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Abstract

This paper re-assesses the problem of general equilibrium models in matching the behaviour of real exchange rate. We do so by developing a two country general equilibrium model with non-traded goods, home bias, incomplete markets and partial degrees of pass through as well as nominal rigidities both in the goods and labour markets. We combine this comprehensive framework with a data consistent shock structure. Our key finding is that presenting an encompassing model structure improves the performance of the model in addressing the behaviour of the real exchange rate but this improvement is at the expense of failing to replicate some other characteristics of the data, such as high consumption volatility and negative cross-country consumption correlation. We argue that the ability of a general equilibrium model to account for the features of the data is closely related to the predominant driving source of the fluctuations.

JEL classifications: F31; F32; F41.

Keywords: Real Exchange Rates, Non-traded goods, Incomplete Asset Markets, Imperfect Exchange Rate Pass Through, Nominal Rigidities.

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1 Introduction

This paper re-asses the lack of success of general equilibrium models to match the behaviour of real exchange rate (RER). We do so by developing an encompassing two-country, two-sector model that contains rich set of imperfections that can deliver deviations from purchasing power parity (PPP), as well as a permanent global productivity shock that is consistent with the empirical findings on the existence of common global factors and balanced growth.

The observed characteristics of the RER contradict the traditional PPP assumption. According to PPP theory the nominal exchange rate is solely determined by the relative prices between countries, so RER should be equal to one. However, in reality the RER is highly persistent and volatile implying a well known failure of the PPP assumption. An important attempt to replicate the behaviour of RER using DSGE models is the paper by Chari, Kehoe, McGrattan (2002, CKM hereafter). Their model accommodates failure of PPP through home bias and local currency pricing (LCP) assumptions. Their main finding is that, in a model with complete markets, separable preferences, high degree of risk aversion and sufficient price stickiness, the monetary shocks can account for the volatility of the RER. Although generating substantial persistence in the model, they still fail to capture the observed persistence of RER. Moreover, in their paper, they try to address the observed lack of international risk sharing. The evidence shows that there is a very low and in most cases even negative relationship between RER and relative consumption. CKM's model fails to generate the negative cross correlation between relative consumption and RER even when they remove the complete markets assumption. This result is known as consumption- RER anomaly which is also known as Backus-Smith puzzle since the early work of Backus and Smith (1993)¹.

Ever since the findings of CKM, there has been ongoing research to capture the stylised facts about RER dynamics in an international real business cycle framework. Our paper offers a re-assessment of the previous models through a comprehensive and encompassing theoretical set-up combined with a rich shock structure that is consistent with the stochastic behaviour of the data. This allows us to further investigate whether replicating RER moments is costly in terms of other moments or not.

There are several papers in which the authors focus on specific puzzles in data. For instance, Corsetti *et al* (2008) shows that a correctly parameterised model with non-tradable distribution services can account for the negative co-movement between the relative consumption and RER. There are others which highlight the critical role of non-traded goods to address the consumption- RER anomaly such as Benigno and Thoenissen (2005), Benigno and Thoenissen (2008) and Tuesta (2013). Their results are in line with the Balassa-Samuelson proposition. According to Balassa-Samuelson proposition, a productivity improvement in the traded sector increases relative consumption while the RER appreciates as a consequence of the increase in non-traded goods prices. There is also growing literature which investigates the importance of multi-sector structure on replicating the behaviour of RER (see for instance, Benigno and Thoenissen (2003), Dotsey and Duarte (2008) or Rabanal and Tuesta (2013) for an estimated model).

Our work is motivated by the mismatch between the findings of general equilibrium models with the features of the data. We are re-assessing the evidence and shortcomings of these models by presenting a comprehensive model calibrated in a standard way. The theoretical framework we develop, contains many features that are proposed as potential sources of RER fluctuations in the literature. The model is a two country general equilibrium model with multiple sectors. We introduce various elements that create persistence and volatility as well as wealth effects to account for the dynamics of the RER and its correlation

 $^{^{1}}$ Kollman (1995) also reported the puzzle and rejected the complete markets assumption at international level.



Figure 1: TFP for Total Industries

with relative consumption.

Our model accommodates deviations from PPP through existence of non-traded goods sector², home bias and imperfect exchange rate pass through³. We also introduce an incomplete asset market structure to be able to address the consumption-RER anomaly thoroughly. The model incorporates nominal rigidities both in goods and labour markets as well as external habit formation in consumption to increase the persistence produced by the model.

One of the characteristics of the model is the rich and data consistent shock structure. The model incorporates preference shocks⁴, monetary shocks, country and sector specific total factor productivity (TFP) shocks as well as a permanent global TFP shock. The assumption of a permanent worldwide TFP improvement is motivated by the empirical evidence. Figure 1 plots the evolution of total factor productivities in Euro Area and US for total industries⁵. It is clear that both series carry a stochastic trend implying permanent effects. There is also evidence on the existence of a global factor which drives the dynamics of aggregate variables (See, for instance, Kose *et al* (2003) or Karadimitropoulou and León-Ledesma (2013)). We capture these two properties of the data by assuming a permanent improvement in the world technology. This permanent shock is also consistent with the necessary existence of a balanced growth path.

A closely related paper to ours is Rabanal and Tuesta (2013) where they estimate a two-country DSGE model with non-traded goods, incomplete markets and nominal rigidities. Their model also incorporate a

 $^{^{2}}$ Betts and Kehoe (2008) show that about one third of RER volatility can be attributable to the relative price of non-traded goods to traded goods.

³The empirical evidence on exchange rate pass through supports the partial exchange rate pass through as a modelling strategy. For instance, Campa and Goldberg (2005) provide evidence in favour of intermediate degrees of pass through among OECD countries especially in the short run. They argue that the exchange rate pass through is gradual as also emphasised by Engel (1999) and Goldberg and Knetter (1997). On the other hand, Obstfeld and Rogoff (200b) highlight the empirical support on the expenditure-switching effect of pass through (implying PCP) while Engel and Rogers (1996) and Gopinath *et al* (2011) provide evidence in favour of pricing to market behaviour in international markets. Moreover, Engel (1999) and CKM argue that the deviations from PPP can be attributable to the international price differentials between traded goods (failure of LoOP).

 $^{^{4}}$ See, Stockman and Tesar (1994), Benigno and Thoenissen (2004) and Corsetti *et al* (2014) for an analysis on the importance of preference shocks to account for the features of the data. Also, Rabanal and Tuesta (2010 & 2013) show that preference shocks have an important role in explaining the fluctuations of RER by estimating a general equilibrium model on the US and Euro Area data.

 $^{^{5}}$ We constructed TFP series from the TFP estimates of EU-Klems for the period 1981-2007. The details on the construction of TFP series can be found in Appendix A.

permanent global TFP shock. Our model differ from theirs in the specification of the global shock, the introduction of nominal rigidities in labour market and the specification of the monetary policy rule. Also, we show the implications of different degrees of exchange rate pass through. The focus of the paper by Rabanal and Tuesta (2013) is mainly the role of non-traded sector and distribution services in explaining the dynamics of RER between the US and the Euro Area. In this paper, we do not focus on one certain puzzle, or a friction, but we rather evaluate the overall performance of the model compared to data. We focus more on how a model with most of the usual real and nominal rigidities performs in terms of matching data moments, and whether fixing one puzzle leads to failures in terms of performance of other moments. That is, if there are trade-offs in the approach of trying to resolve a puzzle at a time.

In this respect, this paper assesses whether fitting some moments of the RER comes at the cost of other unrealistic data moments. We show that the problem of general equilibrium models is not only addressing the behaviour of RER; in fact the structure we present accounts for the observed behaviour of RER to a certain extent. However, this result comes with a trade-off; when the model gets closer with producing the fluctuations of RER, it fails to replicate some other features of the data.

We calibrate the model for the US and Euro Area and we examine the performance of our model compared to data. Our results shows that even a model which incorporates various real and nominal frictions do not replicate the characteristics of the data fully. We show the implications of different pricing regimes as well as market completeness on the behaviour of variables of interest. In our model, the improvement in fitting the moments of RER is result of having preference shocks as predominant source of fluctuations. But this then leads to failure of capturing the high positive consumption correlation across countries and the correlation between relative consumption and RER. In addition, having a global TFP shock, which is motivated by the empirical facts, helps fitting the cross-country correlations of aggregate variables but it brings the cost of high volatility in output. Our analysis also suggest that the performance of our model is conditional on the predominant source of disturbance in the model.

The remainder of the paper is organised as follows: In the next section we present the model. In Section 3 we describe the calibration. In Section 4 we discuss the performance of our model by comparing data and model moments and in Section 5 we present the dynamic adjustment of selected variables following TFP improvements. In Section 6 we present sensitivity analysis and, finally, Section 7 concludes.

2 The Model

The general equilibrium model we present here is a two country model; Home (H) and Foreign (F), which are populated by a mass of infinitely lived households with a fraction of (n) and (1 - n) of the world total, respectively. We will denote the foreign country variables by an asterisk (*).

Households can consume non-traded goods, domestically produced traded goods and imported goods. We define consumption baskets in constant elasticity of substitution (CES) aggregation form as the parameter of elasticity of substitution has important implications on the dynamics of the model. We allow for external habit formation to generate further persistence in the fluctuations. The asset markets are incomplete at the international level. We assume monopolistic competition and nominal rigidities in both goods and labour markets, following Calvo (1983) with indexation. Monetary policy is specified by a Taylor type interest rate feedback rule.

To be able test for the sources of RER fluctuations fully, in our model neither PPP nor LoOP hold. The

multi-sector structure allows us to test for the importance of Balassa-Samuelson proposition considering the controversial results in the literature ⁶. Among tradable goods, we allow a preference bias towards domestically produced tradable goods⁷, thus the terms of trade variability has an impact on RER fluctuations. We also assume that a certain fraction of exporting firms can set the prices in the currency of the buyer as modelled in Betts and Devereux (2000). As in their model, we assume that certain fraction firms can engage in pricing to market; for the rest, the prices are invoiced in the currency of the producer. The limited exchange rate pass through along with the home bias ensures fluctuations in RER arising from the changes in tradable goods prices in line with the findings of Engel (1999).

2.1 Households

The preferences over intertemporal decisions are identical across countries. The representative home household, (i), receives utility from consumption (C_t^i) , and disutility from supplying labour (L_t^i) . Preferences are separable between these two arguments. We allow external habit formation in consumption through hC_{t-1} with h > 0, so the marginal utility of consumption in period t depends on aggregate consumption in (t-1). τ_t represents the preference shock which affects the discount rate hence the intertemporal decision of the household. The representative household's lifetime utility function may be expressed as:

$$U_t^i = E_t \sum_{t=0}^{\infty} \beta^t \tau_t \left[\frac{(C_t^i - hC_{t-1})^{1-\rho}}{1-\rho} - \chi \frac{(L_t^i)^{1+\eta}}{1+\eta} \right], \ 0 < \beta < 1$$
(1)

where E_t denotes the expectations operator conditional on time t information, β is the discount factor, $1/\rho$ is the intertemporal elasticity of substitution and the parameter η is the inverse of the Frisch elasticity of labour supply. The preference shock, τ , follows an AR(1) process in logs:

$$ln(\tau_t) = \rho^{\tau} ln(\tau_{t-1}) + \varepsilon_t^{\tau}, \qquad \qquad \varepsilon_t^{\tau} \sim N(0, \sigma_{\tau}^2)$$
(2)

Households in each country consume a domestically traded good, an imported good and a non-traded good. The consumption index, which consists consumption of non-traded $(C_{N,t})$ and traded goods $(C_{T,t})$, has the following CES aggregation form:

$$C_t^i = \left(\alpha^{\frac{1}{\nu}} (C_{T,t}^i)^{\frac{\nu-1}{\nu}} + (1-\alpha)^{\frac{1}{\nu}} (C_{N,t}^i)^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}}$$
(3)

where α and $(1 - \alpha)$ represent the share of traded goods consumption and non-traded goods consumption in steady state home consumption expenditure respectively, and parameter ν is the elasticity of substitution between tradable and non-tradable goods. The share of traded goods in the foreign consumption basket is different form the home country $\alpha^* \neq \alpha$, while the substitution elasticity is same.

Traded goods consumption, $C_{T,t}$, is divided between home tradable, $C_{H,t}$, and foreign tradable goods consumption, $C_{F,t}$.

$$C_{T,t}^{i} = \left(\omega^{\frac{1}{\theta}} (C_{H,t}^{i})^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} (C_{F,t}^{i})^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}$$
(4)

 $^{^{6}}$ For instance, Benigno and Thoenisson (2003) suggest that a positive productivity shock to tradable sector results in RER depreciation contradicting with Balassa-Samuelson explanation. On the other hand, there are many other contributions in which internal relative price movements cause RER appreciation such as Benigno and Thoenissen (2008) or Tuesta (2013).

⁷See Obstfeld and Rogoff (2000a) for an empirical discussion on home bias.

where ω is the weight of home produced tradable goods, and θ is the elasticity of substitution between home and foreign tradable goods. The elasticity of substitution between the two types is same across countries but the preference weights may differ ($\omega \neq \omega^*$). The value of ω is important, as it shows the degree of home bias in preferences. $\omega > 0.5$ implies a bias towards home produced tradable goods relative to imported goods.

As a result of monopolistic competition, each firm produces differentiated goods. Home country producers are indexed by $h \in [0, 1]$ and foreign country producers are indexed by $f \in [0, 1]$. The consumption sub-indices for each variety of goods are:

$$C_{j,t} = \left[\int_0^1 c_{j,t}(h)^{\frac{\phi-1}{\phi}} dh\right]^{\frac{\phi}{\phi-1}}, \ C_{j^*,t} = \left[\int_0^1 c_{j^*,t}(f)^{\frac{\phi-1}{\phi}} df\right]^{\frac{\phi}{\phi-1}}$$
(5)

where $j = N, H, H^*$, $j^* = N^*, F^*, F$ and $\phi > 1$ is the elasticity of substitution across brands. $c_{j,t}(h)$ represents the consumption of differentiated goods produced in home country in each sector and $c_{j,t}(f)$ represents the consumption of differentiated goods in produced in foreign country in each sector.

The aggregate consumer price index in home country is then:

$$P_t = \left(\alpha (P_{T,t})^{1-\nu} + (1-\alpha)(P_{N,t})^{1-\nu}\right)^{\frac{1}{1-\nu}}$$
(6)

And the price index for tradable goods is given by

$$P_{T,t} = \left(\omega(P_{H,t})^{1-\theta} + (1-\omega)(P_{F,t})^{1-\theta}\right)^{\frac{1}{1-\theta}}$$
(7)

 $P_{H,t}, P_{F,t}, P_{N,t}$ are price indices for home traded, foreign traded and non-traded goods respectively defined as:

$$P_{j,t} = \left[\int_0^1 p_{j,t}(h)^{1-\phi} dh\right]^{\frac{1}{1-\phi}}, \ P_{j^*,t} = \left[\int_0^1 p_{j^*,t}(f)^{1-\phi} df\right]^{\frac{1}{1-\phi}}$$
(8)

where $j=N,H,H^{\ast},\,j^{\ast}=N^{\ast},F^{\ast},F$

The optimal allocation of nominal expenditure of the representative household in home country for each differentiated good yields the following demand functions:

$$c_{N,t}(h) = \left(\frac{p_{N,t}(h)}{P_{N,t}}\right)^{-\phi} C_{N,t} , \qquad (9)$$

$$c_{H,t}(h) = \left(\frac{p_{H,t}(h)}{P_{H,t}}\right)^{-\phi} C_{H,t} , \qquad c_{F,t}(f) = \left(\frac{p_{F,t}(f)}{P_{F,t}}\right)^{-\phi} C_{F,t}$$

where

$$C_{N,t} = (1-\alpha) \left(\frac{P_{N,t}}{P_t}\right)^{-\nu} C_t , \qquad (10)$$

$$C_{H,t} = \omega \left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\theta} C_{T,t} , \qquad C_{F,t} = (1-\omega) \left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\theta} C_{T,t}$$

with

$$C_{T,t} = \alpha \left(\frac{P_{T,t}}{P_t}\right)^{-\nu} C_t$$

2.2 Asset market structure and the budget constraint

Households finance their expenditure through labour income and nominal profits. We assume that labour income is subsidised with a constant rate, κ^w . In addition, it is assumed that the international asset markets are incomplete in the sense that households are able to trade non-state-contingent bonds to purchase consumption goods. Here, we follow Benigno (2001) to model incomplete asset market structure. It is assumed that households in home country can hold two kinds of bonds which are denominated in the units of the home currency and the foreign currency. However, the bonds issued by home country are not traded internationally. As discussed in Schmitt-Grohé and Uribe (2003), we avoid the arising non-stationarity in the model from incomplete market structure by introducing an additional cost for the households in the home country when they trade in the foreign asset market. The cost function $\Theta(.)$ ensures a stationary distribution of wealth between countries.⁸

$$P_{t}C_{t}^{i} + \frac{B_{H,t}^{i}}{(1+i_{t})} + \frac{S_{t}B_{F,t}^{i}}{(1+i_{t}^{*})\Theta(S_{t}B_{F,t}/P_{t})} \leq B_{H,t-1}^{i} + S_{t}B_{F,t-1}^{i} + (1+\kappa^{w})W_{t}^{i}L_{t}^{i} + \int_{0}^{1}\Pi_{t}^{i}di$$

$$(11)$$

 $B_{H,t}^i$ and $B_{F,t}^i$ are the household *i*'s holdings of the domestic and foreign currency denominated nominal risk-free bonds. The nominal interest rates on these bonds in time *t* are i_t and i_t^* respectively. S_t is the nominal exchange rate defined as the home currency price of buying one unit of foreign currency, W_t^i is the nominal wage rate and Π_t^i is the profit income received from the ownership of shares of domestic firms.

The households in home country make the intertemporal decision by maximising (1) subject to (11). The optimum consumption-saving decision is characterised by:

$$\tau_t (C_t - hC_{t-1})^{-\rho} = \beta (1+i_t) E_t \left[\tau_{t+1} (C_{t+1} - hC_t)^{-\rho} \left(\frac{P_t}{P_{t+1}} \right) \right]$$
(12)

In addition, the optimal portfolio choice describes the uncovered interest rate parity (UIP) condition:

$$\tau_t (C_t - hC_{t-1})^{-\rho} = \beta (1 + i_t^*) \Theta \left(\frac{S_t B_{F,t}}{P_t}\right) E_t \left[\tau_{t+1} (C_{t+1} - hC_t)^{-\rho} \left(\frac{P_t}{P_{t+1}}\right) \left(\frac{S_{t+1}}{S_t}\right)\right]$$
(13)

The situation of foreign households is analogous.

2.3 Production technologies and price setting behaviour

There is continuum of monopolistically competitive firms in each country producing differentiated goods for traded and non-traded sectors with labour being the sole production factor. Production has the standard

⁸In order to ensure a well-defined steady state in the model, the following restrictions are imposed on the cost function: $\Theta(.)$ is a differentiable decreasing function in the neighbourhood of steady state level of net foreign assets and when the net foreign assets are in the steady state level $(B_{F,t}^i = 0)$, the cost function is equal to 1 ($\Theta(0) = 1$). See Benigno (2001) for details.

constant returns to scale functional form:

$$Y_{j,t}(h) = A_{j,t} X_t L_{j,t}(h)$$
(14)

where j = H, N. $Y_{j,t}(h)$ is the sectoral output, $A_{j,t}$ is the sector specific exogenous technology shock and $L_{j,t}$ is the total labour employed in each sector. X_t is the global innovation. The stochastic process of the world TFP improvement will be explained in Section 3. The technology in each sector follows an AR(1) process:

$$ln(A_{j,t}) = \rho^{a_j} ln(A_{j,t-1}) + \varepsilon_t^{a_j}$$
(15)

where $\varepsilon_t^{a_j} \sim N(0, \sigma_{a_j}^2)$ and j = H, N.

In each sector, pricing decisions are staggered à la Calvo (1983). It is assumed that in every period $(1 - \xi_p)$ of firms can reset their prices. A non-traded good producer who is able to re-optimise the price in the current period will choose the price $\tilde{p}_t^N(h)$ that maximises the expected present discounted value of profits

$$\max \sum_{k=0}^{\infty} (\beta \xi_p)^k E_t \left[\left(\frac{U_C(C_{t+k})}{U_C(C_t)} \frac{P_t}{P_{t+k}} \right) \tilde{y}_{t,t+k}^{Nd}(h) \left((1+\kappa^p) \tilde{p}_t^N(h) - P_{N,t} M C_{N,t} \right) \right]$$
(16)

subject to the downward sloping demand curve

$$\tilde{y}_{t,t+k}^{Nd}(h) = \left(\frac{\tilde{p}_t^N(h)}{P_{N,t}}\right)^{-\phi} C_{N,t}$$
(17)

where $\tilde{y}_{t,t+k}^{Nd}(h)$ denotes the total individual demand for non-traded goods produced by producer h at time t+k and $MC_{N,t} = \frac{W_t}{A_{N,t}P_{N,t}}$ is the real marginal cost for the non-tradable goods producer.

The first order condition associated with the above profit maximising problem is given by:

$$\sum_{k=0}^{\infty} (\beta \xi_p)^k E_t \left[\left(\frac{U_c(C_{t+k})}{U_c(C_t)} \frac{P_t}{P_{t+k}} \right) \tilde{y}_{t,t+k}^{Nd}(h) \left((1+\kappa^p) \tilde{p}_t^N(h) - \frac{\phi}{\phi-1} P_{N,t} M C_{N,t} \right) \right] = 0$$
(18)

We introduce a constant subsidy, κ^p , which producers receive for their production in each sector. The subsidy ensures a perfectly competitive equilibrium in steady state as we set $(1 + \kappa^p) = \frac{\phi}{\phi - 1}$. Hence when all prices are flexible $(\xi_p \to 0)$, prices will be equal to marginal cost.

Firms producing in the tradable goods sector can sell their products in the domestic country or export them. An important feature of the model is the failure of law of one price in traded goods, such that exports exhibit partial local currency pricing (LCP). Among the exporting firms, a fraction (d) of firms can price discriminate and set the prices in the currency of the buyer while (1 - d) of the firms set the prices in the currency of the seller (PCP). The price index for exports can be written in the following form:

$$P_{H,t}^* = P_{LCP^*,t}^d (P_{H,t}/S_t)^{(1-d)}$$
(19)

Note that in the limiting case of perfect pass-through (d = 0) all firms set prices in the producers' local

currency. Hence LoOP holds for traded goods. With d = 1 on the other hand, all exports are expressed in the currency of the buyer implying complete pricing-to-market behaviour.

In the traded sector, the optimal pricing rule for a firm selling tradable goods in the home country has similar pricing rule with the firm producing non-traded goods. The optimal pricing decision can be written as:

$$\sum_{k=0}^{\infty} (\beta \xi_p)^k E_t \left[\left(\frac{U_c(C_{t+k})}{U_c(C_t)} \frac{P_t}{P_{t+k}} \right) \tilde{y}_{t,t+k}^{Hd}(h) \left((1+\kappa^p) \tilde{p}_t^H(h) - \frac{\phi}{\phi-1} P_{H,t} M C_{H,t} \right) \right] = 0$$
(20)

where $\tilde{y}_{t,t+k}^{Hd}(h)$ denotes the total individual domestic demand for traded goods produced by producer h at time t + k and $MC_{H,t} = \frac{W_t}{A_{H,t}P_{H,t}}$ is the real marginal cost for the tradable goods producer, selling goods in the home country. As the (1-d) part of the traded goods are freely traded by consumers across countries, the optimal price setting of producer currency priced exports is equivalent to the price chosen for the domestic market.

On the other hand, a proportion d of firms can set the prices in the currency of the market they sell the goods. In this case, the LCP firms consider the marginal cost in the foreign currency for foreign country and in home currency for home country and make the pricing decision accordingly. The optimum pricing rule in the traded sector for an exporting firm engaged in LCP is given by:

$$\sum_{k=0}^{\infty} (\beta\xi_p)^k E_t \left[\left(\frac{U_c(C_{t+k})}{U_c(C_t)} \frac{P_t}{P_{t+k}} \right) \tilde{y}_{t,t+k}^{Hd*}(h) \left((1+\kappa^p) \tilde{p}_t^{LCP}(h) - \frac{\phi}{\phi-1} P_{LCP^*,t} M C_{H^*,t} \right) \right] = 0$$
(21)

where $\tilde{y}_{t,t+k}^{Hd*}(h)$ denotes the total individual foreign demand for traded goods produced by producer h at time t + k and $MC_{H^*,t} = \frac{W_t}{A_{H,t}P_{LCP^*,t}S_t}$ is the real marginal cost of exports, priced in the local currency.

Following Gali and Gertler (1999), we assume price indexation through co-existence of two types of firms. A fraction $(1-\varsigma_p)$ of firms re-set prices in a forward looking behaviour, as in the Calvo model described above. The remaining ς_p of firms re-set prices in a backward looking behaviour by using lagged inflation to forecast current inflation. These fractions are assumed to be equal across sectors. The evolution of log-linearised aggregate price setting can be expressed as follows:

$$p_t = \xi_p p_{t-1} + (1 - \xi_p) \breve{p}_t \tag{22}$$

 \breve{p}_t is the index for re-set prices which is in log-linear form given by:

$$\vec{p}_t = (1 - \varsigma_p) p_t^f + \varsigma_p p_t^b \tag{23}$$

where p_t^f is the price set by forward looking firms, p_t^b is the price set by backward looking firms based on the following rule-of-thumb:

$$p_t^b = \breve{p}_{t-1} + \pi_{t-1} \tag{24}$$

This price setting structure generates a higher inflation persistence in the model. The New Keynesian Phillips Curve is not purely forward looking any more; it is a combination of future expected inflation and lagged inflation.

2.4 Labour supply and the wage setting

There is monopolistic competition among households in the labour market, in the sense that households offer differentiated labour services to each sector indifferently. Labour is mobile between non-traded and traded sectors, but not across countries. Hence, the total labour supply is sum of sectoral supply of labour.

$$L = L_H + L_N \tag{25}$$

As described in Erceg *et al* (2000), an "employment agency" combines individual household's supply in the following Dixit-Stiglitz form:

$$L_t = \left[\int_0^1 L_{j,t}(i)^{\frac{\sigma_w - 1}{\sigma_w}} di\right]^{\frac{\sigma_w - 1}{\sigma_w - 1}}$$
(26)

where j = H, N and $\sigma_w > 1$ is the elasticity of substitution among differentiated labour services which is assumed to be same in the two sectors.

Note that, as a result of perfect labour mobility, the nominal wages will be equalised across sectors. The aggregate nominal wage index is defined as:

$$W_t = \left[\int_0^1 W_t(i)^{1-\sigma_w} di\right]^{\frac{1}{1-\sigma_w}}$$
(27)

The cost minimisation problem of producers gives the downward sloping labour demand curve. The total demand for household i's labour services by all firms is:

$$L_t(i) = \left[\frac{W_t(i)}{W_t}\right]^{-\sigma_w} L_t \tag{28}$$

Households set the wages in a Calvo style staggered way. In a given period a constant fraction of $(1 - \xi_w)$ of households are able to adjust their wages. To choose the optimum wage $\widetilde{W}_t(i)$, households maximise the expected lifetime utility (1) subject to the budget constraint (11) and the labour demand curve (28). The first order condition for this nominal wage setting problem is:

$$\sum_{k=0}^{\infty} (\beta\xi_w)^k E_t \left[L_{t+k}(i) U_C(C_{t,t+k}) \left((1+\kappa^w) \frac{\widetilde{W}_t(i)}{P_{t+k}} - \frac{\sigma_w}{\sigma_w - 1} MRS_{t,t+k} \right) \right] = 0$$
(29)

where $MRS_{t,t+k}$ is the marginal rate of substitution between consumption and labour in period t + k for the household resetting the wage in period t, i.e. $MRS_{t,t+k} \equiv -\frac{U_L(L_{t,t+k})}{U_C(C_{t,t+k})}$. When wages are flexible $(\xi_w \to 0)$, the real wage multiplied by the subsidy will be equal to the mark-up over the marginal rate of substitution:

$$(1+\kappa^w)\frac{W_t}{P_t} = \frac{\sigma_w}{\sigma_w - 1}MRS_{t,t+k}$$
(30)

To offset the effect of monopolistic distortion, similar to price setting mechanism, we assume that the amount of subsidy is equal to the monopolistic distortion: $(1 + \kappa^w) = \frac{\sigma_w}{\sigma_w - 1}$. As in the price setting case, it is assumed that in each period, among re-optimising households, a fraction

As in the price setting case, it is assumed that in each period, among re-optimising households, a fraction $(1 - \varsigma_w)$ behave in a forward looking manner while the remaining (ς_w) set wages in a backward looking manner.

The aggregate log-linearised wage index evolves according to:

$$w_t = \xi_w w_{t-1} + (1 - \xi_w) \breve{w}_t \tag{31}$$

 \breve{w}_t is an index for re-set wages which can be defined in log-linear form as:

$$\breve{w}_t = (1 - \varsigma_w) w_t^f + \varsigma_w w_t^b \tag{32}$$

where w_t^f is the wage setted by forward looking households and w_t^b is the wage setted by backward looking households. Backward looking households are rule-of-thumb wage setters such that:

$$w_t^b = \breve{w}_{t-1} + \pi_{t-1}^w \tag{33}$$

2.5 Current account

As the foreign currency denominated bonds can be internationally traded, it is possible to analyse the dynamics of current account by using the budget constraint. The evolution of the net foreign asset position of the home country can be written as:

$$\frac{S_t B_{F,t}}{P_t (1+i_t^*)\Theta(S_t B_{F,t}/P_t)} - \frac{S_t B_{F,t-1}}{P_t} = \frac{P_{H,t} Y_{H,t}^d}{P_t} + \frac{(1-n) S_t P_{H,t}^* Y_{H,t}^{d*}}{n P_t} - \frac{P_{T,t} C_{T,t}}{P_t}$$
(34)

where $Y_{H,t}^d$ represents the domestic demand for home produced traded goods and $Y_{H,t}^{d*}$ is the demand for home produced traded goods of the foreign country consumers.

2.6 Monetary policy

In this model, the monetary authority follows a Taylor rule in each country, that targets consumer price inflation and the deviation of output from its steady state value:

$$i_t = \Gamma_{i-1}i_{t-1} + (1 - \Gamma_{i-1})\Gamma_{\pi_t}\pi_t + (1 - \Gamma_{i-1})\Gamma_{y_t}(y_t - \bar{y}) + \varepsilon_t^m$$
(35)

where $\varepsilon_t^m \sim N(0, \sigma_m^2)$ is the monetary policy shock.

2.7 Equilibrium conditions

In order to have equilibrium in the model demand should be equal supply in each market. This leads to the following market clearing conditions in goods market for home and foreign country respectively:

$$Y_{N,t} = C_{N,t}, \qquad Y_{N,t}^* = C_{N,t}^*$$
(36)

$$Y_{H,t} = C_{H,t} + \left(\frac{1-n}{n}\right)C_{H,t}^*, \qquad Y_{F,t} = C_{F,t}^* + \left(\frac{n}{1-n}\right)C_{F,t}$$
(37)

Therefore, non-traded goods production and consumption will be equal for markets to clear. In traded goods sector, the production will be equal to consumption of traded goods in domestic country and consumption of exports.

Finally, aggregate output can be defined as:

$$Y_t = C_t + \left(\frac{1-n}{n}\right) C_{H,t}^* - C_{F,t},$$
(38)

$$Y_t^* = C_t^* + \left(\frac{n}{1-n}\right)C_{F,t} - C_{H,t}^*$$
(39)

2.8 Some Definitions and Decomposition of RER

As the international variables have a key role for our analysis, we present the definition of some variables we will be using in the following sections.

The home country's terms of trade, the price of imports relative to price of exports of the home country, is defined as:

$$ToT_{t} = \frac{P_{F,t}}{S_{t} P_{H,t}^{*}}$$
(40)

Thus, an increase in ToT implies a deterioration of terms of trade, as imports become more expensive.

Exports of the home country are given by:

$$NX_t = \frac{(1-n) P_{H,t}^* S_t C_{H,t}^* - n P_{F,t} C_{F,t}}{P_t}$$
(41)

The RER is the ratio of foreign CPI to home CPI adjusted by the nominal exchange rate:

$$RER_t = \frac{P_t^* S_t}{P_t} \tag{42}$$

Real exchange rate appreciates when RER falls.

As discussed earlier, there are three channels which cause deviations from PPP in our model: the existence of a non-traded goods sector, home bias, and imperfect exchange rate pass through. In order to show the role of these channels on the movements of RER, the RER can be decomposed into its components. To be tractable, we will show the log-linear form of the components. The following three components sum up to the RER and they are obtained by plugging in the definition of CPI and tradable price index to RER.

$$rerptm_t = (1 - \omega^*)(s_t + p_t^{H^*} - p_t^H) + \omega^*(s_t + p_t^{F^*} - p_t^F)$$
(43)

$$rertot_t = (\omega - (1 - \omega^*))(p_t^F - p_t^H)$$
(44)

$$rerint_t = (1 - \alpha^*)(p_t^{N^*} - p_t^{T^*}) - (1 - \alpha)(p_t^N - p_t^T)$$
(45)

The first channel is *rerptm* which captures the limited pass-through to prices from changes in the exchange rate. Depending on the degree of pass through, nominal exchange rate fluctuations have different implications on the dynamics of the RER. Under PCP, a nominal exchange rate depreciation will decrease the price of exports causing a terms of trade deterioration. This is the classical expenditure switching effect. The magnitude of the change in the terms of trade will depend on the value of the substitution parameter which in turn will affect the RER variability through the home bias. On the other hand, when the prices are set in the currency of the buyer, a nominal exchange rate depreciation will not change the relative prices, implying the failure of LoOP. Terms of trade will improve through the increase in nominal exchange rate. In addition, with LCP, the correlation between RER and nominal exchange rate will be higher as relative prices are not affected from the nominal exchange rate fluctuations. Finally, when the number of firms that engage in PCP and LCP are equal (d = 0.5), the increase in home currency price of imports (from PCP) will be equal to the increase in home currency price of exports (from LCP), leaving the terms of trade at its initial level.

The second component, *rertot*, shows the effects of preference bias towards domestically produced goods. Without home bias in preferences, the terms of trade will have no impact on the fluctuations of RER. The higher the degree of home bias the higher the influence of terms of trade on the variability of RER. The fluctuations of terms of trade depends on the value of elasticity of substitution between home and foreign traded goods.

The final channel, *rerint*, captures the traditional Balassa-Samuelson explanation of the RER fluctuations. The presence of non-traded sector causes failure of the PPP as the non-traded sector is not subject to international arbitrage. When the productivity of traded goods sector and non-traded goods sector are driven by different stochastic processes, TFP improvements will have different effects on RER dynamics. Even if the law of one price holds among traded goods changes in internal relative prices will cause deviations from PPP.

The existence of non-traded goods sector can also account for the lack of international risk sharing. A positive supply side improvement in traded sector at home country, increases the income of that country's consumers raising the relative consumption when there is home bias in preferences. As RER will decrease (Balassa-Samuleson effect) while relative consumption increase, it is possible to obtain a negative relationship between the relative consumption and RER following a positive TFP shock in traded sector. But, notice that the behaviour of the RER depends on where the shock has originated. A TFP improvement in non-traded goods sector would cause RER to depreciate (increase) and relative consumption to increase, implying a positive cross correlation between RER and relative consumption. In addition, the value of elasticity of substitution between home and foreign produced traded goods have crucial implications on the dominance of the Balassa-Samuelson effect ⁹.

 $^{^{9}}$ See, Corsetti *et al* (2011) for an analytical representation of the existence of non-traded goods sector as a possible solution to Backus-Smith puzzle.

3 Calibration

In this section we discuss the baseline calibration of the model which is reported in Table 1. We will discuss the sensitivity of the results to the calibrated values in Section 6. The parameter values are chosen such that the home country is USA and the foreign country is the Euro Area.

We set the discount rate to 0.99 so that steady state real interest rate is 4% per year. The two countries are assumed to have the same size (n = 1 - n = 0.5). Following estimations of Smets and Wouters (2002), the external habit formation parameter h is set to 0.55. As the structure of the global technological shock is allowed to have permanent effects, we set the coefficient of relative risk aversion, ρ , to 1 in order to have balanced growth path in the model. The value of inverse of the Frisch elasticity of labour supply is taken from CKM and set to 1.5.

For the share of traded goods in total consumption basket, we choose a value of 0.4 so that the share of non-traded goods is higher than the share of traded goods 10 . We set the degree of home bias to 0.72 following Corsetti *et al* (2008). The calibration of elasticity of substitution between traded and non-traded goods as well as between home and foreign produced traded goods has important consequences on the dynamics of the model. To be able evaluate the performance of our model, we choose the value of the substitution parameters in accordance with the literature. The elasticity of substitution between traded and non-traded goods is set to 0.44 as in Stockman and Tesar (1995) and the elasticity of substitution between traded goods is set to 1.5 as in CKM.

We take the value of elasticity of substitution among differentiated labour services from Erceg *et al* (2000) implying a 13% mark-up. We assume 4 quarters of contract duration for both wages and prices. We set the proportion of backward looking agents to 0.4 so that still the forward looking behaviour dominates the dynamics of price and wage inflation ¹¹. The Taylor rule coefficients are taken from the estimates of Rabanal and Tuesta (2013). These coefficients have an important role in the dynamics of the model as the model is closed by the monetary policy rule. We follow the work of Rabanal and Tuesta (2013) since their estimations are based on the US and the Euro Area data¹².

Regarding the stochastic processes, the standard deviations of global TFP shock, monetary and preference shocks as well as the autocorrelation coefficient of preference shocks are specified again from Rabanal and Tuesta (2013). We assumed very high persistence for all technology shocks, but slightly higher for traded sectors ($\varepsilon_{aN^*} = \varepsilon_{aN} = 0.9$ and $\varepsilon_{aT^*} = \varepsilon_{aT} = 0.98$). We took the standard deviation of sectoral disturbances from our VAR estimates on TFP series for the home and foreign traded and non-traded sectors. The details of this estimation can be found in Appendix A. Briefly, we took disaggregated data for sectors of the Euro Area and US of about 30 sectors and then aggregated them by large sectors. The data for this calculation is at annual frequency and covers the period 1981-2007. It is assumed that agriculture, mining and manufacturing

 $^{^{10}}$ This value is in line with other studies; for instance Dotsey and Duarte (2008) set the share of traded goods in total consumption basket to 0.44 and similarly Devereux, Lane and Xu (2006) set it to 0.45. For an empirical discussion see Stockman and Tesar (1995).

¹¹It is a standard approach to assume symmetry for preference parameters between US and Euro Area among calibrated papers. But US and Euro area prices and wages are not equally flexible in reality. We also did the experiments by increasing the rigidity in both goods and labour markets for the Euro Area. We found that the results do not show any significant change compared to the results obtained with the benchmark calibration.

¹²Their sample period is 1985:Q2- 2004:Q4 which overlaps with our sample period. The structure of monetary policy rule in our paper is different from the way it is formalised in Rabanal and Tuesta (2013). We work with a more standard monetary policy rule rather than the rule in Rabanal and Tuesta (2013). In their model the monetary policy reacts to the growth rate of output while in our paper it reacts to the difference between current and steady state output. For this reason, we carried out a robustness analysis for a range of values; $\Gamma^Y = \Gamma^{Y^*} = 0.3, 0.75, 1$. We found that the change in the value of the coefficient do not change fundamentally the conclusions of the model.

are traded sectors and the remaining are non-traded sectors. We took the correlations of the shocks from our estimations of VAR as well¹³. We converted the obtained variances from annual to quarterly frequency through dividing them four. In order to specify the evolution of the global technological progress, we used the same data set for total industries. We tested the existence of a unit root in both TFP series of Euro Area and US by conducting an ADF test. They both found to be non-stationary. We further investigated the structure of the TFP series by estimating an AR(1) process. Our estimations showed that the technology process of the US is a unit root while for Euro Area it is persistent in growth rates. Therefore the global technology shock assumed to have the following form in our model:

There is a global TFP unit root shock :

$$ln(G_t) = ln(G_{t-1}) + \varepsilon_{g,t} \tag{46}$$

which is equivalent to the shock process of the home country:

$$ln(X_t) = ln(G_t) \tag{47}$$

However, the technology of the foreign economy adjusts to the global shock slowly:

$$ln(X_t^*) - ln(G_t) = \rho_{x^*}(ln(X_{t-1}^*) - ln(G_{t-1}))$$
(48)

In long run, these two processes are equivalent. One can think this as a co-integrating relationship in which these two processes are sharing the same stochastic trend; the error correction mechanism for the US is zero while for the Euro Area it is found to be 0.86^{14} .

4 Quantitative Properties of the Calibrated Model

We now examine the performance of our simulated model compared with the data. We present the incomplete markets and complete markets versions of different pricing regimes. The LCP case refers to d = 1, PCP refers to d = 0 and partial LCP refers to d = 0.5.

The moments are calculated by assuming the US as home country and the Euro Area as foreign country. Our data covers the period between 1999:Q1 and 2012:Q3. Because the model is log-linearised around the deterministic steady state, we take logs of the data as well so that the results will be consistent. The details on the description of data are given in the Appendix A. In our analysis, the standard deviation of the variables are shown relative to standard deviation of US GDP 15 .

Having permanent global technology shock causes non-stationarity in the model. In order to have a well behaved steady state, we transform the trended variables. The normalised log-linear dynamics of the model can be found in Appendix B. We focus on the performance of the model at business cycle frequencies using the HP filter¹⁶. For the analysis, we simulated the stationary model and then added back the stochastic

¹³We do not take into consideration the values which are very close to zero in the variance-covariance matrix.

 $^{^{14}}$ We calculated the quarterly counterpart of the persistence parameter by taking the 0.25 to the power of the value obtained from the estimations of annual data.

¹⁵The calculated standard deviation of US GDP is 0.57%. This value is quite low as our sample is within the "Great Moderation" period which output volatility has fallen substantially.

 $^{^{16}}$ We choose 1600 for the smoothing parameter of HP filter as we are working with quarterly data.

Parameter	Value	Description
β	0.99	discount factor
n = 1 - n	0.5	relative country size
$h = h^*$	0.55	habit persistence
ρ	1	coefficient of risk aversion
η	1.5	inverse Frisch elasticity of labour supply
δ	0.001	cost of intermediation
$\alpha = \alpha^*$	0.4	share of traded goods in total consumption
$\omega = \omega^*$	0.72	degree of home bias
ν	0.44	elast. of subst.: tradable & non-tradable goods
heta	1.5	elast. of subst.: home & foreign tradable goods
σ_w	4	elast. of substitution across types of labour
$\xi_p = \xi_p^* = \xi_w = \xi_w^*$	0.75	Calvo prices/wages
$\varsigma_p = \varsigma_p^* = \varsigma_w = \varsigma_w^*$	0.4	price/ wage index
Taylor Rule Coefficients		× , C
Γ_i	0.88	interest rate smoothing- US
Γ_{i^*}	0.76	interest rate smoothing- Euro Area
Γ_y	0.90	response to output-US
Γ_{u^*}	0.56	response to output- Euro Area
Γ_{π}	2.05	response to inflation- US
Γ_{π^*}	2.72	response to inflation-Euro Area
AR Coefficients- Shocks		*
$ ho_{ au}$	0.88	Preference- US
$\rho_{ au^*}$	0.87	Preference- Euro Area
ρ_{aT}	0.98	Tradable sector- US
ρ_{aN}	0.9	Nontradable sector- US
ρ_{aT^*}	0.98	Tradable sector- Euro Area
ρ_{aN^*}	0.9	Nontradable sector- Euro Area
ρ_{x^*}	0.86	persist. of global shock- Euro Area
Standard Deviation of Shocks (in percentages)		
$\varepsilon_{ au}$	1.91	Preference- US
$\varepsilon_{ au^*}$	1.89	Preference- Euro Area
ε_m	0.11	Monetary- US
ε_{m^*}	0.16	Monetary- Euro Area
$arepsilon_{aT}$	1.2124	Technology shock: tradable sector- US
ε_{aN}	0.4472	Technology shock: nontradable sector- US
ε_{aT^*}	0.8185	Technology shock: tradable sector- Euro Area
$arepsilon_{aN^*}$	0.2191	Technology shock: nontradable sector- Euro Area
$arepsilon_{aT^*,aN^*}$	0.0010	Covariance of technology shocks: Euro Area
$arepsilon_{aT,aN}$	0.0008	Covariance of technology shocks: US
$arepsilon_{aT^*,aT}$	0.0013	Covariance of technology shocks: Euro Area-US
ε_x	0.36	Permanent Global TFP Shock

Table 1: Calibrated Values for US and Euro Area

trends to the non-stationary variables to obtain simulated non-stationary variables. Then we applied the HP filter to both the simulated data and the real data so that both are compared according to the same metric.

For the business cycle frequencies, the second moments generated by the model in comparison with the US data are reported in Table 2. The statistics obtained from the data shows that both real and nominal exchange rate are very volatile and persistent; they are about 5 times more volatile than GDP with autocorrelation of 0.7. The terms of trade are also highly persistent with autocorrelation of 0.78; but the volatility obtained from our HP filtered data for terms of trade relative to output is less than one. Looking at the business cycle statistics, both consumption and output have autocorrelation about 0.9 indicating very high persistence and consumption is 0.84 as volatile as GDP. We also report the variance decomposition in Table 3 to show the role of shocks in explaining the behaviour of variables of interest.

Looking at the Table 2, an evident results is the indifference of the variable moments to complete/incomplete market structures with minor exceptions. This is a consequence of our model set up, as there are various channels that create income and wealth effects apart from the incomplete market structure. Our findings do not present a substantial difference between pricing regimes either with the only exception being the behaviour of terms of trade. This result confirms the findings of Dotsey and Duarte (2011). In our setting, the high share of non-traded goods in preferences as well as nominal rigidities reduces the impact of the pass through. Even so, the correlation between the terms of trade and other variables will depend on the degree of pass through. In our data, the correlation between terms of trade and RER is positive, confirming both the Mundell-Flemming explanation and the empirical evidence presented by Obstfeld and Rogoff (2000b). Consistently, our model produces positive co-movement under PCP and partial LCP regimes. Despite of matching the correct signs in these two pricing regimes, the model does not perform well in terms of magnitude.

In terms of matching the behaviour of RER, the model performs the best under LCP model. The volatility of RER is higher with LCP and partial LCP regimes. When the prices are invoiced in the currency of the buyer the changes in nominal exchange rate are not corrected by the changes in prices, so with zero exchange rate pass through (LCP) the volatility of all international variables are relatively higher. However, quantitatively, the observed volatility is still higher than what we obtain from our model. As our sample period is part of the period which is known as "Great Moderation", the observed volatility of RER relative to GDP is quite high. So even though, our model does not replicate this feature of the data completely, it still produces substantial volatility compared to the findings of other studies.¹⁷ The LCP model matches with the persistence of RER; in fact our model performs well in terms of producing persistence for all variables. It also captures the high correlation between real and nominal exchange rates.

The model accounts for the negative correlation between net exports and output as well as RER and output. It also generates a very strong negative correlation between RER and relative consumption. These results are not surprising considering the predominant role of preference shocks in explaining the fluctuations of the variables (see table 3). Having preference shocks in the model helps to obtain counter cyclical net exports and RER. In addition, the negative correlation between RER and relative consumption is a natural consequence in a model with preference shocks. It is an exogenous factor which breaks the link between these two variables. As 54% of the fluctuations of consumption is driven by the preference shocks, the model generates too high consumption volatility as well as negative consumption correlation across countries.

We further evaluate the performance of the model by abstracting the model from preference shocks.

¹⁷For instance; the volatility of RER is 1.5 in Dotsey, Duarte (2008) or 1.01 in Tuesta (2013).

	Data	PCP		LCP		Par.LCP	
		Comp.	Incomp.	Comp.	Incomp.	Comp.	Incomp.
Std.dev.Rel.Y							
C	0.8438	1.48	1.39	1.22	1.19	1.37	1.16
RER	5.2801	1.90	1.72	2.58	2.39	2.22	1.79
TOT	0.9116	2.57	2.27	2.63	2.38	1.72	1.30
NER	5.1890	2.52	2.34	2.83	2.67	2.71	2.22
Autocorrelations							
Y	0.88	0.85	0.85	0.84	0.85	0.85	0.85
C	0.91	0.85	0.85	0.85	0.85	0.85	0.85
RER	0.71	0.79	0.79	0.70	0.70	0.75	0.75
TOT	0.78	0.81	0.80	0.73	0.71	0.94	0.94
NER	0.69	0.80	0.81	0.74	0.75	0.77	0.78
Cross-Correlations							
RER-NER	0.9960	0.9731	0.9691	0.9765	0.9718	0.9760	0.9715
NX- Y	-0.5045	-0.2419	-0.2405	-0.3192	-0.3213	-0.2788	-0.2779
RER-Y	-0.1022	-0.2320	-0.2255	-0.3653	-0.3458	-0.2871	-0.2745
C - C^*	0.6332	-0.2325	-0.1471	0.0129	0.0894	-0.1241	-0.0410
Y - Y^*	0.7934	0.5923	0.5994	0.5014	0.5276	0.5657	0.5796
RER-TOT	0.2267	0.8915	0.8633	-0.4822	-0.5407	0.4531	0.4053
RER-Rel.C	0.0091	-0.8611	-0.8292	-0.7654	-0.7281	-0.8410	-0.8067

 Table 2: Selected HP-filtered Moments

Table 3: Variance Decomposition (in percentages)

Vbl. Name	ε_m	ε_{m^*}	$\varepsilon_{ au}$	ε_{τ^*}	ε_{aT}	ε_{aN}	ε_{aT^*}	ε_{aN^*}	ε_a
Y	9.20	0.06	32.19	0.63	4.95	0.83	0.06	0.00	52.09
C	4.39	0.04	54.03	8.42	0.78	0.62	0.47	0.01	31.23
Y^*	0.02	7.57	0.25	28.83	0.66	0.02	5.69	0.40	56.56
C^*	0.04	3.88	8.27	49.45	1.30	0.05	2.13	0.24	34.66
RER	3.08	3.70	43.35	40.36	6.27	0.72	1.41	0.23	0.88
TOT	0.43	0.32	36.78	29.96	19.01	0.39	12.81	0.11	0.20
NER	2.59	3.09	39.41	36.56	11.76	0.41	5.28	0.16	0.75
Rel. C	1.57	1.68	47.18	47.41	0.41	0.43	0.66	0.13	0.53
NX	0.37	0.48	49.19	47.10	2.06	0.13	0.52	0.04	0.09

Note: We report the variance decomposition for the Partial LCP with incomplete markets version of the model.

	Data	PCP	LCP	Par.LCP
Std.dev.Rel.Y				
C	0.8438	0.96	1	0.96
RER	5.2801	0.82	1.22	1
TOT	0.9116	1.18	1.66	1.01
NER	5.1890	1.41	1.59	1.48
Autocorrelations				
Y	0.88	0.86	0.88	0.87
C	0.91	0.88	0.88	0.88
RER	0.71	0.66	0.55	0.61
TOT	0.78	0.72	0.73	0.93
NER	0.69	0.79	0.71	0.75
Cross-Correlations				
RER-NER	0.9960	0.8779	0.8865	0.8833
NX-Y	-0.5045	0.2605	0.1853	0.2309
RER- Y	-0.1022	0.0690	0.0219	0.0562
C - C^*	0.6332	0.8920	0.8203	0.8689
Y- Y *	0.7934	0.7391	0.8165	0.7775
RER-TOT	0.2267	0.2158	-0.8361	-0.3903
RER- $Rel.C$	0.0091	0.6609	0.8006	0.7377

Table 4: Selected HP-filtered Moments: Incomplete Markets Model without Preference Shocks

Quantitative properties of the model without preference shocks are reported in Table 4. We only report the results from the incomplete markets version of the model since the incomplete markets allocation is very similar to the complete markets one. Evidently, the model performs much worse in fitting the dynamics of RER. The volatility of RER falls substantially in this experiment. The model also fails to account for its persistence. The existence of a permanent shock is increasing the volatility of output and since it is global, it cannot be insured away, so consumption is also very volatile. The lack of insurance reduces the reaction of relative prices to the shock, decreasing the volatility of international variables. On the other hand, having a dominant global shock improves the cross country correlations of output and consumption. Table 5 shows that the global shock drives the fluctuations of most of the aggregate variables.

Having a dominant global TFP shock improves the cross country correlations of aggregate variables but this comes with a trade-off; the model fails to capture both volatility and persistence of RER, net exports become pro-cyclical and there is high correlation between RER and relative consumption.

The improvement in fitting the moments of RER in our model is result of having preference shocks as predominant source of fluctuations. But this then leads to failure to capture cross country consumption correlation and the correlation between relative consumption and RER.

Our analysis shows that, presenting a comprehensive theoretical framework is important for fitting the moments of RER to the data. But this result is at the expense of some other features of the data. Our model brings several frictions and rigidities together with a rich shock structure that have previously suggested in the literature to address specific puzzles in the data mostly. When we incorporate these channels to a single theoretical set-up to evaluate the overall performance of these models, we find that there is an improvement in the fit of the RER, but making some other moments worse. We showed that the results are conditional on the predominant source of fluctuations.

Vbl.	ε_m	ε_{m^*}	ε_{aT}	ε_{aN}	ε_{aT^*}	ε_{aN^*}	ε_a
Y	13.69	0.08	7.36	1.24	0.09	0.00	77.53
C	11.69	0.10	2.09	1.65	1.26	0.02	83.19
Y^*	0.03	10.68	0.93	0.03	8.02	0.56	79.75
C^*	0.09	9.17	3.07	0.11	5.05	0.56	81.96
RER	18.9	22.7	38.51	4.40	8.68	1.42	5.39
TOT	1.30	0.95	57.17	1.16	38.51	0.32	0.59
NER	10.77	12.86	48.94	1.69	21.97	0.65	3.12
Rel. C	29.07	31.08	7.52	7.95	12.12	2.45	9.81
NX	10.03	12.95	55.50	3.64	14.16	1.18	2.54

Table 5: Variance Decomposition (in percentages): Incomplete Markets Model without Preference Shocks

Note: We report the variance decomposition for the Partial LCP with incomplete markets version of the model.

5 Transmission of Productivity Shocks

In this section, we look at the dynamic responses of the RER and its components following supply side innovations. By doing so, we will be able to evaluate the transmissions of shocks. We are only focussing on sectoral productivity improvements to be able to evaluate the implications of Balassa-Samuelson proposition. We are using the incomplete markets version of the model with polar cases of pass through (d = 1 and d = 0).

Figure 2 depicts the impulse responses to a productivity improvement in the traded goods sector in home country. Following the shock, the RER appreciates. The source of appreciation is the increase in internal relative prices conforming Balassa-Samuelson proposition as can be seen from the decrease in internal RER. A productivity improvement in the traded sector, on one hand, deteriorates the terms of trade, as exports become cheaper, and causes an increase in home bias component of RER. But on the other hand, it increases relative prices of non-traded goods to traded goods. In our model, the final impact on RER is an appreciation as the change in internal relative prices dominates the terms of trade impact. The incomplete market structure reduces the movements in relative prices through asset accumulation, hence lowering the movements in terms of trade, in addition to the implicit income effect arising from the productivity innovation in traded sector.

The degree of pass through has implications on home bias and pricing to market channels. Notice that, under PCP, the pricing to market channel becomes inconsequential. In the LCP case, however, the pricing to market channel falls when a positive productivity shock in traded sector hits the economy. This is because of the dominant nominal exchange rate appreciation combines with a high degree of home bias and staggered price setting. An exchange rate appreciation decreases the home currency price of exports while relative prices remain the same. As a result, RER appreciates through this channel.

The effects of a positive productivity shock to the non-tradable sector is shown in Figure 3. The RER depreciates when a productivity increase occurs in non-traded sector despite the nominal exchange rate appreciation. This comes from the strong depreciation of the internal RER. Both the terms of trade and the pricing to market channel decrease as the home produced traded goods become more expensive now.

The impulse response analysis we present here, is not only the result of our multi-sector structure but also the strong wealth effects arising from the model set-up and our calibration. RER and internal relative prices move in the same direction as a consequence of relatively high share of non-traded goods in the consumption



Figure 2: Positive technology shock to the US Traded Goods Sector



Figure 3: Positive technology shock to the US Non-Traded Goods Sector

basket, a low elasticity between non-traded and traded goods and a high elasticity between traded goods. Therefore, the overall dynamic adjustment of the RER is in line with Balassa-Samuelson explanation. Similar transmission mechanism has also been presented in several general equilibrium frameworks such as Benigno and Thoenissen (2008) and Selaive and Tuesta (2003). Figures 2 and 3 suggest that the transmission will depend on the relative dominance of the innovation. This result is line with our findings on the quantitative properties of the model. The results we obtain from a general equilibrium model is conditional on the predominant source of fluctuations.

6 Robustness

To test the sensitivity of our results to the calibration, in this section, we are presenting the results of the robustness exercises. We set the parameters between low and high values and obtain a range of model moments. To do so, we run two separate Monte-Carlo experiments. The Monte Carlo simulation generates parameter values by drawing randomly from a uniform distribution over the parameter range. In the first Monte Carlo experiment, we set the preference parameters from low values to high values and maintain the monetary policy parameters and the shock parameters as in our baseline calibration. For the second one, we investigate the importance of parameterisation of the shocks and monetary policy coefficients leaving the other parameters as in our baseline calibration. For both experiments, we report the median values of the calculated moments along with the 10 and 90 percentiles.

The Monte Carlo parameters of the first experiment are listed in the Table 6¹⁸. For the habit parameter, we choose a very low value which almost removes the habit formation from consumption. And as a high value we choose 0.7. We let the degree of pass through to vary between the polar cases of pass through. The share of traded goods are set such that non-traded goods can have higher or lower share in aggregate consumption basket. The degree of home bias ranges from no home bias to really high degrees of home bias. As discussed previously the calibration of elasticity of substitution between home and foreign produced traded goods is controversial. We set it between a very low and a high value to accommodate the different estimations from the literature. The elasticity of substitution between non-traded and traded goods ranges from a low value of 0.13, based on the estimations of Rabanal and Tuesta (2013), to a value above unity, 1.2. We set the range for the proportion of Calvo wage/price setters from almost fully flexible to almost completely rigid within a period. The range for the proportion of backward looking firms are chosen such that the forward looking behaviour still dominates in Phillips Curve. The proportion of backward looking households ranges from almost zero, based on the empirical estimates of Rabanal and Tuesta (2010), to a high value, 0.66 as in Smets and Wouters (2002).

The results obtained from the Monte Carlo experiment on preference parameters are shown in Table 7. Our experiment proves the sensitivity of the results to the calibration of the model. Looking at the intervals, one can realise that the range is quite large for almost all parameters. In fact, the cross correlation of variables in most cases vary from positive values to negative values.

Focussing on the median values, the persistence of RER generated by the model actually perfectly matches with the persistence obtained from the data. Similar to the results from our benchmark, the model generates excess consumption volatility and the volatility of RER is lower than the observed value. Our Monte Carlo experiment shows that the volatility of terms of trade is quite sensitive to the parameterisation of preference

 $^{^{18}\}mathrm{The}$ parameters which are not listed in Table 6 are kept as in the baseline calibration.

Parameter	Range	Description
$h = h^*$	[0.05, 0.7]	habit persistence
$d = d^*$	[0, 1]	degree of pass through
$\alpha = \alpha^*$	[0.2, 0.6]	share of traded goods in total consumption
$\omega=\omega^*$	[0.5, 0.95]	degree of home bias
heta	[0.5, 5]	elasticity of substitution: home and foreign tradable goods
u	[0.13, 1.2]	elasticity of substitution: tradable and non-tradable goods
$\xi_p = \xi_p^*$	[0.01, 0.9]	Calvo prices
$\varsigma_p = \varsigma_p^*$	[0.2, 0.49]	proportion of bwd. looking firms
$\xi_w = \hat{\xi}_w^*$	[0.01, 0.9]	Calvo wages
$\varsigma_w = \varsigma_w^*$	[0.1, 0.66]	proportion of bwd. looking households
Monte Carlo draws	200	

Table 6: Monte Carlo Parameter Ranges: Preferences

 Table 7: Robustness: Preference Parameters

	Auto	ocorrelations	Stand	ard Deviation
	Data	Model	Data	Model
Y	0.88	0.75	0.0057	0.0084
		[0.64, 0.85]		[0.0072, 0.0098]
C	0.91	0.78	0.0044	0.01
		[0.68, 0.87]		[0.0088, 0.012]
RER	0.71	0.73	0.03	0.013
		[0.64, 0.77]		[0.0094, 0.019]
TOT	0.78	0.86	0.0051	0.014
		[0.81, 0.90]		[0.003, 0.021]
NER	0.69	0.79	0.029	0.018
		[0.68, 0.87]		[0.015, 0.025]
	Cross	s-Correlations		
	Data	Model		
RER- NER	0.996	0.8669		
		[0.4659, 0.9448]		
NX-Y	-0.5045	-0.0929		
		[-0.3583, 0.1891]		
RER-Y	-0.1022	-0.1265		
		[-0.2571, 0.1646]		
$C-C^*$	0.6332	-0.1360		
		[-0.3042, 0.1471]		
$Y - Y^*$	0.7934	0.4096		
		[0.2208, 0.5542]		
RER-TOT	0.2267	0.5217		
		[-0.0278, 0.7813]		
RER- $Rel.C$	0.0091	-0.6820		
		[-0.8022, -0.4339]		

Parameter	Range	Description
Taylor Rule Coefficients		
$\Gamma_i = \Gamma_{i^*}$	[0.3, 0.9]	interest rate smoothing
$\Gamma_y = \Gamma_{y^*}$	[0.03, 1.6]	response to output
$\Gamma_{\pi} = \Gamma_{\pi^*}$	[1, 3]	response to inflation
Persistence of Shocks		
$ ho_{ au}= ho_{ au^*}$	[0.1, 0.9]	Preference
$ \rho_{aT} = \rho_{aT^*} $	[0.1, 0.9]	Technology shock: tradable sector
$\rho_{aN} = \rho_{aN^*}$	[0.1, 0.9]	Technology shock: non-tradable sector
$ ho_{x^*}$	[0, 0.9]	persist. of global shock- Euro Area
Standard Deviation of Shocks		
$\varepsilon_{ au} = \varepsilon_{ au^*}$	[0.01, 0.03]	Preference
$\varepsilon_m = \varepsilon_{m^*}$	[0.001, 0.03]	Monetary
$\varepsilon_{aT} = \varepsilon_{aT^*}$	[0.003, 0.03]	Technology shock: tradable sector
$\varepsilon_{aN} = \varepsilon_{aN^*}$	[0.001, 0.01]	Technology shock: non-tradable sector
$arepsilon_a$	[0.003, 0.03]	Global Technology shock
Monte Carlo draws	200	

Table 8: Monte Carlo Parameter Ranges: Shocks and Monetary Policy

parameters considering the large range in between the interval. In fact, the correct volatility coincides into the interval. Our experiment shows that the cross-correlations generated with our benchmark calibration do not differ substantially from the median values. The correlation between the relative consumption and RER is negative and very low as in our benchmark results.

As a result of the sensitivity of the results to the calibration of preference parameters, we further explore the robustness of our results by making a second Monte Carlo experiment on the shock processes and the monetary policy coefficients. We now investigate the sensitivity of our results to the persistence of the shocks as well as their standard deviations¹⁹. The list of parameter values for this Monte Carlo experiment are shown in Table 8. We set the range of the Taylor rule coefficients such that the upper bound refer to a relatively aggressive monetary policy. For the productivity shocks, we set the persistence of the shocks from almost purely transitory to very close to a unit root. The large range of standard deviation of shocks reflects the uncertainty on the actual values of these parameters²⁰.

The results of the Monte Carlo simulation on the shock processes and the monetary policy coefficients are shown in Table 9. Our Monte Carlo simulations on the calibration of the shock processes, presents substantial differences from the results that we obtained from our benchmark calibration proving the important role of the specification of the innovations. The model does worse in matching with the autocorrelation of the variables. The existence of a global permanent TFP shock causes excess output and consumption volatility in this experiment. The driving source of fluctuations is very important in terms of the variability of the variables. As in our first Monte Carlo experiment, the parameter values vary substantially across the range. The cross correlation between nominal and real exchange rate, with this experiment, is consistently high and even the range within the interval is quite small. On the other hand, this correlation obtained from the first

 $^{^{19}{\}rm While}$ doing this exercise we set the covariance between the shocks to zero.

 $^{^{20}}$ There is a big diversity in the literature on the calibration of standard deviations of the shocks. For instance, CKM chose the standard deviation of the monetary shocks in order to match with volatility of the GDP which implies a large standard deviation. On the other hand, Ireland (2002) estimated a relatively low standard deviation of monetary policy shock for US, 0.0022.

	Auto	ocorrelations	Standa	rd Deviation
	Data	Model	Data	Model
Y	0.88	0.80	0.0057	0.03
		[0.73, 0.84]		[0.02, 0.05]
C	0.91	0.82	0.0044	0.03
		[0.73, 0.85]		[0.01, 0.05]
RER	0.71	0.41	0.03	0.038
		[0.3, 0.57]		[0.02, 0.07]
TOT	0.78	0.90	0.0051	0.01
		[0.88, 0.92]		[0.0065, 0.02]
NER	0.69	0.43	0.029	0.04
		[0.31, 0.58]		[0.026, 0.08]
	Cross	s-Correlations		
	Data	Model		
RER- NER	0.996	0.9974		
		[0.9922, 0.9988]		
NX- Y	-0.5045	0.1027		
		[-0.0851, 0.3966]		
RER- Y	-0.1022	0.2736		
		[-0.0432, 0.6594]		
$C-C^*$	0.6332	0.6966		
		[0.3116, 0.8916]		
$Y - Y^*$	0.7934	0.6432		
		[0.1783, 0.8875]		
RER-TOT	0.2267	0.3690		
		[0.1750, 0.4805]		
RER- $Rel.C$	0.0091	0.5629		
		[0.0975, 0.7822]		

Table 9: Robustness: Shocks and Monetary Policy Parameters

Note: We run the simulations for partial LCP (d = 0.5) with incomplete markets version of the model.

experiment was lower (see Table 6). Therefore, the correlation between these two variables are more related to the parameterisation of preference parameters. However, when we look at the correlation between RER and relative consumption and consumption across countries, the situation is the opposite; they both vary only between positive. The sign of cross country consumption correlation is related to the relative dominance of the innovations. When we define loops for standard deviations we obtain positive cross-correlation but when the fluctuations of consumption is driven by preference shocks, as in our first experiment, the crosscountry correlation is negative. This experiment also proves that a theoretical set-up which incorporates non-traded goods and incomplete markets at international level is not sufficient enough to account for the consumption-RER anomaly. Our model's ability to generate a negative cross correlation is conditional on the driving source of fluctuations in the model. Our sensitivity analysis certifies the importance of predominant driving source of the fluctuations for the general equilibrium models to account for the properties of the data.

7 Conclusion

In this paper, we revisited the classical question about whether general equilibrium models are well equipped to match with the observed behaviour of RER within a rich framework that incorporates most of the frictions in the DSGE framework. We did so by developing a two country general equilibrium model with non-traded goods, home bias, incomplete markets, partial degrees of pass through and habit formation in consumption as well as nominal rigidities both in the goods and labour markets. In addition to our rich theoretical model set-up, we analysed the importance of source of fluctuations through incorporating monetary shocks, preference shocks, country and sector specific technology shocks as well as a permanent global technology shock into our framework.

We evaluated the overall performance of the model rather than focussing on a certain puzzle and we investigated whether fixing one puzzle leads to failures in terms of performance of other moments. We argue that the problem of general equilibrium models matching the behaviour of RER can be addressed successfully through a encompassing theoretical set-up and a careful calibration, however this is at the cost of some other moments. The problem is not only replicating the behaviour of RER but it is to fit the moments of RER without compromising from the fit of some other variable moments to data. We show that even with a comprehensive theoretical framework the trade-off remains. In particular, our model successfully address the persistence of RER and also generates substantial volatility. It accounts for the counter-cyclical behaviour of the net exports and RER and in our model real and nominal exchange rates are highly correlated as observed in the data. However it fails to produce the correct consumption volatility, positive cross-country consumption correlation and the correlation between the RER and relative consumption. We further find that the ability of the model to account for the behaviour of macroeconomic variables closely related to the predominant source of fluctuations.

Appendices

A Data Appendix

The time span of the data covers the period 1999:Q1-2012:Q3. The following series for the US and the Euro Area are used for the analysis:

- The nominal exchange rate data is taken from IMF, IFS data base. It is defined as US Dollars per Euro in market rate. It is converted to an index by using 2005 as base year.
- To calculate the RER we used the CPI of the US and the Euro area. The CPI of the US is taken from the OECD's Main Economic Indicators database and the CPI of the Euro Area is taken from the EUROSTAT. Both series are for all items and invoiced in national currency. 2005 is taken as 100. We calculate the bilateral RER between euro and the US dollar by multiplying Euro Area CPI with nominal exchange rate and dividing it by US CPI.
- Terms of trade is obtained from the Bureau of Labour Statistic (BLS) by taking the ratio of import prices from EU to the aggregate export price index of US. Both price indices are in US dollars and as there is no price information about imports from the Euro Area, we use EU as an approximate. We transformed the base year from 2000 to 2005 for both series.
- The series of output and consumption for both US and Euro Area were taken from the OECD's Main Economic Indicators database. We use seasonally adjusted index series (2005=100), in national currency. Output is the GDP by total expenditure and consumption is the private final consumption expenditure.
- Net exports are constructed by dividing the trade balance of US with Euro Area by the nominal GDP of USA. Both series are in US dollars. The GDP series is taken from the OECD's Main Economic Indicators database which is seasonally adjusted and computed by expenditure approach in current prices. The trade balance taken from the BLS. While computing the net exports we kept both series in levels.
- As explained previously, for the productivity innovations we took disaggregated data for sectors of the Euro Area and US of about 30 sectors and then aggregated them by large sectors. The data for this calculation is in annual frequency and covers the period 1981-2007. Through calculating value added shares of sectors, we constructed TFP growth series from the TFP estimates of Euklems. It is assumed that agriculture, mining and manufacturing are traded and the remaining are non-traded. To be consistent with the model specification, we converted the growth rate series into an index. As a next step, we estimated a VAR with a constant and a deterministic trend in order to specify the variance-covariance structure of the shocks as well as to calculate the shock correlations. We restricted the VAR to have international spillovers only through traded sectors. We used the variance-covariance matrix of the residual and converted the obtained shock variances from annual to quarterly frequency through dividing them by four. Finally, we chose the specification of the global permanent TFP shock by using the same data set for total industries. We tested the existence of a unit root in both TFP series of Euro Area and US by conducting an ADF test. They both found to be non-stationary. We

further investigated the structure of the TFP series by estimating an AR(1) process. This allowed us to see whether the shocks are whether a unit root or a growth rate shock. Our estimations showed that the technology process of US is a unit root while for Euro Area it is persistent in growth rates. We calculated the quarterly counterpart of the persistence parameter for Euro Area, by taking the 0.25 to the power of the value obtained from the estimations of annual data.

• The correlation between relative consumption and RER is calculated by using the logged series in the following way:

$$Corr[rer_t = (c_t^{US} - c_t^{EA})]$$

B Log-Linearised model

In this appendix, we list the normalised log-linear equations of the model. The model is log-linearised around the steady state. The stationarised variables will be denoted by a hat²¹. For instance; $\hat{c}_t = c_t - x_t$.

• Euler equation and UIP condition²²:

$$\hat{c}_{t} = \frac{1}{1+h} E_{t} \hat{c}_{t+1} + \frac{h}{1+h} \hat{c}_{t-1} - \frac{1-h}{(1+h)} (i_{t} - E_{t} \pi_{t+1}) - \frac{1-h}{(1+h)} (\tau_{t+1} - \tau_{t}) + \frac{1}{1+h} (x_{t+1} - x_{t}) - \frac{h}{1+h} (x_{t} - x_{t-1})$$
(B.1)

$$\hat{c}_{t}^{*} = \frac{1}{1+h} E_{t} \hat{c}_{t+1}^{*} + \frac{h}{1+h} \hat{c}_{t-1}^{*} - \frac{1-h}{(1+h)} (i_{t}^{*} - E_{t} \pi_{t+1}^{*}) - \frac{1-h}{(1+h)} (\tau_{t+1}^{*} - \tau_{t}^{*}) + \frac{1}{1+h} (x_{t+1} - x_{t}) - \frac{h}{1+h} (x_{t} - x_{t-1})$$
(B.2)

where $\pi_{t+1} = p_{t+1} - p_t$ and $\pi^*_{t+1} = p^*_{t+1} - p^*_t$

$$s_{t+1} - s_t = i_t - i_t^* + \delta b_t \tag{B.3}$$

• Demand Functions

$$rer_t = \frac{1}{1-h} [(\hat{c}_t - h\hat{c}_{t-1}) - (\hat{c}_t^* - h\hat{c}_{t-1}^*)] - (\tau_t - \tau_t^*)$$

Note that, the permanent global TFP improvement does not enter into the risk sharing condition as we assumed symmetry in habit persistence (i.e. $h = h^*$).

²¹To have balanced growth in the model, we set the risk aversion parameter to one ($\rho = 1$).

 $^{^{22}}$ Note that when only state-contingent nominal bonds are traded, i.e. international markets are complete, through households intertemporal decision we obtain a perfect risk sharing condition:

$$\hat{c}_t^N = \nu(p_t - p_t^N) + \hat{c}_t \tag{B.4}$$

$$\hat{c}_t^T = \nu(p_t - p_t^T) + \hat{c}_t \tag{B.5}$$

$$\hat{c}_t^H = \theta(p_t^T - p_t^H) + \hat{c}_t^T \tag{B.6}$$

$$\hat{c}_t^F = \theta(p_t^T - p_t^F) + \hat{c}_t^T \tag{B.7}$$

$$\hat{c}_t^{N^*} = \nu(p_t^* - p_t^{N^*}) + \hat{c}_t^*$$
(B.8)

$$\hat{c}_t^{T^*} = \nu(p_t^* - p_t^{T^*}) + \hat{c}_t^*$$
(B.9)

$$\hat{c}_t^{H^*} = \theta(p_t^{T^*} - p_t^{H^*}) + \hat{c}_t^{T^*}$$
(B.10)

$$\hat{c}_t^{F^*} = \theta(p_t^{T^*} - p_t^{F^*}) + \hat{c}_t^{T^*}$$
(B.11)

- The Price Indices
 - Consumer Price Indices

$$p_t = \alpha p_t^T + (1 - \alpha) p_t^N \tag{B.12}$$

$$p_t^* = \alpha^* p_t^{T^*} + (1 - \alpha^*) p_t^{N^*}$$
(B.13)

– Price Indices for Tradable Goods

$$p_t^T = \omega p_t^H + (1 - \omega) p_t^F \tag{B.14}$$

$$p_t^{T^*} = \omega^* p_t^{F^*} + (1 - \omega^*) p_t^{H^*}$$
(B.15)

Export Prices

As some firms engage in local currency pricing, some exports are home currency priced and some are foreign currency priced. The export price indices have the following form:

$$p_t^F = d^* p_t^{LCP} + (1 - d^*) (p_t^{F^*} + s_t)$$
(B.16)

$$p_t^{H^*} = d \, p_t^{LCP^*} + (1 - d) \, (p_t^H - s_t) \tag{B.17}$$

(B.18)

• Production Functions

$$\hat{y}_t^H = a_t^H + l_t^H, \quad \hat{y}_t^N = a_t^N + l_t^N$$
 (B.19)

$$\hat{y}_t^F = a_t^F + l_t^F + x_t^* - x_t, \quad \hat{y}_t^{N^*} = a_t^{N^*} + l_t^{N^*} + x_t^* - x_t \tag{B.20}$$

• Price Setting

Let's define the inflation variables: $\pi_t^N = p_t^N - p_{t-1}^N$, $\pi_t^H = p_t^H - p_{t-1}^H$, $\pi_t^{N^*} = p_t^{N^*} - p_{t-1}^{N^*}$, $\pi_t^{F^*} = p_t^{F^*} - p_{t-1}^{F^*}$, $\pi_t^{LCP} = p_t^{LCP} - p_{t-1}^{LCP}$, $\pi_t^{LCP^*} = p_t^{LCP^*} - p_{t-1}^{LCP^*}$

- home country non-traded sector

$$\pi_t^N = \gamma^f \, \pi_{t+1}^N \, + \, \gamma^b \, \pi_{t-1}^N \, + \, \lambda \, \hat{mc}_t^N \tag{B.21}$$

where

$$\hat{mc}_t^N = \hat{w}_t + p_t - p_t^N - a_t^N$$

where \hat{w} is the normalised real wage.

- home country traded sector

$$\pi_t^H = \gamma^f \, \pi_{t+1}^H + \, \gamma^b \, \pi_{t-1}^H + \lambda \, \hat{mc}_t^H \tag{B.22}$$

where

$$\hat{m}c_t^H = \hat{w}_t + p_t - p_t^H - a_t^H$$

- home country locally priced imports

$$\pi_t^{LCP} = \gamma^{f^*} \pi_{t+1}^{LCP} + \gamma^{b^*} \pi_{t-1}^{LCP} + \lambda^* \hat{mc}_t^F$$
(B.23)

where

$$\hat{mc}_{t}^{F} = \hat{w}_{t}^{*} + p_{t}^{*} - p_{t}^{LCP} - a_{t}^{F} + s_{t} - x_{t}^{*} + x_{t}$$

- foreign country non-traded sector

$$\pi_t^{N^*} = \gamma^{f^*} \pi_{t+1}^{N^*} + \gamma^{b^*} \pi_{t-1}^{N^*} + \lambda^* \hat{m} c_t^{N^*}$$
(B.24)

where

$$\hat{m}c_t^{N^*} = \hat{w}_t^* + p_t^* - p_t^{N^*} - a_t^{N^*} - x_t^* + x_t$$

- foreign country traded sector

$$\pi_t^{F^*} = \gamma^{f^*} \pi_{t+1}^{F^*} + \gamma^{b^*} \pi_{t-1}^{F^*} + \lambda^* \hat{mc}_t^{F^*}$$
(B.25)

where

$$\hat{mc}_t^{F^*} = \hat{w}_t^* + p_t^* - p_t^{F^*} - a_t^F - x_t^* + x_t$$

- foreign country locally priced imports

$$\pi_t^{LCP^*} = \gamma^f \, \pi_{t+1}^{LCP^*} + \gamma^b \, \pi_{t-1}^{LCP^*} + \lambda^* \, \hat{mc}_t^{H^*} \tag{B.26}$$

where

$$\hat{mc}_{t}^{H^{*}} = \hat{w}_{t} + p_{t} - p_{t}^{LCP^{*}} - a_{t}^{H} - s_{t}$$

The corresponding composite parameters are:

$$\gamma^{f} = \frac{\beta \xi_{p}}{\xi_{p} + (\varsigma_{p}(1 - \xi_{p}(1 - \beta)))},$$
$$\gamma^{b} = \frac{\varsigma_{p}}{\xi_{p} + (\varsigma_{p}(1 - \xi_{p}(1 - \beta)))},$$
$$\lambda = \frac{(1 - \beta \xi_{p})(1 - \xi_{p}(1 - \beta))}{\xi_{p} + (\varsigma_{p}(1 - \xi_{p}(1 - \beta)))}$$

$$\gamma^{f^*} = \frac{\beta \, \zeta_{p^*}}{\xi_{p^*} + (\zeta_{p^*}(1 - \xi_{p^*}(1 - \beta)))},$$
$$\gamma^{b^*} = \frac{\zeta_{p^*}}{\xi_{p^*} + (\zeta_{p^*}(1 - \xi_{p^*}(1 - \beta)))},$$
$$\lambda^* = \frac{(1 - \beta \, \xi_{p^*})(1 - \xi_{p^*})(1 - \zeta_{p^*})}{\xi_{p^*} + (\zeta_{p^*}(1 - \xi_{p^*}(1 - \beta)))}$$

• Wage Setting

The wage inflation is:

- wage dynamics in home country

$$\pi_t^w = \gamma^{f,w} \, \pi_{t+1}^w + \gamma^{b,w} \, \pi_{t-1}^w - \lambda^w \, \widehat{mrs}_t \tag{B.27}$$

where

$$\widehat{mrs}_{t} = \hat{w}_{t} - \eta \, l_{t} - \left(\frac{1}{1-h}\right) \left(\hat{c}_{t} - h \, \hat{c}_{t-1}\right) - \frac{h}{1-h} \left(x_{t} - x_{t-1}\right)$$

– wage dynamics in foreign country

$$\pi_t^{w^*} = \gamma^{f,w^*} \pi_{t+1}^{w^*} + \gamma^{b,w^*} \pi_{t-1}^{w^*} - \lambda^{w^*} \widehat{mrs}_t^*$$
(B.28)

where

where

$$\widehat{mrs}_{t}^{*} = \hat{w}_{t}^{*} - p_{t}^{*} - \eta l_{t}^{*} - (\frac{1}{1-h}) \left(\hat{c}_{t}^{*} - h \hat{c}_{t-1}^{*}\right) - \frac{h}{1-h} \left(x_{t} - x_{t-1}\right)$$
with $\pi_{t}^{w} = \hat{w}_{t} - \hat{w}_{t-1} + \pi_{t} + (x_{t} - x_{t-1})$ and $\pi_{t}^{w^{*}} = \hat{w}_{t}^{*} + \pi_{t}^{*} - \hat{w}_{t-1}^{*} + (x_{t} - x_{t-1}).$

The corresponding composite parameters are:

$$\gamma^{f,w} = \frac{\beta \xi_w}{((1+\eta \,\sigma_w)\xi_w) + (\varsigma_w(1-\xi_w(1-\beta)))},$$
$$\gamma^{b,w} = \frac{\varsigma_w}{((1+\eta \,\sigma_w)\xi_w) + (\varsigma_w(1-\xi_w(1-\beta)))},$$
$$\lambda^w = \frac{(1-\beta \,\xi_w)(1-\xi_w)(1-\varsigma_w)}{((1+\eta \,\sigma_w)\xi_w) + (\varsigma_w(1-\xi_w(1-\beta)))}$$

$$\gamma^{f,w^*} = \frac{\beta \,\xi_{w^*}}{\left((1+\eta \,\sigma_w)\xi_{w^*}\right) + \left(\varsigma_{w^*}(1-\xi_{w^*}(1-\beta))\right)},$$

$$\gamma^{b,w^*} = \frac{\varsigma_{w^*}}{\left((1+\eta\,\sigma_w)\xi_{w^*}\right) + \left(\varsigma_{w^*}(1-\xi_{w^*}(1-\beta))\right)},$$

$$\lambda^{w^*} = \frac{(1 - \beta \,\xi_{w^*})(1 - \xi_{w^*})(1 - \varsigma_{w^*})}{((1 + \eta \,\sigma_w)\xi_{w^*}) + (\varsigma_{w^*}(1 - \xi_{w^*}(1 - \beta)))}$$

• Current Account

$$\beta b_t - b_{t-1} = (1 - \omega^*) \alpha^* \frac{(1-n)}{n} (\hat{c}_t^{H^*} + p_t^{H^*} + s_t - p_t) + \omega \alpha (\hat{c}_t^H + p_t^H - p_t) - \alpha (\hat{c}_t^T + p_t^T - p_t)$$
(B.29)

• Monetary Policy

$$i_{t} = \Gamma_{i-1}i_{t-1} + (1 - \Gamma_{i-1})\Gamma_{\pi_{t}}\pi_{t} + (1 - \Gamma_{i-1})\Gamma_{y_{t}}\hat{y}_{t} + \varepsilon_{m,t}$$
(B.30)

$$i_t^* = \Gamma_{i-1}^* i_{t-1}^* + (1 - \Gamma_{i-1}^*) \Gamma_{\pi_t}^* \pi_t^* + (1 - \Gamma_{i-1}^*) \Gamma_{y_t}^* \hat{y}_t^* + \varepsilon_{m,t}^*$$
(B.31)

where

$$\varepsilon_{m,t} \sim N(0, \sigma_m^2), \qquad \varepsilon_{m,t}^* \sim N(0, \sigma_{m^*}^2)$$

- Market Clearing
 - Non-traded sector

$$\hat{y}_t^N = \hat{c}_t^N, \qquad \qquad \hat{y}_t^{N^*} = \hat{c}_t^{N^*}$$
(B.32)

Traded sector

$$\hat{y}_t^H = \omega \, \hat{c}_t^H + \frac{(1-n)(1-\omega^*)\alpha^*}{n\,\alpha} \, \hat{c}_t^{H^*} \tag{B.33}$$

$$\hat{y}_t^F = \omega^* \hat{c}_t^{F^*} + \frac{n(1-\omega)\alpha}{(1-n)\alpha^*} \hat{c}_t^F$$
(B.34)

• Output and Labour supply

- output

$$\hat{y}_t = \hat{c}_t + \frac{(1-\omega^*)\,\alpha^*\,(1-n)}{n}\,\hat{c}_t^{H^*} - (1-\omega)\alpha\,\hat{c}_t^F \tag{B.35}$$

$$\hat{y}_t^* = \hat{c}_t^* + \frac{(1-\omega)\,\alpha\,n}{1-n}\,\hat{c}_t^F - (1-\omega^*)\alpha^*\,\hat{c}_t^{H^*} \tag{B.36}$$

- labour supply

$$l_t = \alpha l_t^H + (1 - \alpha) l_t^N \tag{B.37}$$

$$l_t^* = \alpha^* l_t^F + (1 - \alpha^*) l_t^{N^*}$$
(B.38)

- Definition of International Variables
 - net exports

$$nx_{t} = \left(\frac{(1-\omega^{*})\alpha^{*}(1-n)}{n}\right) (\hat{c}_{t}^{H^{*}} + p_{t}^{H^{*}} + s_{t} - p_{t}) - (1-\omega)\alpha (\hat{c}_{t}^{F} + p_{t}^{F} - p_{t})$$
(B.39)

- terms of trade

$$tot_t = p_t^F - p_t^{H^*} - s_t (B.40)$$

- real exchange rate

$$rer_t = p_t^* + s_t - p_t \tag{B.41}$$

- decomposition of real exchange rate
 - * internal relative price movements channel

$$rerint_t = (1 - \alpha^*)(p_t^{N^*} - p_t^{T^*}) - (1 - \alpha)(p_t^N - p_t^T)$$
(B.42)

* pricing to market channel

$$rerptm_t = (1 - \omega^*)(s_t + p_t^{H^*} - p_t^H) + \omega^*(s_t + p_t^{F^*} - p_t^F)$$
(B.43)

* home bias channel

$$rertot_t = (\omega - (1 - \omega^*))(p_t^F - p_t^H)$$
(B.44)

- Shock Processes
 - preference shocks

$$ln(\tau_t) = \rho^{\tau} ln(\tau_{t-1}) + \varepsilon_t^{\tau} \tag{B.45}$$

$$ln(\tau_t^*) = \rho^{\tau^*} ln(\tau_{t-1}^*) + \varepsilon_t^{\tau^*}$$
(B.46)

technology shocks

$$a_t^N = \rho^{a^N} a_{t-1}^N + \varepsilon_t^{a^N} \tag{B.47}$$

$$a_t^H = \rho^{a^T} a_{t-1}^H + \varepsilon_t^{a^H} \tag{B.48}$$

$$a_t^{N^*} = \rho^{a^{N^*}} a_{t-1}^{N^*} + \varepsilon_t^{a^{N^*}}$$
(B.49)

$$a_t^F = \rho^{a^r} a_{t-1}^F + \varepsilon_t^{a^r}$$
(B.50)

- global technology shock

$$ln(G_t) = ln(G_{t-1}) + \varepsilon_{g,t} \tag{B.51}$$

$$ln(X_t) = ln(G_t) \tag{B.52}$$

$$ln(X_t^*) - ln(G_t) = \rho_{x^*}(ln(X_{t-1}^*) - ln(G_{t-1}))$$
(B.53)

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