

Jump, Robert

Working Paper

A Fair Wage Explanation of Labour Market Volatility

School of Economics Discussion Papers, No. 1413

Provided in Cooperation with:

University of Kent, School of Economics

Suggested Citation: Jump, Robert (2014) : A Fair Wage Explanation of Labour Market Volatility, School of Economics Discussion Papers, No. 1413, University of Kent, School of Economics, Canterbury

This Version is available at:

<https://hdl.handle.net/10419/105715>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

University of Kent
School of Economics Discussion Papers

A Fair Wage Explanation of Labour Market Volatility

Robert Jump

September 2014

KDPE 1413



A Fair Wage Explanation of Labour Market Volatility*

Robert Jump[†]

School of Economics, University of Kent

September 2014

Abstract

This paper proposes an explanation for observed differences in the business cycle volatility of employment and unemployment across a sample of OECD countries. Using an incomplete markets variant of the fair wage real business cycle model, increases in the gross replacement rate of public unemployment insurance are shown to increase the volatility of employment, and decrease the volatility of real wages, *ceteris paribus*. For a sample of 14 OECD countries over the period 1985-2005, the gross replacement rate is found to be positively correlated with the business cycle volatility of hours worked, lending support to the argument. A secondary contribution, which may be of some use in the incomplete markets literature, is the simple manner in which unemployment is endogenised in the model.

Keywords: Fair Wages, Unemployment, Incomplete Markets.

JEL Codes: E24, E32, J64, J65.

*The research for this paper was undertaken at the School of Economics, University of Kent, with funding provided by the ESRC (grant number: ES/J500148/1). I am indebted to Mathan Satchi for his ongoing help, and Jagjit Chadha, Paul Levine, and Keisuke Otsu for comments on an earlier draft that have greatly improved the finished article. I would also like to thank seminar participants at the University of Kent and WEHIA 2014 for their helpful comments and suggestions.

[†]School of Economics, University Of Kent, Canterbury CT2 7NP, UK. Email: rgj4@kent.ac.uk, tel: +44 (0)1227 827952.

1 Introduction

This paper proposes an explanation for observed differences in the business cycle volatility of employment and unemployment across a sample of OECD countries, based on an incomplete markets variant of the fair wage real business cycle model. There are two justifications for a study of this sort. First, the volatility of the employment and unemployment rates are intrinsically interesting - unemployment remains a significant problem across the OECD, and results in significant welfare losses for those affected by it (see, for instance, Di Tella et al 2001). Volatility in the employment and unemployment rates over the business cycle will, in all likelihood, lead to greater uncertainty in regards to individual unemployment histories, which may be expected to aggravate associated welfare losses. Second, whilst an extensive literature on this topic in the context of search and matching models has emerged over the past decade, there is little consensus on the determinants of labour market volatility, or their empirical relevance. This paper adds to the literature from a different viewpoint, using the fair wage theory of Solow (1979), Akerlof (1982), Danthine and Donaldson (1990), and Collard and de la Croix (2000).

The model presented here predicts that employment and unemployment business cycle volatility should increase with the gross replacement rate of public unemployment insurance. Usually, in a model in which employment is adjusted along the extensive margin, some sort of perfect insurance assumption must be made to avoid individual unemployment histories affecting individual consumption and savings decisions. In the fair wage literature, this is usually achieved by some sort of “large family” assumption, or efficient unemployment insurance markets (see below). In this case, however, there is no role for public unemployment insurance, and as a result it is usually not an object of study. Given that the purpose of the present study is to analyse the relationship between public unemployment insurance and labour market outcomes, the “large family” or private insurance assumptions must be abandoned, and individual unemployment histories will affect savings decisions. For this reason, the present model is an incomplete markets model along the lines of Krusell and Smith (1998), and the entire distribution of wealth must be computed using numerical methods. Whilst the distribution of wealth is not of direct interest here, and approximate aggregation appears to hold in the sense of Krusell and Smith (1998), the simplicity with which unemployment is endogenised via the fair wage labour market may be of some interest to incomplete markets real business cycle theorists.

Section 2 provides an overview of the fair wage real business cycle literature, particularly Collard and de la Croix (2000) and the subsequent literature that this paper inspired. The incomplete markets literature following Krusell and Smith (1998) is not considered, but the reader is referred to Heathcote et al (2009) for an extensive literature review, and the *Journal of Economic Dynamics & Control* special edition on solution methods for an overview of the computational issues raised by this class of model¹. Section 3 describes the model, which is essentially a marriage of Collard and de la Croix (2000) and Krusell and Smith (1998), with the addition of public unemployment insurance. The computational procedure is also described in this section, which closely follows the Euler equation approach of Maliar et al (2010). The main results of the study are presented in section 4. First, approximate aggregation holds to a high degree, and labour market moments accord well with both the representative agent efficiency wage model and OECD data. The chief result is a prediction that the absolute standard deviation of employment, the relative standard deviation of em-

¹Volume 34 Issue 1; see Den Haan et al (2010).

ployment to output, and the correlation of employment and output should all increase with the generosity of public unemployment insurance, measured by the gross replacement rate of benefits relative to wages. The mechanism behind this result is straightforward. In short, worker effort depends positively on both real wage growth and unemployment (via a wage norm in which the benefit income of the unemployed enters alongside the unemployment rate). Real wage rigidity, resulting from the dependence of effort on wage growth, means that fluctuations in labour costs have to be partially accounted for by fluctuations in employment. In effect, any change in wages serves to drive effort away from its optimal level; as a result, the unemployment rate must change to compensate. Unemployment insurance, in turn, reduces the impact of unemployment on worker effort. As a result, increased benefit generosity means that unemployment fluctuations have to increase in order to perform the same compensatory role as previously.

There exists a substantial literature studying the cyclical volatility of employment and unemployment in the context of search and matching models, following the observations of Shimer (2005) that the basic Mortensen-Pissarides model struggles to match the data along this dimension. Mortensen and Nagypál (2007) provides a good overview of the possible solutions to this problem, an important class of which are amplification mechanisms operating through the match surplus (eg Hagedorn and Manovskii 2008). Interestingly, in this case, the same result as the present paper is arrived at - increases in benefit generosity should increase the cyclical volatility of unemployment - through an entirely different mechanism. Despite this shared prediction, the limited empirical work that exists is ambivalent, at best, in regards to the relation between unemployment insurance and unemployment volatility. Faccini and Bondibene (2012) use a panel data approach to examine the impact of different labour market institutions on unemployment volatility in 20 OECD countries. Although labour market institutions such as union density and union coverage are found to be important, the benefit replacement ratio is found to be insignificant (with a negative effect on unemployment volatility). Given this, section 4.2 of the present paper provides descriptive statistics to justify the results. Using the dataset of OECD labour market time series provided by Ohanian and Raffo (2012), significantly positive correlation coefficients are found between the absolute standard deviation of total hours worked and gross replacement rates. In addition, significantly positive rank correlation statistics are found between the absolute standard deviation of harmonised unemployment rates and gross replacement rates. Whilst this constitutes a rudimentary analysis in comparison with Faccini and Bondibene (2012), it indicates that a positive relation between public unemployment insurance and labour market volatility is not completely absent in the data. Finally, section 5 concludes, and suggests avenues for future research.

2 Fair Wage Models of the Business Cycle

All of the variants of efficiency wage theory, pioneered by Solow (1979), Akerlof (1982), and Shapiro and Stiglitz (1984), share in common the proposition that the real wage at which competitive firms maximise profit is higher than the real wage at which full employment would be attained². The key component of efficiency wage models, and in particular the fair wage model, is the inclusion of effort in the production function:

²See Bowles (1985) for a general discussion, and Akerlof and Yellen (1986) for the major references.

$$Y_t = F(A_t, K_t, e_t L_t).$$

Here, A , K , and L are standard, and refer to total factor productivity, the stock of fixed capital, and person hours, respectively. Actual labour performed per person hour, e_t , is then postulated to be increasing in the real wage and some reference wage: $e_t = e(w_t, w_t^r)$, with $e' > 0$ and $e'' < 0$. That this is a realistic proposition is fairly basic, and can be traced back to Marx's observation that labour power (or labour time) is the commodity that workers sell in the labour market; actual labour performed is not (Marx 1967 [1867]: 589 *et seq.*). More importantly, the theory has received considerable empirical support. Particularly, after an extensive qualitative research project into wage setting by firms during the early 1990s recession, Bewley (1999) concluded that, although not without their difficulties, the fair wage models of Solow (1979) and Akerlof (1982) were mostly supported by the interview evidence gathered (Bewley 1999: 415). The main result of the partial equilibrium model is that, as labour productivity is a function of the wage paid by firms, the wage becomes a choice variable and is no longer set in a Walrasian labour market. The wage rate which induces optimal effort on the part of workers might not, in that case, coincide with the wage that clears the labour market, resulting in involuntary unemployment.

Whilst this is a compelling explanation of steady state unemployment, and is consistent with the traditional Keynesian explanation, the basic model does not resolve the "labour market puzzle": the high volatility of employment over the business cycle, and the relatively acyclical behaviour of real wages. This was demonstrated initially in Danthine and Donaldson (1990), which incorporated both shirking and fair wage labour markets into the standard real business cycle model. However, as per Collard and de la Croix (2000), and following observations in Bewley (1999), an augmented effort function $e_t = e(w_t, w_t^r, w_{t-1})$ resolves this problem by the introduction of real rigidity. The justification for this is based upon worker morale - a decrease in the real wage, in particular, should be expected to decrease morale and motivation, and thereby labour productivity. The result is that it is optimal for firms to reduce their total labour costs by laying off some workers, keeping the wages of others relatively constant, instead of retaining the entire labour force with reduced wages.

Subsequent papers, particularly Tripier (2006) and Danthine and Kurmann (2004, 2007, 2010), explored the basic model's empirical performance in more depth, and extended the real business cycle framework to a full New Keynesian DSGE model. Related papers, including Gomme (1999), Alexopoulos (2004, 2007), Grandmont (2008), Nakajima (2010), Brecher et al (2010), and Givens (2011), build on the Shapiro and Stiglitz (1984) shirking model to examine similar phenomena. However, most of the aforementioned papers assume either full unemployment insurance via private markets or a "large family" argument. These assumptions are made to maintain computational tractability by the use of a representative agent construct, as without them, households' idiosyncratic unemployment risk would affect their savings decisions, and individual shocks and the distribution of wealth would have to be tracked as per the incomplete markets model. The exceptions to this rule are Gomme (1999) and Grandmont (2008); the former assumes that workers cannot save, whilst the latter solves the problem by a combination of generational saving and relative patience assumptions.

Given that full unemployment insurance is generally not provided by the market, and that the public provision of unemployment insurance should be expected to have significant effects on labour market outcomes, the behaviour of the basic fair wage real business cycle model when unemployment insurance markets are absent is an interesting object of study.

The papers with which the present study has most in common are Alexopoulos (2004) and Givens (2011). Both of these papers analyse a real business cycle model with a labour market related to the Shapiro and Stiglitz (1984) shirking model. Whilst a large family assumption is made to divorce the capital accumulation decision from idiosyncratic unemployment histories, the implications of partial private consumption insurance (provided by the family) and partial public insurance (provided by the state) are analysed. Particularly, Givens (2011) demonstrates that partial insurance increases the effects of technology and policy shocks and reduces the relative volatility of real wages compared to full insurance. In a sense this result is the opposite to that demonstrated in this paper, where increasing the replacement rate of public insurance increases the volatility of employment and reduces the volatility of the real wage. However, the two models are very different; Givens (2011) analyses private and public insurance in a shirking labour market representative agent model, whilst the present paper analyses public insurance in a fair wage labour market heterogeneous agent model. Again, as the empirical work on this topic is by no means conclusive, it is worth noting theoretical disagreement at the outset, in the hope that consensus will emerge with continued research.

3 The Model

The model is essentially a marriage of Collard and de la Croix (2000) and Krusell and Smith (1998), with the addition of public unemployment insurance. First, the population of households and the household problem are described, and the effort function is derived as a first order condition. The firm problem is then described, after which the dynamic process for individual unemployment history can be defined. Finally, the computational strategy is outlined (further details are provided in the appendix).

3.1 Households

The economy is populated by a large population (measure one) of infinitely lived households, indexed by i . There is only one output good, which can be consumed or used as fixed capital, and preferences over the consumption and effort streams of each household are given by the following:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}, e_{it}). \quad (1)$$

Here, c denotes consumption of the output good, and e denotes effort. As with the efficiency wage theories of Solow (1979) and Akerlof (1982), and as opposed to, for instance, the shirking theory of Shapiro and Stiglitz (1984), e is defined in an analogous manner to labour productivity, as described above. The intra-period utility function u is specified as follows:

$$u(c_{it}, e_{it}) = v(c_{it}) - \epsilon_{it} \left[e_{it} - \phi + \gamma \log \left(\frac{w_{it}}{w_{it}^a} \right) + \psi \log \left(\frac{w_{it}}{w_{it}^s} \right) \right]^2, \quad (2)$$

$$v(c_{it}) = \begin{cases} \ln(c_{it}) & \text{if } \sigma = 1 \\ \frac{c_{it}^{1-\sigma} - 1}{1-\sigma} & \text{if } \sigma \neq 1. \end{cases} \quad (3)$$

Here, w^a and w^s denote reference wages, and, as households are assumed to supply one unit of labour inelastically each period, ϵ is an indicator variable that takes the value 1 if the household is employed, and 0 otherwise. As suggested by eqs 1 - 3, both consumption and employment states will be heterogeneous across households in equilibrium. Effort conditional on being employed is potentially heterogeneous in equilibrium, inasmuch the reference wages w^a and w^s are dependent on employment history. In this case, the real wage w will then be heterogeneous across employed households. In order to simplify the computation, w^a and w^s are assumed to be identical across all households, such that in equilibrium the effort level and the real wage will be identical across employed households (see below). As such, in what follows, household specific subscripts will be suppressed for the variables e , w , w^a , and w^s .

Given eqs 2 and 3, each household maximises eq 1 subject to the Markov processes for the unemployment and total factor productivity shocks (defined below), and the flow budget constraint and borrowing constraint defined as follows:

$$k_{it+1} = (1 - \tau_t)w_t\epsilon_{it} + \mu w_t(1 - \epsilon_{it}) + (1 + r_t)k_{it} - c_{it}, \quad (4)$$

$$k_{it+1} \geq 0. \quad (5)$$

Here, k denotes claims on the aggregate capital stock K , τ denotes a flat tax on labour income, and μ denotes the replacement rate for public unemployment insurance. With eqs 1 - 5, the household first order conditions are given by the following:

$$v'(c_{it}) - \kappa_{it} = \beta E_t[v'(c_{it+1})(1 + r_{t+1})], \quad (6)$$

$$e_t = \phi + \gamma \log\left(\frac{w_t}{w_t^a}\right) + \psi \log\left(\frac{w_t}{w_t^s}\right). \quad (7)$$

Here, κ is the Lagrangian multiplier associated with the non-negativity constraint on individual capital. Eq 6 is the consumption Euler equation, and eq 7 is the effort function for employed households. The latter is identical to the effort function in Collard and de la Croix (2000) and Tripier (2006), with the reference wages w^a and w^s defined as follows:

$$w_t^a = L_t w_t + (1 - L_t)\mu w_t, \quad (8)$$

$$w_t^s = w_{t-1}. \quad (9)$$

The reference wage w^a is an arithmetic average of the labour income of employed households and the transfer income of unemployed households, where L denotes aggregate employment (and therefore the employment rate; recall that the population of households is normalised to one). The reference wage w^s is the wage received by employed households in the last period. As noted above, and made explicit in eqs 8 and 9, both w^a and w^s are assumed to be identical across households. Whilst this is an innocuous assumption for w^a , it is a stronger assumption for w^s , as there is little reason to suppose that the wage received by households employed in period $t - 1$ is of relevance in determining the effort level of a household that is newly employed in period t . In this case, however, a newly employed household would have a different effort level to previously employed households, and receive a different wage as a result. This would immediately lead to household specific wages conditional on employment history, greatly complicating the analysis. As such, a sacrifice of realism for tractability is made: effort is assumed to be conditional on the previous wage, via w^s , regardless of when households are hired³. Finally, and further to this end, eqs 7 - 9 ensure that effort is independent of household wealth.

3.2 Firms

A representative firm operates in competitive output and factor markets, and with K_t as aggregate capital at time t , and L_t as aggregate employment at time t , it has access to a Cobb-Douglas technology defined as follows:

$$Y_t = A_t K_t^\alpha (e_t L_t)^{1-\alpha}. \quad (10)$$

The representative firm then maximises total profit ($= Y_t - w_t L_t - (r_t + \delta)K_t$) subject to eq 10 and the effort function defined by eq 7. The first order conditions are as follows:

$$(1 - \alpha) \frac{Y_t}{L_t} = w_t, \quad (11)$$

$$(1 - \alpha) \frac{Y_t}{e_t} \left(\frac{\gamma + \psi}{w_t} \right) = L_t, \quad (12)$$

$$\alpha \frac{Y_t}{K_t} = r_t + \delta. \quad (13)$$

Eqs 11 and 12 combined give the Solow efficiency condition: $e'(w)w = e(w)$, from which it follows that effort is constant through time: $e_t = \gamma + \psi$. That this is uniform across workers follows from the fact that household wealth, which will be heterogeneous in equilibrium, does not enter the effort function, and that w^a and w^s are identical across households, as discussed above.

³The conjecture made in support of this simplification is that the main results are robust to household specific wages; the confirmation/refutation of this is left to future research.

3.3 Macroeconomic Environment

As per Krusell and Smith (1998), total factor productivity A_t is assumed to follow a two state Markov process, where the first state is a low productivity outcome, $A_t = A^L$, and the second state is a high productivity outcome, $A_t = A^H$. The transition probabilities are denoted as follows: $P(A_t = A^{s'} | A_{t-1} = A^s) = \pi^{ss'}$. Given this process, eqs 7, 10, 11, and 13, together with $e_t = \gamma + \psi$, allow the equilibrium employment and unemployment rates, real wage, real interest rate, and output to be computed as a function of the two endogenous state variables K_t and w_t^s and the exogenous state A_t . Denote the equilibrium unemployment rate thus determined $U_t = U(K_t, w_t^s, A_t)$. The flow identity for unemployment is then as follows, where λ^{eu} denotes the rate at which employed households become unemployed, and λ^{ue} denotes the rate at which unemployed households become employed:

$$U_t = \lambda_t^{eu}(1 - U_{t-1}) + (1 - \lambda_t^{ue})U_{t-1}. \quad (14)$$

Given the flow unemployment identity, either λ^{eu} or λ^{ue} has to be specified exogenously in order to close the model. Following the observation in Shimer (2005: 32-33) that the job separation rate is essentially acyclical, this is taken as exogenous in the present model. The job creation rate is then determined as follows:

$$\lambda_t^{ue} = \lambda^{eu} \left(\frac{1 - U_{t-1}}{U_{t-1}} \right) - \frac{U_t - U_{t-1}}{U_{t-1}}. \quad (15)$$

The foregoing allows the endogenous transition probabilities for the two state Markov process for idiosyncratic employment shocks to be determined: $P(\epsilon_t = 0 | \epsilon_{t-1} = 1) = \lambda^{eu}$, $P(\epsilon_t = 1 | \epsilon_{t-1} = 0) = \lambda_t^{ue}$, the latter of which is a function of the aggregate state vector in time periods t and $t - 1$. Finally, the government is assumed to run a balanced budget in every time period, such that total tax revenues are adjusted to the policy parameter μ :

$$\tau_t = \mu \left(\frac{1 - L_t}{L_t} \right). \quad (16)$$

3.4 Computational Strategy

The model described in sections 3.1 - 3.3 is more complex, in a structural sense, than the representative agent efficiency wage models described in section 2, but significantly less complex than the existing incomplete markets models with endogenous unemployment, eg Krusell et al (2010). The strategy followed is an adaptation of the algorithm proposed in Maliar et al (2010), which combines an iterative Euler equation procedure to solve the household problem, given boundedly rational predictors of the endogenous state variables, with a Monte Carlo procedure to simulate the aggregate economy. The predictor functions postulated in the household decision problem can then be checked for accuracy against the simulated time series, and adjusted if necessary. This entire process can then be repeated until the predictor functions are sufficiently accurate.

3.4.1 Solving the Household Problem

The Euler equation method for solving household saving decisions, as proposed by Maliar et al (2010), is both intuitively attractive and considerably more efficient than, for example, value function methods. By substituting the flow budget constraint (eq 4) into the consumption Euler equation (eq 6), the following non-linear equation in individual capital holdings is derived:

$$k_{it+1} = (1 - \tau_t)w_t\epsilon_{it} + \mu w_t(1 - \epsilon_{it}) + (1 + r_t)k_{it} - \left(\kappa_{it} + \beta E_t \left[\frac{1 + r_{t+1}}{((1 - \tau_{t+1})w_{t+1}\epsilon_{it+1} + \mu w_{t+1}(1 - \epsilon_{it+1}) + (1 + r_{t+1})k_{it+1} - k_{it+2})^\sigma} \right] \right)^{-\frac{1}{\sigma}}. \quad (17)$$

Eq 17 can be solved as follows. First, choose a grid for $k \in [0, k_{\max}]$, and grids for the endogenous states $K \in [K_{\min}, K_{\max}]$ and $w^s \in [w_{\min}^s, w_{\max}^s]$, and initialise a decision function $k_{t+1} = \tilde{k}(k_t, K_{t+1}, w_{t+1}^s, \epsilon_t, A_t)$ on the grids. Note, first, that w_{t+1}^s enters the decision function, but as $w_{t+1}^s = w_t$, its prediction is trivial. On the other hand, aggregate future capital K_{t+1} enters the decision function, and as it is the result of integrating over the individual capital holdings of a continuum of households, and therefore a potentially infinite dimensional object, a rational expectations predictor is unlikely to exist. A common solution is to suppose that households use a finite set of the moments of the time t wealth distribution to predict mean capital at time $t + 1$. Following Krusell and Smith (1998) and Maliar et al (2010), the boundedly rational predictor of future capital is specified as a function of the mean of current holdings only, and takes a log-linear form. In addition, experimentation with different functional forms has led to the conclusion that including the remaining endogenous state, w_t^s , significantly improves prediction accuracy. Therefore, the functional form for the predictor is as follows:

$$\ln(K_{t+1}) = \begin{cases} B_0^L + B_1^L \ln(K_t) + B_2^L \ln(w_t^s) & \text{if } A_t = A^L \\ B_0^H + B_1^H \ln(K_t) + B_2^H \ln(w_t^s) & \text{if } A_t = A^H. \end{cases} \quad (18)$$

Given eq 18, the household decision problem is solved as follows. First, specify a guess for the decision function \tilde{k} . Set $\kappa = 0$, and for each grid point, substitute this decision function into the right hand side of eq 17, and compute k_{t+1} on the left hand side (recalling that r_t and w_t can be computed as functions of K_t , w_t^s , and A_t). This is then taken as a new guess for the decision function, \tilde{k}' . This process can then be iterated by substituting the new guess for the decision function (or a weighted average of the new guess and the old guess) back into the right hand side of eq 17, until the maximum difference between the new and old guesses falls below a pre-specified convergence criterion.

3.4.2 Simulating The Economy

Denote the solution to the household decision problem as \bar{k} . The order of events in any given time period is then as follows:

1. Aggregate States:
 - (a) A_t is determined according to $\pi^{ss'}$ given A_{t-1}
 - (b) $K_t = \sum k_{it}$ and w_t^s are inherited from period $t - 1$
2. Idiosyncratic States:
 - (a) K_t , w_t^s , and A_t determine U_t
 - (b) λ_t^{ue} is determined given U_t and U_{t-1} , which allows the idiosyncratic employment shocks ϵ_{it} to be determined given ϵ_{it-1}
 - (c) k_{it} are inherited from period $t - 1$
3. Given the state variables determined above, $k_{it+1} = \bar{k}(k_{it}, K_t, w_t^s, \epsilon_{it}, A_t)$, $w_{t+1}^s = w_t$

Given initial conditions for the aggregate and idiosyncratic states and the decision rule \bar{k} , this intra-period process can be simulated recursively for T periods and N households. Prior to the simulation, the entire history of A_t can be computed using a pseudo-random number generator, but the employment shocks for each of the N households must be generated within each time period of the simulation (as in Krusell et al (2010), but unlike Krusell and Smith (1998) or any similar model in which the unemployment process is exogenous). Given the foregoing, the complete solution algorithm is as follows: first, postulate values for the parameters of the predictor function and solve the household decision problem. Given the solution \bar{k} , simulate the economy with large N and large T , given initial values for the aggregate and endogenous states. Finally, regress the generated time series for the aggregate states on the predictor function. If the implied parameter values are sufficiently different from the previously postulated values, repeat the process using a weighted average of the old and new estimates. This entire process is then repeated until the estimated parameter values of the predictor function have converged⁴.

4 Results

In this section, the results and predictions of the paper are presented. First: that approximate aggregation holds in the model, and the dynamic behaviour of unemployment and the real wage is in line with OECD data. And second: that public provision of unemployment insurance in lieu of insurance markets can have significant effects on labour market outcomes. Particularly, the absolute standard deviation of employment, the relative standard deviation of employment to output, and the correlation of employment and output are all

⁴See the appendix for further details of the algorithm and accuracy checks. I am indebted to Maliar et al for making their code available online; the detailed algorithm described in the appendix is based on this.

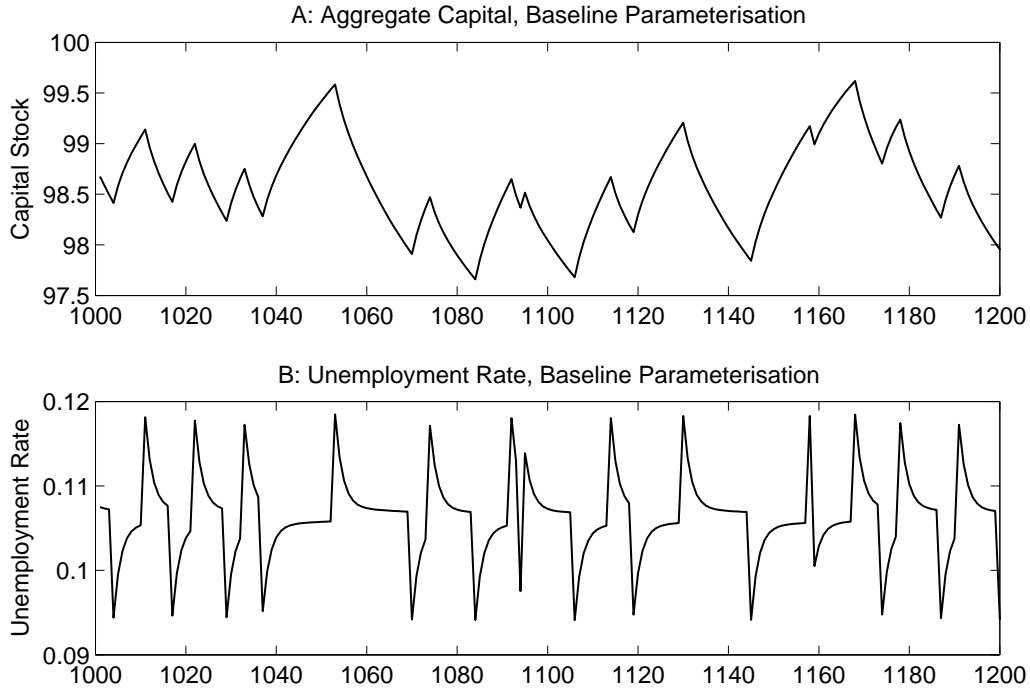


Figure 1: Simulated Time Series: Capital and Unemployment

predicted to increase with the generosity of public unemployment insurance, measured by the gross replacement rate of benefits relative to wages. Using the dataset of OECD labour market time series provided by Ohanian and Raffo (2012), evidence is presented in support of the first of these predictions. Section 5 concludes, and suggests avenues for future research.

4.1 Approximate Aggregation

The baseline parameterisation is standard. Following, for example, Krusell and Smith (1998), Collard and de la Croix (2000), or Den Haan et al (2010), $\beta = 0.99$, $\sigma = 1$, $\alpha = 0.36$, and $\delta = 0.025$, assuming quarterly time periods. Following Krusell and Smith (1998), $A^L = 0.99$, $A^H = 1.01$, and $\pi^{LL} = \pi^{HH} = 0.875$, such that the average duration of periods of high and low productivity is 8 quarters. Following Shimer (2005), the job separation rate $\lambda^{eu} = 0.1$, given an average monthly separation rate reported in the latter of 0.034. In order to keep the model consonant with Collard and de la Croix (2000), the baseline values for the effort function parameters are as follows: $\phi = 2.82$, $\gamma = 0.9$, $\psi = 2$. Varying $\phi - \psi$ from 0.9 to 0.7 varies average unemployment from approximately 0% to 25%; the present parameterisation yields an average unemployment rate of approximately 10%⁵. Finally, $\mu = 0.2$ (the OECD replacement rates for unemployment insurance are quite diverse, and the consequences of varying this parameter will be considered in section 4.2). With the number of households $N = 5000$, and the number of time periods $T = 5500$ (with

⁵OECD unemployment in September 2013 ranged from 3% (South Korea) to 26.6% (Spain). US unemployment was 7.2% and the EU average was 11%; see www.oecd.org/std/labour-stats/HUR.11e13.pdf.

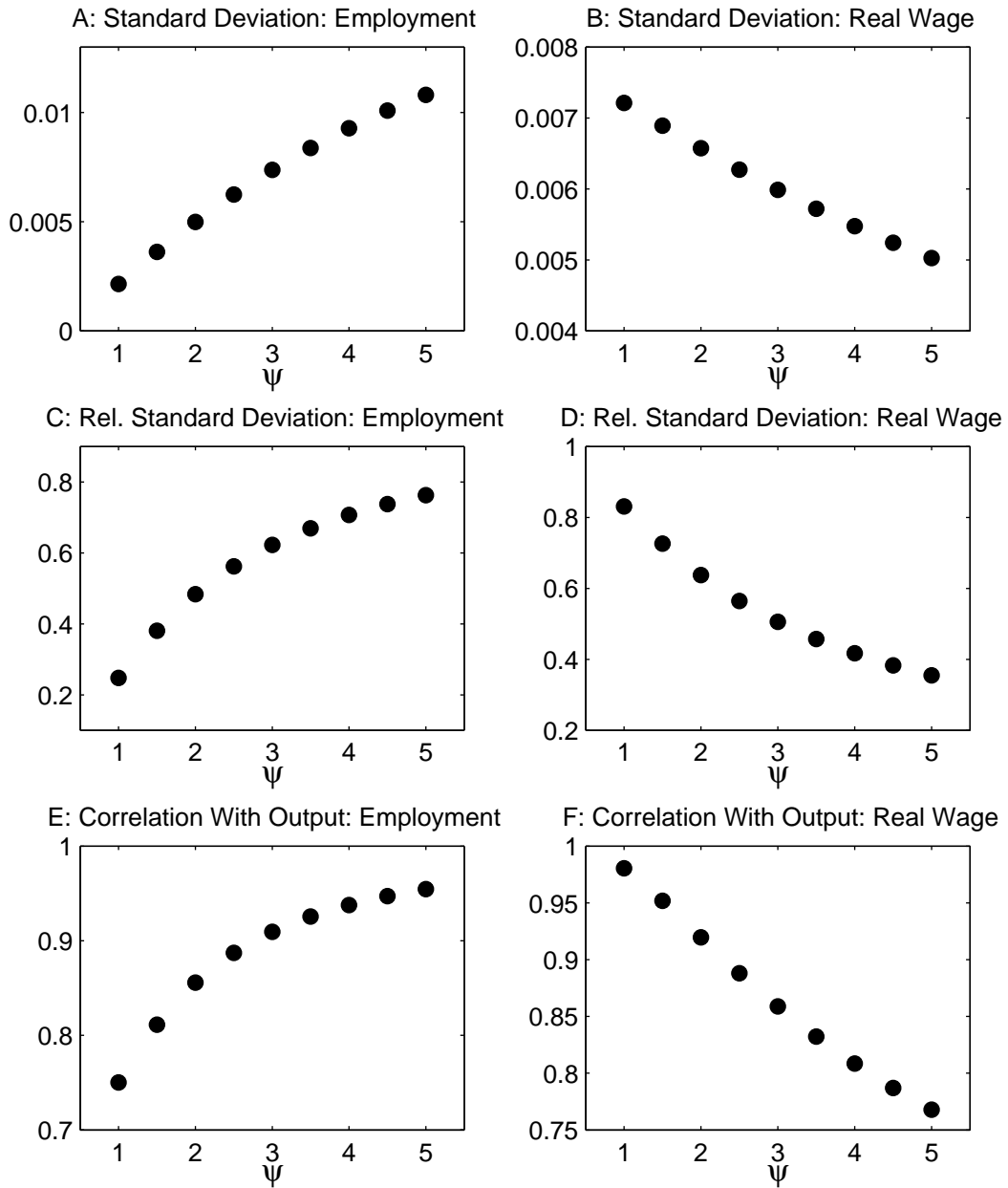


Figure 2: Labour Market Moments Varying ψ (sensitivity of effort to wage growth)

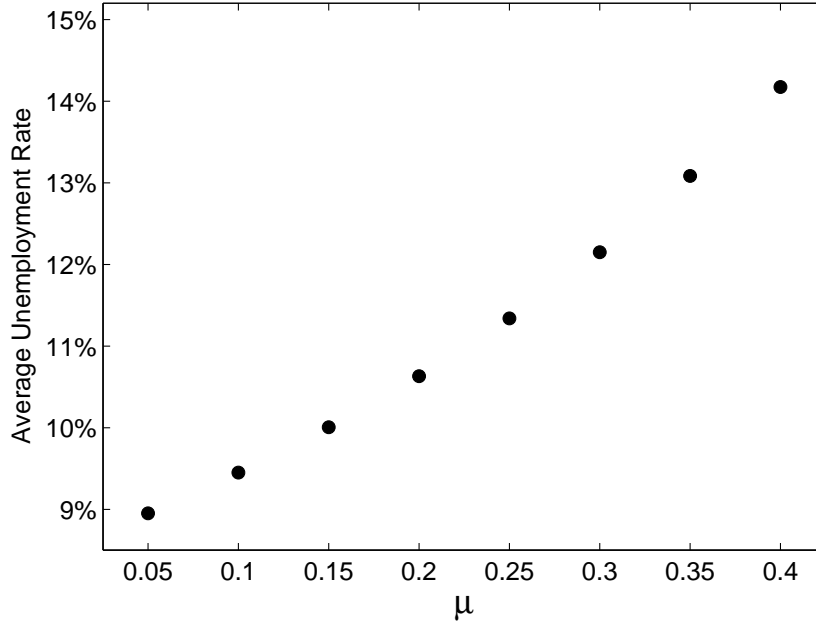


Figure 3: Unemployment Rate Varying μ (benefit gross replacement rate)

the first 500 periods discarded for the predictor regressions), the approximate equilibrium is as follows:

$$\ln(K_{t+1}) = \begin{cases} 0.1789 + 0.9936 \ln(K_t) - 0.0784 \ln(w_t^s); & R^2 = 0.999997 \quad (A_t = A^L) \\ 0.1935 + 0.9916 \ln(K_t) - 0.0795 \ln(w_t^s); & R^2 = 0.999998 \quad (A_t = A^H). \end{cases}$$

The time series behaviour of the aggregate capital stock and unemployment rate for this parameterisation is illustrated in figure 1⁶. The approximate equilibrium is clearly rather accurate, with very high R^2 values, and dynamic Euler equation tests indicate a very low welfare loss from the boundedly rational nature of the predictor function (see the appendix). As higher moments of the wealth distribution are not required in the predictor, approximate aggregation holds in the sense of Krusell and Smith (1998).

4.2 Labour Market Moments

As illustrated in figure 2, the heterogeneous agent model appears to display approximate aggregation in the further sense that the key labour market moments are consistent with the representative agent case, allowing for different stochastic processes for total factor

⁶Note that the unemployment peaks coincide with the capital peaks in figure 1; this occurs when the total factor productivity process changes state. For example, capital peaks, and starts to be de-cumulated, after period 1053. This corresponds to total factor productivity falling from A^H to A^L , which also causes a spike in the unemployment rate as labour demand jumps down. The real wage then starts to fall, and labour is substituted for capital such that the unemployment rate reduces through to period 1070, at which point total factor productivity jumps back up to A^H .

Period	Country	Std. Hours	R.Std. Hours	Corr. Hours	Std. Emp.	R.Std. Emp.	Corr. Emp.
1960 - 1984	USA	0.016	0.84	0.86	0.013	0.69	0.81
	Japan	0.012	0.68	0.62	0.005	0.29	0.46
	Germany	0.011	0.68	0.85	0.010	0.60	0.74
	UK	0.012	0.63	0.71	0.008	0.50	0.47
	France	0.010	0.81	0.35	0.006	0.49	0.45
1985 - 2007	USA	0.011	1.29	0.78	0.008	0.94	0.75
	Japan	0.009	0.72	0.53	0.005	0.37	0.54
	Germany	0.007	0.61	0.64	0.008	0.68	0.77
	UK	0.013	1.26	0.76	0.010	0.93	0.74
	France	0.008	0.87	0.61	0.006	0.68	0.79

Table 1: OECD Labour Market Moments

productivity (consult figure 2 in Collard and de la Croix 2000: 179). Moreover, these moments are consistent with OECD data. Table 1 reports key labour market moments for the five largest OECD countries over two time periods: 1961 - 1984, and 1985 - 2007, where the latter period is a common dating for the “great moderation”. The moments are calculated from the database provided in Ohanian and Raffo (2012), and are fairly dispersed across countries and time periods. Nevertheless, total hours worked and total employment are both highly correlated with output, in general, and are relatively volatile, with Japan being the main exception to this rule. Clearly, by consulting figure 2, the model presented in this paper is consistent with these data⁷. The incomplete markets efficiency wage model, given its structural simplicity, is therefore relatively simple to compute, as described in section 3.4, and yields empirically reasonable aggregate time series moments.

The common assumption of full private unemployment insurance (and zero public insurance) in the efficiency wage real business cycle literature, on the other hand, is not an innocuous assumption. As illustrated in figure 3, when private insurance is absent, the exact degree of public unemployment insurance can have significant effects on labour market outcomes. Particularly, under the baseline parameterisation, increases in the replacement rate μ are associated with considerable increases in the average unemployment rate. This corresponds to an established empirical regularity in OECD countries - see, for instance, Nickell (1997) and Nickell et al (2005). The mechanism in the present model is straightforward, and follows directly from the effort function described by eq 7. Consider, for instance, a steady state situation in which $w_t = w_{t-1}$. The Solow efficiency condition requires effort to be constant and equal to $\gamma + \psi$, reducing the effort function to the following condition:

$$e_t = \gamma + \psi = \phi + \gamma \log \left(\frac{w_t}{w_t^a} \right). \quad (19)$$

Given the above, the following condition holds in the steady state:

⁷The database can be accessed as supplementary material to Ohanian and Raffo (2012) available online. It is log quarterly data, and all of the moments in table 1 are based on deviations from a trend computed using the Hodrick-Prescott filter with multiplier set to 1600. Likewise, the moments presented in figure 2 are calculated from the cyclical component of logged data from simulation output, using the Hodrick-Prescott filter with the same multiplier.

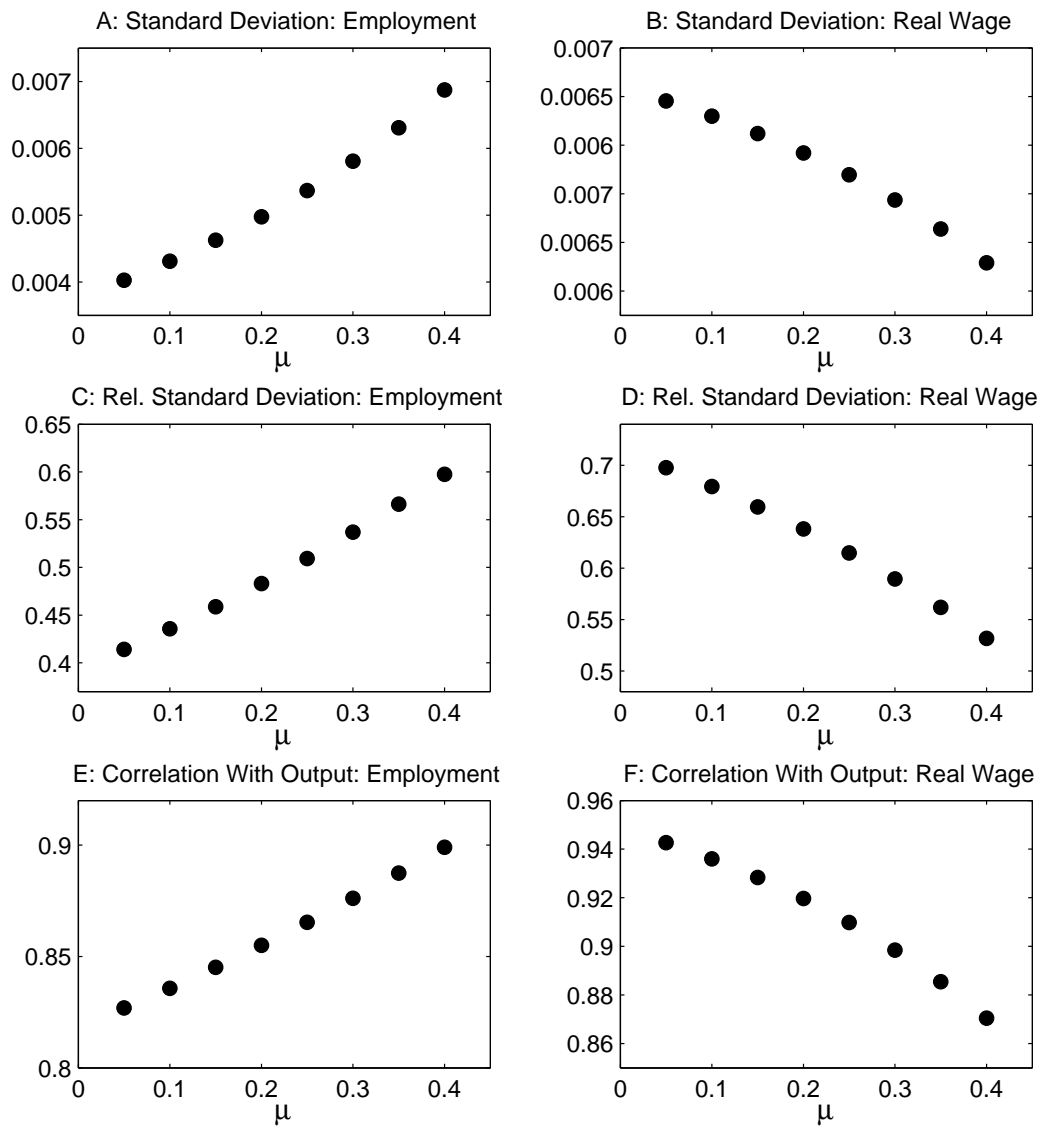


Figure 4: Labour Market Moments Varying μ (benefit gross replacement rate)

$$\frac{w_t}{w_t^a} = \frac{1}{L_t + (1 - L_t)\mu} = \exp\left[\frac{\gamma + \psi - \phi}{\gamma}\right]. \quad (20)$$

In short, due to the specification of the effort function, firms desire constant effort on the part of workers. In the steady state, this implies a constant ratio between w_t and w_t^a , but as w_t^a is a linear function of w_t , this results in eq 20, in which any increase in the replacement rate μ has to be offset by an increase in unemployment. Intuitively, the desire on the part of firms to keep effort constant requires any increases in factors likely to decrease effort - eg generous unemployment insurance - to be offset by increases in factors likely to increase effort - eg the threat of unemployment. In addition to the impact on average unemployment, the model presented here also allows an examination of the effects of public unemployment insurance on the time series properties of labour market statistics. As illustrated in figure 4, these effects are predicted to be quite substantial. Particularly, increases in the replacement rate of public unemployment benefits are associated with increases in the absolute standard deviation of employment, standard deviation of employment relative to output, and correlation of employment and output, and with decreases in the absolute standard deviation of the real wage, standard deviation of the real wage relative to output, and correlation of the real wage and output. Again, the explanation is straightforward, and the mechanism is related to the similar effects of increases in ψ . Consider, for instance, the condition described by eq 20 outside the steady state:

$$\gamma + \psi = \phi + \gamma \log\left(\frac{1}{L_t + (1 - L_t)\mu}\right) + \psi \log\left(\frac{w_t}{w_{t-1}}\right). \quad (21)$$

Utilising the labour share of income described by eq 11, eq 21 yields the following:

$$\log\left(\frac{L_t}{L_{t-1}}\right) - \log\left(\frac{Y_t}{Y_{t-1}}\right) = \frac{\phi - \gamma - \psi}{\psi} - \frac{\gamma}{\psi} \log(L_t + (1 - L_t)\mu). \quad (22)$$

Eq 22 indicates that the relative volatility of employment and output is increasing in both ψ and μ . Intuitively, as ψ increases, the dependence of effort on changes in the real wage increases, and thus fluctuations in employment substitute for fluctuations in the real wage in firms' responses to market conditions, given that they desire constant effort. At the same time, as μ increases, the effect of changes in employment on the reference wage w^a decreases. Thus, as μ increases, the change in employment necessary to keep effort constant, given ψ , must increase, leading to the results presented in figure 4⁸.

All of these predictions are, in principle, testable. However, experimentation with different calculations of the relative standard deviation of employment statistics and output, and correlations between these two variables, indicates that the relation between these moments and gross replacement rates in the OECD is very sensitive to the de-trending method chosen. As such, the results presented here focus solely on absolute standard deviations. Particularly, results are presented for those OECD countries in the Ohanian and Raffo (2012) database for which gross replacement rate data is also available: Australia, Austria, Canada, Finland,

⁸As with figure 2, the moments presented in figure 4 are calculated from the cyclical component of logged data from simulation output, using the Hodrick-Prescott filter with multiplier set to 1600.

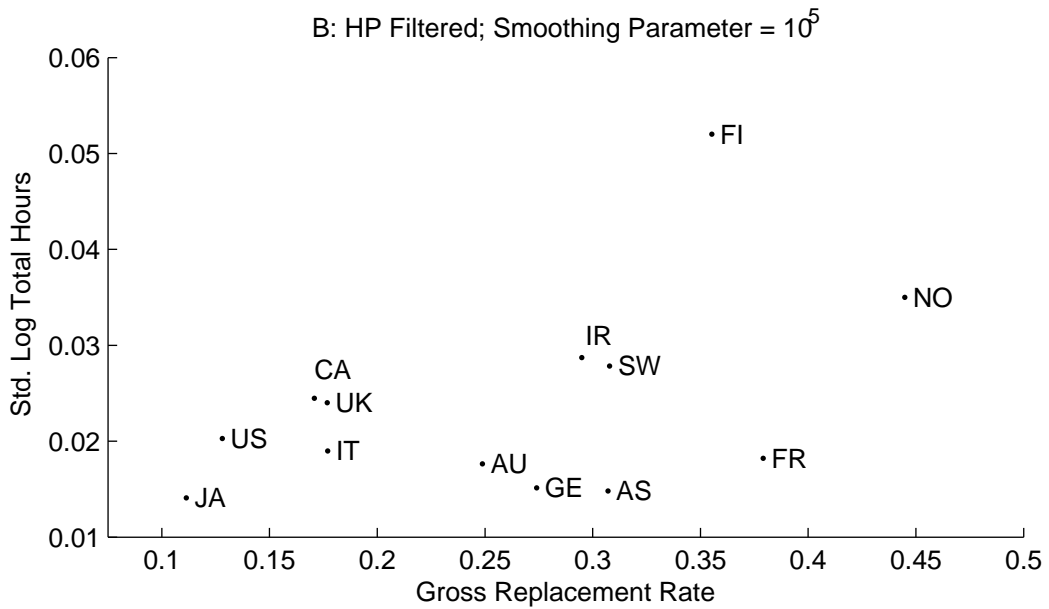
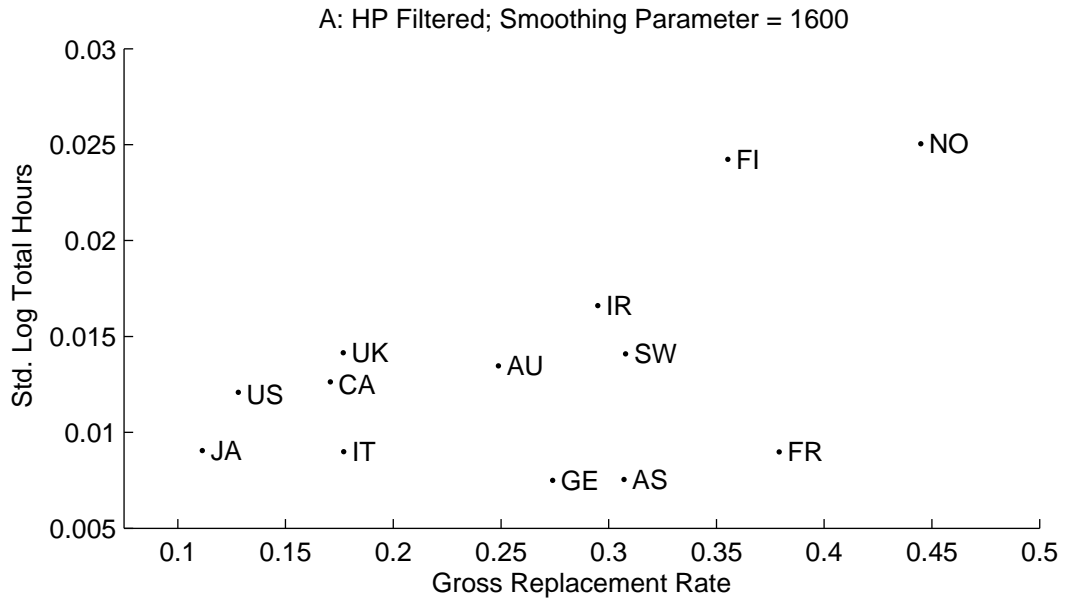


Figure 5: Std. Log Total Hours vs Gross Replacement Rates, OECD

Variable	De-Trending Method	Correlation Measure	Coefficient	P-Value (> 0)
Employment Rate Std.	HP ($\lambda = 1600$)	Pearson	0.3543	0.1070
		Kendall	0.3407	0.0505
		Spearman	0.4415	0.0579
	HP ($\lambda = 10^5$)	Pearson	0.4237	0.0656
		Kendall	0.2967	0.0786
		Spearman	0.4110	0.0729
Unemployment Rate Std.	HP ($\lambda = 1600$)	Pearson	0.3019	0.1471
		Kendall	0.2747	0.0963
		Spearman	0.3011	0.1475
	HP ($\lambda = 10^5$)	Pearson	0.4058	0.0750
		Kendall	0.4286	0.0178
		Spearman	0.4857	0.0405
Log Total Hours Std.	HP ($\lambda = 1600$)	Pearson	0.5186	0.0347
		Kendall	0.2564	0.1259
		Spearman	0.2692	0.1867
	HP ($\lambda = 10^5$)	Pearson	0.4849	0.0465
		Kendall	0.2564	0.1259
		Spearman	0.4231	0.0758

Table 2: Correlations of OECD Employment Stats and Replacement Rates

France, Germany, Ireland, Italy, Japan, Norway, Spain, Sweden, UK, and USA. First, the quarterly employment rate over this period, for each country, is calculated as the employment to working age population ratio, from Q1-1985 to Q4-2005 (the starting date follows Ohanian and Raffo’s dating of the start of the “great moderation”, whilst the end date is dictated by the OECD gross replacement rate data). Quarterly harmonised unemployment rates are taken directly from the OECD labour force statistics database, from Q1-1991 to Q4-2005 (the data for Austria starts in Q1-1993), whilst the quarterly data on total hours worked is taken directly from the Ohanian and Raffo database, again from Q1-1985 to Q4-2005 (Spain, however, is dropped here for lack of hours data prior to 1995; total hours are calculated as hours per worker multiplied by the employment to working age population ratio). Finally, gross replacement rates are taken from the benefits and wages statistics of the OECD Directorate for Employment, Labour and Social Affairs⁹. The replacement rate data runs from 1961 to 2005, for uneven years only (whilst further statistics run to 2011, there is a break in the methodology between 2005 and 2007).

Figure 5 presents scatter plots of the absolute standard deviation of the log of total hours worked, for the sample of OECD countries, against the average gross replacement rate for those countries during the period in question. The hours data in the top panel is de-trended using the HP filter, with the usual smoothing parameter of 1600 for quarterly data. As a robustness test, given uncertainty surrounding the relation of trend and cycle in labour market data, the bottom panel reproduces the plot with the HP smoothing parameter set to 10^5 , following Shimer (2005). Although both plots appear to be rather noisy, there is an identifiably positive relation in both, and the second de-trending procedure does not introduce particularly significant differences into the bottom panel compared to the top panel.

⁹Accessible at www.oecd.org/els/benefitsandwagesstatistics.htm.

In fact, the relative positions of Finland and Norway appear to be the only pronounced changes between panels A and B. Finally, table 2 presents Pearson correlation coefficients and Kendall and Spearman rank correlation coefficients, for the two different de-trending methods, for employment rate standard deviations and gross replacement rates, unemployment rate standard deviations and gross replacement rates, and the standard deviation of the log of total hours worked and gross replacement rates, for the sample of countries described above. Whilst there appears to be no statistically significant relationship between the employment rate standard deviations and gross replacement rates, the Pearson correlation coefficients for the standard deviation of the log of total hours worked are significantly greater than zero at the 5% level, and thus significantly different from zero at the 10% level, for both de-trending procedures. In addition, for the second de-trending procedure, both rank correlation coefficients are significantly greater than zero at the 5% level, and thus significantly different from zero at the 10% level, for the unemployment rate data.

These results are not inconsistent with Faccini and Bondibene (2012). Like the latter paper, no straightforward correlation is found between the benefit replacement rate and either employment or unemployment. Rank correlation is found between the replacement rate and unemployment rate, but this is not a correlation that could be easily explored further, and is sensitive to the detrending procedure chosen. The major result of this section is located in the relation between benefit generosity and the volatility of hours worked, rather than unemployment, and this could certainly be explored further. Particularly, repeating the analysis of Faccini and Bondibene (2012) using hours data rather than unemployment data might prove worthwhile, to see whether or not the results found here are maintained with a more sophisticated analysis. It would also be desirable to test the model against real wage statistics, as well as employment statistics. Real wage statistics, however, are rather difficult to interpret, at least in comparison to employment statistics. First, quarterly data over any significant time span is often confined to the manufacturing industries, which constitute a declining share of employment in the OECD. Second, the cyclicity of real wages are very heterogeneous across OECD countries, and the relevant statistics appear to be sensitive to the dataset chosen¹⁰. Nevertheless, the employment evidence available for the OECD during the “great moderation” does not contradict the predictions of the model presented in this paper, and the statistics calculated here may be taken as supporting evidence.

5 Concluding Remarks

This paper examines the relation between the generosity of public unemployment insurance and labour market volatility. Specifically, the volatility of employment, relative volatility of employment to output, and correlation of employment with output are all predicted to increase with the benefit replacement rate, alongside corresponding decreases in the volatility of real wages, relative volatility of real wages to output, and correlation of real wages with output. Given the efficiency wage framework considered, the mechanism is straightforward. In short, worker effort depends positively on both real wage growth and unemployment (via a wage norm in which the benefit income of the unemployed enters alongside the unemployment rate). Real wage rigidity, resulting from the dependence of effort on wage growth,

¹⁰A number of OECD countries have counter-cyclical wages at business cycle frequencies, for example, and the extent to which this is case depends on the deflator used. See Messina et al (2009) for a good overview of the evidence.

means that fluctuations in labour costs have to be partially accounted for by fluctuations in employment. In effect, any change in wages serves to drive effort away from its optimal level; as a result, the unemployment rate must change to compensate. Unemployment insurance, in turn, reduces the impact of unemployment on worker effort. As a result, increased benefit generosity means that unemployment fluctuations have to increase in order to perform the same compensatory role as previously.

The prediction that employment volatility should increase with the gross replacement rate of benefits relative to wages is shared by the standard search and matching model, as discussed in the introduction. Nevertheless, the existing empirical evidence is ambivalent in regards to the relationship between employment volatility and unemployment benefits. Given this, a set of descriptive statistics were provided as supporting evidence in section 4.2, using OECD data for 14 countries. In particular, while there appears to be no straightforward relation between the volatility of employment and unemployment benefits, and a limited positive relation between the volatility of unemployment and unemployment benefits, there does appear to be a positive relationship between the volatility of hours worked and the benefit replacement rate. The first line of research suggested by the present paper is therefore an application of the considerably more sophisticated econometric approach of Faccini and Bondibene (2012) to the volatility of hours worked, rather than the unemployment rate, to examine whether or not the basic results presented here are still supported. If this is indeed the case, it would appear necessary to augment the basic fair wage business cycle model with a labour supply decision (along the intensive margin), to explore the theoretical mechanism further.

Finally, given that the theoretical mechanism explored here is relatively straightforward, it may be of use to embed it in a model with a less complex structure. At the same time, the structural complexity of the heterogeneous agent real business cycle model allows, in principle, the relation between unemployment, unemployment volatility, and inequality to be examined. Although this was not the focus of the present paper, it constitutes a potentially fruitful line of research going forwards. Given that distributional effects are crucial in determining the total welfare loss to society from unemployment, such a model would permit a welfare analysis to be performed, which might then inform optimal government policy in regards to unemployment insurance.

Appendix

Computational Details

The complete solution algorithm is as follows. The algorithm is an adaptation of the algorithm proposed in Maliar et al (2010), and the Matlab code is based on the Maliar et al Matlab code available online: www.stanford.edu/~maliaarl/Files/Codes.html.

Solution Algorithm:

1. Preliminary Computations:

- (a) Solve the representative agent steady state numerically; denote the capital stock thus computed K^{ss} , and the wage rate thus computed w^{ss} .
- (b) Construct the grid for individual capital: $k \in [0, k_{\max}]$. Here, $k_{\max} = 5K^{ss}$, and the grid has 100 polynomially spaced grid points.
- (c) Construct the grid for mean capital: $K \in [K_{\min}, K_{\max}]$. Here, K_{\min} and K_{\max} depend on the parameterisation; for the baseline parameterisation, $K_{\min} = K^{ss} - 5$ and $K_{\max} = K^{ss} + 5$, and the grid has 4 linearly spaced grid points.
- (d) Construct the grid for the wage norm: $w^s \in [w_{\min}^s, w_{\max}^s]$. Here, $w_{\min}^s = w^{ss} - 0.2$ and $w_{\max}^s = w^{ss} + 0.25$, and the grid has 4 linearly spaced grid points.
- (e) Initialise a decision function $k_{t+1} = \tilde{k}(k_t, K_{t+1}, w_{t+1}^s, \epsilon_t, A_t)$ on the grids thus constructed; in all the computations above, the initial decision function $k_{t+1} = 0.9k_t$ on each grid point.
- (f) Initialise the individual capital distribution and initial w^s . Here, each of the N agents' initial capital is set to K^{ss} , and w^s is set to w^{ss} .
- (g) Initialise B . Here, $B_0^L = B_2^L = B_0^H = B_2^H = 0$, and $B_1^L = B_1^H = 1$.
- (h) Given eqs 7, 10, 11, and 13, together with $e_t = \gamma + \psi$, solve $L_t = L(K_t, w_t^s, A_t)$, $Y_t = Y(K_t, w_t^s, A_t)$, $w_t = w(K_t, w_t^s, A_t)$, $r_t = r(K_t, w_t^s, A_t)$, $U_t = U(K_t, w_t^s, A_t) = 1 - L(K_t, w_t^s, A_t)$ numerically for each of the aggregate state grid points (recall A_t takes the values A^L and A^H).
- (i) Given $\pi^{ss'}$, generate a realisation of aggregate TFP shocks of length T using a pseudo-random number generator.

2. Main Loop:

- (a) Construct a boundedly rational estimate of the transition matrix for the idiosyncratic unemployment states. This involves the construction of a boundedly rational estimate of $\lambda_{t+1}^{ue} = \lambda(U_{t+1}, U_t) = \lambda(K_{t+1}, w_{t+1}^s, A_{t+1}, K_t, w_t^s, A_t)$. As the stochastic process for A_t is exogenous and known, and $w_{t+1}^s = w_t = w(K_t, w_t^s, A_t)$, the only non-trivial part of this problem is the estimation of K_{t+1} , which is achieved given B and the predictor described by eq 18 in the text. Note that, given eq 15 in the text, there are certain combinations of the aggregate states in consecutive time periods for which U_{t+1} and U_t are very different, and this can result in the estimated λ_{t+1}^{ue} being greater than 1. Therefore, the estimated λ_{t+1}^{ue} is restricted to lying between 0 and 1.

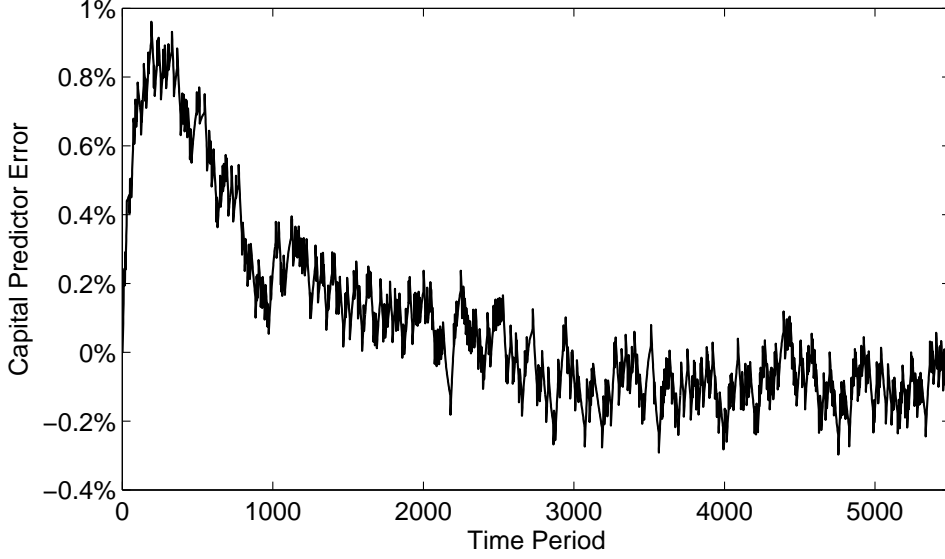


Figure 6: Capital Predictor Accuracy Plot

- (b) Given the estimate of the transition matrix thus calculated, solve the household decision problem. Substitute $L_t = L(K_t, w_t^s, A_t)$, $Y_t = Y(K_t, w_t^s, A_t)$, $w_t = w(K_t, w_t^s, A_t)$, and $r_t = r(K_t, w_t^s, A_t)$ into eq 17 in the text. Substitute the initial capital decision rule $k_{t+1} = \bar{k}(k_t, K_{t+1}, w_{t+1}^s, \epsilon_t, A_t)$ into the right hand side of eq 17 (where $k_{t+2} = \tilde{k}(\tilde{k}(k_t, K_{t+1}, w_{t+1}^s, \epsilon_t, A_t))$), which results in a new decision rule, $k_{t+1} = \tilde{k}'$. This process is iterated, where a weighted average of \tilde{k} and \tilde{k}' is used as the new decision rule to enter the right hand side of eq 17, until the difference between \tilde{k} and \tilde{k}' thus calculated falls below a given degree of precision. Denote this decision rule \bar{k} . Note that, in general, the predicted K_{t+1} and w_{t+1}^s will not generally be on the grid points, so L_{t+1} , w_{t+1} , and r_{t+1} are calculated by cubic spline interpolation of the functions $L_t = L(K_t, w_t^s, A_t)$, $w_t = w(K_t, w_t^s, A_t)$, and $r_t = r(K_t, w_t^s, A_t)$.
- (c) Given the preliminary computations and the solution to the household problem \bar{k} , simulate the aggregate economy for N agents and T time periods. This is as per section 3.4, and is fairly straightforward. Note, however, that λ_t^{ue} has to be computed each period given U_t and U_{t-1} , such that the idiosyncratic unemployment shocks have to be generated within each time period of the simulation. Also, a sub-routine randomly reassigns the employment status of agents each period such that the sampling error between the simulated unemployment rate and $U_t = U(K_t, w_t^s, A_t)$ is minimised given N .
- (d) The simulation thus computed generates time series for the aggregate state variables K and w^s . These are regressed onto the predictor function, yielding new estimates for the parameters B' . Finally, therefore, the entire main loop can be iterated until the difference between B and B' falls below a given degree of precision. The solution to the model is then the law of motion given by B , the household decision rule \bar{k} , and the employment and pricing functions L , w , and r thus computed.

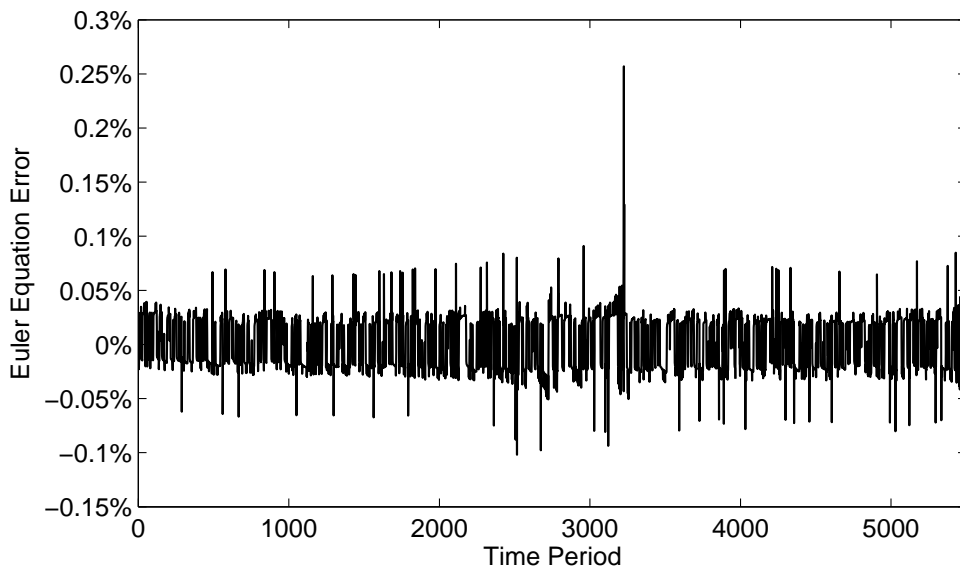


Figure 7: Euler Equation Accuracy Plot

With $N = 5000$, $T = 5500$ (with the first 500 periods discarded for the predictor regressions), the degree of precision for the individual capital function 10^{-12} , and the degree of precision for the aggregate law of motion 10^{-2} , the algorithm described above takes approximately 30 minutes to solve on a Lenovo Thinkpad T430i with a 2.4GHz intel processor. This is correspondingly slower than the solution time for the original Krusell and Smith (1998) model using the Maliar et al (2010) algorithm, given the larger state space of the present model and more involved aggregate simulation procedure.

Accuracy Checks

The 2010 JEDC special edition on solving the standard incomplete markets model (see Den Haan et al 2010) advises testing the perceived law of motion for capital against *ex post* realisations of the aggregate capital stock without updating the forecast each period (that is, iterating the aggregate law of motion from the initial state vector given the time series of total factor productivity shocks). It is not clear, however, that this procedure is exactly relevant to the present model, as the endogenous state vector also includes w^s . As $w_t^s = w_{t-1}$, its prediction is trivial, and it does not seem useful to attempt to generate it from initial conditions using the pricing functions described above. Given this, the adapted capital predictor accuracy test used here is as follows. Given a time series of aggregate shocks, and a corresponding time series of realisations of w_t generated from the solved model, calculate a predicted capital series K^P as follows:

$$\ln(K_{t+1}^P) = \begin{cases} B_0^L + B_1^L \ln(K_t^P) + B_2^L \ln(w_{t-1}) & \text{if } A_t = A^L \\ B_0^H + B_1^H \ln(K_t^P) + B_2^H \ln(w_{t-1}) & \text{if } A_t = A^H \end{cases}$$

The generated time series K^P does not exactly allow the same test that Den Haan et al advise, but it does allow a stronger test of the predictor function accuracy than examination

of R^2 and $\hat{\sigma}$ values alone. Figure 6 plots the percentage error between K^P and the model generated aggregate capital time series for the case $N = 5000$ and $T = 5500$. The accuracy of the predictor tested in this manner is relatively high: the per-period error is always below 1%, and is much lower than this after the time period for which the regressions are calculated (in this example, $T = 500$).

Finally, figure 7 plots the percentage Euler equation errors of a single agent for the baseline parameterisation calculated from a dynamic Euler equation accuracy test. This is slightly more involved than the capital predictor accuracy test. Given the time series of the cross sectional distribution of capital computed in the numerical solution to the model, a single agent's time series for individual capital is chosen. Given this time series, and the employment and pricing functions described above, the optimal level of consumption implied in each period can be calculated using eq 17. This can then be compared to that agent's actual time series of consumption - that is, the level of consumption given the boundedly rational predictor of capital. Again, the computation appears to be fairly accurate, with the Euler equation errors generally less than 0.1% in absolute value.

References

- [1] Akerlof, G. 1982. Labor Contracts As A Partial Gift Exchange. *The Quarterly Journal of Economics*, **97**, 543-569.
- [2] Akerlof, G., & Yellen, J. 1986. *Efficiency Wage Models Of The Labor Market*. Cambridge: Cambridge University Press.
- [3] Alexopoulos, M. 2004. Unemployment And The Business Cycle. *Journal of Monetary Economics*, **51**, 277-298.
- [4] Alexopoulos, M. 2007. A Monetary Business Cycle Model With Unemployment. *Journal of Economic Dynamics & Control*, **31**, 3904-3940.
- [5] Bewley, T. 1999. *Why Wage Don't Fall During A Recession*. Cambridge, MA: Harvard University Press.
- [6] Brecher, R., Chen, Z., & Choudhri, E. 2010. A Dynamic Model Of Shirking And Unemployment: Private Saving, Public Debt, And Optimal Taxation. *Journal of Economic Dynamics & Control*, **34**, 1392-1402.
- [7] Collard, F., & de la Croix, D. 2000. Gift Exchange And The Business Cycle: The Fair Wage Strikes Back. *Review Of Economic Dynamics*, **3**, 166-193.
- [8] Danthine, J., & Donaldson, B. 1990. Efficiency Wages And The Business Cycle Puzzle. *European Economic Review*, **34**, 1275-1301.
- [9] Danthine, J., & Kurmann, A. 2004. Fair Wages In A New Keynesian Model Of The Business Cycle. *Review Of Economic Dynamics*, **7**, 107-142.
- [10] Danthine, J., & Kurmann, A. 2007. The Macroeconomic Consequences Of Reciprocity In Labor Relations. *The Scandinavian Journal Of Economics*, **109**, 857-881.
- [11] Danthine, J., & Kurmann, A. 2010. The Business Cycle Implications Of Reciprocity In Labor Relations. *Journal of Monetary Economics*, **57**, 837-850.
- [12] Den Haan, W., Judd, K., & Juillard, M. 2010. Computational Suite Of Models With Heterogeneous Agents: Incomplete Markets And Aggregate Uncertainty. *Journal of Economic Dynamics & Control*, **34**, 1-3.
- [13] Di Tella, R., MacCulloch, R., & Oswald, A. 2001. Preferences Over Inflation And Unemployment: Evidence From Surveys Of Happiness. *American Economic Review*, **91**, 335-341.
- [14] Faccini, R., & Bondibene, C. 2012. Labour Market Institutions And Unemployment Volatility: Evidence From OECD Countries. Bank of England Working Paper No. 461.
- [15] Givens, G. 2011. Unemployment Insurance In A Sticky-Price Model With Worker Moral Hazard. *Journal of Economic Dynamics & Control*, **35**, 1192-1214.
- [16] Gomme, P. 1999. Shirking, Unemployment And Aggregate Fluctuations. *International Economic Review*, **40**, 3-21.

- [17] Grandmont, J. 2008. Negishi-Solow Efficiency Wages, Unemployment Insurance and Dynamic Deterministic Indeterminacy. *International Journal of Economic Theory*, **4**, 247-272.
- [18] Hagedorn, M., & Manovskii, I. 2008. The Cyclical Behavior Of Equilibrium Unemployment And Vacancies Revisited. *American Economic Review*, **98**, 1692-1706.
- [19] Heathcote, J., Storesletten, K., & Violante, G. 2009. Quantitative Macroeconomics With Heterogeneous Households. Federal Reserve Bank of Minneapolis Research Department Staff Report 420.
- [20] Krusell, P., Mukoyama, T., & Şahin, A. 2010. Labour-Market Matching With Precautionary Savings And Aggregate Fluctuations. *Review of Economic Studies*, **77**, 1477-1507.
- [21] Krusell, P., & Smith, A. 1998. Income And Wealth Heterogeneity In The Macroeconomy. *Journal of Political Economy*, **106**, 867-896.
- [22] Maliar, L., Maliar, S., & Valli, F. 2010. Solving The Incomplete Markets Model With Aggregate Uncertainty Using The Krusell-Smith Algorithm. *Journal of Economic Dynamics & Control*, **34**, 42-49.
- [23] Marx, K. 1967 [1867]. *Capital*. London: Lawrence & Wishart.
- [24] Messina, J., Strozzi, C., & Turunen, J. 2009. Real Wages Over The Business Cycle: OECD Evidence From The Time And Frequency Domains. *Journal of Economic Dynamics & Control*, **33**, 1183-1200.
- [25] Mortensen, D., & Nagypál, E. More On Unemployment And Vacancy Fluctuations. *Review of Economic Dynamics*, **10**, 327-347.
- [26] Nakajima, T. 2010. Optimal Monetary Policy With Imperfect Unemployment Insurance. *Journal of Economic Dynamics & Control*, **34**, 365-387.
- [27] Nickell, S. 1997. Unemployment And Labor Market Rigidities: Europe Versus North America. *The Journal of Economic Perspectives*, **11**, 55-74.
- [28] Nickell, S., Nunziata, L., & Ochel, W. 2005. Unemployment In The OECD Since The 1960s: What Do We Know? *The Economic Journal*, **115**, 1-27.
- [29] Ohanian, L., & Raffo, A. 2012. Aggregate Hours Worked In OECD Countries: New Measurement And Implications For Business Cycles. *Journal of Monetary Economics*, **59**, 40-56.
- [30] Shapiro, C., & Stiglitz, J. 1984. Equilibrium Unemployment As A Worker Discipline Device. *The American Economic Review*, **74**, 433-444.
- [31] Shimer, R. 2005. The Cyclical Behavior Of Equilibrium Unemployment And Vacancies. *The American Economic Review*, **95**, 25-49.
- [32] Solow, R. 1979. Another Possible Source Of Wage Stickiness. *Journal of Macroeconomics*, **1**, 79-82.

- [33] Tripier, F. 2006. Sticky Prices, Fair Wages, And The Co-Movements Of Unemployment And Labor Productivity Growth. *Journal of Economic Dynamics & Control*, **30**, 2749-2774.
- [34] Wesselbaum, D. 2013. Reciprocity And Matching Frictions. *International Review of Economics*, **60**, 247-268.