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Working Paper

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Real-time Markov Switching and Leading Indicators in Times of the Financial Crisis

Abstract
This paper uses several macroeconomic and financial indicators within a Markov Switching (MS) framework to predict the turning points of the business cycle. The presented model is applied to monthly German real-time data covering the recession and the recovery after the financial crisis. We show how to take advantage of combining single MS forecasts and of changing the number of regimes on the real-time path, where both leads to a higher forecast accuracy. Changing the number of regimes implies a distinction for recessions representing either a normal or an extraordinary one, which particularly means to determine as early as possible the point in time, from which the last recession structurally exceeded the previous ones. In fact it turns out that the Markov Switching model can signal quite early whether a conventional recession happens or whether an economic downturn will be more substantial.

Keywords: Business Cycle, Leading Indicators, Macroeconomic Forecasting, Markov Switching

JEL ref.: EL CLASSIFICATION SYSTEM: C24, C53, E37

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1 Introduction

As a consequence of the financial crisis in the years 2008 and partly 2009, Germany suffered from its strongest decline in GDP since the global economic crisis of 1929. More than ever it seems to be worthwhile to predict the business cycle and its turning points in order to react properly to recessions by counter-cyclical economic policy. This paper [1] delivers predictions within a Markov Switching (MS) framework. In this class of models it is also possible to analyze the impact of an additional regressor by Markov Switching Autoregressive Models with Exogenous Variables (MSARX). For instance this was used by Lee, Liang and Chou (2009) for regressing the real estate cycle on its lags as well as on a composite leading index. But still most of the MS business cycle literature concentrates on purely autoregressive estimations following the famous Markov Switching Mean Model (MSM) by Hamilton (1989) or as stated by Boldin (1996): 'Because the estimated parameters of relatively simple MSM specifications match many stylized facts about the business cycle, this framework has become an important alternative to linear, autoregressive structures.' Yet, contrary to the linear case, a straightforward set of specification tests for MS models, in particular covering a highly parameterized design and clearly preferring MSM, is not available, also see Breunig, Najarian and Pagan (2003). In contrast to a purely autoregressive MSM the inclusion of leading indicators as explanatory variables can be motivated by the promise to deliver additional information for a policy maker. Thus, this paper considers these indicators in univariate MS regressions with lagged dependent and lagged leading variables to draw conclusions about their significance and prediction performance according to their (real-time) characteristics. These characteristics comprise the dimension of an indicator being either subject to a publication lag or not and the dimension of an indicator belonging to the class of financial or real economy variables. Apart from such an analysis about the leading indicators the timely and safe recognition of business cycle regimes still represents the central feature of the model.

Back in the 70ies research efforts were mainly focused on exact dating of business cycle turning points culminating in the fundamental book of Bry and Boschan (1971), where they developed a solid working non parametric dating algorithm. Nowadays the focus

[1] I would like to thank the whole IMK research team, especially Sven Schreiber, Christian Proaño and Sabine Stephan, for many helpful comments and suggestions, as well as Daniel Detzer for excellent data assistance. The paper also benefited from the comments of conference participants *Statistische Woche 2011*.

has turned to real-time business cycle predictions ahead from the publication point in time. Therefore it is crucial to deal with two questions. Firstly what estimation procedure to use and secondly what indicators related to the business cycle are to be included. *Inter alia* the development of estimation procedures was fostered by Chauvet and Potter (2005) using different specifications of the probit model, by Stock and Watson (1989) introducing dynamic factor models for the business cycle and by Hamilton (1989) proposing the Markov Switching model, which this paper applies to monthly German real-time data. Simultaneously within the development of different prediction procedures the set of leading real economy indicators was extended by financial ones such as spreads from the term structure of interest rates, e.g. by Estrella and Hardouvelis (1991) and the spread between corporate and public issuers, e.g. by Friedman and Kuttner (1992). Most recently the connection between the corporate spread and the economic development was analyzed by Gilchrist, Yankov and Zakrajsek (2009) as well as by Meeks (2011). As the rapid expansion of the credit derivative market is seen as one of the reasons for the extent of the last crisis, credit growth may arise as a predictor of the business cycle. Here the general connection was described by Biggs, Mayer and Pick (2009), whereas the presented model concentrates on credit growth as it is reported in the balance sheets of monetary financial institutions.

Disparity between the characteristics of financial and real economy indicators becomes especially essential with real-time forecasts. While financial data, at least with monthly frequency, is provided immediately and is not subject to revisions, this is the case for most of macroeconomic variables. As Diebold and Rudebusch (1991) pointed out for the U.S. Composite Leading Index, revisions and in fact also the lagged data availability substantially affect the predictive power of leading indicators. That is why this paper considers real-time data and additionally contributes to the literature by adapting the MS model while proceeding on the real-time path. For this purpose a proposal by Hamilton (2011) is elaborated by 'averaging the inference from alternative specifications'. Whereas this has been done so far for other methods, see e.g. Proaño (2010), little Markov Switching literature follows such an approach. Indeed it is the basis to show how a data sample dependent change in the number of regimes can stabilize real-time forecasts.

The paper at hand is structured as follows. Section 2 briefly repeats the main steps of the filter introduced by Hamilton (1989, 1990) and stresses the fact that exogenous variables have an impact on the state probabilities when using an expectation-maximization (EM) algorithm for the MS model. Section 3 describes input and in-sample results of the model.

First it presents the data. Then the best working monthly proxy of the business cycle, i.e. the reference series, is selected. Moreover this part of section 3 explains the selection of time series representing leading indicators. For both reference series and indicator series real-time characteristics are provided. Afterwards model specification is discussed in detail, thereby expressing how the structure of the model results as a compromise between information needs and available real-time data records. While the former suggests a highly parameterized design the latter clearly restricts the parameter space. As a next stage a benchmark algorithm, based on the work of Bry and Boschan (1971) as well as Harding and Pagan (2002) is presented in order to provide an evaluation tool for the performance of the MS model. The next part of section 3 describes the in-sample results for the publication on November 2010, which confirm the selected model specification. In a last part of this section goodness-of-fit measures in the case of more than two regimes are discussed. Based on these measures it is analyzed to see, if in general those leading indicators perform better which are not subject to a data availability lag. Section 4 deals with the predictive output of the model. First the real-time features of the forecast are given. This in particular incorporates how to change the number of regimes in real-time and how the sensitivities of financial versus real economy indicators have developed at the point in time, when introducing new regimes. Afterwards out-of-sample results for industrial production as the monthly proxy of overall economic activity are presented. Finally the procedure is repeated for German data of the Composite Index of Leading Indicators (CLI), which is provided by OECD to represent a leading proxy for the growth rates of the reference series. Section 5 concludes.

2 Markov Switching and Exogenous Variables

When starting business cycle modeling it is useful to look for a well-defined and generally acknowledged borderline between recessions and expansions, such as it is given by the National Bureau of Economic Research, NBER (2011) for the U.S.:

> A recession is a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales. A recession begins just after the economy reaches a peak of activity and ends as the economy reaches its trough. Between trough and peak, the economy is in an expansion.

Although this definition might suggest a five-dimensional MSVAR model, as introduced by Krolzig (1997), a different approach is taken here. The main motive for that is the fact that a MSVAR model would additionally enlarge the parameter space, which is difficult to align with the available German real-time data records. For the same reason one-dimensional equations are arranged to include only two kinds of regressors, lags of the dependent variable and one exogenous variable including its lags. Moreover only the coefficient of the most recent lag is chosen to switch in order to minimize the number of parameters that have to be estimated. This leads to the following form

$$\begin{aligned} y_t = \beta_0^{S_t} + \beta_{1,y}^{S_t} y_{t-f-D_y} + \sum_{j=2}^{p} \beta_{j,y} y_{t-j+1-f-D_y} \\ + \beta_{1,x}^{S_t} x_{t-f-D_x} + \sum_{j=2}^{q} \beta_{j,x} x_{t-j+1-f-D_x} + u_t, \\ u_t \sim N\left(0, \sigma^{S_t}\right), \quad t = 1, \ldots, T, \end{aligned} \tag{1}$$

where f represents the forecasting horizon and D_y, D_x the data availability lag of the dependent and independent variable. S_t stands for the latent states that generate the total process of the observed time series y_t. In the following Hamilton (1989, 1990)'s filter is reproduced with the data availability lag being set to 0, the forecasting horizon to 1 and we turn to vector notation for simplification. So (1) is rewritten as

$$\begin{aligned} y_t &= z_t' \beta^{S_t} + u_t, \quad u_t \sim N\left(0, \sigma^{S_t}\right) \quad \text{with} \\ z_t' &= \left(1, y_{t-1}, y_{t-2}, \ldots, y_{t-p}, x_{t-1}, x_{t-2}, \ldots, x_{t-q}\right), \\ \beta^{S_t\prime} &= \left(\beta_0^{S_t}, \beta_{1,y}^{S_t}, \beta_{2,y}, \ldots, \beta_{p,y}, \beta_{1,x}^{S_t}, \beta_{2,x}, \ldots, \beta_{q,x}\right), \\ \theta' &= \left(\beta^{S_t\prime}, \sigma^{S_t}\right). \end{aligned} \tag{2}$$

The central characteristic of the Markov Switching model is the fact that the hidden states of the dependent variable are generated by a first order Markov chain, whose transition matrix for a two regime setting looks like

$$P = (p_{ij}) = \begin{pmatrix} \mathcal{P}\left(S_t = 1 | S_{t-1} = 1\right) & \mathcal{P}\left(S_t = 1 | S_{t-1} = 2\right) \\ \mathcal{P}\left(S_t = 2 | S_{t-1} = 1\right) & \mathcal{P}\left(S_t = 2 | S_{t-1} = 2\right) \end{pmatrix}. \tag{3}$$

Later on the model will be extended to four regimes so that alone for the transition matrix 12 parameters have to be estimated. This is where the available real-time data records become relevant. But to keep notation simple the case of two regimes is considered here.

The second element to make a Markov chain unique is its starting distribution, i.e.

$$\hat{\xi}_{1|0} = \begin{pmatrix} \mathcal{P}\left(S_1 = 1|y_0\right) \\ \mathcal{P}\left(S_1 = 2|y_0\right) \end{pmatrix}. \tag{4}$$

Considering the observation period in (1), y_0 cannot be observed so that $\hat{}$-notation hints at an initial guess for the starting distribution. In fact at the beginning of the maximization algorithm both entries of the transition matrix and starting distribution are chosen uniformly, which later plays a role when deciding which regimes are related to recessions and which to expansions of the business cycle. The same applies to the entries of θ. The normality assumption delivers the following density vector

$$\eta_t = \begin{pmatrix} f\left(y_t|S_t = 1, z_t, \theta\right) = \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\left(\frac{y_t - z_t'\beta^1}{2\sigma^1}\right)^2\right) \\ f\left(y_t|S_t = 2, z_t, \theta\right) = \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\left(\frac{y_t - z_t'\beta^2}{2\sigma^2}\right)^2\right) \end{pmatrix}. \tag{5}$$

Applying the proposition of total probability and denoting an element wise multiplication by $\odot$ one obtains the following result of Hamilton (1994, p.692)

$$\hat{\xi}_{t|t} = \frac{\left(\hat{\xi}_{t|t-1} \odot \eta_t\right)}{\mathbf{1}'\left(\hat{\xi}_{t|t-1} \odot \eta_t\right)}, \tag{6}$$

which for the beginning of the series means nothing else than that filtered probabilities

$$\hat{\xi}_{1|1} = \begin{pmatrix} \mathcal{P}\left(S_1 = 1|y_1, \hat{\theta}\right) \\ \mathcal{P}\left(S_1 = 2|y_1, \hat{\theta}\right) \end{pmatrix} \tag{7}$$

have been calculated. The important implication for this paper is that, whenever estimates of θ change with different exogenous variables, $\hat{\xi}_{1|1}$ and in general all filtered probabilities will also do. By the equivalence of $S_t = i \Leftrightarrow \xi_t = e_i$, where the last one represents the i-th unity vector, the preliminary probability for the second observation can be computed by

$$\hat{\xi}_{2|1} = \hat{P}\hat{\xi}_{1|1}. \tag{8}$$

Here P stands for the transition matrix. Kim (1994) showed how to compute smoothed state probabilities by

$$\hat{\xi}_{t|T} = \hat{\xi}_{t|t} \odot \left(P'\left(\hat{\xi}_{t+1|T} \oslash \hat{\xi}_{t+1|t}\right)\right), \tag{9}$$

where $\oslash$ denotes an element wise division. Repeating the procedure recursively finally delivers (smoothed) state probabilities for all observations. What is left is to concretize

the maximization process. As a manipulation of standard maximum likelihood approach Hamilton (1994, p.696) developed for equations of the form (2) the following target function

$$\sum_{t=1}^{T}\sum_{i=1}^{r=2}\mathcal{P}\left(S_t=i|Z_T\right)\log f\left(Y_t|S_t=i,Z_t,\theta\right). \tag{10}$$

Thus, writing the necessary condition of maximization in vector notation leads to

$$\sum_{t=1}^{T}\left(\frac{\partial\log\eta_t}{\partial\theta'}\right)'\hat{\xi}_{t|T}=0. \tag{11}$$

For simplification restrict θ' to $\left(\beta_0^{S_t},\beta_{1,y}^{S_t},\beta_{1,x}^{S_t},\sigma^{S_t}\right)$. Then (11) is equivalent to the following non-linear system of equations

$$\begin{aligned}
&\sum_{t=1}^{T}\mathcal{P}\left(S_t=1|z_T\right)\frac{\partial\log\eta_t^1}{\partial\beta_0^1}+\mathcal{P}\left(S_t=2|z_T\right)\frac{\partial\log\eta_t^2}{\partial\beta_0^2}=0\\
&\sum_{t=1}^{T}\mathcal{P}\left(S_t=1|z_T\right)\frac{\partial\log\eta_t^1}{\partial\beta_{1,y}^1}+\mathcal{P}\left(S_t=2|z_T\right)\frac{\partial\log\eta_t^2}{\partial\beta_{1,y}^2}=0\\
&\sum_{t=1}^{T}\mathcal{P}\left(S_t=1|z_T\right)\frac{\partial\log\eta_t^1}{\partial\beta_{1,x}^1}+\mathcal{P}\left(S_t=2|z_T\right)\frac{\partial\log\eta_t^2}{\partial\beta_{1,x}^2}=0\\
&\sum_{t=1}^{T}\mathcal{P}\left(S_t=1|z_T\right)\frac{\partial\log\eta_t^1}{\partial\sigma^1}+\mathcal{P}\left(S_t=2|z_T\right)\frac{\partial\log\eta_t^2}{\partial\sigma^2}=0.
\end{aligned} \tag{12}$$

Its nonlinearity arises from the derivative with respect to σ^{S_t}. Perlin (2009) used gradient ascent method to obtain $\hat{\theta}$ as a solution of (12). Moreover Hamilton (1990) derived estimates for the transition probabilities by

$$\hat{p}_{ij}=\frac{\sum_{t=2}^{T}\mathcal{P}\left(S_t=j,S_{t-1}=i|z_T,\hat{\theta}\right)}{\sum_{t=2}^{T}\mathcal{P}\left(S_{t-1}=i|z_T,\hat{\theta}\right)}.^{1} \tag{13}$$

With $\hat{\theta}$, $\hat{\xi}_{1,T}$ and $\hat{p}_{ij}$ replacing the randomly chosen initial values the process reaches its second turn. Here again state probabilities are computed and the whole procedure iterates up to the convergence of the likelihood function. The process is described in detail to stress the fact that with different exogenous explanatory variables also the solution of the

[1] One could include $(\hat{p}_{ij})$ in $\hat{\theta}$, but to facilitate some remarks in this section as well as in the one about the LR test transition probabilities are separated by the selected notation.

non-linear system of equations in (12) changes. Thus it is the double effect [2] of different entries in z_t' and in β^{S_t} during the whole maximization process that ultimately generates different state probabilities.

At first sight this may sound trivial, but the impact of it becomes evident, when considering that the model does not comprise one equation of type (1). Instead of this the model is constructed in the spirit of Timmermann (2006) assuming that each equation of type (1) may be subject to a 'misspecification bias of unknown form' and that simple averaging can lower the effect of such a bias. As mentioned above one reason for the bias could be the limit of the parameter space given the available real-time data records. In the following each equation will be represented by its exogenous regressor. Against the background of achieving proper forecasts these regressors will be leading indicators of the business cycle.

3 Model Specification and In-sample Evaluation

3.1 Data Selection

The intention of this paper is to use monthly data in order to make a statement about business cycle regimes between the quarterly publications of GDP. Fritsche and Stephan (2002) point out that the highest correlated monthly proxy of overall economic activity is industrial production capturing the volatile parts of GDP. These are the main parts of investment and exports. In the database of Deutsche Bundesbank (2011*a*,*b*,*c*), which represents the data source for most of the data [3], even a slightly broader defined production index (IP1AA020) can be found. So this is taken as the monthly reference series. At the time of work the corresponding data availability lag covers two months.

When considering the growth rates of the reference series in figure 1, the high volatility of this frequency delivers several months with a relatively high positive value even in recessions and with a relatively low negative value even in expansions, e.g. $\pm$ 2%. As it turns out, this short term contrary dynamic cannot be captured by the autoregressive terms, which results in the MS model switching inconsistently with the business cycle phases. In this case the filtered regimes seem to reflect the asymmetry between positive and negative rates exceeding a certain absolute value. But, as it is stated by the NBER definition, this does

[2] The most intuitive term to see this is the conditional expectation $z_t'\beta^{S_t}$ in the density vector (5).

[3] Other sources are ifo Institut für Wirtschaftsforschung (2011) and OECD (2011).

Growth rates of backwards smoothed industrial production

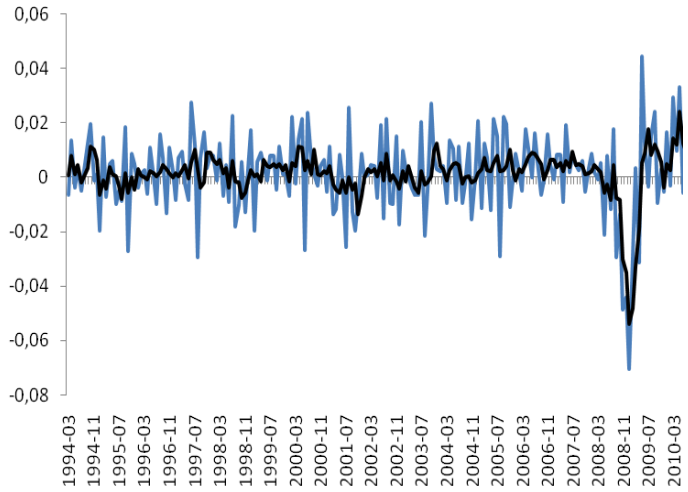


Figure 1: The blue line illustrates the growth rates of monthly industrial production on publication November 2010 for observations from March 1994 to September 2010, which lead to the MS model switching inconsistently with the business cycle phases. This changes when taking a backwards smoothed version (black line) of the industrial production.

BDS Test for growth rates of smoothed industrial production

Dimension	BDS Statistic	Std. Error	z-Statistic	Normal Prob.	Bootstrap Prob.
2	0.027095	0.005469	4.954080	0.0000	0.0000
3	0.056127	0.008721	6.435694	0.0000	0.0000
4	0.067283	0.010419	6.457709	0.0000	0.0000
5	0.069270	0.010894	6.358438	0.0000	0.0000
6	0.062452	0.010539	5.925663	0.0000	0.0000

Raw epsilon		0.006371		
Pairs within epsilon		27118.00	V-Statistic	0.705904
Triples within epsilon		4039498.	V-Statistic	0.536487

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	9877.000	0.522178	13309.00	0.703621	0.495083
3	7530.000	0.402222	13144.00	0.702099	0.346095
4	5726.000	0.309046	12992.00	0.701209	0.241763
5	4417.000	0.240892	12889.00	0.702934	0.171622
6	3420.000	0.188482	12848.00	0.708074	0.126029

Figure 2: The test result with a clear rejection of the i.i.d - hypothesis confirms the appropriateness of a non-linear model. When computing ϵ/σ this lies in the interval of $[0.5, 2]$. This as well as a maximum embedded dimension of $m = 5$ represent the relevant range for i.i.d - series according to Brock et al. (1996). Since N/m does not reach the usual size in addition bootstrapped p-values are calculated.

not automatically correspond to recessions and expansions, since the dynamic in the surrounding, in particular the duration of more than a few months and the significance of a change, should be taken into account. One way to handle this is by smoothing the reference series backwards by a moving average of order 3. In doing so we obtain the coincidence between iterated regimes and business cycle phases found by Hamilton (1989), who runs his model on quarterly data. Despite the necessary smoothing it is not useless to take monthly data since this enables one to make a statement between the quarterly publications. As for example discussed by Krolzig (1997, p.20) MS after all represents a non-linear model. Thus before starting estimation it is reasonable to apply a nonlinearity test to the reference series. A widely used test for nonlinearity is the one developed by Brock et al. (1996). Thereby it is analyzed if the residuals of an ARMA estimation obey

$$H_0 : \text{The time series is i.i.d.}$$

As figure 2 shows, H_0 is rejected on every regular level of significance, which in fact suggests to apply a non-linear method to the data.

As a next stage leading indicators are introduced. These are foreign and domestic orders, construction permissions, CDAX stock index, a spread between corporate and public issuer's current yield, the 3-month EURIBOR interest rate, the ifo business climate index, credit growth according to the European System of Central Banks (ESCB) statistics, a maturity spread between 10-year federal bonds and 3-month EURIBOR as well as job vacancies. Apart from the corporate spread, the credit growth and job vacancies, for which the introduction contains relevant literature, a similar information set contributes to the U.S. Composite Index of Leading Indicators.

The lead of most of the indicators is obvious since they reflect pre-stages to the production process, such as orders or construction permissions, or expectations to the economic development, such as business climate or job vacancies. But as it turns out later on, the last one only delivers weak significance. Nevertheless its purpose is also to include at least one variable from the labor market. The intuition behind the corporate spread is that whenever a recession approaches, this will lead to higher default rates of companies, whereas federal bonds remain a safe haven. Since short term interest rates react more sensitively to the current economic situation, the spread between long-term and short-term maturity embodies predictive power - even sometimes ending up with an inverse yield curve. The role of credit growth will be dealt with in detail later on.

In general financial and survey variables are not subject to revisions and to lagged data availability, whereas macroeconomic variables are. For all macroeconomic variables used in this model the data availability lag is two months - except for job vacancies, which are provided immediately. The only financial variable, for which it takes some time (one month) until it is published in the reports of the ESCB, is credit growth.

3.2 Model Specification

The usual trade-off between improving the overall fit by additional significant regressors and making it worse by over-specification particularly arises with MS models. Whereas in a standard linear estimation an equation with only two kinds of regressors and restricted lag selection may lead to an omitted variable bias, in MS models this is only obvious, when the bias occurs in each of the regime depending equations. However extending the equations with additional variables or switching parameters in such a way that optimization described in section 2 only finds local maxima seems to be the greater mistake, Boldin (1996). As it can be seen due to the degree of freedom reported in table 1 there are at most four switching coefficients in a single equation in order to guarantee the numerical robustness of the approach. Nevertheless restricted real-time data records still remains a problem.

Table 1 illustrates the shape of each equation according to a general-to-specific lag choice by information criteria. Lags can be selected up to a maximum of 5 when in addition a minimum of state probabilities agrees with the benchmark method described in section 3.3. In a former version lag choice was implemented to be renewed for each publication on the real-time path. But because of an exploding running time the lag structure in table 1 was fixed for all real-time estimations.[4]

A natural benchmark for MS business cycle applications was introduced by Hamilton (1989):

$$y_t - \beta_{S_t} = \sum_{j=1}^{4} \phi_j \left(y_{t-j} - \beta_{S_{t-j}} \right) + \epsilon_t, \quad \epsilon_t \sim N\left(0, \sigma^2\right), \quad S_t = 1, 2.$$

[4]First of all lag choice is based on the publication on January 2007, i.e. the last before real-time forecasts of section 4.2 start, but at this point in time a higher regime design is only hypothetically assumed and not introduced according to the criteria (19). In-sample results presented in section 3.4 therefore check if the selected lag structure still works for a publication (November 2010) after the number of regimes has really changed.

Although this model is not used to evaluate the results of our MS model, it is helpful to decide if the intercept in each equation of type (1) should switch or not. Perlin (2009) showed that estimating the following version of the Hamilton model

$$y_t = \beta_{S_t} + \epsilon_{a,t}, \quad \epsilon_{a,t} \sim N\left(0, \sigma_a^2\right), \quad S_t = 1, 2$$
$$\epsilon_{a,t} = \sum_{j=1}^{4} \phi_j \epsilon_{a,t-j} + \epsilon_{b,t}, \quad \epsilon_{b,t} \sim N\left(0, \sigma_b^2\right)$$

delivers similar values for the coefficients linked to the state probabilities. This identifies the switching intercept as the most relevant part for the iteration of the state probabilities. So it is included in the model presented here.

But as mentioned before there are more switching coefficients in each equation. Reasons for that are as follows: Firstly a switching variance of the error term allows applying the Welch test in order to identify different normal distributions when turning to a four regime setting. Secondly at least one switching coefficient of the embedded leading indicator allows measuring the change between regimes representing different intensities of the same business cycle phase (weak and strong recession as well as weak and strong expansion). See table 4 in section 4.1.

3.3 The Benchmark Model

In order to evaluate the results of the MS model an ex-post-dating algorithm based on the work of Bry and Boschan (1971) as well as Harding and Pagan (2002) was employed. Coming back to the definition in section 2 this algorithm should find recessions between peaks and troughs of the reference series (and expansions vice versa). Rewriting the reference series in levels, in a first step, candidates for the turning points of the business cycle have to be recognized. Local extrema can be found by

$$\{y_t : y_t > y_{t\pm k}, k = 1, \ldots, 5\} \cup \{y_t : y_t < y_{t\pm k}, k = 1, \ldots, 5\}. \tag{14}$$

Not each local extremum automatically represents a turning point since one might be confronted with the phenomenon of extra cycles or non-alternating extrema. That is why (14) just reflects the necessary condition for the dating algorithm. The sufficient condition is introduced according to the triangle approximation of a recession, Harding and Pagan

(2002). It consist of the product of duration and relative amplitude exceeding a certain limit, i.e. for recessions

$$\Delta = \frac{1}{2}\left(t_{peak} - t_{trough}\right)\frac{y_{peak} - y_{trough}}{y_{peak}} > 0.025. \tag{15}$$

There is no unique value in the literature for the right hand side of the inequality, so it has been chosen to be quite selective. Imagine a recession lasting 5 months. Then the decline of the overall economic downturn must add up to 1% of the base level, i.e. the value at the last peak. If the duration of the recession is 10 months, a decline of 0.5% of the base level will be sufficient. The resulting binary series, generated by the benchmark algorithm and detecting six recessions (respectively periods of stagnation) between 1994 and 2010, is shown in each of the sub-figures of figure 3. Regarding the criterion (14) this dating algorithm can only be used for expost analysis, namely after 5 months plus the data availability lag have expired. This should be in mind when in figures 3, 4 and 5 MS results are compared with the benchmark.

3.4 In-sample Evaluation

Tables 1 and 2 as well as figure 3 present in-sample results for the different MS specifications. In section 3.2 their design was developed. Table 1 deals with a four regime setting and contains p-values for each of the parameters in brackets. Standard errors have been calculated by the method of Perlin (2009) to be robust to heteroskedasticity and serial correlation according to Newey and West (1987). The only systematic insignificance is the rejection of third regime's parameters when running the specification with job vacancies. This means that in this case it cannot be excluded that each of the coefficients and with this the contribution to the fit could be equal to 0. With respect to the consistency of the whole system and to the intention to include at least one variable from the labor market these results were accepted anyway. However it must be mentioned that these are p-values for the publication on November 2010. Indeed p-values may change for all real-time estimations, but it would go beyond the scope of this paper to consider each of the publications for the in-sample analysis.

Generally, one would link an extended credit flow, which reflects better access to finance investments, with an upturn of the economic situation.

Sample: 1994:03 – 2010:09 Publication: 2010:11	Switching Intercept				Switching Endogenous Lag				Non Switch Endogenous
Purely Autoregressive	0.0101 (0.00)	0.0035 (0.00)	-0.0022 (0.05)	-0.0395 (0.00)	- (-)	- (-)	- (-)	- (-)	-0.1128 (0.00)
Foreign Orders	0.0054 (0.00)	-0.0001 (0.59)	-0.0203 (0.00)	-0.0341 (0.00)	-0.0919 (0.00)	-0.1320 (0.00)	-1.2808 (0.00)	0.3485 (0.00)	- (-)
Domestic Orders	0.0134 (0.00)	0.0032 (0.00)	-0.0018 (0.00)	-0.0326 (0.00)	- (-)	- (-)	- (-)	- (-)	- (-)
Construction Permissions [5]	0.0049 (0.00)	0.0048 (0.00)	-0.0015 (0.00)	-0.0397 (0.00)	-0.0099 (0.00)	0.4554 (0.00)	-0.1212 (0.00)	-0.0690 (0.00)	-0.0171 (0.18)
CDAX	0.0059 (0.00)	-0.0005 (0.03)	-0.0097 (0.00)	-0.0311 (0.00)	-0.1006 (0.00)	-0.0944 (0.00)	0.2203 (0.00)	0.9772 (0.00)	- (-)
Corporate Spread	0.0097 (0.00)	0.0034 (0.00)	-0.0034 (0.12)	-0.0538 (0.00)	-0.0574 (0.00)	-0.0894 (0.00)	-0.1222 (0.00)	-0.4204 (0.00)	- (-)
Euribor - 3M	0.0117 (0.00)	0.0035 (0.00)	-0.0025 (0.00)	-0.0427 (0.00)	- (-)	- (-)	- (-)	- (-)	- (-)
ifo Business Climate	0.0097 (0.00)	0.0085 (0.00)	0.0055 (0.00)	-0.0001 (0.00)	0.2738 (0.00)	1.0877 (0.00)	-0.0854 (0.04)	-0.1269 (0.00)	- (-)
Credit Growth [6]	0.0103 (0.00)	0,0047 (0.85)	-0.0025 (0.16)	-0.0431 (0.00)	-0.1169 (0.08)	-0.1221 (0.00)	-0.1279 (0.05)	-0.2321 (0.25)	- (-)
Maturity Spread	0.0099 (0.00)	0.0030 (0.00)	-0.0020 (0.00)	-0.0270 (0.00)	- (-)	- (-)	- (-)	- (-)	- (-)
Job Vacancies	0.0115 (0.01)	0.0028 (0.03)	-0.0024 (0.71)	-0.0162 (0.00)	-0.1226 (0.01)	-0.0953 (0.02)	-0.1751 (0.30)	-0.3057 (0.00)	- (-)

Sample: 1994:03 – 2010:09 Publication: 2010:11	Switching Exogenous Lag				Non Switch Exogenous		Degree of Freedom
Purely Autoregressive	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	162
Foreign Orders	0.0152 (0.00)	0.0446 (0.00)	0.7366 (0.00)	-0.0778 (0.00)	0.0320 (0.00)	- (-)	154
Domestic Orders	0.1592 (0.00)	0.0520 (0.00)	0.0508 (0.04)	0.0531 (0.00)	0.0360 (0.01)	- (-)	158
Construction Permissions [5]	-0.0025 (0.45)	0.1594 (0.00)	-0.0028 (0.10)	-0.2093 (0.00)	-0.0023 (0.45)	-0.0038 (0.08)	148
CDAX	-0.0071 (0.00)	-0.0047 (0.00)	0.1356 (0.00)	0.2885 (0.00)	-0.0045 (0.00)	- (-)	154
Corporate Spread	0.0414 (0.00)	0.0176 (0.04)	-0.1345 (0.00)	0.1974 (0.00)	- (-)	- (-)	155
Euribor - 3M	0.6540 (0.00)	0.0429 (0.00)	-0.1018 (0.00)	1.3878 (0.00)	- (-)	- (-)	159
ifo Business Climate	1.5692 (0.00)	0.2059 (0.00)	0.0883 (0.00)	0.1032 (0.00)	- (-)	- (-)	155
Credit Growth [6]	0.0253 (0.02)	0.0085 (0.07)	0.0116 (0.28)	0.0587 (0.34)	0.0011 (0.70)	-0.0097 (0.00)	93
Maturity Spread	-0.0432 (0.16)	0.0178 (0.17)	0.0061 (0.24)	-0.2036 (0.00)	- (-)	- (-)	159
Job Vacancies	0.0682 (0.00)	0.0201 (0.19)	0.0399 (0.70)	0.8619 (0.00)	0.0481 (0.03)	0.0188 (0.12)	153

Table 1: Summary of MS regressions with 4 regimes for publication on November 2010. Each equation is represented by its embedded leading indicator. The upper table deals with the endogenous parts of the equations and the lower with the exogenous parts. The order between the columns containing switching coefficients is linked to the magnitude of the corresponding intercept.

This means that the coefficients of the regime with the largest intercept stand on the left and the coefficients of the regime with the lowest intercept on the right and explains the structure of the columns *Switching Intercept*, *Switching Endogenous Lag* and *Switching Exogenous Lag*. Since only the most recent lag of each kind of regressors is chosen to switch, remaining lags due to the lag choice of section 3.2 are listed in the columns *Non Switch Endogenous* and *Non Switch Exogenous*. The latter one has the youngest lag standing on the left hand side and the oldest one on the right hand side. Model specification, as described in section 3.2, leads to a slightly different lag structure for each of the equations. Thus the degrees of freedom differ, namely between 153 and 162 - with the exception of construction permissions and credit growth, where only shorter observation periods [5] [6] are available. P-values for each of the coefficients are reported in brackets.

Against this spirit Biggs et al. (2009) call it 'a stylized fact that after financial crises economic activity recovers without a rebound in credit', but they do not exclude this connection for the flow of credit. In fact when considering credit growth rates as reported in the balance sheets of monetary financial institutions this leaves another impression, figure 3. The middles of the last recessions as detected by the benchmark model seem to coincide with the peaks of credit growth. This coherency becomes especially evident when including credit flows to other monetary financial institutions. Thus this might be seen as a reflection of an unhealthy credit growth or a credit bubble. Regarding the non-stationarity of the reference series, see Levanon (2010), the MS model is based on growth rates, which means to include the growth of credit growth as a regressor. Running such a specification one only receives overall significance when including interbank deals, table 1. Yet, the relation must not be overvalued since the series of credit growth is only available from the start of the ESCB, i.e. the end of 1998. Additionally most credit derivatives, which could serve as a possible explanation, were declared as off-balance transactions before the financial crisis.

When turning to the state probabilities, figure 3 illustrates that they change with different MS specifications, as it was theoretically expressed in section 2. Concerning this matter it is reasonable to assume that the predictive power of a single indicator varies in different recessions. Whereas the benchmark method detects six recessions (respectively periods of stagnation) not each MS specification ends up with the same number. But since most of the MS specifications interpret the same periods as recessions, there are six of them, when

[5] Sample 1994:07 - 2010:09 for table (1), (2), (3) and sample 1994:07 - 2008:06 for table (4).
[6] Sample 1999:03 - 2010:09 for table (1), (2), (3) and sample 1999:03 - 2008:06 for table (4).

averaging the regime probabilities of the different MS specifications.

Each specification identifies parts of the recession linked to financial crisis as an additional regime. Thus it has to be expressed how different regimes, in particular in the case of more than two, can be interpreted as recessions or expansions. First it is natural to relate regimes to recessions or expansions, which determine the business cycle, since with the MS model these regimes generate the reference series. Technically this is possible regarding the switching intercept. Each iteration starts with uniformly distributed initial values, so it is not necessary that the same label, for example regime 1, represents a recession for all estimations. So it makes sense that the regime leading to the most negative growth rate is taken as the one related to the financial crisis. This will be dealt with in detail later on when analyzing forecasting results.

An interesting question arising with different MS specifications is if a leading indicator can bequeath its predictive power to the corresponding specification. Hints of such a relation can be found. For example the assumed continuous recession between February 2001 and September 2003 in the specifications of CDAX and ifo business climate reflects the course of the corresponding indicators. Another example would be the leading start of the last recession reflecting the predictive power of foreign orders. Nevertheless there are also counterexamples, e.g. the development of corporate spread and job vacancies clearly fits the downturn from August 2002 to September 2003, whereas the corresponding specifications do not report a recession. But even if such an intuitive relation would not be given, so that benefits from averaging in terms of Timmermann (2006) just appear due to white noise, it makes sense to use leading indicators for generating the differences between the single forecasts.

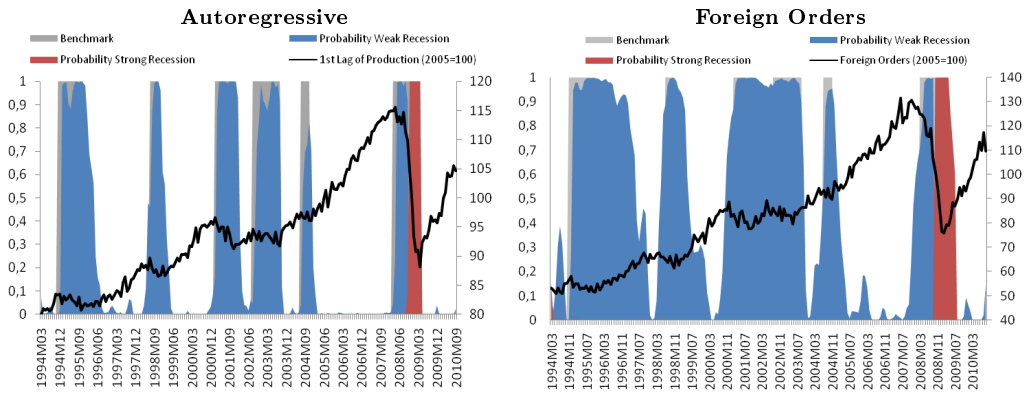
Later on OECD Composite Index of Leading Indicators (CLI) will be used as the explained variable in the MS regression. One could also think of using CLI as an explanatory variable. But as it can be seen in last sub-figure of figure 3 we do not find persistent regimes in such a regression. Thus this approach is left out, whenever in the following the industrial production is taken as the dependent variable.

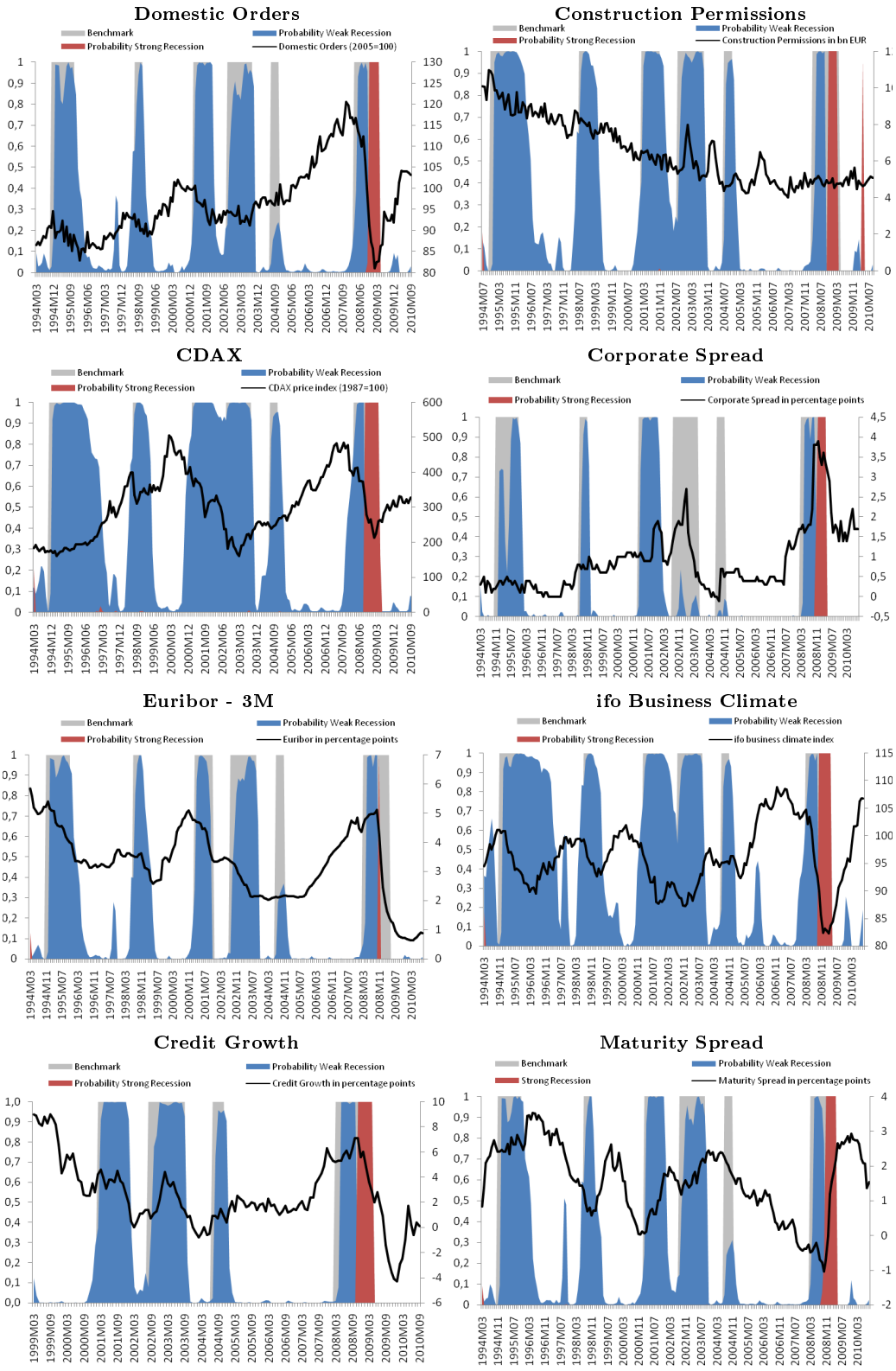
Table 2 summarizes the number of correct recessions and expansions for each specification. As pointed out in section 3.2 the purely autoregressive estimation can mostly be compared with the Hamilton model. Both specifications with a higher number of correct recessions (e.g. CDAX) and with a higher number of correct expansions (e.g. corporate spread) as

well as a specification with a higher number of total correctness (domestic orders) can be found. Naturally there is a trade-off between correct recession and expansion regimes. Again, it is appealing to look for averaging effects when turning to out-of-sample forecasts. In addition to the regime probabilities table 2 also contains standard measures which are linked to the fitted growth rates of the reference series. These are the adjusted R^2 and the root mean squared error. If regime probabilities are already filtered and the state dependent coefficients are estimated, fitted values can be computed from the different state equations. Therefore each state equation is weighted by the corresponding regime probability.

Except the specification with ifo business climate index each of the MS regressions reaches values above 0.7 for the adjusted R^2. In this way they outperform a simple AR(1) benchmark. In fact the low value in the regression with ifo business climate might be based on a local maximum found by the algorithm presented in section 2.

Thus, for the certain publication on November 2010, this could reveal one of the eleven regressions to be subject to a misspecification bias. An instrument to handle this problem is averaging the single forecasts since Timmermann (2006) finds that particularly simple combinations of forecasts often reach better results than the ex-ante best one. A similar impression arises after computing the root mean squared error. For the way of computation consider equation (22) and replace the probability forecast by the fit of the growth rates. Again it is the regression with ifo business climate reaching a far higher value than the others. Finally in table 2 there are also measures, which are linked to the log likelihood function of the single estimations. Here none of the results is conspicuously out of range. More details for the interpretation of the results are provided in the next sections.




Domestic Orders
Benchmark
Probability Weak Recession
Probability Strong Recession
Domestic Orders (2005=100)
Construction Permissions
Benchmark
Probability Weak Recession
Probability Strong Recession
Construction Permissions in bn EUR
CDAX
Benchmark
Probability Weak Recession
Probability Strong Recession
CDAX price index (1987=100)
Corporate Spread
Benchmark
Probability Weak Recession
Probability Strong Recession
Corporate Spread in percentage points
Euribor - 3M
Benchmark
Probability Weak Recession
Probability Strong Recession
Euribor in percentage points
ifo Business Climate
Benchmark
Probability Weak Recession
Probability Strong Recession
ifo business climate index
Credit Growth
Benchmark
Probability Weak Recession
Probability Strong Recession
Credit Growth in percentage points
Maturity Spread
Benchmark
Probability Weak Recession
Strong Recession
Maturity Spread in percentage points

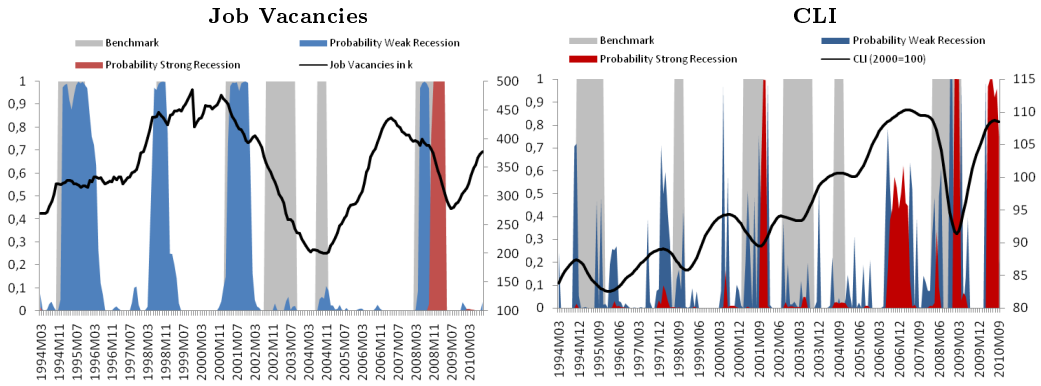


Figure 3: Recession probabilities of MS specifications 1 - 12 (left axis) and the embedded indicators (right axis). Whereas for the illustration the later ones are given in levels, for the regression the growth rates of the reference series are regressed on the growth rates (differences) of the leading indicators. All data are calendar and seasonally adjusted. Weak and strong expansions are colored white, but in most cases strong expansion fits the recovery after the financial crisis. Not each MS specification recognizes six recessions (respectively periods of stagnation) in the observation period, whereas the benchmark method does. But since most of the MS specifications do, the number is the same, if the state probabilities are averaged. A regression with the CLI as an explanatory variable does not deliver persistent regimes (last graph), so this approach is left out in the following.

Sample: 1994:03 – 2010:09	recession		expansion		total		$\overline{R}^2$	RMSE	SIC	LR_T
	#	%	#	%	#	%				
Purely Autoregressive	52	85.25	127	92.03	179	89.95	0.8234	0.0033	-1467.39	260.54
Foreign Orders	59	96.72	89	44.72	148	74.37	0.7664	0.0037	-1440.76	228.32
Domestic Orders	50	81.97	133	96.38	183	91.96	0.7435	0.0039	-1437.91	187.09
Construction Permissions [5]	52	85.25	115	85.82	167	85.64	0.8131	0.0033	-1369.53	228.73
CDAX	58	95.08	98	71.01	156	78.39	0.7825	0.0035	-1440.37	250.77
Corporate Spread	33	54.10	131	94.93	164	82.41	0.7803	0.0036	-1438.46	265.84
Euribor - 3M	42	68.85	127	92.03	169	84.92	0.7528	0.0038	-1446.56	192.57
ifo Business Climate	58	95.08	91	65.94	149	74.87	0.0980	0.0072	-1447.86	247.49
Credit Growth [6]	37	86.05	90	93.75	127	91.37	0.8023	0.0037	- 957.64	219.76
Maturity Spread	49	80.33	124	89.86	173	86.93	0.8409	0.0031	-1432.48	231.48
Job Vacancies	37	60.66	125	90.58	162	81.41	0.7944	0.0034	-1452.31	268.24
AR(1)	-	-	-	-	-	-	0.5916	0.0055	-	-

Table 2: Goodness of fit for different MS specifications. The first six columns are linked to the MS regime probabilities and compare their allocation to recessions and expansions with the result of the benchmark method. The adjusted R^2 as well as the root mean squared error (RMSE) are calculated regarding the fitted growth rates. With respect to these measures also a comparison with values from an AR(1) estimation was included. Information criteria as well as the likelihood ratio test statistic are linked to the log likelihood function as described in section 2 (10). In order to compute the latter one, for each of the MS specifications a linear version without switching coefficients was estimated and the corresponding log likelihood was taken into account, see (16).

3.5 Likelihood Ratio Test in a 4 Regime MS Model

The last column of table 2 contains the test statistic for the likelihood ratio test between the unconstrained MS model and the linear constrained versions of each of its equations, where no parameter switch, i.e.

$$LR_T = -2\left(Q_T^{\text{unconstrained}}\left(\hat{\theta},(\hat{p}_{ij})\right) - Q_T^{\text{constrained}}\left(\tilde{\theta}\right)\right). \tag{16}$$

Here Q_T represents the log likelihood, where T in this special case stands for the last observation of the publication on November 2010. A careful reading of the notation makes it obvious that there are parameters, namely the entries of the transition matrix, which are arbitrary under the null hypothesis of a true linear model. This is what Hansen (1996) calls 'inference when a nuisance parameter is not identified under the null'. Choosing the following linear and unique decomposition of the switching parameters in equation (1),

$$\begin{aligned} y_t = &\left(\alpha_0^0 + \left(\alpha_0^1 + \alpha_0^2 + \alpha_0^3\right) S_t\right) + \left(\alpha_{1,y}^0 + \left(\alpha_{1,y}^1 + \alpha_{1,y}^2 + \alpha_{1,y}^3\right) S_t\right) y_{t-f-D_y} \\ &+ \sum_{j=2}^{p} \beta_{j,y} y_{t-j+1-f-D_y} + \left(\alpha_{1,x}^0 + \left(\alpha_{1,x}^1 + \alpha_{1,x}^2 + \alpha_{1,x}^3\right) S_t\right) x_{t-f-D_x} \\ &+ \sum_{j=2}^{q} \beta_{j,x} x_{t-j+1-f-D_x} + \left(\sigma^0 + \left(\sigma^1 + \sigma^2 + \sigma^3\right) S_t\right) \epsilon_t, \quad \epsilon_t \sim N(0,1), \quad t = 1,\ldots,T, \end{aligned}$$

the null hypothesis can be written as

$$H_0 : \left(\alpha_0^1, \alpha_0^2, \alpha_0^3, \alpha_{1,y}^1, \alpha_{1,y}^2, \alpha_{1,y}^3, \alpha_{1,x}^1, \alpha_{1,x}^2, \alpha_{1,x}^3, \sigma^1, \sigma^2, \sigma^3\right)' = 0. \tag{17}$$

to obtain a nested design. Because of the nuisance parameters, which are only available under H_1, the (asymptotic) LR-distribution will not conform to the standard χ^2_{d-c}-distribution, where the degree of freedom is equal to the difference in the number of estimated parameters. Although Garcia (1998) as well as Carrasco, Hu and Ploberger (2005) develop asymptotic null distributions for such a test problem, their results do not fit our requirements since both only consider the special case of 2 regimes [7] and a lower number of switching parameters. [8] It remains for future research to extend their test procedures explicitly to the case of 4 regimes and to a higher parameterized design. Nevertheless the test

[7] This is the comfortable case, where $E(S_t = i) = \frac{1-\mathcal{P}(S_t=j|S_{t-1}=j)}{2-\mathcal{P}(S_t=i|S_{t-1}=i)-\mathcal{P}(S_t=j|S_{t-1}=j)}$, $i,j = 1,2$, which is used as an entry in all covariance matrices for the asymptotic χ^2-process developed by Garcia (1998).

[8] Another problem with the approach of Carrasco et al. (2005) is that the test result is obtained without estimating the model under the alternative. In doing so, it will be hard to capture the differences between the single MS specifications, which are at heart of this paper.

statistics in table 2 provide some (heuristic) insights without simulating the appropriate distribution of LR_T.

Firstly, when creating a ranking of the leading indicators in the next section test statistics can be compared without knowing the exact test result since the asymptotic null distribution for each of the MS specifications will mainly differ because of including a switching coefficient with the most recent autoregressive lag or not. [9] Secondly, analyzing critical values, as they are provided by Garcia (1998), shows that they are higher as in the standard case and that they increase with a higher parameterized design, but that they are still below 40 on an one percent level of significance - even in the case of a model with at least a switching intercept and a switching variance as well. Hence, when considering the distance to the test statistics of table 2, a rejection of the null seems to be likely for all the MS specifications. [10]

3.6 Ranking Leading Indicators

Our MS framework is constructed in a way to include all the information given by the leading indicators listed in section 3.1. This also fits the idea that their predictive power (and implicitly the impact of a potential misspecification bias) may change over time. A simple but effective way to deal with such a time-variant behavior is by using equal weights when averaging the single forecasts, Timmermann (2006).

Nevertheless there might be the need to create a ranking of the indicators, e.g. to answer the question, if, in general, indicators do better when they are not subject to a data availability lag. Again we restrict ourselves to the publication on November 2010 and as criteria the measures introduced in the previous sections are used. Since they belong to the corresponding MS regressions, this means to take the indicators as representatives of the equations, where they are embedded.

[9] Compared to the impact of the three other switching coefficients in all equations, i.e. the switching intercept, the switching exogenous lag and the switching variance of the error term, this is assumed to be subordinated.

[10] This statement is based on the fact that in the case of 4 regimes a supremum-type test (Hansen (1996) theorem 3, Garcia (1998) theorem 1), i.e. $LR_T = \sup_{p_{ij} \in \Gamma} LR_T(p_{ij})$, will still be feasible and converge to some χ^2-process, but that it is necessary to modify the corresponding covariance matrix to some degree, which goes beyond the scope of this paper. Extending the dimension of the vector in (17) there are at least 18 derivatives to compute corresponding to a 18×18 covariance matrix in most of the specifications. This would allow for simulating the whole range of possible transition probabilities.

Sample: 1994:03 – 2010:09	1. Position	2. Position	3. Position
Total Correctness	Domestic Orders	Maturity Spread	Euribor
Adjusted R^2	Maturity Spread	Construction Permissions [5]	Credit Growth [6]
Root Mean Squared Error	Maturity Spread	Construction Permissions [5]	Job Vacancies
LR Test Statistic	Job Vacancies	Corporate Spread	CDAX

Table 3: A ranking of leading indicators due to the measures of table 2. Taking the indicators as representatives of the corresponding regressions, the first three indicators are listed that reach the best values according to each of the measures. Results from the purely autoregressive estimation are neglected here.

Table 3 lists the first three indicators that reach the best values according to the measures of table 2. The consistency between the adjusted R^2 and $RMSE$ can be explained by both depending on the sum of squared residuals. Thus one of them could be removed if the R^2 would not additionally consider the number of estimated parameters. Regarding the overview in table 3 the best performer in the set of indicators is the maturity spread - occurring three times and available without a publication lag. But although there are more such indicators mentioned than those, which are subject to a data availability lag, 8 versus 4, the result is not clear enough to conclude that in general they will perform better in a business cycle analysis.

4 Out-of-sample Evaluation

4.1 Model Adaption in Real-time

Forecasts with MS models are produced similar to equation (8), i.e.

$$\hat{\xi}_{T+f|T} = \hat{P}^f \hat{\xi}_{T|T}, \tag{18}$$

where the exponent represents the real [11] forecast horizon. Thus the future state probability to be in a certain regime comes from the regime probabilities of the last observation. These are weighted by the probability to change to the certain regime. Note that this model follows a standard approach insofar that transition probabilities do not depend on the amplitude or the duration of the last regime, as e.g. introduced by Durland and McCurdy (1994). It is obvious that the idea to include a third and fourth regime arose under

[11] Given a data availability lag of 2 months the real forecast horizon for 1 month ahead is 3 months.

the economic downturn of the financial crisis and the recovery hereafter. But real-time estimation cannot be based on information given later on. That is why a criterion has to be developed, which explains when to introduce new regimes in real-time.

According to figure 3 the probability of the additional regimes has only been allocated since the financial crisis. But these are in-sample results for publication on November 2010. With real-time forecasts it is clear enough that whenever new regimes are introduced some probability will be allocated to them. The essential question is how much probability and the answer to this question lies at the heart of the criteria. With an increasing number of regimes extremer events can be reproduced. This enables someone to distinguish between a strong and a weak intensity of the same kind of business cycle phase. Considering that a first month may represent an outlier it makes sense to change the number of regimes whenever the probability of the strong regime exceeds the one of the weak regime for two consecutive months, i.e.

$$\begin{aligned} &\mathcal{P}\left(S_{T+t+1} = \text{strong}|y_{T+t}\right) > \mathcal{P}\left(S_{T+t+1} = \text{weak}|y_{T+t}\right) \\ \wedge \quad &\mathcal{P}\left(S_{T+t+2} = \text{strong}|y_{T+t+1}\right) > \mathcal{P}\left(S_{T+t+2} = \text{weak}|y_{T+t+1}\right), \quad t = 0, 1, \ldots, \end{aligned} \tag{19}$$

where $T+1$ stands for the beginning of the out-of-sample forecasts. From an operational point of view in order to apply this criterion it is necessary to run both in parallel - the setting with less regimes and the one with more regimes.

Furthermore the question may arise whether to increase the number of regimes by one or two. In this paper a symmetric approach is taken, which means that the number of all regimes can only change from two to four. The reason is quite simple. When introducing a third regime in real-time it will not be clear without laborious computation whether to allocate its probability to a recession or an expansion.

When embedding two more regimes it makes sense that two of the four regimes will lead to higher growth rates in absolute values. Thus approximately the regime with the most positive intercept will be allocated to strong expansion, the one with the most negative intercept to strong recession. The remaining then form weak expansion and weak recession [12]. Naturally this approximation is only feasible in the case that the switching intercept is identified as the most relevant part for the iteration of the state probabilities, see section

[12] In doing so a possible misallocation cannot be excluded categorically since with a single estimation there might occur one expansion and three recession regimes or vice versa, but a symmetric approach fits the main empirical finding of section 3.4.

3.2. Moreover Welch test results given in table 4 confirm this approach except for one case (construction permissions). Changing the number of regimes due to criterion (19) the hypothesis

$$\begin{aligned} H_0: \quad & \text{Regimes chosen to stand for different intensities of a recession or an expansion} \\ & \text{can have the same (normal) distribution.} \\ \Leftrightarrow \quad & \mu^{\text{strong}}_{\text{expansion}} | \sigma^{\text{strong}}_{\text{expansion}} = \mu^{\text{weak}}_{\text{expansion}} | \sigma^{\text{weak}}_{\text{expansion}} \\ \wedge \quad & \mu^{\text{strong}}_{\text{recession}} | \sigma^{\text{strong}}_{\text{recession}} = \mu^{\text{weak}}_{\text{recession}} | \sigma^{\text{weak}}_{\text{recession}} \end{aligned} \tag{20}$$

can be rejected for the publication on August 2008 [13] [14]. This confirms a change to four regimes, although at this point in time Germany's recovery had not started yet.

After the decision has been made that four regimes are appropriate, another interesting question can be analyzed. Obviously the strong downturn of the last recession is linked to turbulences that occurred on financial markets. Thus one could suggest that in this situation financial variables would have revealed a higher predictive power. Transferring this statement to the model one should expect the (relative) change between coefficients of weak and strong recession regimes to be higher with financial than with real economy variables. Table 4 contains the corresponding results. Although the largest change occurs with a financial variable (3-month EURIBOR interest rate), the statement above cannot be confirmed in general. Certainly financial variables are important for business cycle predictions. But the result in table 4 points to the fact that whenever one runs different specifications the impact of financial variables should not be overestimated relative to real economy variables.

[13] The hypothesis means to check under the given difference of empirical variances, if the means can be equal. Because the normal distribution is involved a rejection of equal expectations is taken as a rejection of the same distribution.

[14] Taking S_t as a filtration calculus of the conditional expectation shows how to approximate parameter μ by the intercept and the exogenous parts of the equation:

$$\begin{aligned} E(y_t|S_t) &= E(E(y_t|S_t)|S_{t-1}) \\ &= E\left(\beta_0^{S_t} + \beta_{1,y}^{S_t} E(y_{t-1}|S_t) + \beta_{1,x}^{S_t} x_{t-1}|S_{t-1}\right) \\ &= \beta_0^{S_t} + \beta_{1,x}^{S_t} x_{t-1} + \beta_{1,y}^{S_t} E(y_{t-1}|S_{t-1}) \\ &= \beta_0^{S_t} + \beta_{1,x}^{S_t} x_{t-1} + \beta_{1,y}^{S_t} \left(\beta_0^{S_{t-1}} + \beta_{1,x}^{S_{t-1}} x_{t-1} + \beta_{1,y}^{S_{t-1}} E(y_{t-2}|S_{t-2})\right). \end{aligned}$$

Since for each β-term one has $\beta << 1$ the last term consisting of products of βs may be neglected. A similar assessment identifies the error's σ^2 as the essential part of the conditional variance.

Sample: 1994:03 – 2008:06 Publication: 2008:08	Switching Intercept	Switching Exogenous	Error's σ	Change of the Intercept	Change of the Exogenous	Welch Test Result
Autoregressive	0.0060	-	0.0026	45.80%	-	T=8.44
	0.0032	-	0.0010			rejected
	-0.0001	-	0.0012	97.50%	-	T=24.73
	-0.0054	-	0.0019			rejected
Foreign Orders	0.0065	0.0041	0.0023	64.89%	-701.24%	T=18.57
	0.0023	0.0327	0.0018			rejected
	-0.0009	0.0801	0.0002	70.17%	-230.28%	T=1.97
	-0.0029	0.0242	0.0028			rejected
Domestic Orders	0.0099	0.0423	0.0013	69.07%	73.26%	T=21.76
	0.0031	0.0113	0.0027			rejected
	0.0001	0.1650	0.0015	101.58%	239.02%	T=21.13
	-0.0041	-0.1187	0.0024			rejected
Construction Permissions [5]	0.0113	-0.0889	0.0000	54.33%	94.34%	T=17.92
	0.0052	-0.0050	0.0025			rejected
	-0.0015	-0.0317	0.0011	45.07%	259.60%	T=2.20
	-0.0027	0.0198	0.0033			not rejected
CDAX	0.0064	-0.0118	0.0025	38.44%	16.06%	T=9.63
	0.0039	-0.0099	0.0010			rejected
	0.0002	-0.0042	0.0013	103.26%	30.05%	T=19.19
	-0.0052	-0.0061	0.0021			rejected
Corporate Spread	0.0050	0.0263	0.0028	71.20%	-61.62%	T=12.73
	0.0014	0.0425	0.0017			rejected
	-0.0019	0.0487	0.0024	67.75%	187.37%	T=14.72
	-0.0059	-0.0558	0.0012			rejected
Euribor - 3M	0.0065	0.2096	0.0034	47.22%	93.89%	T=7.51
	0.0034	0.0128	0.0027			rejected
	0.0026	0.4194	0.0011	199.79%	619.07%	T=14.49
	-0.0026	-0.0808	0.0034			rejected
ifo Business Climate	0.0057	0.0805	0.0025	26.09%	227.80%	T=15.68
	0.0042	-0.1028	0.0007			rejected
	0.0006	0.1260	0.0015	112.68%	-233.53%	T=8.07
	-0.0044	0.0378	0.0025			rejected
Credit Growth [6]	0.0075	-0.0485	0.0006	28.99%	122.55%	T=4.37
	0.0053	0.0109	0.0021			rejected
	0.0002	0.0029	0.0021	102.95%	84.84%	T=29.81
	-0.0073	0.0193	0.0010			rejected
Maturity Spread	0.0041	0.0062	0.0032	101.85%	3107.29%	T=3.81
	-0.0001	-0.1861	0.0002			rejected
	-0.0018	0.0017	0.0035	86.00%	98.97%	T=41.57
	-0.0128	0.1674	0.0001			rejected
Job Vacancies	0.0052	0.0232	0.0031	34.37%	-39.98%	T=4.56
	0.0034	0.0325	0.0012			rejected
	-0.0006	0.0765	0.0005	84.27%	9.70%	T=12.24
	-0.0036	0.0847	0.0022			rejected

Table 4: Change of coefficients and Welch test results for the publication on August 2008. At this point in time the model changes from 2 to 4 regimes so that the relative change between the coefficient allocated to the weak and the one allocated to the strong intensity of a recession or an expansion can be measured for each of the MS specification. Then the order within each row is as follows: Coefficients of the regime with the largest intercept stand on the top and the ones with the lowest intercept on the bottom. Welch test results are also related to the comparison of weak and strong recession or expansion, namely to the difference in the distribution.

4.2 Real-time Forecasts with the Industrial Production

This section deals with one month ahead real-time forecasts generated for industrial production as the monthly proxy of overall economic activity. The methodology was described in the previous section. Table 5 contains measures of forecast accuracy for the different MS specifications and for their average. MAE stands for the mean absolute value, i.e.

$$MAE = \frac{1}{h} \sum_{t=T+1}^{T+h} |\mathcal{P}_{t|t-1} - b_t|, \tag{21}$$

RMSE for the root mean squared error, i.e.

$$RMSE = \sqrt{\frac{1}{h} \sum_{t=T+1}^{T+h} \left(\mathcal{P}_{t|t-1} - b_t\right)^2} \tag{22}$$

and Theil for the Theil coefficient, i.e.

$$Theil = \frac{\sqrt{\sum_{t=T+1}^{T+h} (\mathcal{P}_{t|t-1} - b_t)^2 / h}}{\sqrt{\sum_{t=T+1}^{T+h} (\mathcal{P}_{t|t-1})^2 / h} + \sqrt{\sum_{t=T+1}^{T+h} b_t^2 / h}}, \tag{23}$$

where the last one is normalized to the unit interval with 0 representing a perfect fit. b_t is the binary variable reporting the state of the business cycle with the benchmark algorithm, while $\mathcal{P}_{t|t-1}$ aggregates the MS state probabilities belonging to a recession.

Table 5 shows that the average outperforms each of the single forecasts. In this context Timmermann (2006) mentions that 'simple combinations that ignore correlations between forecast errors often dominate more refined' ones. Yet, the average, as listed in table 5, does not only achieve the best values because of the different specifications, but also because of the fact that the model changes its number of embedded regimes due to criterion (19). This change takes place with the forecast for September 2008 [15]. One reason for the Theil coefficient not reaching lower values than 40% can be seen in figure 4 [16].

[15] The title of the Gemeinschaftsdiagnose, the professional opinion of important German economic research institutes, in the autumn 2008, is 'Germany on the edge of a recession'. In contrast to this report the real-time introduction of new regimes for September 2008 can be interpreted as if Germany was no longer on the edge, but in the middle of a recession becoming deeper as recessions before.

[16] Another reason is that compared to the binary benchmark for the averaged MS probability it is likely to cover only a certain range between 0 and 1. Thus the measures of forecast accuracy could be improved just by generating a binary variable out of the MS probability according to the 0.5 threshold.

Sample: 2007:02 – 2010:12	MAE	RMSE	Theil
Average	0.2948	0.3898	0.4060
Autoregressive	0.3070	0.4439	0.4309
Foreign Orders	0.3651	0.5007	0.4340
Domestic Orders	0.3141	0.4385	0.4203
Construction Permissions	0.3050	0.4394	0.4112
CDAX	0.3234	0.4569	0.4421
Corporate Spread	0.3609	0.4958	0.4921
Euribor - 3M	0.3132	0.4485	0.4537
ifo Business Climate	0.3564	0.4991	0.4860
Credit Growth	0.2559	0.4395	0.4128
Maturity Spread	0.2987	0.4585	0.4279
Job Vacancies	0.3774	0.5038	0.4564

Table 5: Measures of forecast accuracy for different MS specifications. Averaging the forecasts and including a change of the embedded regimes due to criterion (19) leads to the best values. But with the model recognizing the recession after its actual beginning the Theil coefficient cannot reach values below 40%.

Compared to the recession start in March 2008, as it is reported by the benchmark model in October 2008 (7 months later), the MS recession probability exceeds the 0.5 threshold in August 2008. On the one hand this represents a delay of 5 months. On the other hand the recession is recognized earlier by the MS model than by the benchmark method. Considering that the forecast for August is made in July (one month forecasting horizon), the time in advance between the MS and the benchmark model covers 3 months.

Additionally, the recession probability forecast for July 2008 is above 30%. Such an indication of a recession cannot be provided by the nonparametric benchmark method since this focuses on a binary decision (1=recession, 0=expansion). Both models MS and benchmark continuously announce the last recession between August 2008 and April 2009. Compared to the end of the recession in April 2009, as it is reported by the benchmark method in November 2009, the MS forecast for July 2009 is the last above the 0.5 threshold (delay of 3 months). Again considering the point in time, when the information about the end of the recession is provided, this is 4 months earlier with the MS model as with the benchmark method.[17] Thus the forerun of the MS is even longer in the case of the recession end than in the case of the recession beginning.

[17] The end of the recession has to be seen technically. For example this does not mean that at this point in time the former output level was already reached again.

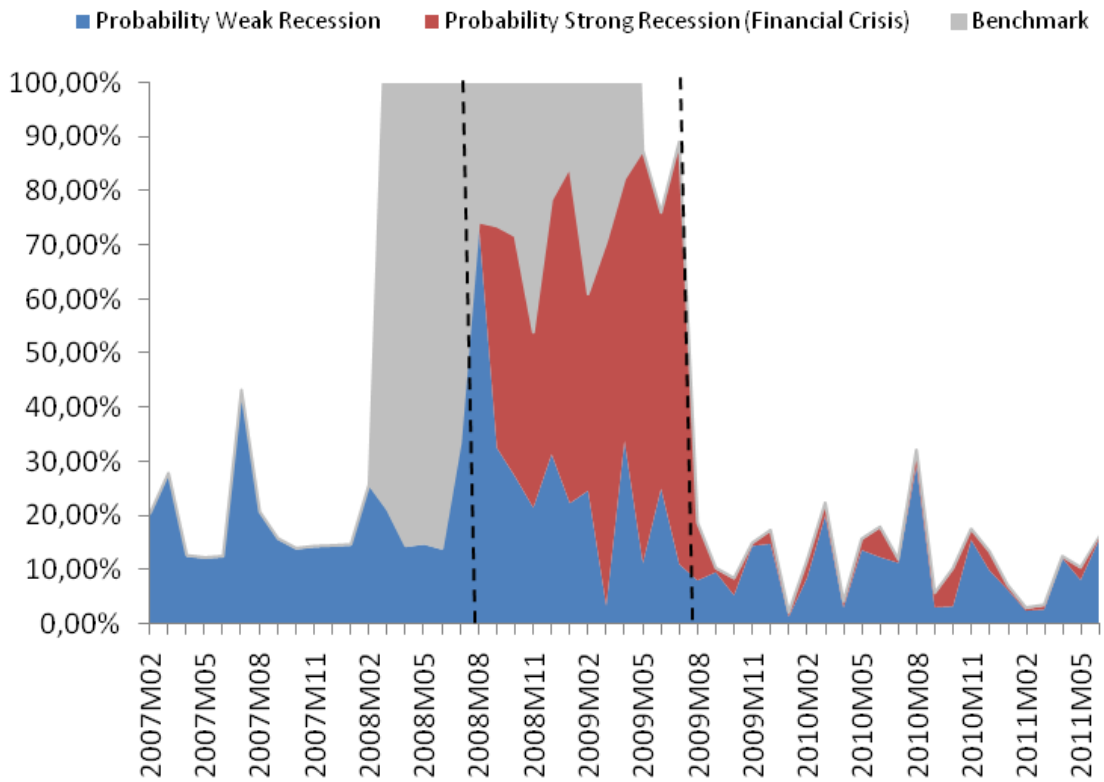


Figure 4: The regime probabilities are averaged one month ahead real-time forecasts over the different MS specifications. Probabilities of weak and strong recession can be added to a total recession probability. For September 2008 the model changes from 2 to 4 regimes according to the criteria described in section 4.1, where the new regime clearly points to the magnitude of the economic decline. When defining a recession on a 0.5 threshold, the downturn is predicted continuously between August 2008 and July 2009 (dashed lines). Actually this represents a delay to the recession start, as it is reported later on by the benchmark method. But considering the point in time, when the recession is recognized, this is 3 months earlier with the MS model as with the benchmark method.

Together with the fact that the aggregated recession probability does not reach values above 90% this reveals a certain restraint towards an erroneous recession declaration, which may be functional with respect to the forecast accuracy. Indeed there is no extra period in the out-of-sample evaluation, where the economic situation is misinterpreted as a recession.

4.3 Real-time Forecasts with the CLI

Although the MS model outperforms the ex-post benchmark model, the question arises, if there was any alternative to recognize the recession in advance (with the MS model). As Lahiri and Wang (1994) showed it is also possible to apply the MS model to the Composite Index of Leading Economic Indicators (CLI). Such monthly data for Germany is provided by OECD (2011). The idea behind the CLI is to generate a synthetic series that represents a lead to the business cycle and anticipates its turning points. To achieve this, leading indicators - similar as they are used as regressors here - are aggregated. That is why the CLI is also subject to revisions. Before aggregation the data is seasonally adjusted, outliers are eliminated, trends are removed and filters for smoothing and normalization are applied in order to obtain homogenized cyclical amplitudes for each of the component series. It is not the topic of this paper to discuss the OECD methods in detail, but it turns out that the procedure above leads to the CLI often behaving relatively undecided between up- and downturn on the current edge. Nevertheless with the Hamilton filter generating the state probabilities endogenously out of the observations and with the result of lagged recession recognition in the case of the industrial production, it is quite appealing to run a specification, where the reference series is substituted by the CLI.

In doing so, some differences to the previous MS regressions have to be considered. Firstly, smoothing backwards by a moving average is no longer necessary since the series is already smoothed. Secondly, the lag choice, described in section 3.2, only makes sense for a purely autoregressive estimation since there must be a bias with leading indicators standing on both sides of the equation in a different manner (aggregated versus disaggregated). In fact the outcome of the lag choice is to use no autoregression so that the right hand side of the equation only consists of a switching intercept and error term. As a consequence of this parsimonious design it is not sufficient to choose the regimes with the lowest intercepts to stand for recessions, but to request that these intercepts have to be negative. In figure 5 the MS regression with the CLI delivers a correct early signal for the recession linked to the financial crisis, but forecasts are volatile. Among the real-time out-of-sample predictions between February 2006 and June 2011 there are three periods (8 months), in which the economic situation is misinterpreted as a recession. An early signal is given six months in advance, whereas a timely signal would have to come one month ahead from the publication point in time. This reveals a general problem with CLI data: Given the high number of

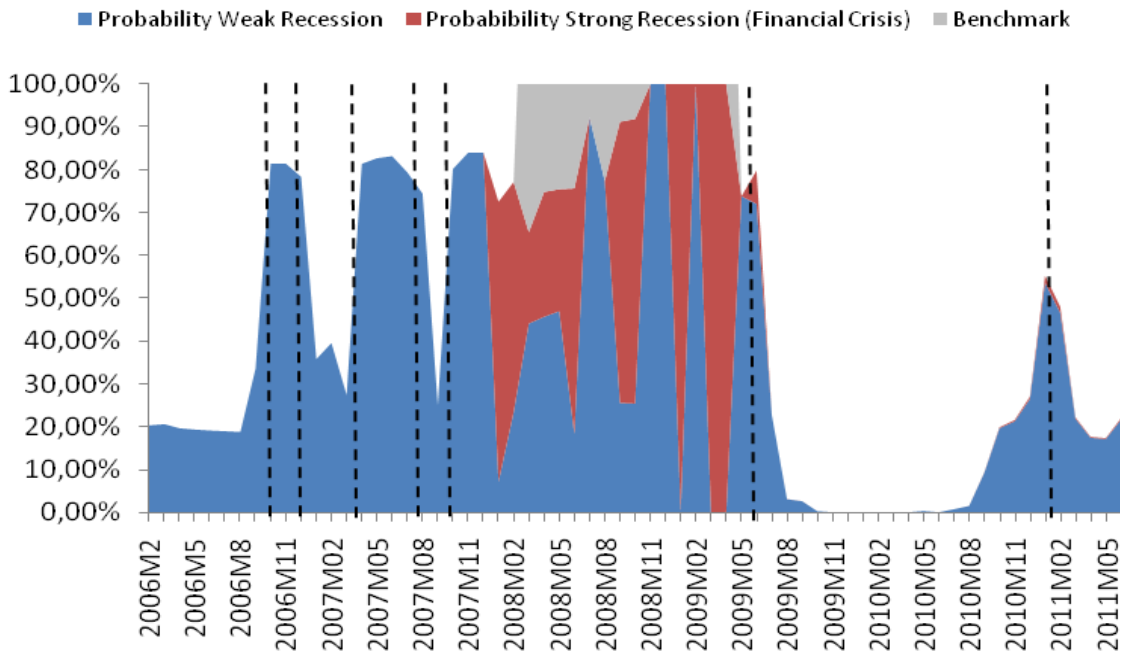


Figure 5: The regime probabilities are one month ahead real-time forecasts with no autoregression. Probabilities of weak and strong recession can be added to a total recession probability. For December 2007 the model changes from 2 to 4 regimes according to the criteria described in section 4.1, where the new regime clearly points to the magnitude of the economic decline. When defining a recession on a 0.5 threshold, there are three periods (October 2006 - December 2006, April 2007 - August 2007 and January 2011), in which the economic situation is misinterpreted as a recession. Nevertheless a correct early signal to the approaching recession is also given, namely in October 2007. Compared to the beginning of the recession in March 2008, as it is reported by the benchmark method, this represents a lead of 6 months and fits the average lead of the CLI, as it is claimed by the OECD. The recession is predicted continuously between October 2007 and June 2009 (dashed lines).

misinterpreted recessions it is not clear whether the forerun is stable and always equal to 6 months as it is claimed by the OECD. Nevertheless - from an operational point of view - both MS regressions on the CLI and on the industrial production can complement very well for the real-time prediction of recessions. While the former might signal the recession in advance, the latter one can confirm it a short time after its beginning.

5 Conclusion

This paper uses a Markov Switching framework applied to German monthly real-time data. While the appropriateness of the method for business cycle applications is well-known since Hamilton's innovation in 1989, based on current literature there are some new insights from this study, for which it is interesting to attest for monthly German real-time data and which are helpful from an operational point of view:

Given limited data records it is appealing to connect Timmermann (2006)'s idea of a single forecast being subject to a misspecification bias with the Markov Switching model generating each of the single forecasts. In order to reduce the bias, forecasting results are averaged. When generating the forecasts the way mentioned above, several macroeconomic and financial leading indicators serve as exogenous variables in univariate MS ARX regressions. This design also opens room for the evaluation of the predictive properties of a single indicator or a group of indicators. In the paper at hand such statements are:

Credit growth, as reported in the ESCB statistics from the balance sheets of monetary financial institutions, turns out to be significant in the MS regression, when including interbank deals. Thus it deserves further consideration as a potential predictor of the business cycle.

In general, we do not find leading indicators which are available immediately to perform better than those which are subject to a data availability lag.

Although intuition tells us that the last recession had its origin on financial markets, regarding its real-time prediction we do not find financial variables reacting more sensitively than real economy variables. This stresses the fact that in a business cycle model the role of financial variables should not be overestimated, e.g. by including a similar number of financial and real economy time series.

Allowing the MS model to change the number of embedded regimes in real-time stabilizes forecasting results. By introducing a criterion for the real-time regime change it is also possible to determine the point in time, from which the recession after the financial crisis structurally exceeded the previous ones. In our analysis this turns out to be for September 2008, where the forecast is made in August - one month before the investment bank Lehman Brothers declared bankruptcy.

When selecting the industrial production as a proxy of overall economic activity, six recessions (respectively periods of stagnation) can be found in the observation period from March 1994 to September 2010. All in all this fits and extends the suggestion for a German business cycle chronology by Schirwitz (2009), when considering the disaggregated results of each of her methods and accepting little time shifts since she used quarterly GDP instead of monthly industrial production.

When forecasting industrial production in real-time from February 2007 to Mai 2011 the MS model outperforms a non-parametric ex-post-dating method based on the work of Bry and Boschan (1971) as well as Harding and Pagan (2002), while revealing similar characteristics: On the one hand recession start and end are recognized too late (ex-post), while the delay for the end of the recession is considerably shorter. On the other hand, at least when considering different specifications and changing the number of regimes, no business cycle phase is misinterpreted as a recession. This fits the fact that Hamilton (2011) compares his MS results with the time of the NBER announcements, which are usually also made several months after the beginning of the recession in order to provide the official dating as accurately as possible.

In order to balance the above mentioned inertia of the MS model in the case of the industrial production, in general, it is appealing to apply it to the OECD Composite Index of Leading Indicators, which tries to be a leading proxy of the business cycle. In doing so the finding for the combination of the MS and a leading index is ambivalent. On the one hand it confirms the result by Lahiri and Wang (1994) that it is possible to obtain a correct early signal for the next recession. But on the other hand, predictions are quite uncertain and several times recessions are mistakenly declared. Nevertheless this approach serves as a reasonable complement to the MS regression on monthly industrial production. Finally several extensions of our MS framework are possible, such as to include different forecast horizons.

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