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Working Paper

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Tinbergen Institute Discussion Paper, No. 14-092/IV/DSF77

Provided in Cooperation with:
Tinbergen Institute, Amsterdam and Rotterdam

Suggested Citation: Lucas, André; Zhang, Xin (2014) : Score Driven exponentially Weighted Moving Average and Value-at-Risk Forecasting, Tinbergen Institute Discussion Paper, No. 14-092/IV/DSF77, Tinbergen Institute, Amsterdam and Rotterdam

This Version is available at:
https://hdl.handle.net/10419/107806

Duisenberg school of finance - Tinbergen Institute Discussion Paper

TI 14-092/IV/DSF77

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Score Driven Exponentially Weighted Moving Averages and Value-at-Risk Forecasting[☆]

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Abstract

We present a simple new methodology to allow for time variation in volatilities using a recursive updating scheme similar to the familiar RiskMetrics™ approach. We update parameters using the score of the forecasting distribution rather than squared lagged observations. This allows the parameter dynamics to adapt automatically to any non-normal data features and robustifies the subsequent volatility estimates. Our new approach nests several extensions to the exponentially weighted moving average (EWMA) scheme as proposed earlier. Our approach also easily handles extensions to dynamic higher-order moments or other choices of the preferred forecasting distribution. We apply our method to Value-at-Risk forecasting with Student's t distributions and a time varying degrees of freedom parameter and show that the new method is competitive to or better than earlier methods for volatility forecasting of individual stock returns and exchange rates.

Keywords: dynamic volatilities, dynamic degrees of freedom, time varying higher order moments, integrated generalized autoregressive score models, Exponential Weighted Moving Average (EWMA), Value-at-Risk (VaR).

JEL: C51, C52, C53, G15

[☆] André Lucas thanks the Dutch National Science Foundation (NWO; grant VICI453-09-005) for financial support. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Executive Board of Sveriges Riksbank.

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1. Introduction

Time variation in second and higher order moments is important for assessing (tail) risk, constructing hedge strategies, and asset pricing. We develop a new empirical methodology to improve the performance of Exponentially Weighted Moving Average (EWMA) methods. Our framework exploits the higher moment properties of the forecasting distribution to drive the dynamics of volatilities. By doing so, the new method is robust to outliers if we use a non-normal forecasting distribution, as is typical for forecasting financial returns. Moreover, the method is easy to implement and remains close in spirit to the highly familiar RiskMetrics^{TM} approach. It also serves as a natural framework to introduce time variation in higher order moments.

As our main contribution, we introduce the score driven EWMA (SD-EWMA) model, which is built on the generalized autoregressive score (GAS) dynamics in Creal et al. [2011, 2013] and Harvey [2013]. We consider an integrated version of the GAS dynamics. The analogy is simple: the RiskMetrics^{TM} approach is a special case of the IGARCH(1,1) model of Engle [1982] and Bollerslev [1986], which are special cases of the IGAS(1,1) model of Creal et al. [2013]. The key feature of this approach is that the volatility dynamics are driven by the score of the forecasting distribution. Empirical evidence of the usefulness of score driven dynamics is provided in for example Creal et al. [2014], Lucas et al. [2014], and Harvey and Luati [2014]. Blasques et al. [2014a] show that score driven updates are optimal from a local information theoretic point of view.

A typical example is the case of forecasting the time varying variance of a fat-tailed distribution. If one uses the standard RiskMetrics^{TM}approach, a large absolute return has a major impact on next period's estimated variance. Given the integrated nature of the RiskMetrics^{TM} dynamics, this impact also affects a large number of subsequent volatility estimates. If one accounts for the fat-tailedness of the return distribution by using a score driven propagation mechanism for the volatilities, the impact of incidental tail observations is substantially mitigated; see the discussions in Creal et al. [2011] and

Harvey [2013]. This mitigation or robustifying mechanism is particularly important in our current context with integrated (infinite memory) dynamics.

Our methodology is computationally simple and very close in spirit to the well-known standard EWMA approach. We also show that the SD-EWMA approach encompasses other proposals from the literature, such as the normal based standard EWMA, the robust EWMA of Guermat and Harris [2002] based on the Laplace distribution, and the skewed EWMA of Gerlach et al. [2013] based on the asymmetric Laplace distribution. Given that we are interested in modeling the time variation in financial risk measures, we explicitly develop an SD-EWMA model based on the fat-tailed Student's t distribution; see for example Poon and Granger [2003] for stylized facts about financial returns. It is clear, however, that the modeler can easily substitute his/her own favorite forecasting distribution instead, such as the skewed Student's t, the normal inverse Gaussian (NIG), or the generalized hyperbolic (GH) distribution.

We apply our approach to forecasting Value-at-Risk (VaR) for individual stock returns and foreign exchange rate returns. It turns out that the Student's t based SD-EWMA schemes work better for most of the series considered. For some series with heavy rather than fat tails, the SD-EWMA scheme based on the heavy-tailed Laplace distribution works better. All SD-EWMA methods improve uniformly on the normal based EWMA method.

Compared to previous methods, such as Jensen and Lunde [2001] and Wilhelmsson [2009], the SD-EWMA approach provides a unifying framework that embeds previous proposals from the literature, such as Guermat and Harris [2002] and Gerlach et al. [2013]. In addition, the generality of the SD-EWMA approach also allows for a straightforward generalization to higher dimensions, estimating score driven versions of both volatilities, covariances and correlations, or even higher order moments. To illustrate this, we also provide an explicit SD-EWMA scheme for a Student's t time varying volatility model with a time varying degrees of freedom parameter.

The remainder of the paper is set up as follows. In Section 2, we introduce the basic methodology and convey the main intuition using the Student's t distribution as a leading example. In Section 3, we briefly review the tests used in our forecasting experiment to assess the performance of quantile forecasts. In Section 4, we provide our empirical application to Value-at-Risk forecasting. Section 5 concludes.

2. Score Driven Exponentially Weighted Moving Averages

2.1. Standard RiskMetrics^{TM} approach

Consider a time series $y_t \in \mathbb{R}$ observed over the sample period $t = 1, \ldots, T$. In our setting, y_t typically holds financial returns, such as stock returns or foreign exchange rate returns. We assume that y_t has a time varying conditional distribution $\mathbf{p}(y_t|\mathcal{F}_{t-1}; f_t, \theta)$, where $\mathcal{F}_{t-1}$ is the information set available at time $t-1$, f_t is a vector of time varying parameters, and θ is a vector of static parameters. For example, $\mathcal{F}_{t-1}$ may include lags of y_t and of exogenous variables, and f_t may include time varying means and/or volatilities, while θ may hold the remaining parameters characterizing the distribution, such as skewness and excess kurtosis parameters.

In the RiskMetrics$^{\text{TM}}$ approach, f_t takes the form of the time varying daily variance of y_t, denoted as $f_t = \sigma_t^2$. This is a key variable in determining the Value-at-Risk; see also the application in Section 4. RiskMetrics$^{\text{TM}}$ uses a standard exponentially weighted moving average (EWMA) scheme to determine the time varying volatility. The scheme is based on a conditional Gaussian distribution and given by

$$\sigma_{t+1}^2 = \lambda \sigma_t^2 + (1-\lambda) y_t^2, \qquad 0 < \lambda < 1. \tag{1}$$

The EWMA scheme in (1) corresponds to a zero-intercept IGARCH model,

$$\sigma_{t+1}^2 = \omega + \alpha y_t^2 + \beta \sigma_t^2 = \omega + \alpha(y_t^2 - \sigma_t^2) + (\alpha + \beta)\sigma_t^2, \tag{2}$$

with $\omega = 0$, $\beta = \lambda$, and $\alpha = 1 - \beta$, such that $\alpha + \beta = 1$. The volatility is thus a weighted sum of past squared observations, $\sigma^2_{t+1} = (1-\lambda)\sum_{i=0}^{\infty} \lambda^i y^2_{t-i}$. The intuition for the precise form of (1) derives directly from the Gaussian distribution: large squared realizations indicate a recent increase in volatility. If the observations y_t are conditionally fat-tailed, however, using squared observations may not be optimal as large realizations of y_t may occur regularly due to the fat-tailedness of the data, even though the variance has not changed substantially. If not properly accounted for, large realizations due to fat-tailedness may bias the estimates of the true underlying volatility. Due to the long memory of the integrated GARCH model (2), the bias may persist for a long time and affect a large number of subsequent volatility estimates.

2.2. Score Driven EWMA

To account for the shape of the conditional distribution in constructing a sensible EWMA scheme, we use the Generalized Autoregressive Score (GAS) framework of Creal et al. [2011, 2013]; see also Harvey [2013]. The theoretical results in Blasques et al. [2014b] show that time variation driven by the score of the conditional distribution results in locally optimal improvements from a Kullback-Leibler information theoretic perspective. The GAS(1,1) dynamics for the time varying parameter f_t are given by

$$f_{t+1} = \omega + A s_t + B f_t, \qquad s_t = \mathcal{S}_t \cdot \partial \ell_t / \partial f_t, \qquad \ell_t = \ln \mathbf{p}(y_t | \mathcal{F}_{t-1}; f_t, \theta), \tag{3}$$

where $\mathcal{S}_t = \mathcal{S}(f_t, \mathcal{F}_{t-1}; \theta)$ is an $\mathcal{F}_{t-1}$-measurable scaling function. Note that the scaled score s_t is a function of y_t, f_t, and $\mathcal{F}_{t-1}$. The time varying parameter f_t as specified in (3) is thus observation driven in the classification of Cox [1981]. More complicated dynamics than the one specified in equation (3) can be added to the specification; see for example Janus et al. [2011], Creal et al. [2013], and Harvey and Luati [2014]. For our current purposes, however, the GAS(1,1) dynamics suffice. For the scaling matrix, we propose the inverse Fisher conditional information matrix to account for the local

curvature of the score,

$$\mathcal{S}_t = \mathcal{I}_{t|t-1}^{-1} = \mathrm{E}_{t-1}\left[\left(\ell_t/\partial f_t\right) \left(\ell_t/\partial f_t\right)' \right]^{-1}.$$

For this form of scaling, the GAS dynamics embed the standard GARCH dynamics; see Creal et al. [2013] for more details. Though other forms of scaling are possible as well, the link to the GARCH dynamics enables us to construct a GAS or Score Driven EWMA (SD-EWMA) scheme by building on the analogy of the EWMA scheme in equation (1) and the IGARCH specification in (2). In particular, similar to (2) our SD-EWMA uses the integrated GAS dynamics

$$f_{t+1} = As_t + f_t. \tag{4}$$

This corresponds to an integrated GAS specification as in equation (3) with $\omega = 0$ and $B = 1$. For example, if $\mathbf{p}(y_t|\mathcal{F}_{t-1}; f_t, \theta)$ is the Gaussian distribution with zero mean and variance $f_t = \sigma_t^2$, Creal et al. [2013] show that (4) reduces precisely to the standard EWMA scheme in (1) if we set $A = 1 - \lambda$.

There is, however, no particular need to restrict oneself to the normal distribution. As it is well established that financial returns are typically fat-tailed, it makes much more sense to use an SD-EWMA scheme based upon a fat-tailed distribution. In this paper we follow Creal et al. [2011, 2013] and Harvey [2013] and use the Student's t distribution with ν degrees of freedom,

$$\mathbf{p}(y_t|\mathcal{F}_{t-1}; f_t, \theta) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{(\nu-2)\pi\sigma_t^2}} \left(1 + \frac{y_t^2}{(\nu-2)\sigma_t^2}\right)^{-\frac{\nu+1}{2}}, \tag{5}$$

with $f_t = \sigma_t^2$ and $\theta = \nu > 2$. The corresponding SD-EWMA scheme is given by

$$\sigma_{t+1}^2 = \sigma_t^2 + A \cdot (1 + 3\nu^{-1}) \cdot \left(\frac{\nu+1}{\nu - 2 + y_t^2/f_t} \cdot y_t^2 - f_t \right) = (1-\lambda)\sigma_t^2 + \lambda \cdot \frac{\nu+1}{\nu - 2 + y_t^2/f_t} \cdot y_t^2, \tag{6}$$

with $\lambda = A \cdot (1 + 3\nu^{-1})$. One can either fix ν at a predetermined value such as 5 for robustness purposes, or estimate it using an initial estimation sample.

As discussed in Creal et al. [2013] and Harvey [2013], the weight factor in front of y_t^2 in equation (6) has a robustifying effect on the volatility dynamics. If y_t lies in the tails of the conditional distribution at time t, the volatility is increased, but not by the full y_t^2. Part of the effect is attributed to the fat-tailedness of the Student's t distribution as can be seen from the division by $(\nu - 2 + y_t^2/\sigma_t^2)$. As the SD-EWMA scheme has the same integrated dynamics as the original EWMA scheme, a more robust estimate of the volatility at time t has a persistent effect on subsequent volatility estimates as well.

Interestingly, the SD-EWMA approach also encompasses previous adaptations of the EWMA scheme proposed in the literature. For example, Guermat and Harris [2002] introduce a robust-EWMA scheme

$$\sigma_{t+1} = \lambda\sigma_t + (1 - \lambda)\sqrt{2}|y_t|, \tag{7}$$

which is driven by absolute rather than squared observations. The authors relate their model to the GARCH type models of Taylor [1986] and Schwert [1990]. However, (7) can also be seen as a special case of the SD-EWMA scheme in (4). To see this, consider the Laplace density

$$\mathbf{p}(y_t|\mathcal{F}_{t-1}; f_t, \theta) = \frac{1}{\sqrt{2}\sigma_t} \exp(-\sqrt{2}\,|y_t|/\sigma_t). \tag{8}$$

As for the standard EWMA, we set $f_t = \sigma_t^2$. The IGAS(1,1) for the Laplace distribution is

$$f_{t+1} = \omega + 2A \cdot \sqrt{2}|y_t|\sigma_t + (B - 2A)f_t \;\Leftrightarrow\; \sigma_{t+1}^2 = \lambda\sigma_t^2 + \sigma_t \cdot (1 - \lambda)\sqrt{2}|y_t|, \tag{9}$$

if we set $\omega = 0$, $A = (1 - \lambda)/2$, and $B = 1$. Except for the multiplication by σ_t, which is due to the parameterization $f_t = \sigma_t^2$ rather than $f_t = \sigma_t$, (9) is the same as (7).[1] The robust-

[1]The robust-EWMA and Laplace based SD-EWMA model would be exactly identical if we set $f_t = \sigma_t$.

EWMA or Laplace based SD-EWMA model produces a modest increase in volatility for large values of $|y_t|$ compared to the standard EWMA (1). The derivation above reveals that the scheme can be motivated as a score driven approach based on the heavy-tailed Laplace distribution rather than the fat-tailed Student's t distribution in (6).

2.3. Further extensions

The SD-EWMA scheme introduced in Section 2.2 is very flexible. We can use it to accommodate the forecaster's favorite conditional distribution $\mathbf{p}(y_t|\mathcal{F}_{t-1}; f_t, \theta)$. As long as the conditional density has a parametric form, we can compute the score and construct the SD-EWMA scheme. The scheme also works for asymmetric distributions. For example, Gerlach et al. [2013] introduces an EWMA scheme based on the asymmetric Laplace distribution

$$\mathbf{p}(y_t|\mathcal{F}_{t-1}; f_t, \theta) = \frac{k_t}{\sigma_t} \exp\left(- \left(\frac{1}{1-p_t}\mathbf{1}[y_t > 0] + \frac{1}{p_t}\mathbf{1}[y_t < 0] \right) \frac{k_t\,|y_t|}{\sigma_t} \right), \tag{10}$$

with $f_t = (\sigma_t, p_t)$, and $k_t = (p_t^2 + (1-p_t)^2)^{1/2}$. Gerlach et al. [2013] introduce EWMA type time variation in both σ_t and p_t, specified by the recursions

$$\begin{aligned}
\sigma_{t+1} &= \lambda\sigma_t + (1-\lambda)\left(\frac{k_t}{1-p_t}\mathbf{1}[y_t > 0] + \frac{k_t}{p_t}\mathbf{1}[y_t < 0] \right) |y_t|, \qquad (11)\\
u_{t+1} &= \beta_u u_t + (1-\beta_u)|y_t|\mathbf{1}[y_t > 0],\\
v_{t+1} &= \beta_v v_t + (1-\beta_v)|y_t|\mathbf{1}[y_t < 0],\\
p_{t+1} &= \left(1 + \sqrt{u_{t+1}/v_{t+1}}\right)^{-1}. \qquad (12)
\end{aligned}$$

We can also derive the IGAS(1,1) dynamics for σ_t^2 using $f_t = \sigma_t^2$ directly from (10) and obtain

$$\sigma_{t+1}^2 = \lambda\sigma_t^2 + \sigma_t \cdot (1-\lambda)\left(\frac{k_t}{1-p_t}\mathbf{1}[y_t > 0] + \frac{k_t}{p_t}\mathbf{1}[y_t < 0] \right) |y_t|, \tag{13}$$

with $\lambda = 1 - 2A$. Again we notice from (13) that the original robust and asymmetric EWMA scheme of Gerlach et al. [2013] can be interpreted as an SD-EWMA update if we set $f_t = \sigma_t$ rather than $f_t = \sigma_t^2$ as in the original EWMA.

Though the SD-EWMA approach adapts itself to any parametric distribution, there is a trade-off to be considered. If the conditional distribution depends on more parameters than the time varying parameter f_t only, e.g., the degrees of freedom parameter ν or the skewness parameter p, these parameters need to be estimated before the SD-EWMA scheme can be operationalized. An attractive feature of the EWMA approach for volatility filtering and forecasting is precisely that no off-line estimation is needed. One way to achieve this is to estimate the auxiliary parameters on an estimation sample and to update them only infrequently. For the Student's t SD-EWMA scheme this approach works well and better than a number of competing schemes for a range of foreign exchange rate and stock returns; see the application in Section 4. For other distributions, however, more care may be needed.

Finally, we note the flexibility of the SD-EWMA approach to account for other dynamic parameters beyond the volatility context. For example, the model can easily be extended to handle both volatilities and covariances, or volatilities and correlations, using the recursions in Creal et al. [2011] and the integrated GAS(1,1) specification in (4). In addition, the approach can be further generalized to handle time variation in higher order moments, such as skewness and kurtosis, by putting the appropriate parameters into f_t rather than θ. An example that we use in our subsequent empirical analysis is a new SD-EWMA model with a time varying degrees of freedom parameter. For this, we take the likelihood in equation (5) and set $f_t' = (f_{1,t}, f_{2,t})$ with $\sigma_t^2 = f_{1,t}$ and $\nu_t = 2 + \exp(f_{2,t})$.

Using inverse Fisher information scaling, we obtain the following recursion for ν_t,

$$f_{2,t+1} = f_{2,t} - A_\nu \frac{2}{\nu_t - 2}\left[\gamma''\left(\frac{\nu_t+1}{2}\right) - \gamma''\left(\frac{\nu_t}{2}\right) + \frac{2(\nu_t+4)(\nu_t-3)}{(\nu_t+1)(\nu_t+3)(\nu_t-2)^2}\right]^{-1} \left[\gamma'\left(\frac{\nu_t+1}{2}\right) - \gamma'\left(\frac{\nu_t}{2}\right) - \frac{1}{\nu_t-2} - \ln\left(1+\frac{y_t^2}{(\nu_t-2)\,\sigma_t^2}\right) + \frac{\nu_t+1}{\nu_t-2}\cdot\frac{y_t^2}{(\nu_t-2)\sigma_t^2+y_t^2}\right], \tag{14}$$

where $A_\nu > 0$ is a scalar tuning parameter similar to the parameter A used for the volatility dynamics in (6), and $\gamma'(\cdot)$ and $\gamma''(\cdot)$ are the first and second order derivatives of $\gamma(\cdot) = \ln\Gamma(\cdot)$. The derivation of this result is provided in the appendix. The reparameterization $\nu_t = 2 + \exp(f_{2,t})$ automatically ensures that the degrees of freedom parameter ν_t is always larger than 2, such that the variance of the Student's t distribution always exists. The score based recursions automatically account for this reparameterization via the chain rule used in the score calculations; see the appendix for further details.

Though the shape of the recursion for ν_t in (14) may look complicated at first sight, it is actually easy to implement. Interestingly, it does not directly use fourth order moments as one may have expected for the dynamics of a tail-shape parameter. Rather, it only uses second order moments combined with the explicit information embedded in the tail shape of the Student's t distribution. An advantage of using the recursion in (14) is that it typically results in a much more stable path of the degrees of freedom parameter. Fourth order moments of the data, by contrast, are notoriously unstable. The composition of squared data and the gamma functions and their derivatives in (14) circumvent this problem of instability. We provide some typical shapes of the news impact curves related to equation (14) for several values of ν_t in Figure 1. The curves are re-centered and re-scaled to be comparable within one figure. We also plot a fourth order polynomial $-z_t^4$ as a benchmark.

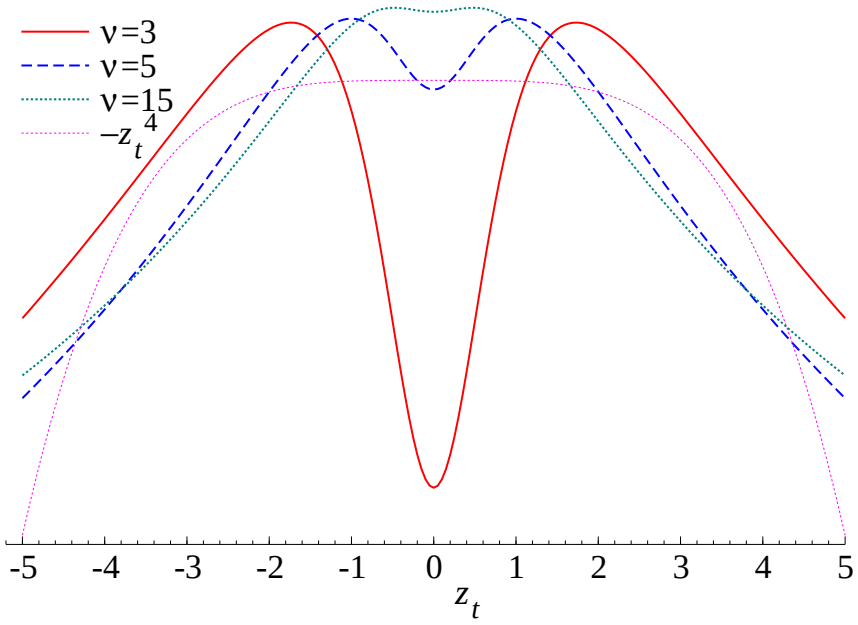

Figure 1: News impact curves for the time varying degrees of freedom recursion (14)

Scaled and recentered news impact curves (14) as a function of $z_t = y_t^2/((\nu_t - 2)\sigma_t^2)$ for different values of ν_t. The (rescaled and recentered) curve of fourth order powers $-z_t^4$ is also shown as a benchmark.

Figure 1 shows that large values of $|z_t|$ result in a downward adjustment of ν_{t+1} for all curves considered. This is intuitive, as large values of $|z_t|$ can be associated with tails being fat. The decline in (14) for large values of z_t is comparable for different values of ν_t. Interestingly, the sensitivity of the GAS based news impact curves for ν_{t+1} is much lower than that of the fourth order polynomial curve $-z_t^4$. This provides the SD-EWMA recursion for ν_t with its robustness feature. Also note that for fatter tailed distributions such as $\nu_t = 3$, values z_t near zero also result in smaller values of ν_{t+1}. This is a consequence of the fact that fat-tails for the Student's t distribution go hand in hand with leptokurtosis, i.e., 'peaked-ness' at the center of the distribution. The less leptokurtic the distribution, the smaller the downward effect of observations near zero compared to near, say, -1 or -2. The informativeness of observations in the center compared to tail observations only really becomes clear if the distribution is already fat-tailed, i.e., if ν_t is low. For higher values of ν_t, downward signals for ν_{t+1} predominantly must come from

tail observations.

As a final remark, we note that the smoothing parameter A_ν for the ν_t recursion is typically smaller than that of the volatility recursion. Starting values for the estimation of A_ν for empirical data in the range of 0.001 work quite well. The low values of A_ν underline the stable path dynamics for ν_t described by (14). We show in Section 4 that allowing for a time varying degrees of freedom parameter helps to further improve the accuracy of tail probability estimates for fat-tailed data.

3. Value-at-Risk and backtesting

We evaluate the performance of the SD-EWMA scheme for forecasting Value-at-Risk (VaR). VaR is a classical risk management tool in the finance industry. It is defined as the quantile of the profit and loss distribution of the bank. In terms of returns we obtain the $\text{VaR} = -Y_a$ at confidence level $(1-a)$ as

$$Y_a = \sup\left\{Y^* \;\middle|\; \mathrm{P}[Y < Y^*] \leq a\right\}.$$

The value of Y_a hinges tightly together with the distributional assumptions for Y; see Chen and Lu [2012] for a recent survey. There is a trade-off between the fat-tailness of the distribution of Y, and the transition dynamics of the volatility updating mechanism. In the Student's t based SD-EWMA framework, the volatility updates are less responsive to extreme realized returns compared to the standard Gaussian EWMA scheme. This makes the computed VaR less responsive to abrupt volatility changes. By contrast, if there are incidental tail observations, the Student's t based SD-EWMA scheme provides a much better and robust estimate of the volatility at time t. Moreover, the fat-tailedness of the conditional Student's t distrubution pushes the VaR levels farther out into the tails compared to the Gaussian distribution for a fixed confidence level $(1-a)$. The trade-off between all these forces results in the relative performance of the different methods for

forecasting, which can only be investigated empirically across different confidence levels $(1-a)$ and different datasets.

To assess the performance of alternative (SD)-EWMA methods, we consider a number of standard tests for the quality of tail probability forecasts: the Unconditional Coverage test, the Independence test, the Conditional Coverage test, the Dynamic Quantile test, and the tail shape test of Berkowitz [2001]. The first three tests and the last test are Likelihood Ratio (LR) based tests, whereas the Dynamic Quantile test is a regression-based test. A good VaR model should be consistent in that the fraction of VaR violations, i.e. events $\{y_t < -VaR_t\}$, should equal a in large samples. Define the violation indicator

$$I_t = \mathbf{1}\{y_t < -VaR_t\},$$

and the number of violation $N = \sum_{t=1}^{T} I_t$ out of T time periods. Following Christoffersen [1998], good VaR models produce serially independent I_ts. Our backtesting methods are all related to good coverage, serial independence, or both.

Kupiec [1995] tests the Unconditional Coverage (UC) of the VaR model using

$$LR_u = 2(\ln L_N - \ln L_\alpha) \sim \chi^2(1), \quad T \to \infty; \tag{15}$$

where $L_N = (1-N/T)^{T-N}(N/T)^N$, and $L_\alpha = (1-\alpha)^{T-N}\alpha^N$. Christoffersen [1998] proposes the Independence (IN) test for the VaR violation indicators I_t. The transition matrix of the corresponding first-order Markov Chain is

$$\Pi = \begin{pmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{pmatrix}, \qquad \pi_{ij} = P(I_t = j \mid I_{t-1} = i) = T_{ij}/(T_{i0} + T_{i1}),$$

with T_{ij} recording the times of transition from state i to j, where $i, j \in \{0, 1\}$. The LR test

for independence is

$$LR_{in} = 2(\ln L_A - \ln L_0) \sim \chi^2(1), \quad T \to \infty, \tag{16}$$

where $L_A = \pi_{00}^{T_{00}} \pi_{01}^{T_{01}} \pi_{10}^{T_{10}} \pi_{11}^{T_{11}}$ and $L_\alpha = (1-\alpha)^{T_{01}+T_{11}} \alpha^{T_{00}+T_{10}}$. The simultaneous test for Unconditional Coverage and Independence, namely the correct Conditional Coverage (CC) test, is

$$LR_c = LR_u + LR_{in} \sim \chi^2(2), \quad T \to \infty. \tag{17}$$

Finally, Engle and Manganelli [2004] proposed the Dynamic Quantile (DQ) test. This is a regression based joint test for correct coverage and independence. Let $H_t = I_t - a$. Then the DQ test regresses H_t on its own lags H_{t-j}, lagged VaR forecasts, and some other regressors. The test statistic is the regression's standard F-test, which is asymptotically $\chi^2(c)$ distributed, where c is the number of (non-constant) regressors. Following Engle and Manganelli [2004], we select lagged H_{t-j} with $j = 1, 2, 3, 4$ and the VaR_t as the regressors.

In practice, risk managers are not only concerned with the number of VaR failures, but also with the accuracy of the model for the tail shape beyond the VaR. This is relevant for assessing the potential magnitude of losses in the tail, and relates to the general shift in the industry and in regulation from VaR to Expected Loss (or Conditional VaR) computations. To test for the general tail shape, we adopt the test proposed by Berkowitz [2001]. The test operates on an inverse standard normal transformation of the probability integral transforms of the data, i.e.,

$$z_t = \Phi^{-1}\big(\hat{F}_t(y_t)\big), \tag{18}$$

where $\hat{F}_t(\cdot)$ denotes the estimated cumulative distribution function applicable at time t using the postulated VaR model, such as the Laplace, Asymmetric Laplace, or Student's t

distribution, and $\Phi^{-1}(\cdot)$ denotes the inverse standard normal distribution function. The variable of interest is constructed by truncating the variable z_t at the threshold $\Phi^{-1}(a) = -\text{VaR}$, such that $z_t = -\text{VaR}$ if $z_t \geq -\text{VaR}$. Estimating the mean and variance for a censored normal random variable can be achieved by maximizing the likelihood function

$$L(\mu, \sigma^2) = \sum_{z_t < -\text{VaR}} \left(-\frac{1}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}(z_t - \mu)^2 \right) + \sum_{z_t \geq -\text{VaR}} \ln\left(1 - \Phi\left(\frac{-\text{VaR} - \mu}{\sigma}\right)\right). \tag{19}$$

The Berkowitz [2001] test uses the maximum likelihood estimates to compute a likelihood ratio (LR) test for the null hypothesis $\mu = 0$ and $\sigma^2 = 1$. The corresponding LR test is

$$LR = -2(L(0,1) - L(\hat{\mu}, \hat{\sigma}^2)),$$

which is asymptotically $\chi^2(2)$ distributed.

4. Empirical results

4.1. Data and descriptive statistics

In this section, we compare the performance of different SD-EWMA schemes. Note that for the normal distribution, the SD-EWMA scheme coincides with the standard EWMA for volatility modeling. As explained in Section 2, the SD-EWMA updating schemes (9) and (13) based on the Laplace and asymmetric Laplace distribution, respectively, are very close to the robust EWMA scheme (7) of Guermat and Harris [2002], and the skewed EWMA scheme(11) of Gerlach et al. [2013], respectively. For the dynamic asymmetric Laplace, we use the same dynamics for p_t in (12) as used in Gerlach et al. [2013]. As Gerlach et al. [2013] show that the GARCH and GJR-GARCH based on a normal or Student's t distribution do not outperform the skewed EWMA models, we do not include them in our current study.

We use 12 financial time series over the period January 5, 1999 to November 1, 2013.

Table 1: Summary Statistics

The descriptive statistics present the centered moments of the financial time series considered. The sample period is January 5, 1999 to November 1, 2013. We split the sample into an in-sample estimation period and out-of-sample forecasting period. The sample mean is multiplied with 100. A standard deviation (SD) of 1.28 denotes 1.28% per day. SK and KT denote skewness and kurtosis, respectively.

Data	In-sample: 1999-2006				Out-of-sample: 2007-2013			
	Mean	SD	KT	SK	Mean	SD	KT	SK
	exchange rate returns							
GBP	0.010	0.51	0.41	0.00	-0.007	0.64	3.21	-0.47
AUD	0.013	0.68	1.22	-0.29	0.011	1.06	10.55	-0.47
JPY	-0.002	0.62	1.94	0.11	0.010	0.74	5.29	-0.14
CAD	0.014	0.44	0.65	0.01	0.008	0.70	2.54	-0.20
SEK	0.011	0.68	0.46	-0.04	0.006	0.93	2.52	-0.04
EUR	0.007	0.64	0.51	-0.01	0.004	0.68	1.85	0.06
	equity returns							
AA	0.032	2.34	2.50	0.22	-0.061	3.19	6.99	-0.18
BA	0.056	2.07	5.85	-0.41	0.033	2.00	4.23	0.10
GE	0.014	1.86	4.07	0.06	-0.005	2.27	8.50	0.00
IBM	0.006	2.06	7.83	-0.15	0.042	1.50	5.15	-0.04
KO	-0.009	1.60	5.22	-0.10	0.040	1.29	11.78	0.60
T	-0.004	2.02	3.46	-0.06	0.022	1.56	10.68	0.58

The dataset contains 6 exchange rate log returns and 6 equity log returns. The exchange rates are always vis-à-vis the US Dollar. We consider the Australian Dollar, the Canadian Dollar, the Euro, British Pound, Japanese Yen, and Swedish Kroner, denoted as AUD, CAD, EUR, GBP, JPY, and SEK, respectively. The stocks considered represent different industries and are all listed at the New York Stock Exchange: Alcoa Inc., Boeing Co., General Electric, IBM, Coca-Cola and AT&T, denoted as AA, BA, GE, IBM, KO, and T. The number of observations for different time series can differ due to the variation of national holidays.

From the descriptive statistics in Table 1, it is obvious that all series exhibit non-normal

features such as non-zero skewness and excess kurtosis. We thus expect the robust (Laplace based) SD-EWMA and Student's t SD-EWMA schemes to provide particular advantages compared to the standard EWMA scheme. We use the same distributional assumptions to set up the SD-EWMA recursions and to compute the VaR.

We split the sample into two subsamples. We use the sample from January 5, 1999 to December 29, 2006 (in-sample) for estimation of the static parameters. In particular, for all models we estimate the optimal smoothing parameter A using the estimation sample. We also estimate any remaining static parameters needed, such as the degrees of freedom parameter ν for the Student's t distribution, or the skewness parameter p for the asymmetric Laplace distribution if the skewness is not time varying. For the asymmetric Laplace with time varying skewness, we estimate an additional separate smoothing parameter λ_p. Also for the SD-EWMA scheme with time varying degrees of freedom, we estimate the additional parameter A_ν. In all cases, the estimated parameters are kept fixed over the entire forecasting period. This results in a computationally fast procedure. We also carry out a robustness check by recursively updating all tuning parameters over the entire forecasting sample; see the discussion in Ardia and Hoogerheide [2014] for the potential benefits of such an approach.

4.2. Full results for the Euro-Dollar rate

For the Euro-Dollar exchange rate, we report the full results for all tests in Table 2. The bad performance of the normal based standard EWMA scheme is not surprising. We also confirm the results of Gerlach et al. [2013] that there appears to be little evidence for skewness: the results for $p = 0.5$ and p estimated are quite similar. The models based on the Laplace distribution result in acceptable diagnostics in terms of UC, ID, CC, and DQ statistics, especially considering VaR at 5% level. The Berkowitz test, however, strongly rejects the Laplace based model in that it fails to provide an adequate description of the tail shape beyond the VaR level.

Table 2: Full SD-EWMA Results for the Euro-Dollar Exchange Rate

The test statistics correspond to the unconditional coverage (UC) test of Kupiec [1995], the independence (ID) and Conditional Coverage (CC) test of Christoffersen [1998], the Dynamic Quantile (DQ) regression test of Engle and Manganelli [2004], and the the Berkowitz [2001] test (BE). We use a confidence level for the VaR equal to $1 - a = 0.995/0.99/0.95$. Critical values (χ^2_{cv}) at a 1% significance level are also displayed, as are the Hit Rate (HR) N/T of N VaR violations out of T observations, multiplied with 100. Static parameters are estimated over Jan 5, 1999 to Dec 29, 2006, and held fixed over the forecast evaluation period Jan 3, 2007 to Nov 1, 2013. The SD-EWMA schemes use the normal distribution (N), Laplace distribution (L) with skewness parameter 0.5, p, or p_t, and Student's and distribution (t) with degrees of freedom parameter ν, ν_t, or 5. We separate the results for models with and without updated parameters in two different panels.

	No parameter updating						With parameter updating					
	CC	UC	IN	DQ	HR	BE	CC	UC	IN	DQ	HR	BE
						$a = 0.5\%$						
N	**3.1**	**2.8**	**0.2**	**5.3**	0.81	9.4	**3.1**	**2.8**	**0.2**	**5.5**	0.81	**6.7**
L(0.5)	**7.4**	7.4	**0.0**	**5.1**	0.12	27.7	**7.4**	7.4	**0.0**	**5.1**	0.12	26.5
L(p_t)	11.0	11.0	**0.0**	**6.8**	0.06	25.3	11.0	11.0	**0.0**	**6.8**	0.06	25.2
L(p)	**7.4**	7.4	**0.0**	**5.1**	0.12	24.8	**7.4**	7.4	**0.0**	**5.1**	0.12	26.2
t(ν_t)	**0.4**	**0.3**	**0.1**	**0.4**	0.41	**1.8**	**0.1**	**0.0**	**0.1**	**0.6**	0.52	**1.0**
t(ν)	**0.3**	**0.2**	**0.1**	**1.2**	0.58	**1.3**	**0.8**	**0.6**	**0.1**	**2.4**	0.64	**1.4**
t(5)	**7.4**	7.4	**0.0**	**5.1**	0.12	21.4	**7.4**	7.4	**0.0**	**5.1**	0.12	21.0
χ^2_{cv}/true	9.2	6.6	6.6	16.8	0.50	9.2	9.2	6.6	6.6	16.8	0.50	9.2
						$a = 1\%$						
N	**7.2**	6.7	**0.4**	**13.5**	1.68	9.6	9.3	9.0	**0.3**	16.8	1.80	9.6
L(0.5)	9.9	9.9	**0.0**	**7.4**	0.35	33.3	12.2	12.2	**0.0**	**9.1**	0.29	32.2
L(p_t)	12.2	12.2	**0.0**	**9.1**	0.29	33.9	12.2	12.2	**0.0**	**9.1**	0.29	33.1
L(p)	14.9	14.9	**0.0**	**10.4**	0.23	34.1	12.2	12.2	**0.0**	**9.1**	0.29	32.0
t(ν_t)	**1.3**	**0.8**	**0.5**	**11.6**	1.22	**2.1**	**3.1**	**2.4**	**0.7**	**10.7**	1.39	**2.3**
t(ν)	**3.1**	**2.4**	**0.7**	**10.5**	1.39	**1.7**	**4.6**	**3.9**	**0.7**	**11.7**	1.51	**2.3**
t(5)	**6.3**	**6.2**	**0.1**	**5.1**	0.46	21.2	**4.9**	**4.8**	**0.1**	**4.3**	0.52	20.3
χ^2_{cv}/true	9.2	6.6	6.6	16.8	1.00	9.2	9.2	6.6	6.6	16.8	1.00	9.2
						$a = 5\%$						
N	**2.8**	**1.6**	**1.1**	**7.6**	5.69	**8.4**	**2.9**	**1.9**	**1.0**	**9.6**	5.75	**7.9**
L(0.5)	**6.7**	**1.3**	**5.4**	19.1	4.41	37.3	**3.7**	**1.6**	**2.1**	**11.9**	4.35	35.7
L(p_t)	**5.2**	**2.6**	**2.7**	**16.4**	4.18	34.3	**5.2**	**2.6**	**2.7**	**11.4**	4.18	33.3
L(p)	**5.2**	**2.6**	**2.7**	**8.6**	4.18	36.6	**3.7**	**1.6**	**2.1**	**11.8**	4.35	36.3
t(ν_t)	**3.1**	**2.2**	**0.9**	**10.4**	5.80	**4.9**	**3.4**	**2.6**	**0.8**	**8.9**	5.86	**4.4**
t(ν)	**2.9**	**1.9**	**1.0**	**11.2**	5.75	**2.1**	**3.1**	**2.2**	**0.9**	**9.1**	5.80	**2.4**
t(5)	**2.3**	**0.6**	**1.8**	**11.2**	5.40	27.2	**2.4**	**0.9**	**1.5**	**10.6**	5.51	28.1
χ^2_{cv}/true	9.2	6.6	6.6	16.8	5.00	9.2	9.2	6.6	6.6	16.8	5.00	9.2

Turning to the model with SD-EWMA dynamics based on the Student's t distribution, we see that the performance of the model is much better. Exchange rate returns are typically characterized by fat-tails such as Student's t tails rather than by heavy tails such Laplace tails. The Student's t based SD-EWMA approach, therefore, provides a useful additional tool for VaR forecasting. The gain from using the Student's t based SD-EWMA is stronger if we look at more extreme tail outcomes, such as the 1% VaR levels. The Student's t based SD-EWMA scheme has the best overall performance, particularly if we allow the degrees of freedom parameter ν_t to follow the dynamics described in equation (14). The Berkowitz test and the Conditional Coverage test do not reject the Student's t based SD-EWMA for any of the VaR confidence levels considered. Moreover, the Hit Rate for the model with time varying ν_t is closest to the true value for most series.

4.3. Full results: all series

To investigate the robustness of the results, we extend our analysis to other exchange rates as well as to individual stock returns. We present the results in Tables 3 and 4. We focus on the intended coverage levels $a = 1\%, 5\%$, and only include the conditional coverage (CC) and Berkowitz (BE) tests. Additional results can be found in the online appendix.

We first note that for $a = 5\%$, the conditional coverage (CC) test does not really differentiate between the alternative models. With a few exceptions for the Laplace based schemes, all models pass the test. The same holds for the equity returns if we consider more extreme tail areas ($a = 1\%$). For the exchange rate returns, however, going deeper into the tails provides a more mixed result. In particular, some of the models are rejected for a number of series. Interestingly, the Student's t based model with time varying degrees of freedom parameter works well, except for the UK pound and Australian dollar exchange rates. For the other exchange rates, this model typically gives the lowest value for the CC test.

Table 3: VaR Forecasting Results, No Parameter Updating

The table contains the values of the the conditional coverage test of Christoffersen [1998] and the Berkowitz [2001] test. Test values *below* the 1% critical value of 9.2 are put in bold. Tests are carried out at two levels for the tail area, namely $a = 1\%$ and $a = 5\%$. The estimation sample is Jan 5, 1999 to Dec 29, 2006, and the forecasting period Jan 3, 2007 to Nov 1, 2013. Parameters are estimated over the estimation sample, and not updated afterwards. The exchange rates are always vis-à-vis the US Dollar. We consider the Australian Dollar (AUD), the Canadian Dollar (CAD), the Euro (EUR), British Pound (GBP), Japanese Yen (JPY), and Swedish Kroner (SEK). The individual stocks are Alcoa Inc. (AA), Boeing Co. (BA), General Electric (GE), IBM, Coca-Cola (KO), and AT&T (T). The SD-EWMA schemes use the normal distribution (N), Laplace distribution (L) with skewness parameter 0.5, p, or p_t, and Student's and distribution (t) with degrees of freedom parameter ν, ν_t, or 5.

	Exchange rate						Equity returns					
	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
	conditional coverage test, $a = 1\%$											
N	**7.7**	29.6	**3.3**	**7.2**	13.5	**7.2**	9.5	14.4	**8.2**	**8.2**	11.4	11.7
L(0.5)	**6.3**	**1.8**	**2.8**	**8.0**	**6.3**	9.9	**6.3**	**0.4**	**0.9**	**1.8**	**2.0**	**4.9**
L(p_t)	9.9	**1.8**	**3.7**	**8.0**	**6.3**	12.2	**6.3**	**0.4**	**0.4**	**5.8**	**1.3**	**3.7**
L(p)	12.2	**3.3**	**3.7**	12.2	**8.0**	14.9	9.9	**0.4**	**1.3**	**2.3**	**2.0**	**3.7**
t(ν_t)	9.5	13.1	**0.9**	**2.8**	**2.4**	**1.3**	**2.0**	**2.4**	**3.1**	**6.5**	**1.8**	**2.4**
t(ν)	11.3	21.2	**2.4**	9.3	9.5	**3.1**	**1.8**	**5.6**	**7.2**	**6.5**	**1.3**	**2.8**
t(5)	**8.0**	**6.5**	**1.8**	**2.0**	**2.0**	**6.3**	**6.2**	**0.6**	**1.3**	**6.5**	**0.9**	**0.6**
	conditional coverage test, $a = 5\%$											
N	**6.6**	**6.6**	**0.6**	**7.8**	**4.5**	**2.8**	**4.9**	**3.8**	**1.1**	**1.3**	**2.7**	**0.6**
L(0.5)	**0.6**	**2.6**	**2.6**	**2.7**	**2.1**	**6.7**	**7.4**	**1.3**	**2.0**	**2.8**	13.3	**4.6**
L(p_t)	**0.8**	**2.7**	**4.9**	**2.0**	**0.9**	**5.2**	11.8	**0.7**	**0.5**	**1.8**	9.3	**3.6**
L(p)	**0.4**	**1.4**	**6.0**	**1.4**	**1.4**	**5.2**	**5.0**	**0.7**	**0.4**	**1.8**	**2.9**	**2.5**
t(ν_t)	**7.1**	**6.6**	**4.0**	**4.5**	**4.2**	**3.1**	**6.5**	**5.8**	**3.8**	**0.8**	**5.1**	**0.5**
t(ν)	**7.6**	**8.1**	**4.0**	**4.1**	**4.5**	**2.9**	**8.5**	**6.2**	**2.7**	**0.8**	**5.0**	**0.5**
t(5)	**4.5**	**5.2**	**2.6**	**1.4**	**0.8**	**2.3**	**6.4**	**5.1**	**2.9**	**0.8**	**5.4**	**1.3**
	Berkowitz test, $a = 1\%$											
N	12.0	97.1	71.1	**7.5**	18.1	9.6	73.9	80.6	107.0	330.7	138.5	72.7
L(0.5)	34.1	**3.7**	14.0	32.9	24.3	33.3	18.0	**4.7**	9.4	16.5	10.2	11.2
L(p_t)	37.4	**2.0**	17.2	30.8	24.0	33.9	14.1	**3.3**	**5.1**	17.5	**8.3**	11.4
L(p)	37.3	**6.6**	18.3	35.2	28.9	34.1	23.0	**4.7**	10.3	17.5	**8.7**	**7.5**
t(ν_t)	**2.8**	**9.1**	**0.2**	**0.1**	**0.3**	**2.1**	**1.6**	**1.3**	**4.4**	**6.8**	**2.1**	**0.2**
t(ν)	**8.0**	22.2	**1.4**	**5.6**	**3.2**	**1.7**	**4.0**	**4.1**	9.8	9.7	**0.9**	**1.0**
t(5)	24.2	**0.2**	**1.5**	21.0	15.1	21.2	**6.9**	**0.3**	**0.5**	**6.3**	**0.0**	**2.4**
	Berkowitz test, $a = 5\%$											
N	12.3	83.6	58.9	10.8	19.5	**8.4**	61.7	73.2	79.0	302.8	127.8	73.5
L(0.5)	40.0	**4.0**	13.4	40.8	35.9	37.3	13.7	9.9	16.4	15.5	14.8	14.3
L(p_t)	37.8	**2.2**	19.8	39.2	34.7	34.3	11.1	**8.1**	10.2	15.3	21.7	15.3
L(p)	40.0	**6.9**	17.1	45.9	36.5	36.6	13.5	9.9	11.7	11.5	13.5	11.5
t(ν_t)	**5.6**	**8.5**	**2.5**	**2.9**	**4.8**	**4.9**	**0.3**	**1.2**	**1.0**	**4.7**	**1.7**	**0.6**
t(ν)	**7.1**	20.1	**0.9**	**8.5**	**5.6**	**2.1**	**1.6**	**3.2**	**4.6**	**7.0**	**0.5**	**1.6**
t(5)	21.4	**1.5**	**5.0**	18.0	19.2	27.2	**2.7**	**0.7**	**2.0**	**4.0**	**1.0**	**2.4**

If we turn to the results of the Berkowitz test, the results are much more interesting. As mentioned earlier, the Berkowitz test is particularly interesting because it tests the adequacy of the model as a description of the size and magnitude of *all* tail observations. At the 5% level for both the exchange rate returns and individual stock returns, we see that all models are rejected except the Student's t SD-EWMA model. Particularly the model with time varying degrees of freedom ν_t is never rejected at the 1% significance level for this (95%) confidence level VaR. In line with our previous results, the Australian dollar exchange rate return is better described by the heavy-tailed Laplace rather than the fat-tailed Student's t distribution. For the UK pound exchange rate, however, the Student's t SD-EWMA model with time varying ν_t performs well in terms of the Berkowitz test.

If we go deeper into the tails ($a = 1\%$), these results are confirmed. Again we find the SD-EWMA model with time varying degrees of freedom to perform best with acceptable Berkowitz test values for all series. Interestingly, the Student's t model with plug-in value $\nu = 5$ performs well for the individual stock returns, but not for the foreign exchange rates. The Laplace based models only perform well for the Australian dollar exchange rate, and for the return on Boeing. The Gaussian based standard EWMA does not perform well for any of the series in our sample.

The results are robust to a forecasting analysis in which we recursively update the static parameters in the model over the forecasting period; see Table 4. The results in Table 4 confirm that the Student's t based SD-EWMA with time varying degrees of freedom performs well for almost all series. The results are in general close to those in Table 3. We also note that recursively updating the static parameters does not unequivocally improve the forecasting results. Though the results are better for some series and some tests, the converse holds for other combinations. In general, however, the results remain quite close for the series considered in this paper.

Table 4: VaR Forecasting Results, With Parameter Updating

The table contains the values of the the conditional coverage test of Christoffersen [1998] and the Berkowitz [2001] test. Test values *below* the 1% critical value of 9.2 are put in bold. Tests are carried out at two levels for the tail area, namely $a = 1\%$ and $a = 5\%$. The estimation sample is Jan 5, 1999 to Dec 29, 2006, and the forecasting period Jan 3, 2007 to Nov 1, 2013. Parameters are estimated over the estimation sample, and recursively updated afterwards. The exchange rates are always vis-à-vis the US Dollar. We consider the Australian Dollar (AUD), the Canadian Dollar (CAD), the Euro (EUR), British Pound (GBP), Japanese Yen (JPY), and Swedish Kroner (SEK). The individual stocks are Alcoa Inc. (AA), Boeing Co. (BA), General Electric (GE), IBM, Coca-Cola (KO), and AT&T (T). The SD-EWMA schemes use the normal distribution (N), Laplace distribution (L) with skewness parameter 0.5, p, or p_t, and Student's and distribution (t) with degrees of freedom parameter ν, ν_t, or 5.

	Exchange rate						Equity returns					
	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
	conditional coverage test, $a = 1\%$											
N	9.3	29.6	**3.3**	9.3	17.5	9.3	**8.7**	13.0	10.4	11.7	**8.9**	10.4
L(0.5)	**8.0**	**2.3**	**3.7**	12.2	**8.0**	12.2	**6.3**	**0.4**	**1.3**	**1.8**	**1.3**	**2.8**
L(p_t)	9.9	**2.3**	**3.7**	9.9	**8.0**	12.2	**6.3**	**0.4**	**1.3**	**1.8**	**0.6**	**3.7**
L(p)	**8.0**	**2.7**	**3.7**	12.2	**8.0**	12.2	**6.3**	**0.6**	**2.0**	**2.0**	**2.8**	**3.7**
t(ν_t)	**5.4**	16.5	**2.1**	**2.4**	9.5	**3.1**	**1.8**	**3.8**	**4.7**	**7.4**	**2.4**	**3.9**
t(ν)	**6.2**	18.6	**2.4**	9.3	**8.0**	**4.6**	**1.8**	**3.8**	**4.7**	**7.4**	**1.3**	**3.9**
t(5)	**2.8**	**6.1**	**0.4**	**2.8**	**5.1**	**4.9**	**5.1**	**0.6**	**0.6**	**7.4**	**0.9**	**0.9**
	conditional coverage test, $a = 5\%$											
N	**6.6**	**5.2**	**0.9**	**8.1**	**5.4**	**2.9**	**8.1**	**4.0**	**0.6**	**1.1**	**5.8**	**1.3**
L(0.5)	**0.1**	**1.3**	**1.9**	**2.7**	**0.7**	**3.7**	**4.7**	**1.3**	**0.4**	**1.5**	**5.5**	**4.1**
L(p_t)	**0.3**	**2.3**	**6.6**	**1.3**	**0.3**	**5.2**	**5.0**	**0.7**	**0.3**	**1.3**	**4.2**	**3.6**
L(p)	**0.1**	**1.3**	**6.0**	**2.1**	**1.2**	**3.7**	**6.7**	**1.3**	**0.5**	**2.8**	**5.2**	**3.6**
t(ν_t)	**7.6**	**7.6**	**1.0**	**8.7**	**5.4**	**3.4**	**8.3**	**5.8**	**4.4**	**0.9**	**5.0**	**1.4**
t(ν)	**7.6**	**7.6**	**1.0**	10.0	**5.0**	**3.1**	**8.5**	**5.8**	**4.9**	**0.9**	**5.1**	**1.5**
t(5)	**3.8**	**4.9**	**0.6**	**3.1**	**0.9**	**2.4**	**6.6**	**5.1**	**2.6**	**1.0**	**5.2**	**0.5**
	Berkowitz test, $a = 1\%$											
N	13.3	103.0	69.8	**7.1**	19.4	9.6	74.7	82.0	96.7	330.0	136.5	69.1
L(0.5)	36.2	**6.3**	16.5	35.9	26.4	32.2	17.9	**4.7**	11.1	17.4	**8.3**	**8.6**
L(p_t)	37.9	**3.3**	17.6	31.5	26.1	33.1	14.1	**3.3**	**8.9**	18.6	**5.4**	11.4
L(p)	35.6	**7.2**	18.2	36.3	26.5	32.0	17.9	**4.7**	12.8	16.5	10.6	9.9
t(ν_t)	**1.0**	10.3	**0.0**	**0.2**	**1.9**	**2.3**	**1.5**	**1.6**	**3.4**	**5.6**	**2.3**	**0.9**
t(ν)	**5.7**	18.2	**0.8**	**5.1**	**2.8**	**2.3**	**3.0**	**3.6**	**6.9**	**8.5**	**0.8**	**1.8**
t(5)	23.5	**0.2**	**1.9**	21.7	15.3	20.3	**5.8**	**0.3**	**1.2**	**5.3**	**0.0**	**2.4**
	Berkowitz test, $a = 5\%$											
N	12.2	94.7	61.1	10.8	19.8	**7.9**	61.0	76.3	82.6	306.4	124.3	66.2
L(0.5)	43.5	**4.5**	12.9	40.6	36.4	35.7	13.1	9.4	13.3	12.5	12.1	13.6
L(p_t)	38.5	**2.1**	20.0	36.9	33.6	33.3	11.1	**7.8**	9.9	13.9	23.3	15.1
L(p)	42.0	**5.5**	17.5	42.4	37.2	36.3	12.5	9.8	13.4	14.4	17.3	13.8
t(ν_t)	**5.7**	11.1	**0.6**	**5.2**	**5.7**	**4.4**	**0.4**	**1.4**	**1.0**	**4.5**	**2.1**	**0.9**
t(ν)	**7.0**	18.7	**0.3**	**8.8**	**6.6**	**2.4**	**1.2**	**3.0**	**2.3**	**7.0**	**0.8**	**1.7**
t(5)	22.5	**1.4**	**3.8**	22.9	19.3	28.1	**2.8**	**0.5**	**1.5**	**3.8**	**0.8**	**2.1**

5. Conclusion

We developed a range of simple EWMA refinements that build on the recent literature on score driven dynamics for time varying parameters in non-normal models. We showed that the standard EWMA and the robust Laplace based EWMA can all be seen as special cases of the new score driven EWMA (SD-EWMA) approach. In particular, as financial return series may typically be fat-tailed rather than heavy-tailed (such as Laplace), we developed a score driven EWMA scheme based on the Student's t distribution. As the score driven approach is not limited to time variation in volatilities only, we also developed a new SD-EWMA scheme for the simultaneous time series dynamics of the volatility and the degrees of freedom parameters in a Student's t distribution. The new schemes exhibit interesting robustness features for the time varying parameter dynamics that make them particularly suited in a context with fat-tailed observations.

We applied the new methods to forecast Value-at-Risk (VaR) for exchange rate and stock return data. We found that the robust Student's t based SD-EWMA model with time varying volatility and degrees of freedom parameter had the best overall performance for different series and different VaR confidence levels. If we accounted for both the event and the magnitude of a tail observation using the test of Berkowitz [2001], the dynamic Student's t based SD-EWMA schemes performed particularly well for most series. Next in line in terms of performance were the SD-EWMA schemes based on the heavy-tailed Laplace rather than fat-tailed Student's t distribution. We showed that these SD-EWMA schemes based on the (skewed) Laplace are very close to the robust EWMA schemes proposed by Guermat and Harris [2002] and Gerlach et al. [2013]. The new score driven SD-EWMA approach thus provides a unified and flexible tool for risk forecasting.

The score driven EWMA approach can easily be adapted to accommodate the researcher's preferred choice of forecasting distribution. For example, the ideas could be generalized further to semi-parametric approaches, such as the Gram-Charlier expansion of Gabrielsen et al. [2012]. Also note that the score driven EWMA can be adapted to han-

dle multivariate observations; see for example Creal et al. [2011] and Lucas et al. [2014]. Both of these possible extensions open up an interesting avenue for further research.

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A. Time varying ν_t derivations

Define $\gamma(x) = \ln\Gamma(x)$, with first and second order derivatives $\gamma'(x)$ and $\gamma''(x)$, respectively. Given the density of the Student's t distribtution with variance σ^2,

$$\ell_t(\sigma^2,\nu) = \gamma\left(\frac{\nu+1}{2}\right) - \gamma\left(\frac{\nu}{2}\right) - \frac{1}{2}\ln\left((\nu-2)\pi\sigma^2\right) - \frac{1}{2}(\nu+1)\ln\left(1+\frac{y_t^2}{(\nu-2)\sigma^2}\right), \tag{A1}$$

we obtain

$$\frac{\partial \ell_t(\sigma^2,\nu)}{\partial \nu} = \frac{1}{2}\gamma'\left(\frac{\nu+1}{2}\right) - \frac{1}{2}\gamma'\left(\frac{\nu}{2}\right) - \frac{1}{2}\frac{1}{\nu-2} - \frac{1}{2}\ln\left(1+\frac{y_t^2}{(\nu-2)\sigma^2}\right) + \frac{1}{2}(\nu+1)\frac{\frac{y_t^2}{(\nu-2)^2\sigma^2}}{1+\frac{y_t^2}{(\nu-2)\sigma^2}}, \tag{A2}$$

with $\mathrm{E}[\partial \ell_t(\sigma^2,\nu)/\partial\nu] = 0$. Taking further derivatives, we obtain

$$\begin{aligned}
\frac{\partial^2 \ell_t(\sigma^2,\nu)}{(\partial\nu)^2} &= \frac{1}{4}\gamma''\left(\frac{\nu+1}{2}\right) - \frac{1}{4}\gamma''\left(\frac{\nu}{2}\right) + \frac{1}{2}\frac{1}{(\nu-2)^2} + \frac{\frac{y_t^2}{(\nu-2)^2\sigma^2}}{1+\frac{y_t^2}{(\nu-2)\sigma^2}} \\
&\quad + \frac{1}{2}(\nu+1)\frac{-2\left(1+\frac{y_t^2}{(\nu-2)\sigma^2}\right)\frac{y_t^2}{(\nu-2)^3\sigma^2} + \frac{y_t^2}{(\nu-2)^2\sigma^2}\left(\frac{y_t^2}{(\nu-2)^2\sigma^2}\right)}{\left(1+\frac{y_t^2}{(\nu-2)\sigma^2}\right)^2} \\
&= \frac{1}{4}\gamma''\left(\frac{\nu+1}{2}\right) - \frac{1}{4}\gamma''\left(\frac{\nu}{2}\right) + \frac{1}{2}\frac{1}{(\nu-2)^2} + \frac{\frac{y_t^2}{(\nu-2)^2\sigma^2}}{1+\frac{y_t^2}{(\nu-2)\sigma^2}} \\
&\quad - \frac{1}{2}(\nu+1)\frac{2\frac{y_t^2}{(\nu-2)^3\sigma^2} + \frac{y_t^4}{(\nu-2)^4\sigma^4}}{\left(1+\frac{y_t^2}{(\nu-2)\sigma^2}\right)^2},
\end{aligned}$$

Using the transformation of variables $\nu^{1/2}(\nu-2)^{-1/2}y/\sigma \to y$, we have that for some $a, b > 0$

$$\begin{aligned}
q(a,b,\nu) &= \mathrm{E}\left[\frac{\left(\frac{y_t^2}{(\nu-2)\sigma^2}\right)^a}{\left(1+\frac{y_t^2}{(\nu-2)\sigma^2}\right)^b}\right] \\
&= \int \frac{\left(\frac{y^2}{(\nu-2)\sigma^2}\right)^a}{\left(1+\frac{y^2}{(\nu-2)\sigma^2}\right)^b} \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{(\nu-2)\sigma^2\pi}} \frac{1}{\left(1+\frac{y^2}{(\nu-2)\sigma^2}\right)^{(\nu+1)/2}} \mathrm{d}y \\
&= \int \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{(\nu-2)\sigma^2\pi}} \frac{\left(\frac{y^2}{(\nu-2)\sigma^2}\right)^a}{\left(1+\frac{y^2}{(\nu-2)\sigma^2}\right)^{(\nu+2b+1)/2}} \mathrm{d}y \\
&= \int \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\nu\pi}} \frac{(y^2/\nu)^a}{\left(1+\frac{y^2}{\nu}\right)^{(\nu+2b+1)/2}} \mathrm{d}y \\
&= \frac{\Gamma\left(\frac{\nu+1}{2}\right)\Gamma\left(\frac{\nu+2b}{2}\right)(\nu+2b)^a}{\Gamma\left(\frac{\nu+2b+1}{2}\right)\Gamma\left(\frac{\nu}{2}\right)\nu^a} \mathrm{E}[z_1^{2a} z_2^{-a}],
\end{aligned}$$

with $z_1 \sim \mathrm{N}(0, \nu/(\nu+2b))$, $z_2 \sim \chi^2(\nu+2b)$, $z_3 = (\nu+2b)^{1/2} z_1/\nu^{1/2} \sim \mathrm{N}(0,1)$, and z_1 and z_2 independent. Note that if $z_4 \sim \chi^2(\nu)$, then

$$\mathrm{E}[z_4^a] = \int \frac{z^a}{\Gamma\left(\frac{\nu}{2}\right)2^{\nu/2}} z^{(\nu/2)-1} e^{-z/2} \mathrm{d}z = \frac{\Gamma\left(\frac{\nu}{2}+a\right)2^a}{\Gamma\left(\frac{\nu}{2}\right)},$$

such that

$$\begin{aligned}
q(a,b,\nu) &= \frac{\Gamma\left(\frac{\nu+1}{2}\right)\Gamma\left(\frac{\nu+2b}{2}\right)}{\Gamma\left(\frac{\nu+2b+1}{2}\right)\Gamma\left(\frac{\nu}{2}\right)}\mathrm{E}[z_3^{2a}z_2^{-a}] \\
&= \frac{\Gamma\left(\frac{\nu+1}{2}\right)\Gamma\left(\frac{\nu+2b}{2}\right)\Gamma\left(\frac{1}{2}+a\right)\Gamma\left(\frac{\nu+2b}{2}-a\right)}{\Gamma\left(\frac{\nu+2b+1}{2}\right)\Gamma\left(\frac{\nu}{2}\right)\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{\nu+2b}{2}\right)} \\
&= \frac{\Gamma\left(\frac{\nu+1}{2}\right)\Gamma\left(\frac{1}{2}+a\right)\Gamma\left(\frac{\nu+2b}{2}-a\right)}{\Gamma\left(\frac{\nu+2b+1}{2}\right)\Gamma\left(\frac{\nu}{2}\right)\Gamma\left(\frac{1}{2}\right)} = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu+1}{2}+b\right)}\,\frac{\Gamma\left(\frac{\nu}{2}+b-a\right)}{\Gamma\left(\frac{\nu}{2}\right)}\,\frac{\Gamma\left(\frac{1}{2}+a\right)}{\Gamma\left(\frac{1}{2}\right)}.
\end{aligned}$$

We use the expression for $q(a,b,\nu)$ to rewrite

$$\begin{aligned}
\mathrm{E}\left[\frac{\partial^2\ell_t(\sigma^2,\nu)}{(\partial\nu)^2}\right] &= \frac{1}{4}\gamma''\left(\frac{\nu+1}{2}\right)-\frac{1}{4}\gamma''\left(\frac{\nu}{2}\right)+\frac{1}{2}\frac{1}{(\nu-2)^2}+\frac{q(1,1,\nu)}{\nu-2} \\
&\qquad -\frac{1}{2}(\nu+1)\left(\frac{2\,q(1,2,\nu)+q(2,2,\nu)}{(\nu-2)^2}\right) \\
&= \frac{1}{4}\gamma''\left(\frac{\nu+1}{2}\right)-\frac{1}{4}\gamma''\left(\frac{\nu}{2}\right)+\frac{1}{2}\frac{1}{(\nu-2)^2}+\frac{1}{(\nu+1)(\nu-2)} \\
&\qquad -\frac{1}{2}(\nu+1)\left(\frac{\frac{2\nu}{(\nu+1)(\nu+3)}+\frac{3}{(\nu+1)(\nu+3)}}{(\nu-2)^2}\right) \\
&= \frac{1}{4}\gamma''\left(\frac{\nu+1}{2}\right)-\frac{1}{4}\gamma''\left(\frac{\nu}{2}\right)+\frac{1}{2}\frac{1}{(\nu-2)^2(\nu+1)(\nu+3)}\times \\
&\qquad ((\nu+1)(\nu+3)+2(\nu+3)(\nu-2)-(\nu+1)(2\nu+3)) \\
&= \frac{1}{4}\gamma''\left(\frac{\nu+1}{2}\right)-\frac{1}{4}\gamma''\left(\frac{\nu}{2}\right)+\frac{1}{2}\frac{1}{(\nu-2)^2(\nu+1)(\nu+3)}\times \\
&\qquad \left(\nu^2+4\nu+3+2\nu^2+2\nu-12-2\nu^2-5\nu-3\right) \\
&= \frac{1}{4}\gamma''\left(\frac{\nu+1}{2}\right)-\frac{1}{4}\gamma''\left(\frac{\nu}{2}\right)+\frac{1}{2}\frac{\nu^2+\nu-12}{(\nu-2)^2(\nu+1)(\nu+3)} \\
&= \frac{1}{4}\gamma''\left(\frac{\nu+1}{2}\right)-\frac{1}{4}\gamma''\left(\frac{\nu}{2}\right)+\frac{1}{2}\frac{(\nu+4)(\nu-3)}{(\nu-2)^2(\nu+1)(\nu+3)}
\end{aligned}$$

Note that if we use the parameterization $\nu(f)$ with first and second derivatives $\dot{\nu} =$

$\dot{\nu}(f) = \partial\nu(f)/\partial f$ and $\ddot{\nu} = \partial\dot{\nu}(f)/\partial f$, respectively, we have

$$\begin{aligned}
\mathrm{E}\left[\frac{\partial\ \ell_t(\sigma^2,\nu)}{\partial f}\right] &= \dot{\nu}\,\mathrm{E}\left[\frac{\partial\ell_t(\sigma^2,\nu)}{\partial\nu}\right] = 0, \\
\mathrm{E}\left[\frac{\partial^2\ell_t(\sigma^2,\nu)}{(\partial f)^2}\right] &= \ddot{\nu}\,\mathrm{E}\left[\frac{\partial\ell_t(\sigma^2,\nu)}{\partial\nu}\right] + (\dot{\nu})^2\,\mathrm{E}\left[\frac{\partial^2\ell_t(\sigma^2,\nu)}{(\partial\nu)^2}\right] = (\dot{\nu})^2\,\mathrm{E}\left[\frac{\partial^2\ell_t(\sigma^2,\nu)}{(\partial\nu)^2}\right].
\end{aligned}$$

With inverse Fisher information scaling and thus using minus the expected hessian, we obtain the steps

$$-(\dot{\nu})^{-1}\ \left(\mathrm{E}\left[\frac{\partial^2\ell_t(\sigma^2,\nu)}{(\partial\nu)^2}\right]\right)^{-1}\ \frac{\partial\ell_t(\sigma^2,\nu)}{\partial\nu}.$$

Online Appendix: Additional results

Distribution	Updating	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
		Conditional Coverage test, $a = 0.5\%$											
N	No	15.2	21.4	11.0	**5.4**	11.0	**3.1**	9.6	17.1	**8.2**	17.5	11.5	14.0
L(0.5)	No	**4.9**	**4.4**	**0.9**	**4.9**	**3.1**	**7.4**	**3.1**	**0.9**	**0.1**	**4.5**	**1.8**	**1.8**
L(p_t)	No	**4.9**	**4.4**	**0.9**	**4.9**	**3.1**	11.0	**1.8**	**0.1**	**0.1**	**4.9**	**1.8**	**1.8**
L(p)	No	**7.4**	**4.4**	**0.4**	11.0	**3.1**	**7.4**	**3.1**	**0.4**	**0.1**	**4.5**	**3.1**	**3.1**
t(ν_t)	No	**3.1**	15.1	**0.1**	**0.3**	**0.3**	**0.4**	**0.4**	**5.4**	**1.4**	**8.4**	**0.1**	**0.1**
t(ν)	No	**5.4**	22.6	**0.8**	**5.4**	**2.1**	**0.3**	**0.1**	**8.2**	9.8	9.6	**0.9**	**1.4**
t(5)	No	**3.1**	**4.5**	**0.1**	**3.1**	**1.8**	**7.4**	**0.9**	**0.8**	**0.1**	**8.4**	**0.9**	**1.8**
N	Yes	15.2	26.4	11.0	**5.4**	9.6	**3.1**	11.0	17.1	9.8	17.5	17.1	15.7
L(0.5)	Yes	**4.9**	**4.2**	**0.4**	**4.9**	**3.1**	**7.4**	**3.1**	**0.4**	**0.1**	**4.2**	**3.1**	**1.8**
L(p_t)	Yes	**7.4**	**4.2**	**0.4**	**4.9**	**3.1**	11.0	**1.8**	**0.1**	**0.1**	**4.5**	**3.1**	**1.8**
L(p)	Yes	**4.9**	**4.2**	**0.4**	**7.4**	**3.1**	**7.4**	**3.1**	**0.4**	**0.1**	**4.2**	**3.1**	**1.8**
t(ν_t)	Yes	**6.7**	16.4	**0.3**	**0.1**	**1.4**	**0.1**	**0.1**	**6.7**	**1.4**	**7.3**	**0.1**	**0.8**
t(ν)	Yes	**6.7**	16.4	**0.3**	**5.4**	**6.3**	**0.8**	**0.1**	**5.4**	**1.4**	**8.4**	**0.1**	**0.8**
t(5)	Yes	**3.1**	**4.2**	**0.1**	**1.8**	**0.9**	**7.4**	**0.9**	**1.4**	**0.1**	**7.3**	**0.9**	**0.9**

Distribution	Updating	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
		Unconditional Coverage test, $a = 0.5\%$											
N	No	14.6	20.7	9.3	**5.1**	9.3	**2.8**	7.8	16.5	7.8	16.5	11.0	12.7
L(0.5)	No	**4.9**	**0.0**	**0.9**	**4.9**	**3.1**	7.4	**3.1**	**0.9**	**0.0**	**1.2**	**1.8**	**1.8**
L(p_t)	No	**4.9**	**0.0**	**0.9**	**4.9**	**3.1**	11.0	**1.8**	**0.0**	**0.0**	**1.9**	**1.8**	**1.8**
L(p)	No	7.4	**0.0**	**0.3**	11.0	**3.1**	7.4	**3.1**	**0.3**	**0.0**	**1.2**	**3.1**	**3.1**
t(ν_t)	No	**2.8**	9.3	**0.0**	**0.2**	**0.2**	**0.3**	**0.3**	**5.1**	**1.2**	**6.4**	**0.0**	**0.0**
t(ν)	No	**5.1**	18.5	**0.6**	**5.1**	**1.9**	**0.2**	**0.0**	7.8	9.3	7.8	**0.9**	**1.2**
t(5)	No	**3.1**	**1.2**	**0.0**	**3.1**	**1.8**	7.4	**0.9**	**0.6**	**0.0**	**6.4**	**0.9**	**1.8**
N	Yes	14.6	22.8	9.3	**5.1**	7.8	**2.8**	9.3	16.5	9.3	16.5	16.5	14.6
L(0.5)	Yes	**4.9**	**0.2**	**0.3**	**4.9**	**3.1**	7.4	**3.1**	**0.3**	**0.0**	**0.6**	**3.1**	**1.8**
L(p_t)	Yes	7.4	**0.2**	**0.3**	**4.9**	**3.1**	11.0	**1.8**	**0.0**	**0.0**	**1.2**	**3.1**	**1.8**
L(p)	Yes	**4.9**	**0.2**	**0.3**	7.4	**3.1**	7.4	**3.1**	**0.3**	**0.0**	**0.6**	**3.1**	**1.8**
t(ν_t)	Yes	**6.4**	11.0	**0.2**	**0.0**	**1.2**	**0.0**	**0.0**	**6.4**	**1.2**	**5.1**	**0.0**	**0.6**
t(ν)	Yes	**6.4**	11.0	**0.2**	**5.1**	**3.9**	**0.6**	**0.0**	**5.1**	**1.2**	**6.4**	**0.0**	**0.6**
t(5)	Yes	**3.1**	**0.2**	**0.0**	**1.8**	**0.9**	7.4	**0.9**	**1.2**	**0.0**	**5.1**	**0.9**	**0.9**

Distribution	Updating	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
		Independence test, $a = 0.5\%$											
N	No	**0.6**	**0.8**	**1.6**	**0.3**	**1.6**	**0.2**	**1.8**	**0.6**	**0.4**	**1.0**	**0.5**	**1.3**
L(0.5)	No	**0.0**	**4.4**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.1**	**3.3**	**0.0**	**0.0**
L(p_t)	No	**0.0**	**4.4**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.1**	**0.1**	**3.0**	**0.0**	**0.0**
L(p)	No	**0.0**	**4.4**	**0.1**	**0.0**	**0.0**	**0.0**	**0.0**	**0.1**	**0.1**	**3.3**	**0.0**	**0.0**
t(ν_t)	No	**0.2**	**5.8**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.3**	**0.2**	**2.0**	**0.1**	**0.1**
t(ν)	No	**0.3**	**4.1**	**0.1**	**0.3**	**0.2**	**0.1**	**0.1**	**0.4**	**0.4**	**1.8**	**0.0**	**0.2**
t(5)	No	**0.0**	**3.3**	**0.1**	**0.0**	**0.0**	**0.0**	**0.0**	**0.1**	**0.1**	**2.0**	**0.0**	**0.0**
N	Yes	**0.6**	**3.5**	**1.6**	**0.3**	**1.8**	**0.2**	**1.6**	**0.6**	**0.4**	**1.0**	**0.6**	**1.1**
L(0.5)	Yes	**0.0**	**4.0**	**0.1**	**0.0**	**0.0**	**0.0**	**0.0**	**0.1**	**0.1**	**3.6**	**0.0**	**0.0**
L(p_t)	Yes	**0.0**	**4.0**	**0.1**	**0.0**	**0.0**	**0.0**	**0.0**	**0.1**	**0.1**	**3.3**	**0.0**	**0.0**
L(p)	Yes	**0.0**	**4.0**	**0.1**	**0.0**	**0.0**	**0.0**	**0.0**	**0.1**	**0.1**	**3.6**	**0.0**	**0.0**
t(ν_t)	Yes	**0.3**	**5.4**	**0.1**	**0.1**	**0.2**	**0.1**	**0.1**	**0.3**	**0.2**	**2.2**	**0.1**	**0.1**
t(ν)	Yes	**0.3**	**5.4**	**0.1**	**0.3**	**2.4**	**0.1**	**0.1**	**0.3**	**0.2**	**2.0**	**0.1**	**0.1**
t(5)	Yes	**0.0**	**4.0**	**0.1**	**0.0**	**0.0**	**0.0**	**0.0**	**0.2**	**0.1**	**2.2**	**0.0**	**0.0**

Distribution	Updating	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
		Dynamic Quantile test, $a = 0.5\%$											
N	No	26.1	100.3	32.2	**7.6**	31.6	**5.3**	25.8	112.3	82.5	46.0	25.2	36.9
L(0.5)	No	**3.8**	38.8	**0.8**	**3.9**	52.1	**5.1**	55.6	34.8	25.3	34.8	40.8	**1.8**
L(p_t)	No	**3.8**	38.7	**0.9**	**4.0**	52.1	**6.8**	42.9	25.6	25.3	33.3	40.7	**1.8**
L(p)	No	**5.2**	38.4	**0.4**	**6.9**	51.9	**5.1**	55.6	29.4	22.2	20.7	**2.5**	**3.2**
t(ν_t)	No	**3.8**	80.8	**0.2**	**1.2**	36.8	**0.4**	28.6	59.0	172.6	68.3	23.6	22.1
t(ν)	No	**8.4**	159.1	**1.9**	**7.6**	73.3	**1.2**	21.1	126.7	215.5	66.3	33.2	**16.5**
t(5)	No	**2.6**	85.8	**0.2**	**2.6**	81.9	**5.1**	34.2	78.3	25.3	68.3	33.2	**1.8**
N	Yes	26.3	116.1	31.8	**7.8**	33.4	**5.5**	36.5	113.4	120.9	45.0	34.3	38.3
L(0.5)	Yes	**3.7**	34.6	**0.4**	**3.9**	52.0	**5.1**	55.6	29.5	25.1	21.2	**2.5**	**1.8**
L(p_t)	Yes	**5.4**	34.5	**0.4**	**3.9**	52.0	**6.8**	42.9	26.8	25.1	34.4	**2.5**	**1.8**
L(p)	Yes	**4.0**	34.5	**0.4**	**5.2**	52.0	**5.1**	55.6	29.4	25.1	21.2	**2.5**	**1.8**
t(ν_t)	Yes	**8.5**	126.1	**0.6**	**0.3**	30.5	**0.6**	24.2	58.2	110.5	69.7	20.8	18.8
t(ν)	Yes	**10.1**	125.8	**1.1**	**7.8**	34.6	**2.4**	24.2	60.4	110.5	67.5	23.6	18.7
t(5)	Yes	**2.5**	34.7	**0.2**	**1.8**	33.5	**5.1**	34.2	72.1	22.1	69.7	33.1	**1.4**

Distribution	Updating	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
		Berkowitz test, $a = 0.5\%$											
N	No	10.7	96.5	70.8	**4.5**	17.6	9.4	74.7	82.8	113.4	331.1	144.4	74.8
L(0.5)	No	27.3	**0.5**	14.5	22.1	14.3	27.7	**2.9**	**5.5**	**3.5**	16.0	**6.2**	**3.7**
L(p_t)	No	25.1	**1.0**	13.4	20.9	14.2	25.3	**2.0**	**2.0**	**4.4**	15.8	**5.5**	**3.8**
L(p)	No	30.2	**1.0**	11.8	29.6	15.1	24.8	**3.0**	**3.8**	**1.9**	11.8	10.0	**5.1**
t(ν_t)	No	**2.8**	9.7	**2.2**	**0.6**	**2.3**	**1.8**	**5.5**	**1.5**	**4.6**	**6.7**	**3.8**	**0.3**
t(ν)	No	**3.1**	22.4	**3.0**	**3.7**	**3.0**	**1.3**	**6.8**	**5.3**	9.9	9.3	**3.2**	**1.1**
t(5)	No	20.4	**0.1**	**3.0**	17.8	10.2	21.4	**1.3**	**0.5**	**1.9**	**6.0**	**1.4**	**2.7**
N	Yes	11.6	106.1	70.4	**5.5**	18.8	**6.7**	72.5	84.4	104.9	335.3	133.7	71.9
L(0.5)	Yes	33.4	**0.2**	12.2	21.1	14.6	26.5	**2.9**	**4.0**	**3.4**	17.4	**7.4**	**3.7**
L(p_t)	Yes	26.9	**0.3**	11.1	20.1	14.4	25.2	**2.0**	**2.0**	**4.1**	17.2	**6.6**	**3.8**
L(p)	Yes	31.3	**0.3**	11.8	22.7	14.8	26.2	**2.9**	**3.9**	**3.3**	15.6	**6.6**	**3.7**
t(ν_t)	Yes	**4.7**	11.0	**1.6**	**1.4**	**1.5**	**1.0**	**4.6**	**2.0**	**5.0**	**6.2**	**3.1**	**0.9**
t(ν)	Yes	**4.5**	17.4	**2.2**	**2.5**	**2.5**	**1.4**	**6.4**	**3.9**	**8.3**	**8.7**	**1.5**	**1.7**
t(5)	Yes	21.1	**0.2**	**3.0**	17.5	9.8	21.0	**1.3**	**0.9**	**1.1**	**5.8**	**1.1**	**2.0**

Distribution	Updating	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
		Conditional Coverage test, $a = 1\%$											
N	No	**7.7**	29.6	**3.3**	**7.2**	13.5	**7.2**	9.5	14.4	**8.2**	**8.2**	11.4	11.7
L(0.5)	No	**6.3**	**1.8**	**2.8**	**8.0**	**6.3**	9.9	**6.3**	**0.4**	**0.9**	**1.8**	**2.0**	**4.9**
L(p_t)	No	9.9	**1.8**	**3.7**	**8.0**	**6.3**	12.2	**6.3**	**0.4**	**0.4**	**5.8**	**1.3**	**3.7**
L(p)	No	12.2	**3.3**	**3.7**	12.2	**8.0**	14.9	9.9	**0.4**	**1.3**	**2.3**	**2.0**	**3.7**
t(ν_t)	No	9.5	13.1	**0.9**	**2.8**	**2.4**	**1.3**	**2.0**	**2.4**	**3.1**	**6.5**	**1.8**	**2.4**
t(ν)	No	11.3	21.2	**2.4**	9.3	9.5	**3.1**	**1.8**	**5.6**	**7.2**	**6.5**	**1.3**	**2.8**
t(5)	No	**8.0**	**6.5**	**1.8**	**2.0**	**2.0**	**6.3**	**6.2**	**0.6**	**1.3**	**6.5**	**0.9**	**0.6**
N	Yes	9.3	29.6	**3.3**	9.3	17.5	9.3	**8.7**	13.0	10.4	11.7	**8.9**	10.4
L(0.5)	Yes	**8.0**	**2.3**	**3.7**	12.2	**8.0**	12.2	**6.3**	**0.4**	**1.3**	**1.8**	**1.3**	**2.8**
L(p_t)	Yes	9.9	**2.3**	**3.7**	9.9	**8.0**	12.2	**6.3**	**0.4**	**1.3**	**1.8**	**0.6**	**3.7**
L(p)	Yes	**8.0**	**2.7**	**3.7**	12.2	**8.0**	12.2	**6.3**	**0.6**	**2.0**	**2.0**	**2.8**	**3.7**
t(ν_t)	Yes	**5.4**	16.5	**2.1**	**2.4**	9.5	**3.1**	**1.8**	**3.8**	**4.7**	**7.4**	**2.4**	**3.9**
t(ν)	Yes	**6.2**	18.6	**2.4**	9.3	**8.0**	**4.6**	**1.8**	**3.8**	**4.7**	**7.4**	**1.3**	**3.9**
t(5)	Yes	**2.8**	**6.1**	**0.4**	**2.8**	**5.1**	**4.9**	**5.1**	**0.6**	**0.6**	**7.4**	**0.9**	**0.9**

Distribution	Updating	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
		Unconditional Coverage test, $a = 1\%$											
N	No	6.7	23.9	**2.4**	6.7	11.5	6.7	6.7	14.3	7.8	7.8	10.2	11.5
L(0.5)	No	**6.2**	**0.2**	**2.6**	7.9	**6.2**	9.9	**6.2**	**0.1**	**0.7**	**0.2**	**1.8**	**4.8**
L(p_t)	No	9.9	**0.0**	**3.6**	7.9	**6.2**	12.2	**6.2**	**0.1**	**0.1**	**0.4**	**1.1**	**3.6**
L(p)	No	12.2	**0.7**	**3.6**	12.2	7.9	14.9	9.9	**0.1**	**1.1**	**0.1**	**1.8**	**3.6**
t(ν_t)	No	6.7	6.7	**0.4**	**1.8**	**1.8**	**0.8**	**0.0**	**1.8**	**2.4**	**2.4**	**1.2**	**1.2**
t(ν)	No	9.0	12.8	**1.2**	9.0	6.7	**2.4**	**0.0**	**4.8**	6.7	**2.4**	**0.8**	**1.8**
t(5)	No	**0.3**	**2.4**	**0.2**	**1.8**	**1.8**	**6.2**	**2.6**	**0.2**	**0.8**	**2.4**	**0.4**	**0.3**
N	Yes	9.0	23.9	**2.4**	9.0	12.8	9.0	**5.7**	12.8	10.2	11.5	7.8	10.2
L(0.5)	Yes	7.9	**0.1**	**3.6**	12.2	7.9	12.2	**6.2**	**0.1**	**1.1**	**0.0**	**1.1**	**2.6**
L(p_t)	Yes	9.9	**0.1**	**3.6**	9.9	7.9	12.2	**6.2**	**0.1**	**1.1**	**0.2**	**0.3**	**3.6**
L(p)	Yes	7.9	**0.3**	**3.6**	12.2	7.9	12.2	**6.2**	**0.3**	**1.8**	**0.0**	**2.6**	**3.6**
t(ν_t)	Yes	**4.8**	11.5	**0.8**	**1.2**	6.7	**2.4**	**0.0**	**3.1**	**3.9**	**3.9**	**1.8**	**3.1**
t(ν)	Yes	**5.7**	14.3	**1.2**	9.0	**4.8**	**3.9**	**0.0**	**3.1**	**3.9**	**3.9**	**0.8**	**3.1**
t(5)	Yes	**2.6**	**1.8**	**0.1**	**2.6**	**1.8**	**4.8**	**1.8**	**0.2**	**0.2**	**3.9**	**0.4**	**0.7**

Distribution	Updating	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
		Conditional Independence test, $a = 1\%$											
N	No	**1.0**	**5.7**	**0.9**	**0.4**	**2.0**	**0.4**	**2.8**	**0.1**	**0.4**	**0.4**	**1.2**	**0.2**
L(0.5)	No	**0.1**	**1.6**	**0.1**	**0.1**	**0.1**	**0.0**	**0.1**	**0.3**	**0.2**	**1.6**	**0.2**	**0.1**
L(p_t)	No	**0.0**	**1.8**	**0.1**	**0.1**	**0.1**	**0.0**	**0.1**	**0.3**	**0.3**	**5.4**	**0.2**	**0.1**
L(p)	No	**0.0**	**2.7**	**0.1**	**0.0**	**0.1**	**0.0**	**0.0**	**0.3**	**0.2**	**2.2**	**0.2**	**0.1**
t(ν_t)	No	**2.8**	**6.3**	**0.5**	**1.0**	**0.6**	**0.5**	**2.0**	**0.6**	**0.7**	**4.1**	**0.6**	**1.1**
t(ν)	No	**2.4**	8.3	**1.1**	**0.3**	**2.8**	**0.7**	**1.8**	**0.9**	**0.4**	**4.1**	**0.5**	**1.0**
t(5)	No	7.7	**4.1**	**1.6**	**0.2**	**0.2**	**0.1**	**3.6**	**0.4**	**0.5**	**4.1**	**0.5**	**0.3**
N	Yes	**0.3**	**5.7**	**0.9**	**0.3**	**4.7**	**0.3**	**3.0**	**0.1**	**0.2**	**0.2**	**1.1**	**0.2**
L(0.5)	Yes	**0.1**	**2.2**	**0.1**	**0.0**	**0.1**	**0.0**	**0.1**	**0.3**	**0.2**	**1.8**	**0.2**	**0.1**
L(p_t)	Yes	**0.0**	**2.2**	**0.1**	**0.0**	**0.1**	**0.0**	**0.1**	**0.3**	**0.2**	**1.6**	**0.3**	**0.1**
L(p)	Yes	**0.1**	**2.4**	**0.1**	**0.0**	**0.1**	**0.0**	**0.1**	**0.3**	**0.2**	**2.0**	**0.1**	**0.1**
t(ν_t)	Yes	**0.6**	**5.0**	**1.3**	**1.1**	**2.8**	**0.7**	**1.8**	**0.7**	**0.8**	**3.5**	**0.6**	**0.8**
t(ν)	Yes	**0.5**	**4.4**	**1.1**	**0.3**	**3.2**	**0.7**	**1.8**	**0.7**	**0.8**	**3.5**	**0.5**	**0.8**
t(5)	Yes	**0.1**	**4.4**	**0.3**	**0.1**	**3.3**	**0.1**	**3.3**	**0.4**	**0.4**	**3.5**	**0.5**	**0.2**

Distribution	Updating	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
		Dynamic Quantile test, $a = 1\%$											
N	No	**10.9**	99.5	23.5	**13.4**	61.0	**13.5**	31.6	46.3	74.1	39.9	21.1	18.0
L(0.5)	No	**5.2**	47.1	**3.0**	**6.1**	28.9	**7.4**	17.1	66.3	74.6	27.0	**9.2**	**4.0**
L(p_t)	No	**7.4**	50.1	**3.4**	**6.3**	28.9	**9.1**	17.2	66.1	101.2	37.8	**8.0**	**3.9**
L(p)	No	**8.8**	33.1	**3.6**	**8.8**	19.9	**10.4**	24.1	66.2	52.1	**12.7**	**9.1**	**3.8**
t(ν_t)	No	39.7	74.1	**6.5**	**7.2**	52.5	**11.6**	**8.5**	47.6	84.2	32.6	**7.3**	50.2
t(ν)	No	40.4	110.7	**10.2**	66.8	61.3	**10.5**	**7.8**	45.7	77.4	32.6	**7.3**	47.6
t(5)	No	47.9	68.9	**11.0**	**9.0**	40.7	**5.1**	17.6	55.8	95.8	32.6	**7.4**	42.6
N	Yes	**13.3**	125.4	22.2	67.0	86.7	16.8	32.3	47.0	86.7	42.1	17.2	18.3
L(0.5)	Yes	**6.2**	27.9	**3.6**	**8.9**	20.0	**9.1**	17.1	66.3	52.0	28.3	**7.9**	**2.6**
L(p_t)	Yes	**7.5**	27.8	**3.6**	**7.5**	20.0	**9.1**	17.2	66.1	52.0	27.6	**6.3**	**3.9**
L(p)	Yes	**6.2**	30.4	**3.6**	**8.9**	20.0	**9.1**	17.1	29.0	56.5	30.0	**10.6**	**3.8**
t(ν_t)	Yes	**9.0**	99.3	**6.6**	**7.0**	61.3	**10.7**	**7.8**	46.9	77.8	32.2	**7.4**	28.2
t(ν)	Yes	18.4	95.2	**9.3**	67.0	63.9	**11.7**	**7.8**	46.7	77.8	32.1	**6.0**	28.2
t(5)	Yes	**2.5**	72.2	**2.5**	**2.4**	21.5	**4.3**	**15.4**	55.8	54.7	32.2	**5.7**	30.2

Distribution	Updating	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
		Berkowitz test, $a = 1\%$											
N	No	12.0	97.1	71.1	**7.5**	18.1	9.6	73.9	80.6	107.0	330.7	138.5	72.7
L(0.5)	No	34.1	**3.7**	14.0	32.9	24.3	33.3	18.0	**4.7**	9.4	16.5	10.2	11.2
L(p_t)	No	37.4	**2.0**	17.2	30.8	24.0	33.9	14.1	**3.3**	**5.1**	17.5	**8.3**	11.4
L(p)	No	37.3	**6.6**	18.3	35.2	28.9	34.1	23.0	**4.7**	10.3	17.5	**8.7**	**7.5**
t(ν_t)	No	**2.8**	**9.1**	**0.2**	**0.1**	**0.3**	**2.1**	**1.6**	**1.3**	**4.4**	**6.8**	**2.1**	**0.2**
t(ν)	No	**8.0**	22.2	**1.4**	**5.6**	**3.2**	**1.7**	**4.0**	**4.1**	9.8	9.7	**0.9**	**1.0**
t(5)	No	24.2	**0.2**	**1.5**	21.0	15.1	21.2	**6.9**	**0.3**	**0.5**	**6.3**	**0.0**	**2.4**
N	Yes	13.3	103.0	69.8	**7.1**	19.4	9.6	74.7	82.0	96.7	330.0	136.5	69.1
L(0.5)	Yes	36.2	**6.3**	16.5	35.9	26.4	32.2	17.9	**4.7**	11.1	17.4	**8.3**	**8.6**
L(p_t)	Yes	37.9	**3.3**	17.6	31.5	26.1	33.1	14.1	**3.3**	**8.9**	18.6	**5.4**	11.4
L(p)	Yes	35.6	**7.2**	18.2	36.3	26.5	32.0	17.9	**4.7**	12.8	16.5	10.6	9.9
t(ν_t)	Yes	**1.0**	10.3	**0.0**	**0.2**	**1.9**	**2.3**	**1.5**	**1.6**	**3.4**	**5.6**	**2.3**	**0.9**
t(ν)	Yes	**5.7**	18.2	**0.8**	**5.1**	**2.8**	**2.3**	**3.0**	**3.6**	**6.9**	**8.5**	**0.8**	**1.8**
t(5)	Yes	23.5	**0.2**	**1.9**	21.7	15.3	20.3	**5.8**	**0.3**	**1.2**	**5.3**	**0.0**	**2.4**

Distribution	Updating	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
		Conditional Coverage test, $a = 5\%$											
N	No	**6.6**	**6.6**	**0.6**	**7.8**	**4.5**	**2.8**	**4.9**	**3.8**	**1.1**	**1.3**	**2.7**	**0.6**
L(0.5)	No	**0.6**	**2.6**	**2.6**	**2.7**	**2.1**	**6.7**	**7.4**	**1.3**	**2.0**	**2.8**	13.3	**4.6**
L(p_t)	No	**0.8**	**2.7**	**4.9**	**2.0**	**0.9**	**5.2**	11.8	**0.7**	**0.5**	**1.8**	9.3	**3.6**
L(p)	No	**0.4**	**1.4**	**6.0**	**1.4**	**1.4**	**5.2**	**5.0**	**0.7**	**0.4**	**1.8**	**2.9**	**2.5**
t(ν_t)	No	**7.1**	**6.6**	**4.0**	**4.5**	**4.2**	**3.1**	**6.5**	**5.8**	**3.8**	**0.8**	**5.1**	**0.5**
t(ν)	No	**7.6**	**8.1**	**4.0**	**4.1**	**4.5**	**2.9**	**8.5**	**6.2**	**2.7**	**0.8**	**5.0**	**0.5**
t(5)	No	**4.5**	**5.2**	**2.6**	**1.4**	**0.8**	**2.3**	**6.4**	**5.1**	**2.9**	**0.8**	**5.4**	**1.3**
N	Yes	**6.6**	**5.2**	**0.9**	**8.1**	**5.4**	**2.9**	**8.1**	**4.0**	**0.6**	**1.1**	**5.8**	**1.3**
L(0.5)	Yes	**0.1**	**1.3**	**1.9**	**2.7**	**0.7**	**3.7**	**4.7**	**1.3**	**0.4**	**1.5**	**5.5**	**4.1**
L(p_t)	Yes	**0.3**	**2.3**	**6.6**	**1.3**	**0.3**	**5.2**	**5.0**	**0.7**	**0.3**	**1.3**	**4.2**	**3.6**
L(p)	Yes	**0.1**	**1.3**	**6.0**	**2.1**	**1.2**	**3.7**	**6.7**	**1.3**	**0.5**	**2.8**	**5.2**	**3.6**
t(ν_t)	Yes	**7.6**	**7.6**	**1.0**	**8.7**	**5.4**	**3.4**	**8.3**	**5.8**	**4.4**	**0.9**	**5.0**	**1.4**
t(ν)	Yes	**7.6**	**7.6**	**1.0**	10.0	**5.0**	**3.1**	**8.5**	**5.8**	**4.9**	**0.9**	**5.1**	**1.5**
t(5)	Yes	**3.8**	**4.9**	**0.6**	**3.1**	**0.9**	**2.4**	**6.6**	**5.1**	**2.6**	**1.0**	**5.2**	**0.5**

Distribution	Updating	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
		Unconditional Coverage test, $a = 5\%$											
N	No	**6.4**	**5.9**	**0.5**	7.5	**4.5**	**1.6**	**0.9**	**1.9**	**0.3**	**0.2**	**0.0**	**0.0**
L(0.5)	No	**0.0**	**0.0**	**2.6**	**0.3**	**0.6**	**1.3**	**0.1**	**0.2**	**1.3**	**1.1**	**2.2**	**2.2**
L(p_t)	No	**0.0**	**0.0**	**4.8**	**0.1**	**0.1**	**2.6**	**0.6**	**0.0**	**0.5**	**0.5**	**0.6**	**1.6**
L(p)	No	**0.0**	**0.0**	**6.0**	**1.3**	**0.8**	**2.6**	**0.1**	**0.4**	**0.1**	**0.5**	**1.9**	**1.6**
t(ν_t)	No	6.9	**6.4**	**2.2**	**4.5**	**3.7**	**2.2**	**2.2**	**4.9**	**2.9**	**0.6**	**0.4**	**0.2**
t(ν)	No	7.5	8.1	**2.2**	**4.1**	**4.1**	**1.9**	**3.7**	**5.4**	**2.6**	**0.6**	**0.6**	**0.2**
t(5)	No	**4.1**	**4.9**	**1.4**	**0.7**	**0.6**	**0.6**	**1.9**	**4.1**	**2.9**	**0.6**	**0.2**	**0.2**
N	Yes	**6.4**	**4.9**	**0.8**	8.1	**5.4**	**1.9**	**1.6**	**2.2**	**0.4**	**0.1**	**0.0**	**0.3**
L(0.5)	Yes	**0.1**	**0.4**	**1.9**	**0.3**	**0.3**	**1.6**	**0.0**	**0.3**	**0.1**	**0.3**	**0.8**	**1.9**
L(p_t)	Yes	**0.1**	**0.3**	**6.6**	**0.2**	**0.1**	**2.6**	**0.1**	**0.0**	**0.3**	**0.2**	**1.1**	**1.6**
L(p)	Yes	**0.0**	**0.3**	**6.0**	**0.6**	**0.6**	**1.6**	**0.0**	**0.2**	**0.2**	**1.1**	**1.6**	**1.6**
t(ν_t)	Yes	7.5	7.5	**0.9**	8.6	**5.4**	**2.6**	**3.3**	**4.9**	**4.1**	**0.7**	**0.6**	**0.7**
t(ν)	Yes	7.5	7.5	**0.9**	9.9	**4.9**	**2.2**	**3.7**	**4.9**	**4.5**	**0.7**	**0.4**	**0.9**
t(5)	Yes	**3.7**	**4.5**	**0.6**	**2.9**	**0.7**	**0.9**	**2.6**	**4.1**	**1.9**	**0.9**	**0.3**	**0.1**

Distribution	Updating	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
		Independence test, $a = 5\%$											
N	No	**0.1**	**0.7**	**0.2**	**0.3**	**0.0**	**1.1**	**3.9**	**1.9**	**0.9**	**1.1**	**2.7**	**0.5**
L(0.5)	No	**0.6**	**2.5**	**0.0**	**2.4**	**1.5**	**5.4**	7.3	**1.1**	**0.7**	**1.7**	11.1	**2.4**
L(p_t)	No	**0.8**	**2.7**	**0.1**	**1.9**	**0.9**	**2.7**	11.2	**0.7**	**0.0**	**1.3**	8.7	**2.0**
L(p)	No	**0.4**	**1.3**	**0.1**	**0.0**	**0.6**	**2.7**	**5.0**	**0.3**	**0.3**	**1.3**	**1.0**	**0.9**
t(ν_t)	No	**0.1**	**0.1**	**1.7**	**0.0**	**0.5**	**0.9**	**4.3**	**0.9**	**0.9**	**0.2**	**4.7**	**0.4**
t(ν)	No	**0.1**	**0.1**	**1.7**	**0.1**	**0.4**	**1.0**	**4.8**	**0.8**	**0.2**	**0.2**	**4.4**	**0.4**
t(5)	No	**0.4**	**0.3**	**1.2**	**0.7**	**0.2**	**1.8**	**4.5**	**1.1**	**0.0**	**0.2**	**5.2**	**1.1**
N	Yes	**0.1**	**0.3**	**0.1**	**0.0**	**0.0**	**1.0**	**6.5**	**1.7**	**0.2**	**1.0**	**5.7**	**1.0**
L(0.5)	Yes	**0.0**	**0.9**	**0.0**	**2.4**	**0.4**	**2.1**	**4.7**	**1.0**	**0.3**	**1.2**	**4.7**	**2.2**
L(p_t)	Yes	**0.2**	**2.0**	**0.1**	**1.1**	**0.2**	**2.7**	**5.0**	**0.7**	**0.0**	**1.1**	**3.2**	**2.0**
L(p)	Yes	**0.1**	**1.0**	**0.1**	**1.5**	**0.5**	**2.1**	6.6	**1.1**	**0.2**	**1.7**	**3.6**	**2.0**
t(ν_t)	Yes	**0.1**	**0.1**	**0.1**	**0.0**	**0.0**	**0.8**	**5.1**	**0.9**	**0.4**	**0.2**	**4.4**	**0.7**
t(ν)	Yes	**0.1**	**0.1**	**0.1**	**0.1**	**0.0**	**0.9**	**4.8**	**0.9**	**0.4**	**0.2**	**4.7**	**0.6**
t(5)	Yes	**0.1**	**0.4**	**0.0**	**0.2**	**0.2**	**1.5**	**4.0**	**1.1**	**0.6**	**0.1**	**4.9**	**0.4**

Distribution	Updating	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
		Dynamic Quantile test, $a = 5\%$											
N	No	**8.1**	25.2	**11.6**	42.5	**9.0**	**7.6**	**16.5**	**8.5**	**15.0**	**8.9**	**5.8**	**9.5**
L(0.5)	No	**6.8**	32.6	**12.1**	44.4	**10.3**	19.1	23.0	**6.8**	26.2	**12.8**	23.6	18.0
L(p_t)	No	**4.7**	24.2	**16.7**	46.6	**9.8**	**16.4**	27.6	**10.2**	26.0	**11.0**	21.4	18.8
L(p)	No	**1.1**	**13.4**	17.4	37.4	**8.5**	**8.6**	**14.6**	**5.2**	**16.4**	**8.0**	**5.5**	**16.6**
t(ν_t)	No	21.8	32.3	**15.7**	50.0	**13.7**	**10.4**	19.7	**15.3**	29.0	18.0	**12.3**	**11.6**
t(ν)	No	19.1	31.4	**15.6**	47.8	**13.7**	**11.2**	26.8	**15.4**	17.5	18.0	**11.9**	**11.6**
t(5)	No	17.3	27.8	**15.0**	40.5	**6.8**	**11.2**	20.2	**12.2**	27.8	18.0	**13.3**	**15.0**
N	Yes	**8.1**	22.0	**10.5**	48.3	**12.2**	**9.6**	25.8	**9.0**	**13.4**	**10.2**	**11.6**	**11.9**
L(0.5)	Yes	**1.4**	18.6	**13.2**	40.6	**7.4**	**11.9**	17.5	**7.3**	**15.9**	**10.4**	**13.2**	20.0
L(p_t)	Yes	**2.4**	17.6	21.8	43.9	**6.9**	**11.4**	18.7	**10.3**	17.9	**12.1**	**9.3**	19.2
L(p)	Yes	**1.1**	19.1	17.9	37.3	**7.7**	**11.8**	17.2	**7.0**	17.0	**10.0**	**7.8**	18.9
t(ν_t)	Yes	**9.5**	25.7	**12.2**	43.6	**13.0**	**8.9**	24.5	**15.4**	19.8	**14.9**	**12.2**	**10.1**
t(ν)	Yes	**9.5**	27.0	**12.1**	46.3	**11.1**	**9.1**	26.4	**15.4**	19.6	**14.9**	**13.3**	**9.8**
t(5)	Yes	**5.6**	19.4	**12.6**	42.0	**5.1**	**10.6**	19.3	**15.5**	21.8	17.0	**13.7**	**10.8**

Distribution	Updating	GBP	AUD	JPY	CAD	SEK	EUR	AA	BA	GE	IBM	KO	T
		Berkowitz test, $a = 5\%$											
N	No	12.3	83.6	58.9	10.8	19.5	**8.4**	61.7	73.2	79.0	302.8	127.8	73.5
L(0.5)	No	40.0	**4.0**	13.4	40.8	35.9	37.3	13.7	9.9	16.4	15.5	14.8	14.3
L(p_t)	No	37.8	**2.2**	19.8	39.2	34.7	34.3	11.1	**8.1**	10.2	15.3	21.7	15.3
L(p)	No	40.0	**6.9**	17.1	45.9	36.5	36.6	13.5	9.9	11.7	11.5	13.5	11.5
t(ν_t)	No	**5.6**	**8.5**	**2.5**	**2.9**	**4.8**	**4.9**	**0.3**	**1.2**	**1.0**	**4.7**	**1.7**	**0.6**
t(ν)	No	**7.1**	20.1	**0.9**	**8.5**	**5.6**	**2.1**	**1.6**	**3.2**	**4.6**	**7.0**	**0.5**	**1.6**
t(5)	No	21.4	**1.5**	**5.0**	18.0	19.2	27.2	**2.7**	**0.7**	**2.0**	**4.0**	**1.0**	**2.4**
N	Yes	12.2	94.7	61.1	10.8	19.8	**7.9**	61.0	76.3	82.6	306.4	124.3	66.2
L(0.5)	Yes	43.5	**4.5**	12.9	40.6	36.4	35.7	13.1	9.4	13.3	12.5	12.1	13.6
L(p_t)	Yes	38.5	**2.1**	20.0	36.9	33.6	33.3	11.1	**7.8**	9.9	13.9	23.3	15.1
L(p)	Yes	42.0	**5.5**	17.5	42.4	37.2	36.3	12.5	9.8	13.4	14.4	17.3	13.8
t(ν_t)	Yes	**5.7**	11.1	**0.6**	**5.2**	**5.7**	**4.4**	**0.4**	**1.4**	**1.0**	**4.5**	**2.1**	**0.9**
t(ν)	Yes	**7.0**	18.7	**0.3**	**8.8**	**6.6**	**2.4**	**1.2**	**3.0**	**2.3**	**7.0**	**0.8**	**1.7**
t(5)	Yes	22.5	**1.4**	**3.8**	22.9	19.3	28.1	**2.8**	**0.5**	**1.5**	**3.8**	**0.8**	**2.1**