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**Working Paper**

## Carbon Lock-In: The Role of Expectations

Tinbergen Institute Discussion Paper, No. 14-100/VIII

**Provided in Cooperation with:**

Tinbergen Institute, Amsterdam and Rotterdam

*Suggested Citation:* van der Meijden, Gerard; Smulders, Sjak (2014) : Carbon Lock-In: The Role of Expectations, Tinbergen Institute Discussion Paper, No. 14-100/VIII, Tinbergen Institute, Amsterdam and Rotterdam

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TI 2014-100/VIII  
Tinbergen Institute Discussion Paper



# Carbon Lock-In: The Role of Expectations

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# Carbon Lock-In: The Role of Expectations\*

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August 1, 2014

## Abstract

We argue that expectations about future energy use affect the transition from fossil fuels to renewable substitutes, because of an interaction between innovation and resource scarcity. The paper presents a model of directed technical change to study this interaction. We find that resource-saving technical change erodes the incentives to implement the substitute. Conversely, the anticipation of the substitute being implemented in the future diminishes the incentives to invest in resource-saving technology. As a result, two dynamic equilibria may arise, one with a transition to the substitute and with low resource efficiency, and one without the substitute and with fast efficiency improvements. Expectations determine which equilibrium arises in the decentralized market equilibrium. If multiple equilibria exist, the transition to the substitute generates higher welfare.

**JEL codes:** O30, Q32, Q42, Q55

**Keywords:** Directed technical change, energy transition, multiple equilibria

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\*The authors would like to thank Christian Bogmans, Reyer Gerlagh, David Hemous, Jenny Ligthart, Rick van der Ploeg, Francesco Ricci, Luc Rouge, Cees Withagen, Aart de Zeeuw, Gijsbert Zwart, and participants of the Annual Conference of the European Association of Environmental and Resource Economists (Toulouse, June 2013), the INSEAD Research Seminar (Fontainebleau, December 2013), and the LSE Research Seminar (London, March 2014) for helpful comments. We gratefully acknowledge financial support from FP7-IDEAS-ERC Grant No. 269788.

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# 1 Introduction

For many decades a major engine of growth in the world economy has been the reliable supply of fossil energy resources. Policy makers have been forced, however, to rethink the dominant role of fossil fuels in energy supply, when facing the challenge of combating climate change and the global concern about the sustainability of current living standards. Part of the solution to both the climate change and the sustainability problem may be a phasing out of non-renewable natural resources like fossil fuels and the implementation of backstop technologies that provide renewable substitutes. A more incremental solution would arise from improving resource efficiency and slowing depletion of fossil resources. The question arises how market parties respond to the challenges and which incentives arise over time to invest in resource saving and energy transition. We argue that the energy future of a growing economy is crucially shaped by a two-way interaction between innovation decisions and energy supply decisions. Prospects about future energy generation technologies may affect the time path of fossil fuel consumption, but also the pace and direction of technical progress. Conversely, the speed and direction of technical progress are crucial for the transition from fossil fuels to backstop technologies.

Since our question concerns the dynamics of energy use and technology in a growing economy, we naturally frame our analysis in a growth model with natural resources and endogenous technical change. Our starting point is the Dasgupta-Heal-Solow-Stiglitz (DHSS) model<sup>1</sup> in which a scarce non-renewable resource (fossil) is an essential input in production. We allow fossil energy to be replaced by renewable energy that can be generated at a constant cost, the so-called backstop technology (cf. Nordhaus, 1973). As is well known, in the DHSS model growth cannot be sustained unless resource-augmenting technical change offsets the negative growth impact of declining availability of the non-renewable resources. At the same time, labor-augmenting technical change fuels growth and boosts the demand for energy. Energy demand thus results from the balance between two types of innovation, resource-augmenting and labour-augmenting technical change. We incorporate both types in our analysis and allow profit incentives to guide innovators how much and in which direction to innovate. Thus, we merge the DHSS model with a model of directed technical change.<sup>2</sup>

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<sup>1</sup>See Dasgupta and Heal (1974), Solow (1974a,b), Stiglitz (1974a,b), Benckekroun and Withagen (2011), and Van der Ploeg and Withagen (2014).

<sup>2</sup>The literature on induced innovations was introduced by Hicks (1932) and more recently formalized in the

Our main finding is that the replacement of fossil resources might require a coordination of expectations. If the costs of generating energy with the backstop technology are sufficiently low, it is a viable alternative to fossil fuels in the long run. However, investment in resource-saving technical change can make fossil effectively cheaper to use than the backstop. Whether in equilibrium fossil is phased out or not then depends on the expectations of fossil suppliers and innovators. A self-fulfilling prophecy arises since when it is expected that the backstop will be implemented, the market for resource-saving inventions will be small and innovation incentives will be eroded; this makes the backstop relatively more attractive and thus justifies the expectation that the backstop will be implemented. Conversely, when no future backstop deployment is expected, resource-saving technical change becomes more profitable, thus making the resource indeed relatively more attractive in the long run. Only when the backstop cost is below a certain threshold, it will always be deployed in the long run.

We also find that different energy transition patterns can emerge that have markedly different impacts on the economy. First, without a transition to the backstop, fossil use typically peaks, i.e. resource use declines over time in later stages of the growth process. However, with a transition to the backstop, resource use is typically rising for a long period. Second, the pattern of innovation differs as well. Along the transitional path leading to the implementation of the backstop technology, resource-augmenting technical change stops well before the backstop is introduced. This is in contrast with the equilibrium without backstop, in which long-run growth goes together with resource-augmenting technical change.

Our results imply that it might be hard to steer the economy away from the current dependence on fossil fuels because the economy is “locked into fossil” (cf. Unruh, 2000). Lock-in is studied in the literature in several settings.<sup>3</sup> In the context of energy use, Acemoglu, Aghion, Bursztyn, and Hemous (2012) study lock-in that arises from initial conditions or “history”, viz. innovation in pollution/energy-intensive sectors in the past. Our analysis is complementary to theirs in that we focus on lock-in that arises from expectations rather than history.<sup>4</sup> Moreover, we adopt a different view of technical change in which society has to choose between incremental change that cannot make scarce resource inputs redundant (because of

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directed technical change models of Acemoglu (1998; 2002; 2003) and Kiley (1999). We choose for investment in knowledge instead of in physical capital to orient our analysis towards the long run, when technical change rather than capital accumulation is the determinant of output growth.

<sup>3</sup>Arthur (1989) and David (1985) introduced the notion of lock-in into economics.

<sup>4</sup>Krugman (1991) formalized the distinction between history and expectations as driving force of lock-in.

poor substitution) and radical change in the form of the transition to the backstop. Also in the context of energy use, Cheikbossian and Ricci (2013) consider a game between a resource owner and an R&D firm and show that depending on expectations one out of two equilibria is selected, one with high R&D and slow depletion, and one with low R&D and high depletion. Their two-period framework cannot explicitly address the link to economic growth and ignores the possibility of a radical technology change in the form of a backstop, which is the focus of our study. In a growth context, existing studies of self-fulfilling expectations and technology choice are restricted to a one-factor setting and thus abstract from directed technical change (e.g., Chen and Shimomura, 1998; Cozzi, 2007).

Directed technical change has been studied in the context of energy scarcity in several studies, with Smulders and de Nooij (2003) as an early example. A key question in this literature concerns the role of resource-augmenting technical change relative to other types of technical change. With resource inputs growing at a lower rate than other inputs, resource-augmenting technical change dominates along a balanced growth path, provided that substitution possibilities are poor, as shown in e.g. André and Smulders (2014). With good substitution, however, resources are not essential for growth and growth can be sustained without technical change in the resource sector, as in Acemoglu, Aghion, Bursztyn, and Hemous (2012). In the model of Di Maria and Valente (2008), in which a non-renewable resource and physical capital are both essential for production, there may be capital-augmenting technical progress in the short run, but technical change will be purely resource-augmenting along any balanced growth path. Pittel and Bretschger (2010) find that technical change is biased towards the resource-intensive sector at the balanced growth equilibrium of their model economy in which sectors are heterogeneous with respect to the intensity of natural resource use. We complement these studies by allowing for a regime shift in energy usage after which the value of accumulated knowledge in the resource sector vanishes.

The remainder of the paper is structured as follows. Section 2 describes the model. Section 3 discusses the dynamics of the model and the energy transition. Section 4 provides a numerical analysis to quantify the results. Section 5 performs a welfare analysis and compares the decentralized equilibrium with the social optimum. Finally, Section 6 concludes.

## 2 The Model

The model describes a closed economy in which final output is produced with labor and energy services according to a constant elasticity of substitution (CES) production function. In line with the empirical evidence in Koetse, de Groot, and Florax (2008) and Van der Werf (2008), energy and man-made factors of production are poor substitutes, i.e. the elasticity of substitution between them is smaller than unity. Labor services are produced with labor and a set of specific intermediate goods. Energy services are either derived from the non-renewable resource combined with another set of intermediate goods, or generated by the backstop technology. The economy is endowed with a finite stock of the non-renewable resource, which can be extracted without costs. The production of intermediate goods and energy generation with the backstop technology both use final output. Technical progress is driven by labor allocated to R&D, which is undertaken by the firms in the two intermediate goods sectors to improve the quality of their products, as in Acemoglu (1998). As a result, there are two types of technical change in the model: labor-augmenting and resource-augmenting technical change. Although the model has three state variables, we can analyze the dynamics and regime shifts by using phase diagrams. Infinitely lived households own the resource stock and the firms and they derive utility from consumption. The remainder of this section describes the different production sectors, energy generation, the process of research and development, and the household sector in more detail. Mathematical derivations can be found in the appendix.

### 2.1 Production

#### 2.1.1 Final Output

Final output  $Y$  is produced using labor services  $Y_L$  and energy services  $Y_E$  according to the following constant elasticity of substitution (CES) specification:<sup>5</sup>

$$Y = \left[ \gamma Y_L^{\frac{\sigma-1}{\sigma}} + (1-\gamma) Y_E^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (1)$$

where  $\gamma$  regulates the relative productivity of the inputs and  $\sigma \in (0, 1)$  denotes the elasticity of substitution between labor and energy services. Profit maximization under perfect

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<sup>5</sup>The time argument  $t$  is omitted if there is no possibility of confusion.



competition gives rise to the following relative factor demand function:

$$\frac{\gamma}{1-\gamma} \left( \frac{Y_L}{Y_E} \right)^{-\frac{1}{\sigma}} = \frac{p_{YL}}{p_{YE}}, \quad (2)$$

where  $p_{YL}$  and  $p_{YE}$  are the prices of labor and energy services, respectively.

### 2.1.2 Energy Generation

Energy can be derived from resource services  $Y_R$  or generated by the backstop technology sector  $Y_H$ :  $Y_E = Y_R + Y_H$ . The generation of energy by the backstop technology requires the final good as input, according to

$$Y_H = \eta H, \quad (3)$$

where  $\eta > 1$  is a productivity parameter and  $H$  denotes the input of the final good.

### 2.1.3 Service Sector

Labor and resource services are produced according to the following Cobb-Douglas specification:

$$Y_i = Z_i^\beta \int_0^1 q_{ik} x_{ik}^{1-\beta} dk, \quad (4)$$

where  $i = \{L, R\}$ , and  $Z_L = L$  and  $Z_R = R$  denote the inputs of labor and the resource, respectively. The amount and quality of intermediate good variety  $k$  used in sector  $i$  are indicated by  $x_{ik}$  and  $q_{ik}$ , respectively, and the mass of different intermediate goods varieties in each sector is normalized to unity. The resource can be extracted from the initial resource stock  $S_0$ , without extraction costs:

$$\dot{S} = -R, \quad R \geq 0, \quad \int_0^\infty R(t) dt \leq S_0. \quad (5)$$

Producers in the perfectly competitive service sectors take factor remunerations  $w_i$  and intermediate goods prices  $p_{xik}$  as given. Their resulting demand for primary inputs and

intermediate goods follows from

$$p_{Y_i} \frac{\partial Y_i}{\partial Z_i} = w_i, \quad (6a)$$

$$p_{Y_i} \frac{\partial Y_i}{\partial x_{ik}} = p_{x_{ik}}. \quad (6b)$$

### 2.1.4 Intermediate Goods Sector

Each firm in the monopolistically competitive intermediate goods sector produces a unique variety and faces a demand function from the service sector, according to (6b). Per unit production costs are equal to  $q_{ik}$  units of the final good, so that production costs increase proportionally with quality. Firms invest in R&D to increase the quality of their products, according to the following specification:<sup>6</sup>

$$q_{ik} = \xi_i Q_i D_{ik}, \quad (7)$$

where  $\xi_i$  is a productivity parameter,  $Q_i \equiv \int_0^1 q_{ik} dk$  is the aggregate quality level in sector  $i$ , and  $D_{ik}$  is labor allocated to R&D by firm  $k$  in sector  $i$  at unit cost  $w_D$ . The producer of each variety chooses how much to produce and how much to spend on in-house R&D in order to maximize the net present value of its profits, giving rise to the following optimality conditions:

$$p_{x_{ik}} = \frac{q_{ik} p_Y}{1 - \beta}, \quad (8a)$$

$$\lambda_{ik} \xi_i Q_i \leq w_D \quad \text{with equality if } D_{ik} > 0, \quad (8b)$$

$$\frac{\beta}{1 - \beta} x_{ik} p_Y = -\dot{\lambda}_{ik} + r \lambda_{ik}, \quad (8c)$$

where  $p_Y$  denotes the price of the final good,  $r$  is the nominal interest rate, and the  $\lambda_{ik}$ 's are shadow prices of quality in sector  $i$ . Price setting equation (8a) shows that firms charge a mark-up over marginal costs. Condition (8b) requires that, at an interior solution, the marginal revenue of improving quality is equal to its marginal costs. Equation (8c) describes the evolution of the shadow prices of quality. We combine the supply function (8a) with the

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<sup>6</sup>Dots above a variable denote time derivatives, i.e.  $\dot{x} = dx/dt$ , and hats denote growth rates, i.e.  $\hat{x} = \frac{dx/dt}{x}$ .

demand for intermediate goods varieties (6b) and the production function (4) to find

$$x_{ik} = x_i = \frac{\theta_i Y (1 - \beta)^2}{Q_i}, \quad (9)$$

where  $i = \{L, R\}$ , and the  $\theta_i$ 's denote the incomes shares of labor and resource services:  $\theta_i \equiv p_{Y_i} Y_i / (p_Y Y)$ . This expression implies that all intermediate goods producers within the same sector produce the same output level  $x_i$ . Combining (8b) with (8c) and (9), we get:

$$r = \beta(1 - \beta)\xi_i \theta_i \frac{Y p_Y}{w_D} + \hat{w}_D - \hat{Q}_i, \quad \text{if } D_{ik} > 0. \quad (10)$$

Equation (10) can be interpreted as a no-arbitrage condition that requires firms to earn the market interest rate on investment in quality improvements. This return depends positively on the relevant income shares  $\theta_i$  (price effect: quality improvements of relatively scarce factors are more valuable) and on the rate of change in the cost of quality improvements  $\hat{w}_D - \hat{Q}_i$  (capital gain effect: increasing research costs make current improvements more valuable in the future). The transversality conditions associated with the problem of firms in the intermediate goods sector are:

$$\lim_{t \rightarrow \infty} \lambda_L(t) Q_L(t) e^{-\int_0^t r(s) ds} = 0, \quad (11a)$$

$$\lambda_R(T) Q_R(T) e^{-\int_0^T r(s) ds} = 0 \Rightarrow \lambda_R(T) = 0, \quad (11b)$$

where  $T$  denotes the time at which the economy switches from using the non-renewable resource to using the backstop technology. Transversality condition (11a) requires that the shadow price of quality in the labor service sector vanishes if time goes to infinity, and (11b) requires the shadow price of quality in the resource service sector to be zero at the moment the economy switches from the resource to the backstop.<sup>7</sup>

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<sup>7</sup>It follows from (8c)-(9) and the transversality conditions that firms in the same sector face the same shadow prices of quality. Hence, the subscript  $k$  is left out in (11a)-(11b).

## 2.2 Goods and Factor Market Equilibrium

The goods market equilibrium condition is given by:

$$Y = C + \int_0^1 q_{Lk} x_{Lk} dk + \int_0^1 q_{Rk} x_{Rk} dk + H = \frac{C + H}{1 - [1 - \theta_E \omega_H](1 - \beta)^2}, \quad (12)$$

where  $\theta_E \equiv p_{YE} Y_E / (p_Y Y)$ ,  $\omega_H \equiv p_{YH} Y_H / (p_{YE} Y_E)$ , and the second equality uses (9). Labor market equilibrium requires that labor supply  $L^S$  equals labor demand from the labor service sector and from R&D:

$$L^S = L + D, \quad (13)$$

where  $D \equiv D_L + D_R$  and  $D_i \equiv \int_0^1 D_{ik} dk$  is aggregate research effort in sector  $i$ . Labor is perfectly mobile between the production and the research sector, which gives rise to a uniform wage rate in equilibrium:  $w_D = w_L$ . We use the labor market equilibrium condition together with (6a) to obtain

$$D = L^S - \beta(1 - \theta_E) \frac{p_Y Y}{w_L}. \quad (14)$$

## 2.3 Households

The representative household lives forever, derives utility from consumption of the final good, and inelastically supplies  $L^S$  units of labor at each moment. It owns the resource stock and all equity in intermediate goods firms. The household chooses the intertemporal consumption profile to maximize lifetime utility  $U(t) = \int_t^\infty \ln C(z) e^{-\rho(z-t)} dz$ , subject to its flow budget constraint  $\dot{V} = rV + w_R R + w L^S - p_Y C$ , and transversality conditions  $\lim_{t \rightarrow \infty} \phi_V(t) V(t) e^{-\rho t} = 0$  and  $\lim_{t \rightarrow \infty} \phi_S(t) S(t) e^{-\rho t} = 0$ , where  $\rho$  denotes the pure rate of time preference,  $V \equiv \lambda_L Q_L + \lambda_R Q_R$ , and  $\phi_V$  and  $\phi_S$  are the shadow prices of firm and resource wealth, respectively. Optimizing behavior of the household gives rise to the following two familiar conditions:

$$\hat{C} = r - \hat{p}_Y - \rho, \quad (15a)$$

$$\hat{w}_R = r. \quad (15b)$$

Condition (15a) is the Ramsey rule, which relates the growth rate of consumption to the difference between the real interest rate and the pure rate of time preference. Condition (15b) is the Hotelling rule, which requires the resource price to grow at the interest rate so that resource owners are indifferent between extracting and conserving an additional unit of the resource.

### 3 Dynamics of the Model

This section discusses the dynamics of the model. Because the resource and the backstop are perfect substitutes, only the relatively cheapest one will be deployed in the energy sector. As a result, different regimes of energy usage will exist. We will discuss the fossil and backstop regime in turn.

#### 3.1 The Fossil Economy

In the fossil economy, the backstop technology will not be used, implying that  $\omega_H = Y_H = 0$  and  $\theta_E = \theta_R$ . Imposing this in the relative factor demand from the final good sector (2) and using (4), (6a), and (9), we find

$$\frac{\theta_R}{1 - \theta_R} = \left( \frac{w_R Q_L}{w_L Q_R} \right)^{1-\nu} \left( \frac{1 - \gamma}{\gamma} \right)^\sigma, \quad (16)$$

where  $\nu \equiv 1 - \beta(1 - \sigma)$  so that  $\nu \in (0, 1)$  because  $\sigma \in (0, 1)$ . Converting (16) into growth rates and using the Hotelling rule (15b), we obtain:

$$\hat{\theta}_R = (1 - \nu)(1 - \theta_R) \left[ r - \hat{w}_L + \hat{Q}_L - \hat{Q}_R \right]. \quad (17)$$

Equation (17) implies that the resource income share increases if, after correcting for relative productivity changes, the resource price grows faster than the wage rate. By using (6a) and imposing  $\omega_H = 0$ , resource extraction growth can be expressed as:

$$\hat{R} = \hat{\theta}_R - \rho. \quad (18)$$

The fossil economy may be characterized by different technological regimes. We continue by first discussing the internal solution to the model in which there is both labor- and resource-

augmenting technical change. Afterwards, we will investigate the corner solution without resource-augmenting technical change. It will become clear that this corner solution is of particular interest for our purposes. We leave out a discussion of the corner solution without labor-augmenting technical change, because such a solution only applies if the resource stock is small from the beginning, whereas we are interested in the energy transition from an initially abundant and therefore cheap non-renewable resource to an expensive substitute.

### 3.1.1 Dynamics with both types of technical change

Condition (8b) holds with equality in both intermediate goods sectors if  $D_L > 0$  and  $D_R > 0$ . Therefore, the return to quality improvements in both sectors is described by (10). Aggregating (7) over all firms in the sector, we find  $\hat{Q}_i = \xi_i D_i$ . Combining this expression with (10) and  $D = D_L + D_R$ , we get an equation that relates the rate of return in the fossil economy to the aggregate research level:

$$r - \hat{w}_L = L^S \frac{\beta(1-\beta)}{\mu_L} - D, \quad (19)$$

where we have defined the overall labor income share (i.e., including labor income both from production and research activities) as  $\mu_L \equiv wL^S/(p_Y Y)$ . Substituting this condition into (10), we find that the two rates of technical change and the resulting endogenous bias in the direction of technical change are given by, respectively

$$\hat{Q}_i = \frac{\beta(1-\beta)L^S}{\mu_L} (\theta_i - \xi_i^{-1}) + D; \quad i = L, R, \quad (20)$$

$$\hat{Q}_R - \hat{Q}_L = \frac{\beta(1-\beta)L^S}{\mu_L} \xi_L \xi_R (\theta_R - \xi_R^{-1}), \quad (21)$$

where we have used the normalization  $\xi_L^{-1} + \xi_R^{-1} = 1$ , implying that the parameters  $\xi_i$  reflect *relative* productivity of labor employed in R&D in both sectors. The level of aggregate research is governed by labor supply  $L^S$ . We will use this normalization throughout. Equation (21) shows that the bias in technical progress depends on the resource income share: if the resource is scarce and therefore the resource income share is large, technical change will be relatively resource-augmenting and *vice versa*.

Differentiating  $\mu_L$  with respect to time and using the Ramsey rule (15a), we obtain

$$\dot{\mu}_L = \mu_L(D + \rho) - L^S \beta(1 - \beta),$$

showing that the overall labor income share is increasing if aggregate research is large enough.

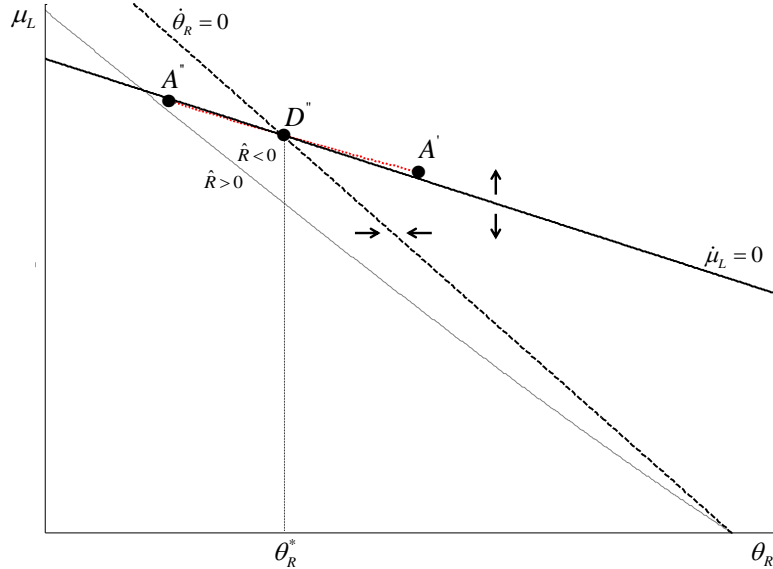
Combining the results of this subsection with (14) and (17), we find that the dynamics of the fossil fuel economy with both types of research occurring are described by the system in Lemma 1.

**Lemma 1** *The dynamics of the fossil economy with labor- and resource-augmenting technical change are described by the following two-dimensional system of first-order differential equations in  $\mu_L$  and  $\theta_R$ :*

$$\dot{\mu}_L = (L^S + \rho) \mu_L - L^S \beta(2 - \beta - \theta_R), \quad (22a)$$

$$\dot{\theta}_R = \frac{\beta L^S \theta_R (1 - \theta_R) (1 - \nu)}{\mu_L} \left\{ 1 + (1 - \beta)(1 + \xi_L) - \frac{\mu_L}{\beta} - [1 + (1 - \beta)\xi_L \xi_R] \theta_R \right\}. \quad (22b)$$

Figure 1: Phase Diagram - Both Types of Technical Change



*Notes:* The solid and dashed black lines represent the  $\dot{\mu}_L = 0$  and  $\dot{\theta}_R = 0$  loci, respectively. The dotted line is the extraction isocline. The fat dots represent the saddle path that starts either from point A' or point A'' and leads to the steady state at point D''. The steady state value of  $\theta_R$  is denoted by  $\theta_R^*$ .

Figure 1 shows the phase diagram in the  $(\theta_R, \mu_L)$ -plane for the dynamic system of the fossil economy with both types of research. The linear  $\dot{\mu}_L = 0$  and  $\dot{\theta}_R = 0$  loci are derived from (22a)-(22b). Still restricting attention to internal solutions with both types of technical progress throughout, the intersection of these two loci determines the unique steady state of the fossil economy. Along the saddle path starting from A' or A'' and leading to the steady state at point D'', two counteracting forces affect the resource income share. On the one hand, increasing physical scarcity of the resource puts upward pressure on the resource income share. On the other hand, the resource income share is negatively affected by induced resource-augmenting technical change. These opposing effects exactly offset each other in the steady state equilibrium, resulting the constant long-run resource income share

$$\theta_R^* = \xi_R^{-1} + \frac{(1 - \beta + \xi_L^{-1})\rho}{\psi}, \quad (23)$$

with  $\psi \equiv \xi_L \xi_R (1 - \beta)(L^S + \rho) + \rho$ . The figure also shows the  $\hat{R} = 0$  isocline, which is located below the  $\dot{\theta}_R = 0$  locus. At points below (above) the  $\hat{R} = 0$  isocline, resource extraction is growing (declining) over time. Hence, an economy moving along the saddle path necessarily exhibits decreasing resource use in the long run.

### 3.1.2 Dynamics with purely labor-augmenting technical change

If condition (8b) holds with inequality in the resource service sector, marginal costs are larger than marginal benefits of quality improvements in this sector, so that resource-augmenting technical change will not occur. Here we discuss this case, in which technical change will be purely labor-augmenting, i.e.  $D_L > 0$  and  $D_R = 0$ . Regimes without resource-augmenting technical change may exist for two reasons. First, if the income share of resource services,  $\theta_R$ , is low, demand for intermediate goods varieties in the resource service sector is small according to (9), so that the return to quality improvements is relatively low, as can be seen from (10). Second, if the backstop technology will be implemented soon, the remaining time during which firms benefit from the quality improvements is small, so that the marginal benefit of quality improvements in present value terms (as measured by the shadow price  $\lambda_R$ ) will be relatively low. This follows from the transversality condition (11b) with  $t$  smaller than, but close to  $T$ .



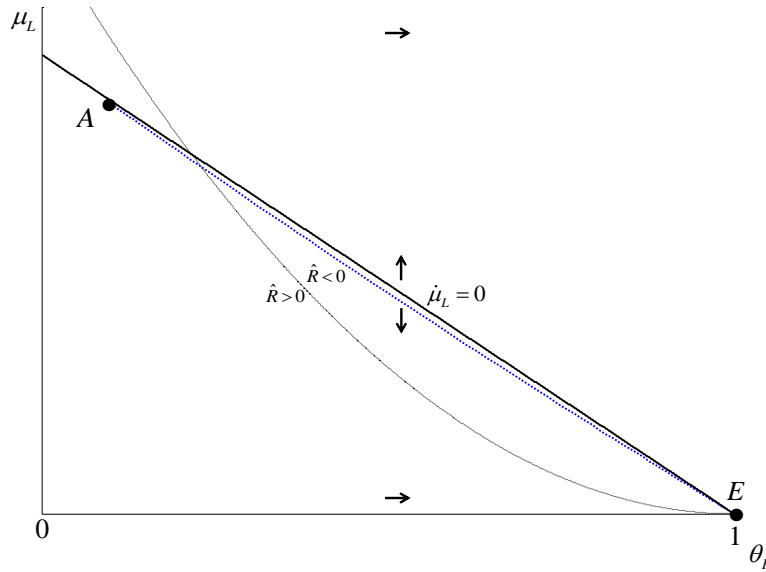
By using (7) and imposing  $D_R = 0$  in (10) and (17), we find that the dynamics of the fossil economy with purely labor-augmenting technical change are described by the system in Lemma 2:

**Lemma 2** *The dynamics of the fossil economy with purely labor-augmenting technical change are described by the following two-dimensional system of first-order differential equations in  $\mu_L$  and  $\theta_R$ :*

$$\dot{\mu}_L = (\xi_L L^S + \rho) \mu_L - \xi_L L^S \beta (2 - \beta) (1 - \theta_R), \quad (24a)$$

$$\dot{\theta}_R = \xi_L L^S \theta_R (1 - \theta_R)^2 (1 - \nu) \frac{\beta (1 - \beta)}{\mu_L}. \quad (24b)$$

Figure 2: Phase Diagram - Purely Labor-Augmenting Technical Change



*Notes:* The solid black line represents the  $\dot{\mu}_L = 0$  locus. The dotted line is the extraction isocline. The fat dots represent the saddle path that leads from point A to the steady state at point E.

Figure 2 shows the phase diagram in the  $(\theta_R, \mu_L)$ -plane for the dynamic system of the fossil economy with purely labor-augmenting technical change. The linear  $\dot{\mu}_L = 0$  locus is derived from (24a). The  $\dot{\theta}_R = 0$  locus is left out, because  $\dot{\theta}_R > 0$  if  $\theta_R \in (0, 1)$ . Disregarding the existence of the backstop technology for the moment and restricting attention to the corner solution of the fossil economy without resource-augmenting technical change, the unique steady state is located at the intersection of the  $\dot{\mu}_L = 0$  locus and the vertical  $\theta_R = 1$

line denoted by point E in the figure. Along the equilibrium path from point A to point E the energy income share is increasing over time, because of increasing physical scarcity of the resource. Without resource-augmenting technical change, there is no offsetting force, implying that the energy income share tends to unity in the long run. The figure also shows the  $\hat{R} = 0$  isocline, which hits the steady state at point E. At points below (above) the  $\hat{R} = 0$  isocline, resource extraction is growing (declining) over time. The slope of this isocline is zero at the steady state. Hence, an economy moving along the equilibrium path necessarily exhibits decreasing resource use in the long run, which is intuitively obvious.

### 3.2 The Backstop Economy

Once the economy has switched from fossil fuels to the backstop technology, we have  $Y_R = D_R = 0$ ,  $\omega_H = 1$ , and  $\theta_E = \theta_H$  with  $\theta_H \equiv p_{YH}Y_H/(p_Y Y)$ . Energy generation with the backstop technology takes place according to (3). Perfect competition implies that the price of energy generated with the backstop technology is equal to its marginal production cost:  $p_{YH} = \eta^{-1}p_Y$ . Using this equality in  $p_{YH} = p_Y \partial Y / \partial Y_H$  together with the factor demand equation (2), we obtain

$$\theta_H = (1 - \gamma)^\sigma \eta^{\sigma-1}. \quad (25)$$

Hence, the energy income share is constant over time. Because the backstop economy is characterized by purely labor-augmenting technical progress, (24a) is still valid. Combining (24a) and (25) we obtain the constant overall labor income share in the backstop economy:

$$\mu_L = \frac{\xi_L L^S \beta (2 - \beta) [1 - (1 - \gamma)^\sigma \eta^{\sigma-1}]}{\xi_L L^S + \rho}. \quad (26)$$

Energy generation with the backstop technology uses the final good  $Y$  as input. As a result, labor is the only primary factor of production, implying that technical change is effectively neutral in the backstop economy. It follows that the economy immediately settles down at its steady state equilibrium with constant income shares and a constant amount of research.

### 3.3 The Energy Transition

Perfect substitutability between resource services and the backstop technology implies that only the cheapest of the two will be deployed at a particular moment in time.<sup>8</sup> It follows from (2) and (25) that the resource will be used if and only if  $\theta_R < (1 - \gamma)^\sigma \eta^{\sigma-1}$ . Given that extraction costs are zero, the resource stock will be exhausted before the economy switches to the backstop technology. As a result, the model generates a potential transition from fossil fuel to the backstop technology. Moreover, the economy might shift between the different technological regimes discussed in Sections 3.1.1 and 3.1.2. We assume throughout the discussion in this section that the initial resource stock is large enough to warrant an initial equilibrium resource income share that is smaller than the steady-state resource income share of the fossil economy, i.e.  $\theta_R(0) < \theta_R^*$ .

Panels (a) and (b) of Figure 3 show the phase diagrams of the two technological regimes in the fossil economy. The figures contain lines for the  $\dot{\mu}_L = 0$  and  $\dot{\theta}_R = 0$  loci, and fat dots that represent the equilibrium paths. Panel (a) also shows the non-negativity constraint  $D_R = 0$  below which resource-augmenting technical change equals zero.<sup>9</sup> In panel (b), zero resource-augmenting technical change is imposed, so that only labor-augmenting technical change takes place and the dynamic system described in Lemma 2 applies.

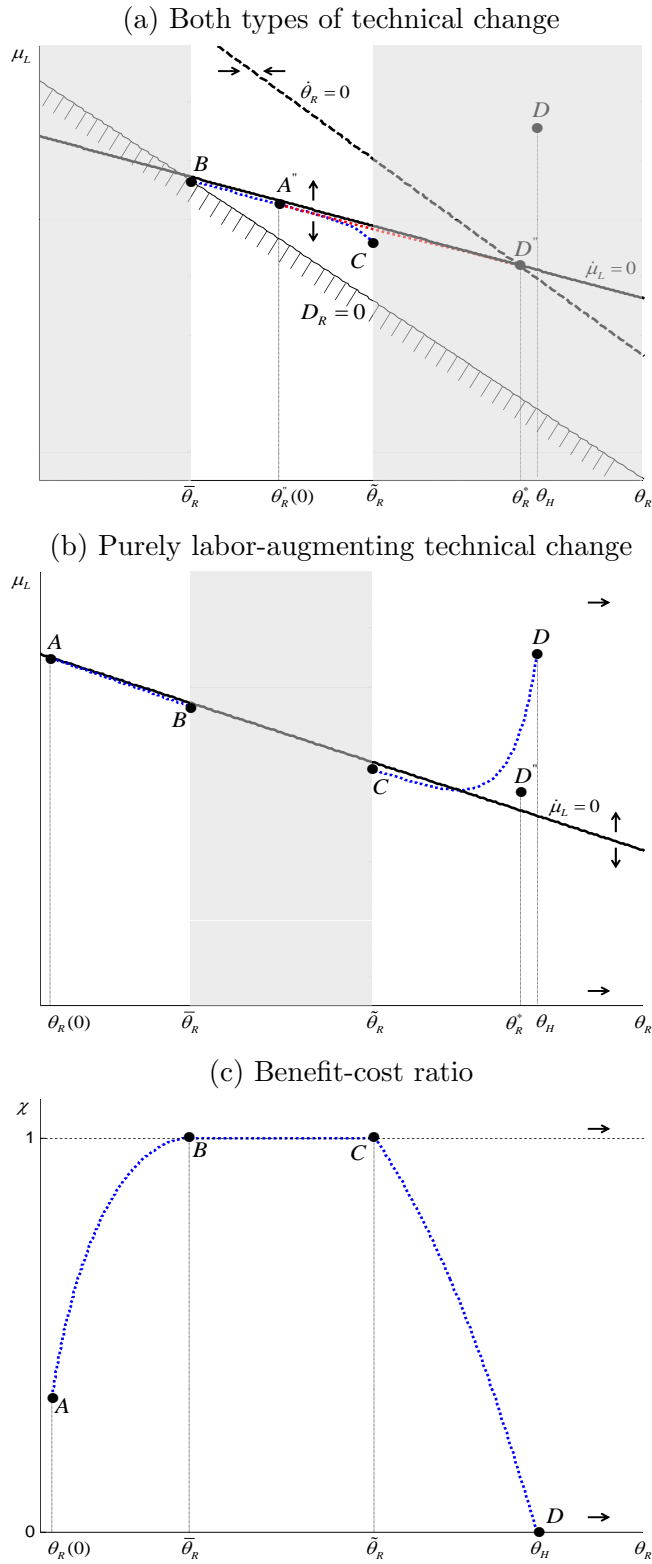
The message to be taken from the figure is that two equilibrium paths can be distinguished. The first path starts at A'' and leads to the steady state of the fossil economy at D'', both in panel (a). Along this path, resource-augmenting technical change still occurs in the long run and energy generation will rely upon fossil fuels forever. Note that the steady state resource income share,  $\theta_R^*$  is smaller than  $\theta_H$ , so that the resource remains cheaper than the backstop technology. The second equilibrium path starts at point A in panel (b), then moves along the fat dots shown in the same panel to point B, continues along the fat dots in panel (a) from point B to point C and finally moves on along the fat dots in panel (b) from point C to point D. At point D, the resource income share has reached  $\theta_H$ , so that the economy will shift to the backstop technology. The determination of points B, C, and D uses backward induction, as will be discussed below.<sup>10</sup>

<sup>8</sup>Appendix A.1.3 shows that simultaneous use will not take place.

<sup>9</sup>By imposing  $\hat{Q}_R = \xi_R D_R = 0$  in (10) and combining the result with (19), we can use (14) to derive the expression underlying the  $D_R = 0$ -line in panel (a) of Figure 3.

<sup>10</sup>The location of points A and A'' depends on the initial stock of fossil fuels and will be discussed in Section 3.4.

Figure 3: Phase Diagrams - Multiple Equilibria



*Notes:* Dynamics in the shaded areas are not applicable to the scenario in which the backstop technology will be implemented.

To understand how the equilibrium moves between regimes and hence between panels (a) and (b), note that an internal solution with both types of technical progress occurring at a particular moment in time requires (8b) to hold with equality. By defining the benefit-cost ratio  $\chi \equiv \xi_R Q_R \lambda_R / w_L \leq 1$ , (8b) implies that resource-augmenting technical change is positive (zero) if  $\chi = (<)1$ . Take the time derivative of  $\chi$  and substitute (8c) to obtain the following differential equation:

$$\dot{\chi} = (r - \hat{w}_L)\chi - \xi_R \beta (1 - \beta) \theta_R L^S \mu_L^{-1}. \quad (27)$$

At the moment of the switch to the backstop technology  $\chi$  equals zero, because resource-augmenting technology is worthless from that moment onwards. The differential equation (27) can be used together with (10) to find the equilibrium path in panel (c) of Figure 3 that leads to the point  $(\theta_H, 0)$ , where  $\theta_R$  and  $\mu_L$  should be taken from the solution to the dynamical system in Lemma 1 (2) if  $\chi = (<)1$ . The resulting equilibrium path in panel (c) shows that  $\chi$  is smaller than unity in between point A and point B, equal to unity on the path from point B to point C, and again smaller than unity along the path from point C to point D. Accordingly, during the transition from fossil fuels to the backstop technology, resource-augmenting technical change is positive only on the equilibrium path between points B and C. At points to the left of  $\bar{\theta}_R$ , fossil fuel is relatively too cheap to warrant investments in resource-saving. At points to the right of  $\tilde{\theta}_R$ , the switch from fossil fuel to the backstop technology occurs soon, so that investing in resource-saving technical change is not profitable anymore. This explains why the transition in the  $(\theta_R, \mu_L)$ -plane starts in panel (b), continues in panel (a), and finally switches back to panel (b).

The locations of the points B, C, and D in  $(\theta_R, \mu_L)$ -space can be determined by using continuity arguments and backward induction. Along a dynamic equilibrium path, all (shadow) prices should be continuous. Moreover, the Ramsey rule (15a) implies that consumption is continuous as long as the rate of interest is finite, which is the case according to (10) with  $i = L$ . Point D can be found by noting that the overall labor income share  $\mu_L$  in the backstop economy is given by (26). At the moment of the energy switch, energy generation with the backstop technology jumps up from zero to a positive level. Consequently, final output needs to jump up in order to prevent a discontinuity in consumption. Using the goods market equilibrium condition (12) and denoting the switching time by  $T$ , we need

$Y(T^-) = (1 - \theta_H)Y(T^+)$ . From this condition, we find that employment in the labor sector jumps up so that the overall labor income share jumps down according to (14), as stated in the following lemma.

**Lemma 3** *At  $T$ , the moment of the switch from fossil fuels to the backstop technology, the overall labor income share jumps down from  $\mu_L^- = \frac{\xi_L \beta (2-\beta) L^S}{\xi_L L^S + \rho}$  to  $\mu_L^+ = (1 - \theta_H)\mu_L^-$ .*

**Proof.** See Appendix A.1.4.  $\square$

The result in Lemma 3 immediately fixes the location of point D,  $(\theta_H, \mu_L^-(T))$ . Given the continuity of both  $\theta_R$  and  $\mu_L$  at all other moments in time, the equilibrium path leading from point A to point D in Figure 3 is uniquely determined.

Summarizing, we have established that Figure 3 shows two equilibrium paths. The first one is characterized by both types of technical change in the long run, so that the backstop technology will not become competitive as the resource income share converges to  $\theta_R^* < \theta_H$ . Along the second equilibrium path, resource-augmenting technical change will drop to zero eventually, implying that the resource income share continues growing beyond  $\theta_R^*$  until it reaches  $\theta_H$  at the moment the backstop technology is introduced. Therefore, Figure 3 shows a situation with multiple equilibria in which the introduction of the backstop technology becomes a self-fulfilling prophecy. The existence of multiple equilibria crucially depends on the difference between  $\theta_R^*$  and  $\theta_H$ , as stated in the following Lemma.

**Lemma 4** *A necessary condition for the existence of multiple equilibria is  $\theta_H \geq \theta_R^*$ .*

**Proof.** If  $\theta_H < \theta_R^*$ , the path leading to the steady state of the fossil economy  $(\theta_H^*, \mu_L^*)$  along which fossil fuels are used forever cannot be a competitive equilibrium, because in the interval with  $\theta_R > \theta_H$ , the backstop technology is relatively cheaper than fossil fuels so that a shift to the backstop technology must occur.  $\square$

The energy income share in the backstop regime depends negatively on  $\eta$ . Hence, the result in Lemma 4 implies that the transition to the backstop technology will certainly take place if  $\eta$  is large enough. If  $\eta$  takes a value below a certain threshold, on the other hand, point C may be located to the right of the  $\dot{\theta}_R = 0$  locus in Figure 3. In that case,  $\theta_R$  does not drop below  $\tilde{\theta}_R$  during the energy transition, meaning that the resource price remains high and cumulative

extraction is low during the energy transition. Hence, the path leading to the implementation of the backstop technology can only be an equilibrium if the initial resource stock is small enough. For higher initial resource stocks, the transition to the backstop technology can be excluded as an equilibrium, as will be further discussed in Section 4. In the remaining case with intermediate values of  $\eta$ , there may exist multiple equilibria so that a self-fulfilling prophecy arises. The numerical analysis below indicates the area for which this is actually the case.

The results of this section are summarized in the following proposition.

**Proposition 1** *Assuming that the initial resource stock  $S_0$  is large enough to get  $\theta_R(0) < \theta_R^*$ , the following two scenarios can be distinguished.<sup>11</sup>*

(i) *If  $\theta_H < \theta_R^*$ , the unique equilibrium in a decentralized economy is a transition from fossil fuels to the backstop technology. Resource-augmenting technical change stops before the energy transition is completed.*

(ii) *If  $\theta_H \geq \theta_R^*$ , there exist potentially two equilibria in a decentralized economy.*

- *The first equilibrium is a transition from fossil fuels to the backstop technology, where resource-augmenting technical change stops before the energy transition is completed.*
- *The second equilibrium is a transition to the internal state, implying that the backstop technology will never be implemented, fossil fuels will be used forever, and resource-augmenting technical change will take place in the long run.*

*Expectations of market participants determine which equilibrium actually arises.*

### 3.4 Initial Condition

To complete the solution to the model, we use the size of the initial stock to determine the initial point in the  $(\theta_R, \mu_L)$ -plane (i.e., the location of points A and A" in Figure 3). By using (4) and the relative factor demand (16), we obtain a relationship between the resource

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<sup>11</sup>The dependence between the initial stock of fossil fuels and the initial resource income share will be discussed in Section 3.4. The expressions for  $\theta_R^*$  and  $\theta_H$  are given in (23) and (25), respectively.

income share and the reserve-to-extraction rate  $y \equiv S/R$ :

$$\frac{\theta_R(t)}{1 - \theta_R(t)} = \left( \frac{1 - \gamma}{\gamma} \right)^{\frac{\sigma}{\nu}} \left( \frac{S(t)\mu_L(t)}{y(t)\beta L^S [1 - \theta_R(t)]} \frac{Q_R(t)}{Q_L(t)} \right)^{\frac{\nu-1}{\nu}}. \quad (28)$$

Total differentiation of this expression gives a differential equation that describes the evolution of the reserve-to-extraction rate:

$$\dot{y}(t) = -y(t)(1 - \nu)[1 - \theta_R(t)] \left\{ r(t) - \hat{w}_L(t) - [\hat{Q}_R(t) - \hat{Q}_L(t)] \right\} + \rho y(t) - 1,$$

where we have used (17). By using (10) and (14), for each of the two technological regimes this differential equation can be expressed in terms of  $\theta_R$ ,  $\mu_L$ , and parameters only. Together with the end conditions  $y(T) = 0$  or  $\lim_{t \rightarrow \infty} y(t) = \rho$ , depending on whether or not the backstop technology will be implemented eventually, the resulting differential equations can be used to calculate the equilibrium path for the reserve-to-extraction rate along which cumulative demand for fossil fuels is equal to the remaining stock. Define  $\mu_L = f(\theta_R)$  as the equilibrium path in  $(\theta_R, \mu_L)$ -space. The initial income share  $\theta_R(0)$  now follows from the intersection of this equilibrium path and (28) with  $\mu_L = f(\theta_R)$  and  $t = 0$  in  $(\theta_R, y)$ -space, where cumulative demand for fossil fuels equals the initial resource stock.<sup>12</sup>

## 4 Numerical Illustration

In this section, we quantify the results of the model by performing a numerical analysis.<sup>13</sup> The aim of this exercise is to show in which scenarios the introduction of the backstop technology actually is a self-fulfilling prophecy and to highlight the differences between the two equilibria in terms of the time profiles of fossil fuel use and resource-augmenting technical progress.

We first calibrate the model to match data on energy income shares and consumption growth in modern industrialized economies. For the elasticity of substitution between labor and resource services, we choose  $\sigma = 0.45$ , which is within the range of 0.17 to 0.61 that Van der Werf (2008) reports. The parameter  $\beta$  is the output elasticity of the primary factors,

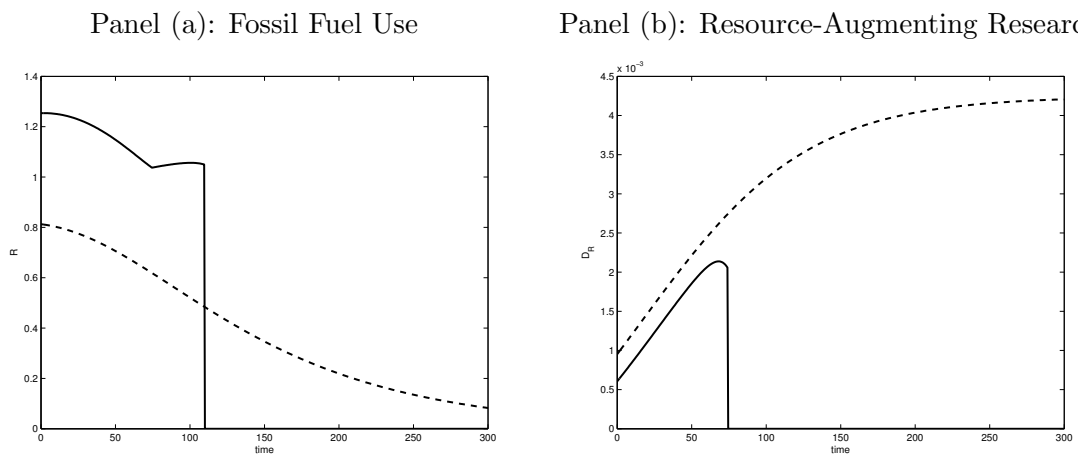
<sup>12</sup>If a certain path in  $(\theta_R, \mu_L)$ -space does not intersect the line implicitly defined by (28), this path cannot be an equilibrium path because in that case cumulative resource demand does not equal the initial resource stock.

<sup>13</sup>For the numerical simulation, we use the relaxation algorithm explained in Trimborn, Koch, and Steger (2008).



labor and fossil fuel, in both service sectors. Our value of 0.8 lies within the range of the labor income shares reported in Gollin (2002) and is in line with the average share of fossil fuel consumption in total energy consumption in the OECD countries, which amounted to 82 percent over the years 2000-2011 (World Bank, 2012). We set the rate of pure time preference  $\rho$  to 0.01 and choose  $\gamma = 0.50$  for the final good production function parameter. The backstop productivity parameter  $\eta$  is fixed at 10, so that  $\theta_R^* = 0.19$ . The initial stocks of quality in both sectors  $Q_{L0}$ ,  $Q_{R0}$  are normalized to unity. We choose an initial non-renewable resource stock of 125 to obtain an initial energy income share of 8.2 and 9.4 percent in the scenarios with and without the transition to the backstop technology, respectively, which is in line with the average energy expenditure share in GDP of 8.8 percent over the period 1970-2009 in the United States (U.S. Energy Information Administration, 2011). In the scenarios with and without the transition to the backstop technology, our implied initial reserve-to-extraction rates are 100 and 150, respectively. The implied initial price of the backstop technology compared to the resource price,  $p_{YH}(0)/p_{YR}(0)$ , amounts to 5.3 and 4.1 in the two scenarios. We fix the labor supply  $L^S$  at 0.14 and the research productivity parameter  $\xi_R$  at  $20/3$  to obtain an initial yearly consumption growth rate of 1.7 percent in both scenarios, which is equal to the average yearly growth rate of GDP per capita in the United States over the period 1970-2011 (The Conference Board, 2011).

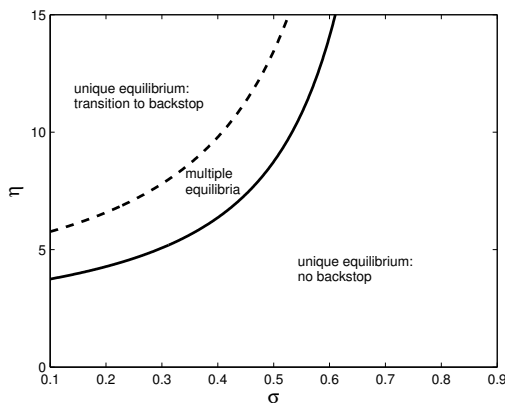
Figure 4: Time Profiles



*Notes:* The solid line represents the transition to the backstop technology, whereas the dashed line shows the scenario in which the backstop technology does not become competitive. Parameters and initial stocks are set at their benchmark values.

In our benchmark calibration, the introduction of the backstop technology is a self-fulfilling prophecy. Figure 4 shows the time profiles for fossil fuel use (panel a) and resource-augmenting technical change (panel b) for the two equilibria. If the backstop technology is expected not to become competitive (see the dashed lines), resource extraction starts out relatively low and is monotonically declining over time. Resource-augmenting technical change is relatively high and monotonically increasing over time. On the contrary, if the future implementation of the backstop technology is expected (see the solid lines), current resource extraction is relatively high, whereas resource-augmenting technical change is modest and drops to zero before the backstop technology actually becomes competitive.

Figure 5: Region with Multiple Equilibria



*Notes:* Above the black line, the transition to the backstop technology always takes place. Below the grey line, the transition never takes place. In between the two lines, multiple equilibria exist so that the future implementation of the backstop technology is a self-fulfilling prophecy. Parameters and initial stocks are set at their benchmark values.

The elasticity of substitution  $\sigma$  is a measure for how difficult it is to replace fossil fuels by conventional factors of production, whereas  $\eta$  measures the productivity of a non-conventional substitute for fossil fuels. Both parameters are important for the characteristics of the energy transition. Figure 5 shows for which combinations of those two parameters there exist multiple equilibria in the model. For all points above the dashed line in the figure, the backstop technology is relatively productive, so that it will always become competitive in the future. This dashed line is directly derived from the result in Lemma 4. It is upward-sloping, because an increase in the elasticity of factor substitution increases the energy income share in the backstop economy, so that a higher productivity is needed to guarantee the implementation of the backstop technology. For all points below the solid line, the productivity of the backstop

technology is so low that it will never become competitive.<sup>14</sup> Intuitively, if the backstop technology has a low productivity, the long-run energy income share  $\theta_B$  will be high. Hence, during the energy transition, the resource income share needs to reach high values as well, in order to come close enough to  $\theta_B$ . A high resource income shares, however, triggers a lot of resource-saving technical change, which puts downward pressure on the resource income share. As a result, the resource will remain effectively cheaper than the backstop technology, so that the backstop technology will never be implemented. The area in between the two lines gives the combinations for which the implementation of the backstop technology is a self-fulfilling prophecy. The next section provides a welfare analysis and discusses the policy implications of the existence of multiple equilibria.

## 5 Welfare Analysis

The aim of this section is twofold. First, we want to compare the decentralized market outcome with the optimum. Second, we want to compare welfare in an economy that experiences a transition to the backstop technology with welfare in an economy in which the backstop technology is never introduced. Appendix A.2 derives the solution to the optimization problem of the social planner. Similar to the decentralized equilibrium, this problem may also have multiple candidate solutions that satisfy the first-order and transversality conditions.<sup>15</sup>

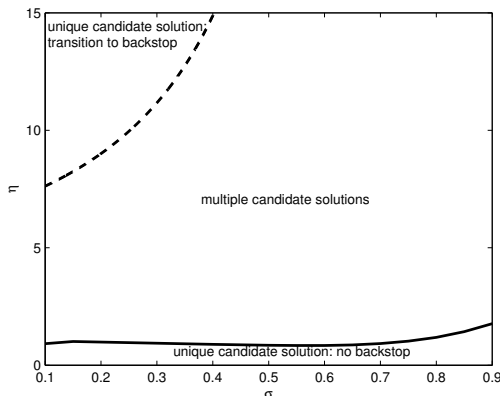
The region within the solid and the dashed line in Figure 6 gives combinations of the backstop technology productivity parameter  $\eta$  and the elasticity of factor substitution  $\sigma$  for which multiple candidate solutions exist: one with and one without the eventual implementation of the backstop technology. For all points above the dashed line, the backstop technology is relatively productive and there is no candidate solution without the implementation of the backstop technology. For all points below the solid line, the productivity of the backstop technology is so low that there is no candidate solution with the implementation of the

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<sup>14</sup>The numerical construction of the solid line uses the fact that we can exclude all equilibria that do not satisfy the initial condition of Section 3.4. For low values of  $\eta$ ,  $\theta_B$  is high and point C in panel (a) of Figure 3 may be located to the right of the income share locus. If this is the case, point C must be approached from the right, so that  $\theta_R$  never drops below  $\tilde{\theta}_R$  during the energy transition. The resource price is now high throughout, so that cumulative demand during the transition to the backstop is small. Therefore, the transition to the backstop technology can be excluded as an equilibrium if  $\eta$  is low and  $S_0$  is high enough.

<sup>15</sup>'Candidate solutions' are defined as trajectories that satisfy the first-order conditions and the transversality conditions.

Figure 6: Planner Solution - Region with Multiple Candidates



*Notes:* Above the dashed line, transition to the backstop technology is the only candidate for an optimum. Below the solid line, the transition to the internal steady state is the only candidate for an optimum. In between, the transition to the backstop technology and to the internal steady state are both candidates for an optimum. Along the solid line, welfare is the same for both candidate solutions. Parameters and initial stocks are set at their benchmark values.

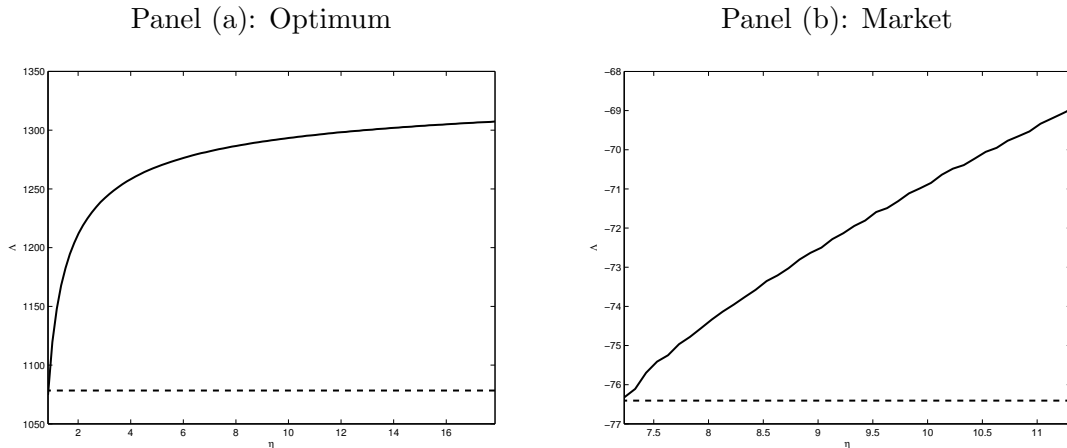
backstop technology.<sup>16</sup>

A comparison of Figures 5 and 6 reveals that in the decentralized market equilibrium, the dashed line is located lower, whereas the solid line is located higher in the  $(\sigma, \eta)$  plane. Intuitively, the shift of the dashed line can be explained as follows: along the path that leads to the internal steady state, the value of knowledge spillovers in the resource sector is relatively large compared to the spillovers in the labor sector, because of the finite availability of the resource. As a result, the market invests relatively too little in resource-saving technical change, so that the range of backstop productivity parameter values for which the market always shifts to the backstop becomes larger. Hence, the dashed line is located lower in the decentralized equilibrium than in the optimum. Similarly, along the path that leads to the implementation of the backstop technology, the value of the knowledge spillovers in the resource sector are relatively small compared to spillovers in the labor sector, because accumulated knowledge in the resource sector becomes worthless from the time of the shift to the backstop technology onwards. As a result, the market relatively overinvests in resource-saving technical change, so that the range of backstop productivity parameter values for which the market never shifts to the backstop technology becomes larger. Hence, the solid

<sup>16</sup>The dashed line is directly derived from the condition  $\xi_R = (1 - \gamma)^{-\sigma} \eta^{1-\sigma}$ . The solid line is determined numerically, by using that all equilibria that do not satisfy the initial condition can be excluded. Details can be found in Appendix A.2.

line is located relatively higher in the decentralized equilibrium. It follows that the multiple candidate solutions region in the centralized economy is larger than the multiple equilibria region in the market economy.

Figure 7: Welfare



*Notes:* The solid line gives the utility associated with the transition to the backstop technology. The dashed line gives the utility associated with transition to the internal steady state, without introduction of the backstop technology. The interval for  $\eta$  in each panel corresponds with the distance between the dashed and solid lines evaluated at  $\sigma = 0.45$  in Figures 5 and 6, respectively. Parameters and initial stocks are set at their benchmark values.

Figure 7 compares the welfare in terms of discounted utility generated by the path leading to the internal steady state (dashed lines) with the welfare generated by the path leading to the backstop technology (solid lines), both for the social planner problem (panel (a)) and the decentralized market equilibrium (panel (b)).<sup>17</sup> The two panels show the interval of the backstop productivity parameter  $\eta$  for which the social planner problem has two candidate solutions and the market economy has two equilibria, where all other parameters are set at their benchmark values. Hence, the intervals for  $\eta$  in panels (a) and (b) correspond with the difference between the dashed and solid lines in Figures 6 and 5, respectively, at  $\sigma = 0.45$ .

Panel (a) shows that at the lowest  $\eta$  for which there are two candidate solutions, the welfare generated by the path heading towards the internal steady state is equal to the welfare generated by the transition to the backstop technology. For higher productivity values, the transition to the backstop technology delivers a strictly higher level of welfare. Therefore, in this interval the social planner will select the path leading to the introduction of the backstop

<sup>17</sup>The welfare levels shown in the figure are equal to the value of the Hamiltonian corresponding to the two paths at time zero, divided by the pure rate of time preference  $\rho$  (cf. Grass, Caulkins, Feichtinger, Tragler, and Behrens, 2008, p. 161).

technology. Panel (b) shows a comparable result for the decentralized market economy: at the lowest  $\eta$  for which there exist multiple equilibria, the welfare generated by the path leading to the internal steady state is equal to the welfare generated by the transition to the backstop technology. If the backstop productivity exceeds this threshold level, the transition to the backstop technology gives strictly higher welfare. It was already clear from Figures 5 and 6 that the lowest  $\eta$  for which there are multiple equilibria in the decentralized equilibrium is higher than the minimum value for  $\eta$  that is required for multiple candidate solutions in the social planner problem.

Summarizing the results in this section, we have learned from the welfare analysis that the region in  $(\sigma, \eta)$ -space for which the backstop will not be implemented (the areas below the solid lines in Figures 5 and 6) is smaller in the centralized economy than in the market economy, because the market generates too much resource-saving technical progress along the transitional path towards the backstop technology. Moreover, the welfare analysis has shown that if there exist multiple equilibria in the benchmark market economy, the equilibrium leading to the implementation of the backstop technology generates (at least weakly) higher welfare.

## 6 Conclusion

This paper has investigated the interaction between innovation and the energy transition from fossil fuels to renewable energy. For this purpose, we have constructed a growth model with a non-renewable resource and a backstop technology in which profit incentives determine both the rate and the direction of technical change endogenously. We take into account that natural resources and man-made factors of production are poor substitutes, that energy generation with the backstop technology is costly, and that resource-augmenting technology becomes worthless after a shift from fossil fuels to renewable energy.

Our main finding is that the future large-scale implementation of renewable energy might be a self-fulfilling prophecy. If it is expected that backstop technologies will become competitive eventually, the market for resource-saving inventions will be small, so that incentives for efficiency improvements in the resource sector will be eroded and renewable energy will indeed become competitive in the long run. Conversely, if investors expect that backstop technologies will not be able to produce renewable energy at competitive prices on a large

scale, the market for resource-saving inventions remains significant and renewable energy will indeed be relatively unattractive in the long run. Only when the backstop productivity is above a certain threshold level, the transition to renewable energy will always take place in a decentralized market economy.

The existence of expectations-driven multiple equilibria has important implications for policy. As is standard in other models of directed technical change, our model includes externalities that can be addressed by policies: the benefits from research are not fully appropriated and there is monopolistic competition. If the coordination of expectations is difficult, additional temporary policies might be needed to steer the economy into the direction of the optimal path. Our welfare analysis suggests that in case of multiple decentralized equilibria, the transition to the backstop technology generates higher utility. The real challenge for policy design is then to know if the market is right or wrong when it bets on fossil fuels: if we would know that we are in the multiple equilibria range and the market expects the economy to use fossil forever, the development of resource-augmenting technology needs to be discouraged (to steer away expectations). However, if we are in the range in which using fossil fuels forever is optimal, we need to subsidize resource-augmenting R&D (because of spillovers). We leave it for future research to extend the model with externalities associated with fossil use and to analyze policy options.

## Appendix

### A.1 Decentralized Market Equilibrium

#### A.1.1 Intermediate Goods Producers

The current-value Hamiltonian associated with the optimization problem of firm  $k$  in the intermediate good sector is given by:

$$\mathcal{H}_{ik} = p_{Yi}(1 - \beta)q_{ik}Z_i^\beta x_{ik}^{1-\beta} - q_{ik}p_Y x_{ik} - w_D D_{ik} + \lambda_{ik}\xi_i Q_i D_i, \quad (\text{A.1})$$

where  $i = Z_i = \{R, L\}$ . According to the Maximum Principle, the necessary first-order conditions for an optimum are given by:

$$\frac{\partial \mathcal{H}_{ik}}{\partial x_{ik}} = 0 \Rightarrow (1 - \beta)^2 p_{Yi} q_{ik} Z_i^\beta x_{ik}^{-\beta} = q_{ik} p_Y, \quad (\text{A.2})$$

$$\frac{\partial \mathcal{H}_{ik}}{\partial D_{ik}} \leq 0 \Rightarrow -w_D + \lambda_{ik}\xi_i Q_i \leq 0, \quad \text{with equality if } D_{ik} > 0, \quad (\text{A.3})$$

$$\frac{\partial \mathcal{H}_{ik}}{\partial q_{ik}} = -\dot{\lambda}_{ik} + r\lambda_{ik} \Rightarrow p_{Yi}(1 - \beta)Z_i^\beta x_{ik}^{1-\beta} - x_{ik}p_Y = -\dot{\lambda}_{ik} + r\lambda_{ik}. \quad (\text{A.4})$$

The transversality conditions are given by (11a)-(11b). Substitution of (6b) in (A.2) gives (8a), (A.3) directly implies (8b), and the combination of (A.2) and (A.4) gives (8c).

#### A.1.2 Households

The current-value Hamiltonian associated with the households' optimization problem reads:

$$\mathcal{H} = \ln C + \phi_V [rV + w_R R + wL^S - p_Y C] - \phi_S R. \quad (\text{A.5})$$

According to the Maximum Principle, the necessary first-order conditions for an optimum are

$$\frac{\partial \mathcal{H}}{\partial C} = 0 \Rightarrow \frac{1}{C} - \phi_V p_Y = 0 \Rightarrow \hat{C} + \hat{p}_Y = -\hat{\phi}_V, \quad (\text{A.6a})$$

$$\frac{\partial \mathcal{H}}{\partial R} = 0 \Rightarrow \phi_V w_R - \phi_S = 0 \Rightarrow \hat{\phi}_V + \hat{w}_R = \hat{\phi}_S, \quad (\text{A.6b})$$

$$\frac{\partial \mathcal{H}}{\partial S} = -\dot{\phi}_S + \rho\phi_S \Rightarrow \dot{\phi}_S + \rho\phi_S = 0 \Rightarrow \hat{\phi}_S = \rho, \quad (\text{A.6c})$$

$$\frac{\partial \mathcal{H}}{\partial V} = -\dot{\phi}_V + \rho\phi_V \Rightarrow \phi_V r = -\dot{\phi}_V + \rho\phi_V \Rightarrow \hat{\phi}_V = \rho - r. \quad (\text{A.6d})$$



Combining (A.6a) and (A.6d) gives the Ramsey rule (15a). The first-order conditions (A.6b)-(A.6d) yield the Hotelling rule (15b).

### A.1.3 Exclusion of Simultaneous Use

We show that it is not possible to have a regime of simultaneous use of the resource and the backstop technology. Simultaneous use requires equal effective prices of the resource and the backstop technology, so that  $p_{YH} = p_{YR} = p_{YE}$ . Using  $p_{YH} = p_Y \partial Y / \partial Y_H = p_Y / \eta$ , this implies  $\theta_E = (1 - \gamma)^\sigma \eta^{\sigma-1}$  and

$$\hat{p}_Y = \hat{p}_{YH} = \hat{p}_{YR} = \hat{p}_{YE}. \quad (\text{A.7})$$

If we combine (4) with (6a), and (6b) to find the price indexes  $p_{YL}$  and  $p_{YR}$ , and subsequently convert the expression into growth rates, we get

$$\hat{p}_{YL} - \hat{p}_Y = \beta(\hat{w}_L - \hat{p}_Y - \hat{Q}_L), \quad (\text{A.8a})$$

$$\hat{p}_{YR} - \hat{p}_Y = \beta(\hat{w}_R - \hat{p}_Y - \hat{Q}_R). \quad (\text{A.8b})$$

Using  $\hat{p}_Y = \theta_E \hat{p}_{YE} + (1 - \theta_E) \hat{p}_{YL}$  together with (A.7), we find  $\hat{p}_{YL} = \hat{p}_Y$ . Substitution of this result into (A.8a) and (A.7) into (A.8b), and using the Hotelling rule (15b), we obtain  $r - \hat{w}_D = \hat{Q}_R - \hat{Q}_L$ . Substitution of (14) into (10), in a regime with purely labor-augmenting technical change (i.e.  $\hat{Q}_L > 0$  and  $\hat{Q}_R = 0$ ) we have  $r - \hat{w}_D = (1 - \beta)\xi_L(L^S - D) - \hat{Q}_L$ . The latter two conditions can only be satisfied jointly if  $D = L^S$ . However, this implies that  $L = Y = 0$ , which cannot hold in equilibrium because it implies  $\hat{C} = \hat{Y} = 0$ , whereas the Ramsey rule (15a) together with (A.8b) gives  $\hat{C} = -\rho$ . Hence, during a regime with purely labor-augmenting technical change, the effective relative price of the resource and the backstop cannot be constant, so that simultaneous use of both energy sources will not occur. As a result, simultaneous use is also impossible in a regime with both resource-augmenting and labor-augmenting technical change. Optimality condition (8b) together with (11b) namely implies that the economy eventually necessarily shifts to a regime without resource-augmenting technical change. Continuity of energy prices requires that  $\theta_E$  is continuous at this regime shift. However, at the beginning of the regime without resource-

augmenting technical change,  $\theta_E < (1-\gamma)^\sigma \eta^{\sigma-1}$ .<sup>18</sup> The jump from a regime with simultaneous use with resource-augmenting and labor-augmenting technical change to a regime with purely labor-augmenting technical change necessarily implies a discontinuity in  $\theta_E$ . Therefore, a regime of simultaneous use cannot exist.

#### A.1.4 Proof of Lemma 3

As argued in the main text, continuity of consumption at  $t = T$  requires  $Y(T^-) = (1 - \theta_H)Y(T^+)$ . By using the income share definitions we rewrite output as:

$$Y = Y_L \left[ \gamma + (1 - \gamma) \left( \frac{\theta_E}{1 - \theta_E} \frac{p_{YL}}{p_{YE}} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}.$$

Using the continuity of prices and income shares, a jump in  $Y$  must be proportional to a jump in  $Y_L$ . Furthermore, it follows from (4) and (9) that a jump in  $Y_L$  is proportional to a jump in  $L$ . Therefore, the jump in  $\mu_L$  must be proportional to the jump in  $Y$ , from which the result in Lemma 3 follows.  $\square$

## A.2 Social Optimum

If  $H = 0$  and  $R > 0$ , which must hold in the optimum as long as  $\theta_R < \theta_H = (1 - \gamma)^\sigma \eta^{\sigma-1}$ , consumption can be written as  $C = F(LQ_L, RQ_R)$ , with

$$F(LQ_L, RQ_R) = \zeta \left[ \gamma^{\frac{\sigma}{\nu}} (LQ_L)^{\frac{\nu-1}{\nu}} + (1 - \gamma)^{\frac{\sigma}{\nu}} (RQ_R)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}},$$

and  $\zeta \equiv \beta(1 - \beta)^{\frac{1-\beta}{\beta}}$ . The social planner maximizes  $\int_0^\infty \ln C(t)e^{-\rho t}$ , subject to

$$\dot{S} = -R, \quad \dot{Q}_L = \xi_L D_L Q_L, \quad \dot{Q}_R = \xi_R D_R Q_R, \quad \dot{L}^S = L + D_L + D_R.$$

The associated current-value Hamiltonian is given by

$$\mathcal{H} = \ln F(LQ_L, RQ_R) - \lambda_S R + \lambda_L \xi_L D_L Q_L + \lambda_R \xi_R D_R Q_R + \lambda_{LME} (L^S - D_L - D_R - L),$$

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<sup>18</sup>This inequality follows from the continuity of  $\mu$ , optimality condition (8b), and (24b).

with the following first-order conditions, according to the Maximum Principle:

$$\frac{\partial \mathcal{H}}{\partial R} = \frac{1}{F(\cdot)} \frac{\partial F}{\partial(Q_R R)} Q_R - \lambda_S = 0 \Rightarrow \frac{\theta_R}{R} = \lambda_S, \quad (\text{A.9})$$

$$\frac{\partial \mathcal{H}}{\partial L} = \frac{1}{F(\cdot)} \frac{\partial F}{\partial(Q_L L)} Q_L - \lambda_{LME} = 0 \Rightarrow \frac{\theta_L}{L} = \lambda_{LME}, \quad (\text{A.10})$$

$$\frac{\partial \mathcal{H}}{\partial D_L} = \frac{-1}{F(\cdot)} \frac{\partial F}{\partial(Q_L L)} Q_L + \lambda_L \xi_L Q_L \geq 0 \Rightarrow \lambda_{LME} \geq \lambda_L \xi_L Q_L, \quad (\text{A.11})$$

$$\frac{\partial \mathcal{H}}{\partial D_R} = \frac{-1}{F(\cdot)} \frac{\partial F}{\partial(Q_L L)} Q_L + \lambda_R \xi_R Q_R \geq 0 \Rightarrow \lambda_{LME} \geq \lambda_R \xi_R Q_R, \quad (\text{A.12})$$

$$\frac{\partial \mathcal{H}}{\partial Q_L} = \frac{-1}{F(\cdot)} \frac{\partial F}{\partial(Q_L L)} L + \lambda_L \xi_L D_L = -\dot{\lambda}_L + \rho \lambda_L \Rightarrow \frac{\theta_L}{\lambda_L Q_L} + \hat{Q}_L = -\hat{\lambda}_L + \rho, \quad (\text{A.13})$$

$$\frac{\partial \mathcal{H}}{\partial Q_R} = \frac{-1}{F(\cdot)} \frac{\partial F}{\partial(Q_R R)} R + \lambda_R \xi_R D_R = -\dot{\lambda}_R + \rho \lambda_R \Rightarrow \frac{\theta_R}{\lambda_R Q_R} + \hat{Q}_R = -\hat{\lambda}_R + \rho, \quad (\text{A.14})$$

$$\frac{\partial \mathcal{H}}{\partial E} = 0 = -\dot{\lambda}_S + \rho \lambda_S \Rightarrow \hat{\lambda}_S = \rho, \quad (\text{A.15})$$

where  $\lambda_S$ ,  $\lambda_L$ , and  $\lambda_R$  denote the co-state variables,  $\lambda_{LME}$  is a Lagrange multiplier, and we have used  $\theta_L = \frac{\partial F}{\partial(Q_L L)} \frac{Q_L L}{F}$  and  $\theta_R = \frac{\partial F}{\partial(Q_R R)} \frac{Q_R R}{F}$ . The transversality conditions are given by

$$\lim_{t \rightarrow \infty} \lambda_S(t) e^{-\rho t} S(t) = 0, \quad \lim_{t \rightarrow \infty} \lambda_L(t) e^{-\rho t} Q_L(t) = 0, \quad \lim_{t \rightarrow \infty} \lambda_R(t) e^{-\rho t} Q_R(t) = 0.$$

By combining (A.9) and (A.10), we obtain

$$\frac{\theta_R}{\theta_L} = \left( \frac{1 - \gamma}{\gamma} \right)^\sigma \left( \frac{Q_R / \lambda_S}{Q_L / \lambda_{LME}} \right)^{\nu - 1}. \quad (\text{A.16})$$

In an internal solution, (A.11)-(A.12) hold with equality, implying  $\lambda_L \xi_L Q_L = \lambda_R \xi_R Q_R$  and  $\hat{\lambda}_L + \hat{Q}_L = \hat{\lambda}_R + \hat{Q}_R$ . Substitution of this result into (A.13)-(A.14) gives  $\theta_R = \frac{1}{\xi_R}$  and  $\mu_L = \theta_L \frac{L^S}{L} = \frac{L^S}{\rho}$ . Hence, an interior solution only occurs in a steady state. The transition to this steady state is characterized by a corner solution with either  $D_L = 0$  or  $D_R = 0$ . The dynamic systems that characterize both corner solutions are given in Lemma A.1 and A.2.

**Lemma A.1** *The dynamic system in the regime with  $D_R = 0$  and  $D_L > 0$  is given by:*

$$\dot{\theta}_R = \theta_R (1 - \theta_R) (1 - \nu) \xi_L L^S, \quad (\text{A.17a})$$

$$\dot{\mu}_L = L^S [\rho \mu_L - (1 - \theta_R) \xi_L L^S], \quad (\text{A.17b})$$

$$\dot{z} = \xi_R \frac{L^S}{\mu_L} \left[ (1 - \theta_R) \frac{\xi_L}{\xi_R} z - \theta_R \right]. \quad (\text{A.17c})$$

**Proof.** If  $z \equiv \frac{\lambda_R \xi_R Q_R}{\lambda_L \xi_L Q_L} < 1$ , the case with  $D_R = 0$  applies and (A.12) holds with inequality. Totally differentiating (A.10), (A.16), and the definition of  $z$ , while using (A.11), (A.13), and (A.15) gives the results in the lemma.  $\square$

**Lemma A.2** *The dynamic system in the regime with  $D_L = 0$  and  $D_R > 0$  is given by:*

$$\dot{\theta}_R = \theta_R(1 - \theta_R)(1 - \nu)\xi_R L^S \frac{1 - \mu_L}{\mu_L}, \quad (\text{A.18a})$$

$$\dot{\mu}_L = L^S (\rho\mu_L - \theta_R \xi_R L^S), \quad (\text{A.18b})$$

$$\dot{z} = \xi_R \frac{L^S}{\mu_L} z \left[ (1 - \theta_R) \frac{\xi_L}{\xi_R} z - \theta_R \right]. \quad (\text{A.18c})$$

**Proof.** If  $z \equiv \frac{\lambda_R \xi_R Q_R}{\lambda_L \xi_L Q_L} > 1$ , the case with  $D_L = 0$  applies and (A.11) holds with inequality. Totally differentiating (A.10), (A.16), and the definition of  $z$ , while using (A.12), (A.14), and (A.15) gives the results in the lemma.  $\square$

The end conditions of the dynamic systems depend on whether the transition leads to the internal steady state, or to the regime with the backstop technology. In the former case,  $\theta_R$  reaches  $\frac{1}{\xi_R}$  at time  $t = T$ . Continuity of the state and co-state variables then implies that the end conditions are  $\theta_R(T) = \frac{1}{\xi_R}$ ,  $\mu_L = \frac{L^S}{\rho}$ , and  $z(T) = 1$ . If  $\theta_R < (>) \frac{1}{\xi_R}$ , the regime with  $D_L = 0$  and  $z > 1$  ( $D_R = 0$  and  $z < 1$ ) applies, because otherwise  $z$  would be increasing (decreasing) and never reach unity, as can be seen from combining (A.13) and (A.14).

If the transition leads to the regime with the backstop technology,  $\theta_R$  converges to  $\theta_H$ , which it reaches at time  $t = T'$ . In the backstop technology regime, (A.10)-(A.11) and (A.13) imply  $\mu_L = \xi_L L^S \frac{1 - (1 - \gamma)^\sigma \eta^{\sigma - 1}}{\rho}$ . Continuity of the state and co-state variables and the transversality condition  $\lambda_R(T') Q_R(T') = 0$  then imply that the end conditions are  $\theta_R(T') = (1 - \gamma)^\sigma \eta^{\sigma - 1}$ ,  $\mu_L(T') = \frac{\xi_L L^S}{\rho}$ ,  $z(T') = 0$ , where the end condition for  $\mu_L$  uses  $C(T'^-) = C(T'^+) \Leftrightarrow L(T'^-) = [1 - (1 - \gamma)^\sigma \eta^{\sigma - 1}] L(T'^+)$ .

The start conditions of the regime with  $D_R = 0$  and  $D_L > 0$  depend on the initial resource stock  $S_0$ . By combining (A.9)-(A.10) and using  $y = S/R$ , we find

$$y = \frac{SQ_R}{LQ_L} \left( \frac{1 - \theta_R}{\theta_R} \frac{1 - \gamma}{\gamma} \right)^{\frac{\nu}{\nu - 1}}. \quad (\text{A.19})$$

Taking the total differential and using (A.17a)-(A.17b) we obtain

$$\dot{y} = -(1 - \theta_R)(1 - \nu)\xi_L L^S y + \rho y - 1. \quad (\text{A.20})$$

The end condition reads  $\lim_{t \rightarrow \infty} y(t) = \rho^{-1}$  if the transition leads to the internal steady state, and  $y(T') = 0$  if the transition leads to the backstop technology regime. In each case, the solution to the differential equation (A.20) yields a relationship between  $y$  and  $\theta_R$ . Condition (A.19) gives a relation between  $y(0)$  and  $\theta_R(0)$ . Combining this relationship with the solution to the differential equation (A.20) gives  $y(0)$  and  $\theta_R(0)$ . The dynamic system described by (A.17a)-(A.17c) can now be used to determine  $\mu_L(0)$  and  $z(0)$ .

The start condition of the regime with  $D_L = 0$  and  $D_R > 0$  is immediately obtained from combining (A.12) and (A.16), yielding

$$\frac{\theta_R(0)}{1 - \theta_R(0)} = \left( \frac{1 - \gamma}{\gamma} \right)^\sigma \left( \frac{Q_R(0)S(0)}{Q_L(0)} \xi_R \right)^{\nu-1}, \quad (\text{A.21})$$

where we have used  $\lambda_R Q_R = \lambda_S S$ , which can be seen by defining  $V_R(t) \equiv e^{-\rho t} \lambda_R(t) Q_R(t)$ , implying

$$\dot{V}_R(t) = -\lambda_S(0)R(t) \Rightarrow V_R(t) = V_R(\infty) + \lambda_S(0) \int_t^\infty R(\tau) d\tau = \lambda_S(0)S(t), \quad (\text{A.22})$$

where the first equality uses (A.9), (A.14), and (A.15) and the second equality uses the transversality condition  $\lim_{t \rightarrow \infty} V_R(t) = 0$  (cf. Amigues and Moreaux, 2008, p. 18).

Trajectories that satisfy the first-order conditions and the transversality condition are labeled ‘candidate solutions’ to the social planner problem. On the basis of our analysis so far, the solution can be stated as in Proposition A.1.

**Proposition A.1** *Assuming that  $\theta_R(0) < \xi_R^{-1}$ , the following two scenarios may occur:*

- (i) *If  $\theta_H < \xi_R^{-1}$  the unique solution to the planner problem is a transition to the backstop technology. Throughout the transition,  $D_R = 0$ .*
- (ii) *If  $\theta_H > \xi_R^{-1}$  there are two candidate solutions to the planner problem: a transition to the backstop technology and a transition to the internal steady state. Along both transitional paths, the  $D_R = 0$  regime occurs. The choice between the two candidate solutions depends on the development of  $z(t)$  along the two paths:*

(a) If  $z(t) < 1$  along the entire path towards the backstop technology, both candidate solutions remain.

(b) If  $z(t) \geq 1$  for some  $t$  along the path towards the backstop technology, the unique solution to the planner problem is a transition to the internal steady state.

**Proof.** To prove part (i), note that the costs per unit of energy generated with the resource cannot be higher than the cost per unit of energy generated with the backstop technology, because they are perfect substitutes in production. Hence, the internal steady state with  $\theta_R = \xi_R^{-1}$  cannot be reached if  $\theta_H < \xi_R^{-1}$ , so that the transitional path leading to the internal steady state can be excluded as a candidate solution. The  $D_L = 0$  regime cannot occur, because it follows from  $z > 1$  that  $\frac{\xi_L}{\xi_R} < \frac{\lambda_R Q_R}{\lambda_L Q_L}$ . Together with  $\frac{\theta_R}{\theta_L} < \frac{\xi_L}{\xi_R}$  this implies  $\frac{\theta_R}{\theta_L} < \frac{\lambda_R Q_R}{\lambda_L Q_L}$ . By using this inequality in combination with (A.13)-(A.14), we find  $\hat{z} = \frac{\theta_L}{\lambda_L Q_L} - \frac{\theta_R}{\lambda_R Q_R} > 0$ . Therefore,  $\dot{z} > 0$ , so that  $z$  will never reach unity. Hence, the  $D_R = 0$  regime must apply.

To prove part (ii), note that the costs per unit of energy generated with the resource are lower than the costs per unit of energy generated with the backstop technology along the entire transition path to the internal steady state. Therefore, this path can no longer be excluded as a candidate solution. The argument from part (i) can be used to prove that the  $D_R = 0$  regime must apply along the transitional path to the internal steady state, along which  $\theta_R < \xi_R^{-1}$ . It follows from the transversality condition  $\lambda_R(T')Q_R(T') = 0$  along the transitional path to the backstop technology that  $z < 1$  just before the switch to the backstop technology, implying that  $D_R = 0$ . If  $z < 1$  throughout the transition to the backstop technology,  $D_R = 0$  from the beginning. However, if  $z$  becomes larger than unity, a preceding  $D_L = 0$  regime must exist. But during this regime, condition (A.21) implies that the energy income share at a particular moment in time for a given  $S$ ,  $Q_R$ , and  $Q_L$  is independent of the backstop productivity parameter  $\eta$ , which is only possible if the economy converges to the internal steady state instead of to the backstop technology. Therefore, if  $z$  reaches unity during the  $D_R = 0$  regime, the transitional path leading to the introduction of the backstop technology can no longer be a candidate solution. The transition towards the internal steady state remains as the unique solution to the planner problem.  $\square$

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