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CAPITAL FORMATION, RISK TAKING, AND TAXATION*

KAI A. KONRAD**

I. INTRODUCTION

Profit taxes and depreciation allowances have effects on capital formation and risk taking. Both effects have been discussed extensively, but within different frameworks.

On the one hand, the incentive effects of regimes of capital income taxation on capital formation are treated under conditions of certainty. Sinn [1985], [1987] developed a disaggregated intertemporal general equilibrium framework to analyse neutrality properties of various tax regimes, covering most of the issues treated in the literature on capital income taxation. The question then arises of whether the results that have been obtained in this work, particularly the neutrality results, carry over to the case of uncertainty.¹

The "supply side" of capital and risk taking under uncertainty has been studied by several approaches. Domar and Musgrave [1944] in a seminal paper consider the portfolio selection problem of an investor who splits a given amount of capital between two assets, one riskless, the other risky. The question of how various taxes change the portfolio structure is asked. The basic problem has been generalized in several directions.² The central mechanisms scrutinized by the risk taking and taxation literature is described, e.g., by Feldstein [1969, p. 755]. A tax on revenues decreases both the yield and the risk of a given portfolio. It reduces the investor's amount of risk bearing and may lower his resistance to assuming more risk. Both effects induce the investor to shift resources from the safe to the risky investment opportunities.

Investment and risk taking may be joint inputs of economic activity and perhaps they can be varied only together and in fixed proportions. Capital and risk involved in production may be perfect complements. In this case a tax on the difference between profits and opportunity costs of capital, i.e., a tax on risk remuneration creates incentives for additional capital formation. This has been shown by Konrad [1989] in an intertemporal general equilibrium. Domar and Musgrave [1944, p. 391] pointed out that "a shift towards a more risky investment (or rather asset combination) may be accomplished by reducing the proportion of the investor's total assets held in cash, that is, by larger total investment, or through a change from less to more risky investments". This suggests that risk taking and investment are two different activities. What, however, will happen, if risk taking and capital are not complements, but instead are close substitutes? This question is the motivation for the present paper. I consider an intertemporal equilibrium with output uncertainty. The approach is similar to my [1989] paper but it takes the suggestion of Domar and Musgrave seriously,

allowing for independent variation of risk taking and capital formation. The analysis is general equilibrium except for taking into account public expenditures.³ The impact of different profit taxes and depreciation allowances on both capital formation and risk taking is analysed, generalizing the neutrality of cash-flow taxes to the case of uncertainty, and generalizing some results about the impact of taxation on risk taking to the intertemporal case. The model allows both depreciation allowances that are equal to true economic depreciation and the case of accelerated depreciation allowances to be considered.

Further, I consider the impact of a tax that is levied only on risk remuneration, i.e., a tax on the difference between returns that accrue in the risky production and those of the safe production. Finally, the results are summarized.

II. AN INTERTEMPORAL EQUILIBRIUM WITH GENERAL PROFIT TAXES

Consider a representative household at time $t = 0$, owning and managing a firm that represents the production sector of the economy. This firm is endowed with an initial capital stock $K(0) > 0$ and with the intertemporally constant flow of a non-capital resource $N(t) = \bar{N}$ that may be called labour, e.g., the labour of the owner-manager. The "firm" consists of two smaller plants with different production technologies, represented by their production functions

$$(1) \quad Y^s(t) = F[K^s(t), N^s(t)],$$

$$(2) \quad Y^u(t) = [1 + \vartheta(t)]F[K^u(t), N^u(t)].$$

In both technologies (or plants) capital (K) and labour (N) are used to produce the same kind of perishable output good that can be invested or consumed. Superscripts s and u denote amounts of inputs and output in the safe way of production that is described by eq. (1) and the uncertain (stochastic) way to produce that is described by eq. (2). Consider first the function F that is the safe production function, but is also part of the risky production function.⁴ F is assumed to be linear homogeneous with partial derivatives $F_K > 0$, $F_N > 0$, $F_{KK} < 0$, $F_{NN} < 0$ and $F_{KN} = F_{NK} > 0$, similar to the ordinary production function in one-sector neoclassical growth models. Consider now the risky production. It differs from the safe way by the factor $1 + \vartheta(t)$. Realizations of the random variable $\vartheta(t)$ are "white noise", i.e., all $\vartheta(t)$ are identically distributed and stochastically independent with

$$(3) \quad E\vartheta(t) = \bar{\vartheta} > 0.$$

Production in the second plant is risky, because $\vartheta(t)$ is assumed not to be degenerate, and it has a higher expected yield (given the same amounts of inputs) than the safe

way of producing, because $E[1 + \vartheta(t)] = 1 + \bar{\vartheta} > 1$. The risky production function (2) is a frequently used special case of Diamond's [1967] stochastic production function.

The modeling of production here is motivated by the remark of Domar and Musgrave [1944] that has been cited in the introduction. To consider the effects of profit taxation on risk taking and capital formation, the representative entrepreneurial household should be able to change these variables independently. If capital and risk taking are perfect complements, a positive shift in both might be due to an incentive to increase capital or an incentive to increase risk taking. Certain taxes might create incentives for risk taking and, at the same time, reduce investment incentives. Therefore, one should allow for independent reactions of the economy with respect to these variables.

Using the assumption of linear homogeneity of F , the technology that is given by (1) and (2) can be transformed into considerably simplified expressions. The total amounts of resources are distributed between both technologies, such that

$$(4) \quad K^s(t) + K^u(t) = K(t)$$

$$(5) \quad N^s(t) + N^u(t) = N(t) = \bar{N} \equiv 1.$$

As $N(t)$ is a constant flow, its amount can be set equal to one when the unit of labour is chosen appropriately. Considering that capital and labour can be moved arbitrarily between plants, technical efficiency requires that capital intensities are the same in the safe and in the risky technology, ⁵ i.e.,

$$(6) \quad K^s(t)/N^s(t) = K^u(t)/N^u(t) = K(t)/N(t) = K(t).$$

Moreover, this allows

$$f(K) \equiv F(K, 1)$$

to be defined, suppressing the time index here and hereafter except for some cases where special emphasis is necessary for clarification. Let $\eta \in [0, 1]$ be the fraction of resources (capital and labour) that is used in the risky industry. In this case, using (4), (5), (6), the linear homogeneity of F and the definition of f , the output of both plants u and s can be rewritten as

$$(7) \quad Y^s = (1 - \eta)f(K)$$

$$(8) \quad Y^u = (1 + \vartheta)\eta f(K),$$

with first and second derivatives $df/dK \equiv f' > 0$ and $d^2f/(dK)^2 \equiv f'' < 0$.

The capital stock K depreciates with a constant rate δ , independently of the technology in which it is used. Therefore, gross investment I^{br} consists of capital replacement δK and net investment $I = I^{br} - \delta K$. The change of capital stock can be described by

$$(9) \quad I = \dot{K} = I^{br} - \delta K$$

(the time derivative of a variable generally will be denoted by a dot, e.g., $dK/dt \equiv \dot{K}$). The consumption-saving decision is made by the choice of $I(t)$. It is assumed here that there is a decision lag. The choice of $I(t)$ has to be made before the true $\vartheta(t)$ of that period is known. The time interval of the decision lag might be infinitely small, but the entrepreneurial household cannot make the decision on $I(t)$ dependent on $\vartheta(t)$. For its choice of $I(t)$, however, a household can take all previous realizations $\vartheta(t - \Delta)$ for $\Delta > 0$ into account.⁶

Together with the exclusion of storage and stochastically independent realizations of ϑ , the assumption of a decision lag breaks down the complexity of the stochastic dynamic optimization problem to be described in (14) and allows a simple analytical solution to be reached. In a more general case the information obtained by the realizations of ϑ may affect the savings decision ex post and some of the risks can be reduced by some kind of "intertemporal smoothing". Moreover, correlation of the ϑ 's may also be used for adjusting the investment path. This may affect the steady-state values of capital stock and of risk taking. However, these effects are at work independently of whether tax rates are higher or not. A priori, there is no reason why allowances for intertemporal smoothing should change the results about the impact of tax rate changes qualitatively.

There are the following taxes and depreciation allowances: all revenues are taxed by a proportional general profit tax. The tax factor is t_p . The taxable revenue at each instant of time is defined here as total output minus depreciation allowances. The resource N is owned by the firm. Deductions of factor costs for N are therefore not allowed. Except for an income effect, allowing deductions of labour costs would not change the results, as the supply of N is exogenous and constant. Deduction of true or imputed interest costs is not allowed, all interest income is subject to taxation.

Deductions for depreciation might be allowed. Depreciation allowances for tax purposes may equal true economic depreciation or may be accelerated. All capital goods, however, can be written off only once. Such accelerated depreciation allowances are described by Sinn [1987, pp. 59n.]. It is assumed that a certain fraction $0 \leq \alpha \leq 1$ of gross investment expenditures can be written-off immediately. The remainder, $(1 - \alpha)$, is written-off geometrically, i.e., proportionally to true economic depreciation. Given a flow of gross investment $\delta K + I$, the flow of immediate depreciation is $\alpha(\delta K + I)$, and, as true economic depreciation is δK , the flow of depreciation on existing assets

is $(1 - \alpha)\delta K$. Hence, the aggregate current flow of depreciation for tax purposes is

$$(10) \quad A = \alpha(I + \delta K) + (1 - \alpha)\delta K = \delta K + \alpha I.$$

In the above framework, the case of an immediate write-off is obtained by setting $\alpha = 1$, while $\alpha = 0$ corresponds to depreciation allowances that are equal to true economic depreciation. Independently of α , this feature satisfies the constraint that each capital good can be written-off only once: accelerated depreciation allowances only allow an earlier deduction than true economic depreciation, but do not mean additional depreciation allowances, i.e., writing off more than the total capital stock, as is typically assumed in many two-period models.⁷

A general profit tax with immediate write-off corresponds to some proposals on capital income tax reforms to implement a cash-flow tax. In particular, it is a description of Sinn's [1984, 1985, 1987, 1989] mixed-system proposal, i.e., a system of capital income taxation with an immediate write-off and equal tax rates on retained profits and of personal taxes on interest income and a dividend tax at least as high as this rate. The approach that is chosen here cannot discriminate between different cash-flow systems, because here no difference is made between equity finance and debt finance. Another extremely relevant tax regime, however, is captured by the case $\alpha = 0$. This case is very similar to the existing system of capital income taxation in the Federal Republic of Germany.

II. A. The Choice Problem of the Entrepreneurial Household

Consider now the consumption possibilities of the household. The flow of consumption goods equals output $Y \equiv Y^u + Y^s$ minus gross investment I^{br} , or, using eqs. (7), (8), and (9),

$$(11) \quad Y - I^{br} = (1 + \eta\vartheta)f - \delta K - I.$$

A proportional tax with tax rate τ_p is levied on output net of depreciation allowances for tax purposes. Using eqs. (10) and (11) yields

$$(12) \quad T_p = \tau_p[(1 + \eta\vartheta)f - \delta K] - \alpha\tau_p I.$$

Tax revenue T_p equals τ_p times the output net of replacement of capital (first term of (12)) minus a partial rebate of taxes on net investment, corresponding to the allowances for an immediate write-off. Consumption net of tax equals the amount of output that is not invested, diminished by the tax payments T_p , i.e., $C = Y - I^{br} - T_p$, or, using eqs. (11) and (12),

$$(13) \quad C = (1 - \tau_p)[(1 + \eta\vartheta)f - \delta K] - (1 - \alpha\tau_p)I.$$

Consider now the calculus of the representative household: at each instant of time t , the household enjoys the flow of consumption goods. The household is interested in expected utility, $EU[C(t)]$ of the random consumption flow, with U a strictly concave von Neumann-Morgenstern utility function, implying that the household is risk averse. It maximizes the sum of all instantaneous expected utility, discounted by the subjective rate of time preference, ρ . The decision variables of the household are I , the investment choice, and η , the choice of the mixture of safe and risky production. The optimization problem is

$$(14a) \quad \max_{\{I(t), \eta(t)\}} \int_0^{\infty} EU[C(t)]e^{-\rho t} dt$$

s.t.

$$(13) \quad C = (1 - \tau_p)[(1 + \eta\vartheta)f - \delta K] - (1 - \alpha\tau_p)I,$$

$$(9) \quad \dot{K}(t) = I(t),$$

$$(14b) \quad K(0) > 0,$$

$$(14c) \quad K(t) \geq 0,$$

$$(14d) \quad \eta \in [0, 1).$$

Eq. (13) determines how consumption depends on production possibilities and the choice of investment (I) and riskiness (η). Eq. (9) is the equation of motion. Eq. (14b) determines the initial capital stock. The non-negativity condition (14c) and the requirement that both technologies can be used only on non-negative activity levels, (14d), will be assumed not to be binding. The household chooses a time path of I and η . Notice that at each instant of time t_0 the household may reconsider the path $\{I(t)\}$ and $\{\eta(t)\}$ for all $t > t_0$, given the restrictions and the household's new information. However, although there is production uncertainty, the stochastic intertemporal problem boils down to an ordinary intertemporal problem, because the household can choose a unique savings plan at time $t = 0$ that stays optimal later. This is due to two reasons. (i) In each $t > 0$ the household has to choose $I(t)$ and $\eta(t)$ before production takes place, because there is a decision lag. The true $\vartheta(t)$ is revealed via production, and, therefore, $I(t)$ and $\eta(t)$ are chosen before the household knows whether t will be a fortunate or an unfortunate date, i.e., whether $\vartheta(t)$ will be high or low. Given the choice of $I(t)$, the change in capital stock is determined and, therefore, the savings-decision for t is irreversibly made. Given $\vartheta(t) > \bar{\vartheta} (< \bar{\vartheta})$, the household would perhaps like to save more (less) than this, but ex post — by virtue of the decision lag — such changes are impossible. Moreover, goods are assumed here not to be storable. Therefore, the household cannot smooth the volatility of the consumption path by intertemporal transfers "ex post", i.e., reacting to $\vartheta(t)$ in the same period.

(ii) While in any period $t_0 > 0$ the investment decision, i.e., the change of the stock of capital in period t_0 cannot be revised after production has occurred and $\vartheta(t)$ has become known, the household in t_0 may deliberately change its savings plan for all points of time $t > t_0$. But why should it? A moment after t_0 , i.e., in $t_0 + dt$, the household has exactly the capital stock as was planned in $t = 0$ to have in this period. Consumption goods that accrued in $t \leq t_0$ could not be transferred to $t_0 + dt$, the actual trajectory of consumption $\{C(t)\}_{t \leq t_0}$ has no impact on the production possibility locus of the household in $t_0 + dt$. The only new information that the household obtains, the realization of $\{C(t)\}_{t \leq t_0}$ is irrelevant for the actual savings decision because the $\vartheta(t)$ are stochastically independent. The "stochastic dynamic optimization problem" therefore can be solved by the simple maximum principle of Pontrjagin. The current-value Hamiltonian⁸ is

$$\mathcal{H} = EU(C) + \lambda I,$$

with the costate variable λ . Using the first-order conditions $\partial \mathcal{H} / \partial I = 0$, $\partial \mathcal{H} / \partial \eta = 0$ and $-\partial \mathcal{H} / \partial K = \dot{\lambda} - \rho \lambda$ one gets

$$(15) \quad E[U'(C)\partial C/\partial I] + \lambda = E\{U'(C)[-(1 - \alpha\tau_p)]\} + \lambda = 0,$$

$$(16) \quad fE[U'(C)\vartheta] = 0$$

and

$$(17) \quad -E\{U'(C)(1 - \tau_p)[(1 + \eta\vartheta)f' - \delta]\} = \dot{\lambda} - \rho\lambda,$$

with $U' \equiv dU/dC$ (and $U'' \equiv d^2U/(dC)^2$ in what follows).

Conditions (15) - (17) characterize the optimal choices $\{\eta(t)\}$, $\{I(t)\}$ and, therefore, the path of the capital stock. They could be used to determine the effects of changes of the tax rate and of depreciation allowances on the optimal adjustment path. The analysis in this section, however, is concerned with the allocative, i.e., the long-run effects of tax rate changes, and, therefore, the steady-state equilibrium values of capital stock and risk taking are considered.

II. B. Properties of the Steady State

A steady-state equilibrium⁹ fulfills the condition $\dot{\lambda} = 0$, implying that (15) and (17) can be transformed to

$$(18) \quad E[U'(C)\{(1 - \tau_p)[(1 + \eta\vartheta)f' - \delta - \rho] - \rho\tau_p\}] = 0,$$

or, using (16),

$$(19) \quad f' - \delta - \rho = \rho\tau_p(1 - \alpha)/(1 - \tau_p).$$

Eq. (19) says that, in the steady state, for given optimally chosen η , the expected utility cannot be increased by an increase or decrease in savings. Some conclusions can be made immediately from (19). In the certainty case ($\eta = 0$) (19) describes the well-known (cf. e.g., Sinn [1987]) steady-state equilibrium condition in a one-sector growth equilibrium with capital income taxation for different depreciation allowances for tax purposes. For an immediate write-off we get the laissez-faire condition: marginal productivity of capital net of depreciation equals the subjective rate of time preference. Moreover, if depreciation allowances are restricted to equal true economic depreciation, (i.e., $\alpha = 0$) the analysis reproduces the well-known distorting properties of a general capital income tax: $(f' - \delta)(1 - \tau_p) = \rho$, i.e., net marginal product of capital after tax equals the subjective rate of time preference.

Proposition 1

The introduction or an increase of a general profit tax with an immediate write-off does not change the aggregate steady-state capital stock.

Proof: Insert $\alpha = 1$ in (19) to get $f' - \delta = \rho$ for any $\tau_p \in (0, 1)$. As $f'' < 0$, the steady-state capital stock that fulfills $f' - \delta = \rho$ is uniquely determined.

Proposition 1 generalizes the neutrality property of a system of capital income taxation with full loss offset without interest deductibility to the case of uncertainty. A look at (19) also reveals:

Proposition 2

Given any $\tau_p \in (0, 1)$, the steady-state capital stock is increasing in α .

For a proof consider (19). $d/d\alpha [f' - \delta - \rho] = -\rho\tau_p/(1 - \tau_p) < 0$, but $f'' < 0$. Proposition 2 shows that the distorting properties of a profit tax with depreciation allowances that are equal to true economic depreciation also appear in the case of uncertainty. An increase of depreciation allowances increases the steady-state capital stock. For the case of certainty, this incentive effect of accelerated depreciation allowances has been shown by Sandmo [1974] and Boadway and Bruce [1979].

Proposition 3

Given that tax laws allow for an immediate write-off, an increase of a general profit tax τ_p increases (decreases, keeps constant) the fraction η of the risky production if relative risk aversion is increasing (decreasing, constant).

The proof is given in Appendix 2. Proposition 3 contains one of the central results of the paper. The risk-taking incentive of profit taxes is well established within the

static portfolio theory. Attempts to generalize the result to the intertemporal case, however, revealed that the income effects of a profit tax make its effect on risk-taking rather indeterminate, except for rather particular assumptions on risk aversion (cf. e.g., Ahsan [1989]). Proposition 3 shows that, — in contrast to this indeterminateness — there is a unique relation between relative risk aversion and risk taking for the case of immediate write-off allowances. Under plausible assumptions profit taxes *increase* risk taking.

If depreciation allowances are smaller than immediate write-off allowances (i.e., $\alpha < 1$), then a profit tax induces a steady-state capital stock that is lower than in the laissez-faire (cf. proposition 2) case. The amount of capital which can be distributed between the two technologies is lower. Income — or wealth effects in this case prevail, and, in general, the effect of an increase of profit taxes on risk taking depends to a large extent on the parameters and results are far less clear.

Notice that in propositions 2 and 3 the perfect substitutability of capital and risk taking turns out to be crucial. Given an immediate write-off ($\alpha = 1$), the profit tax does not alter the steady state capital stock, but increases risk taking. If households are able to choose their amount of risk taking and their amount of capital perfectly independently, then households may use this degree of freedom.

III. THE CASE OF A TAX ON "PURE RISK REMUNERATION"

Suppose now that government does not tax all profits. Instead, all profits that would accrue if production took place only in the safe way of production, are tax exempt. Only those revenues are taxed that exceed safe revenues. Assume that this capital-gains tax, or, probably more precisely, "risk remuneration tax" is strictly proportional, τ_r being the tax rate. The deviation of output from that amount of goods that could be produced if the safe technology only were used, equals $\eta\vartheta f$. The tax on risk remuneration therefore equals

$$(20) \quad T_r = \tau_r \eta \vartheta f.$$

Replacing (12) by (20) yields, instead of (13), consumption as

$$(21) \quad C = [1 + (1 - \tau_r)\eta\vartheta]f - \delta K - I.$$

Replacing restriction (13) with (21), maximization of (14) yields the first order optimality conditions $\partial\mathcal{H}/\partial I = 0$, $\partial\mathcal{H}/\partial\eta = 0$ and $-\partial\mathcal{H}/\partial K = \dot{\lambda} - \rho\lambda$, or, inserting,

$$(22) \quad -EU'(C) + \lambda = 0,$$

$$(23) \quad fE[U'(C)\vartheta] = 0,$$

$$(24) \quad -E\{U'(C)[(1 + (1 - \tau_r)\eta\vartheta)f' - \delta]\} = \dot{\lambda} - \rho\lambda.$$

Consider first the impact of a pure risk remuneration tax on risk taking.

Proposition 4

Given the production technology that is described by (1) and (2), and a pure tax on risk remuneration that is described by (20), an increase of the tax rate induces a change of the fraction η of risky production by $d\eta/d\tau_r = \eta/(1 - \tau_r)$.

Proof: The result of proposition 4 is equivalent to the ordinary Domar-Musgrave result. The proof therefore follows similar lines. Differentiation of eq. (23) yields

$$E\{U''(C)[\vartheta f(1 - \tau_r)\vartheta f]\}d\eta - E\{U''(C)[\vartheta f\eta\vartheta f]\}d\tau_r = 0,$$

or, equivalently,

$$d\eta/d\tau_r = E[U''(C)\vartheta^2 f^2 \eta]/E[U''(C)\vartheta^2 f^2 (1 - \tau_r)] = \eta/(1 - \tau_r).$$

Proposition 4 states that the Domar-Musgrave phenomenon (cf., e.g., Mossin [1968], p. 75 n.) appears also in this intertemporal framework.¹⁰ Notice that for proposition 4 to hold it is not required that the change of tax rates happens in the steady state equilibrium, but it holds generally, even on an adjustment path towards a steady-state.

Consider the introduction of a risk remuneration tax with a small tax rate τ_r (similar reasoning would apply to a small increase of the tax rate). Let η_0 be the optimal choice of η before the tax is introduced. By proposition 4, then, $\eta_{\tau_r} \equiv \eta_0/(1 - \tau_r)$ is the optimal choice of η for small $\tau_r > 0$. Using this result in (21) reveals that the path $\{C(t)\}$ of consumption (after tax) is not altered by the introduction of the tax if, additionally, the household does not alter its investment decisions. As in the atemporal Domar-Musgrave framework, the household can restore its (net of tax) consumption plan in every instant of time. Considering (22) and (24) moreover reveals that, given the reaction $\eta_{\tau_r} \equiv \eta_0/(1 - \tau_r)$, these conditions also reduce to the laissez-faire conditions. And, given the same net consumption path $\{C(t)\}$ as in the laissez-faire, these conditions remain fulfilled for this path, implying that the before-tax-change investment path stays optimal.

Proposition 5

A tax on pure risk remuneration does not distort the intertemporal allocation of capital.

A pure risk remuneration tax is very similar to a general profit tax with an immediate write-off. Propositions 1 and 5 show that both taxes are neutral, even in the case of uncertainty. Both tax regimes induce very similar behavior of households, but tax revenues are quite different: the tax revenues of a tax on pure risk remuneration are

$\tau_r \eta \theta f$. Given equal tax rates $\tau_r = \tau_p$, in the steady state the tax proceeds of a general profit tax with immediate write-off are higher by $\tau_p(f - \delta K)$.

IV. CONCLUSIONS

Do profit taxes induce a higher level of investment under uncertainty? It is shown that in an intertemporal equilibrium profit taxes and taxes on pure risk remuneration do not lead to a steady state with a higher stock of capital, if risk and capital are perfect substitutes. Instead, even under uncertainty, capital formation is determined mainly by the relation between the subjective rate of time preference and marginal productivity of capital.

Some results of the income taxation literature carry over to the case of uncertainty. The distorting properties of profit taxes in particular are similar to the certainty case, leading to a smaller steady-state stock of capital. Moreover, a general income or profit tax with an immediate write-off and a tax on pure risk remuneration have been shown to be neutral under certain conditions.

A minor point of criticism might be that the model here is not disaggregated. The representative firm and the household are identical. Obviously this assumption has been made for simplicity. If the model is disaggregated, under uncertainty problems of unanimity of shareholders and the appropriate aim of the firm have to be considered. Leland [1974] proposes that a reasonable aim of firms is the maximization of the utility of their shareholders. With regard to this assumption, there does not seem to be very much difference between an aggregated and a disaggregated model.

One of the questions that motivated this analysis was whether profit taxes indeed induce a higher level of investment under uncertainty, as Cansier [1989a, 1989b] suggested and also Konrad [1989] derived for the case of complementarity. It is shown here that in an intertemporal equilibrium profit taxes and taxes on pure risk remuneration do not lead to a steady state with a higher stock of capital, if risk and capital are perfect substitutes. Instead, capital formation, even under uncertainty, is determined mainly by the relation between the subjective rate of time preference and marginal productivity of capital. Comparing the results of this paper with the results of Konrad [1989] where fixed proportions of aggregate capital formation and aggregate risk taking were considered, the question arises of what happens in the intermediate case between the extremes of perfect substitutability and perfect complementarity of capital and risk taking. Considering risk taking as a factor of production in Pigou's [1929] and Sinn's [1986] sense, activity-analytical considerations suggest that an increase of wisely chosen risk-bearing may increase the productivity of capital and vice versa. Risk taking and capital in this case are not fully independent but perhaps not perfect complements. A thorough analysis of taxation in this case must be left to future research.

The effect that a tax on pure risk remuneration and a profit tax have on risk taking has also been considered. It has been shown that the result of the Domar-Musgrave literature concerning "capital gains taxes", i.e., taxes on pure risk remuneration, holds in the intertemporal case. While the impact of profit taxes on risk taking is known to be rather indeterminate for the intertemporal case, risk taking could be shown to be encouraged by a general profit tax with an immediate write-off under plausible assumptions about risk aversion.

NOTES

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¹ Cf. Richter [1986] and Fane [1987] for some results on the Johansson-Samuelson theorem in case of uncertainty.

² For an overview of the literature on risk-taking and taxation see Sandmo [1985] and for a unified approach Buchholz [1987].

³ Tax proceeds are assumed to be used in a "neutral" way, here. This assumption is standard in the Domar-Musgrave framework (for exceptions see Stiglitz [1972], Atkinson and Stiglitz [1980], and Gordon [1985]). The results of taking the use of tax proceeds into consideration depends on the stochastic properties of tax proceeds, their use for lump-sum transfers or public goods and on how this use enters the utility of households. For some discussion see Konrad [1991].

⁴ It is crucial for obtaining perfect substitutability of risk taking and capital formation that F is the same function in both plants. Changes in the relative size of the two plants do not affect the relative scarcity of factors, and therefore, the shadow prices of factors. The assumption involves a sacrifice of generality which virtually all one-sector-growth models have in common.

⁵ See Appendix 1 for a formal proof. Efficiency requires the equality of the ratios of marginal productivity of labor and capital in both plants. This can be verified by an arbitrage argument. (6) follows then from linear homogeneity of F . If, e.g., the ratio $(\partial F/\partial K^u)/(\partial F/\partial N^u) > (\partial F/\partial K^s)/(\partial F/\partial N^s)$, then a reallocation of labour and capital is possible, decreasing N^u (increasing N^s) and increasing K^u (decreasing K^s), which does not change Y^u , but increases Y^s .

⁶ In reality there is a decision lag. Firms cannot base their decision to buy a machine on the actual realization of stochastic output of this machine.

⁷ The similarity and difference between additional and accelerated depreciation allowances have been pointed out by Boadway and Bruce [1979].

⁸ For technical details see Feichtinger and Hartl [1986].

⁹ It is well known from the one-sector growth model that a steady-state equilibrium does not necessarily exist. Existence depends on the shapes of $F(K, N)$ and $U(C)$ and appropriate restrictions on the distribution of ϑ but can be assumed here. Moreover, notice that if a capital stock $K^* > 0$ exists such that (15), (16), and

(17) are fulfilled for $I = \dot{\lambda} = 0$, the sufficient conditions (cf. Feichtinger and Hartl [1986], pp. 40 and 43) for K^* to be an intertemporal optimum of the household are also fulfilled, as $K(t) \equiv K^* \geq 0$ and constant, and, therefore, bounded, and using (15), for $\dot{\lambda} = 0$, $\lim_{t \rightarrow \infty} \lambda(t) e^{-\rho t} = (1 - \alpha \tau_p) E U'(C) \lim_{t \rightarrow \infty} e^{-\rho t} = 0$.

¹⁰ For a similar result in the two-period case in a partial analytic framework see Ahsan [1989].

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APPENDIX 1

Let K and N be the available amounts of capital and labour. It is shown that efficiency requires that factor intensities are the same in both technologies. The household solves the problem

$$\max EU(C)$$

s.t.

$$C = (1 + \vartheta)F(K^u, N^u) + F(K^s, N^s) - I^{br},$$

$$K = K^u + K^s,$$

$$N = N^u + N^s.$$

The first-order conditions are

$$E[U'(C)(1 + \vartheta)\partial F(K^u, N^u)/\partial K^u] = E[U'(C)\partial F(K^s, N^s)/\partial K^s] = \mu_1,$$

$$E[U'(C)(1 + \vartheta)\partial F(K^u, N^u)/\partial N^u] = E[U'(C)\partial F(K^s, N^s)/\partial N^s] = \mu_2,$$

with μ_1 and μ_2 being the shadow prices of capital and labour, respectively. Dividing these equations by each other, and taking into account that F is not stochastic, yields

$$[\partial F(K^u, N^u)/\partial K^u]/[\partial F(K^u, N^u)/\partial N^u] = [\partial F(K^s, N^s)/\partial K^s]/[\partial F(K^s, N^s)/\partial N^s] = \mu_1/\mu_2.$$

Given that F is linear homogeneous, these terms only depend on the ratios K^u/N^u and K^s/N^s and they are equal if these ratios are equal. Therefore, the efficient use of factors implies that the factors are used in the same proportion in both the certain and the uncertain production technology.

APPENDIX 2

Proof of proposition 3. In case of an immediate write-off, a general profit tax does not change the steady-state capital stock (see proposition 1). The capital stock K that fulfills (18) of the main text is constant in the steady-state, implying that $I = \dot{K} = 0$. Differentiation of (16) of the main text in the steady state therefore yields

$$E[U''(C)(1 - \tau_p)\vartheta^2 f^2]d\eta = E\{U''(C)\vartheta f[(1 + \eta\vartheta)f - \delta K]\}d\tau_p$$

$$\Leftrightarrow d\eta/d\tau_p = E\{U''(C)f\vartheta[(1 + \eta\vartheta)f - \delta K]\}/E\{U''(C)(1 - \tau_p)\vartheta^2 f^2\}.$$

As $E[U''(C)(1 - \tau_p)\vartheta^2 f^2] < 0$, the sign of $d\eta/d\tau_p$ is the opposite of the sign of the numerator, i.e.,

$$\text{sgn}(d\eta/d\tau_p) = \text{sgn}\{(-1)E[U''(C)f\vartheta((1 + \eta\vartheta)f - \delta K)]\}.$$

Using

$$(A.2.1) \quad (1 - \tau_p)[(1 + \eta\vartheta)f - \delta K] = C$$

in the steady state (cf. eq. (13) of the main text with $I(t) = 0$), $[1/(1 - \tau_p)] > 0$ and $f > 0$, this can be rewritten

$$(A.2.2) \quad \text{sgn}(d\eta/d\tau_p) = \text{sgn}\{(-1)E[U''(C)\vartheta C]\}.$$

With

$$(A.2.3) \quad RRA(C) \equiv (-U''/U')(1 - \tau_p)[(1 + \eta\vartheta)f - \delta K]$$

being the measure of relative risk aversion in the steady state as the steady-state amount of consumption is given by (A.2.1), the following implications can be shown to hold.

$$\text{sgn}(d\eta/d\tau_p) \left\{ \begin{array}{l} \geq \\ \leq \end{array} \right\} 0 \Leftrightarrow \text{sgn}\{E[U''(C)\vartheta C]\} \left\{ \begin{array}{l} \leq \\ > \end{array} \right\} 0 \Leftrightarrow RRA(C) \left\{ \begin{array}{l} \text{increasing} \\ \text{constant} \\ \text{decreasing} \end{array} \right\}.$$

The equivalence (\Leftrightarrow) is due to (A.2.2). The implications (\Leftarrow) are due to Arrow [1970, p. 120]. Arrow's proof is repeated here.

Only the first implication (" $' <'' \Leftarrow$ increasing") is shown here. Let $RRA_0 \equiv (-U''/U')C$ for $\vartheta = 0$. In the case of increasing relative risk aversion it holds that

$$\begin{aligned} & (-U''/U')C < RRA_0 && \text{for } \vartheta < 0, \\ \Leftrightarrow & U''(C)C > -RRA_0 U'(C) && \text{for } \vartheta < 0, \\ \Leftrightarrow & U''(C)C\vartheta < -RRA_0 U'(C)\vartheta && \text{for } \vartheta < 0, \end{aligned}$$

and similarly, it holds that

$$\begin{aligned} & (-U''/U')C > RRA_0 && \text{for } \vartheta > 0, \\ \Leftrightarrow & U''(C)C < -RRA_0 U'(C) && \text{for } \vartheta > 0, \\ \Leftrightarrow & U''(C)C\vartheta < -RRA_0 U'(C)\vartheta && \text{for } \vartheta > 0, \end{aligned}$$

and, taking the expectation of all ϑ , if ϑ is not degenerate (this assumption was made), it can be concluded that

$$E[U''(C)C\vartheta] < -RRA_0 E[U'(C)\vartheta] = 0,$$

as $E[U'(C)\vartheta] = 0$ by eq. (16) of the main text. This completes the proof of proposition 3.

Summary: *Capital Formation, Risk Taking, and Taxation.* — An intertemporal general equilibrium with output uncertainty is considered. It is shown that capital formation under uncertainty is determined mainly by the relation between the subjective rate of time preference and marginal productivity of capital. Some results of the income taxation literature carry over to the case of uncertainty. The distorting properties of profit taxes are similar to the certainty case and lead to a smaller steady-state stock of capital. A general profit tax with immediate write-off and a tax on pure risk remuneration are neutral with regard to the capital stock, but enforce risk taking.

Résumé: *Formation de capital, prise de risque et taxation.* — L'on considère un équilibre général intertemporel avec incertitude sur l'output. Il est montré que la formation de capital sous incertitude est déterminée principalement par la relation entre le taux subjectif de préférence temporel et la productivité marginal du capital. Certains résultats de la littérature sur la taxation du revenu couvrent le cas de l'incertitude. Les propriétés de distorsion des impôts sur le profit sont similaires au cas certain et conduisent à un stock de capital à l'état stationnaire plus faible. Un impôt général sur le profit avec déduction immédiate et un impôt sur la rémunération du risque pur sont neutres eu égard au stock de capital, mais renforcent la prise de risque.

Zusammenfassung: *Kapitalbildung, Risikoübernahme und Steuern.* — Der Aufsatz verallgemeinert einige Ergebnisse zu Fragen nach den Anreizwirkungen von Kapitaleinkommensteuern für die Kapitalbildung und die Risikoübernahme im Rahmen eines allgemeinen intertemporalen Gleichgewichts mit Produktionsunsicherheit. Eine allgemeine Gewinnsteuer dämpft auch bei Produktionsunsicherheit die Kapitalbildung. Allgemeine Gewinnsteuern mit Sofortabschreibung oder die Besteuerung der reinen Risikoentlohnung hingegen sind neutral hinsichtlich der Kapitalbildung, geben aber Anreize zu vermehrter Risikoübernahme.