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# Working Time Accounts and Unemployment 

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#### Abstract

Working time accounts allow firms to smooth their demand for hours employed. Descriptive literature suggests that this reduces turnover and inhibits increase in unemployment during recessions. We model theoretically the optimal choice of hours by a firm with a working time account. We show that working time accounts do not necessarily imply lower turnover. Turnover and unemployment may be inhibited or catalyzed by working time account depending on whether a firm meets economic downturn with surplus or deficit of hours and on how productive this firm is. Adjustment pattern in Germany implies that working time accounts have contributed positively.


JEL Codes: J23, J63, J64
Keywords: Working time accounts, unemployment, Great Recession, Germany

[^0]
## 1 Introduction

[Stylized facts: General] European unemployment has increased dramatically during the recession that followed the global financial crises of 2007-2008 (the so-called Great Recession). Though while a number of major OECD countries have reported soaring unemployment rates, notably the US where unemployment rate has increased by unprecedented 5.5 percentage points reaching $9.9 \%$ at its peak (OECD, 2013), unemployment rate in Germany has shown nearly no changes. ${ }^{2}$ Comparing Germany and the US, although both countries have experienced a sharp decline in real GDP and a substantial reduction in person-hours worked, two important differences can be pointed out. ${ }^{3}$ First, while in the US a wave of firings went through, in Germany instead there was a large-scale decrease in hours worked per person with little job losses. In other words the post-crises adjustment at the German labour market took place on the intensive, rather than on the extensive margin. Second, composition of sectors affected by the crises and patterns of sector-specific post-crises recovery differ widely in the two countries. In Germany it is rather the exporting branch of the manufacturing sector that was hit strongly by the crisis (as measured by the drop in the value added). In the US, to the contrary, housing market, construction, retail services and financial services have suffered most. Germany has recovered faster than the US.
[Policy tool] In a landmark descriptive study Burda and Hunt (2011) look into multiplicity of factors that could help explain the surprisingly weak response of the German unemployment to the crises. Among others, they put forward a particular flexible working hours scheme called working time accounts. Potential of working time accounts is likewise emphasized by Möller (2010) and Rinne and Zimmermann (2013). Working time account is essentially a bookkeeping tool used by firms to track under- and overtime work. Firms that operate working time accounts for their personnel may, for instance, let employees work overtime but do not need to pay for this overtime work. Instead overtime work is written into an account as a "debt" of the firm to its employee, such that at some point in the future the employee may work less, running down overtime hours accumulated on her account. Hourly wage rate as well as per period pay stay constant regardless of whether the employee currently has surplus or deficit on her working time account. There exist limits on the amount of accumulated surplus and deficit of hours. Finally by the end of the pre-specified time interval, called compensation period, the account must be balanced, i.e. both firms' debt to worker and workers' debt to firm, measured in hours, should be equal to zero. ${ }^{4}$
[Stylized facts: Policy tool] Legislative base regulating working time accounts is in place in all the member states of the European Union that acceded the Union prior to eastern enlargement, as well as in some other states of the European Economic Area, e.g. in Norway (Eurofound, 2010). However the actual use of working time accounts is seen only in the

[^1]Nordic countries ${ }^{5}$, Germany and Austria. Such localization is explained by the particular managerial culture in the countries mentioned (Eurofound, 2010). An important part of this culture constitute works councils that enforce functioning of working time accounts at the most disaggregated level, e.g. at the level of a firm or of a department of a firm. Thinking of the German phenomenon, working time accounts in Germany exist for already a long while. Although operating them is not obligatory for firms, their use has become increasingly widespread in the recent past. At the dawn of the financial crises nearly $45 \%$ of all German employees, irrespective of East or West, were possessing such account (Zapf, 2012). ${ }^{6}$
[Conjecture] An interesting fact about working time accounts in Germany relates to dynamics of their balances. While years 2005-2007 saw gradual increase in balances, year 2008 has been marked with their unusual extremely sharp fall (Zapf, 2012). Such dynamics has led the literature (Burda and Hunt, 2011) to suggest the mechanism through which working time accounts may contribute to inhibiting the increase in German unemployment during the Great Recession. It is suggested that by building up surpluses of hours worked in good times and running them down in bad times firms avoid firing workers immediately. A worker will not be fired unless she is compensated for the unpaid overtime hours worked previously. This compensation takes a form of working for a while at reduced hours with no change in workers salary. The latter is consistent with the stylized fact of falling hours worked per person in Germany during the Great Recession. Since the crises in Germany was rather a consequence of a drop in demand for German export goods at the worlds' market, the nature of the negative shock to the economy was temporary. By running down the surplus first, working time accounts postponed job destruction and gave many jobs sufficient time to survive until worlds' demand started showing signs of recovery. Lack of job destruction reflects itself in the absence of increase of the unemployment rate.
[Our findings] In the present paper we show theoretically that operating working time accounts does not necessarily restrain turnover at the firm level when a negative demand shock hits the goods market. The impact of working time accounts crucially depends on two factors: (i) on the productivity of a firm relative to wage cost, and (ii) on whether a firm has a surplus or deficit on its working time accounts in face of a demand downturn. We find that at relatively high-productive firms working time accounts reduce turnover if firms have surplus of hours at their working time accounts and increase turnover if firms have deficit of hours at their working time accounts prior to the adverse demand shock. At relatively low-productive firms converse is true: working time accounts increase turnover if there is surplus of hours at working time accounts and reduce turnover if there is deficit of hours at working time accounts in face of the shock. Thus the general relationship between working time accounts and turnover is ambiguous at best, whereas the conjecture of Burda and Hunt (2011) is just one element at the board of our results. This insight is new to the literature on working time accounts. It demonstrates that countries wishing to implement the accounts after German success need to be aware of their undesirable impacts depending on the state

[^2]of the market and the particular type of the firm.
[Our model] We achieve all our results by constructing a basic intertemporal model of labour demand by a firm that operates a working time account. In this model the firm is a local monopolist that faces uncertainty about future demand at the goods market and chooses working hours subject to constraints imposed by working time account regulations. There is no borrowing, but the firm may invest in a riskless asset. Intertemporal transfer of profits via investment is instrumental for working time accounts to function.

The paper is organized as follows. Section 2 presents the basic model of a firm with a working time account and solves the problem of optimal hours choice. Section 3 discusses properties of the optimal solution and analyses the relationship between working time accounts, turnover and unemployment. Section 4 concludes and sets directions for future research.

## 2 The model

### 2.1 Market structure and characteristics of a firm

- Output and demand at the goods market

A firm is equipped with production technology $Y_{t}=A h_{t}$, where $A$ is the productivity of the firm and $h_{t}$ are actual hours worked per worker. For simplicity we assume that one firm employes just one worker. Let $m_{t}$ denote the demand for produced good. We specify the demand function as in Bentolila and Bertola (1990). We suggest that the firm is a local monopolist, such that the reduced-form demand function is

$$
\begin{equation*}
m_{t}=z_{t} p_{t}^{1 /(\epsilon-1)}, \quad \epsilon \in(0,1) \tag{1}
\end{equation*}
$$

where $p_{t}$ is the price of a good and $\epsilon$ is the inverted price mark-up that reflects the monopoly power of the firm. Similarly to Bentolila and Bertola (1990), scale parameter $z_{t}$ in this demand function is subject to stochastic fluctuations at the goods market. We would generally suggest that $z_{t}$ is a realization of a random variable $Z_{t}$, where $Z_{t} \sim F\left(z_{t}\right)$ and $F$ is stationary. Stochastic fluctuations of $z_{t}$ will constitute the only source of uncertainty influencing the optimal choice of hours employed by the firm in our model.

Assuming that the firm produces a non-storable good, output needs to equal demand at the goods market, implying

$$
\begin{equation*}
m_{t}=A h_{t} \tag{2}
\end{equation*}
$$

- Working hours and working time accounts

Consider now hours employed. We make important distinction between actual hours and contracted hours employed by the firm. Despite a worker has actually worked $h_{t}$ for her firm, the firm does not pay the worker on the basis of $h_{t}$. Wage bill of the firm is calculated on the basis of a contracted amount of hours $\bar{h}$ instead, where $\bar{h}$ does not change over time. At any given $t$ it need not be that $h_{t}=\bar{h}$, such that there may exist either surplus or deficit of actual hours worked relative to contracted hours. Surplus will be viewed as a credit from
worker to firm and deficit will be viewed as a credit from firm to worker. In addition at any given $t$ there exist objective constraints on the actual hours worked, which tell that a person cannot work more than $h^{\max }$ and less than $h_{\min }$, i.e. $h_{\min } \leq h_{t} \leq h^{\max .7}$

At any $t$ the surplus/deficit of hours worked is written into a working time account. Let us denote the balance of the working time account by $b_{t}$. In addition let $b^{\max }$ stand for the upper limit of surplus accumulation, $b^{\max }>0$, and let $b_{\min }$ stand for the lower limit of deficit accumulation, $b_{\min }<0$. At the moment of opening the working time account, which we set to zero, the balance of the account is necessarily zero, $b_{0}=0$. For all dates to follow the balance of the working time account may take any value between $b_{\min }$ and $b^{\max }$. However, it must hold that at the end of each compensation period the account must be balanced, such that total amount of actual hours worked is equal to total amount of contracted hours within each compensation period. Equivalently, at the end of each compensation period all credit from worker to firm must be compensated by the firm as well as all credit from firm to worker must be compensated by the worker. Denoting the length of the compensation period by $\tau$ we therefore require that $b_{j \tau}=0$, where $j=1,2, \ldots{ }^{8}$

Maintaining that time is discrete, the above argument leads us to the law of motion for the balance of the working time account

$$
\begin{equation*}
b_{t}=b_{t-1}+\left(h_{t}-\bar{h}\right), \tag{3}
\end{equation*}
$$

where $b_{\min } \leq b_{t} \leq b^{\max }, b_{j \tau}=0$ with $j=0,1,2, \ldots$ and $t=1,2, \ldots$

- Profit function and borrowing constraints

Consider now the profit function of a firm. Using equations (1) and (2) in Appendix A. 1 we show that profit of a firm reads

$$
\begin{equation*}
\pi_{t}\left(h_{t}\right)=z_{t}^{1-\epsilon}\left[A h_{t}\right]^{\epsilon}-w \bar{h} . \tag{4}
\end{equation*}
$$

A firm operates as long as it is able to pay its wage costs. If in any of the periods wage bill cannot be paid, firm goes bankrupt and disappears from the market immediately. As a result, there arises demand for credit when in a given period $t$ firms' revenues become insufficient to pay workers their contracted wage. This occurs, for instance, when a negative shock hits the goods market. Consistent with the credit crunch during the last recession, we do not allow firms to finance labour costs through borrowing at the financial market. Important, however, is that despite not being able to borrow a firm can still invest its profit into a riskless asset with an interest rate $r$.

### 2.2 Optimal choice of hours

The task of a firm is to choose the sequence of hours that maximizes the sum of expected discounted profits subject to working time accounts regulations and conditions for survival

[^3]of the firm at the market. In what follows we will set up the optimization problem and derive the optimal solution for hours employed.

- Time horizon and uncertainty

We assume that a firm lives only for two periods (i.e. $t=1,2$ ) and the compensation period for a working time account is equal to two model periods (i.e. $\tau=2$ ). This implies the following dynamics of the balance of a working time account: $b_{0}=0, b_{1} \gtreqless 0$ and $b_{2}=0$. In principle a firm may live infinitely long. However, when drafting its optimal demand for hours the firm should respect the length of the compensation period in order to have its working time account balanced at due dates. Therefore it is only interesting what happens within a single compensation interval. For this reason a two-period model where the life of the firm is equal to the length of the compensation period is sufficient to study the effect of a working time account.

The demand level at the goods market reveals itself at the beginning of each period. A firm drafts its optimal demand for hours at the beginning of the first period. Consequently, the firm observes $z_{1}$ but still needs to form expectations about the value of $z_{2}$. These expectations are formed at $t=1$ with respect to $F$.

- Objective function and constraints

Consider the first period. Under the assumption that the firm observes $z_{1}$ we can guarantee that wage bill of all firms active at the market will always be paid in the first period, i.e. $\pi_{1}\left(h_{1}\right) \geq 0$ is always respected in the optimal choice of hours. By the end of the first period the firm possesses $(1+r) \pi_{1}\left(h_{1}\right)$ accumulated by means of investing into a riskless asset with an interest rate $r$.

Consider the second period. If the realized value of $z_{2}$ in the second period is small enough, such that $\pi_{2}\left(h_{2}\right)$ becomes negative, part of the wage bill in the second period will be paid using the profit from the first period together with returns on investing this profit in the riskless asset, $(1+r) \pi_{1}\left(h_{1}\right)$. If the realized value of $z_{2}$ is too small, the necessity to pay the wage bill in the second period may consume the entire amount of $(1+r) \pi_{1}\left(h_{1}\right)$. Should this amount be insufficient to cover the wage bill the firm goes bankrupt and disappears from the market. Thus the most the firm can loose is $(1+r) \pi_{1}\left(h_{1}\right)$, which provides the lower bound on the size of loss in the second period and defines the limit of liability of the firm towards workers. We write the profit in the second period constrained by limited liability of a firm, $\hat{\pi}_{2}$, as

$$
\begin{equation*}
\hat{\pi}_{2}\left(h_{2}\right)=\max \left\{-(1+r) \pi_{1}\left(h_{1}\right), \pi_{2}\left(h_{2}\right)\right\} . \tag{5}
\end{equation*}
$$

Let $\beta \equiv 1 /(1+r)$ denote the period discount factor. Then the value of a firm writes

$$
\begin{equation*}
V=\max _{\left\{h_{1}, h_{2}\right\}}\left\{\pi_{1}\left(h_{1}\right)+\beta E_{1}\left(\hat{\pi}_{2}\left(h_{2}\right)\right)\right\}, \tag{6}
\end{equation*}
$$

where $E_{1}$ is the expectation operator at $t=1$. Note that (5) and (6) imply that $V \geq 0$.
Consider now the working time account regulations. The assumed two-period structure of the model provides an easy characterization of the balance of working time accounts at
the end of each period. Using (3) we can see that

$$
\begin{array}{ll}
t=1: & b_{1}=h_{1}-\bar{h} \gtreqless 0 \\
t=2: & b_{2}=b_{1}+\left(h_{2}-\bar{h}\right)=0, \tag{8}
\end{array}
$$

where $b_{2}=0$ reflects the necessity to balance the account once compensation period is over. From (7)-(8) follows that $h_{2}=2 \bar{h}-h_{1}$. This means that once the choice of hours in the first period is made, it immediately pins down the choice of hours in the second period, so the problem of the firm reduces to choosing $h_{1}: h_{\min } \leq h_{1} \leq h^{\max }$. From (6) it is evident that this choice remains to be influenced by uncertainty about demand level at the goods market in the second period.

- Optimal solution

With all above, the problem of hours choice subject to working time accounts regulations and limited liability of the firm towards workers writes

$$
\begin{equation*}
V=\max _{\left\{h_{1}\right\}}\left\{\pi_{1}\left(h_{1}\right)+\beta E_{1}\left(\hat{\pi}_{2}\left(2 \bar{h}-h_{1}\right)\right)\right\} \tag{9}
\end{equation*}
$$

subject to:

$$
\begin{align*}
& h_{\min } \leq h_{1} \leq h^{\max }  \tag{10}\\
& \pi_{1}\left(h_{1}\right) \geq 0 \tag{11}
\end{align*}
$$

The first order condition for the firms' problem in (9) follows immediately:

$$
\begin{equation*}
\pi_{1}^{\prime}\left(h_{1}\right)-\beta E_{1}\left(\hat{\pi}_{2}^{\prime}\left(2 \bar{h}-h_{1}\right)\right)=0 \tag{12}
\end{equation*}
$$

Economic interpretation of this first order condition is standard. It tells that marginal benefit of a unit of labour today should be equal to the expected discounted marginal benefit of a unit of labour tomorrow.

Given the profit function in (4), after some algebra (see Appendix A.2) we get

$$
\begin{align*}
& h_{1}=\frac{2}{\frac{1}{z_{1}}\left[\beta E_{1}\left(z_{2}^{1-\epsilon}\right)\right]^{1 /(1-\epsilon)}+1} \bar{h}  \tag{13a}\\
& h_{2}=\frac{2}{1+z_{1}\left[\beta E_{1}\left(z_{2}^{1-\epsilon}\right)\right]^{1 /(\epsilon-1)}} \bar{h} \tag{13b}
\end{align*}
$$

where (13b) follows from (13a) due to the necessity of balancing the working time account at the end of the compensation period.

To complete the characterization of the optimal solution we need to make sure that inequality constraints (10)-(11) are always respected. First note that the optimal amount of hours employed in the first period may not be lower than $\tilde{h}=\frac{1}{A}\left(z_{1}^{\epsilon-1}[w \bar{h}]\right)^{1 / \epsilon}$, where $\tilde{h}$ satisfies $\pi_{1}(\tilde{h})=0$. Second, the optimal amount of hours in the first period may not be lower
than $h_{\min }$ and may not be higher than $h^{\max }$. Defining by $h_{1}^{*}$ and $h_{2}^{*}$ the optimal amount of hours in periods one and two, respectively, $h_{1}^{*}$ and $h_{2}^{*}$ become

$$
\begin{align*}
& h_{1}^{*}=\max \left\{\min \left\{h^{\max }, \frac{2}{\frac{1}{z_{1}}\left[\beta E_{1}\left(z_{2}^{1-\epsilon}\right)\right]^{1 /(1-\epsilon)}+1} \bar{h}\right\}, \max \left\{h_{\min }, \tilde{h}\right\}\right\},  \tag{14a}\\
& h_{2}^{*}=2 \bar{h}-h_{1}^{*}, \tag{14b}
\end{align*}
$$

where, as before, (14b) follows from the necessity to balance the working time account. Figure 1 visualizes this solution. We consider it in detail in the following section.

## 3 Hours, profits and impact of a working time account

### 3.1 Determinants of hours

The solution for optimal hours in (14) has several nice analytical properties. First, we can see that no matter the period optimal hours always depend on two variables: the realized value of demand level parameter at the goods market in the first period, $z_{1}$, and the expected value of demand level parameter at the goods market in the second period, $E_{1}\left(z_{2}\right)$. Second, (14b) implies that whenever constraints do not bind a change in any of these two variables will make $h_{1}^{*}$ and $h_{2}^{*}$ move in opposite directions.

Figure 1 represents the optimal choice of hours in both periods as a function of the realized demand parameter $z_{1}$ for a fixed value of $E\left(z_{2}\right)$. Solid line in the left panel illustrates $h_{1}^{*}$ and solid line in the right panel illustrates $h_{2}^{*}$. It is straightforward to show (see Appendix A.3) that optimal hours in the first period increase in $z_{1}$, and hence optimal hours in the second period fall in $z_{1}$, when constraints do not bind. The interpretation is simple: the better is the situation with demand at the goods market today, the more inclined is the firm to produce today, as compared with tomorrow. Binding constraints are reflected by flat lines at $h^{\max }$ and $\max \left\{h_{\text {min }}, \tilde{h}\right\}$.

(b): Second period


Figure 1 Optimal hours

Dependence of optimal hours on the expected value of $z_{2}$ is just the opposite. As shown in Appendix A.3, for any given value of $z_{1}$ optimal hours in the first period decrease in $E\left(z_{2}\right)$ when constraints do not bind. From this follows that optimal hours in the second period increase in the expected value of the demand level in the second period. In Figure 1 this dependence is reflected by a vertical downward shift of the optimal hours curve in the first period (left panel) and a vertical upward shift of the optimal hours curve in the second period (right panel) for an increasing value of $E_{1}\left(z_{2}\right)$. Interpretation is again simple: the better is the expected situation at the goods market tomorrow the less inclined will be the firm to produce today, and so the more production will be shifted into tomorrow, as compared with today. Again, binding constraints are reflected by flat lines at $h^{\max }$ and $\max \left\{h_{\min }, \tilde{h}\right\}$.

The above properties of optimal hours become particularly interesting if placed in the context of expansion/recession. If one associates an above average demand level at the goods market with an expansion and a below average demand at the goods market with a recession, then with values of $z_{1}$ sufficiently higher than $E\left(z_{2}\right)$ the firm will tend to employ more hours in the expansion and with with values of $z_{1}$ sufficiently lower than $E\left(z_{2}\right)$ the firm will tend to employ less hours in the recession. Consequently the optimal solution displays coherence with the observed fact that German firms have accumulated high surpluses on their working time accounts during the expansion and were running down these surpluses during the recession, as noted by Burda and Hunt (2011).

Lastly, both panels of Figure 1 show a horizontal dotted line at $\bar{h}$. This line represents for the sake of comparison the hours choice of an identical firm that, for some exogenous reason, does not operate a working time account. Since the actual hours at such a firm are always equal to contracted hours, we see that $h_{1}=h_{2}=\bar{h}$. Clearly, this choice of hours is independent of $z_{1}$ and $E\left(z_{2}\right)$, as the firm lacks the necessary instrument to react to demand fluctuations at the goods market. The differences $h_{1}^{*}-\bar{h}$ and $h_{2}^{*}-\bar{h}$ reflect the change to the balance of the working time account within each period.

### 3.2 Determinants of profits

Consider now profit levels implied by the optimal choice of hours in presence of a working time account. Being a function of optimal hours, profit of a firm in any period clearly depends on the parameters that determine optimal hours in this period, i.e. on $z_{1}$ and $E_{1}\left(z_{2}\right)$. Apart form these, profit in the first period depends directly on the realized value of a demand level parameter in the first period, $z_{1}$, and profit in the second period depends directly on the realized value of a demand level parameter in the second period, $z_{2}$. Let us introduce the notation $\pi_{1}^{*} \equiv \pi_{1}\left(h_{1}^{*}\right)$ and $\pi_{2}^{*} \equiv \pi_{2}\left(h_{1}^{*}\right)$. Using (4) and the optimal solution for hours it is straightforward to show that $\pi_{1}^{*}$ increases in $z_{1}$ and $\pi_{2}^{*}$ increases in $z_{2}$. This dependence is captured by Figure 2.

For a given value of $E_{1}\left(z_{2}\right)$ solid line in the left panel of Figure 2 plots $\pi_{1}^{*}$ against the realized value of $z_{1}$ and solid line in the right panel of Figure 2 plots $\pi_{2}^{*}$ against the realized value of $z_{2}$. These solid lines differ in shape because there is no indirect dependence of $\pi_{2}^{*}$ on $z_{2}$ via hours. Still both profit functions are increasing, which sends a very simple message: the higher is the demand at the goods market today the higher is the profit made today.

Changes in the expected value of demand level parameter $z_{2}$ that induce changes in hours result into qualitatively similar changes in profits. Since profit function is monotone
in hours, it follows that $\pi_{1}^{*}$ falls in $E_{1}\left(z_{2}\right)$ and $\pi_{2}^{*}$ increases in $E_{1}\left(z_{2}\right)$, ceteris paribus. In Figure 2 the dependence of profits on the expected value of the demand level parameter in the second period is reflected by a vertical downward shift of the solid line in the left panel and a vertical upward shift of the solid line in the right panel as $E_{1}\left(z_{2}\right)$ goes up. This simply tells: the better is the expected situation at the goods market tomorrow, the lower will be firms' profit today and the higher will be the firms' profit tomorrow. Finally, since an increase in $z_{1}$ lowers optimal hours chosen for the second period, there is also a negative dependence between $\pi_{2}^{*}$ and $z_{1}$.



Figure 2 Optimal profits
What makes Figure 2 particularly interesting is the comparison of $\pi_{1}^{*}$ and $\pi_{2}^{*}$ with profits of an identical firm that for some exogenous reason does not operate a working time account. These profits are depicted by a dotted line in the left and in the right panel (denoted by $\pi_{1}$ and $\pi_{2}$, respectively). Since hours employed by such a firm are simply $h_{1}=h_{2}=\bar{h}$, the corresponding profit functions solely depend on realized demand level parameters, and the dependence is strictly positive. Figure 2 shows that it is not always the case, that profits of a firm with a working time account are higher than profits of a firm without such an account. This leads us to question under which circumstances will a firm be ready to open working time account as such.

### 3.3 Adoption of working time account

To see when a firm will choose to open a working time account we need to consider the values of a firm with and without the account. First of all, from (5) and (6), limited liability of a firm towards worker implies that value of a firm is nonnegative no matter if the firm operates a working time account or not. Let $V^{*}$ denote the value of a firm with working time account, i.e. with hours policy $\left\{h_{1}^{*}, h_{2}^{*}\right\}$ as in (14). Let $\bar{V}$ denote the value of an identical firm without working time account, i.e. with hours policy $\{\bar{h}, \bar{h}\}$. Left panel of Figure 3 plots the ratio $V^{*} / \bar{V}$ as a function of $z_{1}$ and right panel of Figure 3 plots the same ratio as a function of $E\left(z_{2}^{1-\epsilon}\right)$. Parameter values for this illustration are reported in Appendix A.4.


Figure 3 Value of a firm

We see that in both cases the value of a firm with working time account always exceeds the value of a firm without the account (except at $h_{1}^{*}=h_{2}^{*}=\bar{h}$ ). Indeed $V^{*} \geq \bar{V}$ should always hold because $\bar{V}$ is the value of a firm obtained under the same set of constraints as $V^{*}$ plus an additional constraint that restricts hours as $h_{1}^{*}=h_{2}^{*}=\bar{h}$. This means that a firm will always choose to open a working time account for its employee.

### 3.4 Working time account and turnover

Given that a firm will always decide to open a working time account it would be tempting to suggest that a firm with working time account will always be able to withstand stronger demand downturns if compared to an identical firm without working time account. As a result, working time account will arguably always reduce turnover. Whether this is true or not shows the following analysis.

Consider a threshold level of the realized demand parameter in the second period that leads to destruction of a firm. Let $z_{2}^{*}$ denote this threshold level for a firm with working time account and let $\bar{z}_{2}$ denote a similar threshold level for a firm without working time account. Then for any realization of $z_{2}$ such that $z_{2}<z_{2}^{*}\left(z_{2}<\bar{z}_{2}\right)$ demand downturn at the goods market leads a firm with (without) a working time account to bankruptcy. In Appendix A. 5 we show that the respective threshold values are given by

$$
\begin{align*}
& z_{2}^{*}=\left(\frac{w \bar{h}-(1+r) \pi_{1}\left(h_{1}^{*}\right)}{\left[A h_{2}^{*}\right]^{\epsilon}}\right)^{1 /(1-\epsilon)}  \tag{15}\\
& \bar{z}_{2}=\left(\frac{w \bar{h}-(1+r) \pi_{1}(\bar{h})}{[A \bar{h}]^{\epsilon}}\right)^{1 /(1-\epsilon)} \tag{16}
\end{align*}
$$

Both thresholds unambiguously increase in wage rate and decrease in productivity, i.e. the higher is the wage rate (the lower is the productivity) the weaker shock is needed to destroy the firm. We also see that in general $z_{2}^{*}$ and $\bar{z}_{2}$ are not equal to each other. The intriguing question therefore is: Is it always true that $z_{2}^{*}<\bar{z}_{2}$ ? If this is the case, then for intermediate realizations of the demand parameter $z_{2}$ such that $z_{2}^{*}<z_{2}<\bar{z}_{2}$ a firm with working time account will survive the demand downturn, whereas an identical firm without working time account will not. Consequently, working time account will contribute to reduction of turnover and hence to restraining the increase of unemployment.

Surprisingly, we find that $z_{2}^{*}<\bar{z}_{2}$ may not always hold. Figure 4 illustrates the ratio of bankruptcy thresholds, $\bar{z}_{2} / z_{2}^{*}$, as a function of a set of model parameters, namely: $z_{1}, E\left(z_{2}^{1-\epsilon}\right)$, $A$ and $w$. Parameter values for this illustration are reported in Appendix A.4. Figure 4 clearly shows that for a range of values of $z_{1}, E\left(z_{2}^{1-\epsilon}\right)$ and $w$ bankruptcy threshold of a firm with working time account exceeds that of a firm without working time account. Consequently, in this range of values for intermediate realization of the demand level parameter $z_{2}$ in the second period such that $\bar{z}_{2}<z_{2}<z_{2}^{*}$ a firm with working time account gets destroyed whereas a firm without working time account survives the downturn. This tells that working time account indeed increases turnover and hence unemployment.


Figure 4 Implications for turnover

Looking at the first row of Figure 4 we can see that the harmful effect of the working time account obtains either when the current state of demand $z_{1}$ is too low, while expected
value of the future state of demand stays unchanged, or when expected value of the future state of demand $E\left(z_{2}^{1-\epsilon}\right)$ is too high, while the current state of demand stays unchanged. Rearranging (14a) we can show that optimal choice of hours in the first period is always less than the contracted amount hours if $z_{1}^{1-\epsilon}<\beta E_{1}\left(z_{2}^{1-\epsilon}\right)$, i.e. if the current state of demand is sufficiently low relative to the expected state of demand in the next period. Thus, the first row of Figure 4 suggests that working time account is likely to enhance turnover when demand downturn at the goods market is met with a deficit of the working time account balance. Looking at the second row of Figure 4 we can see that the ratio of bankruptcy thresholds positively depends on productivity $A$ and negatively depends on hourly wage rate $w$. For high enough values of hourly wage, keeping productivity unchanged, bankruptcy thresholds flip and working time account again contributes to higher turnover and hence higher unemployment.

Observations made with the help of Figure 4 are not coincidental. In fact these are manifestations of the general conditions under which working time account impacts turnover in two different ways. These conditions are provided in Proposition 1.

Proposition 1 When productivity of a firm is sufficiently high relative to its wage cost, working time account reduces turnover if a firm meets demand downturn with surplus of actual hours employed and increases turnover if a firm meets demand downturn with deficit of actual hours employed. Threshold value for the productivity of a firm relative to wage cost is given by

$$
\begin{equation*}
\frac{A^{\epsilon}}{w}>\frac{2+r}{1+r} \frac{\bar{h}^{1-\epsilon}}{z_{1}^{1-\epsilon}} \frac{[\bar{h}]^{\epsilon}-\left[h_{2}^{*}\right]^{\epsilon}}{\left[h_{1}^{*}\right]^{\epsilon}-\left[h_{2}^{*}\right]^{\epsilon}} . \tag{17}
\end{equation*}
$$

## Proof. See Appendix A.6.

Proposition 1 highlights the key finding of our paper. It tells that the general dependence between working time accounts, turnover and unemployment is ambiguous. While the literature existing to this date seems to have emphasized only the positive side of this dependence, namely turnover-reducing effect of a working time account, we show that turnover-enhancing effect is also present. Furthermore, Proposition 1 demonstrates that ambiguity of the effect also depends on the productivity of the firm relative to its wage cost. While for highproductive firms surplus of hours on working time account insures against higher turnover, for low-productive firm the result is completely opposite. The following corollary establishes the claim.

Corollary 1 When productivity of a firm is sufficiently low relative to its wage cost, working time account increases turnover if a firm meets demand downturn with surplus of actual hours employed and reduces turnover if a firm meets demand downturn with deficit of actual hours employed. Threshold value for the productivity of a firm relative to wage cost is given by

$$
\begin{equation*}
\frac{A^{\epsilon}}{w}<\frac{2+r}{1+r} \frac{\bar{h}^{1-\epsilon}}{z_{1}^{1-\epsilon}} \frac{[\bar{h}]^{\epsilon}-\left[h_{2}^{*}\right]^{\epsilon}}{\left[h_{1}^{*}\right]^{\epsilon}-\left[h_{2}^{*}\right]^{\epsilon}} . \tag{18}
\end{equation*}
$$

How working time account affects turnover? The impact goes through the intertemporal shifting of hours and the intertemporal shifting of profits. Two following situations are of interest.

Consider first the situation in which a firm with working time account meets downturn with surplus of hours on the account and compare this firm to an identical firm without working time account. If a firm with working time account meets downturn with surplus of hours, its profit in the first period is higher and its profit in the second period is lower than respective profits of a firm without working time account, due to intertemporal shifting of hours. As there is more profit to invest in the first period, there are more returns to get for the second period than at a firm without working time account, due to intertemporal shifting of profits. Thus, facing downturn in the second period, a firm with working time account has lower direct profit in the second period but higher returns on investment form the first period than an identical firm without working time account. It will be able to withstand a stronger demand downturn only if higher returns on investment in the first period outweigh lower profits due to reduced hours in the second period. The higher is the productivity of a firm relative to wage cost, the higher is the weight of returns on investment, so the result of Proposition 1 applies and the firm with working time account withstands stronger shock than the identical firm without the account. The lower is the productivity of a firm relative to wage cost, the lower is the weight of returns on investment, so according to Corollary 1 the firm with working time account gets destroyed by a weaker shock than an identical firm without the account.

Another situation, in which a firm with working time account meets downturn with deficit of hours on the account, shows just the opposite. If a firm with working time account meets downturn with deficit of hours, its profit in the first period is lower and its profit in the second period is higher than respective profits of a firm without working time account, as implied by intertemporal shifting of hours. Lower profit in the first period means lower investment in the first period and hence lower returns in the second period than at a firm without working time account, as implied by intertemporal shifting of profits. So, facing downturn in the second period, a firm with working time account has higher direct profit in the second period but lower returns on investment form the first period than an identical firm without working time account. To be able to withstand a stronger demand downturn higher profit in the second period need to outweigh lower return on investment in the first period. However, at high-productive firms the weight of investment appears to be too high, so according to Proposition 1 a firm with working time account needs a weaker shock to be destroyed than a firm without working time account. Once productivity is sufficiently low relative to wage cost, direct effect of higher hours acquires more importance than the size of return on investment from the past, so Corollary 1 applies and a firm with working time account withstands stronger demand downturn.

Before we conclude, our analysis was inspired by the developments on the German labour market during the Great Recession. Thinking of how the results of Proposition 1 and Corollary 1 align with what happened in Germany, two claims from the literature provide the complete picture. First, Burda and Hunt (2011) claim that having met the Great Recession with surpluses on their working time accounts German firms have managed to survive this recession without much of job destruction. Second, Möller (2010) states that the recession has primarily hit German exporting firms in manufacturing, which were mostly "strong
firms in economically strong regions". So the crisis in Germany seems to have affected high-productive firms with substantive surpluses, which in line with the first part of our Proposition 1 has reduced turnover and contributed to lack of increase in unemployment. In a more general cross-country context, though, the particular German pre-crises setup and the nature of the crises may not be identical to the rest of the countries that think of implementing working time accounts. In this more general context our paper becomes particularly important, as it demonstrates that the influence of working time accounts on turnover and unemployment can be quite diverse.

## 4 Conclusion

In this paper we suggest a simple yet powerful model of demand for working hours by a local monopolist who operates a working time account. Optimal hours are chosen in face of uncertain demand at the goods market and under consideration of constraints imposed by working time accounts regulations. Firms do not have access to credit, but can save at a riskfree rate. Motivated by the hypothesis of Burda and Hunt (2011) on performance of working time accounts in Germany during the Great Recession we use our model to investigate the connection between working time accounts, turnover and unemployment in recessions.

Contrary to our initial expectations, we find that firms with working time accounts need not necessarily have lower turnover than firms without such accounts. In fact there may appear situations when working time accounts catalyze turnover and unemployment instead of inhibiting them. This occurs, for instance, when a high-productive firm operates deficit of a working time account and expects improvement of demand at the goods market in future. In such situation a firm without working time account will be able to sustain stronger demand downturns than a firm with the account, should the expected improvement of demand fail to materialize. Working time accounts also catalyze turnover and unemployment in a reciprocal situation, when a low-productive firm operates surplus of a working time account and expects deterioration of demand at the goods market in future. In such situation a firm without working time account will again be able to sustain stronger demand downturns than a firm with the account, should the expected fall in demand indeed materialize. Apart from obtaining these surprising results, our model also encompasses the behaviour described by Möller (2010) and Burda and Hunt (2011). We show that when a high-productive firm has surplus on its working time account and expects demand downturn at the goods market in future, it will be able to sustain a stronger realized demand downturn than a firm without working time account.

The main message of this paper is that working time accounts may not be perceived by policy makers with too much optimism. While they are indeed a useful tool for enhancing flexibility of labour demand, their effect on unemployment is ambiguous and strongly depends on the dynamics of the goods market and on the particular productivity type of a firm. Regarding the German example that motivated our study, our model suggests that working time accounts have indeed contributed to restraining the rise of unemployment in Germany during the Great Recession. The exact size of this effect, though, is an empirical question which we leave for future research.

## Appendix

## A. 1 Profit function

Consider the demand function given in (1). Solving (1) for price we get $p_{t}=z_{t}^{1-\epsilon} m_{t}^{\epsilon-1}$. Inserting (2) for output, revenue $p_{t} m_{t}$ becomes

$$
\begin{aligned}
p_{t} m_{t} & =z_{t}^{1-\epsilon} m_{t}^{\epsilon-1} m_{t} \\
& =z_{t}^{1-\epsilon} m_{t}^{\epsilon}=z_{t}^{1-\epsilon}\left[A h_{t}\right]^{\epsilon} .
\end{aligned}
$$

Since wage costs are given by $w \bar{h}$ profit function writes

$$
\pi_{t}\left(h_{t}\right)=z_{t}^{1-\epsilon}\left[A h_{t}\right]^{\epsilon}-w \bar{h} .
$$

Profit function is an explicit function of actual hours worked, $h_{t}$.

## A. 2 Optimal solution for hours

Optimal solution for $h_{1}$ follows form the first order condition (12). We get

$$
\begin{aligned}
\pi_{1}^{\prime}\left(h_{1}\right) & =\beta E_{1}\left(\pi_{2}^{\prime}\left(2 \bar{h}-h_{1}\right)\right) \\
\epsilon z_{1}^{1-\epsilon} A^{\epsilon}\left[h_{1}\right]^{\epsilon-1} & =\beta E_{1}\left(\epsilon z_{2}^{1-\epsilon} A^{\epsilon}\left[2 \bar{h}-h_{1}\right]^{\epsilon-1}\right) \\
z_{1}^{1-\epsilon} h_{1}^{\epsilon-1} & =\left[2 \bar{h}-h_{1}\right]^{\epsilon-1} \beta E_{1}\left(z_{2}^{1-\epsilon}\right) \\
z_{1}^{(1-\epsilon) /(\epsilon-1)} h_{1} & =\left[2 \bar{h}-h_{1}\right]\left[\beta E_{1}\left(z_{2}^{1-\epsilon}\right)\right]^{1 /(\epsilon-1)} \\
z_{1}^{(1-\epsilon) /(\epsilon-1)} h_{1} & =2 \bar{h}\left[\beta E_{1}\left(z_{2}^{1-\epsilon}\right)\right]^{1 /(\epsilon-1)}-h_{1}\left[\beta E_{1}\left(z_{2}^{1-\epsilon}\right)\right]^{1 /(\epsilon-1)},
\end{aligned}
$$

such that

$$
\begin{aligned}
h_{1} & =\frac{2 \bar{h}\left[\beta E_{1}\left(z_{2}^{1-\epsilon}\right)\right]^{1 /(\epsilon-1)}}{z_{1}^{(1-\epsilon) /(\epsilon-1)}+\left[\beta E_{1}\left(z_{2}^{1-\epsilon}\right)\right]^{1 /(\epsilon-1)}}=\frac{2 \bar{h}}{\left[\beta E_{1}\left(z_{2}^{1-\epsilon}\right)\right]^{-1 /(\epsilon-1)} z_{1}^{(1-\epsilon) /(\epsilon-1)}+1} \\
& =\frac{2 \bar{h}}{\left[\beta E_{1}\left(z_{2}^{1-\epsilon}\right)\right]^{1 /(1-\epsilon)}\left[\frac{1}{z_{1}}\right]^{(1-\epsilon) /(1-\epsilon)}+1}=\frac{2 \bar{h}}{\left[\beta \frac{1}{z_{1}^{1-\epsilon}} E_{1}\left(z_{2}^{1-\epsilon}\right)\right]^{1 /(1-\epsilon)}+1},
\end{aligned}
$$

and finally

$$
h_{1}=\frac{2}{\frac{1}{z_{1}}\left[\beta E_{1}\left(z_{2}^{1-\epsilon}\right)\right]^{1 /(1-\epsilon)}+1} \bar{h} .
$$

## A. 3 Properties of optimal hours

- Dependence of $h_{1}^{*}$ on $z_{1}$

Considering (14a) when constraints do not bind,

$$
\frac{\partial h_{1}^{*}}{\partial z_{1}}=\frac{\partial}{\partial z_{1}}\left(\frac{2}{\frac{1}{z_{1}}\left[\beta E_{1}\left(z_{2}^{1-\epsilon}\right)\right]^{1 /(1-\epsilon)}+1} \bar{h}\right)
$$

$$
=\frac{2 \bar{h}}{\left(\frac{1}{z_{1}}\left[\beta E_{1}\left(z_{2}^{1-\epsilon}\right)\right]^{1 /(1-\epsilon)}+1\right)^{2}} \frac{1}{z_{1}^{2}}\left[\beta E_{1}\left(z_{2}^{1-\epsilon}\right)\right]^{1 /(1-\epsilon)}>0
$$

- Dependence of $h_{1}^{*}$ on $E_{1}\left(z_{2}\right)$

Define $E \equiv E_{1}\left(z_{2}{ }^{1-\epsilon}\right)$. Considering (14a) when constraints do not bind,

$$
\begin{aligned}
\frac{\partial h_{1}^{*}}{\partial E} & =\frac{\partial}{\partial E}\left(\frac{2}{\frac{1}{z_{1}}[\beta E]^{1 /(1-\epsilon)}+1} \bar{h}\right) \\
& =-\frac{2 \bar{h}}{(1-\epsilon) z_{1}\left(\frac{1}{z_{1}}[\beta E]^{1 /(1-\epsilon)}+1\right)^{2}} \beta^{1 /(1-\epsilon)} E^{\epsilon /(1-\epsilon)}<0 .
\end{aligned}
$$

Since $h_{1}^{*}$ decreases in $E_{1}\left(z_{2}^{1-\epsilon}\right)$ and $z_{2}^{1-\epsilon}$ is a monotone increasing transformation of $z_{2}, h_{1}^{*}$ decreases in $E_{1}\left(z_{2}\right)$.

## A. 4 Parameters

Figures 3 and 4 are plotted using the following choice of parameters. We assume that the distribution of demand shocks has a unit mean, implying that $E_{1}\left(z_{2}^{1-\epsilon}\right)=1$. We furhter normalize wage rate to unity, which also implies a scaled produtivity measure (in or application $A=4.1$ ). Lastly, $z_{1}$ is set to one as well. Table A. 1 shows the ranges of variation of parameters on the horizontal axis in Figures 3 and 4. Its first block refers to Figure 3 and its seond block to Figure 4. In this table, leading parameter of each row is the parameter on the $x$-axis which we let varying.

|  | Adoption of working time account $\left(V^{*} / \bar{V}\right)$ |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $z_{1}$ | $E_{1}\left(z_{2}^{1-\epsilon}\right)$ | $A$ | $w$ |  |  |  |
|  |  |  |  |  |  |  |
| $z_{1}(0.750,1.125]$ | 1 | 4.1 | 1 |  |  |  |
| $E_{1}\left(z_{2}^{1-\epsilon}\right)$ | 1 | $[0.925,1.125]$ | 4.1 | 1 |  |  |
|  |  | Turnover $\left(z_{2}^{*} / \bar{z}_{2}\right)$ |  |  |  |  |
|  | $z_{1}$ | $E_{1}\left(z_{2}^{1-\epsilon}\right)$ | $A$ | $w$ |  |  |
| $z_{1}$ | $[0.725,1.115]$ | 1 | 4.1 | 1 |  |  |
| $E_{1}\left(z_{2}^{1-\epsilon}\right)$ | 1 | $[0.925,1.125]$ | 4.1 | 1 |  |  |
| $A$ | 1 | 1 | $[3.85,4.15]$ | 1 |  |  |
| $w$ | 1 | 1 | 4.1 | $[0.95,1.25]$ |  |  |

Table A. 1 Parameter values

The rest of the parameters remains invariant all the time. These parameters are chosen to mimic German economy shortly before the Great Recession. We let one period in our model last six months. First, this corresponds to the time window within which the economy may technically enter recession (two consecutive quarters). Second, the lengh of the compensation period in manufacturing frequently lasts up to one year (Zapf and Herzog-Stein, 2015). Period interest rate is set to $r=0.0188$, which corresponds to the average annual long-term interest rate of $3.8 \%$ in 2006-2009 (OECD, 2013). Period amount of hours worked at a firm without working time account, $\bar{h}$, is set to $\bar{h}=670$ based on average annual hours actually worked per worker in dependent employment in 2006-2007 (OECD, 2013). We further set $h^{\max }=1.15 \times \bar{h}$ and, symmetrically, $h_{\min }=0.85 \times \bar{h}$. Lastly, inverted mark-up, $\epsilon$, is set to $\epsilon=1 / 1.19$ which is implied by the estimated price mark-up of $19 \%$ in German manufacturing (Christopoulou and Vermeulen, 2012).

## A. 5 Bankruptcy thresholds

Consider a firm with a working time account. The firm is on the bankruptcy threshold if invested profit from the first period, together with return on this investment, is just sufficient to cover for the loss in the second period. We therefore look for $z_{2}^{*}$ which solves $(1+r) \pi_{1}\left(h_{1}^{*}\right)+\pi_{2}\left(h_{2}^{*}\right)=0$. We get

$$
\begin{gathered}
(1+r) \pi_{1}\left(h_{1}^{*}\right)+\left[z_{2}^{*}\right]^{1-\epsilon}\left[A h_{2}^{*}\right]^{\epsilon}-w \bar{h}=0 \\
{\left[z_{2}^{*}\right]^{1-\epsilon}\left[A h_{2}^{*}\right]^{\epsilon}=w \bar{h}-(1+r) \pi_{1}\left(h_{1}^{*}\right)} \\
z_{2}^{*}=\left(\frac{w \bar{h}-(1+r) \pi_{1}\left(h_{1}^{*}\right)}{\left[A h_{2}^{*}\right]^{\epsilon}}\right)^{1 /(1-\epsilon)} .
\end{gathered}
$$

Similar argument applies to a firm without a working time account. With $\bar{h}$ replacing $h_{1}^{*}$ and $h_{2}^{*}$ we get

$$
\bar{z}_{2}=\left(\frac{w \bar{h}-(1+r) \pi_{1}(\bar{h})}{[A \bar{h}]^{\epsilon}}\right)^{1 /(1-\epsilon)}
$$

## A. 6 Proof of Proposition 1

Proof. Assume that $z_{2}^{*}<\bar{z}_{2}$ holds. Inserting (15) and (16) we get

$$
\begin{array}{r}
\left(\frac{w \bar{h}-(1+r) \pi_{1}\left(h_{1}^{*}\right)}{\left[A h_{2}^{*}\right]^{\epsilon}}\right)^{1 /(1-\epsilon)}<\left(\frac{w \bar{h}-(1+r) \pi_{1}(\bar{h})}{[A \bar{h}]^{\epsilon}}\right)^{1 /(1-\epsilon)} \\
{\left[w \bar{h}-(1+r) \pi_{1}\left(h_{1}^{*}\right)\right]\left[\frac{\bar{h}}{h_{2}^{*}}\right]^{\epsilon}<w \bar{h}-(1+r) \pi_{1}(\bar{h})} \\
{\left[w \bar{h}-(1+r)\left\{z_{1}^{1-\epsilon}\left[A h_{1}^{*}\right]^{\epsilon}-w \bar{h}\right\}\right]\left[\frac{\bar{h}}{h_{2}^{*}}\right]^{\epsilon}<w \bar{h}-(1+r)\left\{z_{1}^{1-\epsilon}[A \bar{h}]^{\epsilon}-w \bar{h}\right\}} \\
(2+r) w \bar{h}\left[\frac{\bar{h}}{h_{2}^{*}}\right]^{\epsilon}-(1+r) z_{1}^{1-\epsilon}\left[A h_{1}^{*}\right]^{\epsilon}\left[\frac{\bar{h}}{h_{2}^{*}}\right]^{\epsilon}<(2+r) w \bar{h}-(1+r) z_{1}^{1-\epsilon}[A \bar{h}]^{\epsilon}
\end{array}
$$

$$
\begin{array}{r}
(2+r) w \bar{h}\left\{\left[\frac{\bar{h}}{h_{2}^{*}}\right]^{\epsilon}-1\right\}<(1+r) z_{1}^{1-\epsilon}[A \bar{h}]^{\epsilon}\left\{\left[\frac{h_{1}^{*}}{h_{2}^{*}}\right]^{\epsilon}-1\right\} \\
{\left[\frac{\bar{h}}{h_{2}^{*}}\right]^{\epsilon}-1<\frac{1+r}{2+r} \frac{z_{1}^{1-\epsilon}[A \bar{h}]^{\epsilon}}{w \bar{h}}\left\{\left[\frac{h_{1}^{*}}{h_{2}^{*}}\right]^{\epsilon}-1\right\}} \\
{[\bar{h}]^{\epsilon}-\left[h_{2}^{*}\right]^{\epsilon}<\frac{1+r}{2+r} \frac{z_{1}^{1-\epsilon}[A \bar{h}]^{\epsilon}}{w \bar{h}}\left\{\left[h_{1}^{*}\right]^{\epsilon}-\left[h_{2}^{*}\right]^{\epsilon}\right\}}
\end{array}
$$

and finally

$$
\begin{equation*}
\left\{[\bar{h}]^{\epsilon}-\left[h_{2}^{*}\right]^{\epsilon}\right\}-\frac{1+r}{2+r} \frac{z_{1}^{1-\epsilon}[A \bar{h}]^{\epsilon}}{w \bar{h}}\left\{\left[h_{1}^{*}\right]^{\epsilon}-\left[h_{2}^{*}\right]^{\epsilon}\right\}<0 . \tag{A.6.1}
\end{equation*}
$$

Consider the first statement of the proposition. Surplus at the working time account in the first period means that $h_{1}^{*}>\bar{h}>h_{2}^{*}$, implying that $\left[h_{1}^{*}\right]^{\epsilon}>[\bar{h}]^{\epsilon}>\left[h_{2}^{*}\right]^{\epsilon}$ and $\left[h_{1}^{*}\right]^{\epsilon}-\left[h_{2}^{*}\right]^{\epsilon}>$ $[\bar{h}]^{\epsilon}-\left[h_{2}^{*}\right]^{\epsilon}>0$. Consequently, (A.6.1) implies that any $\frac{1+r}{2+r} \frac{z_{1}^{1-\epsilon}[A \bar{h}]^{\epsilon}}{w h} \geq 1$ is sufficient for $z_{2}^{*}<\bar{z}_{2}$ to hold. Rearranging (A.6.1), $z_{2}^{*}<\bar{z}_{2}$ holds as long as

$$
\begin{equation*}
\frac{2+r}{1+r} \frac{\bar{h}^{1-\epsilon}}{z_{1}^{1-\epsilon}} \frac{[\bar{h}]^{\epsilon}-\left[h_{2}^{*}\right]^{\epsilon}}{\left[h_{1}^{*}\right]^{\epsilon}-\left[h_{2}^{*}\right]^{\epsilon}}<\frac{A^{\epsilon}}{w} . \tag{A.6.2}
\end{equation*}
$$

Consider the second statement of the proposition. Deficit at the working time account in the first period means that $h_{1}^{*}<\bar{h}<h_{2}^{*}$, implying that $\left[h_{1}^{*}\right]^{\epsilon}<[\bar{h}]^{\epsilon}<\left[h_{2}^{*}\right]^{\epsilon}$ and $\left[h_{1}^{*}\right]^{\epsilon}-\left[h_{2}^{*}\right]^{\epsilon}<$ $[\bar{h}]^{\epsilon}-\left[h_{2}^{*}\right]^{\epsilon}<0$. Consequently, (A.6.1) implies that any $\frac{1+r}{2+r} \frac{z_{1}^{1-\epsilon}[A \bar{h}]^{\epsilon}}{w h} \geq 1$ is sufficient for $z_{2}^{*}<\bar{z}_{2}$ not to hold. Rearranging (A.6.1) again, $z_{2}^{*}<\bar{z}_{2}$ will be violated as long as (A.6.2) holds.

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[^1]:    ${ }^{2}$ In fact German unemployment rate has continued to fall, loosing 0.5 percentage points in the first quarter of the recession. It did not change in the second quarter and started to go up only thereafter, picking 0.7 percentage points during the next two quarters. With the entire recession lasting one year, the economy entered recession with the unemployment rate of $7.7 \%$ and left recession with the unemployment rate of $7.9 \%$. Once the recession was over unemployment rate started falling again (see OECD, 2013).
    ${ }^{3}$ For excellent descriptions of the US and German labour markets during the Great Recession see Eslby et al. (2010) and Burda and Hunt (2011), respectively.
    ${ }^{4}$ See Zapf and Herzog-Stein (2015) for an excellent review of the organization of working time accounts in Germany.

[^2]:    ${ }^{5}$ According to Eurofound (2010), $30 \%$ of Danish, $18 \%$ of Swedish and $13 \%$ of Finnish firms were operating working time accounts for their employees in 2009.
    ${ }^{6}$ Remarkably, in the West Germany this number has nearly doubled over the period between the reunification and the outbreak of the crises. In the East Germany right after reunification working time accounts were nearly nonexistent.

[^3]:    ${ }^{7}$ At the extreme $h_{\min }$ cannot be less than zero hours per day and $h^{\max }$ cannot be more 24 hours per day. Furthermore, with $h_{\min } \leq h_{t} \leq h^{\max }$, clearly it also holds that $h_{\min }<\bar{h}<h^{\max }$.
    ${ }^{8}$ According to Zapf and Herzog-Stein (2015) in 2007 in Germany the average limit of surplus accumulation was equal to +103 hours, the average limit of deficit accumulation was equal to -63 hours and the average duration of compensation period was about 38 weeks.

