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DOES DISTANCE MATTER FOR TECHNOLOGY SPILLOVERS?

by

Marjolein C.J. Caniëls (MERIT)

How does knowledge, i.e. innovations, diffuse over space? Traditional diffusion theory does not consider this question, but instead focuses on diffusion over time. After some early publications such as Gerschenkron (1962) and Abramovitz (1979), research on technology diffusion over countries was deepened during the 1980s. This literature became known as technology gap literature. Main argument of this literature is that technology is regarded to spread to a country due to specific characteristics of that country. The countries between which spillover of knowledge takes place, need not necessarily be geographically close to each other, therefore geographic proximity does not play a substantive role in this literature. The so-called spatial factors like agglomeration economies and growth poles as used by geographers were not addressed in this set of theories. Only the geographic stream of researchers used these concepts. Within the field of geography, diffusion is regarded as a purely geographical process and space is a central concept. Scale effects and proximity effects are considered to be the main vehicles which carry the spread of economic activity.

This paper develops a theoretical model which tries to explain the development of growth poles, by allowing knowledge spillovers to take place across regions. The aim of the paper is to integrate both streams in the literature (technology gap and geography). The lack of space as an explaining factor in technology gap models is addressed by extending a simple technology gap model with the concept of geographical distance. The geographical distance towards another region partly determines the amount of spillovers one region receives from another.

The 'traditional' technology gap models are also extended in a second way by considering more than two regions. With respect to the geographical spheres, three different experiments are set up, using a lattice of honeycombs, a column and a globe respectively. Depending on the geographical sphere that is used, 19, 21 or 32 regions are considered. Several types of experiments are performed on the different geographical spheres to illustrate the behavior of the model.

Topic information: Regional economics, technology gap models, simulation.

JEL classification: R11, R12, O11, E17, O31.



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1. INTRODUCTION

How does knowledge, i.e. innovations, diffuse over space? Traditional diffusion theory does not consider this question, but instead focuses on diffusion over time. Economists were inspired by natural sciences (like epidemiology) and sociological thinking (with concepts like 'spread by contact'), and devoted attention to the study of S-shaped curves as a representation of the diffusion of knowledge over time. After some early publications such as Gerschenkron (1962) and Abramovitz (1979), research on technology diffusion over countries was deepened during the 1980s. This literature became known as technology gap literature. Main argument of this literature is that technology is regarded to spread to a country due to specific characteristics of that country. For example, the presence of a high learning capability which facilitates the implementation of new technologies. The countries between which spillover of knowledge takes place, need not necessarily be geographically close to each other, therefore geographic proximity does not play a substantive role. The so called spatial factors like agglomeration economies and growth poles as used by geographers were not addressed in this set of theories. Only the geographic stream of researchers used these concepts. Within the field of geography, diffusion is regarded as a purely geographical process and space is a central concept. Scale effects and proximity effects are considered to be the main vehicles which carry the spread of economic activity.

The aim of this paper is to integrate both streams in the literature. The lack of space as an explaining factor in technology gap models is addressed in this paper by extending a simple technology gap model with the concept of geographical distance. In order to do so this paper is organized as follows. In section 2, the spillover system is described. Every region gets a certain amount of spillovers from its neighbours according to certain rules. These spillovers are the input to the growth of a region. In section 3, the model is analysed, using different kinds of artificial worlds. The final section will provide conclusions from these experiments.

2. DESCRIPTION OF THE MODEL

The model that is presented in this section will be aimed at incorporating several considerations from economic theory and geographic theory. Analog to other (empirical) catching up literature, the idea will be adopted that technology is the only factor that influences output. In fact, output growth is a linear function of the growth of the knowledge stock.

$$\frac{\dot{Q}_i}{Q_i} = \beta \frac{\dot{K}_i}{K_i} \quad (1)$$

in which Q_i denotes the level of output of region i and K_i points to the level of the knowledge stock of region i . β is a parameter. Dots above variables denote time derivatives.

Next, the equation of the knowledge stock in region i (K_i) is specified. The growth of the knowledge stock in region i is assumed to be a function of output growth (via the Verdoorn effect), spillovers received from surrounding (not necessarily contingent) regions (S_i), as well as an exogenous rate of growth (ρ_i).

$$\frac{\dot{K}_i}{K_i} = \alpha \left(\lambda \frac{\dot{Q}_i}{Q_i} + S_i + \rho_i \right) \quad (2)$$

in which α and λ are parameters.

The growth of the knowledge stock of a region is partly determined by the spillovers received from surrounding regions. For the explanation of the spillover term, it is convenient to first consider two regions, later on this assumption will be relaxed and a multi-region model will be constructed. In the two-region setting it is assumed that there is one technologically advanced region and one backward region. There are two mechanisms by which spillovers between regions take place. First, via technological distance. This is the ‘ordinary’ way of modelling this, theoretically justified by the technology gap literature and analog to the procedure followed in papers by Verspagen. The second mechanism by which spillovers take place is via geographical distance. A small geographical distance facilitates knowledge spillovers between regions. This assumption comes forward from geographical literature. The uniqueness of introducing geographic distance the way it is done in this model, is that there will always be spillovers (although they might be small) from one region to another, due to the geographic distance. This, in contrast with other catch-up literature in which no spillovers exist if the knowledge stocks in the regions are equal.

The modelling of the spillovers takes the following form:

$$S_j = \frac{\delta_j}{\gamma_{ij}} e^{-\left(\frac{1}{\delta_j} G_{ij} - \mu_j\right)^2} \quad (3)$$

in which S_j denotes the spillovers generated by region i and received by region j . G_{ij} points to the technology gap and is also used as an indicator of technological distance. On the exact specification of the gap we will come back later. γ_{ij} points to the geographical distance between two regions. If γ_{ij} increases, the spillover is reduced, because the occurrence of face-to-face interaction between firms will be less intensive¹. δ_j points to the intrinsic learning capability of region j . The assumption, taking from technology gap models, is that regions which have a certain capability to learn (because they have an educated workforce, good educational facilities, and/or a developed knowledge infrastructure) can implement the knowledge from other region more easily. The part of the received knowledge that is valuable for these regions (in the sense that they can implement it straight away) is much larger than for regions which have a low ‘learning capability’. In Equation (3) a high learning capability ascertains that a region will have many spillovers. μ_j can be interpreted as a catching up parameter, it determines the magnitude of the gap at which catch up occurs. We will come back on this factor later on.

Another extension compared to the ‘ordinary’ technology gap models lies in the fact that the model allows spillovers to occur in two directions, either from technological leader to backward region(s), or the other way around. The first direction consists probably of the largest spillover stream, since the backward region can receive many spillovers from the more advanced region. However, spillovers from a backward region to the leader region can also happen, because it could well be possible that the backward region has (developed) complementary knowledge, knowledge that was not yet in the hands of the leader. So there might be a small flow of knowledge from laggard to leader. If the spillover functions of two regions are drawn (all parameters are assumed to be equal between the regions), we get Figure 2.A. As can be seen, the expression for the knowledge spillovers takes the shape of a Bell-function. Left from the y-axis region j is the most advanced region, since region i receives the most spillovers. Right from the y-axis the opposite is the case, i.e. region i is the leader region. As can be seen in the figure, below the graph of the spillover j receives from i (S_j) there lies still a small part of the S_i line, indicating that, however region i is the leader, still spillovers from region j to region i take place, although they might be small. Another point that has to be noted, looking at the figure, is that the

net spillover will be equal to zero in the case where the gap between the two regions is zero (i.e. they have equal knowledge stocks). This emphasizes the unique working of this model. In this situation there are still spillovers (although the knowledge stream from region i to j is as large as the other way around), contrary to regular catch up models, where knowledge spillovers are reduced to zero, if knowledge stocks are equal between regions.

In Figure 2.B the spillovers received by one region for a two-region model are displayed. With the help of this graph a lot of characteristics of the spillover function become clear. Note that the top of each spillover curve lies at a technology gap equal to $\mu_j \delta_j$. The maximal spillover is equal to δ_j / γ_{ij} . We take S1 as the starting point, and we consider what happens to the spillover function under certain conditions. First, an enlargement of the geographical distance between two regions will lead to proportional lower spillovers received by the laggard region, depicted by the thick line (S2). With the same technology gap, less spillovers occur. Second, an increase in the learning capability of the lagging region (δ_j) will cause the spillover function to behave as displayed by the dotted line (S3). The spillovers that region j will receive are now larger. The gap at which maximal spillovers occur is also larger, indicating that not only more spillovers are received, but also at a larger technological distance from the leader region, i.e. the lagger has a larger leaning capability and therefore is able to learn more (magnitude of the spillover function) and more easy, or earlier (being at a larger technological distance). The top of the curve (which is equal to δ_j / γ_{ij}) has been replaced upward and to the right of the original position.

A third characteristic of the expression for the spillover can be shown with the help of Figure 2.C. If μ_j is increased, the curve will shift to the right. This has several effects. First, it is easy to see that S5 shows a curve by which the spillovers are very near to zero in the case of equal knowledge stocks across regions ($G=0$). This indicates that the model under these circumstances resembles a regular catch-up model, which is characterised by zero spillovers if the technological distance is zero. However, the resemblance is only superficial, as will be shown later on, the geographic effect is not nearly reduced to zero. Second, because the top of the curve moves to the right, catch-up does become more profound. At a larger technological distance, it is still possible to receive relatively a lot knowledge spillovers and therefore may be to catch up. The specifics of this mechanism will become more clear after we discussed the net spillover function. For now it is enough to see that μ_j moves the spillover curve in a horizontal way, making catch-up easier or less easy.

To finish the set up of the model, we need to define one more equation, namely the specification of the technology gap (G_{ij}) which is also used as an indicator of technological distance. G_{ij} can be expressed as a function of the knowledge stocks of two regions:

$$G_{ij} = \ln \frac{K_i}{K_j} \quad (4)$$

in which i is the leader region and j is the lagger. A practical feature of the above logarithmic function is that the gap becomes zero when the knowledge stocks of both regions are equal.

To analyse the dynamics of this model, we take the time derivative of the technology gap in Equation (4) and substitute equations (1), (2) and (3). For a two-region model this boils down to:

$$\dot{G}_{ij} = \frac{d}{dt} \ln \frac{K_i}{K_j} = \frac{\dot{K}_i}{K_i} - \frac{\dot{K}_j}{K_j} = \frac{\alpha}{1-\alpha\beta\lambda} ((\rho_i - \rho_j) - (S_j - S_i)), \text{ with } 0 < \alpha\beta\lambda < 1 \quad (5)$$

in which α , β and λ are assumed to have the same value in each region. This expression can be analysed using Figure 2.D.

Two cases can be distinguished, one in which region i is the most advanced, and one in which region j is the leader region. Since the two cases are mirrored, we will restrict ourselves to describing only one, namely the case in which region i is the leader. In Figure 2.D, $S_j - S_i$ represents the difference in spillover generated by the lagging and the leading region (the net knowledge spillover curve). $\rho_i - \rho_j$ displays the difference in the exogenous rate of growth of the knowledge stock between the two regions. If we assume that region i is the most advanced, it is expected that the exogenous rate of growth of the knowledge stock in region i exceeds that of region j , thus $\rho_i > \rho_j$ and therefore $\rho_i - \rho_j > 0$ (as drawn in the Figure). The intersection points of the two curves correspond to points where the time derivative of the technology gap is equal to zero. This means that at these points the system is in equilibrium. The nature of these equilibrium points differs. The intersection point at which the S-curve has a positive slope is stable, whereas the other intersection point is an unstable equilibrium. Therefore the arrows of motion can be drawn as is done in Figure 2.D. When the $S_j - S_i$ curve is below (above) the $\rho_i - \rho_j$ line, that means that the knowledge spillovers received by region j are smaller (larger) than the exogenous increase of the gap, resulting in a net increase (decrease) of the technology gap. Therefore, depending on the net

knowledge spillovers the technology gap either converges to the equilibrium point close to the y-axis, or goes to infinity.

The position of the intersection points differs if one of the two curves, either $\rho_i - \rho_j$. Or $S_j - S_i$, or both, move. In order to distinguish between the different effects we will first consider a variation in the difference in the exogenous rate of growth of the knowledge stock between the two regions, $\rho_i - \rho_j$. If the difference is enlarged in favour of region i , the $\rho_i - \rho_j$ line in Figure 2.D moves upward, meaning that the range of technology gaps at which catch-up occurs becomes smaller, therefore, there are less opportunities for catch-up. If, on the other hand, the exogenous rate of growth of the knowledge stock in the backward region is increased (e.g. by expanding research efforts) up to a level comparable with the advanced region, the technology gap will be closed and the regions have converged.

The second possibility to alter the position of the intersection points, is to change the position of the spillover function $S_j - S_i$, by varying the parameters γ , δ and/or μ . With respect to the influence of a variation in the parameters on the equilibrium points of the system a few things can be said. First, we consider a variation in the geographical distance between two regions. Of course this experiment seems a little odd, since we consider only two regions, and we cannot pick up one region and locate it somewhere else in order to decrease the geographical distance. However, this experiment is carried out in order to show the influence on the net spillover curve of this parameter. A decrease in the geographical distance would have the effect that the spillover curves S_i and S_j would increase (explained by Figure 2.B) and the maximum of the $S_j - S_i$ curve in Figure 2.E would move upwards². The bifurcation diagram would look as displayed in Figure 2.F³. On the horizontal axis of the bifurcation diagram are the values of the geographical distance parameter γ_{ij} . On the vertical axis are the (equilibrium) values of the technology gap. The line E_s shows a stable equilibrium, while the line E_u points to an unstable equilibrium. The line S_{max} represents the top of the net spillover curve in Figure 2.F. This figure shows that for high values of γ_{ij} no equilibrium value of the technology gap exists. Then, for the threshold value of γ_{ij} , one equilibrium appears. This is point B in the figure, the bifurcation point. This equilibrium point is the point of tangency between the $\rho_i - \rho_j$ line and the $S_j - S_i$ curve. For values of γ_{ij} smaller than the threshold level, two equilibria exist, as described by the curves in the bifurcation diagram. As shown in Figure 2.F the value of the stable equilibrium is always closer to zero than the maximum of the knowledge spillover term.

However, the geographical distance is not something that can be influenced by a region. What can be influenced is the intrinsic learning capability of a region. This is the second parameter

which can move the S_j - S_i curve. The effect of an increase in the learning capability of region j (δ) on the $(S_j$ - $S_i)$ -curve is displayed in Figure 2.G. Note that δ has not changed. It can clearly be seen that on the right hand side of the figure the top of the curve has moved to the upper right of the figure. What has happened on the left hand side is a bit more difficult to see. The top has moved downwards and a little bit towards the y-axis. The bifurcation diagram now looks as displayed in Figure 2.H. On the horizontal axis the learning capability is displayed. This figure shows that as the learning capability of the lagging region is increased, a threshold level is reached at which the curves S_j - S_i and $\rho_i - \rho_j$ (in terms of Figure 2.D) are tangent. This is the bifurcation point B. A further increase of the learning capability leads to two equilibria, a stable and an unstable one. Note that the E_s line for the stable equilibrium can even go below the x-axis if the difference in exogenous growth rates of the knowledge stock is small enough.

The final parameter that can be changed and will have an effect on the graph in Figure 2.D is the catch-up variable μ . We will increase μ_j and μ_i simultaneously ($\mu_i = \mu_j$). The effect on the $(S_j$ - $S_i)$ -curve is that the tops will move away from the x-axis and the y-axis, as displayed in Figure 2.D. As can be seen on the right hand side of Figure 2.D, as μ goes to infinity, the top of the spillover will go to δ/γ . As the catch up parameter μ will increase in value, the net spillover function will approach the shape given by Equation (3). Therefore, the top of the net spillover curve will approach the value δ/γ . In the left hand side of the figure the top will approach the value $-\delta/\gamma$ for the same reason. This means that an increase in μ will enlarge the effect of the geographic distance parameter in the model. Since an increase (or a decrease) in γ will decrease (or increase) the top of the net spillover curve proportionally, with a small μ , a variation in γ will have a smaller effect. S_i becomes larger in terms of Figure 2.A, but S_j also, thus the top of S_i - S_j will be lower than δ/γ . If μ is large, an increase (or a decrease) in γ will decrease (or increase) the top of the net spillover curve to its full extent.

The bifurcation diagram now shows $\mu_i (= \mu_j)$ on the x-axis. As can be seen in Figure 2.I, the stable equilibrium first moves towards the y-axis as $\mu_i (= \mu_j)$ is increased. If μ is increased even further, the stable equilibrium starts to move away from the y-axis in a steady pace. The reason for this can be found in Figure 8. Here is shown that the S_{max} - curve approaches the value δ/γ (because the larger μ becomes, the more the S_i and S_j curve resemble a Bell-function), this implies that a further increase of μ leads to only a minimal increase in height of the curve, meaning that the E_s , E_u and S_{max} curves increase with a steady pace in the bifurcation diagram.

From the set up of the model is clear that spillovers take the crucial role in the model. These spillovers take place according several intuitive rules, like the shorter the distance between two

places, the more spillovers will be transferred and the higher the capability of a region to implement new technology, the more spillovers will be accommodated. These simple rules give occasion to an analysis by a simulation for several regions on a plain.

3.2 VARIATIONS IN THE LEARNING CAPABILITY

In this set of experiments a randomly generated learning capability is appointed to every region. The higher the learning capability of a region, the more it is capable to implement the knowledge which will be available from other regions. Regions with a higher leaning capability make more effective use of the knowledge they receive, therefore the potential spillover they can receive increases.

In order to find out the sensitivity to the ranges, several sets of 50 runs with different ranges were generated⁴. Table 3.2.A gives some statistical information over 50 runs in different ranges.

Table 3.2.A

	0 to 2	0 to 10	0 to 50	1 to 10	1 to 2	2 to 3	40 to 41
Average	1.562	1.562	1.562	1.901	0.543	0.543	0.580
Standard Deviation	0.435	0.435	0.435	0.578	0.088	0.078	0.016
Coefficient of Variation	0.278	0.278	0.278	0.304	0.161	0.144	0.028

A number of conclusions can be drawn from this table. A low coefficient of variation in Table 3.2.A. characterizes a situation in which no regions fall behind. The disparity across the gaps towards the leader shows a distribution similar to Figure 3.1.1.B. We see that the range in which the learning capability is allowed to vary should be small in order to have a low coefficient of variation. The interval 1 to 2 gives a lower coefficient of variation than the interval 0 to 2. Secondly, the interval should be small *relative* to the level of the learning capability. If the interval 1 to 2 is compared with 2 to 3 and 40 to 41, can be seen that the coefficient of variation decreases over these intervals.

This is quite intuitive, since a relative large difference in learning capabilities causes that one region cannot implement the knowledge that has spilled over from another region. This is illustrated in Figure 2.G in which a relative large difference in learning capabilities among two regions causes a skewness of the spillover curve in a way that it becomes very difficult for the backward region to catch up.

The sensitivity to the lower boundary for the lattice is shown in Figure 3.2.A. In this figure the lower boundary is increased from 1.8 till 2.0 with steps of 0.01. It has to be noted that in this

range there are no regions that fall behind. For the column (Figure 3.2.B) and the globe (Figure 3.2.C) the same analysis is done. There are a number of striking features looking at these three figures. As the coefficient of variation for the lattice is fan-shaped - dispersed equally to all directions - when the lower boundary is decreased, the coefficient of variation for the column seems to do the same, but exclusively in a downward direction. The coefficient of variation is more dispersed and decreases as the range out of which the learning capability is randomly chosen is enlarged. When the range is limited to only one amount (in this case 2) the only factor that has an influence on the regions is the geographical distance. As we have seen before this is a polarizing factor and leads to a distribution of gaps as displayed in Figure 3.1.1.B. When the range is broadened, a second polarizing factor executes an influence on the regions, namely the learning capability. These two polarizing factors counteract and cause chaos. Higher levels of chaos are generated by allowing the learning capability to have a higher impact (as the range broadens and the lower boundary is decreased). The chaos leads to lower levels of disparity of gaps *within* one run, causing the downward slope. However, it also leads to more disparity *across* the runs. Since there is more room for the learning capability as a second polarizing factor, the outcomes - in terms of coefficient of variation across the gaps - of different runs become more distinct, causing the fan-shape.

This does not yet explain why the column shows a downward slope compared to the lattice. The difference between the two spheres lies mainly in the fact that the column has 3 regions which will become leader at the end of the run, if only geographic forces are considered. In addition to these 3 there are 6 regions which display a very small (0.0094) gap towards the leaders. For the lattice there is only one region which has the same characteristics. The six first ring regions have a gap which is relatively large (0.039). Together with the pattern of gaps of the rest of the regions, the column displays a coefficient of variation which is higher than in the case of the lattice. When a second polarizing factor is introduced in the column-sphere, the coefficient of variation will decrease. Every individual run shows less and less leaders and also less disparity across gaps, since the difference in learning capabilities across regions counteracts the difference in geographic location. Figure 3.2.C illustrates this point. The average gap (over 50 runs) of every region is displayed against an increasing lower boundary. The four groupings are characterized by the four different kinds of geographic locations a region can have (see the pattern in Figure 3.1.2.A). An increase in the lower boundary leads to a decrease of the gaps towards the leader.

The globe does not show such a clear fan-shape when the range is increased. This is due to the characteristics of the globe. Since there are no edges on a globe, no region can show a large gap

towards another, because they are all closely located to each other. This is not totally true, since the largest possible distance on the globe (5) is larger than the one on the column (4) or the lattice (4), however, there occurs much more overtaking of the lead position on the globe, which causes that the gaps do not reach large values. When a backward region develops a relatively large gap, it will probably not take long before a region becomes a leader in the close neighbourhood (by chance) and the backward region can lower its gap again.

What is also peculiar in this figure is the typical curve the coefficient of variation displays when the lower boundary is decreased. From 1.9 downwards there seems to be a horizontal trend. The coefficient of variation does not decrease any further and has reached its lowest value. This is even more peculiar when we realize that this value - about 0.5 - is the same as in the lattice. It seems to be an “emerging property” of the model.

The conclusion that can be drawn from these figures (3.2.B and 3.2.D) is an extraordinary one. Contrary to what is known from traditional technology gap theory it is found that when differences in learning capabilities increase across regions, the disparities in growth decrease. This finding suggests that the geographic distance across regions generates a force which is this important that it counteracts the influence of difference the learning capability, thereby generating lower levels of disparity. It has to be kept in mind, however, that this conclusion holds under strict conditions, as the globe assumes that all regions (of the same shape) have an equal favourable geographic location with respect to receiving knowledge spillovers.

3.3 VARIATIONS IN THE EXOGENOUS RATE OF GROWTH OF THE KNOWLEDGE STOCK

In the following set of experiments the exogenous rate of growth of the knowledge stock is randomly chosen from different ranges, while the knowledge stocks of the regions are initially equal. Compared to the first set of experiments in which the knowledge stock was initially random, this set behaves a little differently. In general, it does matter for final leadership whether a region has an initial advantage. The region with the initial leader position will in most of the cases remain in this position.

There occur two determinants of initial leadership. On the one hand, the initial exogenous rate of growth of the knowledge stock is a very important influence in determining the initial leader. However, the geographical distance to centrally located regions has not to be taken for granted. Since geographic forces rule the amount of spillovers received by all regions. This is the second determining force in the model. Especially in the first time periods of the run it is very important

for a region to have a geographic location which attracts a lot of spillovers. In general, the region which has the highest ρ of all centrally located, first order regions (which receive many spillovers due to their geographic location) will most likely become a leader.

This can be explained by looking again at Equation (5') in section 3.1.1. The term $(\rho_i - \rho_j)$ together with the term $(\sum_r S_{in} - \sum_r S_{jn})$, determine the position of the horizontal line in Figure 3.1.1.C. In the foregoing experiments the term $(\rho_i - \rho_j)$ was equal to zero, in this experiment it is randomly chosen out of different ranges. It can easily be understood that if the difference between the exogenous rates of growth of the knowledge stock across regions is small, the horizontal line in Figure 3.1.1.C will be located close to the x-axis. This implicates that the horizontal line and the S_j - S_i curve intersect at the two equilibrium points of which the right one has a position relatively far to the right. Therefore, a region can catch-up even at a large gap towards the leader. A small difference between the exogenous rates of growth of the knowledge stock therefore implies a high chance on a distribution of gaps as displayed in Figure 3.1.1.B (in which all regions can catch-up a no regions fall behind). However, if every region is able to catch-up, that does not necessarily mean that the pattern of Figure 3.1.1.B comes up. It is very well possible in this experiment that another region than a centrally located region will end up with final leadership, and all other regions will catch-up to their stable equilibrium gap towards the leader region.

In order to see in which way the model is sensitive to changes in the range out of which the exogenous rate of growth was chosen, several sets of 50 runs were carried out - beginning with the lattice - each with a different range. Figure 3.3.1.A shows the results when the lower boundary of the range is increased from 1.8 to 2 with steps of 0.01. The upper boundary is held constant at 2. The vertical axis shows the coefficient of variation over the last period in every run. The darker a spot in the graph, the more observations possess this coefficient of variation. This figure illustrates that only for a very small interval all 50 observations show nearly the same coefficient of variation.

However, the coefficient of variation seems to have a fan-shape (identical to the experiment with the learning capability). All coefficients of variation tend to one amount as the gap between lower and upper boundary decreases. The explanation is the same as in the experiment with the learning capability. Increasing the difference in exogenous rates of growth of the knowledge stock introduces a second polarizing factor, next to the geographic force. The two polarizing factors counteract, thereby causing a larger equality across gaps *within* one run, and a larger disparity across coefficients of variation *across* several (50) runs.

For the column the sensitivity to the lower boundary of the interval is given in Figure 3.3.B. Contrary to the lattice this sphere seems to generate mostly coefficients of variation which are smaller than the one in which 3 leaders appear (Figure 3.1.2.A). Again a fan-shape appears. The explanation for these features is the same as for the case in which the learning capability was varied across ranges. The fan-shape is generated by the growing inequality in the coefficient of variation across runs, as the difference in learning capabilities across regions counteracts the difference in geographic location. The line is downward sloping because within a run the disparity across gaps decreases, due to the characteristics of the column.

The globe shows a curve which is downward sloping and does not show the fan-shape. Contrary to the former experiment with the learning capability, this curve does not show a horizontal part when the gap between lower and upper boundary is large. However, we see that the lowest observation has a coefficient of variation of about 0.85 at a lower boundary of 1.8. A coefficient of variation of about 0.5 is not yet reached. If the lower boundary is further decreased, the figure shows lower coefficients of variation until the magical 0.5 is reached and from then on there is a horizontal trend (these results are not shown in the figure yet).

The conclusion still holds that increasing differences (disparity in the exogenous rates of growth of the knowledge stock) across regions causes a decrease in the disparity of the gaps. This is indicated by the downward trend in the figures.

3.4 THE ROLE OF THE GEOGRAPHICAL DISTANCE

In the foregoing sections an assumption was made with respect to the role of geographical distance for the spillover of knowledge, namely that there is an inverse relationship between geographical distance and knowledge spillovers. Equation 3 shows the spillover function of the model. In this equation, a neighbouring (in the sense that two regions share one border) region is assigned a weight of 1 ($\gamma_{ij} = 1$). Regions which do not share a border with a specific region are given a weight by using the concept of nearest neighbours, which means that a different (lower) weight is attributed to a second order neighbour. A second order neighbour does not share a border with region, but does share a border with a neighbour of the specific region. It is very important to notice that no evaluation of relative importance of the connection between regions, based on ex ante known information (for example the presence of roads and railways) is taken into account. Only geographical distances are reflected. In this way the distance γ_{ij} is determined for every region towards every other region. Now, it is possible to make a matrix of shortest paths in which for each region the order is defined of every other region. After establishing the orders

of all neighbours, the corresponding weights are determined using the inverse of the orders (inverse shortest path, Hagett, Cliff and Frey 1977). Thus the connection between region 2 and 5 of the lattice of honeycombs (see Appendix) gets weight $1/2$. By taking the inverse orders the relation between geographical distance and weight becomes $1/(\gamma_{ij}^x)$ with $x=1$, assuming that the higher the geographical distance, the influence of region on another one becomes increasingly less.

In this paragraph, we consider two other possible assumptions over the influence of geographical distance on knowledge spillovers. First of all, one could assume that distance does not make any difference for the spillover of knowledge. Knowledge can as easily be learned from a place far away than it can be learned from a place nearby. This was assumed under the traditional technology gap theories.

Second, the relationship between geographic distance and knowledge spillovers can be assumed to be linear. This way of assigning weights to geographical distances (borrowed from geography, Hagett, Cliff and Frey 1977) adopts the concept of maximum shortest path minus shortest path. Figure 3.4.A shows the relation between the order of a region and the weight of the spillovers under the three different ways of modelling the geographic distance.

One has to keep in mind that all matrices of weights are scaled to 1000 ($\sum_i \sum_j \gamma_{ij} = 1000$), therefore making it possible to compare the results across methods and across spheres.

With respect to the resulting pattern of the gaps towards the leader region (for a lattice of honeycombs), the inverse order method leads to a pattern as displayed in Figure 3.1.1.A. Knowledge spills over to a region more easily when the distance towards a knowledge intensive region is small. Since region 1 has the most favorable location, it will end up being the leader (all other things equal). We can add more importance to the geographic distance by increasing the value of x in the relation $1/(\gamma_{ij}^x)$. Figure 3.4.B shows the results for the lattice of honeycombs. On the horizontal axis the power x is displayed. The vertical axis shows the coefficient of variation across the gaps towards the leader at the end of the run. By enforcing the geographic influence it is expected that there would occur more polarization and therefore increasingly higher coefficients of variation. Figure 3.4.D shows that initially something else happens. A possible explanation for this could be that the standard deviation increases in another way than the average, this causes at a certain moment ($x=5$) a ratio between the two which is lower than at $x=2$.

The column shows different results. Where the coefficient of variation for the lattice seems to stabilize at a certain level when x is increased, the coefficient of variation for the column increases exponentially after an initial decrease (Figure 3.4.C).

The globe shows even stranger results. Since the globe consists of relatively many regions compared to the lattice and the column, we only succeeded in taking x up to 3. However, the coefficient of variation is exactly the same for every value of x , $\{x \in \mathbb{N} \mid x \geq 1\}$.

If no geographic influence is assumed, this leads to all regions becoming a leader in the end. Since in this model all regions are assumed to be identical and only differ in geographic location, this is bound to happen.

The third method of modelling geographic distance (maximum shortest path minus shortest path) does not differ that much from the inverse order method. Figure 3.4.D shows the resulting gaps.

In this paper is chosen for taking the inverse orders with $x = 1$, based on the idea frequently advocated by geographers that regions which are located further away have increasingly less influence on the rest of the plain.

4 PRELIMINARY CONCLUSIONS AND FUTURE RESEARCH

From the simulations executed up till now we can draw several extraordinary conclusions. The findings for the variation of the learning capability suggests that the polarizing influence of learning capability can be counteracted by geographic forces. In a situation of relative high differences across regions with respect to the initial learning capability, this creates as a result less disparity with respect to levels of knowledge across regions. This conclusion also holds with regard to disparity in the exogenous rate of growth of the knowledge stock.

My future research on this topic concentrates first on varying all parameters at the same time, in order to see the result of all combined influences. Furthermore, I am constructing and testing a more elaborate model which contains more economic variables. A labor market is added to the spillover system as well as capital accumulation. This model will also be more realistic in that it distinguishes regions within countries. In order to make this possible, an exchange rate system is introduced.

This extended model would allow us to explore the influence of different stadia of economic integration on the economy. At first, wages and exchange rates can both stabilize the economy. After the introduction of a single monetary unit, the exchange rates are fixed and the only stabilizing factor in the system are the wages.

FIGURES

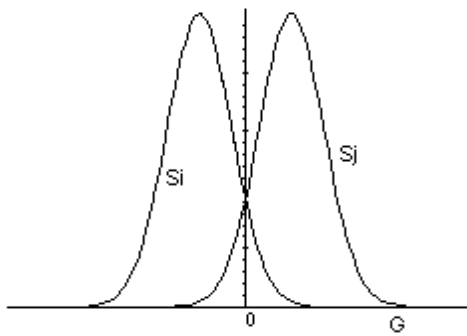


Figure 2.A

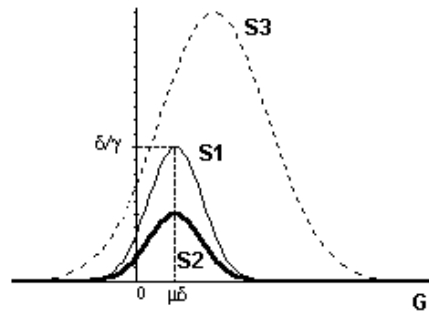


Figure 2.B

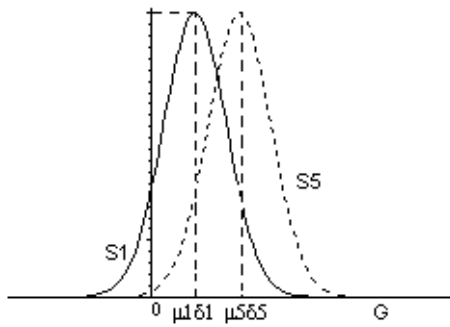


Figure 2.C

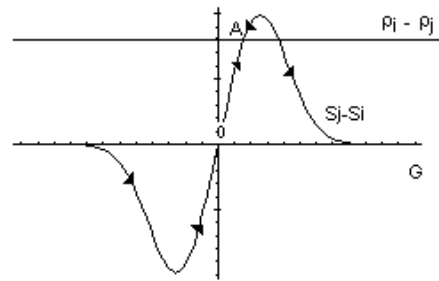


Figure 2.D

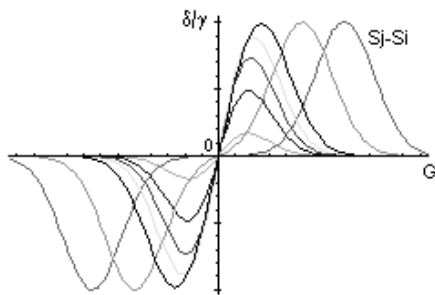


Figure 2.E

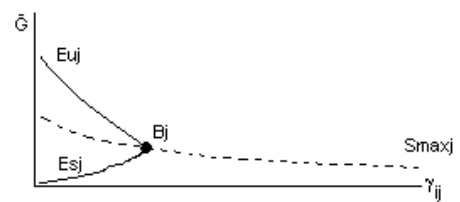


Figure 2.F

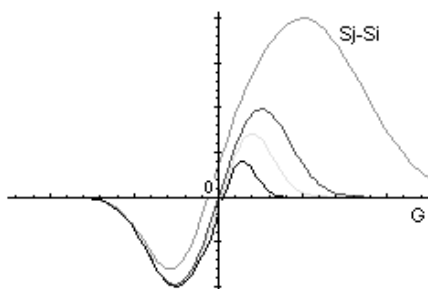


Figure 2.G

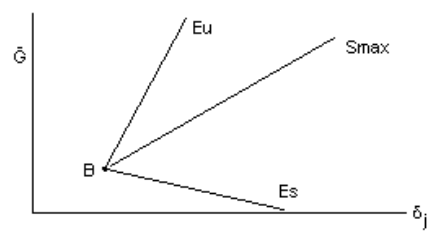


Figure 2.H

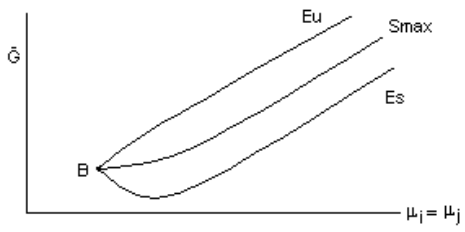


Figure 2.I

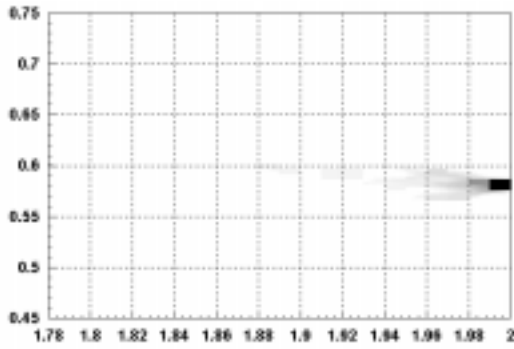


Figure 3.2.A

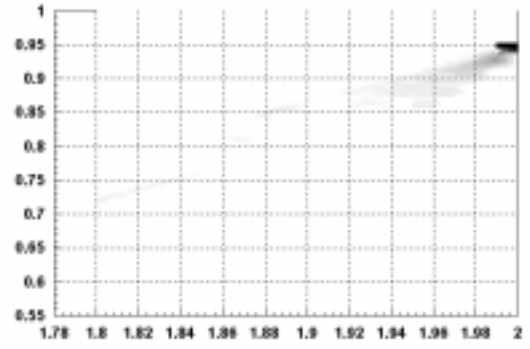


Figure 3.2.B

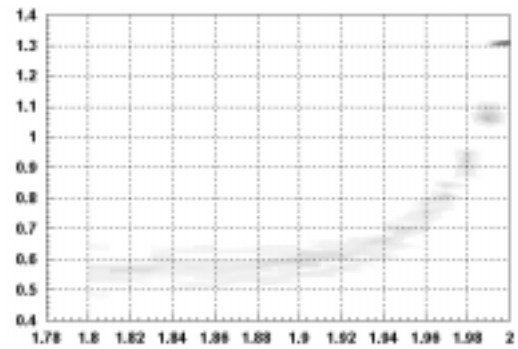
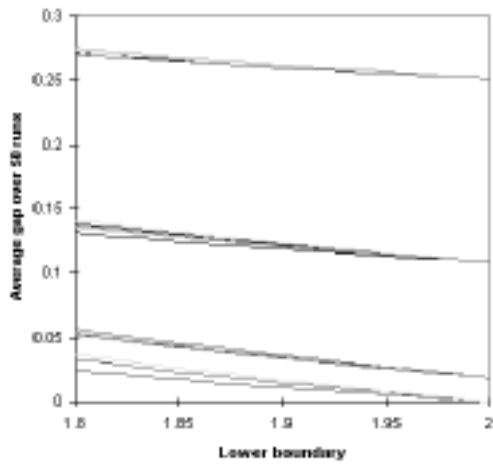


Figure 3.2.D

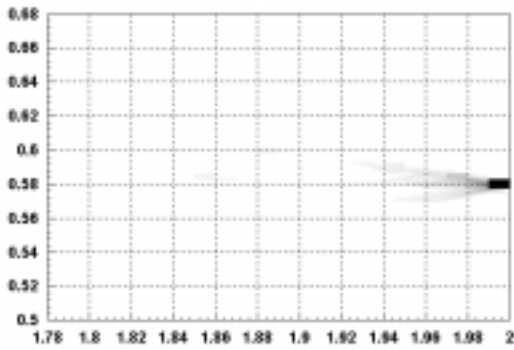


Figure 3.3.A

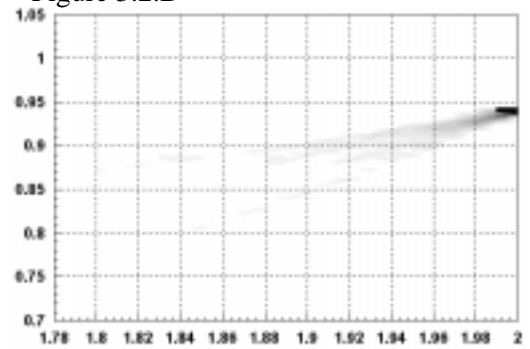


Figure 3.3.B

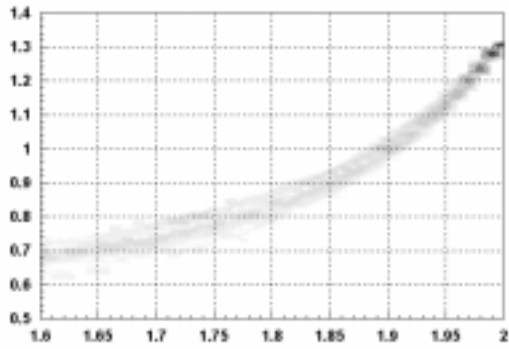


Figure 3.3.C

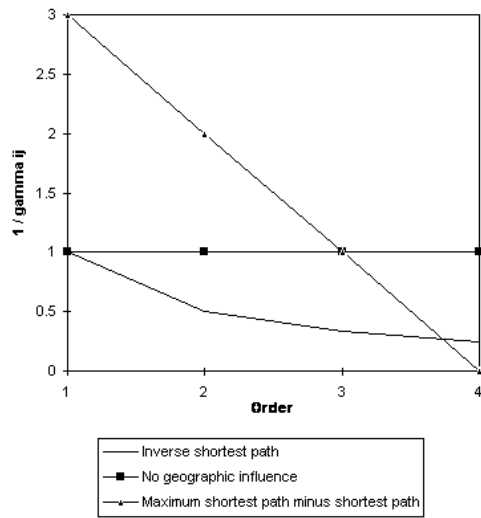


Figure 3.4.A

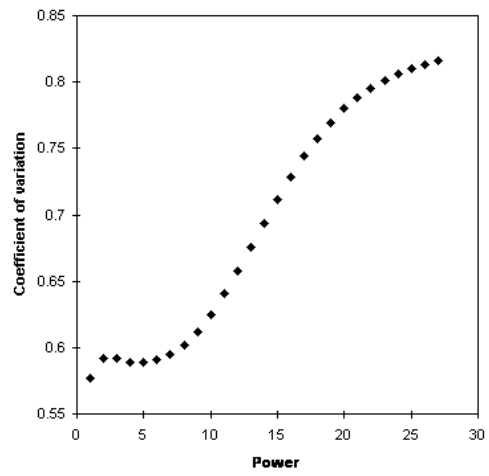


Figure 3.4.B

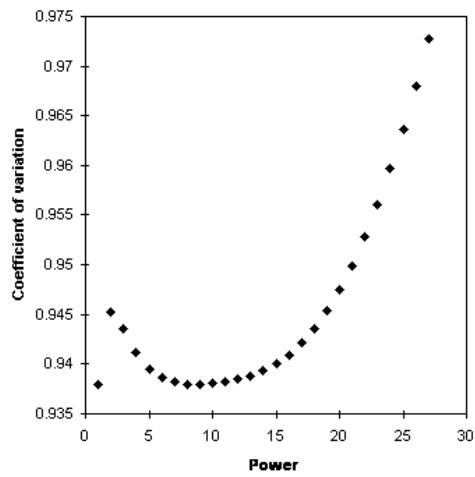


Figure 3.4.C

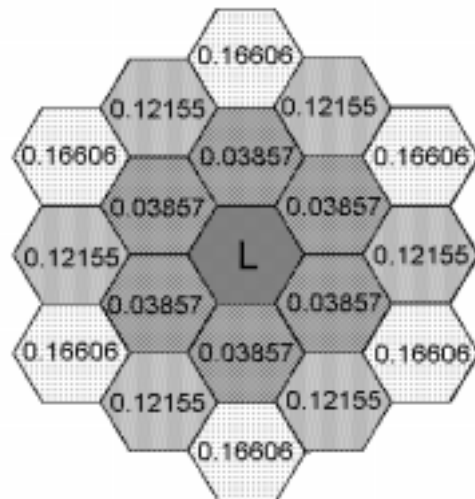


Figure 3.4.D

APPENDIX A

Default levels of the variables and values of the parameters:

10000 (Number of periods, t)

10 (Knowledge stock, K)

1 (Exogenous rate of growth of the knowledge stock, ρ)

1 (Learning Capability δ)

1 (catch-up parameter, μ)

0.005 (β)

0.005 (α)

1 (λ)

γ (geographical distance) is constructed with the help of three different types of distance tables, one for each sphere.

APPENDIX B

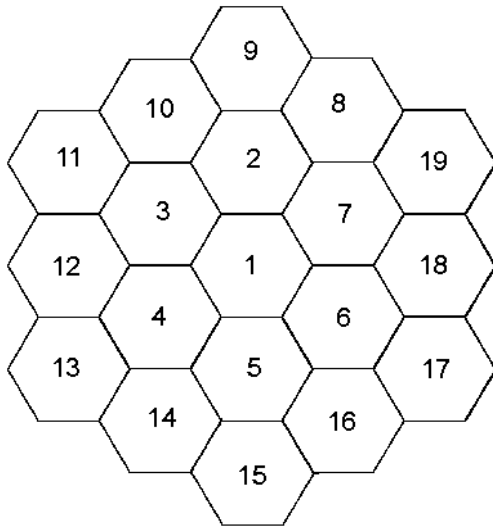


Figure A

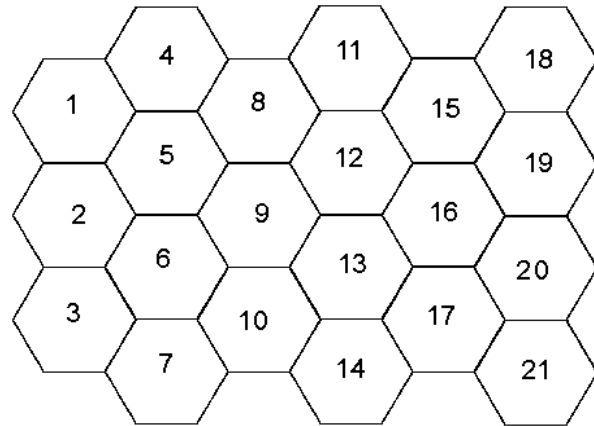


Figure B

Figure A displays the topography of the regions on a lattice of honeycombs. The number within each hexagon was used to establish the geographical distances between all hexagons. Figure B displays the column. Region 1 borders to 18 and 19, region 2 to 19 and 20, and region 3 has 20 and 21 as its direct neighbours. Thus if one would walk from region 20 to the right, one would reappear at the left of the figure in region 2 or 3.

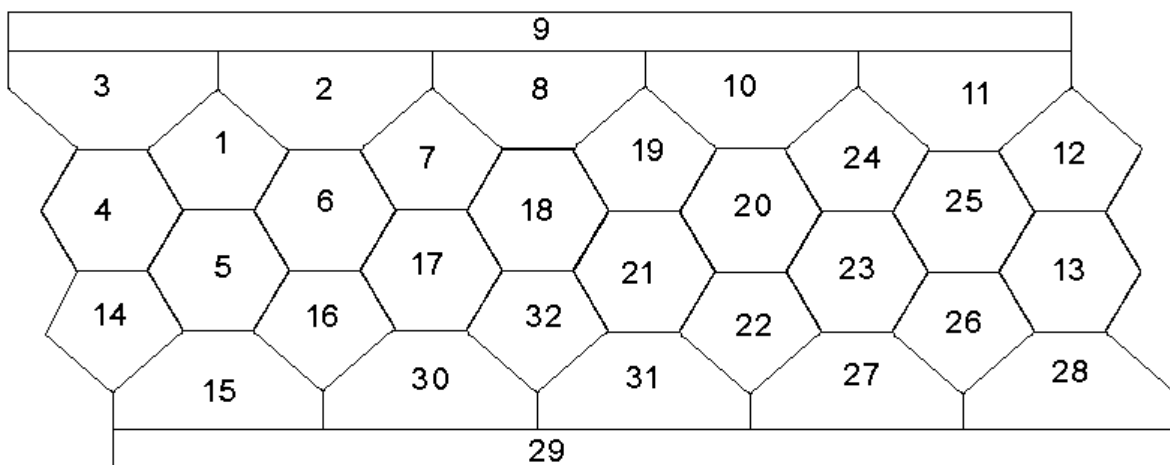


Figure C

The above figure represents a globe with 12 pentagons and 20 hexagons. For the graphical representation we used the sample principle that was applied in making a map of the world. Hence the regions close to the poles look larger as they actually are, while the region around the equator show their true proportions. At the bottom and at the top are regions 29 and 9. These are

pentagons, for example region 9 borders to five regions, namely 3, 2, 8, 10 and 11. Region 29 and 9 are in reality as large as region 1. The graphic representation of a globe had also as a consequence that for example region 3 seems to differ in size from region 6. Again, this is not the case in reality, region 3 is an ordinary hexagon. The same goes for all the other regions bordering 9 or 29. It should also be noted that region 11 borders to 3, 12 borders to 3 and 4, 13 has 4 and 14 as direct neighbours, whereas 28 borders to 14 and 15.

APPENDIX C

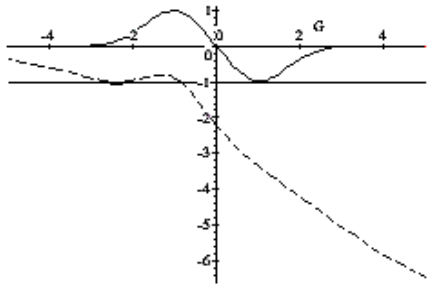


Figure A

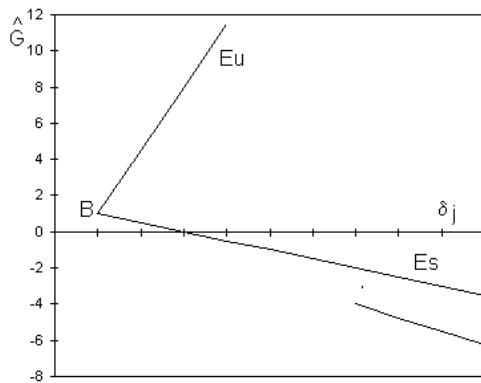


Figure B

When the difference between the learning capability among two regions is large enough, something quite remarkable happens. Figure A shows the case in which the learning capability of one region is 7 (the dotted line), while the learning capability of the other region is only equal to 1.

The dotted line intersects with the horizontal line at three places. This causes the bifurcation diagram to look as displayed in Figure B.

APPENDIX D

TABLE 1: Variation in the initial knowledge stock for the lattice of honeycombs

Lower boundary	-	0.010	0.020	0.030	0.040	0.050	0.060	0.070	0.080	0.090	0.100	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
	3.877	3.868	3.861	3.853	3.842	3.821	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	2.249	2.837	2.834	3.815	3.814	3.810	3.805	3.798	3.785	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	3.814	3.735	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	2.782	2.775	3.803	3.778	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	3.883	3.873	3.865	3.859	3.852	3.845	3.837	3.827	3.805	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	3.820	3.818	3.814	3.810	3.805	3.797	3.784	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	3.896	3.878	3.867	3.859	3.852	3.846	3.840	3.835	3.830	3.826	3.820	3.815	3.808	3.798	3.780	0.577	0.577	0.577	0.577	0.577	0.577
	3.804	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	3.938	3.912	3.899	3.890	3.883	3.877	3.871	3.865	3.859	3.852	3.844	3.827	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	2.819	2.814	2.810	2.807	2.805	2.802	2.799	2.794	3.836	3.832	3.823	3.803	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	3.849	3.846	3.843	3.838	3.833	3.826	3.815	3.783	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	2.259	2.258	2.257	2.255	2.253	2.250	2.776	3.806	3.784	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	2.307	2.254	2.251	2.249	2.247	2.244	2.241	2.768	3.825	3.824	3.817	3.802	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	2.808	2.804	2.798	3.853	3.847	3.836	3.798	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	3.830	3.824	3.816	3.806	3.786	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	3.881	3.877	3.872	3.868	3.864	3.859	3.853	3.846	3.834	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	2.767	3.809	3.803	3.789	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	2.249	2.246	2.767	2.758	3.723	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	3.887	3.879	3.873	3.867	3.860	3.853	3.843	3.825	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	2.845	2.818	2.813	2.810	2.807	2.805	2.803	2.801	2.798	2.793	3.847	3.845	3.842	3.838	3.834	3.828	3.820	3.806	0.577	0.577	0.577
	2.788	2.786	2.784	2.781	2.778	2.774	2.764	3.786	3.763	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	3.831	3.821	3.797	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	3.817	3.792	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577
	3.865	3.858	3.852	3.846	3.839	3.830	3.817	3.761	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577

0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
2.788	2.786	2.783	2.779	2.771	3.807	3.798	3.778	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
3.856	3.849	3.842	3.835	3.828	3.821	3.811	3.795	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
2.813	2.813	3.846	3.838	3.829	3.820	3.808	3.784	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
2.792	2.790	2.787	2.784	2.782	3.850	3.848	3.844	3.840	3.835	3.828	3.819	3.799	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
1.914	1.912	1.910	1.908	1.906	2.270	3.762	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
2.819	2.813	4.007	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
3.842	3.838	3.834	3.830	3.825	3.820	3.815	3.807	3.794	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
3.869	3.860	3.853	3.847	3.841	3.835	3.830	3.823	3.815	3.804	3.781	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
3.853	3.845	3.838	3.832	3.825	3.817	3.806	3.785	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
3.842	3.837	3.830	3.820	3.800	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
3.864	3.858	3.853	3.846	3.837	3.817	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
3.866	3.862	3.858	3.855	3.851	3.847	3.843	3.837	3.831	3.822	3.804	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
2.796	2.792	2.788	2.782	3.808	3.803	3.790	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	0.577	
Average	2.606	2.566	2.511	2.415	2.323	2.177	2.084	1.874	1.464	1.077	1.096	0.966	0.772	0.707	0.707	0.642	0.642	0.642	0.577	0.577	0.577
Standard deviation	1.338	1.363	1.433	1.475	1.513	1.548	1.558	1.557	1.443	1.164	1.201	1.064	0.777	0.642	0.639	0.460	0.459	0.457	0.000	0.000	0.000
Coefficient of variation	0.513	0.531	0.571	0.611	0.651	0.711	0.748	0.831	0.986	1.081	1.096	1.101	1.007	0.907	0.905	0.716	0.714	0.711	0.000	0.000	0

In each cell the coefficient of variation for the final period of a run is displayed. In the first row of Table 1, the lower boundary of the interval out of which the knowledge stock was allowed to vary can be identified. The three bottom rows display the average, standard deviation and coefficient of variation over the 50 runs.

3.144	3.144	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938
0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938
0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938
3.146	3.146	3.146	3.146	3.145	3.145	3.145	3.145	4.551	4.551	4.551	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938
0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938
1.828	1.828	1.828	2.498	2.498	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938
0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938
3.145	3.145	3.145	4.551	4.55	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938
4.551	4.551	4.551	4.551	4.551	4.551	4.551	4.551	4.551	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938
3.146	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	0.938	0.938	0.938	0.938	0.938
4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	4.552	0.938	0.938	0.938	0.938	0.938	0.938
0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938
0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938
4.551	4.551	4.551	4.551	4.551	4.551	4.551	4.551	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938
3.144	3.144	4.543	4.543	4.543	4.543	4.543	4.543	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938
3.144	3.144	3.144	3.144	3.144	3.144	3.144	3.144	3.144	4.543	4.543	4.543	4.543	4.543	0.938	0.938	0.938	0.938	0.938	0.938	0.938
4.551	4.551	4.551	4.551	4.551	4.551	4.551	4.551	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938
3.142	3.142	3.142	3.142	3.141	3.141	4.55	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938
0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938	0.938

Average	3.022	2.946	3.1	2.973	2.97	2.795	2.462	2.069	1.693	1.749	1.604	1.387	1.415	1.127	1.082	1.01	0.938	0.938	0.938	0.938	0.938
Standard deviation	1.412	1.469	1.545	1.603	1.643	1.678	1.674	1.576	1.396	1.48	1.37	1.15	1.21	0.772	0.715	0.511	-	-	-	-	-
Coefficient of variation	2.141	2.006	2.006	1.855	1.808	1.666	1.471	1.313	1.213	1.181	1.171	1.206	1.17	1.459	1.513	1.977	0	0	0	0	0

TABLE 3: Variation in the exogenous rate of growth of the knowledge stock for the lattice

Lower boundary	-	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
0.526	0.538	0.551	0.565	0.56	0.555	0.552	0.55	0.549	0.55	0.554	0.56	0.557	0.547	0.541	0.538	0.538	0.542	0.55	0.562	0.577	
0.492	0.489	0.485	0.482	0.479	0.476	0.474	0.473	0.473	0.473	0.475	0.479	0.486	0.497	0.512	0.533	0.561	0.599	0.648	0.631	0.6	0.577
0.53	0.53	0.53	0.53	0.531	0.533	0.535	0.538	0.543	0.548	0.555	0.565	0.576	0.568	0.562	0.557	0.555	0.556	0.559	0.567	0.577	
0.681	0.674	0.667	0.661	0.654	0.649	0.643	0.639	0.636	0.634	0.634	0.636	0.641	0.649	0.662	0.68	0.694	0.654	0.622	0.597	0.577	
0.405	0.41	0.416	0.422	0.428	0.436	0.444	0.453	0.464	0.476	0.489	0.505	0.524	0.546	0.546	0.547	0.549	0.554	0.559	0.567	0.577	
0.634	0.633	0.631	0.63	0.629	0.628	0.628	0.628	0.63	0.632	0.636	0.642	0.65	0.661	0.675	0.693	0.717	0.726	0.666	0.617	0.577	
0.832	0.827	0.823	0.818	0.813	0.808	0.804	0.799	0.795	0.791	0.788	0.761	0.731	0.704	0.677	0.653	0.632	0.613	0.597	0.585	0.577	
0.55	0.551	0.553	0.556	0.559	0.562	0.566	0.57	0.576	0.582	0.59	0.6	0.611	0.626	0.643	0.665	0.692	0.712	0.66	0.615	0.577	
0.704	0.699	0.695	0.691	0.687	0.684	0.681	0.679	0.677	0.671	0.653	0.636	0.62	0.606	0.594	0.584	0.576	0.571	0.57	0.572	0.577	
0.609	0.609	0.61	0.611	0.612	0.614	0.617	0.62	0.624	0.63	0.637	0.645	0.656	0.669	0.685	0.705	0.726	0.68	0.641	0.607	0.577	
0.502	0.502	0.503	0.504	0.505	0.507	0.509	0.513	0.518	0.524	0.532	0.542	0.551	0.545	0.54	0.539	0.54	0.544	0.551	0.562	0.577	
0.634	0.653	0.662	0.657	0.653	0.649	0.647	0.645	0.645	0.646	0.65	0.655	0.663	0.675	0.69	0.701	0.666	0.637	0.613	0.593	0.577	
0.615	0.602	0.59	0.578	0.566	0.555	0.545	0.535	0.527	0.519	0.513	0.509	0.506	0.505	0.507	0.511	0.518	0.528	0.541	0.557	0.577	
0.537	0.537	0.538	0.54	0.541	0.543	0.546	0.549	0.553	0.558	0.564	0.572	0.583	0.596	0.614	0.636	0.664	0.687	0.643	0.607	0.577	
0.577	0.569	0.562	0.554	0.547	0.54	0.533	0.527	0.521	0.516	0.511	0.508	0.506	0.506	0.507	0.511	0.517	0.526	0.539	0.556	0.577	
0.566	0.56	0.554	0.549	0.544	0.54	0.537	0.535	0.535	0.536	0.54	0.547	0.558	0.573	0.594	0.621	0.63	0.609	0.594	0.584	0.577	
0.43	0.427	0.425	0.423	0.421	0.42	0.42	0.42	0.42	0.422	0.424	0.428	0.433	0.439	0.448	0.458	0.471	0.486	0.504	0.525	0.549	0.577
0.647	0.682	0.662	0.639	0.62	0.604	0.591	0.584	0.58	0.582	0.589	0.6	0.617	0.638	0.63	0.612	0.599	0.59	0.584	0.58	0.577	
0.545	0.545	0.545	0.545	0.546	0.547	0.548	0.544	0.537	0.531	0.526	0.521	0.517	0.515	0.515	0.517	0.522	0.529	0.541	0.557	0.577	
0.766	0.789	0.786	0.78	0.774	0.769	0.764	0.759	0.756	0.753	0.752	0.752	0.753	0.757	0.752	0.713	0.677	0.646	0.619	0.596	0.577	
0.595	0.595	0.596	0.596	0.597	0.598	0.6	0.592	0.584	0.576	0.569	0.562	0.556	0.551	0.547	0.545	0.545	0.548	0.553	0.563	0.577	
0.631	0.622	0.613	0.605	0.597	0.59	0.583	0.577	0.573	0.57	0.57	0.573	0.58	0.591	0.607	0.631	0.612	0.595	0.584	0.579	0.577	
0.664	0.663	0.662	0.661	0.66	0.659	0.659	0.659	0.657	0.643	0.63	0.617	0.606	0.595	0.585	0.577	0.571	0.567	0.567	0.57	0.577	
0.832	0.828	0.819	0.809	0.8	0.79	0.781	0.772	0.764	0.756	0.749	0.744	0.74	0.739	0.741	0.742	0.694	0.654	0.622	0.596	0.577	
0.695	0.682	0.668	0.655	0.642	0.629	0.616	0.603	0.591	0.579	0.569	0.559	0.55	0.544	0.539	0.537	0.537	0.541	0.549	0.561	0.577	
0.519	0.521	0.524	0.528	0.532	0.536	0.542	0.548	0.555	0.564	0.574	0.586	0.6	0.617	0.638	0.663	0.683	0.651	0.623	0.599	0.577	
0.561	0.557	0.554	0.55	0.546	0.543	0.539	0.536	0.533	0.53	0.528	0.526	0.525	0.524	0.525	0.528	0.532	0.538	0.548	0.56	0.577	
0.481	0.483	0.486	0.489	0.493	0.497	0.502	0.507	0.514	0.522	0.532	0.543	0.557	0.574	0.595	0.621	0.63	0.613	0.598	0.586	0.577	
0.529	0.523	0.517	0.512	0.506	0.501	0.496	0.492	0.488	0.485	0.483	0.482	0.482	0.484	0.487	0.493	0.503	0.515	0.531	0.552	0.577	
0.701	0.696	0.692	0.688	0.685	0.682	0.679	0.677	0.676	0.676	0.678	0.681	0.686	0.694	0.676	0.65	0.628	0.61	0.595	0.585	0.577	
0.509	0.506	0.503	0.5	0.497	0.494	0.491	0.488	0.486	0.484	0.483	0.483	0.483	0.485	0.489	0.494	0.503	0.514	0.53	0.551	0.577	

0.559	0.557	0.556	0.555	0.553	0.553	0.552	0.553	0.554	0.556	0.56	0.565	0.574	0.585	0.601	0.619	0.602	0.589	0.58	0.576	0.577
0.46	0.457	0.455	0.452	0.45	0.448	0.446	0.444	0.444	0.444	0.444	0.446	0.449	0.454	0.461	0.471	0.484	0.5	0.52	0.546	0.577
0.552	0.555	0.559	0.563	0.568	0.573	0.578	0.585	0.592	0.6	0.609	0.62	0.633	0.648	0.666	0.677	0.654	0.632	0.612	0.594	0.577
0.467	0.465	0.464	0.462	0.461	0.46	0.459	0.458	0.458	0.459	0.46	0.462	0.465	0.47	0.476	0.484	0.495	0.509	0.527	0.549	0.577
0.651	0.651	0.652	0.653	0.654	0.656	0.659	0.662	0.666	0.671	0.678	0.686	0.696	0.688	0.666	0.646	0.628	0.612	0.599	0.587	0.577
0.637	0.661	0.69	0.725	0.742	0.727	0.713	0.701	0.69	0.681	0.675	0.671	0.672	0.676	0.686	0.702	0.696	0.654	0.621	0.596	0.577
0.638	0.64	0.641	0.644	0.646	0.649	0.652	0.656	0.661	0.666	0.673	0.68	0.69	0.691	0.67	0.65	0.631	0.615	0.6	0.588	0.577
0.408	0.407	0.407	0.406	0.406	0.407	0.409	0.412	0.417	0.423	0.432	0.444	0.46	0.48	0.506	0.54	0.582	0.635	0.632	0.601	0.577
0.602	0.602	0.602	0.603	0.605	0.607	0.61	0.614	0.619	0.625	0.632	0.642	0.651	0.635	0.621	0.608	0.598	0.59	0.583	0.579	0.577
0.504	0.505	0.506	0.508	0.51	0.512	0.516	0.52	0.526	0.533	0.542	0.553	0.567	0.584	0.605	0.631	0.664	0.704	0.659	0.614	0.577
0.515	0.518	0.522	0.526	0.531	0.537	0.543	0.549	0.557	0.566	0.576	0.588	0.602	0.619	0.622	0.61	0.6	0.591	0.584	0.58	0.577
0.733	0.725	0.717	0.709	0.703	0.696	0.691	0.686	0.683	0.681	0.681	0.684	0.688	0.696	0.708	0.724	0.745	0.695	0.647	0.609	0.577
0.577	0.578	0.579	0.581	0.583	0.586	0.59	0.595	0.6	0.607	0.616	0.626	0.624	0.613	0.602	0.594	0.587	0.581	0.578	0.577	0.577
0.603	0.617	0.633	0.652	0.651	0.645	0.641	0.637	0.635	0.634	0.635	0.638	0.645	0.655	0.669	0.688	0.694	0.654	0.622	0.597	0.577
0.616	0.617	0.618	0.62	0.622	0.624	0.627	0.631	0.635	0.64	0.647	0.654	0.644	0.631	0.619	0.608	0.598	0.59	0.583	0.579	0.577
0.697	0.689	0.682	0.675	0.668	0.661	0.656	0.651	0.647	0.644	0.644	0.645	0.65	0.658	0.67	0.687	0.711	0.68	0.637	0.603	0.577
0.754	0.752	0.75	0.749	0.747	0.746	0.743	0.726	0.71	0.694	0.679	0.664	0.65	0.636	0.624	0.613	0.602	0.594	0.586	0.581	0.577
0.554	0.555	0.556	0.557	0.559	0.561	0.563	0.566	0.57	0.575	0.581	0.589	0.599	0.611	0.627	0.648	0.675	0.68	0.64	0.606	0.577
0.572	0.57	0.569	0.568	0.568	0.568	0.569	0.571	0.574	0.579	0.579	0.569	0.56	0.553	0.549	0.547	0.547	0.55	0.556	0.565	0.577

Average	0.592	0.593	0.592	0.591	0.589	0.587	0.586	0.585	0.584	0.585	0.587	0.589	0.593	0.597	0.6	0.605	0.607	0.601	0.589	0.581	0.577
Standard deviation	0.1	0.1	0.1	0.1	0.1	0.09	0.09	0.09	0.09	0.09	0.08	0.08	0.08	0.08	0.08	0.07	0.07	0.06	0.04	0.02	0
Coefficient of variation	0.167	0.167	0.165	0.164	0.162	0.159	0.156	0.152	0.148	0.145	0.141	0.136	0.132	0.128	0.124	0.122	0.118	0.101	0.07	0.03	0

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1. The exact influence of geographical distance on the spillovers is examined in depth in section 3.4.
2. The maximum would also move a little bit away from the y-axis, however this is a very small influence.
3. Note that the figure should show a discontinuous graph (in the model a geographical distance is either 1 or 2, not 1.5), however, for visual reasons the individual points are connected.
4. This calibration was limited to the lattice of honeycombs, but it is not influenced by the choice of a different sphere.