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**Conference Paper**

## Spatial heterogeneity in Danish urban areas: The expansion method philosophy and variable autocorrelated residuals

38th Congress of the European Regional Science Association: "Europe Quo Vadis? - Regional Questions at the Turn of the Century", 28 August - 1 September 1998, Vienna, Austria

**Provided in Cooperation with:**

European Regional Science Association (ERSA)

*Suggested Citation:* Kristensen, Gustav (1998) : Spatial heterogeneity in Danish urban areas: The expansion method philosophy and variable autocorrelated residuals, 38th Congress of the European Regional Science Association: "Europe Quo Vadis? - Regional Questions at the Turn of the Century", 28 August - 1 September 1998, Vienna, Austria, European Regional Science Association (ERSA), Louvain-la-Neuve

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July 10, 1998

## **SPATIAL HETEROGENEITY IN DANISH URBAN AREAS**

The Expansion Method Philosophy and Variable Autocorrelated Residuals

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Paper prepared for:

The 38th Congress of the EUROPEAN REGIONAL SCIENCE ASSOCIATION

Vienna, 28 August -1 September 1998.

## **1. Introduction**

As stated by Alonso (1961) land prices rise towards the city center. The rising land prices create equilibrium in the urban system because in principle they compensate declining transport costs to the city center, so that citizens are economic equal in the entire urban space.

The city center is a mixture of business buildings and housing. In a ring around the core, housing is dominating, and in the periphery manufacturing becomes more frequent. This ring structure creates spatial heterogeneity related to the distance to the city center. Likewise the mixture of land use creates spatial interaction beyond what is described by the distance to the urban center.

Although urban land prices on average significantly tend to rise towards the city center they can due to local spatial interaction also fall towards the city center. Moving towards the city center the standard deviation of land prices rises more rapidly than the average land price and thus indicates a systematic change in the pattern of spatial heterogeneity.

OLS estimated models for urban area demands independent residuals with constant variance. If this condition is not fulfilled OLS is not BEST.

Spatial heterogeneity was discussed by Anselin and Can (1986), and Anselin and Griffith (1988) mention that spatial heterogeneity (autocorrelation) is mostly ignored in the urban literature.

“Urban space” is in this article not seen as “area” but as a “network” where points are connected by lines rather than by the geographical space. In the present model all points are thus only connected with circular and radial lines and the data are registered where a radial line crosses a ring.

This article shows that the autocorrelation in the residual from the estimation of land prices measured along radial- and ring-lines change from positive to negative when passing from the periphery to the center given that the urban center is sufficiently big. The change in the sign of the spatial autocorrelation likewise changes directionally.

The purpose of this article is to discuss spatial autocorrelation in urban space which appears from

a simple urban model. The autocorrelation coefficient is shown to vary systematically over the city space, and is modeled as a variable (expanded) autocorrelation coefficient.

The discussion is divided in two. First an individual city (Copenhagen) is discussed in detail. Then the generality for the entire city system is discussed. This section is in the present paper not yet fully developed. The variable autocorrelation coefficient is discussed in the framework of the expansion method as stated by Casetti (1986).

## 2. The Model

The model basically describe the autocorrelation in the residuals from the estimation of a monocentric urban model for land prices. The model town is described by rings and radial lines. The spatial autocorrelation is supposed to differ from rings to radials.

### 2.1. A Single Town

The model is developed in the following way for a single town. The land price equation is initially given as a third degree polynomial of the distance to the city center

$$LP = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + e \quad (1)$$

where

LP -  $\log(p)$ , the logarithmic value of land prices

t - is distance to the urban center

The urban center is, however, not circular but in relation to that, directional biased (Krakower and Casetti, 1988). This feature can be included in the model by directional varying coefficients

$$\beta_j = \beta_{0j} + \beta_{1j} \cos v + \beta_{2j} \sin v \quad (2)$$

where

$\cos v$  - is the cosine of an angle between an East-West line going through the urban center and a line from the center to the point considered. Similar definition for  $\sin v$ . Thus for north we have:  $\cos v=0$ ,  $\sin v=1$ ). In the expansion model terminology (1) is the primal or initial model while (2) is called the dual model.

This gives the expanded model

$$\begin{aligned}
 P = & (\beta_{00} + \beta_{10}\cos v + \beta_{20}\sin v) \\
 & + (\beta_{01} + \beta_{11}\cos v + \beta_{21}\sin v) t \\
 & + (\beta_{02} + \beta_{12}\cos v + \beta_{22}\sin v) t^2 \\
 & + (\beta_{03} + \beta_{13}\cos v + \beta_{23}\sin v) t^3 \\
 & + e
 \end{aligned} \tag{3}$$

The model is now estimated for the individual town on this form and the residuals are found.

The idea of the *residual* is that it is based on an omitted variable working in the entire urban area. The variable has an impact on the neighboring land price and thus imply spatial autocorrelation. Here only the effect in radial direction and ring direction is considered. A given point is thus in principle influenced by *four* neighboring areas. The autocorrelation is related to all neighbors which in the present ring model is defined to four.

$$e = \alpha_1 EI + \alpha_2 EU + \alpha_3 EL + \alpha_4 ER \tag{4}$$

where

EI - neighbor residual on the radial line towards the center

EU - neighbor residual on the radial line towards the periphery

EL - neighbor residual on the ring to the left

ER - neighbor residual on the ring to the right

EI and EU form the spatial autocorrelation along the radial lines of the urban center, and will most often be fortified by environment connected to radial roads. EL and ER form the autocorrelation on rings. If the radial roads form the environment the autocorrelation will be strongest in connection with EI and EU.

Assuming that the effects from neighbors on the radial line are the same and the effect from neighbors on the ring are the same we get the model

$$e = \alpha_1 EIU + \alpha_3 ELR \tag{5}$$

where

$$ELR = (EL + ER)/2 \tag{6}$$

$$EIU = (EI + EU)/2 \quad (7)$$

The impact of the omitted variable on the price on neighboring land is, however, dependent on the location in the urban space.

$$\begin{aligned}
LP = & (\beta_{00} + \beta_{10}\cos v + \beta_{20}\sin v) \\
& + (\beta_{01} + \beta_{11}\cos v + \beta_{21}\sin v) t \\
& + (\beta_{02} + \beta_{12}\cos v + \beta_{22}\sin v) t^2 \\
& + (\beta_{03} + \beta_{13}\cos v + \beta_{23}\sin v) t^3 \\
& + [(\beta_{00i} + \beta_{10i}\cos v + \beta_{20i}\sin v) \\
& + (\beta_{01i} + \beta_{11i}\cos v + \beta_{21i}\sin v) t \\
& + (\beta_{02i} + \beta_{12i}\cos v + \beta_{22i}\sin v) t^2 \\
& + (\beta_{03i} + \beta_{13i}\cos v + \beta_{23i}\sin v) t^3] * EIU \\
& + [(\beta_{00j} + \beta_{10j}\cos v + \beta_{20j}\sin v) \\
& + (\beta_{01j} + \beta_{11j}\cos v + \beta_{21j}\sin v) t \\
& + (\beta_{02j} + \beta_{12j}\cos v + \beta_{22j}\sin v) t^2 \\
& + (\beta_{03j} + \beta_{13j}\cos v + \beta_{23j}\sin v) t^3] * ELR
\end{aligned} \quad (8)$$

The multilevel structure can be reconsidered on the following form

$$LP = \alpha_0 + \alpha_1 EIU + \alpha_2 ELR \quad (9)$$

$$\alpha_j = \beta_{0j} + \beta_{1j}t + \beta_{2j}t^2 + \beta_{3j}t^3 \quad (10)$$

$$\beta_{ij} = \beta_{0ij} + \beta_{1ij}\cos v + \beta_{2ij}\sin v \quad (11)$$

It is seen that equation (10), now presented as the primal model has no known explaining variables. The justification for this approach is that EIU and ELR represent the effects of an inhomogenous group of local institutions as business centers, railway stations, parks, church yards, etc., who influence the price on neighboring land in a way which even for the same institution can depend on whether you consider the back side or the front side.

The expanded (or variable) autocorrelation coefficient(s) to be estimated is now the coefficients of EIU and ELR.

$$\begin{aligned}
\alpha_2 = \rho_{ring} = & (\beta_{00j} + \beta_{10j}\cos v + \beta_{20j}\sin v) \\
& + (\beta_{01j} + \beta_{11j}\cos v + \beta_{21j}\sin v) t \\
& + (\beta_{02j} + \beta_{12j}\cos v + \beta_{22j}\sin v) t^2 \\
& + (\beta_{03j} + \beta_{13j}\cos v + \beta_{23j}\sin v) t^3
\end{aligned} \tag{12}$$

and similar for the  $\alpha_1 = \rho_{radials}$ .

## 2.2 The City System and the Changing Heterogeneity in Urban Space

In the city system only the general features can be modeled. Because the directional bias from the circular city in the land prices are city specific, and can not be included in an over-all model. This is a severe limitation in the development of the model to include the entire urban system.

## 3. Data and Estimates

This article is based on data from Hansen and Kristensen (1991). The urban land prices are the assessed land values in 1977. Compared to the investigation of Hansen and Kristensen (1991) the urban data is here truncated towards the periphery of the urban area. For the purpose of spatial autocorrelation this has the advantage that the sprawl of the urban outskirts is removed. Besides the collected data were only collected at the interval of 250 meter until 5 kilometers from the urban center. All radial data included here thus have the distance 250 meter. For Copenhagen observations were collected in 16 radial directions. For other cities only in 8 directions. In Hansen and Kristensen (1991) the collected prices was recalculated to prices on untaxed land in order to make comparison possible cross city.

The *missing observations* of EIU and ELR are used zero because zero is the average of each of the two (lagged) variables. (See Pindyck and Rubinfeld (1991) pp.219-221)

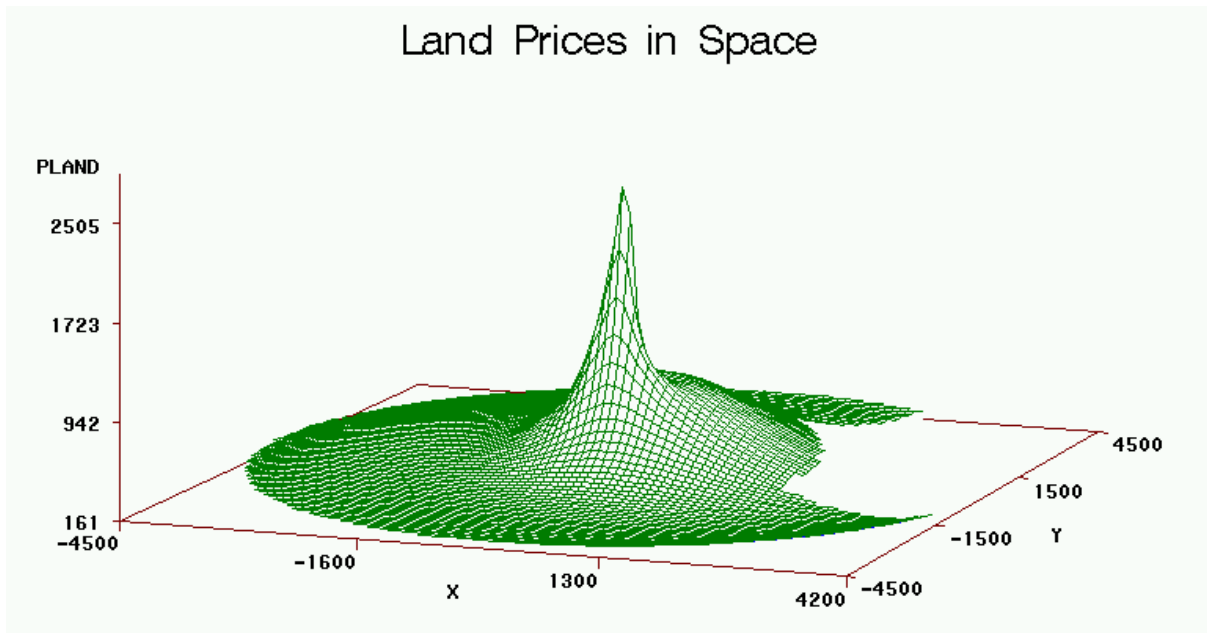
#### 4. Scenarios

An overview for the spatial distributed land prices and spatial distributed autocorrelation coefficients is now given in scenarios.

##### 4.1 A Single City: Copenhagen

The price structure for land in Copenhagen is shown in figure 1.

FIGURE 1. Price structure for urban land in Copenhagen 1977 in a radius of about 5 kilometer



from the center, DKK per square meter.

Figure 1 shows the price structure for urban land in Copenhagen: the price level tops in the center but it is seen that in the north-eastern direction from the center the area close to the center is dominated by low land prices. It is seen that the prices even declines when going very near the center. The figure is truncated towards east-northeast due to the sea.

The model can only grasp the main lines of the urban land price structure that is the distance effect modified directionally. The details are left in the residuals. The spatial variable autocorrelation coefficients  $\rho_{radial}$  and  $\rho_{ring}$  is shown in figure 2 and 3.



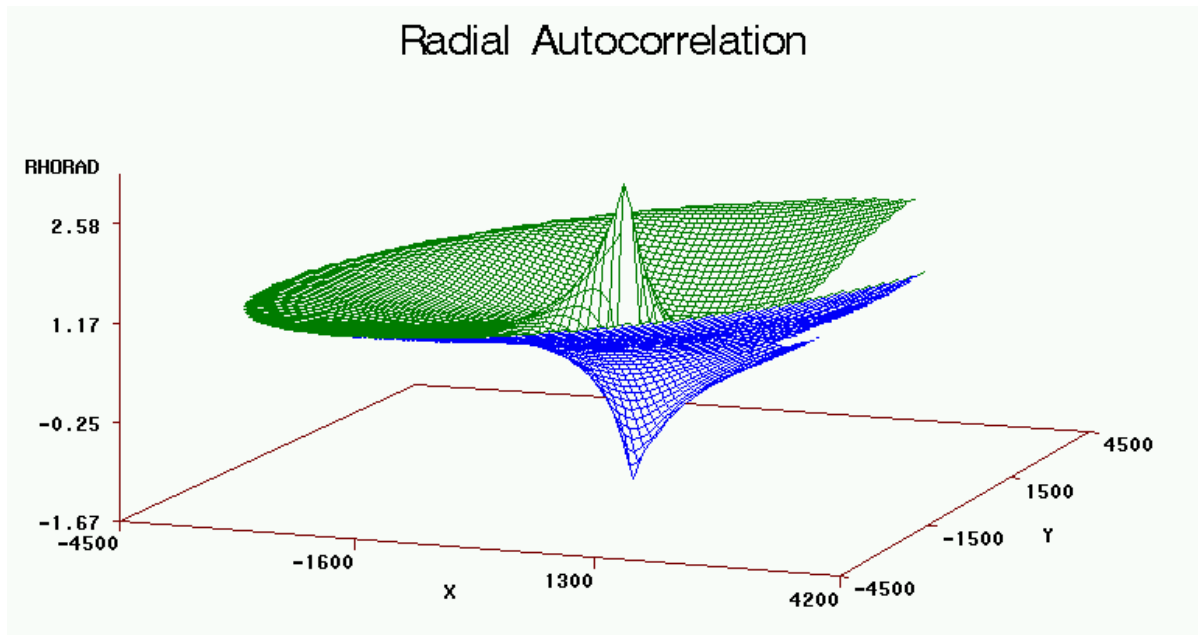


FIGURE 2. The radial based autocorrelation coefficient in Copenhagen 1977 in a radius of 5 kilometer from the center.

The negative autocorrelation will imply that the land prices will be low around a randomly high priced area. Around the center the autocorrelation coefficient becomes erratic.

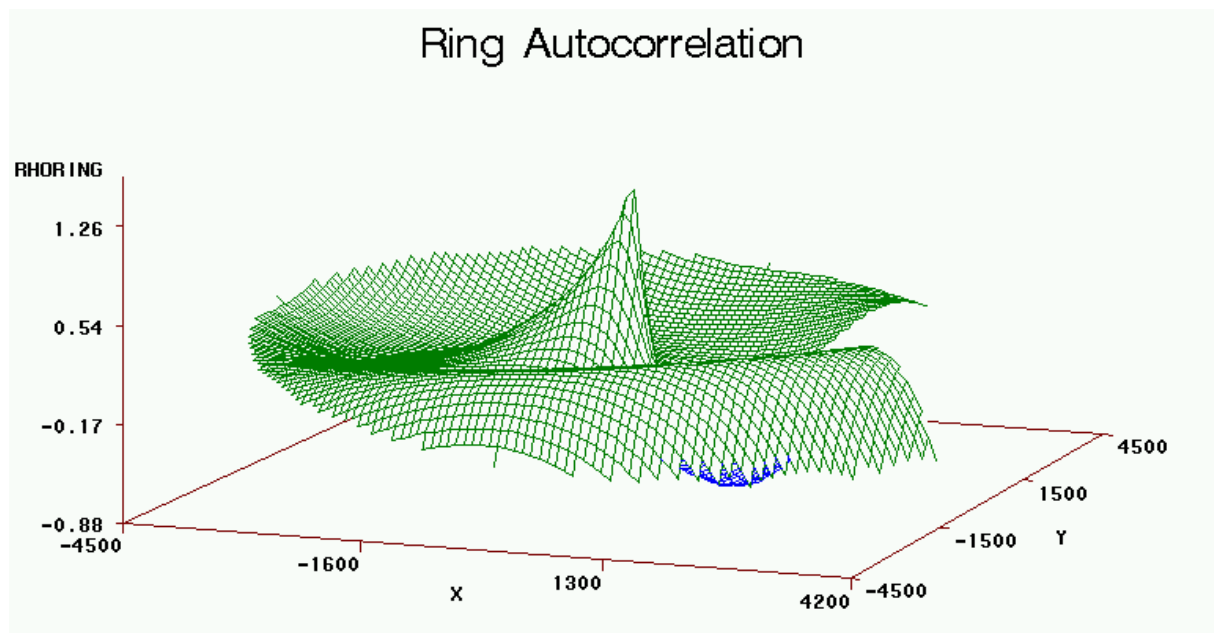


FIGURE 3. The ring based autocorrelation coefficient in Copenhagen 1977 in a radius of 5 kilometer from the center.

#### 4.2 Spatial Autocorrelation and Urban Dynamics

The residual is an unknown omitted variable who influence the neighboring area. We will now see the effect of an initial autonomous price rise or decline on 1 DKK on the urban price structure. The initial rise or decline is originally equal distributed over the urban space.

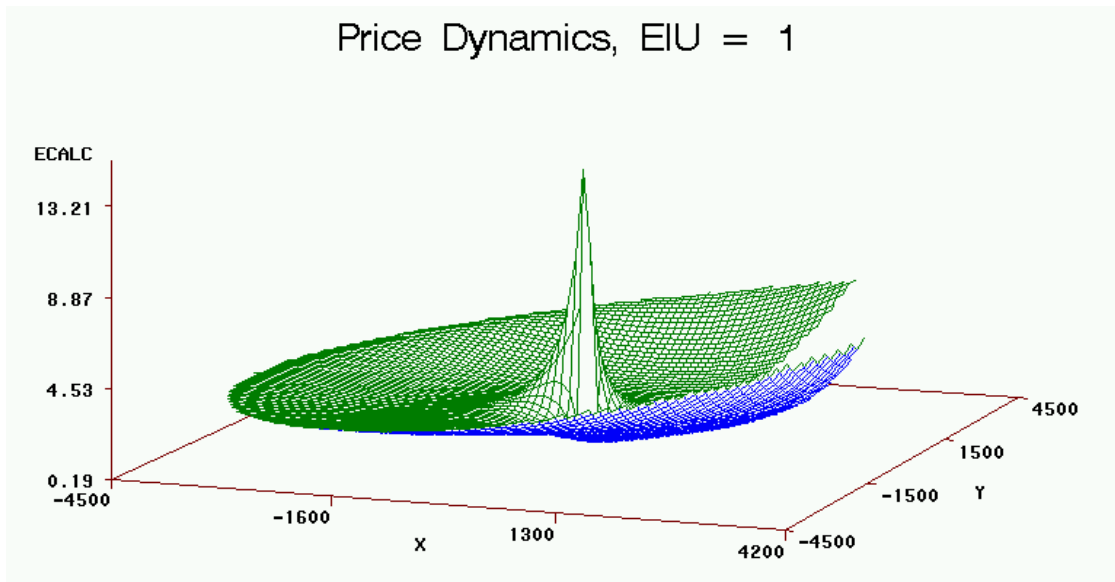


FIGURE 4. The final result of land prices of an initial autonomous rise in square meter price on 1.

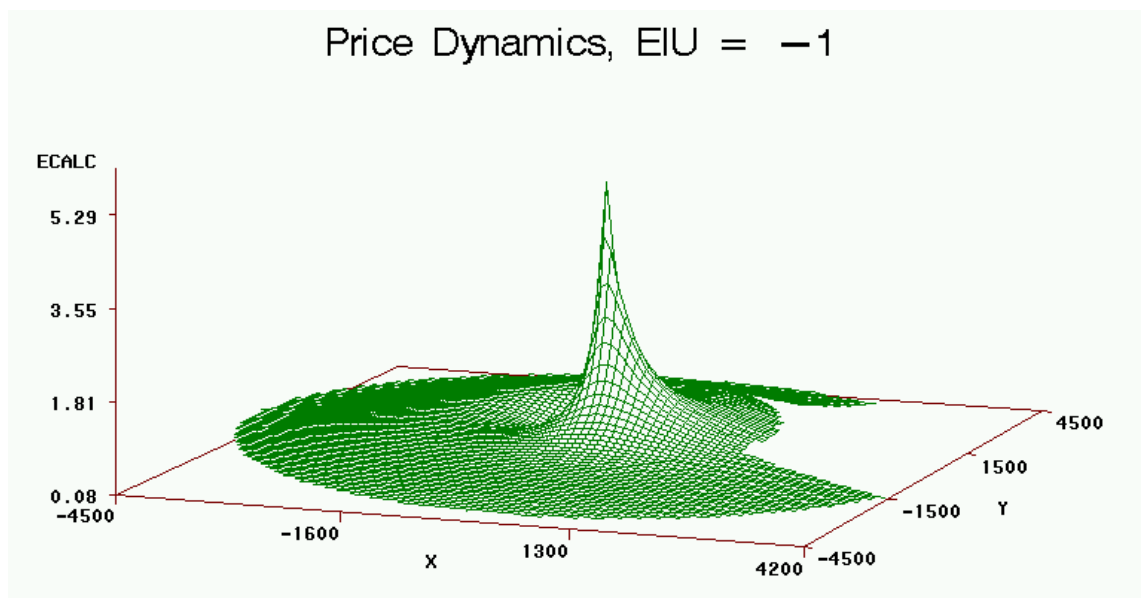


FIGURE 5. The final result on land prices of an initial autonomous decline in square meter price on 1.

The figures 4 and 5 shows the result of a uniform initial price rise, which is a result of the dynamics with origin in the spatial interaction expressed by the spatial autocorrelation coefficient.

#### 4.2 The City System

A summary estimation of a distance varying autocorrelation coefficient can be presented in the following way

**Copenhagen** (pop = 1.337.000)

$$e = .000268 * t * EIU \quad R^2 = .1771 \quad \text{Obs} = 241$$

(7.18)

**Aarhus** (pop = 200.000)

$$e = .000566 * t * EIU \quad R^2 = .0939 \quad \text{Obs} = 58$$

(2.43)

**Odense** (pop = 139.000)

$$e = .000189 * t * EIU \quad R^2 = .0404 \quad \text{Obs} = 61$$

(1.59)

The smaller the town is the more difficult it will be to calculate the spatial autocorrelation as well as the expanded autocorrelation.

Comparing spatial autocorrelation cross-city makes it impossible to include the directional bias. Another difficulty is that the Danish towns rapidly declines in size when the number rises.

### 5. Conclusion

The estimations show that the pattern of heterogeneity and thereby the spatial autocorrelation coefficients vary systematically over space. The variations in the spatial autocorrelation coefficients follow the variables included in the model after the principles of the expansion method.

The estimations shows that in models, which can be presented as expansion models, the *included*

variables from the dual model reveal the dual part of the *omitted* primal variables.(See also appendix 1.)

Spatial autocorrelation express spatial interaction. The spatial varying autocorrelation coefficient shows how the pattern of interaction change (dramatically) towards the urban center. Highly positive spatial autocorrelation exists side by side with highly negative spatial autocorrelation.

## APPENDIX. The Expanded Autocorrelation Coefficient: A Numerical Example

Johnston (1984 p 309) mention that autocorrelation has its origin in an omitted autocorrelated variable. The lagged autocorrelated residual can therefore be understood as the shadow of an omitted variable.

At divine insight the full known model, the expansion model, could be written

$$Y = \beta_0 + \beta_1 * X1 + \beta_2 * X2 \quad (1)$$

$$Y = (\alpha_1 + \alpha_2 Z) + (\alpha_3 + \alpha_4 Z) * X1 + (\alpha_5 + \alpha_6 Z) * X2 \quad (2)$$

With the true coefficients inserted

$$Y = (3 + 4Z) + (5 + 6Z) * X1 + (1 - 3Z) * X2$$

$$\text{Obs} = 1991$$

Z - normal distributed independent random variable with variance 1

X1 - normal distributed independent random variable with variance 1

X2 - autocorrelated variable with autocorrelation coefficient 0.8.

X2 is unknown and following omitted of the equation. The true residual element is:

$$e = (1 - 3Z) * X2$$

$$e(-1) = (1 - 3Z(-1)) * X2(-1)$$

is autocorrelated with the autocorrelation coefficient  $\rho$  well below 0.8.

An intuitive development of the model is:

$$X2 = .8 * X2(-1) + v$$

multiplying through with  $(1 - 3Z)$

$$(1 - 3Z)*X2 \approx .8(1 - 3Z)*X2(-1) + (1 - 3Z)*v$$

Because  $X2(-1)$  can be written  $(1 - 3Z(-1) + 3Z(-1))*X2(-1)$  and inserted

$$(1 - 3Z)*X2 \approx \rho[(1 - 3Z)*(1 - 3Z(-1) + 3Z(-1))*X2(-1)] + (1 - 3Z)*v$$

$$(1 - 3Z)*X2 \approx \rho[(1 - 3Z)*((1 - 3Z(-1))*X2(-1) + 3Z(-1)*X2(-1))] + (1 - 3Z)*v$$

$$e \approx \rho(1 - 3Z)*e(-1) + \rho(1 - 3Z) 3Z(-1)*X2(-1) + (1 - 3Z)*v$$

which when  $Z$  is independent distributed gives the expanded autocorrelation equation

$$e \approx \rho(1 - 3Z)*e(-1) + w$$

Equation (2) is now estimated on a simulated data set by

$$Y = (\alpha_1 + \alpha_2 Z) + (\alpha_3 + \alpha_4 Z)*X1$$

$$Y = (\alpha_1 + \alpha_2 Z) + (\alpha_3 + \alpha_4 Z)*X1 + \beta_5 e(-1)$$

$$Y = (\alpha_1 + \alpha_2 Z) + (\alpha_3 + \alpha_4 Z)*X1 + (\beta_5 + \beta_6 Z)*e(-1)$$

and estimated to

$$Y = (3.103 + 3.948Z) + (4.949 + 5.950Z)*X1$$

(26.96) (34.08) (43.45) (51.57)

$$R^2 = .7493 \quad \text{Adj. } R^2 = .7489 \quad \text{Obs} = 1991 \quad \text{DW} = 1.88$$

$$Y = (3.103 + 3.949Z) + (4.945 + 5.943Z)*X1 + .0556*e(-1)$$

(27.00) (34.13) (43.48) (51.55) (2.48)

$$R^2 = .7501 \quad \text{Adj. } R^2 = .7496 \quad \text{Obs} = 1991 \quad \text{DW} = 2.00$$

$$Y = (3.102 + 3.968Z) + (4.968 + 5.948Z)*X1 + (.0495 - .163Z)*e(-1)$$

(27.33) (34.73) (44.22) (52.25) (2.24) (-7.21)

$$R^2 = .7565 \quad \text{Adj. } R^2 = .7559 \quad \text{Obs} = 1991 \quad \text{DW} = 2.01$$

What is won by the inclusion of the autocorrelation and the expanded autocorrelation can best be evaluated by including the risk function. This point shall, however not be followed here.

## APPENDIX 2.

The estimated coefficients of equation (8)

	Variable	t-value
1	7.4504	55.84
2	-.5270	-2.38
3	-.4876	-2.60
4	-.001050	-4.87
5	.001087	3.24
6	.0009975	3.20
7	1.500e-07	1.54
8	-5.691e-07	-3.95
9	-3.258e-07	-2.32
10	-1.995e-12	-.15
11	7.927e-11	4.37
12	3.049e-11	1.67
13	.5366	1.21
14	-1.5389	-1.97
15	1.9961	2.71
16	-.0006057	-.71
17	.0013985	1.01
18	-.0025488	-2.01
19	3.290e-07	.81
20	-4.155e-07	-.64
21	8.907e-07	1.52
22	-3.348e-11	-.60
23	4.695e-11	.53
24	-9.321e-11	-1.19
25	.6413586	2.03
26	-.23892	-.50
27	-.8739	-2.14
28	-.0014579	-2.12
29	-.0008973	-.94
30	.0012545	1.32
31	6.654e-07	1.90
32	6.226e-07	1.27
33	-5.756e-07	-1.19
34	-8.301e-11	-1.70
35	-9.473e-11	-1.39
36	7.844e-11	1.17

$R^2 = .7719$      $\text{Adj. } R^2 = .7329$      $\text{Obs} = 241$



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tekniske noter

TSP: spaau, spaau1, spaau2

lspahet - København

spaaarh - Århus Odense