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Summary

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JEL Classification: Q30, D81

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Uncertainty and Natural Resources – Prudence facing Doomsday*

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Abstract

This paper studies the optimal extraction of a non-renewable resource under uncertainty using a discrete-time approach in the spirit of the literature on precautionary savings. We find that boundedness of the utility function, in particular the assumption about $U(0)$, gives very different results in the two settings which are often considered as equivalent. For a bounded utility function, we show that in a standard two-period setting, prudence is no longer sufficient to ensure a more conservationist extraction policy than under certainty. If on the other hand we increase the number of periods to infinity, we find that prudence is not anymore not anymore necessary to induce a more conservationist extraction policy and risk aversion is sufficient. These results highlight the importance of the specification of the utility function and its behavior at the point of origin.

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1 Introduction

Uncertainty is ubiquitous in the field of Environmental and Resource Economics. The developments in the optimal resource extraction over time following Hotelling (1931)[1] have introduced uncertainty in this field starting with Kemp (1976), Loury (1978), Gilbert (1979), and Dasgupta and Heal (1979). The size of the stock of the resource was now assumed to be a random variable and these papers characterized the optimal planned extraction path over time. Their results suggest that uncertainty will induce a more precautionary extraction path under relatively general conditions. Gilbert (1979) shows that for isoelastic utility and an exponential distribution of the resource stock, extraction is initially always more conservative than under certainty. Still these results are not very general and not always very intuitive. For instance, Kumar (2005) showed that the optimal extraction path can even increase over time depending on the shape of the distribution of the size of the resource stock S. An intuition for this result is that extraction can anticipate the resolution of uncertainty and therefore provide an incentive for faster extraction.

A more recent strand of the literature builds on decision theoretic results in the expected utility framework, namely the precautionary savings model applying it to the resource extraction problem. Leland (1968), Sandmo (1970), and later Kimball (1990) studied the optimal savings decision when future income is uncertain. When the interest rate is zero, this problem can be interpreted as the resource extraction problem where the agent aims at optimally allocation resource consumption over time. This isomorphism between the consumption problem and optimal resource extraction has been exploited in detail in Lange and Treich (2008). The fundamental result of this literature is that uncertainty induces more savings if and only if the utility function exhibits prudence in the sense of Kimball (1990) or that its third derivative is positive. In the context of resource depletion, this implies that prudence is necessary and sufficient for a more conservationist extraction when facing an uncertain resource stock.

Interestingly, these two strands of the literature obtained different results with respect to the effect of uncertainty on the optimal resource extraction path. The most notable difference is the absence of the role of prudence in the former literature. In this paper, we try to reconcile the results of the two different approaches. We argue that a crucial characteristic of both approaches is the assumption about the

[1] In a more stylized setup, Gale (1967) coined the expression *cake-eating problem* for this problem.

possibility of depletion in finite time. In the classical literature, this possibility is explicitly allowed for since otherwise no solution to the problem exists. In the consumption savings case on the other hand, this possibility is excluded since it is argued that the agent would never prefer to be left with zero consumption in any period.

The possibility of depletion is directly linked to the behavior of the utility function at zero. The classical literature on resource extraction under uncertainty needs to impose a lower bound on $U(0)$ as otherwise no extraction exceeding the certain part of the resource stock will occur. In an infinite horizon model, there will at one point in time arrive a so-called 'moment of sorrow' (Kemp, 1976) or 'doomsday' (Koopmans, 1974) where consumption drops to zero. This lower bound of the utility function has a clear economic intuition in such a partial equilibrium model. Since substitutes of the resource are not considered, having exhausted a particular resource is not likely to lead to the end of humanity. A backstop technology at an arbitrarily high cost will ultimately be able to replace resource consumption thus justifying this assumption.

In the consumption savings problem on the other hand as in Lange and Treich (2008) or Eeckhoudt et al. (2005), the assumption $U(0) = -\infty$ is maintained as a sufficient condition to avoid being left with nothing in the last period. Otherwise expected utility would be minus infinity. In the context of natural resource extraction, however, imposing $U(0) = -\infty$ seems questionable as discussed above. The results can also be seen in the light of recent developments about the crucial behavior of the utility function at zero such as Geweke (2001) or Buchholz and Schymura (2010). In this paper we thus explicitly use a bounded from below utility function. Since we use the expected utility model, the utility function has a cardinal interpretation and we can without loss of generalization set the lower bound to zero ($U(0) = 0$). It is important to distinguish this assumption from the more standard condition on infinite marginal utilita at the origin $\lim_{c \to 0} U'(c) = +\infty$. The assumption of $U(0)$ being bounded below is actually a stronger requirement than infinite marginal utility at zero.

Apart from the substitutability and the behavior of the utility function at zero, another argument for allowing exhaustion follows from the interpretation of a period in the resource context. If one considers the duration of one period in a stylized two period model applied to the expected time span of exhausting a non-renewable resource stock, this suggests the duration of one period consisting in decades or centuries rather than years making exhaustion more plausible within early periods.

2 A stylized model with the possibility of depletion

In this section we study a two period model of resource extraction under the assumption $U(0) = 0$ and thus allowing for exhaustion during the first period. A stock of size S which is random and is distributed according to some distribution $F(s)$ with support $[\underline{S}, \infty[$ is extracted and consumed over two periods. $\underline{S}$ denotes the non-negative lower bound of S or the amount of the resource that is available with certainty. The problem consists in maximizing the expected value of discounted utility

$$V_I(s_1) = U(s_1) + \beta U(S - s_1) \tag{1}$$

where $U(s_t)$ is the standard increasing and concave utility function depending only on s_t, the consumption level of the resource in period t, and we denote by $\beta \leq 1$ the discount factor.[2]

The optimal first-period consumption under uncertainty s_1^u thus can be expressed by the first-order condition $U'(s_1) = \beta E[U'(S - s_1)]$. The second order condition is automatically satisfied if the utility function is concave. If there is no uncertainty, and the expected value of S is available with probability one, the problem becomes maximizing $U(s_1) + \beta U(E[S] - s_1)$. In this case, the optimal solution under certainty s_1^c must satisfy the first-order condition $U'(s_1) = \beta U'(E[S] - s_1)$.

Comparing the two conditions it is easy to show by the Jensen inequality that uncertainty induces a lower first-period consumption ($s_1^u < s_1^c$) for any distribution of S if and only if $U'(s)$ is convex ($U'''(s_t) > 0$). Prudence in the sense of Kimball (1990) is thus a necessary and sufficient condition for more conservationist extraction policy. This important result relies on the assumption that second period consumption is always strictly positive. One way of ensuring that depletion does not occur is by imposing $U(0) = -\infty$, see Lange and Treich (2008).

In the following we instead allow explicitly for the exhaustion of the resource in the first period. We will refer to the model just presented as situation (I). In situation (II), depletion in the first period must be explicitly accounted for. The decision trees for both situations are shown in Figure 1. The timing of the model (II) is as follows: The social planner announces the amount s_1 planned to be extracted and consumed during the first period. If the actual amount available is lower than what was planned for period one, the resource is fully exhausted during the first

[2] This model could also be framed as stylized representation of a continuous time model where the actual size of the resource is learned at time T and s_1 is the amount of the resource that is planned to be extracted and consumed until that date.

period. Otherwise, the available amount is learned at period two and what is left is consumed. The value function $V_{II}(s_1)$ in situation (II) for the agent is more complex

Figure 1: The decision tree for the two models

than the one for the standard case (I). In particular, it has a kink at the endogenous value where $S = s_1$ and it can be written as

$$V_{II}(s_1) = \begin{cases} U(S) + \beta U(0) & if\ S \leq s_1 \\ U(s_1) + \beta U(S - s_1) & if\ S > s_1 \end{cases} \tag{2}$$

The program of maximizing expected discounted utility can now be stated as follows:

$$\max_{s_1} E[U(\min(S, s_1)) + \beta U(\max(S - s_1, 0))] \tag{3}$$

Equivalently, the program can be expressed as

$$\max_{s_1} \int_0^{s_1} \{U(S) + \beta U(0)\} dF + \int_{s_1}^{\infty} \{U(s_1) + \beta U(S - s_1)\} dF \tag{4}$$

First, note that the value function is continuous at $\underline{S}$. Its first derivative with respect to s_1 on the other hand is continuous only if the distribution F is itself continuous. Also, the value function is not guaranteed to be concave since the second term in (3) contains the maximum operator. For typical utility functions and examples we considered, however, concavity of the problem did hold.

Obviously, if $s_1 < \underline{S}$, the model is equivalent to the case (I) since exhaustion will never appear during period one. Now we can characterize the optimal solution s_1^u to program (4). In order to compute the first order condition, we need the assumption $U(0) = 0$ since otherwise $V'_{II}(s_1)$ is not determined. With this assumption, the first-order condition reads

$$\int_{s_1}^{\infty} \{(U'(s_1) - \beta U'(S - s_1)\} dF = 0 \tag{5}$$

which can also be written as

$$(1 - F(s_1))\{U'(s_1) - \beta E[U'(S - s_1)|S > s_1]\} = 0 \tag{6}$$

and looks very similar to condition of the standard case (I). The difference is the *conditional* expectation of second-period marginal utility. What matters for the trade-off between first- and second-period consumption is the conditional expected marginal utility only in the case where depletion does not occur during period one.

The second order condition yields

$$\int_{s_1}^{\infty} \{(U''(s_1) + \beta U''(S - s_1)\} dF - f(s_1)(U'(s_1) - \beta U'(0)) < 0 \tag{7}$$

and it is not necessarily satisfied since the last term is always positive as long as $f(s_1) > 0$. If depletion does not occur through the first period, that is for $s_1 < \underline{S}$, the value function is locally concave. For $s_1 \to \overline{S}$ on the other hand, it is locally convex since the first term tends to zero and the second is positive, while in between these two values it is ambiguous.

Since global concavity of the value function is not ensured in this model, we need to verify that the optimal solution will be interior. Observe that $s_1 = 0$ can be excluded since it is dominated by $s_1 = \overline{S}$ if β is strictly lower than one. The second possible solution, namely $s_1 = \overline{S}$, would imply that the uncertainty is resolved immediately while consumption in period two is zero with probability one. From (6) it is easy to see that the first order condition is satisfied at this point since $F(s_1) = 1$. Moreover, since we saw that the value function is convex towards the right bound of the support of S, the point $s_1 = \overline{S}$ is actually a local minimum. This ensures that the solution will always be interior and we can restrict ourselves to points satisfying (5). Now we can derive the fundamental result of this section:

Lemma 1. *When depletion of the resource stock before the last period is possible, prudence ($U''' > 0$) is necessary but not sufficient to ensure a more conservationist extraction policy under uncertainty than under certainty.*

Proof. It would be sufficient to look at the first example of the following section. Nevertheless, in order to highlight the similarities and difference to the standard proofs for the role of prudence, we can compare the structure of the proofs for both cases. If $\left.\frac{\partial V_{II}}{\partial s_1}\right|_{s_1^c}$ is negative, the value function reaches its maximum before s_1^c and first period consumption under uncertainty would be more conservationist. That is, using the first order condition and substituting $U'(s_1^c)$, we need the condition

$$U'(ES - s_1^c) - E[U'(S - s_1^c)|S > s_1^c] \overset{!}{<} 0 \tag{8}$$

to hold. By the Jensen inequality, the left-hand side of this inequality is smaller than $U'(ES - s_1^c) - U'(E[S - s_1^c|S > s_1^c])$ if and only if the agent is prudent. This term, however is non-negative since the conditional expectation of second period's consumption is higher or equal than the unconditional expectation. Prudence is still necessary for a more conservationist extraction path but it is not anymore sufficient due to the possibility of depletion in period one. □

Compared to the case where running out of the resource is never possible like in Lange and Treich (2008), we need a stronger condition for a more conservative extraction policy when facing uncertainty. This result is somewhat surprising given that it is intuitive that the risk of being left with nothing in the second period could lead to an even more conservationist optimal policy.

One can get an intuition for this result from the first order condition (5). What matters at the margin is only the trade-off between first- and discounted second period expected utility in the case where S is higher than first-period consumption s_1. For the case of depletion in period one, on the other hand, the effect of a marginal increase of s_1 on the increased probability of being left with nothing in period two is exactly offset by the higher conditional expectation $E[U(S)|S < s_1]$ in the case of running out in the first period. Therefore, all that matters for the optimal decision of s_1 is the expected marginal utility in both periods only if consumption is strictly positive. That is, the situation where doomsday arrives is disregarded. This is the what we call the 'doomsday anyway effect', which counteracts the effect of prudence.

3 Two examples

To illustrate the implications of these results we look at three examples. First, consider the case where S takes on the values 0 or 4 with equal probability. Here the assumption of $U(0) > -\infty$ is clearly needed since $\underline{S} = 0$ as otherwise the problem has no solution. Abstracting from discounting ($\beta = 1$), the optimal consumption under certainty for any strictly concave utility function would be $s_1^c = 1$ since $E[S] = 2$. Under uncertainty however, no matter what strictly positive amount is consumed in the first period, depletion occurs with the probability one half. The conditional expected value is 4 for any strictly positive value of s_1 and hence we find the optimal value $s_1^u = 2$, which is larger than s_1^c. Here, the agent considers only the case where there is positive consumption in both period for determining the optimal value s_1^u, that is, she considers only the optimistic case of $S = 4$. Importantly, this result does only depend on the concavity of the utility function and holds independent of the third derivative of U. Even with prudence we get unambiguously $s_1^u > s_1^c$ so that in this case the doomsday anyway effect strictly dominates the prudence effect. This shows clearly the implication of the lemma.

Secondly, consider the situation where the two equally likely values of S are 2 and 10 implying $E[S] = 6$ and thus $s_1^c = 3$ (maintaining $\beta = 1$).[3] Now the lower bound $\underline{S}$ is strictly positive and the value function V^u has a kink at this point. The agent has thus to decide whether or not to run the risk of depletion during the first period. Whenever $s_1^u \epsilon [0, \underline{S}]$, we have the classical case (I) while for a value of s_1^u greater than the lower bound $\underline{S}$, depletion is possible and we have case (II). Denote the maximum on this part of the domain by s_1^{uII} and an interior maximum–if there is one–on the interval $[0, \underline{S}]$ by s_1^{uI}. Now we have to distinguish two cases depending on whether or not there exists an interior maximum s_1^{uI} below $\underline{S}$.

If there is no interior maximum for some s_1 lower than $\underline{S}$, the agent always takes the risk of being left with nothing in period two. This is the case if the agent is not prudent ($U'''(s) \leq 0$) or if she is prudent ($U'''(s) > 0$) but 'not too much' in the sense that his expected utility is monotonically increasing until $\underline{S}$. In these cases, we have that s_1^{uII} is the global maximum of $V^u(s_1)$. In our example, this implies that $s_1^{uII} = \frac{10}{2} > s_1^c = 3$ and we have that first-period consumption is higher under uncertainty than under certainty.

If on the other hand the agent is prudent enough such that there exists an interior maximum $s_1^{uI} < 2$, the value function has two local maxima as depicted in Figure 2. Therefore the result depends on the utility function, and in particular the degree of prudence, and one has to compare the values of $V^u(s_1)$ at the two local maxima s_1^{uI}

[3] This can be interpreted as in Hartwick (1983) as having a certain deposit while a second one is uncertain.

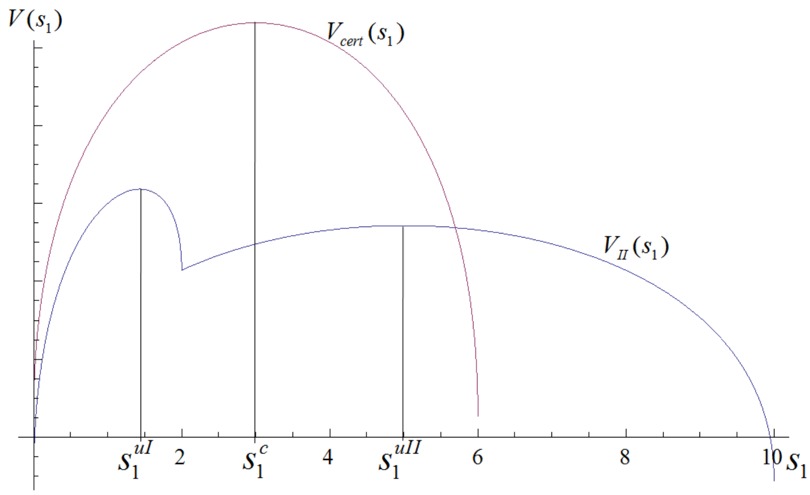

Figure 2: Value function for CRRA utility with $\gamma = .5$, and $\underline{S}$=2, $\overline{S} = 10$, $Pr(\underline{S})$=.5

and s_1^{uII}. For instance, take a CRRA[4] utility function $U(s) = (1-\eta)^{-1}s^{1-\eta}$, which exhibits prudence, e.g., measured by the degree of relative prudence $-\frac{U'''(s)s}{U''(s)} = \eta + 1 > 0$. For our example, we get that for $\eta = .1$ the agent chooses $s_1^u > \underline{S}$ or prefers to take the risk of depletion while for a higher degree of risk aversion and prudence at $\eta = .9$, she will not take the risk. Clearly, in the former case we have the same counter-intuitive result as before that uncertainty induces a less conservationist policy while in the latter we get the opposite. Similar results can be found for a continuous distribution $F(s)$ with CRRA utility where uncertainty thus induces a lower first-period consumption only above a certain threshold of η.

4 More than two periods

When looking at more than two periods, the timing of the decisions is important. We start with three periods where the timing of the problem can be framed as

[4]For CRRA utility, the assumption $U(0) > -\infty$ restricts the parameter η to be less then one. Otherwise the agent would never run the risk of depletion prior to the second period. Alternatively, we could consider a slight variation as $U(s) = (1-\eta)^{-1}(s+\varepsilon)^{1-\eta}$, $\varepsilon > 0$ which is a special case of the Burr utility function (Ikefuji et al., 2010) and is bounded below for all values of η. In both cases, relative risk aversion goes to zero as $s \to 0$.

follows: at date zero, the social planner announces first and second period's planned resource extraction levels whereas in the third period, what is left is extracted if any. Moreover, if in any period the planned consumption plan cannot be realized, the remaining amount of the resource is consumed in the same period. While one could ask whether the planned s_2 could be revised after the first period, it is clear that this is never required given that the plan for the subsequent periods is already based on the conditional expectation of the remaining resource stock for this case. The maximization problem can be written as maximizing $E[V_{II}(s_1, s_2)]$ as

$$\max_{s_1} \max_{s_2} \int_0^{s_1} U(S)dF + \int_{s_1}^{s_1+s_2} \{U(s_1) + \beta U(S - s_1)\}dF$$
$$+ \int_{s_1+s_2}^{\infty} \{U(s_1) + \beta U(s_2) + \beta^2 U(S - s_1 - s_2)\}dF$$

It's first order conditions can be written with respect to s_1 as

$$\int_{s_1}^{s_1+s_2} \{(U'(s_1) - \beta U'(S - s_1)\}dF + \int_{s_1+s_2}^{\infty} \{(U'(s_1) - \beta^2 U'(S - s_1 - s_2)\}dF = 0 \quad (9)$$

and with respect to s_2 as

$$\int_{s_1+s_2}^{\infty} \{(\beta U'(s_2) - \beta^2 U'(S - s_1 - s_2)\}dF = 0. \quad (10)$$

The second condition is equivalent to the first order condition of the two-period case. The condition with respect to first period's consumption on the other hand is more complex given that a change in s_1 affects also the second period due to the possibility of exhaustion.

For the optimal consumption levels under uncertainty $\{s_1^u, s_2^u\}$ to be lower than the levels under certainty $\{s_1^c, s_2^c\}$, we need to show that $V_{II}(s_1, s_2)$ is decreasing in $\{s_1, s_2\}$ at every point where the first order conditions under certainty are satisfied, that is, where the conditions $U'(s_1) = \beta U'(s_2) = \beta^2 U'(E[S] - s_1 - s_2)$ hold.

For the second period, the condition for s_2 is equivalent to the one derived in the two-period case, namely that

$$s_2^u < s_2^c \Leftrightarrow U'(ES - s_1 - s_2) - E[U'(S - s_1 - s_2)|S > s_1 + s_2] < 0. \quad (11)$$

That is, the result from lemma 1 carries over to the second to last period. That is, prudence is a necessary but not anymore sufficient condition for a lower consumption level in the second period compared to the certainty case under certainty.

The case for the first period is more complex. For s_1^u to be less than the level under certainty, s_1^c, we need to show that the left-hand side of (9) is negative at all the points where the first-order conditions under certainty are satisfied. Using these conditions, we can expand the first term in (9) expressing it in terms of second and third period's marginal utilities. After some reformulations, one can show that the condition for $s_1^u < s_1^c$ is equivalent to

$$\int_{s_1}^{s_1+s_2} \beta\{(U'(S-s_1)-U'(s_2)\}dF > \int_{s_1+s_2}^{\infty} \beta^2\{(U'(ES-s_1-s_2)-U'(S-s_1-s_2)\}dF \tag{12}$$

The left-hand side is the difference in marginal utility in the second period if depletion occurs during this period as compared to when it does not occur. This term is always positive due to the concavity of the utility function. The term on the right-hand side is exactly the one in the condition for second period's consumption given by (11) which had to be negative to have $s_2^u < s_2^c$. Thus, the condition for $s_1^u < s_1^c$ is weaker than the one for $s_2^u < s_2^c$.

The last result can be easily generalized for the model with more than three periods. In this case, the respective conditions akin (12) for s_t for any period $t = 1..T-2$ include the differences in marginal utilities for all terms between $j = t+1$ and $j = T-1$ on the left hand side. Denoting by S_j the cumulative consumption until period j, i.e., $S_j \equiv \sum_{i=1}^{j} s_i$, the conditions for $s_t^u < s_t^c$ for any period $t = 1..T-2$ can be written similar to (12) as

$$\sum_{j=t}^{T-2} \int_{S_j}^{S_{j+1}} \beta^j\{U'(S-S_j)-U'(s_{j+1})\}dF > \tag{13}$$
$$\int_{S_{T-1}}^{\infty} \beta^{T-1}\{U'(E[S]-S_{T-1})-U'(S-S_{T-1})\}dF.$$

The summed terms on the left hand side are all non-negative. That is, the earlier the period, the more likely is that consumption in this period is lower than under certainty in the sense that if $s_t^u < s_t^c$ holds for some period t, this is true for all previous periods as well. For earlier periods, all periods until the last matter directly for the decision on its optimal consumption. This is different from the model under certainty or the model (I) where the trade-off shows up directly only between each period and the last one where the realization of S is learned.

The only effect potentially implying a faster extraction than under certainty comes from the second-to-last period with the interpretation we gave in the previous section. If the right hand side of (13) is negative and we therefore have that $s^u_{T-1} < s^c_{T-1}$, this holds for all previous periods as well.

Finally, if we take the limit for $T \to \infty$, we obtain the discrete time equivalent of continuous time models of resource extraction such as Kumar (2005). For a discount factor strictly less than one, we finally get the main result of this section.

Lemma 2. *As the number of periods tends to infinity and for a discount factor less than one, resource consumption under uncertainty is lower for every period up to the second-to-last period for any risk averse decision maker.*

Proof. Given the discount factor $\beta < 1$, the 'doomsday anyway effect' of the last period becomes nil as $T \to \infty$. The right hand side of (13) thus becomes zero. Since in every period j planned consumption j is higher or equal than realized consumption, the left hand side is non- negative and strictly positive if exhaustion occurs in finite time. In this case, the condition (13) is satisfied for all periods t and resource consumption in all periods but the last two are lower under certainty than under certainty. □

As the number of periods tends to infinity, the doomsday anyway effect vanishes and the effect of possible exhaustion in each period dominates implying that the optimal extraction policy is more conservationist than under certainty if the decision maker is risk averse. Now prudence is not anymore necessary for a more conservationist policy and we obtain the classical results as in Kemp (1976) and Kumar (2005). Contrasting this result with the two period case of the last section thus allows to reconcile the two strands of the literatures. Depending on the assumption about $U(0)$, prudence is not necessary for a conservationist resource extraction policy but instead risk aversion is sufficient. A stylized two-period model on the other hand leads to a counter-intuitive result.

5 Conclusion

While the classical literature of resource extraction under uncertainty found that the optimal resource extraction is always more conservationist for any risk averse decision maker than under certainty, the application of the qualitatively equivalent precautionary savings model implies that this is only the case if the decision maker is prudent ($U''' > 0$). We show that the difference between the two results is a crucial assumption about whether or not $U(0)$ is bounded below or in other words, whether or not exhaustion in any period is possible or not.

The results once more suggest that the discussion about the boundedness of $U(0)$ is important when using expected utility models in environmental economics when considering long time horizons. This point has been discussed in recent years also in the context of climate change and catastrophic risks, see Buchholz and Schymura (2010). In the context on non-renewable resources, the substitutability of exhaustible resources, even at a very high cost, indicates that $U(0)$ should be bounded below. Applying a precautionary savings model on the other hand typically imposes $U(0) = -\infty$ in order to prevent zero consumption in any period. However, it is precisely allowing for resource consumption to drop to zero at some point (doomsday), which is needed in the classical resource extraction model to find an optimal extraction path under uncertainty.

When we introduce this possibility in a standard two-period expected utility framework, we find that prudence, while still being necessary, is no longer sufficient for lower first-period consumption than under certainty. The possibility of exhaustion before the last period surprisingly requires *stronger* conditions on the distribution and utility function to ensure a more conservationist extraction policy than under certainty. The intuition behind this result is that the decision maker considers for his decision about today's consumption only the case where depletion does not occur. This 'doomsday anyway effect' works against the effect of prudence.

However, if we extend the number of periods, the condition becomes less stringent the earlier the period since now in each period depletion is possible. With an infinite number of periods and a discount factor strictly less than one, risk aversion is sufficient to ensure that the extraction policy will be more conservationist than under certainty as it was found in the classical Hotelling case under uncertainty.

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