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Research Note on "International Consumption Risk Sharing with Incomplete Goods and Asset Markets"

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Research Note on “International Consumption Risk Sharing with Incomplete Goods and Asset Markets”

Sven Blank

May 31, 2009

Research Note on “International Consumption Risk Sharing with Incomplete Goods and Asset Markets”*

Sven Blank[†]

1 Introduction

Perfect risk sharing requires both, frictionless goods as well as frictionless asset markets. To analyze the consequences of both type of frictions for consumption risk sharing across countries, the model by Ghironi and Melitz (2005) is extended to allow for international trade in equities. The model features fixed costs of exporting as well as variable iceberg costs when shipping goods. Financial markets are incomplete, as only two assets are traded, which cannot span all the uncertainty caused by potential shock scenarios.

In models with incomplete asset markets, two well known problems arise. First, the steady state portfolio allocation in a non-stochastic steady state is indeterminate since assets are perfect substitutes. And, second, as noted by Schmitt-Grohé and Uribe (2003) among others, even transitory shocks may have permanent effects on wealth. This, in turn, may lead to non-stationary responses of the endogenous variables. To deal with these issues, quadratic portfolio costs on asset holdings as in Ghironi, Lee, and Rebucci (2007) are introduced. Besides introducing frictions in asset markets, these costs help to pin down the steady state portfolio allocation and induce model stationarity.

This research note gives technical details on the solution of the model. In the following section, the basic setup of the model as well as the main variables and equilibrium condi-

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tions of the model are briefly summarized. Section 3 solves for the steady state levels of the endogenous variables.

2 Model summary

There are two countries, home and foreign. In each country, there is a unit mass of identical households. The household consumes and receives income out of two sources: First, income from inelastically supplied labor, and, second, income from portfolio investments. The portfolio consists of holdings of a bond that is only traded domestically and home and foreign equity that is traded across borders. In each country, domestic equity is bundled in a mutual fund and issued in the respective market.

When holding home and foreign equity, households have to pay a quadratic portfolio costs to financial intermediaries as in Ghironi, Lee, and Rebucci (2007). These convex costs solve the problem of indeterminacy of the portfolio composition in the deterministic steady state as they pin down portfolio shares and ensure stationarity in response to stochastic shocks. There are no costs on bond holdings. Since these are traded only domestically, bond holdings are zero in the aggregate.

In both countries, there is a continuum of monopolistically competitive firms, each producing one differentiated good. These firms are heterogenous with respect to their productivity level z , which is drawn from a Pareto distribution and constant thereafter. At the beginning of each period, there are prospective entrants in both countries, which face a sunk entry cost $f_{E,t}$ ($f_{E,t}^*$) before entering the home (foreign) market. To keep the number of firms operating in both countries well-behaved, any firm, already operating in the market or just entering, may be hit by a “death” shock that forces market exit and occurs with probability δ .

In principle, each good is tradable. However, exporting is costly. Each home (foreign) firm faces fixed export costs $f_{X,t}$ ($f_{X,t}^*$). Only more productive firms will decide to serve the domestic as well as the market abroad, which leads to an endogenously determined size of the non-traded goods sector. In addition, there are variable iceberg-type trade costs τ_t (τ_t^*), when shipping home (foreign) goods that alter the dynamics of the real exchange rate.

Table 1 gives the notation of key variables in this setup. The main equations and equilibrium conditions for the home as well the foreign country of the aggregated model are summarized in Table 2.

Table 1: Notation summary

<i>Endogenous variables</i>	
C_t, C_t^*	Consumption
w_t, w_t^*	Wage rate
$\tilde{p}_{D,t}, \tilde{p}_{D,t}^*$	Average price for domestic goods
$\tilde{p}_{X,t}, \tilde{p}_{X,t}^*$	Average price for export goods
$N_{D,t}, N_{D,t}^*$	Number of firms
$N_{E,t}, N_{E,t}^*$	Number of entrants
$N_{X,t}, N_{X,t}^*$	Number of exporting firms
$\tilde{z}_{X,t}, \tilde{z}_{X,t}^*$	Average export productivity
$\tilde{d}_t, \tilde{d}_t^*$	Average total profits
$\tilde{d}_{D,t}, \tilde{d}_{D,t}^*$	Average profits from domestic activity
$\tilde{d}_{X,t}, \tilde{d}_{X,t}^*$	Average profits from exporting
$\tilde{v}_t, \tilde{v}_t^*$	Average firm value
$R_{B,t}, R_{B,t}^*$	Return on riskless bond
$R_{H,t}, R_{H,t}^*$	Return on home equity
$R_{F,t}, R_{F,t}^*$	Return on foreign equity
$\mathcal{P}_t, \mathcal{P}_t^*$	Financial wealth
$\alpha_{H,t}, \alpha_{H,t}^*$	Portfolio share on home equity
$\alpha_{F,t}, \alpha_{F,t}^*$	Portfolio share on foreign equity
$x_{H,t}, x_{H,t}^*$	Share in the home mutual fund
$x_{FH,t}, x_{FH,t}^*$	Share in the foreign mutual fund
Q_t	Real exchange rate
<i>Parameters</i>	
β	Subjective time preference factor
γ	Relative risk aversion
θ	Elasticity of substitution
k	Shape parameter of Pareto distribution
z_{\min}	Lower bound of Pareto distribution
$\tilde{z}_D, \tilde{z}_D^*$	Average domestic productivity
τ_t, τ_t^*	Trade costs
$f_{X,t}, f_{X,t}^*$	Fixed export costs
$f_{E,t}, f_{E,t}^*$	Sunk entry costs
δ	Probability of market exit
$\xi_{H,t}, \xi_{H,t}^*$	Home financial market frictions
$\xi_{F,t}, \xi_{F,t}^*$	Foreign financial market frictions
L, L^*	Labor supply
Z_t, Z_t^*	Aggregate labor productivity

Notes: All endogenous variables are defined in real terms. Foreign variables are denoted by an asterisk. Parameters are identical across countries if only one parameter is specified.

Table 2: Model Summary

Price indexes	$N_{D,t} (\tilde{\rho}_{D,t})^{1-\theta} + N_{X,t}^* (\tilde{\rho}_{X,t}^*)^{1-\theta} = 1$ $N_{D,t}^* (\tilde{\rho}_{D,t}^*)^{1-\theta} + N_{X,t} (\tilde{\rho}_{X,t})^{1-\theta} = 1$
Profits	$\tilde{d}_t = \tilde{d}_{D,t} + \frac{N_{X,t}}{N_{D,t}} \tilde{d}_{X,t}$ $\tilde{d}_t^* = \tilde{d}_{D,t}^* + \frac{N_{X,t}^*}{N_{D,t}^*} \tilde{d}_{X,t}^*$
Free entry	$\tilde{v}_t = w_t \frac{f_{E,t}}{Z_t}$ $\tilde{v}_t^* = w_t^* \frac{f_{E,t}^*}{Z_t^*}$
Zero-profit export cutoffs	$\tilde{d}_{X,t} = w_t \frac{f_{X,t}}{Z_t} \frac{\theta-1}{k-(\theta-1)}$ $\tilde{d}_{X,t}^* = w_t^* \frac{f_{X,t}^*}{Z_t^*} \frac{\theta-1}{k-(\theta-1)}$
Share of exporting firms	$\frac{N_{X,t}}{N_{D,t}} = (z_{\min})^k (\tilde{z}_{X,t})^{-k} \left[\frac{k}{k-(\theta-1)} \right]^{\frac{k}{\theta-1}}$ $\frac{N_{X,t}^*}{N_{D,t}^*} = (z_{\min}^*)^k (\tilde{z}_{X,t}^*)^{-k} \left[\frac{k}{k-(\theta-1)} \right]^{\frac{k}{\theta-1}}$
Number of firms	$N_{D,t} = (1 - \delta) (N_{D,t-1} + N_{E,t-1})$ $N_{D,t}^* = (1 - \delta) (N_{D,t-1}^* + N_{E,t-1}^*)$
Euler equations for bonds	$1 = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (R_{B,t+1}) \right]$ $1 = E_t \left[\beta \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\gamma} (R_{B,t+1}^*) \right]$
Euler equations for home equity	$1 = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{R_{H,t+1}}{(1 + \xi_{H,t} \alpha_{H,t+1} \mathcal{P}_{t+1})} \right]$ $1 = E_t \left[\beta \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\gamma} \frac{R_{H,t+1}^*}{(1 + \xi_{H,t}^* \alpha_{H,t+1}^* \mathcal{P}_{t+1}^*)} \right]$
Euler equations for foreign equity	$1 = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{R_{F,t+1}}{(1 + \xi_{F,t} \alpha_{F,t+1} \mathcal{P}_{t+1})} \right]$ $1 = E_t \left[\beta \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\gamma} \frac{R_{F,t+1}^*}{(1 + \xi_{F,t}^* \alpha_{F,t+1}^* \mathcal{P}_{t+1}^*)} \right]$

Continued.

Table 2 continued.

Return on home equity	$R_{H,t+1} = (1 - \delta) \frac{\tilde{v}_{t+1} + \tilde{d}_{t+1}}{\tilde{v}_t}$ $R_{H,t+1}^* = (1 - \delta) \frac{Q_t}{Q_{t+1}} \frac{\tilde{v}_{t+1} + \tilde{d}_{t+1}}{\tilde{v}_t}$
Return on foreign equity	$R_{F,t+1} = (1 - \delta) \frac{Q_{t+1}}{Q_t} \frac{\tilde{v}_{t+1}^* + \tilde{d}_{t+1}^*}{\tilde{v}_t^*}$ $R_{F,t+1}^* = (1 - \delta) \frac{\tilde{v}_{t+1}^* + \tilde{d}_{t+1}^*}{\tilde{v}_t^*}$
Net foreign assets	$\alpha_{F,t+1} \mathcal{P}_{t+1} - Q_t \alpha_{H,t+1}^* \mathcal{P}_{t+1}^* = -\alpha_{H,t+1} \mathcal{P}_{t+1}$ $+ Q_t \alpha_{F,t+1}^* \mathcal{P}_{t+1}^* + (\alpha_{H,t} R_{H,t} + (1 - \alpha_{H,t}) R_{F,t}) \mathcal{P}_t$ $- Q_t (\alpha_{F,t}^* R_{F,t}^* + (1 - \alpha_{F,t}^*) R_{H,t}^*) \mathcal{P}_t^*$ $+ w_t L - Q_t w_t^* L^* - C_t + Q_t C_t^*$
Labor market clearing	$L = \frac{\theta - 1}{w_t} (N_{D,t} \tilde{d}_{D,t} + N_{X,t} \tilde{d}_{X,t}) + Z_t^{-1} (\theta N_{X,t} f_{X,t} + N_{E,t} f_{E,t})$ $L^* = \frac{\theta - 1}{w_t^*} (N_{D,t}^* \tilde{d}_{D,t}^* + N_{X,t}^* \tilde{d}_{X,t}^*) + (Z_t^*)^{-1} (\theta N_{X,t}^* f_{X,t}^* + N_{E,t}^* f_{E,t}^*)$

Notes: See Table 1.

The system comprises 25 endogenous variables: $C_t, C_t^*, w_t, w_t^*, N_{D,t}, N_{D,t}^*, N_{E,t}, N_{E,t}^*, N_{X,t}, N_{X,t}^*, \tilde{z}_{X,t}, \tilde{z}_{X,t}^*, \tilde{v}_t, \tilde{v}_t^*, \tilde{d}_t, \tilde{d}_t^*, R_{B,t}, R_{B,t}^*, \alpha_{H,t}, \alpha_{F,t}, \alpha_{F,t}^*, \alpha_{H,t}^*, \mathcal{P}_t, \mathcal{P}_t^*$, and Q_t . Among these, ten variables are predetermined as of time t : portfolio shares, $\alpha_{H,t}, \alpha_{F,t}, \alpha_{F,t}^*$ and $\alpha_{H,t}^*$, financial wealth, \mathcal{P}_t and \mathcal{P}_t^* , the (gross) return on domestic bonds, $R_{B,t}$ and $R_{B,t}^*$, and the number of firms, $N_{D,t}$ and $N_{D,t}^*$.

3 Steady state

The model is solved around a symmetric non-stochastic steady state, where $f_E = f_E^*, f_X = f_X^*, \tau = \tau^*, L = L^*$, and $Z = Z^* = 1$. It follows that $Q = 1$. Steady state variables are denoted without time subscripts. Symmetry of the steady state implies that $C = C^*, w = w^*, N_D = N_D^*, N_E = N_E^*, N_X = N_X^*, \tilde{z}_X = \tilde{z}_X^*, \tilde{v} = \tilde{v}^*, \tilde{d} = \tilde{d}^*, R_B = R_B^*$, and $\mathcal{P} = \mathcal{P}^*$.

The solution of most endogenous variables is straightforward and follows Ghironi and Melitz (2005). I focus on key variables of the model: portfolio shares, the average export productivity, and the average export price. However, the steady state levels of the other variables of the model will become evident in the course of these derivations.

Solving for portfolio shares

In steady state, the Euler equations for equity holdings of the home household and their foreign analogs are given by

$$\tilde{v} = \frac{\beta(1-\delta)}{1-\beta(1-\delta) + \xi_H \alpha_H \mathcal{P}} \tilde{d} \quad (1)$$

$$\tilde{v} = \frac{\beta(1-\delta)}{1-\beta(1-\delta) + \xi_H^* \alpha_H^* \mathcal{P}^*} \tilde{d} \quad (2)$$

for home equity and

$$\tilde{v}^* = \frac{\beta(1-\delta)}{1-\beta(1-\delta) + \xi_F \alpha_F \mathcal{P}} \tilde{d}^* \quad (3)$$

$$\tilde{v}^* = \frac{\beta(1-\delta)}{1-\beta(1-\delta) + \xi_F^* \alpha_F^* \mathcal{P}^*} \tilde{d}^* \quad (4)$$

for foreign equity. Combining (1) with (2) and (3) with (4), it follows that

$$\frac{\xi_H}{\xi_H^*} \alpha_H = \alpha_H^* \quad (5)$$

$$\frac{\xi_F^*}{\xi_F} \alpha_F^* = \alpha_F. \quad (6)$$

Since bond holdings in both countries are zero in the aggregate, shares of home and foreign equity holdings sum to one

$$\alpha_H + \alpha_F = 1 \quad (7)$$

$$\alpha_H^* + \alpha_F^* = 1. \quad (8)$$

Portfolio shares by home and foreign households are then given by

$$\alpha_H = \frac{\xi_H^*}{\xi_H + \xi_H^*} = \frac{\xi_F}{\xi_F^* + \xi_F}, \quad \alpha_F = \frac{\xi_F^*}{\xi_F^* + \xi_F} = \frac{\xi_H}{\xi_H + \xi_H^*}, \quad (9)$$

$$\alpha_H^* = \frac{\xi_H}{\xi_H + \xi_H^*} = \frac{\xi_F^*}{\xi_F^* + \xi_F}, \quad \alpha_F^* = \frac{\xi_F}{\xi_F^* + \xi_F} = \frac{\xi_H^*}{\xi_H + \xi_H^*}. \quad (10)$$

In a symmetric steady state, home portfolio shares of home equity and foreign portfolio shares of foreign equity are equal, so households in both countries face the same *relative* access to financial markets, $\xi_H = \xi_F^*$ and $\xi_F = \xi_H^*$. If portfolio costs for foreign households

for holdings of foreign equity are zero, $\xi_H^* = 0$, then home holdings of home equity are zero, irrespective of the size of distortion for purchasing home equity, except for the special case where both scaling parameters are zero, $\xi_H = \xi_H^* = 0$. Then the home portfolio share on home equity is indeterminate.

As in Ghironi, Lee, and Rebucci (2007), portfolio shares are determined exogenously in steady state. However, the value of each asset position held in the portfolio is endogenous as in Devereux and Sutherland (2007). Symmetry also implies that net foreign assets are always zero in steady state, but gross foreign assets need not.

Financial wealth at home and abroad equals each other in steady state, $\mathcal{P} = \mathcal{P}^*$, and sum to world equity market capitalization

$$\mathcal{P} + \mathcal{P}^* = \tilde{v}(N_D + N_E) + \tilde{v}^*(N_D^* + N_E^*). \quad (11)$$

Together with the value of home portfolio holdings of home and foreign equity and their foreign counterparts

$$\alpha_H \mathcal{P} = x_H (N_D + N_E) \tilde{v}, \quad \alpha_F \mathcal{P} = x_F (N_D^* + N_E^*) \tilde{v}^*, \quad (12)$$

$$\alpha_H^* \mathcal{P}^* = x_H^* (N_D + N_E) \tilde{v}, \quad \alpha_F^* \mathcal{P}^* = x_F^* (N_D^* + N_E^*) \tilde{v}^*, \quad (13)$$

this implies that in steady state portfolio shares of each asset are equal to the respective shares held in the mutual funds

$$\alpha_H = x_H, \quad \alpha_F = x_F, \quad (14)$$

$$\alpha_H^* = x_H^*, \quad \alpha_F^* = x_F^*. \quad (15)$$

Note that this is essentially the same steady state result as in Ghironi, Lee, and Rebucci (2007) for the symmetric case. In their model, financial fees are charged on shares holdings in the mutual fund (rather than on portfolio shares).

Solving for the average export productivity

The average productivity level of exporting firms, $\tilde{\rho}_X$, in steady state can be derived using total average profits from from selling goods at home and abroad. These are given by

$$\tilde{d} = \tilde{d}_D + \frac{N_X}{N_D} \tilde{d}_X. \quad (16)$$

Solving the steady state profits from export activity

$$\tilde{d}_X = (\tilde{\rho}_X)^{1-\theta} \frac{C}{\theta} - wf_X \quad (17)$$

for $\frac{C}{\theta}$ and substituting the resulting expression into the equation for operating in the home market $\tilde{d}_D = (\tilde{\rho}_D)^{1-\theta} \frac{C}{\theta}$ yields

$$\tilde{d}_D = \left(\frac{\tilde{\rho}_X}{\tilde{\rho}_D} \right)^{\theta-1} (\tilde{d}_X + wf_X). \quad (18)$$

Together with average prices for goods sold at home and abroad, $\tilde{\rho}_D = \frac{\theta}{\theta-1} \frac{w}{\tilde{z}_D}$ and $\tilde{\rho}_X = \frac{\theta}{\theta-1} \frac{w}{\tilde{z}_X} \tau$, respectively, equation (18) can be written as

$$\tilde{d}_D = \left(\frac{\tau \tilde{z}_D}{\tilde{z}_X} \right)^{\theta-1} (\tilde{d}_X + wf_X). \quad (19)$$

The zero profit export cutoff is given by

$$\tilde{d}_X = wf_X \frac{\theta - 1}{k - (\theta - 1)}, \quad (20)$$

so that profits from operating at home can be written as

$$\tilde{d}_D = \left(\frac{\tau \tilde{z}_D}{\tilde{z}_X} \right)^{\theta-1} \left(wf_X \frac{k}{k - (\theta - 1)} \right). \quad (21)$$

Using the expression for total profits (16) and the free entry condition in steady state $\tilde{v} = wf_E$ the Euler equation (1) for home equity can be rearranged as

$$\left(\frac{1 - \beta(1 - \delta) + \xi_H \alpha_H \mathcal{P}}{\beta(1 - \delta)} \right) f_E w = \tilde{d}_D + \frac{N_X}{N_D} \tilde{d}_X. \quad (22)$$

Note that it is not essential which Euler equation for home equity is taken to derive the following results. Since $\xi_H \alpha_H = \xi_H^* \alpha_H^* = \frac{\xi_H \xi_H^*}{\xi_H + \xi_H^*}$, and $\mathcal{P} = \mathcal{P}^*$ Euler equations of home and foreign households for home equity equal each other in steady state. Substituting

(20) and (21) into (22) and rearranging yields

$$\left(\frac{1 - \beta(1 - \delta) + \frac{\xi_H \xi_H^*}{\xi_H + \xi_H^*} \mathcal{P}}{\beta(1 - \delta)} \right) \frac{f_E}{f_X} = \left(\frac{\tau \tilde{z}_D}{\tilde{z}_X} \right)^{\theta-1} \left(\frac{k}{k - (\theta - 1)} \right) + \frac{N_X}{N_D} \left(\frac{\theta - 1}{k - (\theta - 1)} \right). \quad (23)$$

The share of exporting firms in the total number of firms operating in the home market in steady state is given by

$$\frac{N_X}{N_D} = \left(\frac{z_{\min}}{\tilde{z}_X} \right)^k \left(\frac{k}{k - (\theta - 1)} \right)^{\frac{k}{\theta-1}}. \quad (24)$$

Together with the steady state average productivity of all firms producing for the home market $\tilde{z}_D = \left(\frac{k}{k - (\theta - 1)} \right)^{\frac{1}{\theta-1}} z_{\min}$, inserting (24) into (23) gives the solution for \tilde{z}_X :

$$\begin{aligned} \left(\frac{1 - \beta(1 - \delta) + \frac{\xi_H \xi_H^*}{\xi_H + \xi_H^*} \mathcal{P}}{\beta(1 - \delta)} \right) \frac{f_E}{f_X} &= \left(\frac{\tau z_{\min}}{\tilde{z}_X} \right)^{\theta-1} \left(\frac{k}{k - (\theta - 1)} \right)^2 \\ &+ \left(\frac{z_{\min}}{\tilde{z}_X} \right)^k \left(\frac{k}{k - (\theta - 1)} \right)^{\frac{k}{\theta-1}} \frac{\theta - 1}{k - (\theta - 1)}. \end{aligned} \quad (25)$$

As in Ghironi and Melitz (2005), the exact value of \tilde{z}_X is obtained numerically. Note that the average export productivity depends on both, frictions in goods markets as well as frictions in financial markets. Everything else constant, a rise in the sunk entry costs, f_E , leads to a decrease in the average export productivity, since there are more entrants in the home economy with potentially lower productivity that will become exporters. As expected, higher export costs, f_X , and higher iceberg costs of shipping goods, τ , increase \tilde{z}_X as only more productive firms will decide to produce for the market abroad. However, higher impediments to purchase home equity (an increase in ξ_H and ξ_H^*) lower the average export productivity.

Solving for the average export price

To solve for the average export price in steady state, note that the evolution of the number of firms in steady state is given by:

$$N_E = \frac{\delta}{1 - \delta} N_D. \quad (26)$$

As aggregate bond holdings are zero, home household's budget constraint in steady state can be written as

$$C = (\alpha_H R_H + \alpha_F R_F - 1) \mathcal{P} + wL. \quad (27)$$

Since financial wealth is equal across countries in a symmetric steady state, $\mathcal{P} = \mathcal{P}^*$, equation (11) implies that financial wealth can be expressed in terms of the free entry condition and the number of firms

$$\mathcal{P} = \tilde{v}(N_D + N_E) = \frac{w f_E}{1 - \delta} N_D \quad (28)$$

The returns on home and foreign equity are given by

$$R_H = (1 - \delta) \left(\frac{\tilde{d}}{\tilde{v}} + 1 \right), \quad \text{and} \quad (29)$$

$$R_F = (1 - \delta) \left(\frac{\tilde{d}^*}{\tilde{v}^*} + 1 \right). \quad (30)$$

Using the Euler equations (1) and (3) and the home and foreign free entry conditions, the returns on home and foreign equity can be written as

$$R_H = \beta^{-1} \left(1 + \frac{\xi_H \xi_H^*}{\xi_H + \xi_H^*} \mathcal{P} \right), \quad \text{and} \quad (31)$$

$$R_F = \beta^{-1} \left(1 + \frac{\xi_F \xi_F^*}{\xi_F + \xi_F^*} \mathcal{P}^* \right). \quad (32)$$

Because both countries face the same relative access to financial markets, it must hold that

$$\frac{\xi_H \xi_H^*}{\xi_H + \xi_H^*} = \frac{\xi_F \xi_F^*}{\xi_F + \xi_F^*},$$

i.e. returns on home and foreign equity are equal in steady state. Even though this would leave the portfolio composition indeterminate in a model absent transaction costs, frictions in financial markets in this setup pin down steady state portfolio shares as shown above. Note that the returns on home and foreign equity *net of transaction costs* equal the steady state return on the riskless bond in each country

$$R_B = R_B^* = \beta^{-1}.$$

Substituting (31) and (32) as well as the expression for the value of the steady state portfolio (28) into the budget constraint (27) leads to

$$\frac{C}{w} = \frac{1 - \beta + \frac{\xi_H \xi_H^*}{\xi_H + \xi_H^*} \mathcal{P}}{\beta(1 - \delta)} f_E N_D + L. \quad (33)$$

Combining the zero export profit cutoff in (20) with average profits from exporting (17) gives

$$\frac{C}{w} = \frac{\theta k}{k - (\theta - 1)} f_X (\tilde{\rho}_X)^{\theta - 1}. \quad (34)$$

The steady state home price index $1 = N_D (\tilde{\rho}_D)^{1 - \theta} + N_X (\tilde{\rho}_X)^{1 - \theta}$ can be transformed to

$$\frac{(\tilde{\rho}_X)^{\theta - 1}}{N_D} = \left(\frac{\tilde{\rho}_X}{\tilde{\rho}_D} \right)^{\theta - 1} + \frac{N_X}{N_D}. \quad (35)$$

With the relative average price for goods sold at home and in the foreign country $\frac{\tilde{\rho}_X}{\tilde{\rho}_D} = \tau \frac{\tilde{z}_D}{\tilde{z}_X}$ and the steady state share of exporting firms in equation (24), equation (35) can be expressed as

$$\frac{(\tilde{\rho}_X)^{\theta - 1}}{N_D} = \left(\frac{\tau \tilde{z}_D}{\tilde{z}_X} \right)^{\theta - 1} + \left(\frac{z_{\min}}{\tilde{z}_X} \right)^k \left(\frac{k}{k - (\theta - 1)} \right)^{\frac{k}{\theta - 1}}. \quad (36)$$

Now substituting (34) and (36) into (23) and rearranging yields the following solution for $\tilde{\rho}_X$:

$$(\tilde{\rho}_X)^{1-\theta} = L^{-1} \left[\frac{\theta k}{k - (\theta - 1)} f_X - \frac{1 - \beta + \frac{\xi_H \xi_H^*}{\xi_H + \xi_H^*} \mathcal{P}}{\beta(1 - \delta)} f_E \right. \\ \left. \times \left(\left(\frac{\tau \tilde{z}_D}{\tilde{z}_X} \right)^{\theta-1} + \left(\frac{z_{\min}}{\tilde{z}_X} \right)^k \left(\frac{k}{k - (1 - \theta)} \right)^{\frac{k}{\theta-1}} \right)^{-1} \right]. \quad (37)$$

Note that in the above equation, the average export price $\tilde{\rho}_X$ does only depend on exogenous parameters, since the average productivity of exporting firms, \tilde{z}_X , is given by (25) and the average productivity of home firms is given by $\tilde{z}_D = \left(\frac{k}{k - (\theta - 1)} \right)^{\frac{1}{\theta-1}} z_{\min}$. As the steady state level of the average export productivity, the average export price depends on frictions in the goods as well as in the financial markets and on the shape of the Pareto distribution.

Model dynamics

The model is log-linearized around the steady state levels of the endogenous variables derived in the previous subsections. The model allows for various shock scenarios to analyze how the interactions between goods and financial market imperfections affect international consumption risk sharing. Impulse responses show that the degree of financial market integration and the time horizon considered, substantially alter the extent of consumption risk sharing depending on the nature of the underlying shock.

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