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## A Keynesian Dynamic Stochastic Labor-Market Disequilibrium model for business cycle analysis

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Christian Schoder<sup>1</sup>

## A Keynesian Dynamic Stochastic Labor-Market Disequilibrium model for business cycle analysis.\*

August 16, 2015

### Abstract

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**Keywords:** Dynamic stochastic labor-market disequilibrium, dynamic stochastic general equilibrium, post-Keynesian economics, micro-foundations

**JEL Classification System:** B41, E12, J52

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# 1 Introduction

At the core of *aggregative* models in the Cambridge tradition of Kalecki (1971), Robinson (1956, 1962) and Kaldor (1982), here referred to as *Traditional Post-Keynesian* (TPK), is the *principle of effective demand* according to which output is determined by aggregate spending. The labor market exhibits *Keynesian* unemployment resulting from a lack of aggregate demand. Labor market conditions may affect the determination of wage growth (cf. Hein and Stockhammer 2010, Taylor 2004).<sup>1</sup>

TPK models have been criticized, however, for lacking satisfactory micro-foundations (cf. Lucas 1976, Farmer and Foley 2009, Murota and Ono 2010, Skott 1989a, 2012 and Schoder 2015). Behavioral hypotheses such as the Keynesian consumption function are anchored in stylized and highly contested empirical observations and justified by means of verbal argumentation (cf. Lavoie 1992, 2014). This practice has been argued to be subject to inconsistencies along three dimensions by Schoder (2015). (i) *Methodological inconsistency*: Post-Keynesians adhere to the idea of formal modeling when studying the interaction of macroeconomic variables. Yet, behavioral hypotheses postulated on the micro-level are anchored in verbal considerations rather than formal modeling. (ii) *Internal inconsistency*: Micro-considerations provided for different behavioral rules within the very same model are often mutually inconsistent. (iii) *Ontological inconsistency*: The micro-considerations provided to back up the postulated behavioral rules typically imply the agents to exhibit *goal-oriented* behavior, i.e. to pursue objectives. Goal-oriented behavior implies behavioral relations to adjust to possibly endogenous variations in the economic environment. Yet, changes in the micro-environment are not taken into account by the postulated rules which implicitly take agents to be *lethargic*, i.e. unable or unwilling to adapt their behavior to a changing environment. Furthermore, Schoder (2015) argues that PK models rely on strong assumptions regarding the formation of expectations which are perceived as purely backward-looking despite Keynes' emphasis on their forward-looking nature (cf. Keynes 1936, p.152).

Schoder's (2015) main conclusion is that the post-Keynesian research paradigm should be open to various forms of micro-foundations as well as various assumptions regarding expectation formation as long as the model tells a post-Keynesian story. The aim of the present paper is to show that a post-Keynesian economy can even be characterized by a model based on conventional micro-foundations featuring inter-temporal optimization and rational expectations. In particular, we propose a new framework referred to as *Dynamic Stochastic Labor-Market Disequilibrium* (DSLMD) model and compare it, in terms of micro-foundations and economic content, to TPK models as well as to *Dynamic Stochastic General Equilibrium* (DSGE) models in the vein of Woodford (2003). As a stark contrast to the DSLMD model we also consider what we refer to as a *Synthetic Neoclassical* (SNC) model loosely in the vein of Ackley (1978, part ii) and (Marglin 1984, ch.2) which shares the behavioral relations of the TPK model but assumes the nominal wage to clear the labor market.

Table 1 summarizes the core argument of the present paper comparing the DSLMD, TPK, DSGE and SNC model classes along two dimensions: micro-foundations and economic content. Regarding the former, TPK and SNC models share a lack of thereof as they are aggregative models. DSLMD and DSGE models share micro-foundations based on inter-temporal optimization and rational expectations. Yet, regarding the latter, DSGE and SNC models are neoclassical while

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<sup>1</sup>In the long run, unemployment may become structural, i.e. independent of aggregate demand, in some model variants (Skott 1989b, Duménil and Lévy 1999, Shaikh 2009).

Table 1: Comparing DSLMD, DSGE, TPK and SNC models with respect to economic content and micro-foundations.

	<b>Keynesian</b>	<b>Neoclassical</b>
<b>Micro-founded</b>	DSLMD	DSGE
<b>Aggregative</b>	TPK	SNC

DSGE and TPK models are Keynesian.

The crucial difference between neoclassical and Keynesian theory lies in the perception of the labor market (cf. Marglin 1984). In the former, the nominal wage is perceived as an *accommodating* variable. In absence of labor-market imperfections such as search and matching frictions (Mortensen and Pissarides 1994, Gertler et al. 2008) or disequilibrium wages arising from asymmetric information (Shapiro and Stiglitz 1984), nominal wage adjustment clears the labor market for a given price level. Keynesian unemployment arising from a lack of aggregate demand cannot exist. The economy is supply-side determined since factor-market clearing combined with an aggregate production function implies a unique level of output and employment (cf. Clarida et al. 1999, Woodford 2003 and Smets and Wouters 2003). In the latter models, the nominal wage is taken as a *non-accommodating* variable. In the simplest case it is assumed to be constant. In more elaborate variants it is modeled as a policy variable subject to collective wage bargaining. In any case, Keynesian unemployment prevails if the level of aggregate spending is insufficient. Aggregate supply is determined by aggregate demand as long as labor is not fully utilized (cf. Taylor 2004). Hence, the DSGE and DSLMD framework differ only but crucially in the model closure.

The core agents of the models considered are households choosing consumption and labor supply as well as intermediate good firms choosing prices, labor demand, investment and bonds supply. In the DSLMD and DSGE models considered, an uninsurable risk of permanent income loss faced by the household gives rise to precautionary saving motives and, hence, to a Keynesian type of consumption function relating consumption to current income (cf. Carroll 1997, Carroll and Jeanne 2009, Carroll and Toche 2009). Consumption and labor supply are obtained as solutions to an inter-temporal utility optimization problem of the households. The firm's choice of prices, labor demand and investment are derived from an inter-temporal profit optimization problem. In the TPK model, economic behavior is governed by rules. We argue that these rules can be interpreted as rough approximations to the micro-foundation of the DSLMD/DSGE models.

We draw the following main conclusions: First, a post-Keynesian economy can be characterized by a model based on inter-temporal optimization and rational expectations. Orthodox micro-foundation is, to a considerable extent, consistent with the behavioral hypothesis of TPK models. Second, unlike DSGE models, the DSLMD model does not require the interest rate to be equal to the natural rate in the steady state. Nevertheless, the interest rate has to be strictly lower than the growth rate of the economy for a meaningful steady state to exist. Third, the DSLMD model may require a strong monetary policy response to inflation only for low steady-state interest rates. For high interest rates, monetary policy may even be insensitive to changes in the interest rate. Finally, the Keynesian type of models predicts much larger fiscal multipliers than the neoclassical variants. Yet, labor-market feedback on wage formation reduces the expansionary effect of a fiscal policy shock.

The remainder of the paper proceeds as follows: The next section jointly presents the DSLMD, TPK, DSGE and SNC models employed. Since the DSLMD and DSGE models differ only in the labor-market closure, we focus on how the DSLMD/DSGE micro-foundation is related to the TPK/SNC behavioral hypotheses. This section also discusses the aggregation of the model, assumptions regarding expectation formation and model solution strategies. In the third section, we evaluate the models by studying steady state implications, determinacy properties of the solutions and the economic mechanisms characterized by the four models considered. In particular, we analyze how the models predict the economy to evolve after a fiscal policy shock under different wage formation scenarios and monetary policy regimes. The final section draws a few conclusions.

## 2 Motivating the model equations

In terms of micro-foundations the DSLMD and DSGE models considered here are reminiscent of the model with firm-specific capital as proposed by Woodford (2005) and Svein and Weinke (2007, 2009). Yet, in contrast to this literature we assume the household sector to face an uninsurable risk of permanent income loss. This modification does not substantially alter the character of the DSGE model since the nominal wage is still assumed to be accommodating, i.e. labor-market clearing. The DSLMD model is identical to the DSGE model except that the assumption of labor-market clearing is replaced by the assumption of a non-accommodating wage. In particular, we will consider the case of a constant nominal wage and the case of a nominal wage subject to collective bargaining. The SNC and TPK models differ from the DSGE and DSLMD models, respectively, only in the behavioral hypotheses. Nevertheless, we will argue that the postulated rules are not too far away from the DSGE/DSLMD counterparts.<sup>2</sup> In the present paper, we focus on the economic content with only limited use of mathematics.

The economy considered comprises households, a final good firm, intermediate good firms and a policy maker. The capital stock is owned by the household, but managed by the firm. Hence, decisions are made by the firm. Capital is firm-specific and cannot be simply moved to another firm. Hence, there is no spot market for capital services. The population is constant but labor embodied productivity grows at a constant rate.

The complete models are derived step-by-step in Appendix B. Here, we want to briefly characterize the micro-economic problems and discuss how they relate to the Keynesian literature.

### 2.1 Consumption and labor supply

This section discusses how the choices of consumption and labor supply are determined in the models considered. Note that the consumption choice implies the demand for newly issued bonds by the firm sector since we assume the government budget to be balanced. The proposed consumption theory underlying the DSGE and DSLMD framework builds on the precautionary savings model popularized by Carroll (1997), Carroll and Jeanne (2009), Carroll and Toche (2009). The determination of the labor supply follows the convention in the DSGE literature (cf. Smets and Wouters 2003). In the SNC and TPK models, consumption and labor supply are determined similar to the

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<sup>2</sup>Note that all equations of the four models are derived explicitly in Appendix B. Details on the household and firm behavior can also be found in Schoder (2014c). The Dynare codes used for computing the steady-states and the impulse-response functions discussed below can be obtained from the author upon request.

literature on stock-flow consistent modeling (cf. Godley and Lavoie 2012, ch.11). We will argue, however, that the latter rules of choice are rough but not inconsistent approximations of the former.

**The households' problems.** In the DSGE/DSLMD framework, we assume individuals to be born into generations which are constant in size. Each household is born as part of the labor force and supplies labor hours which will be (partly) employed by the firm. We shall call this type of household *active*. Each period, the household may drop out of the labor force with a known probability losing all sources of income (except previous savings) which poses an uninsurable risk. Once the household is *inactive*, i.e. has left the labor force, it cannot return. However, it faces the risk of death with a given probability. As it turns out, the household will accumulate precautionary savings in order to insure against the risk of permanent income loss.

The active household's payoff is utility which increases in consumption and decreases in hours worked. It chooses inter-temporal paths for consumption and labor supply in order to maximize discounted expected life-time utility. The inter-temporal nature of the household's problem arises from the fact that today's saving decision implied by the consumption choice affects tomorrow's income. In particular, real wealth tomorrow is the part saved out of today's wealth and household income plus the interest on it.<sup>3</sup>

In particular, the active household will choose the level of consumption such that the current period's marginal utility of consumption equals the discounted expected marginal utility of consumption in the next period. Yet, this expected marginal utility includes the risk of dropping out of the labor force. Because of the inter-temporal link of consumption today and tomorrow and because of the fact that the active household may become inactive tomorrow, it thinks today about tomorrow's consumption choice for the potential case of an income loss. In this case, consumption would be chosen according to considerations of the inactive household discussed below. Hence, the active household's consumption choice today is affected by the inactive household's consumption choice which may become relevant tomorrow.

We follow Carroll (1997), Carroll and Jeanne (2009), Carroll and Toche (2009) and assume that inactive households who do not receive wage or profit income have access to a perfectly competitive Blanchard (1985) type of insurance market transforming wealth into annuities. Once the inactive household dies, bequests will be transferred to the insurance company which, in turn, distributes this wealth to the inactive households still alive. This modeling device facilitates aggregation as no accidental bequests remain on the aggregate level. It can be shown, that the solution of the inactive household's problem implies consumption to be proportional to real wealth. Since we assume that the active household internalizes this solution of the inactive household's problem, the former's expected marginal utility of consumption discussed above depends on the level of previously accumulated wealth. This is the crucial property of the active household's solution and gives rise to a Keynesian type of consumption function in the steady state.

**Consumption in the DSGE and DSLMD models.** As argued above, the solution for the inactive household's problem implies

$$\tilde{C}_t^i = \kappa \tilde{B}_t^i \tag{1}$$

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<sup>3</sup>As discussed in more detail in Carroll and Jeanne (2009) and Schoder (2014c), we assume a non-distortionary transfer from non-newborn households to newborn households which ensures real wealth to be equal across households and facilitates aggregation.

where  $\tilde{C}_t^i$  and  $\tilde{B}_t^i$  denote for the inactive household detrended aggregate consumption and detrended real wealth in time  $t$ . Note that  $\kappa = 1 - \beta(1 - D)$  with  $\beta$  and  $D$  denoting the discount rate and the death probability respectively. To derive the aggregate budget constraint for the inactive households, note that they do not receive any wage or profit income and that tomorrow's aggregate wealth of inactive households is also contributed to by households which will become inactive tomorrow. Hence, we get

$$\tilde{B}_{t+1}^i = \frac{R_t}{\Pi_{t+1}} \frac{1}{\Gamma} (\tilde{B}_t^i - \tilde{C}_t^i) + U \tilde{B}_{t+1}^a \quad (2)$$

where  $\Gamma$ ,  $U$ ,  $R_t$  and  $\Pi_t$  are the deterministic growth factor of the economy, the probability of permanent income loss, the interest factor and the price inflation factor, respectively. To obtain the aggregate consumption Euler equation for the active households, we substitute (1) into its first order condition with respect to consumption, which then reads

$$\frac{1}{\tilde{C}_t^a} = E_t \beta \frac{R_t}{\Pi_{t+1}} \frac{1}{\Gamma} \left( (1 - U) \frac{1}{\tilde{C}_{t+1}^a} + U \frac{1}{\kappa \tilde{B}_{t+1}^a} \right) \quad (3)$$

where  $E_t$  is the expectations operator. Eq. (3) states that, in the aggregate, the current period's marginal utility of consumption equals the discounted expected marginal utility of the next period. The crucial difference to conventional consumption Euler equations is that the expected marginal costs depends on real wealth. Why is that? Note that, because of (1),  $\kappa \tilde{B}_{t+1}^a$  in (3) is the consumption of a newly inactive household in  $t + 1$ . This is exactly the consumption level the active household expects for the next period with probability  $U$ , i.e. if dropping out of the labor force. Note that (3) collapses to the standard consumption Euler equation if there is no risk of permanent income loss, i.e.  $U = 0$ . Then consumption would be independent of wealth.

Let us now derive the active household's aggregate budget constraint. Note that the overall wealth saved today for tomorrow by active households will be divided in tomorrow's wealth of active households and tomorrow's wealth of newly inactive households. The aggregate budget constraint therefore is

$$\tilde{B}_{t+1}^a = \frac{R_t}{\Pi_{t+1}} \frac{1}{\Gamma} (\tilde{Z}_t + \tilde{B}_t^a - \tilde{C}_t^a) - U \tilde{B}_{t+1}^a \quad (4)$$

where  $\tilde{Z}_t$  is the active household's wage and profit income net of lump-sum taxes.

Let us consider the economy at the steady state, i.e. when  $x_t = E_t x_{t+1} = x$  for any variable  $x_t$ . Then, the two eqs. (3) and (4) feature three variables, i.e.  $C^a$ ,  $B^a$  and  $Z$  knowing that  $R$  and  $\Pi$  are determined elsewhere. Hence, conditional on income, we can compute equilibrium consumption and wealth. Seen from a different angle, we can divide both eqs. (3) and (4) by  $Z^{-1}$  and  $Z$ , respectively. Then we have two equations in the consumption-income ratio and the wealth-income ratio for which the existence of a unique solution can be shown under certain parameter constellations. As we can see, introducing the risk of permanent income loss to the conventional consumer problem implies the existence of an equilibrium consumption-income ratio. With rising income, consumption will increase by a fixed proportion which is very similar to a Keynesian consumption function. The core difference is that in our model, the marginal propensities to consume are endogenous. In particular, they depends on the nominal interest rate and the rate of inflation out of the steady state.

Note that in the conventional case of  $U = 0$  no unique solution for the consumption-income ratio and the wealth-income ratio exists. Even for a given income, consumption is not determined by the household's problem.



**Consumption in the SNC and TPK models.** In the aggregative models no distinction between active and inactive households is made. A behavioral relationship between consumption,  $\tilde{C}_t$ , and income,  $\tilde{Z}_t$ , is typically assumed based on stylized empirical observations (Lavoie 1992, 2014). A common stock-flow-consistent specification proposed by Godley and Lavoie (2012) relates consumption to disposable income and wealth,  $\tilde{B}_t$ , as

$$\tilde{C}_t = c_z \tilde{Z}_t + c_b \tilde{B}_t$$

with the budget constraint

$$\tilde{B}_{t+1} = \frac{R_t}{\Pi_{t+1}} \frac{1}{\Gamma} (\tilde{Z}_t + \tilde{B}_t - \tilde{C}_t)$$

where  $c_z$  and  $c_b$  are the marginal propensities to consume out of income and wealth.

How do the DSLMD and TPK consumption theories differ? As it turns out, combinations of  $c_z$  and  $c_b$  exist such that the consumption-income and wealth-income ratios are the same across the two models at the steady state. To see this, compute the steady states of  $\tilde{C}/\tilde{Z}$  and  $\tilde{B}/\tilde{Z}$ . Note that the TPK/SNC budget constraint is the same as the DSLMD/DSGE budget constraint aggregated over the two household types. Both imply at the steady state that

$$\tilde{B}/\tilde{Z} = \frac{A}{1-A} (1 - \tilde{C}/\tilde{Z})$$

where  $A \equiv \frac{R}{\Pi} \frac{1}{\Gamma}$ . Normalizing the TPK consumption function by income at the steady state, substituting out  $\tilde{B}/\tilde{Z}$  by the budget constraint and solving for  $\tilde{C}/\tilde{Z}$  yields

$$\tilde{C}/\tilde{Z} = \frac{c_z + c_b \frac{A}{1-A}}{1 + \frac{A}{1-A}}.$$

We can now see that for given  $R$  and  $\Pi$  there exists a combination of  $c_z$  and  $c_b$  such that the consumption-income and wealth-income ratios of the aggregative models are equal to the ones of the micro-founded models. Hence, at the steady state the aggregative and micro-founded consumption hypotheses are equivalent.

What about the dynamics out of steady-state? In the aggregative models, these are fully characterized by the constant propensities to consume out of income and wealth. In the micro-founded models, these propensities are endogenous depending on the real interest rate.

**Labor supply.** In the micro-founded models, labor supply will be such that the dis-utility arising from an additional working hour will be equal to the utility this additional working hour allows for by raising income. The active household's problem obviously implies a positive relationship between labor supply and the real wage. In the aggregative models, labor supply is assumed to be constant.

The slope of the labor supply curve, however, is not a crucial property of the labor market for any of the models considered. As long as excess labor supply is associated with a nominal wage higher than at equilibrium, the nominal wage will always adjust to equilibrate supply and demand in the DSGE and SNC models. In the variants of the DSLMD and TPK models with the real wage decreasing in the unemployment rate, stability requires that the unemployment rate increases with the real wage. Again, this holds as long as excess labor supply is associated with a nominal wage higher than the equilibrium wage.

**The ontologies of the micro-founded and aggregative models.** Let us briefly compare the ontologies underlying the consumption and labor supply choice of the TPK and of the DSLMD models. In the former, consumption and labor supply are governed by rules which Schoder (2015) has referred to as *lethargic* as they do not adjust to changes in the economic environment. For instance, a change in the interest rate does not affect the consumption-saving trade-off and a change in the real wage does not affect the labor supply. As argued by Schoder (2015), however, the literature typically assumes households to exhibit *goal-oriented* behavior when it comes to the justification of the consumption and labor supply rule, respectively (Lavoie 1992, 2014). For instance, Lavoie (2014, ch.5.4.1) suggests that households seek to minimize the work load for a given real wage and standard of living which they try to achieve. This is different from the problem of the households in our micro-founded models. Yet, at least, it is the same ontology. With a labor supply independent of the real wage as assumed in most TPK models, Lavoie’s proposal is inconsistent in terms of both economic content and ontology.

In the DSLMD model, households engage in an inter-temporal variant of what Schoder (2015) has called *active choice*. For given expectations about future realizations of the variables relevant for the inter-temporal problem at hand (income, interest rates, inflation rates, unemployment), the payoffs (discounted sum of utilities) of known choice options (range of possible inter-temporal paths for consumption and labor supply) are known and the best option (inter-temporal paths for consumption and labor supply) is chosen. The first element of each of these paths (current consumption and labor supply) is then implemented and the state adjusts until the next period when the household optimizes again.

Due to the inter-temporal nature of the problem, households in period  $t$  have to form  $i$ -periods-ahead expectations  $E_t x_{t+i} \equiv E[x_{t+i} | \Omega_t]$  about future realizations of any variable  $x$  appearing in the optimization problem for  $i \rightarrow \infty$ .  $\Omega_t$  is the information set of the household in  $t$ . Does this imply that expectations are necessarily rational? Even though  $E$  is the mathematical expectations operator, it does not. This is because we have not yet specified what  $\Omega_t$  contains. Hence, expectations could well be *static* or *adaptive* as typically assumed in TPK models and the inter-temporal optimization problem could still be solved. Note, however, that this view on expectation formation is rather strong and lies in stark contrast to Keynes (1936, p.152) who argues that agents assume “that the existing state of affairs will continue indefinitely, except in so far as we have specific reasons to expect a change.” Backward-looking expectation formation ignores completely any information regarding future events which are plausibly contained in  $\Omega_t$ .<sup>4</sup> An equally strong take on expectation formation is to assume *rational expectations*, i.e. that  $\Omega_t$  contains the model and distribution of shocks which the modeler has in mind.

**A behaviorist extension of the household’s problem.** One may agree with the ontology underlying the DSGE/DSLMD framework but may find the specific optimization problem too simplistic. As in Schoder (2014c), the household problem could be extended along the lines of Shefrin and Thaler (1988) to be more realistic. The active household could be assumed to consist of an individual inhabited by two souls: the *doer* and the *planner*. The doer seeks to maximize instantaneous utility by desiring to consume as much as possible without concern about financial or resource constraints. Yet, the planner can enforce willpower which infuses bad conscience to the

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<sup>4</sup>This is the reason why TPK models fail, for instance, in explaining why expected but not yet realized changes in income affect current consumption as documented in the empirical literature (cf. Parker 1999, Souleles 2002, Johnson et al. 2006, Blundell et al. 2008, Shapiro and Slemrod 2009 and Jappelli and Pistaferri 2010).

doers utility function and aims at disciplining the doer.

The doer chooses consumption so as to maximize instantaneous utility which is a function in consumption as well as several variables taken as given by the doer. These include the willpower enforced by the planner, a sensitivity measure by which a given level of willpower generates disutility, and the labor supply of the household. Consumption, willpower and the sensitivity measure affect utility jointly. The higher the level of consumption, the higher is the bad conscience of the doer for a given willpower and sensitivity parameter. What level of consumption does the doer choose? Overall, at low levels of consumption a rise in consumption will increase instantaneous utility for any given level of effective willpower (which is the willpower weighted by the sensitivity measure). Yet, as consumption increases, marginal utility decreases. At some consumption level, the additional bad conscience starts exceeding the direct utility of consumption. Then a rise in consumption will decrease utility. The doer chooses consumption associated with the maximum overall utility.

The willpower sensitivity may depend on many variables such as current income, income or consumption of a reference group, consumption habits or the interest rate. Given the level of willpower enforced by the planner, a rise in current income, for instance, can be expected to reduce the doer's bad conscience of consuming, which can be captured by a decreasing sensitivity measure.

The other inhabitant of the individual, i.e. the planner, knows the doer's consumption choice for any given level of willpower and sensitivity. Internalizing the doer's solution, the planner chooses inter-temporal paths for willpower and labor supply in order to maximize discounted expected lifetime utility. The planner now takes into account not only the inter-temporal budget constraint but also the solution to the doer's problem.

This framework allows for much flexibility in modeling the household's problem. Yet, in order to have our models simple, we do not apply it here.

## 2.2 Prices, labor demand, investment and supply of bonds

The firm sector of our micro-founded framework is similar to DSGE models assuming firm-specific capital as outlined in Woodford (2005) and Sveen and Weinke (2007, 2009). Nevertheless, as we will argue below, it is highly consistent with Keynesian theories of the firm as underlying the SNC and TPK models (cf. Taylor 2004).

In the DSGE/DSLMD framework, there are two types of firms: a perfectly competitive firm aggregating intermediate goods into a final good used for consumption and investment, and a continuum of monopolistically competitive firms producing a differentiated good using capital and labor input. This distinction is used in order to reconcile in a simple way market power in production (due to heterogeneous intermediate goods) and having one single consumption and investment good (due to a homogeneous final good). We further assume capital to be firm specific. It cannot be transferred from one firm to the other and, hence, there is no capital market. Labor is rented from households.

**Final good firm.** A representative final good firm bundles a continuum of differentiated intermediate goods into a final good and sells it on a perfectly competitive market. Taken as given the price of the intermediate good, the elasticity of substitution of inputs given by its technology as well as the overall demand for the final good, its demand for the intermediate good can simply be obtained from cost minimization considerations. The result of this problem is an inverse relationship between the demand for an input and its price for a given output of final goods. This demand

schedule will be assumed to be part of the information set of the intermediate good firm. It will turn out to be important when choosing the optimal price.

**The intermediate good firm’s problem.** There is a continuum of intermediate good firms each producing a differentiated good according to a constant-returns-to-scale Cobb-Douglas production function in capital and labor with labor embodied productivity growing at a deterministic rate.<sup>5</sup> Intermediate goods are sold on a monopolistically competitive market. Facing quadratic adjustment costs the firm purchases investment goods to accumulate capital. These adjustment costs eat up output. We assume Rotemberg (1982) price setting. Price setting is subject to quadratic adjustment costs which are also assumed to destroy output.<sup>6</sup> Taking total output, the overall price level, its capital stock which is predetermined, the nominal wage, the law of motion of capital, the production function, the demand function for intermediate goods and the target debt-capital ratio as given, the firm chooses an inter-temporal path of prices, labor demand, investment and supply of bonds to maximize the discounted sum of expected future distributed profits. The distributed profits are sales minus operating and adjustment costs as well as investment in excess of new bonds.<sup>7</sup>

**Price setting.** What is the optimal price? Without price adjustment costs, the firm would set the price with a mark-up on nominal marginal costs. The mark-up is determined by the elasticity by which the final good firm can substitute intermediate goods to produce a given amount of final goods. Obviously, if the elasticity is high (low), the mark-up will be low (high). With price adjustment costs, prices will be set lower than without. Hence, the imputed mark-up over marginal costs will also decrease. Price adjustment costs ensure that faster wage inflation leads to an under-proportional increase of price inflation and, hence, to a larger real wage. The device of distinguishing between a perfectly competitive final good firm and a monopolistically competitive intermediate good firm allows us to introduce mark-up pricing over marginal costs. The final good firm’s elasticity of substitution between differentiated intermediate goods as inputs is the source of the monopoly power of the intermediate good firms and, hence, the mark-up. This is highly consistent with Kalecki’s (1971) *degree of monopoly* which is typically referred to the Keynesian literature to justify the price mark-up over wage costs.

It is remarkable to note that Kaleckian mark-up pricing can be obtained from our framework by merely assuming that no substitution between capital and labor is possible. Then, marginal costs are proportional to wages and, assuming no price adjustment costs, the Kaleckian mark-up is equal to the imputed mark-up implied by the DSGE/DSLMD models. Two more implications are worth

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<sup>5</sup>Note that the choice of a Cobb-Douglas production function is not crucial for neither the DSGE nor DSLMD model. Any production function with increasing marginal costs in the short run, i.e. at a given capital stock, and constant marginal costs in the long run, i.e. at a fully adjusted capital stock, may be chosen. The production function relates labor input to the output for a given capital stock and a given utilization rate. Causality, however, depends on whether we consider a neoclassical or Keynesian closure. In the former framework, labor demand is determined simultaneously with labor supply by an adjusting nominal wage. Then, output is determined. In the DSLMD framework, output is determined by spending decisions and the production function then implies the labor demand.

<sup>6</sup>We assume price rigidities in the vein of Rotemberg (1982) instead of Calvo (1983), since the former implies that an acceleration of wage inflation increases real marginal costs, and hence the real wage, while the latter does not. Moreover, aggregation is difficult in a model featuring both Calvo price setting and firm-specific capital (cf. Sveen and Weinke 2007, 2009).

<sup>7</sup>Note that we implicitly assume sufficiently large costs of market entry required to keep the number of firms constant despite positive profits in the steady state.

to note: First, introducing price adjustment costs in a conventional firm's profit maximization problem is a viable micro-foundation to obtain a cyclical mark-up which, in traditional Keynesian models of the Kaleckian type, is typically simply postulated (cf. Lavoie 1992, 2014). With stronger wage inflation which moves pro-cyclically in case of labor market tightening feeding back to wage formation, the imputed mark-up of prices over wages decreases. Second, the substitutability as implied by the Cobb-Douglas production function is another source of variation of the imputed mark-up over wages absent in the TPK model.

In the aggregative models, we therefore freeze the labor-capital ratio at the steady-state implied by the micro-founded models which effectively eliminates factor substitution despite a Cobb-Douglas production function and gives rise to a Kaleckian pricing rule.

**Labor demand.** Since the Cobb-Douglas production function allows for input substitution, the labor demand is not proportionally linked to output as it is implied by the TPK model after fixing the labor-capital ratio. With substitution, the optimizing firm will choose labor input such that the marginal revenue product of labor is equal to the marginal cost of labor. The latter is simply the real wage. The former is the marginal product of an additional unit of labor weighted by the marginal cost of a unit of output. To get a better understanding of this result, assume that the real wage is lower than the marginal revenue product of labor. Then, a one-unit rise in labor input would increase output by the marginal product of labor. For a given marginal cost of output, this translates into an increase in costs which exceeds the real wage. Hence, an expansion of labor demand is beneficiary which lowers the marginal product of labor. Note that the marginal product of labor is constant if a production function is assumed which does not allow input substitution which is typically implied by TPK models. Then the relationship between marginal costs of output and the real wage is proportional. Moreover, labor demand is directly linked to the production function in this case.<sup>8</sup>

**Investment and Tobin's  $q$ .** The firm chooses the path of the capital stock taking into consideration the law of motion of capital as well as capital adjustment costs and taking demand as given. The firm's problem is basically the well-known Tobin's (1969)  $q$ -theory of investment. The firm has to consider two questions.

First, in the short-run, what is the optimal response of investment to shocks such as a change in expected sales? The answer lies in the nature of the adjustment costs. Since these are assumed to be quadratic, a sharp adjustment in investment might cause a strong depreciation of the capital stock and, hence, increase costs. A slow adjustment might cause a temporary output-capital ratio which is too high to be cost efficient. As is well known from Tobin's investment theory, the relevant signal for the firm comes from  $q_t$  which is the Lagrangian multiplier for the law of motion constraint in the firm's optimization problem. It measures how much profits the firm would gain by having one more unit of capital installed in the next period. It is the marginal value of an additional unit of capital taking into account capital adjustment costs. The optimality condition for investment then states that, in any period, investment should be chosen such the marginal loss measured in terms of profits due to an one-unit increase in investment is equal to the marginal gain in terms of profits due to an extra unit of capital in the next period implied by a one-unit increase of investment

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<sup>8</sup>Note further that the firm's marginal product of labor depends on its previously chosen capital stock. Hence, the pricing decision is not independent from the capital accumulation decision since capital is not purchased on a spot market.

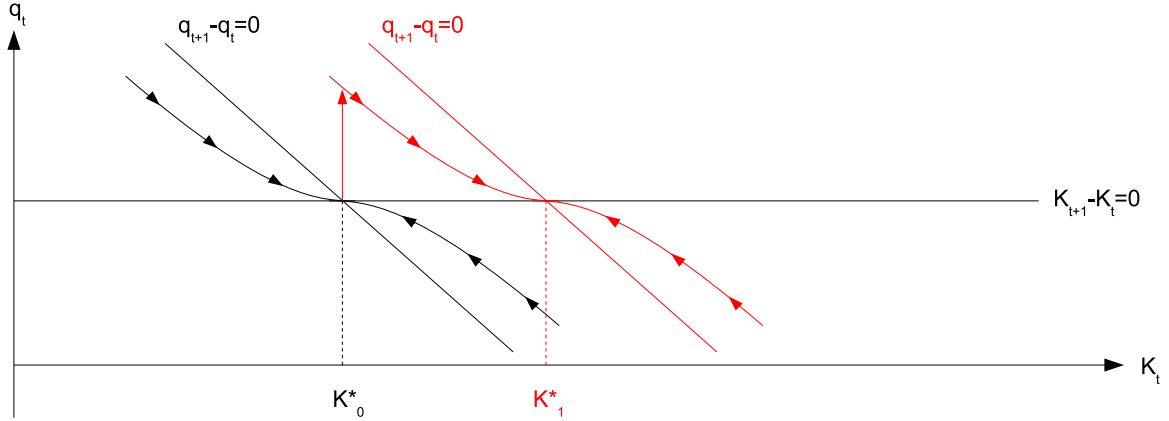


Figure 1: Partial equilibrium investment dynamics

today, i.e.  $q_t$ . Hence, for a given  $q_t$ , the optimality condition for investment tells us what the optimal investment will be in period  $t$ . If  $q_t = 1$ , then the marginal adjustment costs have to be zero which will only be the case when investment only covers depreciation and the capital stock does not change.

Second, in the long-run, what is the optimal capital stock for a given level of output? The answer is implied by the production function. It is the capital stock which minimizes the costs of production, i.e. maximizes profits. In the short-run, the optimality condition for the capital stock requires that the capital stock has to be chosen such that the negative marginal cost of capital, i.e. the marginal revenue product of capital is equal to the opportunity cost of a unit of capital which is the real interest minus capital gains.<sup>9</sup> Given the decision on investment today, which is implied by the optimality condition for investment and today's  $q_t$  and which implies the capital stock for tomorrow, the optimality condition for capital determines tomorrow's  $q_{t+1}$ .

The dynamics are illustrated in Figure 1. The optimality condition for capital implies for the  $k_t$ -nullcline on which  $k_{t+1} - k_t = 0$  and, hence,  $i_t = 0$  that  $q_t = 1$ , where  $k_t$  and  $i_t$  denote the capital stock and investment. In times of  $q_t > 1$ , it is worthwhile to expand the capital stock since the marginal gain in terms of profits from an extra unit of capital exceeds the marginal loss from an extra unit of capital. For points below the curve the reverse holds. The  $q_t$ -nullcline implies an inverse relationship between  $q_t$  and  $k_t$ . For an initial value of output, the steady state capital stock is  $k_0^*$ . Suppose output increases shifting the  $q$ -nullcline to the north.  $q$  jumps upwards immediately causing an expansion of the capital stock converging to a new steady state,  $k_1^*$ .

**Steindl meets Tobin.** Even though the underlying investment theory is based on Tobin's  $q$ , it implies a relation between the rate of capital accumulation and the gap between the current rate of capacity utilization,  $v_t$ , and the so-called *normal* rate of capacity utilization,  $v_t^*$ , which we do not yet assume to be constant. This is a popular behavioral rule for investment in the Keynesian literature (cf. Steindl 1952). The rate of capacity utilization is the ratio between output  $y_t$  and full-capacity output  $y_{c,t}$ . To derive the investment function from Tobin's  $q$  theory, first the notion

<sup>9</sup>Note that in our model the marginal return on capital is not measured by the firm's marginal revenue product of capital due to the absence of a rental market for capital services. Rather, it captures the reduction of nominal labor costs that can be afforded after a one-unit increase in the capital stock in order to produce a given level of output.

of full-capacity output has to be motivated in the context of our firms optimization problem.

Note that the Fed provides data on the rate of capacity utilization as well as its components, production output and full-capacity output. The questionnaire asks: “Full Production Capability - The maximum level of production that this establishment could *reasonably expect to attain under normal and realistic operating conditions fully utilizing the machinery and equipment in place*. In estimating market value at full production capability, consider the following [...] Assume *only the machinery and equipment in place* and ready to operate will be utilized. Do not include facilities or equipment that would require extensive reconditioning before they can be made operable. [...] Assume number of shifts, hours of plant operations, and overtime pay that can be sustained under normal conditions and a realistic work schedule. [...]”

What does this imply for the firms considered here? The maximum production level that can be sustained under normal conditions may be interpreted as the maximum output which still allows profits to be positive for a given capital stock, real wage and interest rate. Hence, we shall define full-capacity output,  $y_{c,t}$ , as the level of output at which average costs equal marginal revenues for a given capital stock, real wage and interest rate and steady-state full-capacity output,  $y_{c,t}^*$ , as full-capacity output with the capital stock, real wage and interest rate at the steady state.

This interpretation is illustrated in Figure 2 depicting marginal revenues, marginal costs and average costs with respect to output. We start at the steady state in time 0. Optimal price setting of the firm implies that the real wage  $\omega_0$  will be such that the marginal costs are equal to marginal revenues at the level of output  $y_0$ . With a given real wage  $\omega_0$  and capital stock  $k_0$ , full-capacity output,  $y_{c,0}$ , is where average costs are equal to marginal revenues. The rate of capacity utilization is equal to the normal rate, i.e.  $v_0^* = y_0/y_{c,0}$ , since the capital stock is fully adjusted. Suppose there is a permanent demand shock with output increasing to  $y_1$  in time 1. With a given capital stock, price setting implies the real wage to fall to  $\omega_1$  such that marginal costs cut marginal revenues at the new level of output. Average costs decrease slightly because of lower real wages for any level of output given the initial capital stock. Hence, full-capacity output increases slightly to  $y_{c,1}$ . Overall, the rate of capacity utilization goes up to  $v_1 = y_1/y_{c,1}$ . The corresponding normal utilization rate is the utilization rate after full adjustment of the capital stock, real wages and the interest rate which is achieved in time 2, i.e.  $v_1^* = y_2/y_{c,2}$  corresponding to the new steady state. A rising capital stock will induce pricing to increase the real wage such that the marginal cost curve remains unchanged from time 1 to 2. The average cost curve however will now cut the marginal revenue curve at a higher level of output due to a higher real wage and a higher capital stock. Overall, the adjustment of the capital stock between time 1 and 2 will be associated with the utilization rate exceeding the normal utilization rate.

Let  $v_t \equiv y_t/y_{c,t}$  and  $v \equiv y/y_c$  where missing time indices indicate steady-state values. Now we assume the relationship between output and capacity output to be always constant at the steady state. The previous considerations imply that a capacity utilization rate exceeding the steady-state capacity utilization rate will be associated with a positive rate of investment. Hence, the firm’s investment behavior can be approximated by

$$\frac{\dot{i}_t}{k_t} = \delta + f(v_t - v)$$

with  $f_v(\cdot) > 0$  and  $f(\cdot) = 0$  for  $v_t = v$ . Note that the real wage (and, therefore, for a given level of productivity also the wage share) as well as the interest rate are captured in the utilization differential through their effect on capacity output. Hence, income distribution as well as monetary policy affects investment through changes in the rate of capacity utilization. Explicitly including

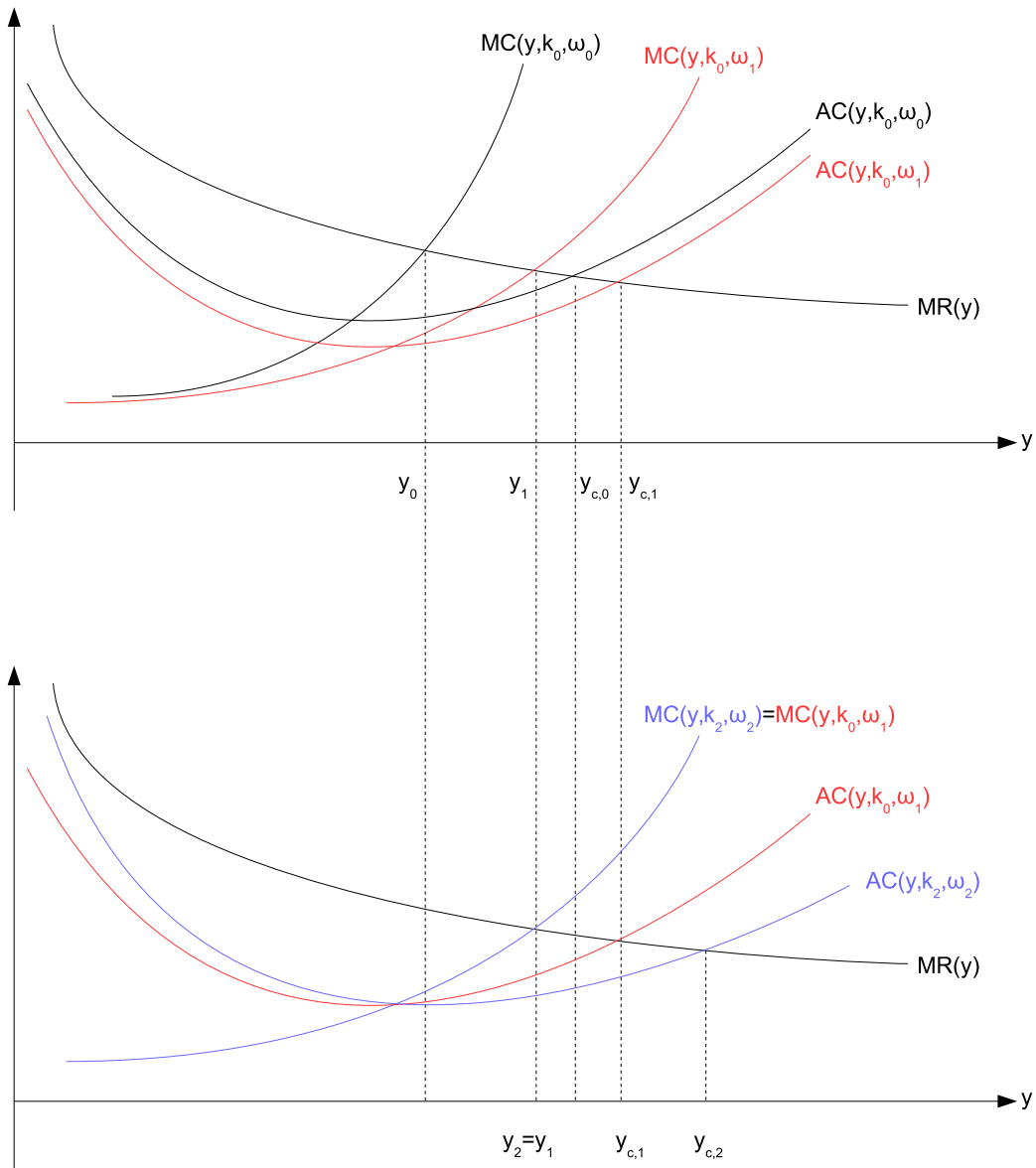


Figure 2: Marginal revenues, marginal costs and average costs after a demand shock in the short run (upper panel) and long run (lower panel)

distribution in the investment function as has become the convention in much of the Keynesian literature since Bhaduri and Marglin (1990) is not required. Note further that the interpretation of the normal rate of utilization is based on cost and profit considerations which differs from the interpretation put forward in parts of the TPK literature which emphasizes the role of idle steady-



state capacity as a means to deter market entry of potential rival firms.<sup>10</sup>

To summarize, the remarkable implication of these considerations is that all relevant information of the firm's investment behavior captured by Tobin's  $q$  is also contained by the utilization differential as long as capacity output is properly defined. Hence, the steady-state rates of investment in the DSGE/DSLMD and SNC/TPK models are the same. Yet, the dynamics out of steady-state differ since, in the former models,  $f(v_t - v)$  is represented by a highly non-linear and dynamic term in Tobin's  $q$  whereas the SNC/TPK specification of investment crudely approximates  $f(v_t - v)$  by a linear function.

**Supply of bonds.** In the DSGE and DSLMD framework, the active household's (creditor) and firm's (debtor) first order conditions with respect to next period's bonds are exactly the same. From the household's and firm's perspective it does not matter whether investment is financed internally or externally which is known as the Modigliani-Miller theorem.

In the conventional DSGE literature without a risk of permanent income loss ( $U = 0$ ), the financial structure of the firm implied by its profit distribution policy, in turn, implied by its supply of bonds, does not make a macroeconomic difference. If the firms decide to increase the supply of bonds, thus financing a larger share of investment externally and leaving more profits for distribution to the households, the households' consumption behavior will not be affected since the additional income will be saved completely.

This is not the case when households face the risk of permanent income loss which implies a relationship between consumption and current income. Any increase in income will trigger a Keynesian multiplier process. Hence increasing the distributed profits by one unit will generally cause the demand for bonds not to increase exactly by one unit. Since the desired financial structure of the firm now makes a macroeconomic difference (even though not a microeconomic one) an assumption has to be made.

An elaborate approach would be to introduce financial frictions along the lines of Bernanke et al. (1999) and Queijo von Heideken (2009). Asymmetric information between debtors and creditors give rise to costly monitoring and bankruptcy costs which increase with financial leverage. An optimal debt-capital ratio exists which is governed by the cost structure assumed. At the steady state this ratio is equal to the desired ratio calibrated according to the empirical counterpart. The advantage of this approach is that the debt-capital ratio varies endogenously over the cycle. The downside for our purpose is that it inflates the model considerably. Hence, we make the simplifying assumption in all models considered that the debt-capital ratio is always at the target.

### 2.3 Policy and market clearing

Fiscal policy involves government consumption and lump-sum taxes. We suppose a balanced budget at all times. Government expenditures are assumed to follow an auto-regressive process.

The monetary authority is assumed to set the interest rate according to the Taylor rule with only inflation stabilization and subject to persistent shocks. Note that the monetary policy rule features a secular interest prevailing at the steady-state. For a given target inflation rate, this secular rate is the interest rate which implies the inflation rate to meet the target in the steady state. Its size and interpretation will depend on the closure of the model, as discussed below.

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<sup>10</sup>Interpreted in the context of the current model, the results obtained by Schoder (2012) for an analysis for US industries suggest that the steady-state rate increases after a positive output shock. That means that the steady-state capacity output increases less-than-proportional to the initial increase in demand.

Our models feature three markets: goods market, bonds market and labor market. Aggregating over the household's budget constraints yields the macroeconomic balance condition stating that output equals expenditures implying the goods market to be cleared. Causality depends on the closure of the model. The bonds market is assumed to clear at all times as bonds issued need to be held by someone. The labor market is cleared in the neoclassical models but not (necessarily) in the Keynesian models.

## 2.4 Model closures

Here, we consider two distinct type of closures of the models: the neoclassical closure assuming labor market clearing through wage adjustment and a Keynesian closure with a non-accommodating nominal wage. In particular we consider two Keynesian closures: first, a constant rate of nominal wage inflation; second, the rate of nominal wage inflation subject to a collective Nash bargaining process between workers' and firms' representatives.

**Neoclassical closure: accommodating nominal wage.** Assuming labor market clearing at all times closes the above model and imposes a general equilibrium. The assumption of labor market clearing lends a neoclassical character to the model. Nominal wage setting ensures that labor supply equals labor demand (even if wage setting rigidities were introduced). Inputs to production are fully employed in equilibrium and output is then determined by the production function. The macroeconomic balance condition now has the interpretation of a resource constraint with total output feeding the demand components. Output is *supply-driven* even in the short run: Along the adjustment to the steady state, e.g. after a fiscal shock, an increase in production can only be achieved since households are willing to provide more labor because of higher real wages. Since the government consumes more output, private consumption will be crowded out.

**Keynesian closure: wage inflation constant or subject to collective bargaining.** The simplest case considered is to assume the rate of nominal wage inflation to be constant. More elaborate is to assume that the rate of wage inflation is subject to a bargaining process between a workers' and a firms' representatives. The respective return functions are crucial for the bargaining game. We take the steady-state real wage,  $\bar{\omega}(\Pi^{\bar{\omega}})$ , as the worker's return and the steady-state profit rate,  $r(\Pi^{\bar{\omega}})$ , as the firm's return. The former can be shown to increase and the latter to decrease in the rate of wage inflation. Hence, we suggest that the bargaining parties are concerned with the long-run implications of the bargaining. Nevertheless, the bargaining game is affected by the short run by assuming that the state of the labor market determines the relative bargaining power.

We consider a Nash solution to the bargaining problem which is the rate of wage inflation  $\Pi_t^{\bar{\omega}^*}$  solving the joint maximization problem

$$\max_{\Pi_t^{\bar{\omega}^*}} [\tilde{\omega}(\Pi_t^{\bar{\omega}^*})]^{v_t} [r(\Pi_t^{\bar{\omega}^*})]^{1-v_t}$$

with

$$v_t = (1 - u_t)\nu V_t \tag{5}$$

where  $\nu$  is a scaling parameter and  $V_t$  is an auto-regressive shock to the bargaining power. The first order optimality condition of this problem characterizes the evolution of the desired rate of

wage inflation,  $\Pi_t^{\tilde{w}^*}$ , i.e.

$$1 = (1 - 1/v_t) \frac{\tilde{\omega}(\Pi_t^{\tilde{w}^*}) r'(\Pi_t^{\tilde{w}^*})}{r(\Pi_t^{\tilde{w}^*}) \tilde{\omega}'(\Pi_t^{\tilde{w}^*})} \quad (6)$$

The evolution of the rate of wage inflation is then assumed to be

$$\Pi_t^{\tilde{w}} = \rho_w \Pi_{t-1}^{\tilde{w}} + (1 - \rho_w) \Pi_t^{\tilde{w}^*}. \quad (7)$$

## 2.5 How to solve the models?

Both the DSGE and the DSLMD model are highly non-linear. To study their characteristics, the model dynamics need to be approximated in the neighborhood of the steady state. Approximation is done by computing the steady state followed by a first-order Taylor expansion around the steady state. The resulting log-linearized first order dynamical system is then solved assuming rational expectations (DSGE and DSLMD models) and static expectations (SNC and TPK models).

Once the steady state is computed, a log-linearized dynamical model can be obtained which we would like to express in auto-regressive state-space form. To do so, let us collect all log-linearized *state* or *predetermined* variables in a  $(n \times 1)$  vector  $X_{1,t}$ . The realizations of these variables are known before the stochastic elements of the model have been realized. To be precise, a *predetermined* variable is a function of only variables being part of the full information set in time  $t$ , hence  $E_t X_{1,t+1} = X_{1,t+1}$ . For instance, the capital stock or wealth in period  $t + 1$  are known already in period  $t$  and are independent of economic choices or shocks in period  $t + 1$ . We also collect all log-linearized *jump* or *forward-looking* variables in a  $(m \times 1)$  vector  $X_{2,t}$ . This vector includes all variables whose realizations are known only once the stochastic shocks have been realized. Hence, a forward-looking variable can be a function of any variable in the information set in  $t + 1$ . These variables include e.g. consumption, Tobin's  $q$ , inflation, etc. Finally, we collect all exogenous variables in a  $(k \times 1)$  vector  $V_t$ . The log-linearized model can then be compactly represented as

$$A \begin{bmatrix} X_{1,t+1} \\ E_t X_{2,t+1} \end{bmatrix} = B \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + C V_t$$

where the  $(n+m) \times (n+m)$  matrices  $A$  and  $B$  as well as the  $(n+m) \times k$  matrix  $C$  collect the model parameters. Note that the expectation operator is not required for  $X_{1,t+1}$  since these variables are known in  $t$ . Note further that so far we have not assumed rational expectations. Each line of the system of equations represented above corresponds basically to a model equation. A solution of this system of equation which includes forward-looking variables is a characterization of the model variables in only predetermined variables since the forward-looking values are not known. One simple way to solve the model is to assume rational expectations, i.e.  $E_t X_{2,t+1} = E_t(X_{2,t+1} | \Omega_t)$  where  $\Omega_t$  is the information set in time  $t$  which includes at least all past and current values of  $X_1$ ,  $X_2$  and  $V$ . In this case, i.e. when we assume that each economic agent knows the entire model, we can pre-multiply both sides by  $A^{-1}$  to obtain

$$\begin{bmatrix} X_{1,t+1} \\ E_t X_{2,t+1} \end{bmatrix} = F \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + H V_t \quad (8)$$

where  $F = A^{-1}B$  and  $H = A^{-1}C$ . Note that one could also assume different ways of expectation formation in this type of models such as adaptive expectations (cf. Sidrauski 1967). In the SNC and

TPK model variants, expectations are assumed to be backward-looking. In particular, we assume  $E_t X_{2,t+1} = X_{2,t}$ . In this case, (8) can be solved easily. In the DSLMD and DSGE models, however, we assume rational expectations. We want to show that neoclassical results can be produced even with backward-looking expectations and that Keynesian results can be produced even with rational expectations.

A solution  $(X_{1,t}, X_{2,t})$  of the model is a sequence of functions of variables in the information set  $\Omega_t$  which is consistent with (8). Blanchard and Kahn (1980) shows how to derive such a solution which shall be skipped here. Note that a unique solution only exists if the number of eigenvalues of  $F$  outside the unit cycle is equal to the number of forward-looking variables, an assumption which holds in both of our models with the calibration chosen.

### 3 Model evaluation

#### 3.1 Calibration

In order to simulate the responses to macroeconomic shocks, the models need to be calibrated.<sup>11</sup> To have our four models as comparable as possible, any parameters appearing in any two models are calibrated to the same value. Hence, the DSGE and DSLMD models as well as the SNC and TPK models are calibrated exactly the same. Calibration follows loosely Smets and Wouters (2003), Carroll and Jeanne (2009), Schoder (2014c,a). Yet, to ensure that for a given set of parameters a steady-state as well as a unique stable solution exist for all four models, the discount rate (0.998) is somewhat higher than in the literature.

The parameters of the aggregative models which are not implied by the steady state of the micro-founded models are the marginal propensities to consume in the consumption function,  $c_z$  and  $c_b$ , and the utilization elasticity of investment,  $\phi_i$ . We set  $c_b = 0.003$ . Then,  $c_z$  is implied by the steady state consumption-income and wealth-income ratios as argued above. The calibration of  $\phi_i$  is crucial for comparing the out-of-steady-state dynamics across models. What  $\phi_i$  allows the aggregative models to mimic the investment behavior of the micro-founded models as close as possible? Total differentiation of the investment function (spelled out in Appendix B) w.r.t. a one-unit increase in a firm's sales and solving for  $\phi_i$  yields  $\phi_i = \Delta i_t / \Delta y_t \Delta y_{c,t} / \Delta k_t$  with  $\Delta y_t = 1$ . Hence,  $\phi_i$  measures how investment responds to a permanent one-unit increase in sales after capacity output and capital have adjusted. We can ask the same in the Tobin's  $q$  framework: How does a permanent one-unit increase in sales affect investment, capacity output and capital after full adjustment? From simulation we get  $\Delta i_t$ ,  $\Delta y_{c,t}$  and  $\Delta k_t$  which we use to compute  $\phi_i = 0.035$ .

All four models share the same steady state. Note that this implies that the Keynesian models do not feature unemployment either in the steady state. The labor supply scaling parameter  $\psi$  is calibrated such that a wage inflation rate of zero is associated with a real wage that clears the labor market. While such a parameter restriction is a necessity in the DSGE/SNC models, it is a special case in the DSLMD/TPK models which we nevertheless consider since we are interested in the out-of-steady-state dynamics rather than the steady states themselves.

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<sup>11</sup>A list of variables and parameters as well as a description and calibration can be found in Appendix A. The equations characterizing the four models considered are discussed in Appendix B.

### 3.2 Steady-state implications for the interest rate

**Is the steady-state interest rate natural?** Let us briefly consider a conventional DSGE model by setting  $U = 0$ . Then, the solution to the consumption problem collapses to the standard Euler equation. The model is then characterized by a set of  $(n+m)$  independent equations with the same number of variables. Does a steady state exist? In general, it does not. For a steady state to exist, the target interest rate in the monetary policy function needs to be consistent with the interest rate required in the consumption Euler equation in order to have constant marginal utilities over time for a given target inflation rate. If this is not the case, then the monetary authority will not be able to settle on an inflation rate which is consistent with its target. A steady state will not exist in general. Hence, what needs to be assumed is that the target interest rate is equal to the *natural* rate of interest which equalizes marginal utilities across time. At the natural rate of interest the monetary policy rule implies that both price and wage inflation are consistent with target inflation.

Let us assume that the target interest rate is at the natural level. To obtain the corresponding DSLMD model, let us now drop the assumption of zero unemployment. We are now short of an equation. A naive approach would be to assume an exogenous wage inflation and take that as an additional equation. Obviously one would have to additionally assume a parameter restriction on the inflation target of the monetary authority in order for it to be consistent with the exogenous wage inflation rate. Unfortunately, this additional equation does not add any information to the system. This is because the interest rate being at its natural level implies that the target inflation rate could be any number. The natural rate is conditional on the inflation target. To see it from a different angle, let the zero unemployment equation still be in place. Assuming additionally that the nominal wage inflation rate is exogenous and consistent with the target inflation rate does not over-identify our system.

Do we have to add yet another equation if we drop the zero unemployment restriction and assume an exogenous rate of wage inflation? No, it will be sufficient to introduce the risk of permanent income loss ( $U > 0$ ) and thus adding the same number of variables and equations to the system (see (1)-(4)). Then, the steady state will be identified even though we drop the zero unemployment restriction and assume constant wage inflation. To see this, recall that in the conventional DSGE model with zero unemployment the target interest rate needs to be consistent with only the household's Euler equation which alone implies the natural rate for a given inflation target. Now, assume  $U > 0$ . How does the restriction on the target interest rate change? With a risk of permanent income loss, the steady state implies the current detrended marginal utility of the active household to be equal to the probability-weighted average of the next period's marginal utility of the active and newly inactive household. Note that in contrast to the conventional DSGE model this does not yet imply the natural rate conditional on the inflation target. The reason for this is that, in contrast to conventional DSGE model, the interest rate affects the marginal utility of the newly inactive household directly since its FOC tells us that consumption depends on previous savings which are obviously affected by the interest rate through the budget constraint. Technically speaking, while the natural rate conditional on the inflation target is implied by a single Euler equation in the DSGE model with  $U = 0$ , it is implied by almost the entire system of equations in the DSGE model with  $U > 0$ . The natural rate which the target rate and, hence, the steady-state rate have to be equal to is such that, among others, the labor market clears in the steady state. Does making the additional assumption of exogenous wage inflation now over-identify the precautionary-saving DSGE model? No, it does not as the natural rate is still conditional on the

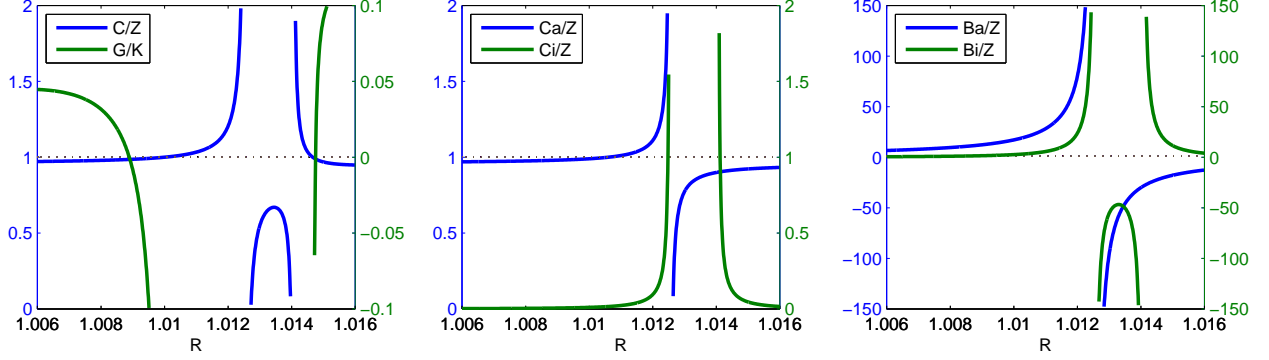


Figure 3: Steady-state values of the government consumption-capital ratio, consumption-income ratios and wealth-income ratios for different steady-state interest rates.

target inflation rate. To obtain the DSLMD model, we can drop the zero unemployment restriction and assume an exogenous wage rate. Again, we are short of one equation. Put differently, the steady state is now conditional on one variable such as the interest rate. We can take the assumption of an exogenous target interest rate as the missing equation closing the system. The steady-state rate will be still equal to the target rate but in general different from the natural rate which clears the labor market. While in the precautionary-saving DSGE model the target interest rate necessarily equals the natural rate, equality is coincidental in the DSLMD model. The only remaining question is why we could not take the assumption of an exogenous interest rate as an additional equation above when we considered the DSLMD model with  $U = 0$ . The answer is that without a precautionary saving motive the steady state is *not* conditional on the interest rate. The interest rate is implied by the household's Euler equation and not by the entire system.

Note that instead of  $R$  we calibrate the labor supply scaling parameter  $\psi$  such that the labor market clears in the steady state in all model variants. The additional degree of freedom is used to calibrate  $R$  freely. Note that  $R$  is still the natural rate.

**Interest rate restrictions.** Despite the endogenous adjustment of  $\psi$ ,  $R$  cannot be set arbitrarily. In general, calibration has to take into account two parameter restrictions in order for a meaningful steady-state to exist. These follow from the precautionary saving motive and are absent in conventional DSGE models: a positive growth-interest rate differential and a low debt-capital ratio.

The positive growth-interest rate differential requires  $R$  to be lower than the deterministic per-capita growth rate, i.e.  $R < \Gamma$ , which is illustrated in Figure 3 for the calibration discussed above. The blue line in the first panel depicts the consumption-income ratio  $\tilde{C}/\tilde{Z}$  at the steady state for different  $R$ . The second and third panels repeat this exercise for the consumption-income and wealth-income ratios for active and inactive households. To get the intuition behind the  $R < \Gamma$  condition it is important to understand why the consumption-income and wealth-income ratios increase with  $R$  when  $R$  is sufficiently low. It is helpful to evaluate (1)-(4) conditional on  $\tilde{Z}$  at the steady state. First, note that the inactive household's FOC implies  $\tilde{C}^i$  to be proportional to  $\tilde{B}^i$ . Second, the inactive household's budget constraint implies that  $\tilde{B}^i$  is proportional to  $\tilde{B}^a$  with the proportionality factor increasing in  $R$ . Third, the active household's budget constraint implies  $\tilde{B}^a$  to be proportional to  $\tilde{Z} - \tilde{C}^a$  with the proportionality factor increasing in  $R$ . Finally, the

active household's FOC implies  $\tilde{B}^a/\tilde{C}^a$  to increase with  $R$  as it will be beneficial to move some additional consumption to the future. Now we can put the pieces together: A rise in  $R$  will cause  $\tilde{B}^a/\tilde{C}^a$  to go up. Since the interest rate induced rise in  $\tilde{B}^a$  through the active household's budget constraint is rather strong,  $\tilde{C}^a$  will have to rise, too, which mitigates the rise in  $\tilde{B}^a/\tilde{C}^a$  directly and through  $\tilde{B}^a$ . Hence, the consumption-income and wealth-income ratios of the active households will increase with  $R$ . As a consequence, the consumption-income and wealth-income ratios of the inactive households will increase as well. For  $R = 1.01$ , which corresponds to the growth rate  $\Gamma$ ,  $\tilde{C}/\tilde{Z}$  can be shown to equal unity. Any  $R > 1.01$  implies at least one of the steady-state ratios to be meaningless.

Moreover, a low debt-capital ratio is required to obtain a positive capital stock  $\tilde{K}$ . To derive this condition normalize the macroeconomic balance condition by the capital stock to obtain  $\tilde{Y}/\tilde{K} = \tilde{C}/\tilde{K} + \tilde{I}/\tilde{K} + \tilde{G}/\tilde{K}$ . What do we know about these ratios in the steady state? Note that  $\tilde{Y}/\tilde{K}$  and  $\tilde{I}/\tilde{K}$  can be obtained from the firm's optimization considerations alone.  $\tilde{C}/\tilde{K}$  can be extended to  $\tilde{C}/\tilde{Z}\tilde{Z}/\tilde{K}$  with  $\tilde{Z}/\tilde{K} = (\tilde{Y}/\tilde{K} - (1 - \lambda)\tilde{I}/\tilde{K} - \tilde{T}/\tilde{K})$  where  $\lambda$  is the debt-capital ratio and  $\tilde{T} = \tilde{G}$  are taxes. Government expenditures,  $\tilde{G}$ , are exogenous.  $\tilde{C}/\tilde{Z}$  is implied by the households' problems alone. We can substitute these steady state values into the balance condition and rearrange to get

$$\tilde{I}/\tilde{K} + \tilde{G}/\tilde{K} = \tilde{Y}/\tilde{K} - \left( \tilde{C}/\tilde{Z}\tilde{Y}/\tilde{K} - \tilde{C}/\tilde{Z}(1 - \lambda)\tilde{I}/\tilde{K} - \tilde{C}/\tilde{Z}\tilde{G}/\tilde{K} \right)$$

where only  $\tilde{G}/\tilde{K}$  is not yet determined. The left hand side collects all demand injections which happen to be independent from the output-capital ratio at the steady state. The right hand side collects all leakages and represents aggregate saving which increases with the output-capital ratio. At what output-capital ratio are injections equal to leakages? The answer depends on the value of  $\tilde{K}$ . In fact,  $\tilde{K}$  will be such that aggregate leakages and aggregate injections align to each other at  $\tilde{Y}/\tilde{K}$  for given  $\tilde{I}/\tilde{K}$ ,  $\tilde{C}/\tilde{Z}$ ,  $\tilde{G}$ , and  $\lambda$ . Solving the above equation for  $\tilde{G}/\tilde{K}$  reveals that  $\tilde{G}/\tilde{K} > 0$  (and hence  $\tilde{K} > 0$ ) requires

$$\tilde{Y}/\tilde{K} - \left( 1 - \tilde{C}/\tilde{Z}(1 - \lambda) \right) / \left( 1 - \tilde{C}/\tilde{Z} \right) \tilde{I}/\tilde{K} > 0.$$

Only when this condition is met the capital stock ensuring that injections and leakages intersect at  $\tilde{Y}/\tilde{K}$  will be positive. Note that  $\lambda$  cannot be too high for the condition to hold. Also, the lower  $\tilde{C}/\tilde{Z}$  the larger is the term on the left hand side. Since  $\tilde{C}/\tilde{Z}$  rises with  $R$ , a low  $R$  is beneficial for the condition to hold. The terms on the left hand side are plotted in the first panel of Figure 3 as a function of  $R$ . Note that with  $\lambda > 0$ ,  $R$  needs to be strictly smaller than  $\Gamma$ .

### 3.3 Equilibrium determinacy

If the number of unstable eigenvalues of  $F$  is lower than the number of forward-looking variables, an infinite number of equilibria exists and the model solution is indeterminate. If it is larger, then no stable solution exists. In conventional DSGE models an over-proportional response of the monetary authority's interest rate to a rise in the inflation rate, i.e. the so-called *Taylor principle*, typically is sufficient to ensure determinacy, i.e. that the number of unstable eigenvalues equals the number of forward-looking variables. To understand this, assume the Taylor principle not to hold. Let inflation expectations be hit by a sunspot, i.e. a shock which is unrelated to fundamentals of inflation. Monetary policy responds with only a weak rise in the interest rate. The real interest rate will fall stimulating economic activity causing marginal costs to rise and inflation

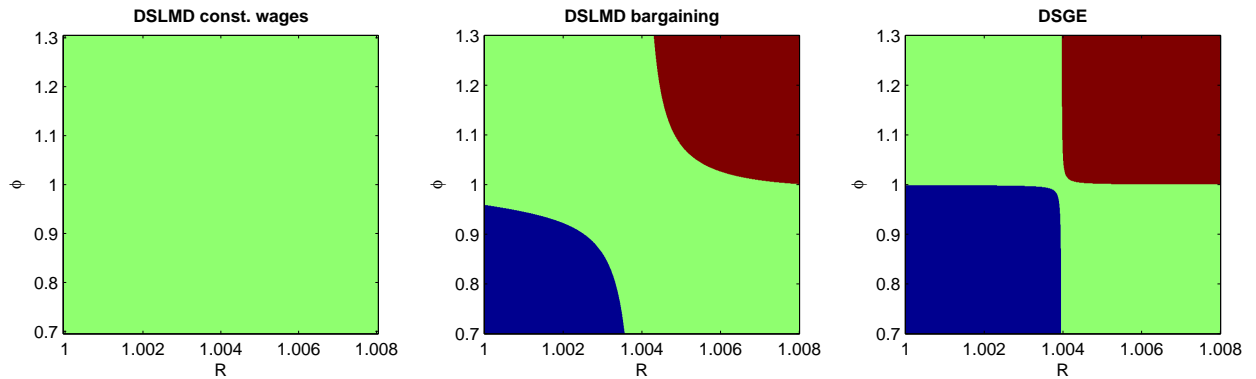


Figure 4: Solution properties for combinations of  $\phi$  and  $R$ : indeterminacy (blue), determinacy (green), and non-existence (red).

to accelerate. Hence, a rise in expected inflation unrelated to its fundamentals initiated a self-fulfilling rise in observed inflation. If the Taylor principle holds, the rise in inflation expectations will induce the nominal interest rate to rise over-proportionally causing the real interest rate to fall and inflation to decrease. Hence, following this principle, the central bank can avoid initiating unnecessary macroeconomic fluctuations.<sup>12</sup>

Let us briefly analyze the solution properties of our models, i.e. the relation between unstable eigenvalues and forward-looking variables for different parameter combinations. Since monetary policy rules are designed to limit macroeconomic fluctuations, we ask what response of the interest rate to a change in inflation,  $\phi$ , is required for the solution to be determinate, at given values of the other parameters. Figure 4 illustrates, for the DSLMD models with constant wages and wage bargaining as well as for the DSGE model, the solution properties for  $\phi = [0.7, 1.3]$  and the steady-state gross interest rate  $R = [1, 1.008]$ . The remaining parameters are calibrated as discussed above. The blue, green, and red areas indicate combinations of  $\phi$  and  $R$  implying the rational expectation equilibrium to be indeterminate, determinate, and non-existent, respectively.

The results are remarkable. In the DSLMD model with constant wages, the monetary policy (MP) response to inflation does not affect the solution properties for different steady-state interest rates. Equilibrium is always determinate. Macroeconomic adjustment implies that a sunspot rise in the expected inflation rate has contractionary effects on the economy regardless of the monetary authority's response. Hence, there is no stabilizing role for monetary policy. In the DSLMD model with a labor-market feedback on the wage formation as well as in the DSGE model, however, passive (active) monetary policy associated with low (high) steady-state interest rates lead to indeterminacy (non-existence). In both models, there exists a threshold interest rate target beyond which one eigenvalue becomes unstable. Hence, there are two monetary policy regimes: For low  $R$  the monetary authority can avoid sunspot equilibria by responding actively to inflation. Yet, for high  $R$  monetary policy needs to be passive for an equilibrium to exist. This is in stark contrast to conventional DSGE models which require sufficiently aggressive monetary policy for the equilibrium to be unique.

Note that the higher the price rigidity, the smoother are the edges of the blue and red regions.

<sup>12</sup>Svein and Weinke (2007) show that the Taylor principle fails to be a sufficient condition for determinacy in case of firm-specific capital as assumed in the DSGE model considered here as well as price and wage rigidities.



Also, for a given level of price rigidity, the connection between the active and passive MP regimes is broader in the DSLMD than in the DSGE model.

### 3.4 Impulse-response analysis of a budget-neutral fiscal policy shock

This section contrasts the proposed DSLMD model with the TPK, SNC and DSGE models.<sup>13</sup> We study the model predictions of the macroeconomic effects of a budget-neutral fiscal policy shock. We analyze how the monetary policy regime and the wage formation process influence the transmission mechanism.

**Comparing the baseline models.** The impulse-response functions (IRFs) for a temporary but persistent 1% budget-neutral government spending shock are plotted in Figure 5. We assume for the sake of simplicity that lump-sum taxes increase by the same amount as government expenditures in order to keep the public budget balanced. In the Keynesian models we constant wage inflation as the baseline. Let us first collect some remarkable observations with the underlying mechanism becoming clear below: First, the multiplier on output is larger for the Keynesian models than for the neoclassical models which due to strong crowding-out of consumption and particularly investment in the latter. Second, nominal adjustment is more pronounced in the neoclassical models than in the Keynesian models. Third, the IRFs of the DSLMD and TPK models are remarkably similar.

In the DSGE model, the impact multiplier of the fiscal expansion is roughly 1.5.<sup>14</sup> The positive short-run output multipliers are due to the crowding-out of consumption. With a lower level of consumption, its marginal utility increases. Hence, households are willing to supply more labor. This affects output through the supply side. The output expansion increases marginal costs which, given the mark-up, pushes inflation above the monetary authority's target which, in turn, increases the interest rate. The jump in the interest rate reduces investment. Note that the nominal wage adjusts immediately to clear the labor market and unemployment stays at zero. Capacity utilization goes up but does not feed back into the model.

Note that consumption adjusts back to the steady state in a sluggish way which is in stark contrast to conventional DSGE models without consumption habits. Our DSGE model predicts such a slow adjustment even after a one-time fiscal policy shock. The reason can be found in the precautionary saving motive. The budget neutral fiscal policy shock immediately reduces income for the active household. Without the risk of permanent income loss, the rational household would simply borrow to smooth out consumption completely. With the risk of becoming inactive tomorrow, the active household will be hesitant to reduce too much the accumulated wealth to compensate for the temporary drop in income. After all, the household may drop out of the labor force tomorrow. In this case its future consumption will depend on its savings. Hence, the rational active household will choose a consumption path to return to its target wealth-income ratio which optimally solves the trade-off between dis-saving to compensate partly the temporary drop in income and maintaining a buffer-stock of wealth to hedge against the risk of a permanent income loss.

In the SNC model, the impact multiplier is 0. The rise in government spending is crowding out consumption and investment to the very same extent. With a predetermined capital stock and a

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<sup>13</sup>The Matlab codes can be obtained from the author upon request.

<sup>14</sup>The initial increase in government expenditures amounts to 0.01 units which is 1% of the steady-state government spending of 1. The impact effect in the DSGE model is 0.5% of the steady state output of roughly 3 which amounts to 0.015 unit increase in output. Hence the impact multiplier is  $0.015/0.01=1.5$ .

given labor supply, output is given. The more the government consumes, the less is available for consumption and investment. Depreciation reduces the capital stock with investment below break-even which explains why output falls persistently below steady state. Because of high demand for labor, the real wage goes up causing a boost in marginal costs, price inflation and the interest rate which reduces Tobin's  $q$  and investment.

In the TPK model, the rise in budget-neutral government spending immediately transfers funds from household's which save part of their income to the government which spends all of it. Further, a jump in capacity utilization causes an increase in investment and hence output and consumption through the accelerator effect. Hence, the impact multiplier is around 3. The impact effect on disposable income is low, which then increases quickly due to the multiplier and phases out again. This explains the rise in consumption. The rise in investment triggered by a rise in the rate of capacity utilization adds to the multiplier effect. The nominal wage remains constant by assumption despite a drop in unemployment.<sup>15</sup> Average costs which in the TPK model equal marginal costs remain unchanged causing the price inflation rate and the interest rate to be constant. The entire adjustment runs through changes in quantities rather than prices.

The mechanisms at work in the DSLMD model are very similar to the TPK model. The crucial difference is that behavioral relations are endogenous in the former. For instance, while the marginal propensities to consume out of income and wealth are invariant to the government shock in the TPK model, they are fully endogenous in the DSLMD model and depend on the interest rate and the inflation rate. Further, while the response of investment to a utilization gap is exogenous in the TPK model, it is endogenous in the DSLMD and DSGE models and represented by Tobin's  $q$ . Similar to the TPK model, the budget-neutral fiscal expansion implies an impact effect on output of around 3. Again the impact effect is due to the transfer of partly saved funds to completely spent funds. In contrast to the TPK model consumption expands in a hump-shaped form since since a rising real interest rate impedes the multiplier effect on consumption. Higher sales raise Tobin's  $q$  despite a higher real interest rate and let investment jump upwards. As output expands and marginal costs increase, prices go up reducing the real wage at a given nominal wage. This reduces the labor supply. Nevertheless, unemployment drops less than in the TPK model. This is because labor is increasingly substituted by capital in the production process.

**The role of the wage formation process.** Above, the rate of wage inflation was assumed to be given, which is consistent with many TPK models taking distribution as exogenous (cf. Bhaduri and Marglin 1990, Stockhammer 5 06, Hein 2007). Here, we discuss how the model dynamics in response to a persistent budget-neutral fiscal expansion change if the wage inflation rate is assumed to be subject to collective bargaining with the relative bargaining power depending on the rate of unemployment. We limit our analysis to the DSLMD model and the case of  $\rho_w = 0$ , i.e. full and immediate adjustment of the nominal wage inflation to labor market conditions. The IRFs are plotted in Figure 6 for the DSLMD models with collective wage bargaining and constant wage inflation (as before). The core difference is that the reduction in unemployment triggered by the fiscal expansion now strengthens the workers' bargaining power. This causes the nominal wage inflation to jump upwards. Marginal costs and, hence, prices and the interest rate, increase more strongly than in the case of constant wages. With a higher real interest rate Tobin's  $q$  now turns negative causing investment to drop and recover slowly. Consumption also drops which is due to

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<sup>15</sup>Note that the fact of zero unemployment in the steady state is of no concern. In a linearized model, the out-of-steady-state dynamics are independent of the steady state.

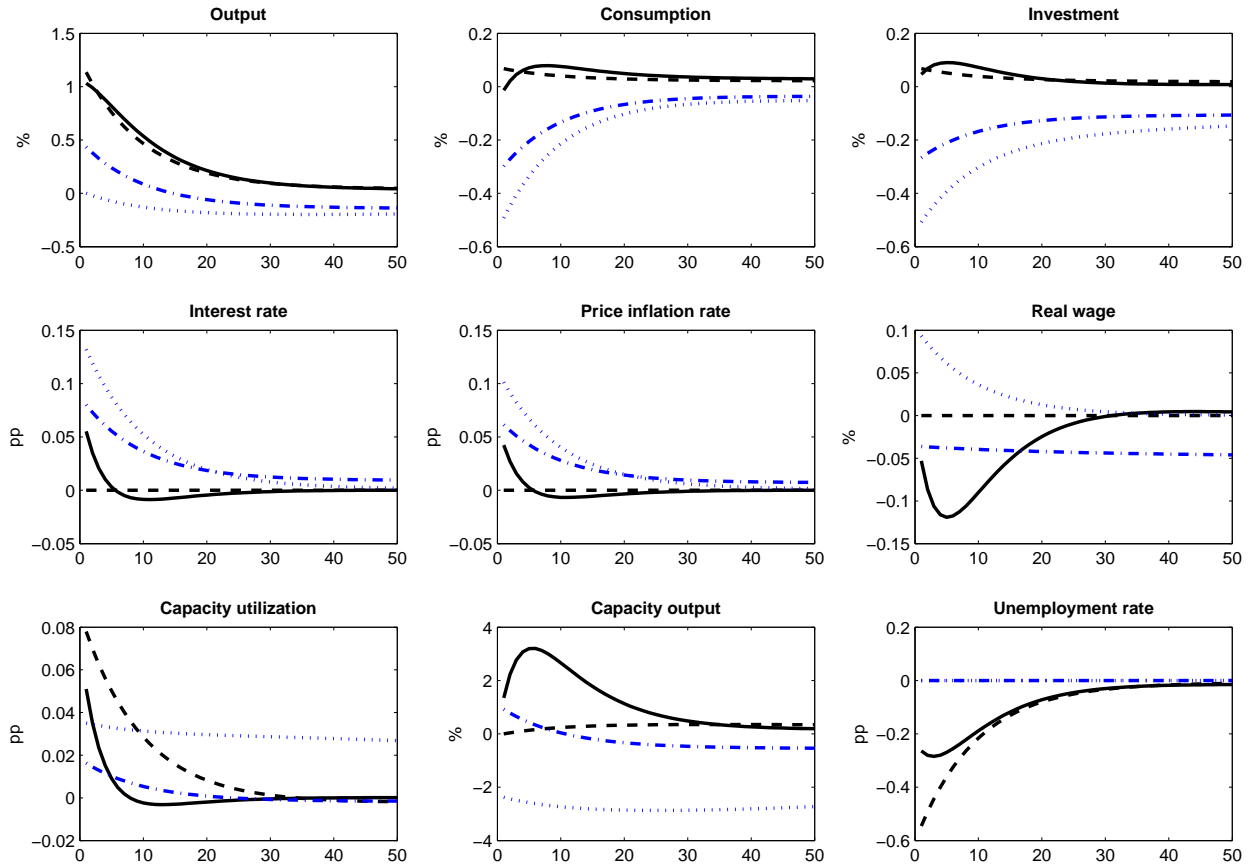


Figure 5: Responses to a 1% persistent budget-neutral government spending shock in the DSLMD (black-solid) and TPK (black-dashed) models with constant wage inflation as well as in the DSGE (blue-dashed-dotted) and SNC (blue-dotted) models as deviations from the steady state.

the jump in the interest rate and drop in disposable income. As discussed above in the context of the DSGE model, active households respond by reducing consumption in order not to use up too much of the buffer-stock savings. Overall, the output multiplier of a fiscal expansion is now lower.<sup>16</sup>

<sup>16</sup>Empirically, there seems to exist strong evidence for Goodwin-type of cycles with the wage share following utilization which has been observed for the US by Barbosa-Filho and Taylor (2006) and Zipperer and Skott (2010) and for European economies by Flaschel (2009). This is because high rates of utilization implying low unemployment and strong trade unions tend to cause profit margins to go up rather than down (cf. Steindl 1979, Kurz 1994). Our DSLMD model is able to generate such a cyclical adjustment to shocks under certain parameter constellations and with a strong persistence in the evolution of nominal wage inflation ( $\rho_w = 0.95$ ). Introducing persistence in the wage formation process causes the economy to readjust to the steady state in cycles. In this case, expansionary fiscal policy immediately reduces unemployment which, now, does not immediately trigger an upward adjustment of the wage inflation rate. Rather, it increases only gradually. Price inflation rises causing the real wage to decrease contributing to the boom phase. Wage inflation accelerates eventually raising the real wage and cutting into consumption and investment. Output starts to fall with real wages rising further and causing output to undershoot at the trough. These *profit-squeeze* dynamics are equivalent to the findings of Taylor (2012) and Schoder (2014b) using aggregative Keynesian models.

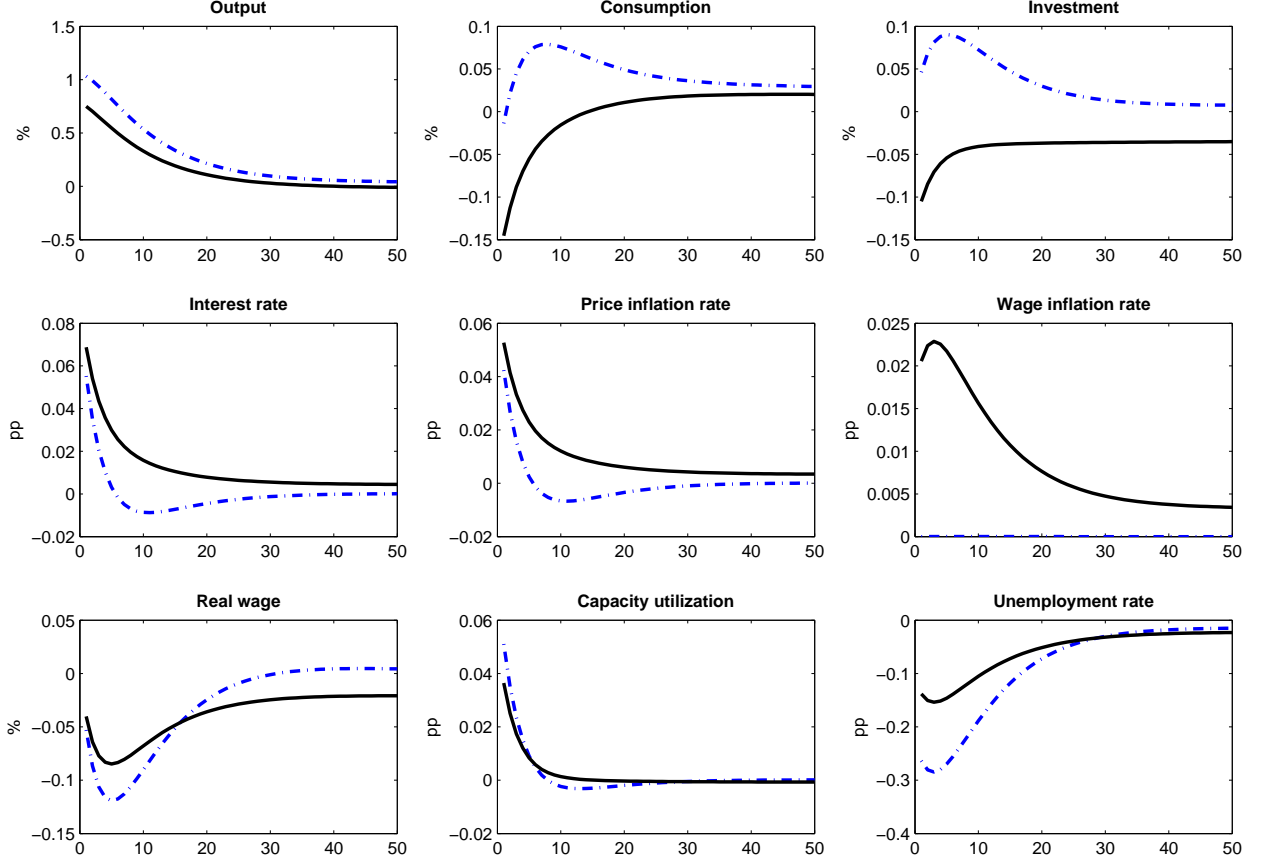


Figure 6: Responses to a 1% persistent budget-neutral government spending shock in the DSLMD model with collective wage bargaining (black-solid) and with constant wage inflation (blue-dashed-dotted) as deviations from the steady state.

**The role of the monetary policy regime.** In DSGE models monetary policy is perceived as the main instrument to achieve macroeconomic stability. As discussed above, the Taylor principle rules out macroeconomic fluctuations which are not due to changes in the fundamentals of the economy. As we have seen in the section discussing the determinacy properties of models featuring precautionary savings, two monetary policy regimes may exist: Active monetary policy, i.e. responding aggressively to inflation, may be required for low steady-state interest rates to ensure determinacy. Passive monetary policy, i.e. responding at most weakly to inflation, may be required for high steady-state interest rates to ensure the existence of a solution. Hence, with a sufficiently high interest rate, the economy does not need stabilization through monetary policy. Additionally to the baseline scenario of  $R = 1.03$  and  $\phi = 1.3$  discussed above, we consider here the macroeconomic effects of a fiscal shock with  $R = 1.04$  and  $\phi = 0$ . Hence, the interest rate is constant. Figure 7 compares the impulse-response functions.

As in the baseline case, the price inflation rate goes up immediately due to the expansionary impact effect of the fiscal shock. Yet, the monetary authority does not respond and leaves the nominal interest rate constant. Hence, the real interest rate drops. The striking result is that this does not trigger a consumption boom. Quite the contrary, consumption decreases. The persistent

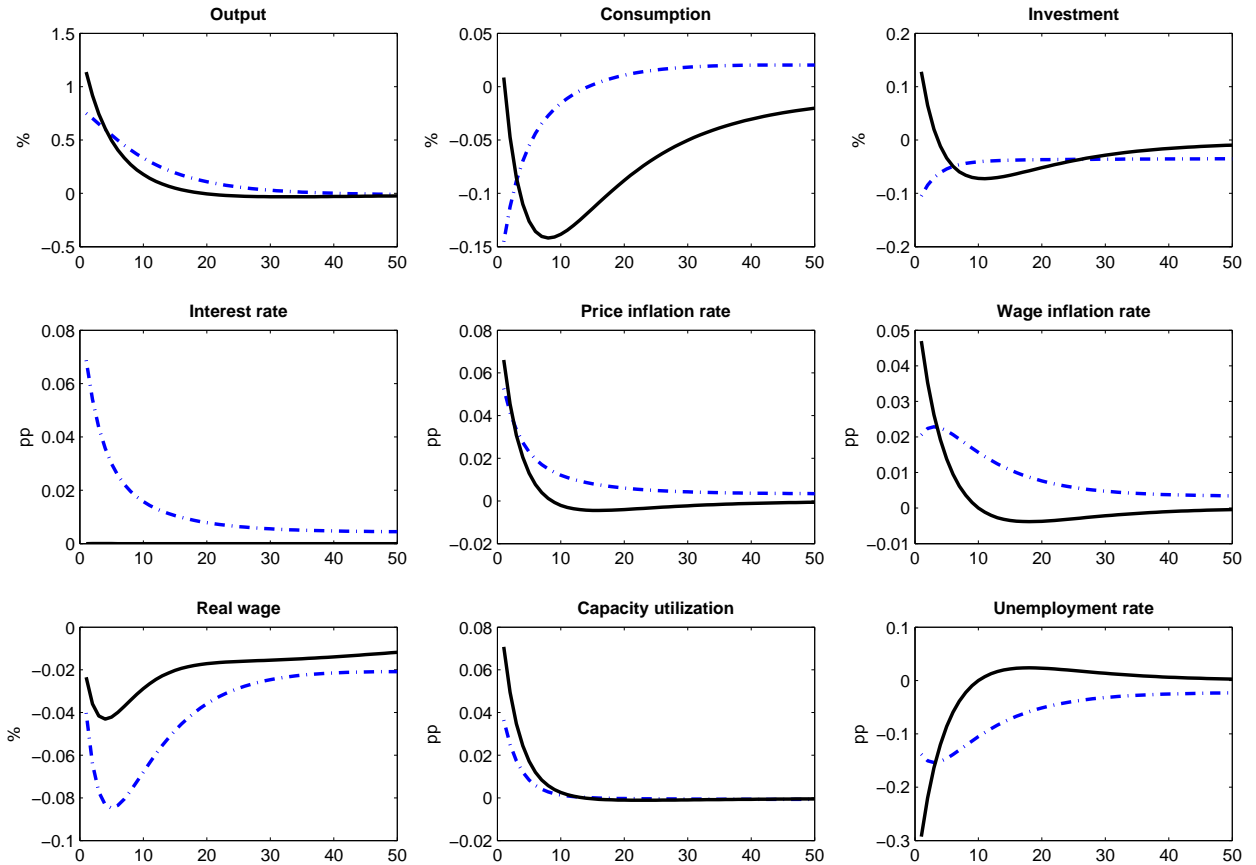


Figure 7: Responses to a 1% persistent budget-neutral government spending shock in the DSLMD model with collective wage bargaining in the passive monetary policy regime ( $R = 1.04$ ,  $\phi = 0$ , black-solid) and in the baseline active monetary policy regime ( $R = 1.03$ ,  $\phi = 1.3$ , blue-dashed-dotted) as deviations from the steady state.

reduction in the real interest rate triggers the optimal wealth-income and consumption-income ratios to drop. This reduces the multiplier and leads to a reduction in consumption and investment. Nevertheless, the impacts on the demand components are modest.

## 4 Concluding remarks

In the present paper we have presented a Dynamic Stochastic Labor-Market Disequilibrium (DSLMD). It shares the micro-foundations of conventional Dynamic Stochastic General Equilibrium (DSGE) models based on inter-temporal optimization and rational expectations and the economic content of aggregative Traditional Post-Keynesian (TPK) models with the principle of effective demand at the core. We have compared these models as well as a Synthetic Neoclassical (SNC) model featuring aggregative behavioral relations and an accommodating nominal wage clearing the labor market in terms of micro-foundation and economic content. We have analyzed how the models predict the economy to evolve after shocks to fiscal policy and productivity and how labor-market feedback into the wage formation affects the model dynamics.

The main conclusion is that a fundamentally Keynesian economy can be characterized by a set of micro-foundations consistent with mainstream methodology. We do not claim that the proposed micro-foundation is the only viable one. In particular, the formation of expectations should be based on a more realistic footing in future research. Yet, explicitly anchoring behavioral relations in goal-oriented considerations of the economic agents can no longer be seen as an obstacle for Keynesian analysis. Quite the contrary, modeling economic behavior rather than postulating it comes with considerable benefits for Keynesian macroeconomics: First, the methodological inconsistency arising from using a *modeling* approach to explain macro-phenomena but a *verbal* approach to explain micro-phenomena can be overcome by consistently modeling economic behavior as well as its interaction. Second, the parameters characterizing behavioral rules in TPK models are highly endogenous to policy which has been known since Lucas (1976). Neglecting Lucas' critique may well have been fueled by the fear among Keynesians that the mainstream rational-expectations, general-equilibrium solution is a necessary implication of his objection. As has been argued in the present paper, however, this fear is ill-founded. The framework proposed demonstrates one possible way of how to address Lucas' critique within the framework of Keynesian economic analysis.

Without the assumption of labor market clearing and with consumption depending on current income through precautionary savings motives, the DSLMD model has a Keynesian character. Since labor is not fully employed and involuntary unemployment persists, the macroeconomic balance condition cannot be interpreted as a resource constraint. Rather, it is a goods market equilibrium condition stating that aggregate output needs to equal aggregate spending. Business fluctuations are *demand-driven*. A demand shock affects output without requiring households to provide more resources, i.e. labor, since unemployed labor can be employed. An accelerator effect is predicted since consumption and investment move in the same direction of the demand shock. Labor market conditions then change, affect the bargaining process over wages and move the economy back to the steady state. The precautionary saving motive implies the existence of an active and a passive monetary policy regime. In the latter, the stability of the economy does not need to rely on monetary policy fighting inflation aggressively.

## References

- Ackley, G. (1978). *Macroeconomic Theory*. Macmillan Library Reference.
- Barbosa-Filho, N. H. and Taylor, L. (2006). Distributive and demand cycles in the us economy—a structuralist goodwin model. *Metroeconomica*, 57(3):389–411.
- Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1, Part 3, pages 1341 – 1393. Elsevier.
- Bhaduri, A. and Marglin, S. (1990). Unemployment and the real wage: the economic basis for contesting political ideologies. *Cambridge Journal of Economics*, 14:375–93.
- Blanchard, O. J. (1985). Debt, deficits, and finite horizons. *The Journal of Political Economy*, 93(2):pp. 223–247.
- Blanchard, O. J. and Kahn, C. M. (1980). The Solution of Linear Difference Models under Rational Expectations. *Econometrica*, 48(5):1305–11.

- Blundell, R., Pistaferri, L., and Preston, I. (2008). Consumption inequality and partial insurance. *American Economic Review*, 98(5):1887–1921.
- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398.
- Carroll, C. D. (1997). Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis. *The Quarterly Journal of Economics*, 112(1):1–55.
- Carroll, C. D. and Jeanne, O. (2009). A Tractable Model of Precautionary Reserves, Net Foreign Assets, or Sovereign Wealth Funds. NBER Working Papers 15228, National Bureau of Economic Research, Inc.
- Carroll, C. D. and Toche, P. (2009). A Tractable Model of Buffer Stock Saving. NBER Working Papers 15265, National Bureau of Economic Research, Inc.
- Clarida, R., Gali, J., and Gertler, M. (1999). The science of monetary policy: A new keynesian perspective. *Journal of Economic Literature*, 37(4):1661–1707.
- Duménil, G. and Lévy, D. (1999). Being keynesian in the short term and classical in the long term: The traverse to classical long-term equilibrium. *The Manchester School*, 67(6):684–716.
- Farmer, J. D. and Foley, D. K. (2009). The economy needs agent-based modelling. *Nature*, 460(6):685–6.
- Flaschel, P. (2009). *The Macrodynamics of Capitalism: Elements for a Synthesis of Marx, Keynes, and Schumpeter*. Springer, Berlin.
- Gertler, M., Sala, L., and Trigari, A. (2008). An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining. *Journal of Money, Credit and Banking*, 40(8):1713–1764.
- Godley, W. and Lavoie, M. (2012). *Monetary Economics: An Integrated Approach to Credit, Money, Income, Production and Wealth*. Palgrave Macmillan, second edition edition.
- Hein, E. (2007). Interest rate, debt, distribution and capital accumulation in a post-kaleckian model. *Metroeconomica*, 58(2):310–39.
- Hein, E. and Stockhammer, E. (2010). Macroeconomic policy mix, employment and inflation in a post-keynesian alternative to the new consensus model. *Review of Political Economy*, 22(3):317–354.
- Hein, E. and Vogel, L. (2008). Distribution and growth reconsidered - empirical results for six oecd countries. *Cambridge Journal of Economics*, 32(3):479–511.
- Jappelli, T. and Pistaferri, L. (2010). The Consumption Response to Income Changes. *Annual Review of Economics*, 2(1):479–506.
- Johnson, D. S., Parker, J. A., and Souleles, N. S. (2006). Household expenditure and the income tax rebates of 2001. *American Economic Review*, 96(5):1589–1610.

- Kaldor, N. (1982). *The Scourge of Monetarism*. Oxford University Press, Oxford.
- Kalecki, M. (1971). *Selected Essays on the Dynamics of the Capitalist Economy*. Cambridge University Press, Cambridge.
- Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money*. Macmillan, London.
- Kurz, H. D. (1994). Growth and distribution. *Review of Political Economy*, 6(4):393–420.
- Lavoie, M. (1992). *Foundations of Post-Keynesian Economic Analysis*. Edward Elgar, Aldershot.
- Lavoie, M. (2014). *Post-Keynesian Economics: New Foundations*. Edward Elgar, Aldershot.
- Lucas, R. (1976). Econometric policy evaluation: A critique. *Carnegie-Rochester Conference Series on Public Policy*, 1:19–46.
- Marglin, S. (1984). *Growth, Distribution and Prices*. Harvard University Press, Cambridge.
- Mortensen, D. T. and Pissarides, C. A. (1994). Job Creation and Job Destruction in the Theory of Unemployment. *Review of Economic Studies*, 61(3):397–415.
- Murota, R. and Ono, Y. (2010). A reinterpretation of the keynesian consumption function and multiplier effect. Discussion paper no. 779, Institute of Social and Economic Research (ISER).
- Naastepad, C. W. M. and Storm, S. (2006-07). Oecd demand regimes (1960-2000). *Journal of Post Keynesian Economics*, 29(2):211–46.
- Onaran, Ö. and Galanis, G. (2012). Is aggregate demand wage-led or profit-led? national and global effects. ILO Working Papers 478623, International Labour Organization.
- Parker, J. A. (1999). The reaction of household consumption to predictable changes in social security taxes. *The American Economic Review*, 89(4):pp. 959–973.
- Queijo von Heideken, V. (2009). How Important are Financial Frictions in the United States and the Euro Area? *Scandinavian Journal of Economics*, 111(3):567–596.
- Robinson, J. V. (1956). *Accumulation of Capital*. Macmillan, London.
- Robinson, J. V. (1962). *Essays in the Theory of Economic Growth*. Macmillan, London.
- Rotemberg, J. J. (1982). Sticky Prices in the United States. *Journal of Political Economy*, 90(6):1187–1211.
- Schoder, C. (2012). Hysteresis in the kaleckian growth model: A bayesian analysis for the us manufacturing sector from 1984 to 2007. *Metroeconomica*, 63(3):542–568.
- Schoder, C. (2014a). An estimated dynamic stochastic labor market disequilibrium model for the euro area and the united states.
- Schoder, C. (2014b). Instability, stationary utilization and effective demand: A structuralist model of endogenous cycles. *Structural Change and Economic Dynamics*, 30(0):10 – 29.



- Schoder, C. (2014c). Precautionary saving motives and inner conflict in an estimated dsge model for the united states and the euro area.
- Schoder, C. (2015). Methodological, internal and ontological inconsistencies in the conventional micro-foundation of post-keynesian theory.
- Shaikh, A. (2009). Economic policy in a growth context: A classical synthesis of keynes and harrod. *Metroeconomica*, 60(3):455–94.
- Shapiro, C. and Stiglitz, J. E. (1984). Equilibrium Unemployment as a Worker Discipline Device. *American Economic Review*, 74(3):433–44.
- Shapiro, M. D. and Slemrod, J. (2009). Did the 2008 tax rebates stimulate spending? *American Economic Review*, 99(2):374–79.
- Shefrin, H. M. and Thaler, R. H. (1988). The behavioral life-cycle hypothesis. *Economic Inquiry*, 26(4):609–43.
- Sidrauski, M. (1967). Inflation and Economic Growth. *Journal of Political Economy*, 75:796.
- Skott, P. (1989a). *Conflict and Effective Demand in Economic Growth*. Cambridge University Press, Cambridge.
- Skott, P. (1989b). Effective demand, class struggle and cyclical growth. *International Economic Review*, 30(1):231–47.
- Skott, P. (2012). Pluralism, the Lucas critique, and the integration of macro and micro. UMASS Amherst Economics Working Papers 2012-04, University of Massachusetts Amherst, Department of Economics.
- Smets, F. and Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association*, 1(5):1123–1175.
- Souleles, N. S. (2002). Consumer response to the reagan tax cuts. *Journal of Public Economics*, 85(1):99 – 120.
- Steindl, J. (1952). *Maturity and Stagnation in American Capitalism*. Basil Blackwell, Oxford.
- Steindl, J. (1979). Stagnation theory and stagnation policy. *Cambridge Journal of Economics*, 3(1):1–14.
- Stockhammer, E. (2005-06). Shareholder value orientation and the investment-profit puzzle. *Cambridge Journal of Economics*, 28:193–215.
- Stockhammer, E., Onaran, Ö., and Ederer, S. (2009). Functional income distribution and aggregate demand in the euro area. *Cambridge Journal of Economics*, 33(1):19–40.
- Stockhammer, E. and Stehrer, R. (2011). Goodwin or kalecki in demand? functional income distribution and aggregate demand in the short run. *Review of Radical Political Economics*, 43(4):506–522.

- Svein, T. and Weinke, L. (2007). Firm-specific capital, nominal rigidities, and the Taylor principle. *Journal of Economic Theory*, 136(1):729–737.
- Svein, T. and Weinke, L. (2009). Firm-Specific Capital and Welfare. *International Journal of Central Banking*, 5(2):147–179.
- Taylor, L. (1985). A stagnationist model of economic growth. *Cambridge Journal of Economics*, 9(4):381–403.
- Taylor, L. (2004). *Reconstructing Macroeconomics: Structuralist Proposals and Critiques of the Mainstream*. Harvard University Press, Cambridge.
- Taylor, L. (2012). Growth, cycles, asset prices and finance. *Metroeconomica*, 63(1):40–63.
- Tobin, J. (1969). A general equilibrium approach to monetary theory. *Journal of Money, Credit and Banking*, 1(1):15–29.
- Woodford, M. (2003). *Interest and prices*. Princeton Univ. Press, Princeton, NJ [u.a.].
- Woodford, M. (2005). Firm-Specific Capital and the New Keynesian Phillips Curve. *International Journal of Central Banking*, 1(2).
- Zipperer, B. and Skott, P. (2010). Cyclical patterns of employment, utilization and profitability. Working Papers 2010-02, University of Massachusetts Amherst, Department of Economics.

## A Description of variables and parameters and calibration

### Variables

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$\tilde{Y}_t$	Real aggregate output detrended by labor embodied productivity growth
$\tilde{Z}_t$	Real detrended aggregate household income net of taxes
$\tilde{C}_t$	Real detrended aggregate consumption
$\tilde{I}_t$	Real detrended aggregate investment
$\tilde{G}_t$	Real detrended government spending
$\tilde{K}_t$	Real detrended aggregate capital stock
$\tilde{T}_t$	Real detrended aggregate lump-sum taxes
$\tilde{C}_t^a$	Real detrended aggregate consumption of active households
$\tilde{C}_t^i$	Real detrended aggregate consumption of inactive households
$\tilde{B}_t^a$	Real detrended aggregate wealth of active households
$\tilde{B}_t^i$	Real detrended aggregate wealth of inactive households
$\tilde{B}_t$	Real detrended aggregate wealth of households
$\tilde{Y}_{c,t}$	Real detrended aggregate capacity output
$L_t$	Aggregate labor demand
$N_t$	Aggregate labor supply
$A_t$	Total factor productivity
$\tilde{w}_t$	Detrended nominal wage per unit of labor
$P_t$	Price level
$p_t$	Price level of the individual firm
$\tilde{\Pi}_t^d$	Real detrended aggregate distributed profits
$R_t$	Nominal interest factor
$\Pi_t$	Factor of price inflation
$\Pi_t^{\tilde{w}}$	Factor of nominal wage inflation net of labor embodied productivity growth
$\Pi_t^{\tilde{w}^*}$	Target factor of nominal wage inflation net of labor embodied productivity growth
$\tau_t$	Non-distortionary transfer between households
$\tilde{\omega}_t$	Detrended real wage per unit of labor
$\Lambda_{t,t+1}$	Stochastic discount factor
$v_t$	Rate of capacity utilization
$\varphi_t$	Real marginal costs
$q_t$	Tobin's $q$
$u_t$	Unemployment rate
$r_t$	Profit rate
$V_t$	Autoregressive shock to the workers' relative bargaining power
$v_t$	Workers' relative bargaining power
$\epsilon_{G,t}$	Innovation to real government spending
$\epsilon_{M,t}$	Innovation to the interest rate
$\epsilon_{A,t}$	Innovation to total factor productivity
$\epsilon_{V,t}$	Innovation to the workers' relative bargaining power

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## Parameters

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### Calibrated parameters in baseline models

$\beta$	Household's discount rate	0.998
$\eta$	Inverse of the Frisch labor supply elasticity	1
$\Gamma$	Growth factor of labor-embodied productivity	1.01
$D$	Inactive household's probability of death	0.002
$c_b$	Marginal propensity to consume out of wealth	0.003
$\delta$	Rate of capital depreciation	0.025
$\epsilon$	Elasticity of substitution of intermediate goods	3
$\lambda$	Target debt-capital ratio	0.05
$\tau_p$	Price adjustment cost scaling parameter	50
$\tau_i$	Investment adjustment cost scaling parameter	20
$\alpha$	Output elasticity of capital	0.333
$\Pi$	Target inflation rate of monetary authority	1
$\phi$	Inflation elasticity of interest factor	1.3
$R$	Interest target of the monetary authority	1.003
$\tilde{G}$	Steady-state government expenditures	1
$\rho_G$	Persistence of a government spending shock	0.9
$\rho_M$	Persistence of a monetary policy shock	0.9
$\rho_A$	Persistence of a total factor productivity shock	0.9
$\rho_V$	Persistence of a bargaining power shock	0.9
$\rho_w$	Persistence of wage inflation	0

### Computed parameters in baseline models

$U$	Active household's probability of income loss	0.0006 (such that old-age dependency ratio is 0.3)
$\theta^a$	Active households' share	0.769 (follows from $D/(U + D)$ )
$\theta^i$	Inactive households' share	0.231 (follows from $U/(U + D)$ )
$\psi$	Scaling parameter of labor supply	0.533 (such that labor market clears for R)
$\kappa$	Consumption-wealth ratio of inactive household	0.004 (follows from $1 - \beta(1 - D)$ )
$c_z$	Marginal propensity to consume out of income	0.95 (follows from $\frac{\tilde{C}}{\tilde{Z}} - \frac{\tilde{B}^i + \tilde{B}^a}{\tilde{Z}} c_b$ )
$\phi_i$	Utilization effect on investment	0.035 (such that the responses of investment to a one-unit change are equal in the TPK and DSLMD model)
$V$	Bargaining power shock steady state	0.792 (such that $\Pi^{\tilde{w}^*} = 1$ )

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## B Model appendix

This appendix derives all aggregated equations characterizing the DSLMD and DSGE models. The corresponding equations for the TPK and SNC models will be stated along the way. Let  $\Gamma$  denote the deterministic growth factor of the economy arising from labor embodied productivity growth. Then, we use the following notation:  $\tilde{X}_t \equiv \frac{X_t}{\Gamma^t}$  for any aggregated variable  $X_t$ . Note that a description of the variables can be found in Appendix A.

Active households are born into generations of constant size and face a per-period risk  $U$  of becoming inactive. Inactive households face a per-period risk  $D$  of dying. Then the share of active households is  $\theta^a = D/(U + D)$  and the share of inactive households is  $\theta^i = U/(U + D)$ .

### B.1 Households

**Inactive households.** Let us derive the inactive household's first order conditions (FOCs) and budget constraint. Inactive households do not obtain labor or profit income, face a per-period probability,  $D$ , of death and have access to a Blanchard (1985) insurance market. The problem reads

$$\begin{aligned} \max_{c_t^i, b_{t+1}^i} \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta(1-D))^t \ln(c_t^i) \\ \text{s.t. } b_{t+1}^i = \frac{R_t}{\Pi_{t+1}} (\tau_t^i + b_t^i - c_t^i) \end{aligned}$$

where the budget constraint is conditional on staying alive and  $\tau_t^i$  is the per-capita payments of the insurance company to the inactive households. The zero-profit condition implies  $0 = D \left( \frac{R_t}{\Pi_{t+1}} (\tau_t^i + b_t^i - c_t^i) \right) - \frac{R_t}{\Pi_{t+1}} \tau_t^i$  where the first term are per-capital accidental bequests transferred to the insurance company and the second term are the payments to the household. Solving for  $\tau_t^i$  and substituting into the inactive household's budget constraint yields

$$b_{t+1}^i = \frac{R_t}{\Pi_{t+1}} \frac{1}{1-D} (b_t^i - c_t^i).$$

The Lagrangian characterizing this problem is

$$L = \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta(1-D))^t \left( \ln(c_t^i) + \lambda_t \left( \frac{R_t}{\Pi_{t+1}} \frac{1}{1-D} (b_t^i - c_t^i) - b_{t+1}^i \right) \right).$$

The FOCs w.r.t. consumption and wealth are

$$\frac{1}{c_t^i} = \lambda_t \mathbb{E}_t \frac{R_t}{\Pi_{t+1}} \frac{1}{1-D}$$

and

$$\lambda_t = \beta(1-D) \mathbb{E}_t \frac{R_t}{\Pi_{t+1}} \frac{1}{1-D} \lambda_{t+1},$$

respectively. Combining the two FOCs leads to

$$\frac{1}{c_t^i} = \beta \mathbb{E}_t \frac{R_t}{\Pi_{t+1}} \frac{1}{c_{t+1}^i}$$

$$\frac{1}{c_t^i} = \beta^n \mathbb{E}_t \prod_{k=0}^{n-1} \frac{R_{t+k}}{\Pi_{t+k+1}} \frac{1}{c_{t+n}^i}$$

The budget constraint can be rearranged and iterated forward and, then, the previous result can be used to get

$$\begin{aligned} b_t^i &= (1-D) \left( \frac{R_t}{\Pi_{t+1}} \right)^{-1} b_{t+1}^i + c_t^i \\ &= \sum_{n=0}^{\infty} (1-D)^n \prod_{k=0}^{n-1} \left( \frac{R_{t+k}}{\Pi_{t+k+1}} \right)^{-1} c_{t+n}^i \\ &= \sum_{n=0}^{\infty} (1-D)^n \prod_{k=0}^{n-1} \left( \frac{R_{t+k}}{\Pi_{t+k+1}} \right)^{-1} c_{t+n}^i \beta^n \mathbb{E}_t \prod_{k=0}^{n-1} \frac{R_{t+k}}{\Pi_{t+k+1}} \frac{1}{c_{t+1}^i} c_t^i \\ &= \sum_{n=0}^{\infty} (\beta(1-D))^n c_t^i \\ &= 1/\kappa c_t^i \\ \tilde{C}_t^i &= \kappa \tilde{B}_t^i \end{aligned} \tag{MF.1}$$

with  $\kappa = 1 - \beta(1-D)$ . To derive the aggregate budget constraint for the inactive households, note the following: First, the budget constraint of the individual inactive household is conditional on staying alive. Hence the aggregation of tomorrow's wealth  $b_{t+1}^i = (1-D)B_{t+1}^i$ . Second, tomorrow's wealth consists of today's savings of inactive households plus interest and the wealth that active households which are going to be inactive tomorrow will bring over. Hence, we get

$$\begin{aligned} b_{t+1}^i &= \frac{R_t}{\Pi_{t+1}} \frac{1}{1-D} (b_t^i - c_t^i) + U b_{t+1}^a \frac{\theta^a}{\theta^i} \\ (1-D)B_{t+1}^i &= \frac{R_t}{\Pi_{t+1}} \frac{1}{1-D} (B_t^i - C_t^i) + U B_{t+1}^a \\ \tilde{B}_{t+1}^i &= \frac{R_t}{\Pi_{t+1}} \frac{1}{\Gamma} (\tilde{B}_t^i - \tilde{C}_t^i) + U \tilde{B}_{t+1}^a. \end{aligned} \tag{MF.2}$$

**Active households.** The active household's problem reads

$$\begin{aligned} \max_{c_t^a, b_{t+1}^a, n_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \ln(c_t^a) - \psi \frac{n_t^{1+\eta}}{1+\eta} \right) \\ s.t. \quad b_{t+1}^a = \frac{R_t}{\Pi_{t+1}} \left( \omega_t(1-u_t)n_t + \pi_t^d + b_t^a - t_t - c_t^a - \tau_t^a \right) \end{aligned}$$

where  $n_t$ ,  $\omega_t$ ,  $u_t$ ,  $\pi_t^d$ ,  $t_t$ ,  $\tau_t^a$ ,  $\psi$ , and  $\eta$  denote labor supply, the real wage, the unemployment rate, distributed profits, the lump-sum government tax, a non-distortionary transfer from active non-newborn households to newborn households, a scaling parameter and the inverse of the Frisch

elasticity, respectively.  $\tau_t^a$  ensures that wealth is distributed equally across each active household at any point in time which facilitates aggregation. Since the household faces a risk  $U$  of permanent income loss, it is convenient to set up the household's problem as a dynamic program:

$$v_t^a(b_t^a) = \max_{c_t^a, b_{t+1}^a, n_t} \left[ \begin{array}{l} \ln(c_t^a - \psi \frac{n_t^{1+\eta}}{1+\eta} + \\ + \beta(1-U)E_t v_{t+1}^a(b_{t+1}^a) + \beta U E_t v_{t+1}^i(b_{t+1}^a) \end{array} \right]$$

$$s.t. \quad b_{t+1}^a = \frac{R_t}{\Pi_{t+1}^p} \left( \omega_t(1-u_t)n_t + \pi_t^d + b_t^a - t_t - c_t^a - \tau_t^a \right)$$

where  $v_t^a(b_t^a)$  is the value function in  $t$ . Note that  $v_{t+1}^i(b_{t+1}^a)$  is the value function for  $t+1$  of a household that became inactive between  $t$  and  $t+1$ . Substituting out  $c_t^a$  using the budget constraint and, the FOC w.r.t. to wealth,  $b_{t+1}$ , implies after applying the envelop condition,

$$v_t^a = \beta E_t \frac{R_t}{\Pi_{t+1}^p} \left( (1-U)v_{t+1}^a + Uv_{t+1}^i \right)$$

$$\tilde{v}_t^a = \beta \Gamma^{-1} E_t \frac{R_t}{\Pi_{t+1}^p} \left( (1-U)\tilde{v}_{t+1}^a + U \left( \frac{\theta^a}{\theta^i} \right)^{-1} \tilde{v}_{t+1}^i \right) \quad (\text{MF.3})$$

where

$$v_t^a = (c_t^a)^{-1}$$

$$\tilde{v}_t^a = (\tilde{C}_t^a)^{-1} \quad (\text{MF.4})$$

and, using the FOC of the inactive household as well as recalling that  $v_t^i$  is the value function of the newly inactive household,

$$v_t^i = (c_t^i)^{-1}$$

$$\tilde{v}_t^i = (\kappa \tilde{B}_t^a)^{-1}. \quad (\text{MF.5})$$

To derive the active household's aggregate budget constraint note that we assume a transfer which ensures that every active household has the same wealth which facilitates aggregation. For details, see Carroll and Jeanne (2009). This transfer sums up to zero over all active households. Note further that a fraction  $U$  of active households will become inactive tomorrow. The aggregate budget constraint can then be obtained as

$$b_{t+1}^a = \frac{R_t}{\Pi_{t+1}^p} \left( \omega_t n_t + \pi_t^d + b_t^a - t_t - c_t^a - \tau_t^a \right) - U b_{t+1}^a$$

$$\tilde{B}_{t+1}^a = \frac{R_t}{\Pi_{t+1}^p} \frac{1}{\Gamma} (\tilde{Z}_t + \tilde{B}_t^a - \tilde{C}_t^a) - U \tilde{B}_{t+1}^a \quad (\text{MF.6})$$

where

$$\tilde{Z}_t = \omega_t L_t + \tilde{\Pi}_t^d - \tilde{T}_t \quad (\text{MF.7/AG.1})$$

is the active households' detrended aggregate real net income. Note that  $L_t = (1-u_t)N_t$ , which is implied by the definition of the unemployment rate

$$u_t = 1 - \frac{\hat{L}_t}{\hat{N}_t}, \quad (\text{MF.8/AG.2})$$

denotes labor input and  $\tilde{\Pi}_t^d$  distributed profits by the intermediate good firms to be specified below. Consumption simply is

$$\tilde{C}_t = \tilde{C}_t^a + \tilde{C}_t^i. \quad (\text{MF.9})$$

The FOC of the active household w.r.t. labor supply implies

$$\begin{aligned} \psi n_t^\eta &= \lambda_t \frac{R_t}{\Pi_{t+1}} \omega(1 - u_t) \\ \psi n_t^\eta &= \frac{1}{c_t^a} \omega(1 - u_t) \\ \psi(\theta^a)^{-(1+\eta)} N_t^\eta &= \frac{1}{\tilde{C}_t^a} \tilde{\omega}_t(1 - u_t). \end{aligned} \quad (\text{MF.10})$$

**Households in the SNC and TPK models.** In the aggregative models, (MF.1)-(MF.6), (MF.9) and (MF.10) are replaced by a consumption function, the law of motion of wealth and a constant labor supply which are specified as

$$\tilde{C}_t = c_z \tilde{Z}_t + c_b \tilde{B}_t, \quad (\text{AG.3})$$

$$\tilde{B}_{t+1} = \frac{R_t}{\Pi_{t+1}} \frac{1}{\Gamma} (\tilde{Z}_t + \tilde{B}_t - \tilde{C}_t) \quad (\text{AG.4})$$

and

$$N_t = N, \quad (\text{AG.5})$$

respectively.

## B.2 Firms

**Final good firms.** Taken as given price  $p_{i,t}$ , the final good firm's demand for the intermediate good  $y_{i,t}$  supplied by intermediate good firm  $i$  can be obtained from the following cost minimization problem:

$$\begin{aligned} \min_{y_{i,t}} & \int_0^1 p_{i,t} y_{i,t} di \\ \text{s.t.} & Y_t = \int_0^1 \left( y_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \end{aligned}$$

where  $\epsilon > 1$  is the elasticity of substitution. Noting that the Lagrangian multiplier of the constraint is equal to the aggregate price index,  $P_t$ , one can show the FOC to read

$$y_{i,t} = \left( \frac{p_{i,t}}{P_t} \right)^{-\epsilon} Y_t.$$



**Intermediate good firms.** Taking as given total output, the overall price level, the capital stock, and the wage rate well as the law of motion of capital, the production function, the demand function for intermediate goods, and the requirement to maintain a debt-capital ratio  $\lambda$ , the firm  $i$  chooses  $\{p_{i,t}, l_{i,t}, i_{i,t}, k_{i,t+1}, d_{i,t+1}\}$  to maximize discounted inter-temporal distributed profits. Dropping the firm index for convenience, the optimization problem reads

$$\begin{aligned} \max_{p_t, l_t, i_t, k_{t+1}, d_{t+1}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{P_0}{P_t} \Lambda_{0,t} \left[ p_t y_t - w_t l_t - P_t i_t - P_t \frac{\tau_i}{2} \left( \frac{i_t}{k_t} - (\Gamma - (1 - \delta)) \right)^2 k_t - \right. \\ & \left. - P_t \frac{\tau_p}{2} \Gamma^t \left( \frac{p_t}{p_{t-1}} - \Pi \right)^2 + P_{t+1} \frac{d_{t+1}}{R_t} - P_t d_t \right] \\ \text{s.t.} \quad & k_{t+1} = i_t + (1 - \delta) k_t \\ & y_t = k_t^\alpha (\Gamma^t l_t)^{1-\alpha} \\ & y_t = \left( \frac{p_t}{P_t} \right)^{-\epsilon} Y_t \\ & \frac{d_t}{R_{t-1}} = \lambda q_t k_t \end{aligned}$$

where

$$\Lambda_{t,t+j} = \beta^j \frac{U_{c^a,t+j}}{U_{c^a,t}}$$

is the stochastic discount factor which expresses the value of a unit real profit in time  $t+j$  in terms of the value of a unit real profit in time  $t$ . After substituting  $y_t = \left( \frac{p_t}{P_t} \right)^{-\epsilon} Y_t$  into the objective function and the production function, the Lagrangian of the intermediate good firm is

$$L = \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{P_0}{P_t} \Lambda_{0,t} \left[ \begin{aligned} & p_t \left( \frac{p_t}{P_t} \right)^{-\epsilon} Y_t - w_t l_t - P_t i_t - P_t \frac{\tau_i}{2} \left( \frac{i_t}{k_t} - (\Gamma - (1 - \delta)) \right)^2 k_t - \\ & - P_t \frac{\tau_p}{2} \Gamma^t \left( \frac{p_t}{p_{t-1}} - \Pi \right)^2 + P_{t+1} \frac{d_{t+1}}{R_t} - P_t d_t + P_t \varphi_t \left( k_t^\alpha (\Gamma^t l_t)^{1-\alpha} - \left( \frac{p_t}{P_t} \right)^{-\epsilon} Y_t \right) + \\ & + P_t q_t (i_t + (1 - \delta) k_t - k_{t+1}) + P_t \mu_t \left( \frac{d_t}{R_{t-1}} - \lambda q_t k_t \right) \end{aligned} \right]$$

Recalling from the active household's problem that  $\frac{R_t}{\Pi_{t+1}} \Lambda_{t,t+1} = 1$ , the FOC w.r.t  $d_{t+1}$  implies

$$\begin{aligned} \frac{P_{t+1}}{R_t} + \mathbb{E}_t \frac{P_t}{P_{t+1}} \Lambda_{t,t+1} \left( -P_{t+1} + P_{t+1} \mu_{t+1} \frac{1}{R_t} \right) &= 0 \\ 1 + \mathbb{E}_t \frac{R_t}{\Pi_{t+1}} \Lambda_{t,t+1} (-1 + \mu_{t+1}) &= 0 \\ \mu_t &= 0. \end{aligned}$$

The financial structure of the firm is irrelevant from the household's perspective. Note that all firms charge the same price,  $p_t = P_t$ , and, hence  $y_t = Y_t$  with a mass one of firms. The FOC w.r.t.  $p_t$  then implies

$$\begin{aligned} (1 - \epsilon) y_t - P_t \tau_p \Gamma^t \left( \frac{p_t}{p_{t-1}} - \Pi \right) \frac{1}{p_{t-1}} + \epsilon P_t \varphi_t \frac{y_t}{p_t} + \mathbb{E}_t \frac{P_t}{P_{t+1}} \Lambda_{t,t+1} P_{t+1} \tau_p \Gamma^{t+1} \left( \frac{p_{t+1}}{p_t} - \Pi \right) \frac{p_{t+1}}{p_t^2} &= 0 \\ (1 - \epsilon) Y_t - \tau_p \Gamma^t (\Pi_t - \Pi) \Pi_t + \epsilon \varphi_t Y_t + \mathbb{E}_t \Lambda_{t,t+1} \tau_p \Gamma^{t+1} (\Pi_{t+1} - \Pi) \Pi_{t+1} &= 0 \\ ((\epsilon - 1) - \epsilon \varphi_t) \tilde{Y}_t + \tau_p (\Pi_t - \Pi) \Pi_t - \mathbb{E}_t \Lambda_{t,t+1} \tau_p \Gamma (\Pi_{t+1} - \Pi) \Pi_{t+1} &= 0 \end{aligned}$$

(MF.11/AG.6)

where

$$\Lambda_{t,t+1} = \beta^j \frac{1}{\Gamma} \frac{U_{\tilde{C}^\alpha,t+1}}{U_{\tilde{C}^\alpha,t}}. \quad (\text{MF.12})$$

To derive the implications of the FOC w.r.t.  $l_t$  first note that the production function can be rewritten as  $\left(\frac{y_t}{k_t}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{k_t}{\Gamma^t l_t}\right)^{-\alpha}$ . Then,

$$\begin{aligned} -w_t + P_t \varphi_t \Gamma^{t(1-\alpha)} (1-\alpha) l_t^{-\alpha} k_t^\alpha &= 0 \\ -w_t + P_t \varphi_t (1+\gamma)^t (1-\alpha) ((1+\gamma)^t l_t)^{-\alpha} k_t^\alpha &= 0 \\ \varphi_t &= \frac{w_t}{P_t} \frac{1}{\Gamma^t} \frac{1}{1-\alpha} \left(\frac{k_t}{\Gamma^t l_t}\right)^{-\alpha} \\ \varphi_t &= \frac{w_t}{P_t} \frac{1}{\Gamma^t} \frac{1}{1-\alpha} \left(\frac{y_t}{k_t}\right)^{\frac{\alpha}{1-\alpha}} \\ \varphi_t &= \tilde{\omega}_t \frac{1}{1-\alpha} \left(\frac{\tilde{Y}_t}{\tilde{K}_t}\right)^{\frac{\alpha}{1-\alpha}}. \end{aligned} \quad (\text{MF.13})$$

The FOC w.r.t to  $i_t$  implies

$$\begin{aligned} -P_t - P_t \tau_i \left(\frac{i_t}{k_t} - (\Gamma - (1-\delta))\right) \frac{1}{k_t} k_t + P_t q_t &= 0 \\ q_t &= 1 + \tau_i \left(\frac{i_t}{k_t} - (\Gamma - (1-\delta))\right) \\ q_t &= 1 + \tau_i \left(\frac{\tilde{I}_t}{\tilde{K}_t} - (\Gamma - (1-\delta))\right) \end{aligned} \quad (\text{MF.14})$$

Recalling that  $\mu_t = 0$  and noting that  $\frac{y_t}{k_t} = \left(\frac{k_t}{\Gamma^t l_t}\right)^{\alpha-1}$ , the FOC w.r.t.  $k_{t+1}$  implies

$$\begin{aligned} P_t q_t &= E_t \frac{P_t}{P_{t+1}} \Lambda_{t,t+1} \left[ -P_{t+1} \left( \tau_i \left( \frac{i_{t+1}}{k_{t+1}} - (\Gamma - (1-\delta)) \right) k_{t+1} + \frac{\tau_i}{2} \left( \frac{i_{t+1}}{k_{t+1}} - (\Gamma - (1-\delta)) \right)^2 \right) + \right. \\ &\quad \left. + P_{t+1} \varphi_{t+1} \alpha k_{t+1}^{\alpha-1} (\Gamma^t l_{t+1})^{1-\alpha} + P_{t+1} q_{t+1} (1-\delta) - P_{t+1} \mu_{t+1} \lambda q_{t+1} \right] \\ q_t &= E_t \Lambda_{t,t+1} \left[ -\tau_i \left( \frac{i_{t+1}}{k_{t+1}} - (\Gamma - (1-\delta)) \right) k_{t+1} - \frac{\tau_i}{2} \left( \frac{i_{t+1}}{k_{t+1}} - (\Gamma - (1-\delta)) \right)^2 + \right. \\ &\quad \left. + \varphi_{t+1} \alpha \left( \frac{k_{t+1}}{\Gamma^t l_{t+1}} \right)^{\alpha-1} + q_{t+1} (1-\delta) \right] \\ q_t &= E_t \Lambda_{t,t+1} \left[ -\tau_i \left( \frac{\tilde{I}_{t+1}}{\tilde{K}_{t+1}} - (\Gamma - (1-\delta)) \right) \tilde{K}_{t+1} - \frac{\tau_i}{2} \left( \frac{\tilde{I}_{t+1}}{\tilde{K}_{t+1}} - (\Gamma - (1-\delta)) \right)^2 + \right. \\ &\quad \left. + \varphi_{t+1} \alpha \frac{\tilde{Y}_{t+1}}{\tilde{K}_{t+1}} + q_{t+1} (1-\delta) \right]. \end{aligned} \quad (\text{MF.15})$$

Aggregating the law of motion of the capital stock leads to

$$\begin{aligned} k_{t+1} &= i_t + (1-\delta)k_t \\ \tilde{K}_{t+1} &= \frac{1}{\Gamma} \left( \tilde{I}_t + (1-\delta)\tilde{K}_t \right) \end{aligned} \quad (\text{MF.16/AG.7})$$

The production function can be aggregated as follows. Note that all firms set the same price,  $p_t = P_t$ .

$$\begin{aligned}
y_t &= k_t^\alpha (\Gamma^t l_t)^{1-\alpha} \\
\left(\frac{p_t}{P_t}\right) Y_t &= k_t^\alpha (\Gamma^t l_t)^{1-\alpha} \\
Y_t &= K_t^\alpha (\Gamma^t L_t)^{1-\alpha} \\
\frac{Y_t}{\Gamma^t} &= \left(\frac{K_t}{\Gamma^t}\right)^\alpha L_t^{1-\alpha} \\
\tilde{Y}_t &= \tilde{K}_t^\alpha L_t^{1-\alpha}.
\end{aligned} \tag{MF.17/AG.8}$$

Recalling that firms maintain a debt-capital ratio of  $\lambda$ , the aggregated detrended real distributed profits are

$$\tilde{\Pi}_t^d = \tilde{Y}_t - \tilde{\omega}_t L_t - (1 - \lambda) \tilde{I}_t - \frac{\tau_i}{2} \left( \frac{\tilde{I}_t}{\tilde{K}_t} - (\Gamma - (1 - \delta)) \right)^2 \tilde{K}_t - \frac{\tau_p}{2} (\Pi_t - \Pi)^2. \tag{MF.18/AG.9}$$

The growth rate of the real wage is linked to wage and price inflation according to

$$\frac{\tilde{\omega}_t}{\tilde{\omega}_{t-1}} - 1 = \Pi_t^{\tilde{\omega}} - \Pi_t. \tag{MF.19/AG.10}$$

**Firms in the aggregative SNC and TPK models.** The firms in the aggregative models set prices according to (AG.5) but instead of (MF.11/AG.6) the discount factor

$$\Lambda_{t,t+1} = \Lambda \tag{AG.11}$$

is constant at the steady state. SNC and TPK firms produce according to (MF.16/AG.7). Capital evolves according to (MF.15). Even though the production function is Cobb-Douglas, firms do not substitute inputs. Hence, instead of (MF.12), marginal costs are

$$\varphi_t = \tilde{\omega}_t \frac{1}{1 - \alpha} \left( \frac{\tilde{Y}_t}{\tilde{K}_t} \right)^{\frac{\alpha}{1-\alpha}}, \tag{AG.12}$$

where the output-capital ratio is fixed at the steady-state level. Then, marginal costs are proportional to wage costs. Instead of (MF.13) and (MF.14), firms invest according to

$$\frac{\tilde{I}_t}{\tilde{K}_t} = \Gamma - (1 - \delta) + \phi_i (v_t - v) \tag{AG.13}$$

where  $v_t$  and  $v$  are the actual and steady-state rates of capacity utilization, respectively, with

$$v_t = \frac{\tilde{Y}_t}{\tilde{Y}_{c,t}} \tag{MF.20/AG.14}$$

Capacity output requires average revenues to be equal to average costs which implies

$$\tilde{Y}_{c,t} = \left( \tilde{\omega}_t \left( \frac{\tilde{Y}_{c,t}}{A_t \tilde{K}_t^\alpha} \right)^{\frac{1}{1-\alpha}} + (1 - \lambda) \delta \tilde{K}_t \right). \tag{MF.21/AG.15}$$

### B.3 Fiscal and monetary policy

For all models we assume the government budget to be balanced at all times and monetary policy to follow a Taylor rule:

$$\tilde{T}_t = \tilde{G}_t \quad (\text{MF.22/AG.16})$$

$$\frac{R_t}{\bar{R}} = \left( \frac{\Pi_t}{\bar{\Pi}} \right)^\phi V_t \quad (\text{MF.23/AG.17})$$

### B.4 Market clearing

In all models, the goods market clears, i.e.

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{G}_t + \frac{\tau_i}{2} \left( \frac{\tilde{I}_t}{\tilde{K}_t} - (\Gamma - (1 - \delta)) \right)^2 \tilde{K}_t + \frac{\tau_p}{2} (\Pi_t - \bar{\Pi})^2. \quad (\text{MF.24/AG.18})$$

The bond-market-clearing conditions for the micro-founded and the aggregative models are

$$B_t^i + B_t^a = D_t \quad (\text{MF.25})$$

and

$$B_t = D_t, \quad (\text{AG.19})$$

respectively.

### B.5 Exogenous processes

Government expenditures, the monetary policy disturbance and the total factor productivity are assumed to evolve according to

$$\tilde{G}_t = \left( \tilde{G}_{t-1} \right)^{\rho_G} \tilde{G}^{1-\rho_G} \exp \varepsilon_{G,t}, \quad (\text{MF.26/AG.20})$$

$$M_t = \left( M_{t-1} \right)^{\rho_M} \exp \varepsilon_{M,t}, \quad (\text{MF.27/AG.21})$$

and

$$A_t = \left( A_{t-1} \right)^{\rho_A} \exp \varepsilon_{A,t}, \quad (\text{MF.28/AG.22})$$

respectively, where  $\varepsilon_{G,t}$ ,  $\varepsilon_{M,t}$  and  $\varepsilon_{A,t}$  are exogenous innovations.

### B.6 Model closures.

The neoclassical DSGE and SNC models are closed by assuming labor market clearing, i.e.

$$u_t = 0. \quad (\text{NC.1})$$

For the Keynesian DSLMD and TPK models, we consider two closures. The first one assumes the detrended rate of wage inflation to be constant, i.e.

$$\Pi^{\tilde{w}} = \bar{\Pi}. \quad (\text{PK1.1})$$

The second one assumes collective bargaining between the worker's and firms's representatives over wage inflation detrended by productivity growth. The Nash bargaining problem reads

$$\max_{\Pi_t^{\tilde{w}^*}} [\tilde{\omega}(\Pi_t^{\tilde{w}^*})]^{v_t} [r(\Pi_t^{\tilde{w}^*})]^{1-v_t} \quad (\text{PK2.1})$$

with

$$v_t = (1 - u_t)V_t \quad (\text{PK2.2})$$

where  $\omega(\cdot)$  and  $r(\cdot)$  denote the steady states of the real wage and the profit rate, respectively, as functions of the target wage inflation.  $V_t$  is an auto-regressive shock to the bargaining power,

$$V_t = (V_{t-1})^{\rho_V} V^{1-\rho_V} \exp \varepsilon_{V,t}, \quad (\text{PK2.3})$$

where  $\varepsilon_{V,t}$  are exogenous innovations. The FOC of this problem determines the desired rate of wage inflation,  $\Pi_t^{\tilde{w}^*}$  and reads

$$1 = (1 - 1/v_t) \frac{\tilde{\omega}(\Pi_t^{\tilde{w}^*})}{r(\Pi_t^{\tilde{w}^*})} \frac{r'(\Pi_t^{\tilde{w}^*})}{\tilde{\omega}'(\Pi_t^{\tilde{w}^*})}. \quad (\text{PK2.4})$$

The evolution of the rate of wage inflation is then assumed to be

$$\Pi_t^{\tilde{w}} = (\Pi_{t-1}^{\tilde{w}})^{\rho_w} (\Pi_t^{\tilde{w}^*})^{1-\rho_w}. \quad (\text{PK2.5})$$

The DSLMD, TPK, DSGE and SNC models are characterized by the following equations:

	model equations	model closure
DSLMD with constant wage inflation	(MF.1)-(MF.28)	(PK1.1)
DSLMD with bargaining	(MF.1)-(MF.28)	(PK2.1)-(PK2.5)
TPK with constant wage inflation	(AG.1)-(AG.22)	(PK1.1)
TPK with bargaining	(AG.1)-(AG.22)	(PK2.1)-(PK2.5)
DSGE	(MF.1)-(MF.28)	(NC.1)
SNC	(AG.1)-(AG.22)	(NC.1)

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