

School of Economics and Finance

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Working Paper No. 721

July 2014

ISSN 1473-0278



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Abstract

We develop a model of relational contracts with moral hazard and asymmetric persistent information about an employee's type. We find that the form of the optimal contract depends on the job characteristics as well as the distribution of employees' talent. Bonus contracts are more likely to be adopted in complex jobs and when high talent is not too common or too rare. Firms with 'normal' jobs are more likely to adopt termination contracts. In labor market equilibrium, different contracts may be adopted by ex ante identical firms. Hence, we offer an explanation for the co-existence of different employment systems within the same industry.

Keywords: Relational contracts, Job characteristics, Employment systems, Labor market segmentation

JEL classification: D82, M5, L14

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1 Introduction

Employment contracts are often incomplete and informal reputational mechanisms are prominently used in organizations, where informal codes of conduct affect agents' behavior. A growing empirical literature documents employers' use of firing, promotion or bonuses based on informal agreements to motivate their employees.¹ MacLeod and Parent (2013) empirically show that reputational mechanisms can provide insight into the form of compensation and that job characteristics affect the nature of the compensation scheme offered to employees.

In this paper, we develop a model of self-enforceable relational contracts when the employee's type is his private information. The model allows us to understand how the characteristics of the job, the labor market, or the firm affect the compensation package used by employers and their hiring and firing policies. We also show that in labor market equilibrium, a priori identical firms may choose to offer different yet equally profitable contracts, with different hiring and firing policies. Hence, the paper also helps explain why different employment systems may co-exist within the same industry. In the US manufacturing industry some firms adopt a flexible organizational structure, with employees' involvement and team work, while others adopt a more rigid job structure oriented towards cost cutting (Osterman, 1994). Hunter (1999) offers evidence of the co-existence of different employment models in the US retail banking sector and Bailey and Sandy (1999) examine the different practices in the apparel industry.

In the model, a continuum of heterogeneous workers and a continuum of homogeneous employers play an infinitely repeated game. Each employee privately observes his talent. Each firm has one job to fill, and jobs are assumed to be in short supply. An employee's job consists of handling one task in each period, and the firm owner's problem is to motivate him to successfully perform this task. To succeed in a task, an employee needs ability and effort. The more difficult a task is, the more able an employee must be to successfully perform it. At the end of each period, the employer observes the employee's performance. However, performance cannot be verified by a third party, and therefore, a contract based on such performance cannot be enforced by a court of law. A performance-based bonus effectively motivates an employee to exert effort only if it is credible that the employer will not renege on its payment.

¹MacLeod and Parent (1999) analyze different incentive schemes used in the US; 14% of the individuals in their data set received informal bonus pay. Hayes and Shaefer (1997) provide evidence for the use of subjective performance measures when boards of directors decide the salaries and bonuses of chief executives. Cappelli and Chauvin (1991) show that termination contracts are used as an incentive device. Altmann et al. (2014) provide experimental evidence of the link between contractual incompleteness and unemployment. They also find that contractual incompleteness leads to the co-existence of different job types, i.e., jobs that are offering different wages and performance requirements.

This requires that the rent of the employer from continuing the relationship with a successful employee is sufficiently high. In our setting, where jobs are in short supply, the only rent for the employer comes from the potential difference in talent between the current employee and the average unemployed person. Consequently, a bonus can only be used to motivate employees with high talent whose talent is sufficiently rare. Therefore, when a firm uses a bonus to motivate an employee, the contract offered is only acceptable for high-type employees. Whenever the firm is matched with a low-type employee, the contract is rejected, and the position remains vacant.

An alternative mechanism to motivate employees to exert effort is to guarantee sufficiently high gains for the employee from continuing the employment relationship. In that case, the employee exerts effort to retain his job. The principal offers a pooling contract with a fixed wage and a performance requirement that the employee must achieve to remain employed. The vacancy is filled in every period; however, whenever an employee with low talent is unable to reach the performance requirement, he is fired. Therefore, the tenure of employees in jobs where an efficiency wage contract is used is shorter than the tenure in jobs in which a bonus contract is offered.

When a principal uses a bonus contract, he cannot motivate a low-type employee to exert effort, and therefore, whenever the employer is matched with such an employee, the position remains vacant. Therefore, the higher the likelihood is to be matched with a high-talent employee, the higher the benefit is from adopting a bonus contract. However, if the fraction of high-type employees is too high, the principal cannot credibly promise to pay the bonus. Therefore, the bonus contract is more likely to be adopted when high talent is not too common or too rare.

For a given distribution of talent, if agents with low talent are able to solve fewer tasks, then a company is more likely to adopt a bonus contract. Indeed, in jobs for which most of the tasks require the expertise of a highly talented employee, the opportunity cost of not hiring low-talent employees is lower. Hence, bonus contracts are more likely to be found in ‘complex jobs’. In ‘normal’ jobs, i.e., those in which low-type employees are able to solve a larger fraction of tasks, an efficiency wage contract is more likely to be used.

The various forms of contracts lead to different levels of selection of employees. With a bonus contract, employees are selected *ex ante*, and the contract is only accepted by high-type employees. With an efficiency wage contract, there is endogenous turnover and on-the-job selection by the employer.² The stationary distribution of talent in the pool of unemployed agents therefore depends on the

²For evidence of on-the-job selection, see, for example, Lazear and Show (2008) and Frederiksen and Takats (2011).

contracts offered by employers. If all firms offer a bonus contract, then the likelihood of finding an employee with high talent in the pool of unemployed workers is relatively low, which makes the adoption of a termination contract a better choice for an individual employer with vacant position. Hence, in labor market equilibrium, different contracts may coexist.

Related literature. The early literature on relational contracts focuses on frameworks with symmetric information (see Shapiro and Stiglitz (1984) and Bull (1987), among others). MacLeod and Malcomson (1989) proposed the first treatment of the problem. They proved that provided the surplus created by the continuation of the employment relationship is sufficiently high, either a performance-based bonus or a termination contract can be sustained in equilibrium.³ MacLeod and Malcomson (1998) go further and stress the importance of labor market conditions in the choice between a termination contract and a bonus. When agents are homogeneous in skills and there is unemployment, the principal can replace an employee with an unemployed agent at no cost. There is no rent for the employer to continue the employment relationship. Thus, the bonus is not credible, and the termination contract is the only way to motivate employees. Conversely, when the latter are in short supply, vacancies cannot be immediately filled, which creates rent from retaining an agent. Hence, whenever the cost of unoccupied jobs is not too high, an equilibrium with full employment and bonus could emerge. Differently from these papers, we consider that employees are heterogeneous in their ability, and this ability is unknown to the employer.

Levin (2003) analyzes relational contracts when the agent's type is private information. However, he assumes that the type is non-persistent, *i.e.*, it changes from period to period, and the focus of his analysis is on the restrictions on revelation due to the self-enforcing character of the agreement. Yang (2012) considers a setting of relational contracts with asymmetric persistent information about the employee's type and non-contractible performance. The focus of his paper is on demonstrating that the optimal pooling contract is non-stationary and on deriving implications in terms of wage progressions over time. In his setting, the separating contract is always optimal, and the arguments against it being used by employers are exogenous to the model. Differently from his paper, we show that the form of the optimal relational contract differs depending on the characteristics of the job and the labor market conditions. Additionally, in the present paper, in labor market equilibrium different types of contracts may coexist.

Halac (2012) studies relational contracts with persistent private information in

³Because both the principal and employees are risk-neutral, the reward is a pure transfer.

a different setting. Her paper considers the dynamics of a relationship in which the principal has private persistent information about her outside option.

We also contribute to the literature by offering an endogenous theoretical explanation of the existence of segmentation in the labor market.⁴ The closest paper to ours is Board and Meyer-ter-Vehn (2013). They also consider a relational contract setting. However, differently from our paper, they assume that employees are homogeneous. In their article, segmentation is driven by the possibility of on-the-job search. In the present paper, we do not allow for on-the-job search, but employees are heterogeneous. Also, in our paper segmentation translates into different employment systems, not only in terms of the level of reward but also in terms of hiring and firing practices.⁵

The rest of the paper is organized as follows. Section 2 presents the setup of the model and a benchmark case in which the principal can tie pay to performance. Section 3 characterizes the optimal relational contract adopted by an individual employer for the given characteristics of the pool of unemployed agents and given the outside option of the employees. Section 4 presents the equilibrium in the labor market and shows that different types of contracts may coexist in equilibrium. In Section 5, we extend the model to consider a continuum of types. Then, we conclude. All proofs can be found in the Appendix.

2 Model

2.1 The setting

We consider a firm (employer, principal) with one occupation (job) and a continuum of potential employees (agents) with a mass of one. The firm and the agents live forever and discount the future at the same rate δ . Time is discrete. Each agent is endowed with ability (talent) θ , where $\theta \in \{\underline{\theta}, \bar{\theta}\}$, with $\bar{\theta} > \underline{\theta}$. An employee is privately informed about his ability. The employer only observes the distribution of types $Prob(\theta = \bar{\theta}) = \bar{p}$ and $Prob(\theta = \underline{\theta}) = 1 - \bar{p}$. For the purpose of this section, we assume that the unconditional distribution of talent in the pool of available employees remains the same over time. The first step in the analysis consists of solving for the optimal relational contract from the prospective of a single employer and

⁴See for example, Albrecht and Vroman (1992) and Burdett and Mortensen (1998), among others.

⁵Bartling et al. (2012) show in an experimental study the existence of complementarity between high wages, effort discretion, and the possibility to observe information about an employee's past performance. Hence, they offer insight on the bundling of employment practices into 'control' vs. 'trust' strategies. However, the co-existence of these practices under a specific experimental treatment results from sub-optimal choices by some of the agents.

taking the employees' outside option and the unemployed agents' talent distribution as given. The labor market equilibrium will be characterized in Section 4.

At the beginning of a period, if a job is vacant, the employer is randomly matched with an agent from the pool of unemployed workers. In each employment period t , the principal offers a contract to the employee. For a newly formed match, $t = 0$. The employee decides to accept or reject the contract. If the contract is rejected, the position remains unoccupied for the period, thus generating a period profit of zero, and the employee receives his inter-temporal outside utility V^{out} . At this point, we assume that V^{out} is independent of the employee's type. In Section 4, we show that in an anonymous labor market, the equilibrium inter-temporal utility of unemployed agents is independent of their talent. The contract comprises a fixed wage w_t paid at the beginning of a period (i.e., independent of a worker's performance) and a bonus b_t that the firm promises to pay if the worker's performance is satisfactory. The contract may also comprise a minimum performance requirement y_t that the employee is expected to achieve to retain his job. The firm is committed to the wage of the current period only.

In order to introduce the idea of task heterogeneity we use a production function in the spirit of Garicano (2000).⁶ Production requires tasks (problems) to be successfully performed. These tasks differ in terms of their difficulty. In each employment period, an employee receives a task with difficulty x , drawn from a commonly known distribution $F(\cdot)$ on the bounded support $[0, \bar{\theta}]$, i.i.d. over time. An employee who receives a task x chooses whether to exert effort e . The choice is binomial; the employee either works $e = \bar{e}$ or shirks $e = \underline{e}$. Working is costly, and the cost function is given by:⁷

$$c(x, \theta, \bar{e}) = \begin{cases} c & \text{if } x \leq \theta \\ +\infty & \text{if } x > \theta \end{cases}$$

The effort-cost function implies that an employee only exerts effort on tasks within his ability set ($x \in [0, \theta]$). An employee who exerts effort succeeds in resolving a problem with a probability e . For simplicity, we assume that $\bar{e} = 1$ and $\underline{e} = 0$. This type of production function describes well for example jobs that require handling clients' requests, such as customer services, loan approval process in a retail bank etc.

The modeling adopted here is appealing for several reasons. First, it enables us to introduce and discuss the impact of job characteristics on the choice of the optimal relational contract. Second, it allows us to simplify the analysis without losing in terms of generality. Throughout the paper and in Section 5, we discuss the

⁶In Garicano (2000) succeeding on a task only requires knowledge and the paper's focus is on the optimal organizational structure.

⁷Shirking is costless: $c(x, \theta, \underline{e}) = 0$.

assumptions and their implications on the generality of the results.

The expected one-period surplus from employing an agent with ability θ is $F(\theta)(1 - c) - (1 - \delta)V^{out}$. For the remainder of the paper, we will focus on the interesting cases in which the surplus created from employing an agent with type $\underline{\theta}$ is positive.

Assumption 1. $F(\underline{\theta})(1 - c) - (1 - \delta)V^{out} > 0$

Assumption 1 implies that it is socially optimal to hire any employee with whom a firm is matched (at least) for the current period.

At the end of each employment period, the employee and the employer observe the difficulty of the task and whether it has been successfully performed. However, this information cannot be verified by a third party such as a court of law, and hence, it is impossible to write a legally binding contract tying pay to performance. Once payments have been made (or not), each party can decide to leave the current relationship. With probability $(1 - \alpha)$, an employer-employee match is ended for exogenous reasons. We assume that α is independent of the number of periods an employee has been in a relationship with the employer, it also does not depend on past performance. $(1 - \alpha)$ can be interpreted as the probability of a match to become unprofitable for exogenous reasons. Consistent with this interpretation, after the separation, the employee joins the pool of unemployed agents and receives his outside option, while the employer has a vacant position to fill.

The timing of one period of employment is summarized in Figure 1 below.

Before characterizing the optimal relational contract in Section 3, we present a benchmark corresponding to the case in which the principal can offer a legally enforceable contract based on an employee's performance.

2.2 Benchmark: Optimal contract with commitment

Let us assume that the principal can credibly offer a contract contingent on the employee's performance in the current period. As we will see, a menu of contracts with short-term commitment implements the socially optimal solution and allows the principal to extract all the surplus created by the employment relationship.

Proposition 1. *The set of optimal menus of contracts is as follows:*

$$\begin{cases} \underline{C} = (w = (1 - \delta)V^{out}, b(x \in [0, \underline{\theta}]) = c, b(x \in [\underline{\theta}, \bar{\theta}]) = 0) \\ \bar{C} = (w = (1 - \delta)V^{out} - (1 - F(\underline{\theta}))\epsilon, b(x \in [0, \underline{\theta}]) = c, b(x \in [\underline{\theta}, \bar{\theta}]) = c + \epsilon), \end{cases}$$

where $\epsilon \geq 0$. An employee with type $\underline{\theta}$ selects \underline{C} , and an employee with type $\bar{\theta}$ selects \bar{C} .

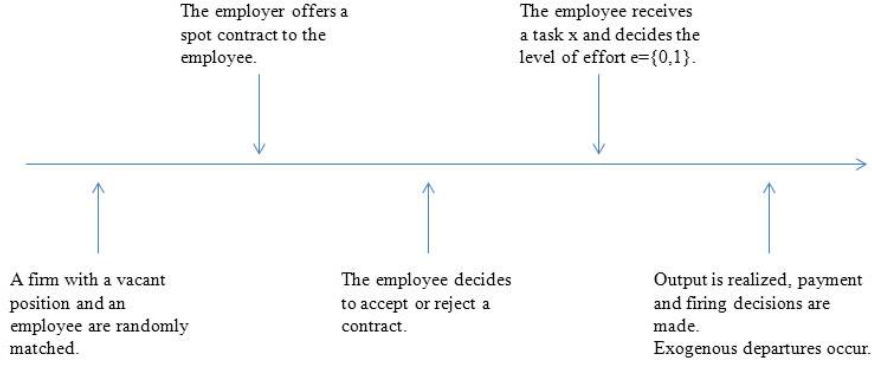


Figure 1: Timing of the employment relationship

The set of contracts described in Proposition 1 allows the principal to satisfy the participation constraint of any employee $\theta \in \{\underline{\theta}, \bar{\theta}\}$ with equality and to guarantee an employee's incentives to exert effort and truthfully reveal his type. The inter-temporal expected profit of the employer under the set of contracts defined in Proposition 1 is independent of ϵ , and we notice that the principal extracts all the surplus created by the employment relationship:

$$(1) \quad \Pi^{com} = \frac{(1 - \bar{p})(F(\underline{\theta})(1 - c) - (1 - \delta)V^{out})(1 - \alpha\delta) + \bar{p}(1 - c - (1 - \delta)V^{out})}{(1 - \delta)(1 - \alpha\delta(1 - \bar{p}))}$$

An employee with revealed talent $\underline{\theta}$ is always fired after one period of employment. The reason is that the inter-temporal expected profit from an employee $\underline{\theta}$ is always lower than the inter-temporal expected profit from the average employee. This holds as long as there is no firm-specific human capital. If we account for the possibility of acquiring firm-specific human capital while on the job, it may become interesting for a company to retain an employee $\underline{\theta}$.

If the principal can commit to a performance-based contract, then in the current setting, despite the presence of moral hazard and adverse selection, the socially

optimal outcome can be implemented. In this setting, any distortion from the social optimum will be driven by the impossibility of writing a legally binding performance-based contract.

3 The optimal relational contract

In what follows, we assume that performance (i.e., the difficulty of a task an employee has been able to solve or not) is observable by the employee and the employer but cannot be verified by a third party. The solution concept is perfect public equilibrium. Public strategies require that each party's strategy be dependent on the public history within the current relationship. A newly hired employee is unaware of the circumstances under which the previous employee working in that position left the firm.⁸ The public history of an employee's performance boils down to knowing whether the employee has been able to solve a problem with difficulty $[0, \underline{\theta}]$ and/or $(\underline{\theta}, \bar{\theta}]$ conditional on the contract that he has been offered.⁹ An employee's strategy consists of deciding whether to accept the contract and, after receiving a task, whether to exert effort. A strategy for the firm specifies whether to fire a worker and what spot contract to offer, with both as a function of public history.

Because performance is not verifiable by third parties, the principal cannot write a legally enforceable contract based on performance. The principal and the agent can agree on a relational contract enforced by the possibility of future actions for each of them. A contract is self-enforceable if it is in both parties' interest to abide by it. As we will show in the following subsections, an employee exerts effort either to receive a performance-based bonus or to retain his current job.

3.1 Bonus contracts and *ex ante* selection

Let us first consider the possibility of using a bonus contract to motivate an employee to exert effort.

Proposition 2. *A bonus contract cannot be used to motivate employees with talent θ to exert effort.*

A bonus effectively motivates an employee to exert effort only if it is credible that the employer will pay it after observing the employee's performance. Such a promise

⁸In Section 4, where we solve for the market equilibrium, we make the symmetric assumption that future employers do not observe the employment history of an agent with whom they are matched.

⁹In general terms, the employee's history can be summarized by the most difficult problem the employee has resolved and the least difficult one he has failed to resolve, subject to the contract that he has been offered.

is only credible if the rent the employer obtains from continuing the relationship with a successful employee is sufficiently high. In our framework, because the principal is matched with an agent from the unemployment pool after firing an employee, the only rent for the employer comes from the potential difference in talent between the current employee and the average unemployed agent. An employee with ability $\underline{\theta}$ is less talented than the average unemployed worker; therefore, the principal cannot credibly promise to pay a bonus to a successful employee with revealed ability $\underline{\theta}$.

We further notice that the principal cannot motivate an employee with talent $\underline{\theta}$ to exert effort by using a pooling bonus contract. In that case, the expected ability of an employee who has successfully solved a problem $x \in [0, \underline{\theta}]$ is $\bar{p}\bar{\theta} + (1 - \bar{p})\underline{\theta}$, which is equal to the expected talent of an unemployed agent. There is no rent, and therefore, the bonus cannot be credibly offered.

To use a bonus to motivate an employee with talent $\bar{\theta}$ to exert effort on all tasks $x \in [0, \bar{\theta}]$, the principal must separate the employees *ex ante* by offering a contract that is only accepted by employees with high talent ($\bar{\theta}$). An employee with talent $\underline{\theta}$ rejects the contract and takes his outside payoff V^{out} .¹⁰

Lemma 1. *When a principal offers a contract from the set $C_b = (b(x \in [0, \bar{\theta}]) = c + \epsilon, w = (1 - \delta)V^{out} - \epsilon)$, where $\epsilon > 0$, subject to being self-enforceable, the contract is accepted by an employee with talent $\bar{\theta}$ and is rejected by an employee with talent $\underline{\theta}$.*

A contract from the set $C_b = (b(x \in [0, \bar{\theta}]) = c + \epsilon, w = (1 - \delta)V^{out} - \epsilon)$ is accepted by high-talent employees and motivates them to exert effort only if it is credible that the principal will pay the bonus once he has observed a high degree of effort by an employee. The self-enforceability constraint must guarantee that the principal is better off by paying the bonus to an employee with talent $\bar{\theta}$ who has successfully performed a task $x \in [0, \bar{\theta}]$ rather than not paying it:

$$(2) \quad \Pi_b(\bar{\theta}) - \Pi_b^{dev} \geq \frac{c + \epsilon}{\alpha\delta}$$

To effectively deter renegeing by the firm, we assume that the employees follow the trigger strategy: exert effort in t only if the company has paid the bonus in period $t - 1$; otherwise, quit immediately. This strategy corresponds to the most severe punishment, and the penalty for the employer is given by the alternative available in the market:

$$(3) \quad \Pi_b^{dev} = \frac{\bar{p}(1 + \epsilon - (1 - \delta)V^{out})}{1 - \delta}$$

¹⁰Full separation is specific to the assumption that an employee's talent can only be of two types. In a model where the talent θ can be of more than two types, the equilibrium will exhibit some degree of pooling.

Note that the higher the bonus is (i.e., the higher ϵ), the more difficult it is to satisfy the self-enforceability constraint.¹¹ Therefore, the largest set of parameters for which the contract of Lemma 1 can be credibly implemented is obtained for $\epsilon \rightarrow 0$. Because information is revealed in the first period of employment, there is no gain from using non-stationary contracts. If the principal were to offer a bonus that changes with the employee's past performance, the self-enforceability constraint that would be the most difficult to satisfy would be that for the highest bonus. Therefore, focusing on a stationary contract $C_b = (b(x \in [0, \bar{\theta}]) = c + \epsilon, w = k - \epsilon)$ with $\epsilon \rightarrow 0$ defines the largest set of parameters for which a self-enforceable bonus contract can be offered to an employee with talent $\bar{\theta}$.¹²

Proposition 3. *If $\bar{p} < \bar{p}^{SE}$ (where $\bar{p}^{SE} \equiv 1 - \frac{c}{\alpha\delta(1-(1-\delta)V^{out})}$), then a contract $C_b = (b(x \in [0, \bar{\theta}]) = c + \epsilon, w = (1 - \delta)V^{out} - \epsilon)$ with $\epsilon \rightarrow 0$ is self-enforceable.*

The bonus can be credibly used to motivate employees $\bar{\theta}$ only if their talent is sufficiently rare. If the principal can be easily matched with a high-type employee, then the rent from continuing the ongoing employment relationship is not sufficiently high, and the principal is likely to renege on the payment of the bonus.

The inter-temporal expected profit for the principal as a result of offering the separating bonus contract C_b when the self-enforceability condition holds is as follows:

$$(4) \quad \Pi_b = \frac{\bar{p}(1 - c - (1 - \delta)V^{out})}{(1 - \delta)(1 - \alpha\delta(1 - \bar{p}))}$$

The inter-temporal expected profit increases with the fraction of employees with talent $\bar{\theta}$. Offering a contract that is only accepted by employees with high talent increases when the employer is more likely to be matched with a high-type employee. However, if there are many high-type employees in the pool of available agents, then the principal cannot credibly promise to pay the bonus.

Notice that when the pure bonus contract is adopted by the principal, there is an inefficiently low employment of agents with talent $\underline{\theta}$. Indeed, those agents never accept the bonus contract described in Lemma 1, and the position remains vacant, even if it is socially optimal to hire an employee with low talent for one period.

¹¹If we were to consider a more general cost function that increases with the difficulty of the task, for example, then the principal could offer a bonus that depends on the level of difficulty of the task performed by an employee, and the strongest self-enforceability constraint would be the one for the highest bonus offered by the principal. In that case, the principal may be able to motivate an agent with talent $\bar{\theta}$ to only solve a subset of tasks for which the bonus is credible.

¹²Considering a non-stationary bonus contract would reduce the set of parameters for which the contract can be credibly used.

3.2 Efficiency wages and *ex post* selection

In the previous section, we demonstrated that in the context of non-verifiable performance, the principal cannot motivate employees with talent $\underline{\theta}$ by promising a bonus contingent on performance and that if \bar{p} is too high, then the bonus cannot be credibly used to motivate employees $\bar{\theta}$ either. An alternative mechanism to motivate employees to exert effort is to guarantee sufficiently high gains for the employee from continuing the employment relationship. In that case, the employee exerts effort to retain his job. The termination contract in our setting is the pair $(w_t(y_t, \theta), y_t)$, where $w_t(y_t, \theta)$ is the fixed wage offered to employees with talent θ at the beginning of period t of the employment relationship and y_t is a minimum performance requirement, i.e., employees are expected to solve any task $x \leq y_t$ that they receive.¹³ If such a task is not resolved, the employee is fired. Firing an employee who has not been able to solve a task $x < \min\{\underline{\theta}, y_t\}$ is in the principal's interest if he believes that the employee will continue shirking in the future.¹⁴ If an employee has failed at a task $x \in (\underline{\theta}, y_t]$, the principal updates his beliefs about the employee, and firing the employee again becomes the best response.

Let $V_t(\theta, w_t(y_t), y_t)$ be the inter-temporal expected utility of an employee with talent θ who exerts effort on tasks $x \leq \min\{\theta, y_t\}$ in period t of the employment relationship, when the efficiency wage contract is $(w_t(y_t, \theta), y_t)$. The incentive compatibility constraint in employment period $t - 1$ of an agent θ who has received a task $x \leq \min\{\theta, y_{t-1}\}$ is as follows:

$$(5) \quad -c + \alpha \delta V_t(\theta, w_t(y_t, \theta), y_t) \geq \alpha \delta V^{out}$$

From equation (5), we notice that in period $t - 1$, an employee is motivated by future rents, i.e., the fixed wage of period t . However, if the firm owner decides to renegotiate and offer a wage w'_t lower than the perceived 'fair wage' w_t in the beginning of t , the worker would respond by shirking. It is indeed in the employee's interest to do so if he anticipates that an employer who reduces the wage in this period will also do so in future periods.

$$(6) \quad \begin{aligned} V_t(\theta, w_t(y_t, \theta), y_t) &= w_t(y_t, \theta) - F(\min\{y_t, \theta\})c + \delta \alpha (F(\max\{y_t, \theta\}) - F(\theta))V_{out} + \delta(1 - \alpha)V_{out} \\ &\quad + \delta \alpha (1 - F(\max\{y_t, \theta\}) + F(\theta))V_{t+1}(\theta, w_{t+1}(y_{t+1}, \theta), y_{t+1}) \end{aligned}$$

Lemma 2. *If a principal wants to motivate both types of agents, he offers a pooling termination contract.*

There is no gain for the principal from using a menu of separating termina-

¹³We allow for the possibility of offering a menu of termination contracts.

¹⁴Any employee is able to solve tasks $x < \min\{\underline{\theta}, y_t\}$.

tion contracts. The intuition is as follows. An agent exerts effort to retain his job. An employee with revealed talent $\underline{\theta}$ is less able than the average unemployed worker; therefore, the principal will credibly retain such an employee only if his inter-temporal expected payoff from an employee $\underline{\theta}$ is the same as his inter-temporal expected payoff from an employee with talent $\bar{\theta}$. This implies that the inter-temporal expected profit of the principal when using a separating menu of contracts will never exceed his expected profit from an agent with talent $\underline{\theta}$. Therefore, a pooling contract non-strictly dominates a menu of separating termination contracts. The formal proof can be found in the Appendix.

The pooling efficiency wage contract is accepted by both types of agents. The principal updates his beliefs based on the employee's performance and decides whether the employee should be retained. For the contract to be incentive-compatible and self-enforceable, it must be in the principal's interest to retain any employee who has succeeded at a task $x < y$ and fire any employee who has failed at $x < y$.

Proposition 4.

- i) The minimum performance requirement y_t is never lower than $\underline{\theta}$.*
- ii) If $\bar{p} > c$, then the optimal efficiency wage contract is stationary, the performance requirement is $y = \bar{\theta}$, and the fixed wage is $w(\bar{\theta}) = \frac{c}{\alpha\delta} + (1 - \delta)V^{out}$.*

The condition $\bar{p} > c$ is sufficient and not necessary. When \bar{p} becomes very low, the optimal efficiency wage contract is still stationary, but the minimum performance requirement is $y = \underline{\theta}$ and $w(\underline{\theta}) = \frac{c(1 - \alpha\delta)}{\alpha\delta} + F(\underline{\theta})c + (1 - \delta)V^{out}$. The stationarity of the optimal termination (and bonus) contract is driven by the specifics of the model, particularly the fact that talent can only be of two types. If we consider a setting with more than two types, the optimal contract may exhibit some non-stationarity, as will be discussed in Section 5 and in Appendix 8.11.

The inter-temporal expected payoff to the principal when he offers an efficiency wage contract $(w(y), y)$ is as follows:

$$(7) \quad \Pi(w(y)) = \frac{\bar{p} \frac{F(y) - w(y)}{1 - \alpha\delta} + (1 - \bar{p}) \frac{F(\underline{\theta}) - w(y)}{1 - \alpha\delta(1 - F(y) + F(\underline{\theta}))}}{1 - \bar{p} \frac{\delta(1 - \alpha)}{1 - \alpha\delta} - (1 - \bar{p}) \frac{(1 - \alpha)\delta + \alpha\delta(F(y) - F(\underline{\theta}))}{1 - \alpha\delta(1 - F(y) + F(\underline{\theta}))}}$$

With the efficiency wage contract $(w(y), y)$, an employee with ability $\underline{\theta}$ is not fired after the first period of employment but only when (if at all) the principal discovers that his talent is $\underline{\theta}$, i.e., when the employee fails to solve a task $x \in (\underline{\theta}, y]$. This, in turn, implies that there is too much retention of employees with low levels of ability compared to the social optimum.

3.3 The optimal contract

We have characterized the types of relational contracts available to the principal to motivate an employee to exert effort, and we have defined the self-enforceability conditions for the different types of contracts. In the following, we first establish a necessary condition to be able to elicit some effort in equilibrium, and we then characterize the conditions under which the different contracts are likely to be adopted by a profit-maximizing employer.

Lemma 3. *If $c \geq \alpha\delta(1 - (1 - \delta)V^{out})$, then no effort is exerted in equilibrium.*

If the cost of effort is high and/or the value of the future payoffs is low because of a low probability of staying in the relationship or because of a low discount rate, then the principal cannot achieve high effort in equilibrium. If $c \geq \alpha\delta(1 - (1 - \delta)V^{out})$, then, on the one hand, the bonus contract is not self-enforceable, and, on the other, the rent that must be offered to motivate an employee to exert effort is too high and makes employment unprofitable for the firm. Below, we focus on the cases where $c < \alpha\delta(1 - (1 - \delta)V^{out})$.

Proposition 5. *The optimal relational contract as a function of the characteristics of the pool of available employees is as follows:*

- i) For $\bar{p} \in [\bar{p}^{SE}, 1]$, the optimal relational contract is $(w(\bar{\theta}), \bar{\theta})$.*
- ii) For $\bar{p} \in [\bar{p}'', \bar{p}^{SE}]$, the optimal relational contract is a bonus accepted only by the employees with talent $\bar{\theta}$.*
- iii) For $\bar{p} \in [\bar{p}', \bar{p}'']$, the optimal relational contract is $(w(\bar{\theta}), \bar{\theta})$.*
- iv) For $\bar{p} \in [0, \bar{p}']$, the optimal relational contract is $(w(\underline{\theta}), \underline{\theta})$.*

The threshold \bar{p}'' is given by the following:

$$(8) \quad \Pi(w(\bar{\theta})) = \Pi_b$$

and the threshold \bar{p}' is obtained from the following:

$$(9) \quad \Pi(w(\bar{\theta})) = \Pi(w(\underline{\theta}))$$

The choice of the optimal relational contract depends not only on the distribution of talent but also on the characteristics of the job that a firm offers. Proposition 5 characterizes the choice of the optimal relational contract depending on the likelihood of being matched with a high-type employee. Hence, a bonus contract is more likely to be used in a context where employees with high talent are neither too rare nor too common.

Corollary 1. \bar{p}' and \bar{p}'' increase with the fraction of tasks that can be solved by an employee with talent $\underline{\theta}$.

For a given distribution of talent in the unemployment pool, if an employee with talent $\underline{\theta}$ is able to solve a lower fraction of tasks, then a company is more likely to adopt a bonus contract. Indeed, the cost of using a bonus contract relative to an efficiency wage contract is that employees with talent $\underline{\theta}$ are never hired. A low $F(\underline{\theta})$ corresponds to a low productivity of the employees with talent $\underline{\theta}$. Therefore, the opportunity cost of not hiring low-talent employees is lower, and a bonus contract is more likely to be adopted by an employer. The opposite is true for jobs where $F(\underline{\theta})$ is high. In that case, a termination contract is more likely to be offered by the principal.

$F(\theta)$ captures both the characteristics of the job and the characteristics of the population of agents. For example, for a given talent of employees $\underline{\theta}$, a higher $F(\underline{\theta})$ would correspond to a job with a higher fraction of basic tasks, while a lower $F(\underline{\theta})$ would imply that the job is characterized by a prevalence of exceptional tasks. For a given distribution of tasks, a lower $F(\underline{\theta})$ corresponds to a higher ability gap between skilled and unskilled agents.¹⁵ Hence, a bonus contract is more likely to be adopted in complex jobs and when the skill gap between employees is higher. A termination contract is more likely to be observed in ‘normal’ jobs and when the ability gap is not too high.

The choice of the employment contract also has implications on the employer’s hiring and firing practices. In ‘normal’ jobs for which a termination contract is adopted, vacancies are filled more quickly, but the turnover is higher because employees are selected on-the-job. In contrast, in complex jobs for which a bonus contract is used, filling a vacancy takes longer, but the turnover rate is lower. MacLeod and Parent (2013) provide empirical support for the existence of a longer tenure in complex jobs for which bonus contracts are used.

In our model the distinction between ‘normal’ vs ‘complex’ jobs and therefore the optimal relational contract can evolve with changes in the technology, and in the characteristics and supply of skills. If, for example, a new technology affects the distribution of tasks by decreasing $F(\underline{\theta})$, this would affect the nature of the job and the employer could move from termination to bonus contract. Identifying such changes could allow to empirically test some of our theoretical results.

When introducing the model, we made the assumption that all tasks are equally valuable. This assumption could be easily relaxed. Depending on which ‘easy’ or ‘difficult’ tasks are more valuable to the firm, the choice of a termination contract will be more likely in the former case, and the choice of a bonus contract will be

¹⁵In the latter case, the lower $F(\underline{\theta})$ is only driven by a decrease in $\underline{\theta}$.

more likely in the latter.

4 Market equilibrium

In this section, we endogenize the outside options for employees and for the firm, and we characterize the stationary labor market equilibrium. There is a continuum of identical firms with size n .¹⁶ Each employer has one job to fill.

Assumption 2. *Jobs are in short supply $n < 1$.*

If employees were in short supply ($n > 1$), there would be rent from retaining an employee due to the scarcity of the workforce. The impact of this effect on the form of the contract is discussed by MacLeod and Malcomson (1998). Hence, we focus on the cases of unemployment where any rent from retaining an employee is related to the gap between his and the average unemployed agent's expected productivity.

Assumption 3. *The labor market is anonymous.*

The reasons a particular employer-employee match is terminated are not observed by any party in the labor market, whether employers or employees. The idea behind this is that monitoring the reasons for a separation is difficult.¹⁷ The implication of Assumption 3 is that it is impossible to build an external reputation. If either party is allowed to build an external reputation, then not respecting the contract leads to the loss of the cooperation of current and future partners. This potential loss of cooperation would increase the contractor's incentives to respect the terms of the agreement.

No principal is 'large enough' to have an impact on market conditions. Hence, when maximizing his profit – under the incentive compatibility and self-enforceability constraints – the employer takes the employees' outside option V^{out} and the outsiders' distribution \bar{p} as given, i.e., the decision problem of an individual employer is as described in Section 3. The endogenous values for V^{out} and \bar{p} are consistently determined in market equilibrium.

Employees' outside option. An unemployed agent receives an unemployment benefit k for each period of unemployment, and at the beginning of each period, he is matched with an employer offering a vacant position with the endogenous re-employment probability λ . Hence, the inter-temporal expected utility of an unemployed agent is as follows:

$$(10) \quad V^{out}(\theta) = k + \delta\lambda V_C(\theta) + \delta(1 - \lambda)V^{out}(\theta),$$

¹⁶Let us reiterate that the mass of available employees is 1.

¹⁷It is difficult to collect information about an employee from previous employers.

where $V_C(\theta)$ is the inter-temporal expected utility of an employee with talent θ who is employed and offered a contract C .

The re-employment probability λ guarantees the equilibrium of flows between the firm and the labor market. The steady-state flow into the unemployment pool is $n \times X$, where X is the fraction of jobs that become vacant during the period. This fraction of jobs can be decomposed as follows: $X = (X_R + X_S)$. X_R is the fraction of agents who enter the unemployment pool at the beginning of the period because they have rejected the contract that has been offered by the employers with which they have been matched; X_S is the fraction of agents who enter the pool at the end of the period due to exogenous or endogenous separations. These fractions depend on the type of contract that is adopted by the firms and are formally defined in Table 1 below.

Without a loss of generality, we assume that an agent cannot be immediately reemployed and spends one period in the unemployment pool. An agent who, at the beginning of period T , is matched with a company with a vacant position and who decides to reject the employer's offer immediately reenters the pool of unemployed. He receives the unemployment benefit of k in period T , and at the beginning of $T + 1$ with probability λ , he is matched with an employer offering a vacant position. An employee whose relationship with the current employer is terminated at the end of T remains unemployed for a period and can be matched again with an employer at the beginning of $T + 2$. Hence, the flow out of the unemployment pool is $\lambda(1 - n + n \times X_R)$, *i.e.*, a proportion λ of the $(1 - n + n \times X_R)$ unemployed agents who find a job, and therefore, the reemployment probability is as follows:

$$(11) \quad \lambda = \frac{n \times X}{1 - n + n \times X_R}$$

Talent distribution. The initial distribution of talent in the economy is split into two stationary distributions: inside the firm and in the unemployment pool. The fraction of high-type employees in the overall population is \bar{s} . Let \bar{q} be the fraction of firms employing an agent with talent $\bar{\theta}$. Let \bar{g} be the fraction of agents with talent $\bar{\theta}$ in the pool of unemployed at the beginning of period T . Once some agents decide to reject the contract they have been offered, they return to the unemployment pool and are available to be reemployed at the beginning of $T + 1$. The probability of finding an agent with talent $\bar{\theta}$ among those available for employment at the beginning of $T + 1$ is denoted \bar{p} . Note that if no one rejects the contract offered by the companies, then $\bar{g} = \bar{p}$. We can now define the fractions of agents who leave at the beginning or at the end of a period conditionally on the contract offered by employers. In Table 2, we characterize these fractions for the case where all firms adopt the same contract.

	X_R	X_S
bonus contract	$(1 - \bar{q})$	$(1 - \alpha)\bar{q}$
termination contract with $y = \bar{\theta}$	0	$(1 - \alpha) + (1 - \bar{q})\alpha(1 - F(\underline{\theta}))$
termination contract with $y = \underline{\theta}$	0	$(1 - \alpha)$

Table 1: The fractions of employees returning to the unemployment pool during a given period of time conditional on the contract adopted by the mass of employers.

In stationary equilibrium, the fraction of $\bar{\theta}$ agents in the pool of employed workers and in the pool of unemployed agents should be the same at the beginning and at the end of a period.¹⁸

	T	$T + 1$
employed at the beginning of the period	\bar{q}	$\alpha\bar{q} + \bar{p}X$
unemployed at the beginning of the period	\bar{g}	$(1 - \alpha)\bar{q} + \bar{g} - \bar{p}X$
unemployed at the end of the period	\bar{p}	$((1 - \alpha)\bar{q} + \bar{g} - \bar{p}X) \frac{1-n}{1-n+nX_R}$

Table 2: The fractions of employees $\bar{\theta}$ in conditions of employment and unemployment.

The stationarity conditions are as follows:

$$(12) \quad \begin{cases} n \times \bar{q} + (1 - n) \times \bar{g} = \bar{s} \\ \bar{p} = \frac{(1-\alpha)\bar{q}}{X} \\ \bar{p} = \frac{1-n}{1-n+nX_R} \bar{g} \end{cases}$$

A critical variable for an employer's decision regarding the optimal contract is the probability of being matched with an employee with talent $\bar{\theta}$. Hence, we focus on the endogenously obtained value for \bar{p} . Let \bar{p}_C be the value of \bar{p} when all employers adopt contract C , where $C = \{b, w(\bar{\theta}), w(\underline{\theta})\}$.

Lemma 4. $\bar{p}_b < \bar{p}_{w(\bar{\theta})} < \bar{p}_{w(\underline{\theta})}$

¹⁸Among the employees who reject the principal's contract, there are no agents with talent $\bar{\theta}$.

If all employers adopt the pure bonus contract, then agents with talent $\underline{\theta}$ reject the employers' offers and return to the unemployment pool; therefore, with this type of contract, we obtain the highest degree of selection, and the fraction of employees with talent $\bar{\theta}$ in the pool of agents available to be hired is the lowest. At the other extreme is the case in which all firms offer the efficiency wage contract $(w(\underline{\theta}), \underline{\theta})$. In that case, agents with talent $\underline{\theta}$ and $\bar{\theta}$ are equally valuable to a firm, and there is no selection of employees; therefore, the fraction of agents with talent $\bar{\theta}$ in the pool of unemployed workers is equal to the fraction of employees with talent $\bar{\theta}$, i.e., $\bar{p}_{w(\underline{\theta})} = \bar{q}_{w(\underline{\theta})} = \bar{s}$.

Proposition 6.

- i) If $\bar{p}_b < \bar{p}' < \bar{p}_{w(\bar{\theta})}$, then the contracts C_b and $C_{w(\bar{\theta})}$ coexist in equilibrium.*
- ii) If $\bar{p}_{w(\bar{\theta})} < \bar{p}' < \bar{p}_{w(\underline{\theta})}$, then the contracts $C_{w(\bar{\theta})}$ and $C_{w(\underline{\theta})}$ coexist in equilibrium.*

If all firms adopt a bonus contract, then the selection of employees is very strong; therefore, the fraction of employees with high talent in the pool of unemployed workers is very low. Because the probability of being matched with a high-type employee is low, it may be in the interest of an individual employer with a vacant position to offer a termination contract that motivates both types of agents to exert effort.

Proposition 6 shows that a priori identical companies may adopt different employment systems in equilibrium. Hence, while some firms adopt strong *ex ante* selection, bonus payments and low turn-over; others are less selective, offer fixed wages and experience high turnover. Notice that the *ex ante* inter-temporal expected profit of the employers are the same independently of which of the contracts is adopted. However, conditionally on the position being filled the firms offering a bonus contract are having a higher inter-temporal profit than firms offering a termination contract.

Osterman (1994), Hunter (1999), and Bailey and Sandy (1999) investigate the adoption of high-performance work systems in different US industries. They all show that this adoption is not generalized and different employment systems do co-exist. Hunter (1999), for example, studies the US retail banking industry. He shows that two employment systems co-exist: inclusive and segmented. The inclusive system is characterized with better selected employees, dealing with a broader set of tasks, more likely to be involved in team work with high powered incentives. The segmented system is characterized by a large set of employees specialised in dealing with a narrow set of tasks, low powered incentives with rare promotion possibilities, and high turnover.

One of the justifications for the co-existence of employment systems offered in the above mentioned papers is that firms have different strategies. If in our paper there was heterogeneity among firms in terms of the distribution of tasks that their employees have to perform, then this would certainly drive the adoption of different employment systems. However, setting aside this level of heterogeneity allows us to emphasize the importance of the employees' characteristics for the adoption of a specific employment system, or for the co-existence of different types of jobs.

5 A continuum of types

Extending the model to the case where an employee's talent is drawn from a continuum on the support $[\underline{\theta}, \bar{\theta}]$ considerably increases the complexity of the analysis. In this section, we do not fully characterize the optimal relational contract; however, we do analyze the characteristics of the bonus and termination contracts and claim that our key findings will not be affected by the consideration of a more general distribution of employees' talent.

We consider the problem of an individual employer facing a pool of available employees with a mass of 1. Each agent has a privately observable talent θ . Talent is distributed on the bounded support $[\underline{\theta}, \bar{\theta}]$. The cumulative distribution function $Q(\cdot)$ is common knowledge. We assume that at any point in time, the employer faces a pool of employees with the same distribution of talent, i.e., $Q(\cdot)$ remains constant over time. To maintain consistency with the two-types case, we assume that when $F(\underline{\theta})(1 - c) - (1 - \delta)V^{out} > 0$, the principal is better off by hiring any employee rather than holding the job vacant.

The public history of an employee's performance boils down to knowing the most difficult task in which the employee has succeeded and the easiest task in which the employee has failed.

Bonus contract: A bonus contract can credibly be offered only to employees with high talent. The intuition is as before. The bonus will only be paid if the principal's rent from retaining an employee is sufficiently high. The rent will be sufficiently high only for employees whose talent is beyond a certain threshold $\tilde{\theta}_t$. The optimal bonus contract may not be stationary, and the offer made to an employee in each period may depend on the employee's employment history. A more formal discussion is offered in the Appendix 8.11. However, we note that, as in the case of the two-types model, the downside of a bonus contract is that it only motivates the very best employees to exert effort, and whenever the principal is matched with an employee with talent below the threshold $\tilde{\theta}_0$, the position remains vacant.

Termination contract: If the principal is willing to motivate any employee with whom he is matched to perform any task $x \leq \min\{\theta, y_t\}$ in each employment period t , he offers a termination contract $(y_t, w(y_t))$ in any t . The optimal termination contract may be non-stationary, and the performance standard as well as the fixed wage offered by the employer may evolve contingent on the employee's past performance. A formal discussion of the termination contract is offered in the Appendix 8.11. On the one hand, the cost of the termination contract relative to the bonus contract is that the principal must offer rent to employees to elicit high effort. On the other hand, the benefit is that the vacancy is always filled, and the principal can motivate any employee to exert effort on tasks $x \leq \min\{\theta, y_t\}$.

The preference for one contract over the other will therefore depend on the distribution of talent and the nature of the job that the principal offers. In jobs for which most tasks can be solved by low-type employees, the cost of adopting a bonus contract is too high, and therefore, a termination contract is most likely to be used. In jobs for which most of the tasks require high levels of talent, a bonus contract is more likely to be adopted.

6 Conclusion

Informal reputational mechanisms play a crucial role in a large range of contractual relationships. The aim of this paper is to show how the characteristics of the job, the labor market, or the firm affect the compensation package used by employers and their hiring and firing policies. We did this in a setting with heterogeneous employees whose ability is persistent over time and is the employee's private information. We showed that in bonus jobs, selection occurs *ex ante*, and hence, a vacant position may remain unoccupied for a longer period; however, once an employee has been hired, he will remain for a longer period in that position. In contrast, firms offering efficiency wage contracts fill their vacant positions more quickly. However, because employees are selected on-the-job, the tenure of the average employee will be shorter. We have also offered a theoretical justification for the adoption of different contracts and hiring/firing policies by identical firms operating in the same industry.

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8 Appendix

8.1 Proof of Proposition 1

We characterize the stationary contracts. Since information is revealed in the first employment period, focusing on stationary contracts is without loss of generality.

The principal's inter-temporal expected payoff with commitment is as follows:

$$(13) \quad \Pi^{com} = (1 - \bar{p})\Pi^{com}(\underline{\theta}) + \bar{p}\Pi^{com}(\bar{\theta}),$$

where

$$(14) \quad \begin{cases} \Pi^{com}(\underline{\theta}) = F(\underline{\theta})(1 - \underline{b}(x \in [0, \underline{\theta}])) - \underline{w} + \delta\Pi^{com} \\ \Pi^{com}(\bar{\theta}) = 1 - (1 - F(\underline{\theta}))\bar{b}(x \in [\underline{\theta}, \bar{\theta}]) - F(\underline{\theta})\bar{b}(x \in [0, \underline{\theta}]) - \bar{w} + \alpha\delta\Pi^{com}(\bar{\theta}) + (1 - \alpha)\delta\Pi^{com} \end{cases}$$

The principal's programme is as follows:

$$(15) \quad \max_{\underline{b}, \bar{b}, \underline{w}, \bar{w}} \Pi^{com} = \frac{(1 - \bar{p})(F(\underline{\theta})(1 - \underline{b}(x \in [0, \underline{\theta}])) - \underline{w}) + \bar{p} \frac{(1 - F(\underline{\theta}))(1 - \bar{b}(x \in [\underline{\theta}, \bar{\theta}])) + F(\underline{\theta})(1 - \bar{b}(x \in [0, \underline{\theta}])) - \bar{w}}{1 - \alpha\delta}}{1 - \delta(1 - \alpha\bar{p})}$$

$$(16) \quad \begin{cases} F(\underline{\theta})(\underline{b} - c) + \underline{w} \geq (1 - \delta)V^{out} & \underline{IR} \\ (1 - F(\underline{\theta}))(\bar{b}(x \in [\underline{\theta}, \bar{\theta}]) - c) + F(\underline{\theta})(\bar{b}(x \in [0, \underline{\theta}]) - c) + \bar{w} \geq (1 - \delta)V^{out} & \bar{IR} \\ F(\underline{\theta})(\underline{b}(x \in [0, \underline{\theta}]) - c) + \underline{w} \geq F(\underline{\theta})(\bar{b}(x \in [0, \underline{\theta}]) - c) + \bar{w} & \underline{IC}_S \\ (1 - F(\underline{\theta}))(\bar{b}(x \in [\underline{\theta}, \bar{\theta}]) - c) + F(\underline{\theta})(\bar{b}(x \in [0, \underline{\theta}]) - c) + \bar{w} \geq & \bar{IC}_S \\ (1 - F(\underline{\theta}))(\underline{b}(x \in [\underline{\theta}, \bar{\theta}]) - c) + F(\underline{\theta})(\underline{b}(x \in [0, \underline{\theta}]) - c) + \underline{w} & \\ \underline{b}(x \in [0, \underline{\theta}]) - c \geq 0 & \underline{IC}_M \\ \bar{b}(x \in [0, \underline{\theta}]) - c \geq 0, \bar{b}(x \in [\underline{\theta}, \bar{\theta}]) - c \geq 0 & \bar{IC}_M \end{cases}$$

Since employees with talent $\underline{\theta}$ cannot solve tasks $x > \underline{\theta}$, the bonus contingent on those tasks offered to employees with talent $\underline{\theta}$ is zero, i.e. $\underline{b}(x \in]\underline{\theta}, \bar{\theta}]) = 0$. The principal's profit decreases in the bonus and the fixed wage offered to the employees, therefore he is aiming to offer the lowest possible combination of rewards compatible with the employee's participation and incentive compatibility constraints. If \underline{IR} and \bar{IR} hold with equality, then \bar{IC}_S also holds. Using that \underline{IR} and \bar{IR} hold with equality, we can rewrite \underline{IC}_S as follows:

$$(17) \quad (1 - \delta)V^{out} \geq (1 - \delta)V^{out} - (1 - F(\underline{\theta}))(b(x \in]\underline{\theta}, \bar{\theta}]) - c$$

From equation (17) it is immediate to see that if $b(x \in]\underline{\theta}, \bar{\theta}]) - c > 0$, then \underline{IC}_S

always holds.

Below we define the set of optimal menus of contracts:

$$\begin{cases} \underline{C} = (w = (1 - \delta)V^{out}, b(x \in [0, \underline{\theta}]) = c, b(x \in [\underline{\theta}, \bar{\theta}]) = 0) \\ \bar{C} = (w = (1 - \delta)V^{out} - (1 - F(\underline{\theta}))\epsilon, b(x \in [0, \underline{\theta}]) = c, b(x \in [\underline{\theta}, \bar{\theta}]) = c + \epsilon) \end{cases}$$

where $\epsilon > 0$, satisfies all the constraints and leaves no rent to the employees. This set of menus of contracts implements the social optimum. All the surplus created by the employment relationship is captured by the principal.

8.2 Proof of Proposition 2

If the principal offers a fully separating menu of contracts, then in order to credibly motivate an employee with talent $\underline{\theta}$ to exert effort it must be credible that the principal will pay the bonus after an employee with talent $\underline{\theta}$ has successfully performed a task $x \in [0, \underline{\theta}]$. This requires the payoff from keeping an employee with revealed talent $\underline{\theta}$ to be sufficiently higher than the expected payoff from hiring a new employee. Since this is never the case and the productivity of the average unemployed ($\bar{p}F(\bar{\theta}) + (1 - \bar{p})F(\underline{\theta})$) is higher than the productivity of an employee with talent $\underline{\theta}$, a separating bonus contract cannot be credibly offered to employees with talent $\underline{\theta}$.

If the principal offers a pooling bonus contract, then the bonus cannot be credibly used to motivate an employee to solve a task $x \leq \underline{\theta}$. Indeed, the expected talent from an employee who has solved a task $x \leq \underline{\theta}$ is $\bar{p}\bar{\theta} + (1 - \bar{p})\underline{\theta}$, i.e. the same as the expected talent from a newly hired employee. There is no rent for the principal from keeping the successful employee and therefore the bonus cannot be credibly promised.

8.3 Proof of Lemma 1 and Proposition 3

We first characterize the set of contracts that are only acceptable for employees with talent $\bar{\theta}$, then we derive the self-enforceability condition.

Lemma 1 The principal maximizes his expected payoff:

$$(18) \quad \max_{b,w} \Pi_b = \frac{\bar{p}(1 - b - w)}{(1 - \delta)(1 - \alpha\delta(1 - \bar{p}))}$$

subject to

$$(19) \quad \begin{cases} b - c + w \geq (1 - \delta)V^{out} & \overline{IR} \\ b \geq c & \overline{IC}_M \\ F(\underline{\theta})(b - c) + w \leq (1 - \delta)V^{out} & \underline{IC}_S \end{cases}$$

It is in the principal's interest to set the lowest possible values for b and w , satisfying the participation constraint of a high type employee. If \overline{IR} holds with equality then (\underline{IC}_S) can be rewritten as follows:

$$(1 - \delta)V^{out} - (1 - F(\underline{\theta}))(b - c) \leq (1 - \delta)V^{out}$$

This condition holds for any $b > c$. Therefore, a contract from the set $C_b = (b(x \in [0, \bar{\theta}]) = c + \epsilon, w = (1 - \delta)V^{out} - \epsilon)$, where $\epsilon > 0$, is only accepted by employees $\bar{\theta}$, subject to being self-enforceable.

Proposition 3 We now derive the condition that needs to hold for the contract to be self-enforceable:

$$(20) \quad \Pi_b(\bar{\theta}) - \Pi^{dev}(\bar{\theta}) \geq \frac{c + \epsilon}{\alpha\delta}$$

this is equivalent to:

$$\frac{1 - c - (1 - \delta)V^{out}}{1 - \delta} \frac{1 - \delta(1 - \bar{p})}{1 - \alpha\delta(1 - \bar{p})} - \bar{p} \frac{(1 + \epsilon - (1 - \delta)V^{out})}{1 - \delta} \geq \frac{c + \epsilon}{\alpha\delta}$$

The higher ϵ the more difficult it is to satisfy the self-enforceability constraint. Therefore the largest set of parameters for which the bonus contract can be credibly promised is obtained for $\epsilon \rightarrow 0$.

After simplification the self-enforceability constraint becomes:

$$(21) \quad \bar{p} \leq 1 - \frac{c}{\alpha\delta(1 - (1 - \delta)V^{out})}$$

8.4 Proof of Lemma 2

We prove that a pooling efficiency wage contract dominates a menu of separating efficiency wage contracts.

If the information is revealed immediately there is no loss of generality to focus on a menu of stationary contracts: $\underline{C} = (w(\underline{y}), \underline{y})$ and $\overline{C} = (w(\overline{y}), \overline{y})$. The principal's program is as follows:

$$(22) \quad \max_{w(\underline{y}), \underline{y}, w(\bar{y}), \bar{y}} \Pi^S = \bar{p}\Pi(\bar{y}) + (1 - \bar{p})\Pi(\underline{y})$$

subject to

$$(23) \quad \begin{cases} V(\underline{\theta}, \underline{y}) - V^{out} \geq \frac{c}{\alpha\delta} & \underline{IC}_M \\ V(\bar{\theta}, \bar{y}) - V^{out} \geq \frac{c}{\alpha\delta} & \overline{IC}_M \\ V(\underline{\theta}, \underline{y}) \geq V(\underline{\theta}, \bar{y}) & \underline{IC}_S \\ V(\bar{\theta}, \bar{y}) \geq V(\bar{\theta}, \underline{y}) & \overline{IC}_S \\ \Pi(\underline{y}) = \Pi(\bar{y}) & SE \end{cases}$$

where

$$(24) \quad \begin{cases} \Pi(\underline{y}) = \frac{F(\underline{y}) - w(\underline{y})}{1 - \delta} \\ \Pi(\bar{y}) = \frac{F(\bar{y}) - w(\bar{y})}{1 - \delta} \\ V(\underline{\theta}, \underline{y}) = \frac{w(\underline{y}) - F(\underline{y})c + (1 - \alpha)\delta V^{out}}{1 - \alpha\delta} \\ V(\bar{\theta}, \bar{y}) = \frac{w(\bar{y}) - F(\bar{y})c + (1 - \alpha)\delta V^{out}}{1 - \alpha\delta} \\ V(\underline{\theta}, \bar{y}) = \frac{w(\bar{y}) - F(\underline{\theta})c + (1 - \alpha + \alpha(F(\bar{y}) - F(\underline{\theta})))\delta V^{out}}{1 - \alpha\delta + \alpha\delta(F(\bar{y}) - F(\underline{\theta}))} \\ V(\bar{\theta}, \underline{y}) = \frac{w(\underline{y}) - F(\underline{y})c + (1 - \alpha)\delta V^{out}}{1 - \alpha\delta} \end{cases}$$

The constraint \underline{IC}_M (*resp.* \overline{IC}_M) guarantees the incentives to exert effort for an employee $\underline{\theta}$ (*resp.* $\bar{\theta}$). The constraint \underline{IC}_S (*resp.* \overline{IC}_S) guarantees that an employee $\underline{\theta}$ (*resp.* $\bar{\theta}$) will truthfully reveal his type.

An employee is motivated to exert effort in order to keep his job if the rent from working compared to being unemployed is sufficiently high. If an employee anticipates that he will be fired at the end of the period independently from his performance, the employee will not exert effort. Therefore, an employee with talent $\underline{\theta}$ would be motivated to exert effort only if it is credible that he would not be fired after succeeding at a task $x \in [0, \underline{y}]$. This will only be credible if the inter-temporal expected profit from a low-type employee is the same as the inter-temporal expected profit from a high-type employee (constraint SE).¹⁹

The efficiency wage that satisfies \underline{IC}_M is $w(\underline{y}) = \frac{c(1 - \alpha\delta)}{\alpha\delta} + cF(\underline{y}) + (1 - \delta)V^{out}$. From SE $w(\bar{y}) = w(\underline{y}) + F(\bar{y}) - F(\underline{y})$, it follows that $w(\underline{y}) = \frac{c(1 - \alpha\delta)}{\alpha\delta} + cF(\underline{y}) + (1 -$

¹⁹An alternative way of writing the self enforceability constraint is: $\Pi(\underline{y}) \geq \Pi^S$ and $\Pi(\bar{y}) \geq \Pi^S$. This implies that the inter-temporal expected profit from offering a separating menu of termination contracts to a newly hired employee cannot exceed the inter-temporal expected profit from hiring an employee with talent $\underline{\theta}$. This is valid even without the stationarity argument.

$\delta)V^{out}$ only satisfies \underline{IC}_S for $\bar{y} = \underline{y}$, where $\underline{y} \leq \underline{\theta}$. It follows that:

$$(25) \quad \Pi^S \leq \max\left\{\frac{F(\underline{y}) - \frac{c(1-\alpha\delta)}{\alpha\delta} - cF(\underline{y}) - (1-\delta)V^{out}}{1-\delta}\right\}$$

A stationary pooling contract maximizes:²⁰

$$(26) \quad \max_{w(y), y} \Pi^P = \frac{\bar{p}\frac{F(y)-w(y)}{1-\alpha\delta} + (1-\bar{p})\frac{F(\theta)-w(y)}{1-\alpha\delta(1-F(y)+F(\theta))}}{1 - \bar{p}\frac{\delta(1-\alpha)}{1-\alpha\delta} - (1-\bar{p})\frac{(1-\alpha)\delta + \alpha\delta(F(y)-F(\theta))}{1-\alpha\delta(1-F(y)+F(\theta))}}$$

subject to

$$(27) \quad \begin{cases} V(\underline{\theta}, y) - V^{out} \geq \frac{c}{\alpha\delta} \underline{IC}_M \\ V(\bar{\theta}, y) - V^{out} \geq \frac{c}{\alpha\delta} \overline{IC}_M \end{cases}$$

where

$$(28) \quad V(\theta, y) = \frac{w(y) - F(\min\{y, \theta\}) + \delta(1-\alpha + \alpha(F(\max\{y, \theta\}) - F(\theta)))}{1 - \alpha\delta + \alpha\delta(F(\max\{y, \theta\}) - F(\theta))}$$

The wage $w(y) = \frac{c(1-\alpha\delta)}{\alpha\delta} + cF(y) + (1-\delta)V^{out}$ satisfies \underline{IC}_M and \overline{IC}_M for any y . If the principal sets $y = \underline{y} < \underline{\theta}$, then the pooling profit is $\frac{F(\underline{y}) - \frac{c(1-\alpha\delta)}{\alpha\delta} - cF(\underline{y}) - (1-\delta)V^{out}}{1-\delta}$. It follows that

$$(29) \quad \Pi^P \geq \max\left\{\frac{F(\underline{y}) - \frac{c(1-\alpha\delta)}{\alpha\delta} - cF(\underline{y}) - (1-\delta)V^{out}}{1-\delta}\right\}$$

From (25) and (29) it follows that $\Pi^P \geq \Pi^S$.

8.5 Proof of Proposition 4

i) If the principal offers a contract (y_t, w_t) with $y_t \leq \underline{\theta}$ to an employee in period t of the employment relationship, then the principal's program is as follows:

$$(30) \quad \max_{y_t} \Pi_t = F(y_t) - w(y_t) + \alpha\delta\Pi_{t+1} + (1-\alpha)\delta\Pi_0$$

subject to

$$(31) \quad w(y_t) \geq \frac{c}{\alpha\delta} + cF(y_t) + (1-\delta)V^{out} - \alpha\delta(V_{t+1}(y_{t+1}, \theta) - V^{out})$$

²⁰At this stage focusing on a stationary pooling contract is without loss of generality. If there is a non-stationary contract that does better than the stationary one, it would only make our argument stronger.

The first order condition with respect to y_t is as follows:²¹

$$(32) \quad F'(y_t)(1 - c) > 0$$

From the first order condition it follows that it is never in the principal's interest to offer a contract with $y_t < \underline{\theta}$.

- ii) If the principal offers a contract (y_t, w_t) with $y_t \geq \underline{\theta}$ to an employee in period t of the employment relationship, then his inter-temporal expected profit is as follows:

$$(33) \quad \begin{aligned} \max_{y_t} \Pi_t = & (1 - \bar{p}_t)F(\underline{\theta}) + \bar{p}_t F(y_t) - w(y_t) + \alpha \delta \bar{p}_t \Pi_{t+1}(\bar{\theta}) \\ & + \alpha \delta (1 - \bar{p})_t (1 - F(y_t) + F(\underline{\theta})) \Pi_{t+1}(\underline{\theta}) \\ & + \alpha \delta (1 - \bar{p})_t (F(y_t) - F(\underline{\theta})) \Pi_0 + \delta (1 - \alpha) \Pi_0 \end{aligned}$$

subject to

$$(34) \quad \begin{cases} V(\underline{\theta}, y_t) - V^{out} \geq \frac{c}{\alpha \delta} & (\underline{IC}_t) \\ V(\bar{\theta}, y_t) - V^{out} \geq \frac{c}{\alpha \delta} & (\overline{IC}_t) \end{cases}$$

where \bar{p}_t depends on the employee's history and the incentive compatibility constraints are equivalent to:

$$(35) \quad \begin{cases} w(y_t) \geq \frac{c}{\alpha \delta} + cF(\underline{\theta}) + (1 - \delta)V^{out} - \alpha \delta (1 - F(y_t) + F(\underline{\theta}))(V(\underline{\theta}, y_{t+1}) - V^{out}) & (\underline{IC}_t) \\ w(y_t) \geq \frac{c}{\alpha \delta} + cF(y_t) + (1 - \delta)V^{out} - \alpha \delta (V(\bar{\theta}, y_{t+1}) - V^{out}) & (\overline{IC}_t) \end{cases}$$

We notice that:

$$(36) \quad V(\bar{\theta}, y_t) - V(\underline{\theta}, y_t) = \alpha \delta (V(\bar{\theta}, y_{t+1}) - V(\underline{\theta}, y_{t+1})) + \alpha \delta (F(y_t) - F(\underline{\theta}))(V(\underline{\theta}, y_{t+1}) - V^{out} - \frac{c}{\alpha \delta})$$

It follows that if (\underline{IC}_t) holds, then (\overline{IC}_t) also holds.

Let us first consider the principal's decision to offer a wage pattern that increases over time. Increasing the wage of $t + 1$ by Δ , implies a cost in period t corresponding to the expected present value of the increase: $\alpha \delta \Delta$. Increasing the wage of $t + 1$ by Δ implies that: $V(\underline{\theta}, y_{t+1}) - V^{out} = \frac{c}{\alpha \delta} + \Delta$. Since an employee with talent $\underline{\theta}$ only enjoys the future rent with probability $\alpha(1 - F(y_t) + F(\underline{\theta}))$, this implies that the period t wage can be reduced by $\alpha \delta (1 - F(y_t) + F(\underline{\theta})) \Delta$. Since the cost outweighs the benefit, it is always in

²¹The future rent of an employee who keeps his job is independent from the performance standard of the current period.

the principal's interest to satisfy $V(\underline{\theta}, y_t) - V^{out} \geq \frac{c}{\alpha\delta}$ with equality in any t . Now we can derive the first order condition of the principal in choosing y_t .

$$(37) \quad \frac{\partial \Pi_t}{\partial y_t} = F'(y_t)(\bar{p}_t - c + \alpha\delta(1 - \bar{p})(\Pi_0 - \Pi_{t+1}(\underline{\theta})))$$

Since $\Pi_0 \geq \Pi_{t+1}(\underline{\theta})$, a sufficient condition for the above equation to be always positive is $\bar{p}_t > c$. \bar{p}_t depends on the employee's past performances. In the case with two types, $\bar{p}_t = \{\bar{p}, 1\}$. Therefore the most constraining condition for equation (37) to be positive is $\bar{p} > c$.

- iii) It is not in the principal's interest to use a mix of termination and bonus. Let us consider the possibility for a principal to offer a mix of termination and bonus. The contract is $(w(y_t), y_t, b(x \in [y_t, \bar{\theta}]))$. The incentive compatibility constraint of an employee with talent $\underline{\theta}$ writes:

$$(38) \quad V(\underline{\theta}, y_t) - V^{out} \geq \frac{c}{\alpha\delta} \Leftrightarrow w(y_t) \geq \frac{c}{\alpha\delta} + cF(\underline{\theta}) + (1 - \delta)V^{out} - \alpha\delta F(\underline{\theta})(V(\underline{\theta}, y_{t+1}) - V^{out})$$

From before, we know that satisfying this constraint with equality is sufficient to motivate an employee with talent $\bar{\theta}$ to solve any task $x \in [0, \bar{\theta}]$. Therefore offering a bonus would only duplicate the incentives for the high type employees to exert effort on tasks $x \in [\underline{\theta}, \bar{\theta}]$. This is costly for the principal and therefore bonus and termination contract would not be adopted to motivate a specific employee.

8.6 Proof of Lemma 3

Since the self-enforceability condition for the bonus writes:

$$(39) \quad \bar{p} \leq 1 - \frac{c}{(1 - (1 - \delta)V^{out})}$$

It follows immediately that if $c > \alpha\delta(1 - (1 - \delta)V^{out})$, the principal cannot credibly use a bonus contract to motivate an employee to exert effort.

From $c > \alpha\delta(1 - (1 - \delta)V^{out})$ it follows that $c > \frac{\alpha\delta(F(\underline{\theta}) - (1 - \delta)V^{out})}{1 - \alpha\delta(1 - F(\underline{\theta}))}$. These two inequalities imply that $1 - w(\bar{\theta}) < 0$ and $F(\underline{\theta}) - w(\underline{\theta}) < 0$, therefore the efficiency wage contract always leads to negative profits and therefore is not adopted by the employer.

8.7 Proof of Proposition 5 and Corollary 1

Proposition 5:

\bar{p}' is given by the following equality:

$$(40) \quad \Pi_b = \Pi(w(\bar{\theta}))$$

After simplification the equality becomes:

$$(41) \quad \frac{\bar{p}(1 - c - (1 - \delta)V^{out})}{1 - \alpha\delta(1 - \bar{p})} = \frac{\bar{p}(1 - F(\underline{\theta}))(1 - \alpha\delta w(\bar{\theta})) + (F(\underline{\theta}) - w(\bar{\theta}))(1 - \alpha\delta)}{1 - \alpha\delta(1 - \bar{p}(1 - F(\underline{\theta})))}$$

We notice that both sides of (41) are increasing in \bar{p} . If $F(\underline{\theta}) < w(\bar{\theta})$, then the bonus contract always dominates the efficiency wage contract. The case where $F(\underline{\theta}) > w(\bar{\theta})$ is represented on Figure 8.7 below.

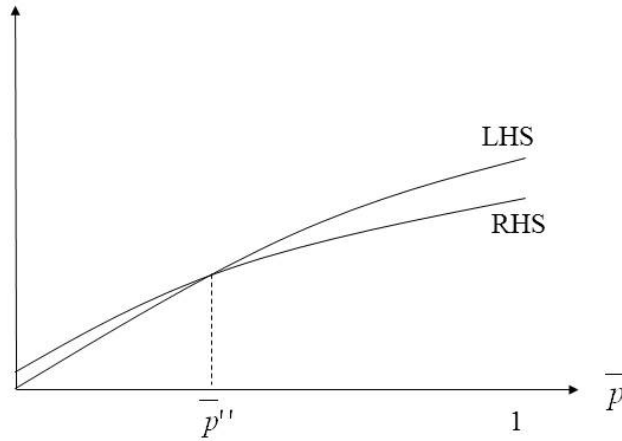


Figure 2: The threshold for \bar{p} below which a termination contract is preferred to a bonus contract.

\bar{p}' is given by the following equality:

$$(42) \quad \Pi(w(\underline{\theta})) = \Pi(w(\bar{\theta}))$$

after simplification, we obtain:

$$(43) \quad \bar{p}' = \frac{c(1 - \alpha\delta)}{1 - \alpha\delta F(\underline{\theta}) - c\alpha\delta(1 - F(\underline{\theta}))}$$

We notice that $\bar{p}' \leq c$.

For $c < \frac{1}{1 + \frac{1}{\alpha\delta(1-(1-\delta)V^{out})}}$, $\bar{p}^{SE} > c$, which implies that whenever the bonus contract is not self-enforceable the optimal contract offered by an individual employer is $(w(\bar{\theta}), \bar{\theta})$.

Corollary 1:

From equation (43), we can rewrite $\bar{p}' = \frac{c(1 - \alpha\delta)}{1 - c\alpha\delta - \alpha\delta F(\underline{\theta})(1 - c)}$ and it is immediate to see that if the value of $F(\underline{\theta})$ increases, then \bar{p}' .

From (41), we notice that the right hand side of the equation is higher for higher values of $F(\underline{\theta})$. Hence, from Figure 8.7 we see that \bar{p}'' is also increasing in the value of $F(\underline{\theta})$.

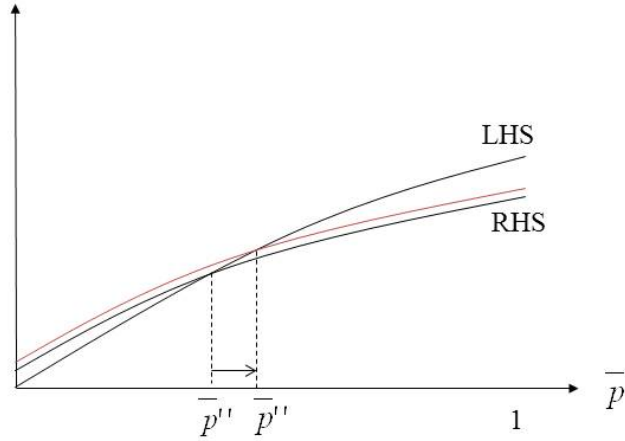


Figure 3: The impact of an increase in $\underline{\theta}$ on \bar{p}'' .

8.8 Proof that $V^{out}(\underline{\theta}) = V^{out}(\bar{\theta})$ in the labor market equilibrium

If all firms adopt a bonus contract

$$(44) \quad V^{out}(\underline{\theta}) = k + \delta V^{out}(\underline{\theta}) \Leftrightarrow V^{out}(\underline{\theta}) = \frac{k}{1-\delta}$$

Even if an employee $\underline{\theta}$ is matched with an employer offering a bonus contract, the employee rejects the offer and immediately returns in the pool of unemployed.

$$(45) \quad V^{out}(\bar{\theta}) = k + \delta(1-\lambda)V^{out}(\bar{\theta}) + \delta\lambda V(\bar{\theta})$$

where,

$$(46) \quad V(\bar{\theta}) = b - c + w + \alpha\delta V(\bar{\theta}) + (1-\alpha)\delta V^{out}(\bar{\theta})$$

The principal's program becomes:

$$(47) \quad \max_{b,w} \Pi_b = \frac{\bar{p}(1-b-w)}{(1-\delta)(1-\alpha\delta(1-\bar{p}))}$$

subject to:

$$(48) \quad \begin{cases} V(\bar{\theta}) \geq V^{out}(\bar{\theta}) \Leftrightarrow b - c + w \geq k & (\overline{IR}) \\ b \geq c & (\overline{IC}_M) \\ F(\underline{\theta})(b - c) + w \leq k & (\underline{IC}_S) \end{cases}$$

The contract is such that $V(\bar{\theta}) = V^{out}(\bar{\theta})$, which combined with equation (45) implies that $V^{out}(\bar{\theta}) = \frac{k}{1-\delta} = V^{out}(\underline{\theta})$.

If all firms adopt a termination contract $(y, w(y))$

$$(49) \quad V(\bar{\theta}, y) = w(y) - F(y)c + \alpha\delta V(\bar{\theta}, y) + (1-\alpha)\delta V^{out}(\bar{\theta}, y)$$

$$(50) \quad V^{out}(\bar{\theta}, y) = k + \lambda\delta V(\bar{\theta}, y) + (1-\lambda)\delta V^{out}(\bar{\theta}, y)$$

$$(51)$$

$$V(\underline{\theta}, y) = w(y) - F(\underline{\theta})c + \alpha\delta(1-F(y)+F(\underline{\theta}))V(\underline{\theta}, y) + \alpha\delta(F(y)-F(\underline{\theta}))V^{out}(\underline{\theta}, y) + (1-\alpha)\delta V^{out}(\underline{\theta}, y)$$

$$(52) \quad V^{out}(\underline{\theta}, y) = k + \lambda\delta V(\underline{\theta}, y) + (1 - \lambda)\delta V^{out}(\underline{\theta}, y)$$

The incentive-compatibility constraints write:

$$(53) \quad \begin{cases} V(\bar{\theta}, y) - V^{out}(\bar{\theta}, y) \geq \frac{c}{\alpha\delta} & \Leftrightarrow w(y) \geq \frac{c(1-\alpha\delta)}{\alpha\delta} + cF(y) + k + \frac{c\lambda}{\alpha} \\ V(\underline{\theta}, y) - V^{out}(\underline{\theta}, y) \geq \frac{c}{\alpha\delta} & \Leftrightarrow w(y) \geq \frac{c(1-\alpha\delta)}{\alpha\delta} + cF(y) + k + \frac{c\lambda}{\alpha} \end{cases}$$

It follows that both constraints hold with equality, i.e., $V(\bar{\theta}, y) - V^{out}(\bar{\theta}, y) = \frac{c}{\alpha\delta}$ and $V(\underline{\theta}, y) - V^{out}(\underline{\theta}, y) = \frac{c}{\alpha\delta}$. Replacing in equations (50) and (52), we obtain:

$$(54) \quad V^{out}(\bar{\theta}, y) = V^{out}(\underline{\theta}, y) = \frac{k + \frac{c\lambda}{\alpha}}{1 - \delta} = V^{out}$$

8.9 Proof of Lemma 4

Simplifying 12, we obtain:

$$(55) \quad \bar{p}\left(\frac{nX}{1 - \alpha} + (1 - n + nX_R)\right) = \bar{s}$$

We use the values for X_S and X_R in Table 1, corresponding to the three types of contracts and we derive the expressions for \bar{p} as a function of \bar{s}

If all firms offer $C_{w(\underline{\theta})} = (w(\underline{\theta}), \underline{\theta})$ then $X_R = 0$ and $X = 1 - \alpha$. From (55) it is immediate to see that $\bar{p}_{w(\underline{\theta})} = \bar{s}$

If all firms offer $C_{w(\bar{\theta})} = (w(\bar{\theta}), \bar{\theta})$ then $X_R = 0$ and $X = 1 - \alpha\bar{q} - \alpha F(\underline{\theta})(1 - q)$. We replace in (55) and use that $\bar{q} = \frac{\bar{p}(1 - \alpha F(\underline{\theta}))}{1 - \alpha + \bar{p}\alpha(1 - F(\underline{\theta}))}$, we find that $\bar{p}_{w(\bar{\theta})}$ is given by the following equation:

$$(56) \quad \bar{p}_{w(\bar{\theta})} \left(\frac{n\alpha(1 - F(\underline{\theta}))(1 - \bar{p}_{w(\bar{\theta})})}{1 - \alpha + \bar{p}_{w(\bar{\theta})}\alpha(1 - F(\underline{\theta}))} + 1 \right) = \bar{s} \Leftrightarrow \bar{p}_{w(\bar{\theta})} = \frac{\bar{s}}{\left(\frac{n\alpha(1 - F(\underline{\theta}))(1 - \bar{p}_{w(\bar{\theta})})}{1 - \alpha + \bar{p}_{w(\bar{\theta})}\alpha(1 - F(\underline{\theta}))} + 1 \right)}$$

If all firms offer $C_b = (w = (1 - \delta)V^{out}, b = c)$ then $X_R = (1 - \bar{q})$ and $X = 1 - \alpha\bar{q}$. We replace in (55) and use that $\bar{q} = \frac{\bar{p}}{1 - \alpha + \bar{p}\alpha}$, we find that \bar{p}_b is given by the following equation:

$$(57) \quad \bar{p}_b \left(\frac{n(1 - \bar{p}_b)}{1 - \alpha + \bar{p}_b\alpha} + 1 \right) = \bar{s} \Leftrightarrow \bar{p}_b = \frac{\bar{s}}{\left(\frac{n(1 - \bar{p}_b)}{1 - \alpha + \bar{p}_b\alpha} + 1 \right)}$$

Proof that $\bar{p}_{w(\bar{\theta})} \leq \bar{p}_{w(\underline{\theta})}$ Since $\frac{n\alpha(1-F(\underline{\theta}))(1-\bar{p}_{w(\bar{\theta})})}{1-\alpha+\bar{p}_{w(\bar{\theta})}\alpha(1-F(\underline{\theta}))} \geq 1$, it follows that

$$\frac{\bar{s}}{\left(\frac{n\alpha(1-F(\underline{\theta}))(1-\bar{p}_{w(\bar{\theta})})}{1-\alpha+\bar{p}_{w(\bar{\theta})}\alpha(1-F(\underline{\theta}))} + 1\right)} \leq \bar{s}$$

and therefore $\bar{p}_{w(\bar{\theta})} \leq \bar{p}_{w(\underline{\theta})}$.

Proof that $\bar{p}_b \leq \bar{p}_{w(\bar{\theta})}$ Since $\frac{n\alpha(1-F(\underline{\theta}))(1-\bar{p})}{1-\alpha+\bar{p}\alpha(1-F(\underline{\theta}))} \leq \frac{n(1-\bar{p})}{1-\alpha+\bar{p}\alpha}$ for any \bar{p} , it follows that $\frac{\bar{s}}{\left(\frac{n\alpha(1-F(\underline{\theta}))(1-\bar{p})}{1-\alpha+\bar{p}\alpha(1-F(\underline{\theta}))} + 1\right)} \geq \frac{\bar{s}}{\left(\frac{n(1-\bar{p})}{1-\alpha+\bar{p}\alpha} + 1\right)}$ for any \bar{p} . This implies, as illustrated on Figure 4 that $\bar{p}_b \leq \bar{p}_{w(\bar{\theta})}$.

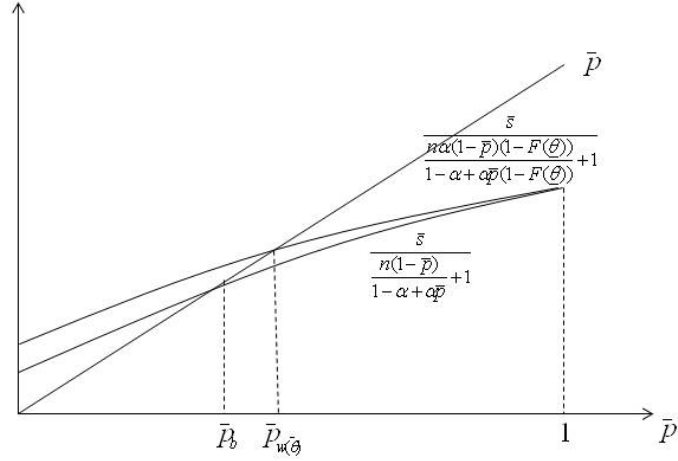


Figure 4: \bar{p}_b and $\bar{p}_{w(\bar{\theta})}$

8.10 Proof of Proposition 6

Let us assume that all firms adopt a bonus contract and let us consider the existence of profitable deviations for an individual employer. If $\bar{p}_b \in [\bar{p}', \bar{p}'']$, then it is in the

interest of an individual employer to deviate and adopt an efficiency wage contract $(w(\bar{\theta}), \bar{\theta})$. Therefore, an equilibrium where all firms adopt a bonus contract cannot exist.

Now we assume that all firms adopt a termination contract $(w(\bar{\theta}), \bar{\theta})$ and consider the existence of profitable deviations for an individual employer. If $\bar{p}_{w(\bar{\theta})} \in [\bar{p}'', \bar{p}^{SE}]$, then it is in the interest of an individual employer to deviate and adopt a bonus contract. Therefore, an equilibrium where all firms adopt a termination contract $(w(\bar{\theta}), \bar{\theta})$ contract cannot exist.

In equilibrium there will be a fraction of employers β adopting the bonus contract and a fraction of employers $(1 - \beta)$ adopting the termination contract, such that $\bar{p}(\beta) = \bar{p}''(\beta)$, where $\bar{p}(\beta)$ is the endogenous probability of being matched with an employee with talent $\bar{\theta}$, when a fraction β of the employers adopt a bonus contract and a fraction $(1 - \beta)$ adopt $(w(\bar{\theta}), \bar{\theta})$.

In that case $X_R = \beta(1 - \bar{q})$ and $X = (1 - \alpha\bar{q}) - (1 - \beta)F(\underline{\theta})\alpha(1 - \bar{q})$. We replace and simplify 12, and we obtain that $\bar{p}(\beta)$ is given by the following expression:

$$(58) \quad \bar{p}(\beta) \left(1 + \frac{n(1 - \bar{p}(\beta))(\alpha + (1 - \alpha)\beta - n(1 - \beta)\alpha F(\underline{\theta}))}{1 - \alpha + \alpha\bar{p}(\beta)(1 - (1 - \beta)F(\underline{\theta}))} \right) = \bar{s}$$

We can also derive \bar{q} as a function of $\bar{p}(\beta)$:

$$(59) \quad \bar{q} = \frac{\bar{s} - (1 - n + n\beta)\bar{p}(\beta)}{n(1 - \beta\bar{p}(\beta))}$$

$\bar{p}''(\beta)$ is given by: $\Pi_b(\beta) = \Pi(w(\bar{\theta}, \beta))$, which after simplification writes:

$$(60) \quad \frac{\bar{p}(1 - c - (1 - \delta)V^{out}(\beta))}{1 - \alpha\delta(1 - \bar{p})} = \frac{\bar{p}(1 - F(\underline{\theta}))(1 - \alpha\delta w(\bar{\theta}, \beta)) + (F(\underline{\theta}) - w(\bar{\theta}, \beta))(1 - \alpha\delta)}{1 - \alpha\delta(1 - \bar{p}(1 - F(\underline{\theta})))}$$

where $w(\bar{\theta}) = \frac{c}{\alpha\delta} + (1 - \delta)V^{out}(\beta)$.

$$(61) \quad V^{out}(\beta) = k + \delta V^{out}(\beta) + \lambda(\beta)\delta(V - V^{out})$$

If an employee is matched with a company offering a bonus contract then $V - V^{out} = 0$, If the employee is matched with a company offering a termination contract then $V_e - V^{out} = \frac{c}{\alpha\delta}$. After replacing in (61) we obtain:

$$(62) \quad V^{out}(\beta) = \frac{\alpha k + \lambda_{w(\bar{\theta})}(\beta)c}{\alpha(1 - \delta)}$$

The probability to be matched with a company offering a termination contract

$\lambda_{w(\bar{\theta})}(\beta)$ is as follows:

$$(63) \quad \lambda_{w(\bar{\theta})}(\beta) = \frac{n(1-\beta)(1-\alpha + (1-\bar{q})\alpha(1-F(\underline{\theta})))}{1-n+n\beta(1-\bar{q})}$$

8.11 Continuum of types

The history of past performances of an employee, in period t of the employment relationship is summarised by the most difficult task that the employee has successfully performed $\underline{x}_{t-1} \equiv \max\{x_{\tau}^S\}$ where $\tau = \{0, 1, \dots, t-1\}$ and the least difficult task that the employee has failed and for which he was motivated to exert effort $\bar{x}_{t-1} \equiv \min\{x_{\tau}^F\}$ where $\tau = \{0, 1, \dots, t-1\}$. In period $t-1$ of the employment relationship \underline{x}_{t-1} corresponds to the lower bound of an employee's talent and \bar{x}_{t-1} corresponds to the upper bound for an employee's talent. If an employee has never failed at a task the upper bound for the talent is $\bar{\theta}$.

Bonus contract. Since the bonus is only paid once an employee has successfully performed a task and since $1 > c$, it is in the principal's interest to offer the bonus to the largest set of employees for whom the offer is credible, i.e., it is credible that the principal will pay the bonus once he has observed a high performance from an employee. The bonus cannot be credibly offered to employees with low talent and the intuition is the same as in the two types case. The bonus can only be offered credibly to employees with talent $\tilde{\theta}_t$. The self enforceability constraint in period t of the employment relationship writes:

$$(64) \quad \Pi(\theta \in [\max\{\tilde{\theta}_t, \underline{x}_t\}, \bar{x}_{t-1}]) - \Pi^{out} \geq \frac{b}{\alpha\delta}$$

where Π^{out} is the inter-temporal expected profit from hiring new employee from the pool of unemployed.

The most difficult to satisfy self enforceability constraint is for $\underline{x}_t \leq \tilde{\theta}_t$:

$$(65) \quad \Pi(\theta \in [\tilde{\theta}_t, \bar{x}_{t-1}]) - \Pi^{out} \geq \frac{b}{\alpha\delta}$$

The principal offers a new bonus contract to the employee in place at the beginning of each period. Hence, the offer he makes may be such that some employees would prefer to reject it and return into the unemployment pool. The threshold for talent $\tilde{\theta}_t$ is changing over time and will depend on the employee's past performance.

(66)

$$\begin{aligned} \Pi(\theta \in [\tilde{\theta}_t, \bar{x}_{t-1}]) &= \int_{\tilde{\theta}_t}^{\bar{x}_{t-1}} (F(\theta)(1-c) - k)g_t(\theta)d\theta + (1-\alpha)\delta\Pi^{out} \\ &+ \alpha\delta \int_{\tilde{\theta}_t}^{\tilde{x}_t} (F(\theta) + 1 - F(\tilde{x}_t))\Pi(\theta)g_t(\theta)d\theta \\ &+ \alpha\delta \int_{\tilde{\theta}_t}^{\tilde{x}_t} (F(\tilde{x}_t) - F(\theta))\Pi^{out}g_t(\theta)d\theta \\ &+ \alpha\delta \int_{\tilde{x}_t}^{\bar{x}_{t-1}} \Pi(\theta)g_t(\theta)d\theta \end{aligned}$$

Termination contract. In every period t of the employment relationship, the principal chooses the performance requirement y_t and the fixed wage $w(y_t)$ by maximizing the following program:

(67)

$$\begin{aligned} \max \Pi_t(y_t) &= F(y_t) - \int_{\underline{x}_{t-1}}^{y_t} (F(y_t) - F(\theta))g_t(\theta)d\theta - w(y_t) \\ &+ \alpha\delta \int_{\underline{x}_{t-1}}^{y_t} (1 - F(y_t) + F(\theta))\Pi(y_{t+1}, \theta)g_t(\theta)d\theta \\ &+ \alpha\delta \int_{y_t}^{\bar{\theta}} \Pi(y_{t+1}, \theta)g_t(\theta)d\theta + ((1-\alpha)\delta + \alpha\delta \int_{\underline{x}_{t-1}}^{y_t} (F(y_t) - F(\theta))g_t(\theta)d\theta)\Pi_0(y_0) \end{aligned}$$

subject to

$$(68) \quad \begin{cases} V(\theta, y_t) - V^{out} \geq \frac{c}{\alpha\delta} \text{ for any } \theta & IC(\theta) \\ \Pi_{t+1}(y_{t+1} | \theta \in [\underline{x}_{t-1}, y_t]) \leq \Pi_0(y_0) & SE \end{cases}$$

The density function of the distribution of talent is $g_t(\theta) = \frac{g(\theta)}{1-G(\underline{x}_{t-1})}$.

The SE constraint states that it must be credible that an employee who fails at the most difficult task of the performance requirement will be fired. What motivates an employee to exert effort is the fear of desire to keep his job. If the employee anticipates that he will keep his job even if he fails at a task within the performance requirement, then he will not exert effort on that task.

The first order condition of the programme with respect to y_t writes:

(69)

$$1 - \int_{\underline{x}_{t-1}}^{y_t} g_t(\theta)d\theta - \frac{w'(y_t)}{f(y_t)} + \alpha\delta \int_{\underline{x}_{t-1}}^{y_t} (\Pi(y_0) - \Pi(y_{t+1}, \theta))g_t(\theta)d\theta - \frac{\lambda}{f(y_t)} \frac{\partial \Pi_{t+1}(y_{t+1} | \theta \in [\underline{x}_{t-1}, y_t])}{\partial y_t} = 0$$

where λ is the Lagrange multiplier of the SE constraint.

We notice that, from the SE constraint it follows that $\int_{\underline{x}_{t-1}}^{y_t} (\Pi(y_0) - \Pi(y_{t+1}, \theta))g_t(\theta)d\theta > 0$.

The first order condition can be rewritten as follows:

$$(70) \quad 1 - G(y_t) - \frac{w'(y_t)}{f(y_t)}(1 - G(\underline{x}_{t-1})) + \alpha\delta \int_{\underline{x}_{t-1}}^{y_t} (\Pi(y_0) - \Pi(y_{t+1}, \theta))g(\theta)d\theta - \frac{\lambda(1-G(\underline{x}_{t-1}))}{f(y_t)} \frac{\partial \Pi_{t+1}(y_{t+1} | \theta \in [\underline{x}_{t-1}, y_t])}{\partial y_t} = 0$$

The choice of y_t is likely to be non-stationary and will be evolving with the

employee's past performance.

**This working paper has been produced by
the School of Economics and Finance at
Queen Mary University of London**

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