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Rural-Urban Migration with Behavioral Preferences*

Dafeng Xu[†]

Abstract

This paper incorporates the behavioral labor economic perspective into the traditional Harris-Todaro rural-urban migration model. We study cases in which one's utility is not only determined by his own wage, but also by wages of others. In our theoretical models, rural-urban migration is driven by the expected utility differential, instead of the expected wage differential. We find that the conventional "Harris-Todaro equilibrium" (in which the rural wage equals the expected urban wage) always exists, but there are two other possible equilibria, in which the rural wage can be either greater or less than the expected urban wage. Moreover, we show that if lower-income players gain utility from the wage differential, i.e., they are altruistic towards higher-income players, and the degree of altruism is sufficiently large, then the two equilibria in which the rural wage differs from the expected urban wage are stable, and the Harris-Todaro equilibrium is not stable.

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1 Introduction

Rural-urban migration has long been studied by economists and demographers both theoretically and empirically since early works of Harris and Todaro (Todaro, 1969; Haris and Todaro, 1970). In their classical model, migration is driven by the expected rural-urban wage differential (adjusted for urban unemployment). In the equilibrium, the rural wage equals the expected urban wage. From then on, this classical model has been expanded from various perspectives. For example, Fields (1972) generalizes the job search process and focuses on the more complicated employment status (the “murky sector”) in the city. Salvatore (1981) revisits this model by focusing on the relative rural-urban wage differential. Stark and Yitzhaki (1988) model migration as the response to relative deprivation, showing alternative conditions under which migration happens.

In this paper, we learn from recent development of the behavioral account of labor economics (e.g., Fehr et al., 2008), and incorporate the behavioral perspective into the conventional framework of the rural-urban migration theory. More specifically, we change the classical setting, in which an individual cares only about on his own wage; instead, individual’s utility function is not only based on his wage, but also on wages of others in the society. Based on this idea, the unselfish attitude plays an important role: a person is altruistic towards anyone whose wage is lower than his own wage; moreover, a person can even be spiteful towards anyone whose wage is higher than his (Levine, 1998; Fehr and Schmidt, 1999, 2005). In both cases, people suffer utility loss if the wage differential exists. This new way of constructing the utility function is used in this paper.

Taking personal attitudes into consideration, we ask following questions: what is the equilibrium of migration in this new story? Can we still get the “Harris-Todaro equilibrium”, in which the rural wage equals the expected urban wage? More interestingly, can we find additional equilibria in which the rural wage differs from

the expected urban wage? To answer these questions, we start by a baseline model without considering unemployment in the urban area. Migration is driven by the utility differential instead of the wage differentials between the urban and rural area, and migration ceases when people in the rural and urban area obtain the same level of utility. We show that there are three possible equilibria, in which the urban wage may be equal to the rural wage, but may also be greater or less than the rural wage. We further investigate the stability of these equilibria. We show that the stability of these equilibria depends on the type and level of individual attitudes: the Harris-Todaro equilibrium remains the only stable equilibrium in some cases; however, if lower-income players gain utility from the rural-urban wage differential, i.e., they are even altruistic towards higher-income players, and the degree of such altruism is adequately large, then we show that two equilibria other than the Harris-Todaro equilibrium are stable, in which the expected urban wage differs from the rural wage; on the contrary, in this case the conventional Harris-Todaro equilibrium is unstable.

This paper based on previous research on attitudes other than selfishness, and in particular, altruism. Altruism of animals have been long studied by biologists (e.g., Trivers, 1971; Wilkinson, 1984 and 1988; Stephens, 1996). There are also many related psychological research (e.g., Underwood et al., 1977; Hunter and Linn, 1980; Rushton et al., 1986; Oman et al., 1999). Early economic studies on personal attitudes and especially altruism include Becker (1974) and Simon (1993). The mainstream theoretical and empirical concentration following earlier theory of altruism is the altruistic behavior between parents and children within a family (see, for instance, Altonji et al, 1992, 1997; Fernandes, 2008; Schwarze and Winkelmann, 2011). Related to our theoretical model, there are also studies on the effect of altruism on migration, but their main interests lie in demographic changes. For example, Canton and Meijdam (1996) the dependence of macroeconomic effects of demographic changes on the degree of altruism. Gaumont and Mesnard (1999) study how the difference in degrees

of altruism affects the labor migration pattern using a two-country model.

In addition, there are also vast economic experiments studying how personal attitudes have behavioral effects. Isaac and Walker (1988) study the mechanism of voluntary contribution among a group of people. Fehr et al. (1993) investigate the experimental result of the effect of fairness on market prices. More related to this paper, Levine (1998) uses a simple altruistic model and experiments to explain why some players would contribute in public goods contribution games.

The remainder of the paper is organized as follows. Section 2 describes the baseline model. Section 3 presents the general model with the consideration of unemployment in the urban area. Section 4 concludes.

2 The Baseline Model

2.1 Setup

We consider a continuum of players, of mass $\hat{L} > 1$, who are *ex ante* identical, i.e., with the same socioeconomic characteristics or other personal features. In the beginning, all players live in the rural area, earning the *rural wage*, denoted as $w_r(\cdot)$. Denote L_r and L_u as the rural labor and the urban labor, respectively. $L_r + L_u = \hat{L}$.

Unlike the setting of the “agricultural wage” and the “manufacturing wage” in the Harris-Todaro model (Harris and Todaro, 1970), in this section we do not make any structural assumptions of the wage for the baseline model. However, some assumptions about the basic mathematical properties of the wage function and the utility function are still needed, as shown below.

Assumption 1 The rural wage $w_r(L_r)$ is a decreasing continuous function of the rural labor, i.e., $w'_r(L_r) < 0$. Similarly, the urban wage $w_u(L_u)$ is a decreasing continuous function of the urban labor, i.e., $w'_u(L_u) < 0$. Moreover, all players in the

rural area have the identical wage, and all players in the rural area have the identical wage.

Next, we assume that the utility function is not only decided by a player's own wage, but also by wages of others.

Assumption 2 The utility of players in the urban area is

$$u_u(w_u) = w_u - \alpha_u f_u(L_r) \max\{w_r - w_u, 0\} - \beta_u f_u(L_r) \max\{w_u - w_r, 0\} \quad (1)$$

where α_u and β_u are parameters measuring personal attitudes; $f_u(L_r)$ is a continuous function describing the weight of players in the rural area, where $f_u(L_r) > 0$ and $\partial f_u(L_r)/\partial L_r \geq 0$.

Clearly, when $\alpha_u = \beta_u = 0$, we have $u_u = w_u$, indicating that players in the urban area are selfish, which follows the standard setting in the classical economic literature. In the case that $\alpha_u \neq 0$ or $\beta_u \neq 0$, the player does not only consider his own wage. The sign and the absolute value of α_u and β_u describe the attitude type and the degree. For instance, Fehr and Schmidt (1999) assume that $0 \leq \beta_u \leq \alpha_u$ and $0 \leq \beta_u < 1$, i.e., a player loses utility in both cases when his wage is relatively higher as $\beta_u \geq 0$ (altruism case) or relatively lower as $\alpha_u \geq 0$ (envy case), but the degree of altruism is no greater than the degree of envy. In our model, however, we do not impose these restrictions. In other words, we allow that $\alpha_u < 0$ and $\beta_u < 0$, i.e., a player is likely to gain utility even if his wage is relatively lower or higher.

Similarly the utility of players in the rural area is assumed as

$$u_r(w_r) = w_r - \alpha_r f_r(L_u) \max\{w_u - w_r, 0\} - \beta_r f_r(L_u) \max\{w_r - w_u, 0\} \quad (2)$$

Subsequently, we define the labor mobility function.

Assumption 3 Let $\dot{L}_u = \phi(\Delta u)$ be the function of labor mobility, where $\Delta u = u_u - u_r$ is the utility differential between urban and rural area. $\phi(\Delta u)$ is an increasing function and $\phi(0) = 0$.

For simplicity, we assume that $\phi(\Delta u)$ is a smooth function. In particular, $\phi(\Delta u)$ is second-order differentiable in the neighborhood around the point $\Delta u = 0$. Finally, we assume the existence of the population allocation such that the urban wage equals the rural wage.

Assumption 4 There exists a pair (\hat{L}_r, \hat{L}_u) such that $\hat{L}_r + \hat{L}_u = \hat{L}$, and $w_r(\hat{L}_r) = w_u(\hat{L}_u)$.

Due to the monotonicity of $w_r(L_r)$ and $w_u(L_u)$, it is obvious that (\hat{L}_r, \hat{L}_u) is unique.

2.2 Labor Market Equilibrium: Definition

We define the labor market equilibrium by the free-entry condition: in the equilibrium, no player in the rural area will migrate to the urban area, and no player in the urban area will migrate back to the rural area. Using the notation of the utility functions defined before, we have the following definition.

Definition 1 Define the labor market equilibrium by the population allocation (L_r, L_u) (where L_r is the rural labor and L_u is the urban labor, and $L_r + L_u = \hat{L}$) such that $\dot{L}_u = \phi(\Delta u) = 0$.

We now introduce two simple examples to understand this definition.

Example 2.2.1 (Selfish Society) Assume that $\alpha_r = \beta_r = \alpha_u = \beta_u = 0$, i.e., all players are selfish. Hence $u_r(w_r) = w_r$ and $u_u(w_u) = w_u$. By Assumption 3, the equilibrium is (\hat{L}_r, \hat{L}_u) , and in this equilibrium we have $w_r = w_u$.

Example 2.2.2 We now relax the assumption in Example 2.2.1, allowing that $\alpha_r, \beta_r, \alpha_u, \beta_u \neq 0$. Assume that $f_u(L_r) \equiv f_r(L_u) \equiv 1$. This indicates that players care about fairness in the labor market, but do not consider the number of players in either areas. Let $\Delta u = u_r(w_r) = u_u(w_u)$, we have

$$w_r - \alpha_r \max\{w_u - w_r, 0\} - \beta_r \max\{w_r - w_u, 0\} = w_u - \alpha_u \max\{w_r - w_u, 0\} - \beta_u \max\{w_u - w_r, 0\} \quad (3)$$

which yields three possible equilibria. The first equilibrium is (\hat{L}_r, \hat{L}_u) , which is exactly the same equilibrium discussed in Example 2.2.1, and $w_r = w_u$ in this equilibrium. However, there are two other types of equilibria in which rural and urban wages are not equal. Consider one situation in which $w_r > w_u$, the equilibrium condition is $w_r - \beta_r(w_r - w_u) = w_u - \alpha_u(w_r - w_u)$, i.e., $\beta_r = \alpha_u$. Similarly, when $w_u > w_r$, the corresponding condition is $\beta_u = \alpha_r$. The latter two types of equilibria exist if the level of altruism of players in the high-wage area equals the level of envy of players in the low-wage area.

We now proceed to the equilibrium analysis of the standard setting of social relationships (Montgomery, 1991; Fehr and Schmidt, 1999). Embedded within a social network, players care about the share of population in the area other than theirs. Let

$$f_r(L_u) = \frac{L_u}{\hat{L}}, f_u(L_r) = \frac{L_r}{\hat{L}} \quad (4)$$

i.e., $f_r(L_u)$ and $f_u(L_r)$ are not constant, but each player in the society has the

same weight. Therefore we can rewrite utility functions as

$$u_r(w_r) = w_r - \alpha_r \frac{L_u}{\hat{L}} \max\{w_u - w_r, 0\} - \beta_r \frac{L_u}{\hat{L}} \max\{w_r - w_u, 0\} \quad (5)$$

and

$$u_u(w_u) = w_u - \alpha_u \frac{L_r}{\hat{L}} \max\{w_r - w_u, 0\} - \beta_u \frac{L_r}{\hat{L}} \max\{w_u - w_r, 0\} \quad (6)$$

There is one point worth mentioning. Unlike the utility function used by Fehr and Schmidt (1999) where $(\hat{L} - 1)$ is in denominators, we here use \hat{L} instead of $(\hat{L} - 1)$. This is because that the utility function used by Fehr and Schmidt is for economic experiments, and the number of players is thus discrete; however, in this paper the total number of players is assumed to be continuous. Therefore, we use \hat{L} in our utility functions.

2.3 Solution of the Model

We now turn to solve the model in the case that $\alpha_r, \beta_r, \alpha_u, \beta_u$ are non-zero. Let the equilibrium condition holds, i.e., $\Delta_u = 0$, we have

$$\begin{aligned} & w_r - \alpha_r \frac{L_u}{\hat{L}} \max\{w_u - w_r, 0\} - \beta_r \frac{L_u}{\hat{L}} \max\{w_r - w_u, 0\} \\ &= w_u - \alpha_u \frac{L_r}{\hat{L}} \max\{w_r - w_u, 0\} - \beta_u \frac{L_r}{\hat{L}} \max\{w_u - w_r, 0\} \end{aligned} \quad (7)$$

Note that we can clearly obtain a labor market equilibrium when the population allocation is such that $w_r = w_u$. This is also the equilibrium for the selfish society.

Equilibrium 0 (the Harris-Todaro equilibrium). (\hat{L}_r, \hat{L}_u) is a labor market equilibrium, in which $w_r = w_u$.

Will the labor market reaches equilibrium in the case $w_r \neq w_u$? We first focus

on the case where $w_r < w_u$, and in this case $L_u < \hat{L}_u$. We show that there is indeed another labor market equilibrium $(\underline{L}_r, \underline{L}_u)$, in which $\underline{L}_u < \hat{L}_u$, and thus $w_r < w_u$.

Equilibrium 1. In the case where $w_u > w_r$, $(\underline{L}_r, \underline{L}_u)$ is a labor market equilibrium, where

$$\underline{L}_u = \frac{(\beta_u - 1)\hat{L}}{\alpha_r + \beta_u} \quad (8)$$

and $\underline{L}_r = \hat{L} - \underline{L}_u$.

To derive this equilibrium, we let $w_r < w_u$, and therefore the equilibrium equation can be rewritten as

$$w_r - \alpha_r \frac{L_u}{\hat{L}}(w_u - w_r) = w_u - \beta_u \frac{L_r}{\hat{L}}(w_u - w_r) \quad (9)$$

which yields

$$w_u - w_r = \beta_u \frac{L_r}{\hat{L}}(w_u - w_r) - \alpha_r \frac{L_u}{\hat{L}}(w_u - w_r) \quad (10)$$

Since $w_u - w_r \neq 0$, the equilibrium exists if $\beta_u L_r - \alpha_r L_u = \hat{L}$. Note that $L_r + L_u = \hat{L}$, which means that when the labor market reaches equilibrium under the condition that $w_r < w_u$, we have

$$\underline{L}_u = \frac{(\beta_u - 1)\hat{L}}{\alpha_r + \beta_u} \quad (11)$$

To make this labor market equilibrium valid, we impose two conditions on it: $\underline{L}_u < \hat{L}$ and $\underline{L}_u > 0$. These two validity conditions are not weak¹. For instance, given the sufficiently large population \hat{L} , this equilibrium does not exist in a model with the setting of Fehr and Schmidt (1999), because it is assumed that $0 \leq \beta_u < 1$

¹A more precise condition for $\underline{L}_u < \hat{L}$ is that $\underline{L}_u < \hat{L}_u$. However, because we impose no structural assumption about the wage function, for any interval $(\underline{L}_u, \hat{L})$ we can always find wage functions $w_r(L_r), w_u(L_u)$ such that $\hat{L}_u \in (\underline{L}_u, \hat{L})$. Therefore we here do not require this condition.

and $\beta_u < \alpha_r$, which makes $\underline{L}_u < 0$. In order to make this equilibrium valid, we now present two cases in which the necessary condition for validity of this equilibrium is fulfilled:

Case 1 (upper-tail altruism). $\beta_u > 1, \alpha_r > -1$ (which also indicates $\alpha_r + \beta_u > 0$).

Case 2 (lower-tail altruism). $\beta_u < 1, \alpha_r < -1$ (which also indicates $\alpha_r + \beta_u < 0$).

Note that in both cases, players in the rural area can also show altruism even if they are victims of unfairness. Indeed, a negative α_r represents that a rural player gains utility if the urban wage is relatively higher. A possible interpretation is the phenomenon of “reciprocal altruism” that rural players expect urban players in the social network send part of their urban wages back to the rural area.

We call Case 1 as the “upper-tail altruism” because players in the upper tail of the earnings distribution (in this case, the players in the urban area) suffer great loss from unfairness. On the contrary we call Case 2 as the “lower-tail altruism” because players in the lower tail of the earnings distribution (in this case, the players in the rural area) appear to be altruistic even facing unfairness, as α_r has a negative upper bound -1 .

We now turn to the case $w_r > w_u$, i.e., the rural wage exceeds the urban wage, and in this case $L_u > \hat{L}_u$. We show that there is another labor market equilibrium at which $w_r > w_u$.

Equilibrium 2. In the case where $w_u < w_r$, $(\overline{L}_r, \overline{L}_u)$ is a labor market equilibrium, where

$$\overline{L}_u = \frac{(1 + \alpha_u)\hat{L}}{\beta_r + \alpha_u} \quad (12)$$

and $\overline{L}_r = \hat{L} - \overline{L}_u$.

To derive this equilibrium, we let $w_r > w_u$, and therefore the equilibrium equation

can be rewritten as

$$w_r - \beta_r \frac{L_u}{\hat{L}}(w_r - w_u) = w_u - \alpha_u \frac{L_r}{\hat{L}}(w_r - w_u) \quad (13)$$

which yields

$$w_r - w_u = \beta_r \frac{L_u}{\hat{L}}(w_r - w_u) - \alpha_u \frac{L_r}{\hat{L}}(w_r - w_u) \quad (14)$$

Since $w_u - w_r \neq 0$, the equilibrium exists if $\beta_r L_u - \alpha_u L_r = \hat{L}$. Note that $L_r + L_u = \hat{L}$, which means that when the labor market reaches equilibrium under the condition that $w_r < w_u$, we have

$$\overline{L_u} = \frac{(1 + \alpha_u)\hat{L}}{\beta_r + \alpha_u} \quad (15)$$

We similarly find two cases such that this equilibrium is valid. Let $0 < \overline{L_u} < \hat{L}$, we have

Case 1 (upper-tail altruism). $\alpha_u > -1$, $\beta_r > 1$ (which also indicates $\beta_r + \alpha_u > 0$).

Case 2 (lower-tail altruism). $\alpha_u < -1$, $\beta_r < 1$ (which also indicates $\beta_r + \alpha_u < 0$).

There are also two cases such that this equilibrium is valid. Again in Case 1, players in the upper tail of the earnings distribution (in this case, players in the rural area) suffer great loss from unfairness. On the contrary Case 2 is the case of “lower-tail altruism” because players in the lower tail of the earnings distribution (in this case, the players in the urban area) appear to be altruistic even facing unfairness. For Equilibrium 2, this shows the symmetric structure to that discussed in Equilibrium 1.

A final check is that Equilibrium 1 and 2 are valid only if $\underline{L_u} < \overline{L_u}$. This yields

the additional condition that

$$\frac{(\beta_u - 1)\hat{L}}{\alpha_r + \beta_u} < \frac{(1 + \alpha_u)\hat{L}}{\beta_r + \alpha_u} \quad (16)$$

which can be further simplified as $(\beta_u - 1)(\beta_r - 1) < (\alpha_u + 1)(\alpha_r + 1)$ in the upper-tail altruism case and $(\beta_u - 1)(\beta_r - 1) > (\alpha_u + 1)(\alpha_r + 1)$ in the lower-tail altruism case.

2.4 Stability

We have shown that under certain circumstances of altruism, the above three labor market equilibria are all theoretically existed. However, it remains unknown whether these equilibria are stable, and under what circumstances they are stable. Assuming that these equilibria do exist, we now proceed to check the stability of them.

To do so, we look at the condition that a player in the rural area will move to the urban area, i.e., whether $\dot{L}_u = \phi(\Delta u)$ is positive. We have

$$\Delta u = u_u(w_u) - u_r(w_r) = \frac{w_u - w_r}{\hat{L}} [(1 - \beta_u)\hat{L} + (\alpha_r + \beta_u)L_u] \quad (17)$$

when $L_u < \hat{L}_u$ (i.e., the urban wage is relatively higher), and

$$\Delta u = u_u(w_u) - u_r(w_r) = \frac{w_r - w_u}{\hat{L} - 1} [(\beta_r + \alpha_u)L_u - (1 + \alpha_u)\hat{L}] \quad (18)$$

when $L_u > \hat{L}_u$ (i.e., the urban wage is relatively lower).

To check the migration process, we discuss two situations separately in which the urban wage is relatively higher and lower, respectively. We start by checking the first situation, in which the urban wage is higher, and the rural is less than \hat{L}_u , i.e., the equilibrium urban labor in the Harris-Todaro model. We have

$$\frac{\partial \dot{L}_u}{\partial L_u} = \phi'(\cdot) \frac{\partial \Delta u}{\partial L_u}, L_u < \hat{L}_u \quad (19)$$

where $\phi'(\cdot) > 0$ and

$$\frac{\partial \Delta u}{\partial L_u} = \frac{w_u - w_r}{\hat{L}} (\beta_r + \alpha_u) + \frac{w'_u + w'_r}{\hat{L}} [(1 - \beta_u)\hat{L} + (\alpha_r + \beta_u)L_u], L_u < \hat{L}_u \quad (20)$$

Similarly in the second situation in which the urban wage is relatively lower, we obtain that

$$\frac{\partial \dot{L}_u}{\partial L_u} = \phi'(\cdot) \frac{\partial \Delta u}{\partial L_u}, L_u > \hat{L}_u \quad (21)$$

where $\phi'(\cdot) > 0$ and

$$\frac{\partial \Delta u}{\partial L_u} = \frac{w_r - w_u}{\hat{L}} (\beta_r + \alpha_u) + \frac{w'_u + w'_r}{\hat{L}} [(\beta_r + \alpha_u)L_u - (1 + \alpha_u)\hat{L}], L_u > \hat{L}_u \quad (22)$$

We subsequently focus on the stability of three equilibria respectively. For the case of upper-tail altruism, we have the following proposition of stability:

Proposition 1. In the upper-tail altruism case (i.e., $\alpha_r + \beta_u > 0$ and $\beta_r + \alpha_u > 0$), (\hat{L}_r, \hat{L}_u) is the only stable equilibrium, and both $(\underline{L}_r, \underline{L}_u)$ and $(\overline{L}_r, \overline{L}_u)$ are unstable.

Proof. We first check the derivative of \dot{L}_u at $L_u = \underline{L}_u$. At this point,

$$\left. \frac{\partial \dot{L}_u}{\partial L_u} \right|_{L_u = \underline{L}_u} = \phi'(\Delta u) \left. \frac{\partial \Delta u}{\partial L_u} \right|_{L_u = \underline{L}_u} \quad (23)$$

where $\phi'(\cdot) > 0$ and

$$\left. \frac{\partial \Delta u}{\partial L_u} \right|_{L_u = \underline{L}_u} = \frac{w_u(\underline{L}_u) - w_r(\hat{L} - \underline{L}_u)}{\hat{L}} (\beta_r + \alpha_u) + \frac{w'_u(\underline{L}_u) + w'_r(\hat{L} - \underline{L}_u)}{\hat{L}} [(1 - \beta_u)\hat{L} + (\alpha_r + \beta_u)\underline{L}_u] \quad (24)$$

It is easy to check that $(1 - \beta_u)\hat{L} + (\alpha_r + \beta_u)\underline{L}_u = 0$. Moreover, since $\underline{L}_u < \hat{L}_u$, $w_u(\underline{L}_u) - w_r(\hat{L} - \underline{L}_u) > 0$. Given $\beta_r + \alpha_u > 0$, this yields that $(\partial\dot{L}_u/\partial L_u)|_{L_u=\underline{L}_u} > 0$. In particular, on both sides of the point $L_u = \underline{L}_u$, $(\partial\dot{L}_u/\partial L_u)$ is positive if L_u is sufficiently close to \underline{L}_u .

Because $\Delta u(\underline{L}_u) = 0$, this result further indicates that for any initial population level $0 < L_u < \underline{L}_u$, $\Delta u(L_u) < 0$. Starting from this population, players in rural are will be unwilling to move to the urban area. Similarly we can find that $\Delta u(L_u) > 0$ for any initial population level $\underline{L}_u < L_u < \hat{L}_u$. Starting from this interval, players in the urban area are not willing to return to the rural area. Therefore, $(\underline{L}_r, \underline{L}_u)$ is not a stable equilibrium.

Note that the sign of $\partial\dot{L}_u/\partial L_u$ in the whole interval $(\underline{L}_u, \hat{L}_u)$ is difficult to analyze; however, we can observe $\partial\dot{L}_u/\partial L_u$ in small neighborhoods around $L_u = \underline{L}_u$ and $L_u = \hat{L}_u$. Specifically, we have

$$\lim_{L_u \rightarrow \underline{L}_u^+} \frac{\partial\dot{L}_u}{\partial L_u} = \lim_{L_u \rightarrow \underline{L}_u^-} \frac{\partial\dot{L}_u}{\partial L_u} > 0, \quad \lim_{L_u \rightarrow \hat{L}_u^-} \frac{\partial\dot{L}_u}{\partial L_u} < 0 \quad (25)$$

We now check the equilibrium (\hat{L}_u, \hat{L}_r) . Because Δu has different function forms on two sides of the point $L_u = \hat{L}_u$, it is not guaranteed that Δu (and thus \dot{L}_u) is differentiable at $L_u = \hat{L}_u$, though they are clearly continuous at $L_u = \hat{L}_u$. However, we can still observe the sign of the derivative of \dot{L}_u on two sides of $L_u = \hat{L}_u$. To do so, note that we have previously shown that $\Delta u(L_u) > 0$ in the interval $(\underline{L}_u, \hat{L}_u)$; furthermore, $\partial\dot{L}_u/\partial L_u < 0$ if L_u is sufficiently close to \hat{L}_u . We similarly obtain that $\Delta u(L_u) < 0$ in the interval (\hat{L}_u, \bar{L}_u) , and it is still true that $\partial\dot{L}_u/\partial L_u < 0$ if L_u is sufficiently close to \hat{L}_u , i.e.,

$$\lim_{L_u \rightarrow \hat{L}_u^-} \frac{\partial\dot{L}_u}{\partial L_u} < 0, \quad \lim_{L_u \rightarrow \hat{L}_u^+} \frac{\partial\dot{L}_u}{\partial L_u} < 0 \quad (26)$$

We immediately get that (\hat{L}_r, \hat{L}_u) is a stable equilibrium. Repeat this exercise

and we can similarly prove that $(\overline{L_r}, \overline{L_u})$ is not a stable equilibrium.

Q.E.D.

Table 1: The Sign of \dot{L}_u

	$(0, \underline{L}_u)$	$(\underline{L}_u, \hat{L}_u)$	$(\hat{L}_u, \overline{L}_u)$	$(\overline{L}_u, \hat{L})$
Sign	-	+	-	+

Upper-tail altruism: $\alpha_r + \beta_u > 0$ and $\beta_r + \alpha_u > 0$.

Table 1 shows the sign of \dot{L}_u in four intervals splitted by three possible equilibria. This straightforwardly shows that the only stable equilibrium of migration is (\hat{L}_r, \hat{L}_u) , i.e., in the upper-tail altruism case, although there are two new possible equilibria besides the conventional Harris-Todaro equilibrium, these new equilibria are unstable. This of course leads to another question: is it possible that the equilibria in which the rural wage differs from the urban wage appear to be stable, and the Harris-Todaro equilibrium is unstable? Proposition 2 investigates the lower-tail altruism, and show that this is possible exactly in this case.

Proposition 2. In the lower-tail altruism case (i.e., $\alpha_r + \beta_u < 0$ and $\beta_r + \alpha_u < 0$) $(\underline{L}_r, \underline{L}_u)$ and $(\overline{L}_r, \overline{L}_u)$ are stable equilibria, and (\hat{L}_r, \hat{L}_u) is the unstable equilibrium.

Proof. The proof follows similar steps as those in Proposition 1. Under the setting $\alpha_r + \beta_u < 0$ and $\beta_r + \alpha_u < 0$, we have

$$\left. \frac{\partial \dot{L}_u}{\partial L_u} \right|_{L_u = \underline{L}_u} < 0, \quad \left. \frac{\partial \dot{L}_u}{\partial L_u} \right|_{L_u = \overline{L}_u} < 0 \quad (27)$$

Again, it is not guaranteed that the second-order derivative exists at $L_u = \hat{L}_u$. However, we can obtain that

$$\lim_{L_u \rightarrow \hat{L}_u^-} \frac{\partial \dot{L}_u}{\partial L_u} > 0, \quad \lim_{L_u \rightarrow \hat{L}_u^+} \frac{\partial \dot{L}_u}{\partial L_u} > 0 \quad (28)$$

A similar table presenting the sign of \dot{L}_u is which indicates that both $(\underline{L}_r, \underline{L}_u)$ and $(\overline{L}_r, \overline{L}_u)$ are stable equilibria, but (\hat{L}_r, \hat{L}_u)

Table 2: The Sign of \dot{L}_u

	$(0, \underline{L}_u)$	$(\underline{L}_u, \hat{L}_u)$	$(\hat{L}_u, \overline{L}_u)$	$(\overline{L}_u, \hat{L})$
Sign	+	-	+	-

Lower-tail altruism: $\alpha_r + \beta_u < 0$ and $\beta_r + \alpha_u < 0$.

is not a stable equilibrium.

Q.E.D.

This proposition shows a different scenario of labor market equilibria than any scenario that we have observed before. When the labor market reaches equilibrium, the urban wage is either greater or less than the rural wage, which depends on the initial population allocation; the conventional Harris-Todaro equilibrium still exists, but is no longer stable.

2.5 Additional Notes

We conclude the baseline model by some additional notes on the labor mobility function \hat{L}_u . A question is whether \dot{L}_u has good mathematical properties. For instance, we have shown our concern that \dot{L}_u may not be differentiable at the point $L_u = \hat{L}_u$. Based on our previous assumption, it is clear that 1) \dot{L}_u is continuous at $L_u = \hat{L}_u$, and 2) \dot{L}_u is differentiable at any point other than $L_u = \hat{L}_u$. However, whether \dot{L}_u is differentiable at \hat{L}_u is still unknown. To start our analysis, we first introduce a lemma concerning the ratio \hat{L}_u/\hat{L}_r .

Lemma 1. The following relationship holds if \dot{L}_u is differentiable at $L_u = \hat{L}_u$:

$$\frac{\hat{L}_u}{\hat{L}_r} = \frac{\alpha_u + \beta_u}{\alpha_r + \beta_r} \tag{29}$$

Proof. If \dot{L}_u is differentiable at $L_u = \hat{L}_u$, then

$$\lim_{L_u \rightarrow \hat{L}_u^-} \frac{\partial \dot{L}_u}{\partial L_u} = \lim_{L_u \rightarrow \hat{L}_u^+} \frac{\partial \dot{L}_u}{\partial L_u} \quad (30)$$

Note that based on our previous assumptions, $\partial \dot{L}_u / \partial \dot{u}$ is differentiable at $L_u = \hat{L}_u$. Therefore, we only need to check the left and right derivative of \dot{u} with respect to L_u . If $\partial \dot{L}_u / \partial \dot{u}$ is differentiable at $L_u = \hat{L}_u$, the left derivative of \dot{u} with respect to L_u at \hat{L}_u is equal to the right derivative of \dot{u} with respect to L_u at \hat{L}_u . Note that $w_u(\hat{L}_u) = w_r(\hat{L}_r)$, hence simplifying the equation of the left and derivative of \dot{u} with respect to L_u at \hat{L}_u we have

$$\begin{aligned} & \frac{\left. \frac{\partial w_u}{\partial L_u} \right|_{L_u = \hat{L}_u} + \left. \frac{\partial w_r}{\partial L_r} \right|_{L_r = \hat{L} - \hat{L}_u}}{\hat{L} - 1} [(1 - \beta_u)\hat{L} + (\alpha_r + \beta_u)\hat{L}_u] \\ &= \frac{\left. -\frac{\partial w_u}{\partial L_u} \right|_{L_u = \hat{L}_u} - \left. \frac{\partial w_r}{\partial L_r} \right|_{L_r = \hat{L} - \hat{L}_u}}{\hat{L} - 1} [(\alpha_u + \beta_r)\hat{L}_u - (1 + \alpha_u)\hat{L}] \end{aligned} \quad (31)$$

which further implies that

$$(1 - \beta_u)\hat{L} + (\alpha_r + \beta_u)\hat{L}_u = (1 + \alpha_u)\hat{L} - (\alpha_u + \beta_r)\hat{L}_u \quad (32)$$

i.e., $(\alpha_u + \beta_u)\hat{L} = (\alpha_r + \beta_u + \alpha_u + \beta_r)\hat{L}_u$. Note that $\hat{L} = \hat{L}_u + \hat{L}_r$, we thus have $(\alpha_u + \beta_u)\hat{L}_r = (\alpha_r + \beta_r)\hat{L}_u$. Furthermore, this immediately implies that $\hat{L}_u / \hat{L}_r = (\alpha_u + \beta_u) / (\alpha_r + \beta_r)$. Q.E.D.

This lemma presents the proportion of populations \hat{L}_u and \hat{L}_r , regardless of the stability of the equilibrium (\hat{L}_r, \hat{L}_u) . Now we proceed to prove that at least under some circumstances, this condition cannot hold, and thus \dot{L}_u is not always differentiable $L_u = \hat{L}_u$. This is done by constructing a contradiction. In fact, it is difficult to analyze differentiability if we have no information about parameters and wage func-

tions. Therefore, we mainly focus on a special case where $\alpha_r = \beta_r$ and $\alpha_u = \beta_u$ (i.e., for a player in one area, he has the identical “qualitative and quantitative” attitudes towards unfairness). To conclude the discussion of our baseline model, we show that in the lower-tail altruism case (i.e., $\alpha_r + \beta_u < 0$ and $\beta_r + \alpha_u < 0$), \dot{L}_u is not differentiable at the point $L_u = \hat{L}_u$.

Proposition 3. Assume that $\alpha_r = \beta_r = \delta_r$ and $\alpha_u = \beta_u = \delta_u$. In the case of lower-tail altruism, i.e., $\delta_r + \delta_u < 0$, \dot{L}_u is not differentiable at \hat{L}_u .

Proof. Suppose that \dot{L}_u is differentiable at $L_u = \hat{L}_u$. By Lemma 1 we have $\hat{L}_u/\hat{L}_r = \delta_u/\delta_r$. Clearly, because $w_r(\underline{L}_r) < w_u(\underline{L}_u)$, $(\underline{L}_r, \underline{L}_u)$ is valid only if $\underline{L}_u/\hat{L} \leq \hat{L}_u/\hat{L}$. By Lemma 1, this implies that

$$\frac{(\beta_u - 1)\hat{L}}{(\alpha_r + \beta_u)\hat{L}} \leq \frac{\alpha_u + \beta_u}{(\alpha_u + \beta_u) + (\alpha_r + \beta_r)} \quad (33)$$

Note that $\alpha_r = \beta_r = \delta_r$ and $\alpha_u = \beta_u = \delta_u$. This further implies that

$$\frac{(\delta_u - 1)\hat{L}}{(\delta_r + \delta_u)\hat{L}} \leq \frac{\delta_u}{\delta_u + \delta_r} \quad (34)$$

In the case that $\delta_r + \delta_u < 0$, this implies that $\delta_u - 1 \geq \delta_u$, a contradiction². Hence, the condition in Lemma 1 is not met, and \dot{L}_u is not differentiable at \hat{L}_u . Q.E.D.

3 A General Model with Unemployment

In this section, we attempt to embed our baseline model within the conventional setting of the Harris-Todaro model, in which unemployment in the urban area is considered. In 3.1 we first reintroduce the setup by bringing a two-sector structure used in the paper of Harris and Todaro. In 3.2 we solve the model and find labor

²Note that in the case that $\delta_r + \delta_u > 0$, this yields $\hat{L} \geq 1$, which does not create the contradictory conclusion.

market equilibria. We conclude this section by the stability analysis of labor market equilibria.

3.1 Setup

Similar with the setup in the baseline model, we consider a continuum of *ex-ante* identical players, of mass $\hat{L} > 1$. There are also two areas, the rural area and the urban area. L_r and L_u are the rural and urban labor, respectively, and $L_r + L_u = \hat{L}$ always holds. Following the conventional formulation in the Harris-Todaro model (1970), we now reintroduce the agricultural production function and the manufacturing production function defined below.

$$\text{Agricultural Production Function : } X_A = q(L_r, \bar{L}, \bar{K}_A), q' > 0, q'' < 0 \quad (35)$$

where X_A is the output of the agricultural good, L_r is the rural labor, \bar{L} is the fixed availability of land, and \bar{K}_A is the fixed capital stock. Note that this agricultural production function is an increasing and concave function of the rural labor L_r . We also define the manufacturing production function below.

$$\text{Manufacturing Production Function : } X_M = f(L_M, \bar{K}_M), f' > 0, f'' < 0 \quad (36)$$

where X_M is the output of the manufactured good, L_M is the part of the urban labor required to produce this output, and \bar{K}_M is fixed capital stock. In this paper we allow L_M to be flexible with the change of L_u (and L_r).

In addition, we assume the price determination by setting

$$P = \rho\left(\frac{X_M}{X_A}\right), \rho' > 0 \quad (37)$$

where P is the price of the agricultural good (in terms of the manufactured good),

i.e., the manufactured good serves as numeraire. Based on this agricultural price, the rural wage can be determined as:

$$w_r^* = P \cdot q' = \rho \left(\frac{f(L_M, \bar{K}_M)}{q(L_r, \bar{L}, \bar{K}_a)} \right) \cdot q' \quad (38)$$

where w_r is the rural wage. Note that if we assume that $\partial L_M / \partial L_r = 0$, which will be explained later, it is clear that the rural wage is decreasing in the rural labor³. This result is the same as the setting in the baseline model, in which we assume that $w_r'(L_r) < 0$. To keep consistency for models in Section 2 and 3, we thus have the following assumption.

Subsequently we determine the manufacturing wage:

$$w_M = f' \geq \bar{w}_M \quad (41)$$

where w_M is the manufacturing wage. This means that the manufacturing wage equals the marginal product of manufacturing labor. Moreover, there is a constraint that f' should be no less than \bar{w}_M , the minimum urban wage. However, following the idea in the Harris-Todaro model, we only deal with the case in which $f' = \bar{w}_M$, i.e., there is no excess demand for labor at \bar{w}_M . In other words, given the fixed number of

³It is easy to prove this argument, as

$$\frac{\partial w_r^*}{\partial L_r} = \rho \left(\frac{f(L_M, \bar{K}_M)}{q(L_r, \bar{L}, \bar{K}_a)} \right) \cdot q'' + \frac{\partial \rho(f(L_M, \bar{K}_M)/q(L_r, \bar{L}, \bar{K}_a))}{\partial L_r} \cdot q' \quad (39)$$

As the price $P = \rho(X_M/X_A)$ is positive and q'' is negative, the first term is negative. For the second term, we have

$$\frac{\partial \rho(f(L_M, \bar{K}_M)/q(L_r, \bar{L}, \bar{K}_a))}{\partial L_r} = \rho \cdot \frac{(\partial f / \partial L_M)(\partial L_M / \partial L_r) \cdot q - f \cdot q'}{q^2(L_r, \bar{L}, \bar{K}_a)} \quad (40)$$

Note that $q, f, \rho > 0$, $q', f' = \partial f / \partial L_M > 0$. It is clearly that the second term, i.e, the derivative of ρ with respect to L_r , is also negative if we assume that $\partial L_M / \partial L_r = 0$. Therefore, under this assumption we have $\partial w_r^* / \partial L_r < 0$, which is exactly the setting in the baseline model. Without this assumption, however, the sign of $\partial w_r^* / \partial L_r$ will be much more complicated.

manufacturing workers in the urban area, we have $\partial L_M / \partial L_r = \partial L_M / \partial L_u = 0$. Based on this, we finally obtain the expected wage in the urban area by considering that not all players in urban area are involved in manufacturing production. The urban expected wage is determined as

$$w_u^* = \frac{\bar{w}_M L_M}{L_u}, \frac{L_M}{L_u} \leq 1 \quad (42)$$

where \bar{w}_M is the minimum wage, which is fixed. This indicates that among all L_u players in the urban area, only L_M of them are employed, where $L_M \leq L_u$. The wage of other unemployed players is assumed to be zero. In other words, the expected wage in the urban area equals the minimum wage \bar{w}_M only in the case of full employment.

3.2 Labor Market Equilibrium and Stability

We now proceed to the equilibrium analysis. Again the labor mobility function $\dot{L}_u = \phi(\Delta u)$, where $\phi(0) = 0$, $\phi'(\Delta u) > 0$ and $\Delta u = u_u - u_r$. This is consistent with the setting in our baseline model, while the only difference is that the urban wage is the expected wage in the urban area, considering that only L_M of all urban labor L_u . This is also consistent with the setting in the Harris-Todaro model, while the only difference is that the variable of the labor mobility function is the expected rural-urban utility differential in this paper, instead of the expected wage differential in the Harris-Todaro model.

Still, utility functions are defined as below:

$$u_r(w_r^*) = w_r^* - \alpha_r \frac{L_u}{\hat{L}} \max\{w_u^* - w_r^*, 0\} - \beta_r \frac{L_u}{\hat{L}} \max\{w_r^* - w_u^*, 0\} \quad (43)$$

and

$$u_u(w_u^*) = w_u^* - \alpha_u \frac{L_r}{\hat{L}} \max\{w_r^* - w_u^*, 0\} - \beta_u \frac{L_r}{\hat{L}} \max\{w_u^* - w_r^*, 0\} \quad (44)$$

where again, w_u^* is the expected urban wage with the consideration of unemployment. More specifically, following our setting in 3.1, we have

$$w_r^* = P \cdot q' = \rho \left(\frac{f(L_M, \bar{K}_M)}{q(L_r, \bar{L}, \bar{K}_a)} \right) \cdot q' \quad (45)$$

and

$$w_u^* = \frac{\bar{w}_M L_M}{L_u}, \frac{L_M}{L_u} \leq 1 \quad (46)$$

Clearly, migration ceases only if the expected rural-urban utility differential equals zero, i.e., $\Delta u = u_u(w_u^*) - u_r(w_r^*) = 0$. If we again assume that there is a population allocation $(\hat{L}_r^*, \hat{L}_u^*)$ such that $w_r^* = w_u^*$, then this immediately leads to the result that the labor market equilibrium is reached if the rural wage equals the expected urban wage, and again we call this equilibrium as the Harris-Todaro equilibrium.

Equilibrium 0 (the Harris-Todaro equilibrium) $(\hat{L}_r^*, \hat{L}_u^*)$ is a labor market equilibrium, in which $w_r^* = w_u^*$.

Similar with the baseline model, in this general model we can also obtain labor market equilibria in which the expected urban wage differs from the rural wage. Indeed, there is an equilibrium each of the case in which $w_u^* > w_r^*$ and $w_u^* < w_r^*$ respectively. However, clearly two equilibria equal to the corresponding equilibria in the baseline model, i.e., we have:

Equilibrium 1. In the case where $w_u^* > w_r^*$, $(\underline{L}_r^*, \underline{L}_u^*)$ is a labor market equilibrium, where

$$\underline{L}_u^* = \frac{(\beta_u - 1)\hat{L}}{\alpha_r + \beta_u} \quad (47)$$

and $\underline{L}_r^* = \hat{L} - \underline{L}_u^*$. Note that compared with the equilibrium $(\underline{L}_r, \underline{L}_u)$ in the baseline model, we have $\underline{L}_u^* = \underline{L}_u$, and also $\underline{L}_r^* = \underline{L}_r$.

Equilibrium 2. In the case where $w_u^* < w_r^*$, $(\bar{L}_r^*, \bar{L}_u^*)$ is a labor market equilibrium,

where

$$\overline{L}_u^* = \frac{(1 + \alpha_u)\hat{L}}{\beta_r + \alpha_u} \quad (48)$$

and $\overline{L}_r^* = \hat{L} - \overline{L}_u^*$. Note that compared with the equilibrium $(\overline{L}_r, \overline{L}_u)$ in the baseline model, we have $\overline{L}_u^* = \overline{L}_u$, and also $\overline{L}_r^* = \overline{L}_r$.

We now turn to the stability analysis on three equilibria. Following the analysis in Section 2, we discuss the upper-tail altruism case and the lower-tail altruism case respectively. We first discuss the upper-tail altruism and have the following proposition:

Proposition 4. In the upper-tail altruism case (i.e., $\alpha_r + \beta_u > 0$ and $\beta_r + \alpha_u > 0$), (\hat{L}_r, \hat{L}_u) is the only stable equilibrium, and both $(\underline{L}_r^*, \underline{L}_u^*)$ and $(\overline{L}_r^*, \overline{L}_u^*)$ are unstable.

Proof. We first check the derivative of \dot{L}_u at $L_u = \underline{L}_u^*$. At this point,

$$\left. \frac{\partial \dot{L}_u}{\partial L_u} \right|_{L_u = \underline{L}_u^*} = \phi'(\Delta u) \left. \frac{\partial \Delta u}{\partial L_u} \right|_{L_u = \underline{L}_u^*} \quad (49)$$

where $\phi'(\cdot) > 0$ and

$$\left. \frac{\partial \Delta u}{\partial L_u} \right|_{L_u = \underline{L}_u^*} = \frac{w_u^*(\underline{L}_u^*) - w_r^*(\hat{L} - \underline{L}_u^*)}{\hat{L}} (\beta_r + \alpha_u) + \frac{\frac{\partial w_u^*}{\partial L_u} + \frac{\partial w_r^*}{\partial L_r}}{\hat{L}} [(1 - \beta_u)\hat{L} + (\alpha_r + \beta_u)\underline{L}_u^*] \quad (50)$$

where

$$w_u^* = \frac{\bar{w}_M L_M}{L_u^*}, \quad \frac{\partial w_u^*}{\partial L_u} = -\frac{\bar{w}_M L_M}{L_u^2} \quad (51)$$

and

$$w_r^* = P \cdot q', \quad \frac{\partial w_r^*}{\partial L_r} = P \cdot q'' + \frac{\partial P}{\partial X_A} (q')^2 \quad (52)$$

Clearly $\partial w_u^*/\partial L_u < 0$. In previous subsection we have also proven that $\partial w_r^*/\partial L_r < 0$. However, note that $(1 - \beta_u)\hat{L} + (\alpha_r + \beta_u)\underline{L}_u^* = 0$ and $w_u^*(\underline{L}_u^*) - w_r^*(\hat{L} - \underline{L}_u^*) > 0$,

indicating that $\partial\Delta u/\partial L_u > 0$ at $L_u = \underline{L}_u^*$, and furthermore, $\partial\dot{L}_u/\partial L_u > 0$ at $L_u = \underline{L}_u^*$.

Because $\Delta u(\underline{L}_u^*) = 0$, this result further indicates that for any initial population level $0 < L_u < \underline{L}_u^*$, $\Delta u(L_u) < 0$. Starting from this population, players in rural area are unwilling to move to the urban area. Similarly we can find that $\Delta u(L_u) > 0$ for any initial population level $\underline{L}_u^* < L_u < \hat{L}_u$. Starting from this interval, players in the urban area are not willing to return to the rural area. Therefore, $(\underline{L}_r^*, \underline{L}_u^*)$ is not a stable equilibrium.

We now check the equilibrium (\hat{L}_u, \hat{L}_r) . Because Δu has different function forms on two sides of the point $L_u = \hat{L}_u$, it is not guaranteed that Δu (and thus \dot{L}_u) is differentiable at $L_u = \hat{L}_u$, though they are clearly continuous at $L_u = \hat{L}_u$. However, we can still observe the sign of the derivative of \dot{L}_u on two sides of $L_u = \hat{L}_u$. To do so, note that we have previously shown that $\Delta u(L_u) > 0$ in the interval $(\underline{L}_u^*, \hat{L}_u)$; furthermore, $\partial\dot{L}_u/\partial L_u < 0$ if L_u is sufficiently close to \hat{L}_u . We similarly obtain that $\Delta u(L_u) < 0$ in the interval $(\hat{L}_u, \overline{L}_u^*)$, and it is still true that $\partial\dot{L}_u/\partial L_u < 0$ if L_u is sufficiently close to \hat{L}_u , i.e.,

$$\lim_{L_u \rightarrow \hat{L}_u^-} \frac{\partial\dot{L}_u}{\partial L_u} < 0, \quad \lim_{L_u \rightarrow \hat{L}_u^+} \frac{\partial\dot{L}_u}{\partial L_u} < 0 \quad (53)$$

We immediately get that (\hat{L}_r, \hat{L}_u) is a stable equilibrium. Repeat this exercise and we can similarly prove that $(\overline{L}_r^*, \overline{L}_u^*)$ is not a stable equilibrium.

Table 3: The Sign of \dot{L}_u

	$(0, \underline{L}_u^*)$	$(\underline{L}_u^*, \hat{L}_u)$	$(\hat{L}_u, \overline{L}_u^*)$	$(\overline{L}_u^*, \hat{L}_u)$
Sign	-	+	-	+

Case 1: $\alpha_r + \beta_u > 0$ and $\beta_r + \alpha_u > 0$.

Table 3 shows the sign of \dot{L}_u in four intervals splitted by three possible equilibria. This presents the same pattern of labor mobility in the baseline model, and indicates that the only stable equilibrium is (\hat{L}_r, \hat{L}_u) ., Q.E.D.

We proceed to the lower-tail altruism case and the similar result is again obtained

in Proposition 5:

Proposition 5. In the lower-tail altruism case (i.e., $\alpha_r + \beta_u < 0$ and $\beta_r + \alpha_u < 0$) $(\underline{L}_r^*, \underline{L}_u^*)$ and $(\overline{L}_r^*, \overline{L}_u^*)$ are stable equilibria, and (\hat{L}_r, \hat{L}_u) is the unstable equilibrium.

Proof. Under the setting $\alpha_r + \beta_u < 0$ and $\beta_r + \alpha_u < 0$, we have

$$\left. \frac{\partial \dot{L}_u}{\partial L_u} \right|_{L_u = \underline{L}_u^*} < 0, \quad \left. \frac{\partial \dot{L}_u}{\partial L_u} \right|_{L_u = \overline{L}_u^*} < 0 \quad (54)$$

Note that \dot{L}_u equals zero at $L_u = \underline{L}_u^*$ and \overline{L}_u^* . Therefore, it indicates that starting with any initial population allocation (L_r, L_u) where $L_u \in (0, \underline{L}_u^*) \cup (\underline{L}_u^*, \hat{L}_u)$, the population allocation will converge to $(\underline{L}_r^*, \underline{L}_u^*)$. Similarly, starting with any initial population allocation (L_r, L_u) where $L_u \in (\hat{L}_u, \overline{L}_u^*) \cup (\overline{L}_u^*, \hat{L}_u)$, the population allocation will converge to $(\overline{L}_r^*, \overline{L}_u^*)$. Therefore, $(\underline{L}_r^*, \underline{L}_u^*)$ and $(\overline{L}_r^*, \overline{L}_u^*)$ are stable.

We now discuss the stability of the equilibrium (\hat{L}_r, \hat{L}_u) . Again, it is not guaranteed that the second-order derivative exists at $L_u = \hat{L}_u$. However, we can obtain that

$$\lim_{L_u \rightarrow \hat{L}_u^-} \frac{\partial \dot{L}_u}{\partial L_u} > 0, \quad \lim_{L_u \rightarrow \hat{L}_u^+} \frac{\partial \dot{L}_u}{\partial L_u} > 0 \quad (55)$$

Note that $\dot{L}_u = 0$ at $L_u = \hat{L}_u$, this shows that the Harris-Todaro equilibrium (\hat{L}_r, \hat{L}_u) is not stable.

A similar table presenting the sign of \dot{L}_u is

Table 4: The Sign of \dot{L}_u				
	$(0, \underline{L}_u)$	$(\underline{L}_u, \hat{L}_u)$	$(\hat{L}_u, \overline{L}_u)$	$(\overline{L}_u, \hat{L}_u)$
Sign	+	-	+	-

Case 2: $\alpha_r + \beta_u < 0$ and $\beta_r + \alpha_u < 0$.

which straightforwardly indicates that both $(\underline{L}_r^*, \underline{L}_u^*)$ and $(\overline{L}_r^*, \overline{L}_u^*)$ are stable equilibria, but (\hat{L}_r, \hat{L}_u) is not a stable equilibrium. Q.E.D.

Proposition 4 and 5 show that the results of the baseline model remain in this general model with the consideration of unemployment: in the case of lower-tail altruism, the original Harris-Todaro equilibrium is stable, while two new equilibria is unstable; however, in the case of upper-tail altruism, we find two stable equilibria in which the rural wage differs from the expected urban wage, and the Harris-Todaro equilibrium, in which the rural wage equals the expected urban wage, is unstable.

4 Conclusion

In this paper, we expand the classical Harris-Todaro model by taking fairness into account. For each player, the utility is not only determined by his own wage, but is also affected by others' earnings. We follow the utility function by Fehr and Schmidt (1999, 2005), in which there are terms for utility gain or loss in cases in which the rural-urban wage differential exists. Unlike in the Harris-Todaro model in which labor mobility is determined by the wage differential, in this paper labor mobility is determined by the utility differential.

We present two models of rural-urban migration. The baseline model describes a simple scenario, in which the urban wage is decreasing in the urban labor and the rural wage is decreasing in the rural labor, but no other structural assumption is made. Subsequently we create a more general model. Following the modeling idea of the Harris-Todaro model (Harris and Todaro, 1970), we assume an agricultural sector in the rural area and a manufacturing sector in the urban area, respectively. Moreover, we assume that the labor required to produce the manufactured good is fixed, i.e., urban unemployment is considered, and the expected rural-urban wage differential (instead of the rural-urban wage differential) affects the utility. However, we draw the similar conclusion in both model: there always exists the conventional Harris-Todaro equilibrium, in which the rural wage equals the expected urban wage;

however, there are also two possible equilibria, in which the rural wage can be either greater or less than the expected urban wage. Moreover, we find the case that the Harris-Todaro equilibrium is unstable, while the two new equilibria is stable. The case is named as “lower-tail altruism” by us in this paper, i.e., relatively lower-income players gain high utility even if their income is lower.

One possible interpretation is the reciprocal altruistic behavior, i.e., players with higher income in one area will provide some monetary supports to another area where players have less money. This is usually seen when rural migrants in cities send some of their money back to their original hometown. The component of reciprocal altruism in the theoretical migration model and the empirical evidence of reciprocal altruism are two interesting topics worthy of subsequent works.

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