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# Knowledge Accumulation in Cities: The Role of Imitation and Innovation

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## Abstract

The aim of this paper is to explicitly model the role of different types of knowledge spillovers (namely imitation and innovation) on agglomeration forces. We use a static general equilibrium framework with two asymmetric locations which represent the city and the periphery, where only the city provides agents with the opportunity to exchange knowledge in face-to-face meetings. These meetings result in the build-up of personal skills and non-excludable innovations. The spatial equilibrium is determined by the interplay of agglomeration and dispersion forces in our model economy. People move to the city until the crowding effect on the urban housing market and decreasing returns to labor exceed the agglomeration effect from having the advantage of the build-up of skills in face-to-face meetings. We can show that people in the city choose an inefficient variety of people they interact with, which in turn leads to cities that are smaller than socially optimal.

*Keywords:* Knowledge exchange, Innovation, Learning, Agglomeration

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## 1. Introduction

The fundamental result of the Spatial Impossibility Theorem (Starrett, 1978) is the fact that without heterogeneity of land, indivisibilities and local aggregate increasing returns, no competitive equilibrium involving transportation costs can exist. This is not even close to what we observe in reality: Despite the existence of urban congestion costs like pollution, crime and commuting costs, almost the entire U.S. population is crowded in only about 2 percent of the overall land area with the rest being very sparsely populated (Burchfield et al., 2006). Furthermore, according to the United Nations World Urbanization Prospects from 2011 we know that 84.2 percent of the entire U.S. population is located in metropolitan areas. Why do individuals accept all these costs only to locate close to each other in cities? Assuming that natural advantages and indivisibilities are not sufficient to justify the extent of this measured concentration in space, this substantial localization must be interpreted as an indicator of agglomeration economies at work. There are mainly three mechanisms explaining agglomeration at the micro level: sharing, matching and knowledge spillovers. Beside the importance of sharing and matching there are several reasons to believe that knowledge spillovers across individuals occurring in a dense urban environment are the main forces behind local aggregate increasing returns. Cities are the places that give individuals the opportunity to exchange their knowledge via face-to-face meetings. People can benefit through these meetings by learning from each other or, if ideas are combined, new innovations arise from these meetings.

Critics like Cairncross (2001) might argue that new communication technologies lead to the “death of distance” making face-to-face contacts to communicate ideas and information irrelevant. If this is true, then cities, as places that in particular cause these contacts, would be harmed because people could avoid the costs of urban living and still benefit from information flows, by just using new communication methods. Leamer and Storper (2001) and Storper and Venables (2004) show arguments against the hypothesis by Cairncross. New communication methods may facilitate the transmission of codifiable information with a stable meaning expressed in a standardized system of symbols, but the exchange of complex uncodifiable knowledge brings along somewhat different challenges. Face-to-face contacts give people the opportunity to use all different forms of communication techniques at the same time. All these different techniques are needed to achieve the most efficient transmission of complex knowledge. Furthermore face-to-face contacts can reduce the moral hazard problem that arises when tacit knowledge is exchanged by offering partners the chance to readily observe and interpret each others behavior. All these properties of face-to-face

meetings cannot be completely replaced by new communication technologies. As long as the transmission of complex uncodified knowledge is of great importance for the economy there is no reason to believe that cities will lose their function as a meeting point for people to exchange their information and ideas. Von Hippel (1994) also demonstrates that frequently happening face-to-face meetings between agents are the best way to exchange what he refers to as sticky knowledge. There is also strong empirical evidence in the literature indicating that knowledge spillovers continue to be restricted by distance. Audretsch and Feldmann (1996) show that more knowledge intensive industries, measured by R&D expenditures and number of skilled workers in that industry, exhibit a significantly stronger geographic concentration. They also find evidence for the great importance of cities in the process of creating new ideas. More than 95 percent of product innovations occur in metropolitan areas and more than 45 percent of these innovations come from the four metropolitan areas New York City, Los Angeles, San Francisco or Boston. Jaffe et al. (1993) find that new patents disproportionately often cite patents that were invented in the same city or region. Their approach directly shows the spatial restriction of knowledge flows. Another argument that is directly linked with the importance of face-to-face contacts comes from Hall (1998), who shows that long-distance travel has grown faster than output and trade. Given the high opportunity costs of business travel, this persistent fact can only be explained by a sustained importance of face-to-face contacts.

To analyze knowledge spillovers occurring in cities, we have to distinguish between two different types. First of all cities give individuals the opportunity to learn from each other through the process of imitation. An untrained worker observes the work of a trained worker and learns by just seeing and imitating his techniques. The process of learning leads to a faster accumulation of human capital in dense urban agglomerations since there are more workers to learn from. A theoretical model explaining the process of learning in cities is established by Glaeser (1999) showing that increasing the city size increases the speed of interactions which in turn accelerates the accumulation of skills. The empirical evidence is provided by Glaeser and Maré (2001). They find that workers who move to cities do not receive the urban wage premium immediately, but experience a disproportionately high wage growth during the time in the city and no decline in wages once they leave the city and move back to the rural area. This empirical observation is most appropriately explained by a process of learning in an urban environment. Jacobs (1968) however claims that cities are the places where new ideas are created when diverse individuals come together and share their knowledge via face-to-face contacts. The process of innovation, which is the second type of knowledge spillover, does not lead to

the accumulation of human capital for a specific worker but increases the rate of technological change in the city, which makes every worker affected by the new innovation more productive. One major difference between the two different types of knowledge spillovers is apparent: Workers directly benefit from the process of learning. The accumulation of human capital makes them more productive what in turn increases their nominal wage. On the contrary innovations are in a sense non-excludable and the contribution to the emergence of innovations is often not directly credited to the inventor and thus not fully compensated. This fact leads to social inefficiencies when it comes to the question, how people should interact in face-to-face meetings.

Since knowledge spillovers are spatially restricted, they directly affect location decisions of economic agents. Individuals prefer living in the city over living in a rural area as long as the advantage arising from local knowledge spillovers transmitted via face-to-face meetings in cities outweigh the disadvantages of urban congestion costs.

So far no theoretical model in the literature incorporates both types of knowledge spillovers. We are the first to explicitly model the impact of imitation and innovation through face-to-face meetings in an urban environment. We use a model economy with two asymmetric locations, called the city and the periphery. The city provides people with the opportunity to exchange their knowledge in local face-to-face meetings, the periphery does not. Agents in our model can choose the range of other people in the city they are willing to interact with in order to exchange information. Since agents do not recognize the impact of these meetings on the rate of technological change in the economy, they only accept a range of matches, that is smaller than socially optimal. We can also show that this additionally leads to a city size, which is smaller than socially optimal.

Section 2 reviews the literature on the topic of local knowledge spillovers. In section 3 we present our model economy. We explicitly model the micro-foundations determining the outcome of face-to-face meetings among agents in the city. We show how people are allocated across the two regions in the spatial equilibrium and we address social inefficiencies arising in our model economy. Section 4 concludes.

## **2. Review of the Literature**

If it is true that spatial proximity accelerates and facilitates the exchange of information and ideas, we have to consider the question which type of face-

to-face contact is best suited for the creation of new ideas and brings about innovation. Does the transmission of information between people with the same knowledge background enhance technological change or is it the combination of ideas coming from different fields of the economy that generate subsequent economic growth? This question is closely related to the discussion about the optimal composition of economic activities in cities. Glaeser et al. (1992) distinguish mainly between two different views of the world. What Glaeser et al. (1992) call the Marshall-Arrow-Romer Model suggests that an increased concentration of a particular industry in a city facilitates the exchange of knowledge between individuals working in this industry and leads to the highest possible innovative outcome. This view relies on the idea that sharing the same knowledge background makes it easier for individuals to communicate specific problems in their field. A common knowledge background is required in order to understand each others' questions and recommendations in face-to-face contacts and is essential for innovations. The Marshall-Arrow-Romer Model suggests that those innovative meetings are particularly promoted by cities which are specialized in one specific industry. Silicon Valley, known for their role as pioneer in computer technology, would be the most famous example for such a highly specialized and innovative city. In contrast, Jacobs (1968) argues that new innovations arise when knowledge coming from completely different industries is combined. In her view the highest innovative outcome is achieved in meetings between highly diverse individuals, sharing ideas that seem unrelated at first glance, but emerge to be innovative when combined. According to Jacobs the most innovative city would be a place where highly diversified individuals have the opportunity to get in contact in face-to-face meetings. Therefore this view would favor diversified cities with no particular specialization in one industry. Glaeser et al. (1992) quote the story of the emergence of the financial industry in New York, where grain and cotton merchants saw the need for national and international financial transactions. It was only that need that gave rise to invent the industry of financial services. Glaeser et al. (1992) and Feldmann and Audretsch (1999) both find empirical evidence for so called Jacobs externalities, i.e. diversity and not specialization of economic activities enhance economic growth in cities. Glaeser et al. (1992) use data on employment growth between 1956 and 1987 of large industries in 170 U.S. cities. They find that industry-employment growth is significantly positively related to urban diversity of industries and negatively related to urban specialization of industries. Feldmann and Audretsch (1994) use a more direct approach to measure the connection between innovative output and the composition of economic activities in a city. Using the United States Small Business Administration's Innovation Data Base (SBIDB) they can directly observe innovative activity across cities by looking

at the number of product introductions across U.S. cities. Their results coincide with Glaeser et al. (1992). Urban specialization of economic activities does not have a positive impact on innovative output, but urban diversity has. Both empirical investigations, differing in methodology, favor Jacobs's perspective, that cities as a meeting point for individuals with different knowledge background are best suited for the creation of new ideas. Those cities are the places where diverse individuals can interact and exchange knowledge via face-to-face contacts. It is true that the same knowledge background facilitates to communicate specific problems and helps to understand each other, but if individuals are too much alike, less innovations will be obtained when relatively similar knowledge is combined.

So far only knowledge spillovers and their effect on innovative output have been discussed. Beside the fact, that knowledge combined in face-to-face meetings leads to the creation of new ideas and thus to a faster rate of technological change, workers can also use these meetings to learn from each other and accumulate human capital at the individual level. There is a wide range of empirical evidence showing that in fact cities are the places that offer the best learning opportunities for workers. Glaeser and Maré (2001) show that urban workers increase the wage differential over non-urban workers during the time they work and live in the city. This urban wage premium is not lost when they move out of the city, supporting the story of skill acquisition in the city. Once workers move out of the urban environment, they keep their skills (and hence their productivity) and therefore continue to earn the same nominal wage in the rural area. Glaeser and Resseger (2010) find that these learning opportunities are especially strong in cities with a surpassing level of skills, indicating that the contact between highly educated individuals accelerates the accumulation of human capital. The city promoting the optimal opportunities for individuals to learn is different from a city promoting the optimal conditions to innovate. Having a different knowledge background might be an advantage in creating new ideas but learning from each other in face-to-face meetings requires at least a related body of knowledge. Glaeser (1999) assumes that agents can enhance their level of skills when meeting other agents who work in a similar field. This assumption indicates that the personal benefit from exchanging knowledge with someone else is higher when agents are relatively alike. Working in different fields makes the exchange of information more difficult and it also gets more complicated to apply someone else's techniques for the enhancement of one's own productivity. The process of learning from somebody in this way can most appropriately be described as the process of imitating somebody. Obviously the imitation of someone else is simplified if the diversity in knowledge background is as small as possible.

So far there are only models focusing on either the role of imitation (or learning) or the role of innovation in an urban context, but up to this point there exists no theoretical model incorporating both types of knowledge spillovers. For instance Glaeser (1999) only addresses the role of learning opportunities in his theoretical model. Berliant et al. (2006) remain silent on the type of knowledge spillovers happening in face-to-face meetings in their model, but their story apparently emphasizes how innovations can arise in urban environment. We in turn explicitly model the effect of the agent's knowledge structure on the effectiveness of learning and on innovative output in regional face-to-face meetings.

### 3. Model Economy

#### 3.1. The Production Function

We consider an economy with two asymmetric locations  $r = C, P$ , where  $C$  represents the city (or core region) and  $P$  represents the periphery. The two regions differ as only the city provides people with the opportunity to get in contact via face-to-face meetings. The underlying assumption is that only the dense environment in cities induces face-to-face contacts of their residents. Two types of workers, the highly educated  $H_r$  and the less educated  $L_r$  are used as inputs to produce the perfectly tradable homogeneous good  $Y_r$  in both locations  $r = C, P$ . Highly educated workers in the city supply  $h_c$  units of labor, less educated workers in the city and both types of workers in the periphery supply one unit of labor. The total number of workers per type in our model economy is given by  $H_c + H_p = \bar{H}$  and  $L_c + L_p = \bar{L}$ . We simplify the treatment of the less educated workers as much as possible in order to focus on the role of highly educated workers. First, less educated workers are immobile and evenly split between both regions, which means that the number of less educated workers in each region is equal to  $\frac{\bar{L}}{2}$ . On the contrary, highly educated workers can choose their location freely. Second, only highly educated workers who are located in the city are involved in the process of knowledge creation and exchange, their face-to-face meetings determine the urban level of technology as well as their individual efficiency. Ignoring the knowledge spillovers for less educated workers can be justified as they rely mostly on codifiable knowledge and techniques that do not differ substantially between city and periphery. Physical capital is not considered as an input to production, this simplification does not affect the results as long as capital is mobile between the regions.



The output of the tradable good in the city, denoted by  $Y_c$ , is produced according to the following Cobb-Douglas production function:

$$Y_c = A_c \left( \sum_{H_c} e_{i,c} h_{i,c} \right)^\alpha L_c^{1-\alpha}, \quad 0 < \alpha < 1. \quad (1)$$

$A_c$  denotes the total factor productivity in the city. As we describe in detail in section 3.2., this total factor productivity  $A_c$  is determined by the knowledge structure of highly educated individuals in the city. The personal effectiveness  $e_{i,c}$  of an highly educated individual  $i$  in the city also critically depends on the chosen knowledge structure, which influences the individual learning outcome in face-to-face meetings with other highly educated individuals. The expression  $\sum_{H_c} e_{i,c} h_{i,c}$  denotes the overall effective input of highly educated labor in the city, where  $H_c$  is the number of the highly educated working in the city and  $h_{i,c}$  indicates the individual supply of labor a highly educated worker offers.  $L_c$  denotes the input of less educated labor in the city. The parameter  $\alpha$  states the relative importance of highly educated and less educated labor in the production process.

The output of the tradable good in the periphery, denoted by  $Y_p$ , is likewise determined by a Cobb-Douglas production function:

$$Y_p = A_p H_p^\alpha L_p^{1-\alpha}, \quad 0 < \alpha < 1. \quad (2)$$

In contrast to the case of the city, no transfer of knowledge between highly educated workers via face-to-face contacts is possible in the periphery. We assume that the periphery is too spacious to allow people to come into contact in personal meetings. While this assumption is restrictive, it allows us to demonstrate the model's main mechanisms most clearly. The main results still hold if we merely assume the quality or quantity of face-to-face contacts to increase in the density of highly educated workers. The individual effectiveness of each highly educated worker in the periphery ( $e_{i,p}$ ) is set equal to one. As in the city the two inputs used for production of the tradable good  $Y_p$  is highly educated labor  $H_p$  and less educated labor  $L_p$ . We assume that the periphery displays the same relative importance of inputs in the production process as the city, meaning that  $\alpha_p = \alpha_c = \alpha$ .

For the moment we remain silent about how the total factor productivity in the city ( $A_c$ ) and in the periphery ( $A_p$ ) emerges. As mentioned above, face-to-face meetings between highly educated individuals play a crucial role in the city. However, these meetings are not possible in the periphery.

### 3.2. Knowledge Spillovers in Face-to-Face Contacts

Our model embeds three different types of knowledge. In the following, the term "education" refers to the worker's type ( $H$  or  $L$ ), the term "skill" refers to the worker's individual effectiveness ( $e_{i,r}$ ) and the term "innovative output" refers to the non-excludable innovations that determine the total factor productivity in the two regions ( $A_r$ ).

The education of the economy's overall population is treated as exogenous and is thus not affected by the structure of the two regions. However as the highly educated are mobile between the two locations, the proportion of highly educated workers in the city is endogenous. The accumulation of the two other forms of knowledge in our economy, skill and innovative output, is determined by local face-to-face interactions of the highly educated in the city.

The city gives highly educated workers the chance to build-up their skills, in the following also referred to as learning. Highly educated workers interact with each other in the city and in the process learn from the other's insights and techniques. However, the extent of this transfer of knowledge depends critically on the match of the two workers who meet. If both work in a similar field, it seems likely, that they find a way to enhance each other's skill set. If they do not work in similar fields, they find it hard to exchange any information of relevance to their work.

The emergence of innovative output in the city also results from face-to-face interactions of the highly educated. Again, the knowledge background of the interacting workers is relevant: the more diverse their type of knowledge, the more likely it is, that their respective knowledge is combined into a new idea which is the source of innovations.

We simplify the treatment of the less educated workers  $L_r$  in both regions as much as possible. As noted in section 3.1. less educated workers are immobile and exogenously split up between the city and the periphery ( $L_c = L_p = \frac{1}{2}$ ). Neither in face-to-face contacts with other less educated workers, nor in face-to-face contacts with highly educated workers, any knowledge spillovers are achieved, independent of their location. Only a match between highly educated workers in the city leads to the accumulation of skills and innovative output.

In our approach, the variety of the economy's knowledge base is displayed by the unit circle. The representation of the economy's knowledge base using a unit circle is adopted from Helsley and Strange (1990) and was also used by Berliant et al. (2006) and Brueckner et al. (2002). Each highly educated worker's

knowledge type is exogenously determined and represented by its position  $k$  on the circle's circumference. The circumference  $K$  can be interpreted as the economy's knowledge space representing all possible knowledge types. The location  $k \in K$  on the unit circle is drawn from a unit distribution and is assigned to each highly educated worker in the economy when born. So each location  $k \in K$  has the same probability of being assigned to a highly educated worker. The position  $k$  on the unit circle is only of relevance when located in the city. In the periphery knowledge type  $k$  is completely irrelevant for the highly educated worker.

Given the position  $k$  on the unit circle, each highly educated individual  $i$  in the city chooses its own knowledge spread  $\delta_i$ , which determines the range of highly educated workers, individual  $i$  is able to communicate with. The knowledge spread  $\delta_i$  is geometrically represented by the arc around knowledge type  $k$  leading to a knowledge horizon for individual  $i$  given by  $[k - \frac{\delta_i}{2}, k + \frac{\delta_i}{2}]$ . The area characterizes the set of disciplines, individual  $i$  has at least some elementary knowledge about. This elementary knowledge is indispensable to enable individual  $i$  to communicate with a worker with knowledge type  $k' \in [k - \frac{\delta_i}{2}, k + \frac{\delta_i}{2}]$  in order to learn from him or to combine their knowledge to create innovative output. One can also interpret the area  $[k - \frac{\delta_i}{2}, k + \frac{\delta_i}{2}]$  as the set of acceptable matches for individual  $i$ .

The unit circle represented in figure 1 displays the knowledge space  $K$  in our model economy. The knowledge type  $k'$  of the highly educated worker  $A$  is located within the knowledge horizon of individual  $i$  with knowledge type  $k$ , which enables  $i$  to communicate with  $A$ . This is the minimum requirement for  $i$  to use the meeting for the build-up of skills (learning) which is accompanied by the creation of innovative output. The knowledge type  $k''$  of individual  $B$  is situated outside the knowledge horizon of individual  $i$ . Consequently no exchange of knowledge can occur between these two highly educated workers.

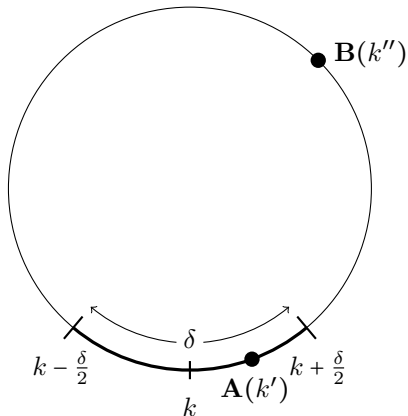


Figure 1: Knowledge Space of the Model Economy

We formalize face-to-face interactions as random meetings between the highly educated in the city. Each individual is randomly matched with one highly educated living in the city. We simplify by assuming that each highly educated agent faces only one potential meeting in his life. Beside the process of learning, which raises individual effectiveness, every meeting in the city generates some form of innovative output at the same time. We can think of these meetings very broadly as any kind of interaction that transfers knowledge. Most plausibly, it can be interpreted as some combination of observing techniques and verbal communication. Formally, we look at the learning output and the innovative output for each individual in each meeting. As described above, individual  $i$  can only learn from his partner  $i'$  if this agent's knowledge type is within the knowledge horizon of  $i$ . However, as we allow for different knowledge horizons across individuals, it is possible, that agent  $i'$  is within the knowledge horizon of  $i$ , but  $i$  is not within the knowledge horizon of  $i'$ . Thus, it is possible, that  $i$  generates positive learning and innovative output from the meeting, while his partner does not. We can imagine such a one-sided meeting where only one agent observes the other and asks him about his expertise, which gives him new information that enhances his skills and the opportunity to combine the new input with his existing knowledge in order to innovate. While our setup allows for this kind of asymmetry, it will be of no concern in the symmetric equilibrium. In a two-sided meeting, both agents learn symmetrically from each other and combine their knowledge into new ideas. It should be noted that our matching mechanism does not assume increasing matching frequency in the number of agents.

As mentioned above, the chosen knowledge spread  $\delta_i$  has crucial impact on the extent of knowledge spillovers occurring in urban face-to-face meetings.

First, we want to look at the influence of the knowledge spread  $\delta_i$  on the process of learning. It is important to stress, that learning improves a highly educated worker's effectiveness  $e_{i,c}$  and thus his level of skill. Skill is excludable in the sense that any agent's learning only affects his own productivity. The knowledge spread  $\delta_i$  influences the highly educated worker's effectiveness  $e_{i,c}$  through two channels. First, increasing  $\delta_i$  increases the range of highly educated workers individual  $i$  is able to learn from. As described above at least some basic knowledge about knowledge type  $k'$  of individual  $i'$  is required for individual  $i$  to learn from  $i'$ . However, increasing the knowledge spread  $\delta_i$  has a second effect: The expected distance  $d(k, k')$  of knowledge type  $k$  and the partner's knowledge type  $k'$  in accepted matches is increasing. Working in diverse fields makes the exchange of information and thus the process of learning more difficult. A higher  $\delta_i$  leads to more acceptable matches, but the average outcome of the learning process is diminished. The personal effectiveness  $e_{i,c}$  of a highly educated worker  $i$  in the city can be described by the following equation:

$$e_{i,c} = 1 + \delta_i \left\{ [b_0 - b_1 E[d(k, k')]] \right\} = 1 + \delta_i \left( b_0 - b_1 \frac{\delta_i}{4} \right), \quad b_0, b_1 \geq 0. \quad (3)$$

Independently of whether the highly educated worker  $i$  is successfully matched with another highly educated in the city, he can always maintain a personal effectiveness which is equal to one. One can think of this outside option as acquiring skills from solitary learning. The expression  $\delta_i(b_0 - b_1 \frac{\delta_i}{4})$  indicates the expected outcome from learning when successfully matched with a partner. The knowledge spread  $\delta_i$  captures the probability of being matched with a suitable partner. The parameter  $b_0$  describes the maximum learning outcome of individual  $i$  when matched with a worker of exactly the same knowledge type. Without loss of generality, this outcome is normalized to one. The sensitivity of learning with respect to the knowledge distance  $d(k, k')$  is represented by the parameter  $b_1$ . By construction, the expected knowledge distance  $d(k, k')$  in all accepted matches is equal to  $\frac{\delta_i}{4}$ . We allow for learning outcomes to be negative by setting  $b_1 > 2$ . If we think of the outcome of a learning match as relative to the outside option of solitary learning, it makes sense to assume that there are partners from whom one can acquire less information of relevance to one's job than from solitary study. Consequently, agents will choose their knowledge spread such that they don't interact with partners that would cause a negative learning output and would lead to a personal effectiveness smaller than one.

The second form of knowledge accumulation we want to capture is the creation

of new ideas resulting in innovations. As mentioned before, this form of knowledge accumulation is seen as a by-product of the learning process happening in face-to-face contacts in the city. In contrast to the process of learning, which benefits from proximity in knowledge types, groundbreaking innovations are more likely to occur in meetings between diverse individuals. This means that the innovative output from a meeting is increasing in the distance between the two worker's knowledge types  $k$  and  $k'$ . However, to be able to communicate knowledge in order to create innovative output, the matched worker's knowledge type  $k'$  again has to fall into the knowledge horizon  $[k - \frac{\delta_i}{2}, k + \frac{\delta_i}{2}]$  of individual  $i$ , otherwise there is no chance to communicate with each other. The greater the difference in the knowledge background, the higher the innovative output from a face-to-face contact between two highly educated workers. The underlying assumption is that the creation of new ideas is positively influenced when experts from different fields meet to exchange their knowledge that is unrelated at first glance but turns out to be complementary and productive when combined into new ideas. This assumption is closely related to Jacobs's view of the world, who claims that innovations arise when people from different branches of the economy come into contact. The minimum requirement for such an innovative meeting is just the ability to understand each other.

The individual innovative output of a highly educated worker  $i$  in the city depends on the expected distance between his knowledge type  $k$  and knowledge type  $k'$  of the matched worker denoted by  $d(k, k')$ . Again the knowledge spread  $\delta_i$  influences the individual innovative output in two ways: A higher  $\delta_i$  increases the range of highly educated workers, individual  $i$  is able to communicate with and thus the probability of being matched with a suitable partner. Secondly, extending the knowledge spread  $\delta_i$  leads to a higher expected distance between the two matched workers, which enhances the expected innovativeness of such a meeting. The individual innovative output  $I_i$  from urban face-to-face contacts is described by the following equation:

$$I_{i,c} = 1 + \delta_i \left\{ a_o + a_1 E[d(k, k')] \right\} = 1 + \delta_i \left( a_o + a_1 \frac{\delta_i}{4} \right), \quad a_o, a_1 \geq 0. \quad (4)$$

Again, independently of whether the highly educated worker  $i$  is successfully matched with another worker in the city, he always creates an individual innovative output equal to one. This innovative output can be raised by getting in contact and exchanging knowledge with other highly educated workers in the city. The additional innovative output arising from face-to-face contacts in the city is described by the expression  $\delta_i(a_o + a_1 \frac{\delta_i}{4})$ . The knowledge spread  $\delta_i$  captures the probability of being matched with a suitable partner and the term

in brackets describes the expected quality of the innovation when successfully matched. Increasing the knowledge horizon  $\delta_i$  raises the probability of being able to interact and additionally increases the expected knowledge distance in meetings between a worker with knowledge type  $k$  and a matched worker with knowledge type  $k'$ . This in turn leads to meetings that are more innovative and increase the individual innovation output. For the sake of simplicity we set the parameter  $a_0 = 0$ . In accordance with Jacobs's view, we assume that the most important factor for the creation of innovation is the diversity of knowledge that is combined in face-to-face meetings.

Figure 2 shows how the distance between knowledge type  $k$  and knowledge type  $k'$  influences both the individual innovative output and the personal effectiveness of highly educated workers in the city. The red line (personal effectiveness) is decreasing with the distance between the two knowledge types  $k$  and  $k'$ . The blue line (individual innovative output) is increasing with the distance  $d(k, k')$ . Individual  $i$  only accepts matches with individuals making him better off (in terms of his personal effectiveness) compared to the outside option of solitary study.

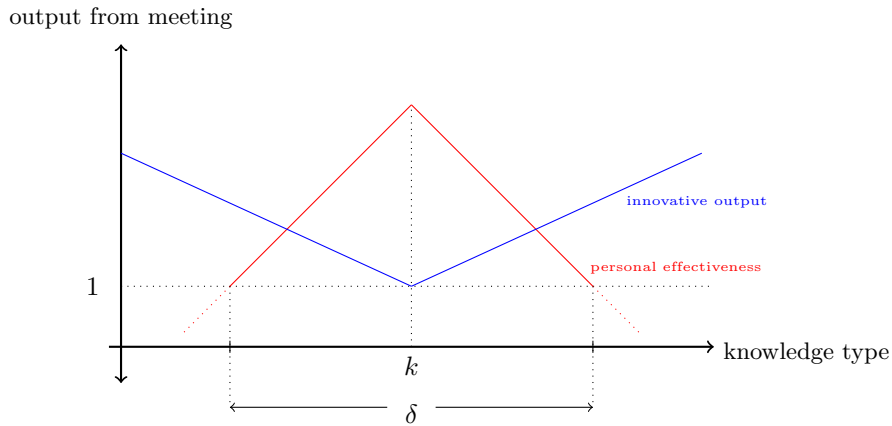


Figure 2: Individual Output from a meeting

The overall level of innovative output that is created in the city is the sum of each highly educated individual's innovative output, which is captured in the

following equation:

$$A_c = \sum_{H_c} \left( 1 + \delta_i a_1 \frac{\delta_i}{4} \right) = \sum_{H_c} \left( 1 + a_1 \frac{\delta_i^2}{4} \right) \quad (5)$$

where  $H_c$  is the total supply of highly educated work in the city.

Contrary to the city, the periphery does not provide the opportunity for highly educated workers to get into contact via face-to-face meetings. The level of the total factor productivity  $A_p$  is determined just by the number of highly educated workers living in the periphery. As indicated by equation (4) each highly educated worker generates innovative output equal to one for his location. The chance for increasing this innovative output through knowledge exchange in face-to-face meetings is not provided in the periphery. Therefore we know that

$$A_p = \sum_{H_p} 1 = H_p. \quad (6)$$

In the following we assume that firms and highly educated workers in the city are not aware of how the technological rate of change in the economy emerges. The reason for highly educated workers to communicate knowledge with other workers is the increase of their personal effectiveness. To be matched with someone who is in one's own knowledge horizon directly increases ones's productivity and thus the wage. The emergence of innovative output in such a meeting is only interpreted as a by-product of the learning process. The more diverse matched individuals are in their type of knowledge, the more complicated the process of learning from each other becomes. But exactly this cumbersome procedure of trying to bring each other closer to the own branch generates the innovative output in our model. The meetings between very diverse individuals, who have trouble to understand each other's point of view and are thus in a vivid give-and-take of information, are the most innovative meetings, in which very different types of knowledge are combined. Highly educated agents in the city are assumed to not fully understand how this process of learning is also translated into innovative output and in turn into a higher technological rate of change in the economy. Therefore they disregard in the following the impact of the chosen knowledge spread on the innovative output of an accepted match. We also assume that firms are not capable of accounting innovative output directly to individuals and are thus also not able to compensate for it.



### 3.3. Wages and Utility

The general equilibrium framework builds upon the model introduced by Peri (2001).

We have four types of workers in our model economy. The highly educated working in the city ( $H_c$ ), the highly educated working in the periphery ( $H_p$ ), the less educated working in the city ( $L_c$ ) and the less educated working in the periphery ( $L_p$ ). The wages  $w_H^c, w_H^p, w_L^c$  and  $w_L^p$  of these different types are determined by their marginal product in their respective location.

$$w_H^c = \frac{\partial Y_c}{\partial h_{i,c}} = \alpha A_c \left( \sum_{H_c} e_{i,c} h_{i,c} \right)^{\alpha-1} e_{i,c} L_c^{1-\alpha} \quad \forall i \in H_c \quad (7)$$

$$w_H^p = \frac{\partial Y_p}{\partial H_p} = \alpha A_p H_p^{\alpha-1} L_p^{1-\alpha} \quad (8)$$

$$w_L^c = \frac{\partial Y_c}{\partial L_c} = (1-\alpha) A_c \left( \sum_{H_c} e_{i,c} h_{i,c} \right)^{\alpha} L_c^{-\alpha} \quad (9)$$

$$w_L^p = \frac{\partial Y_p}{\partial L_p} = (1-\alpha) A_p H_p^{\alpha} L_p^{-\alpha} \quad (10)$$

In the following we will assume that the highly educated in the city will also supply one unit of labor ( $h_{i,c} = 1$ ).

Each agent in the model economy derives utility from consuming the tradable good  $G$  at price  $p_G$  and by consuming housing services  $T$  at price  $p_T$ . The preferences of each agent can be described by a Cobb-Douglas utility function, given by

$$U = G^{\theta} T^{1-\theta}, \quad 0 < \theta < 1. \quad (11)$$

We assume that both locations  $C$  and  $P$  offer the same supply of land and normalize this amount to one in both the city and the periphery. The rents for those housing services in location  $r$ , denoted by  $p_T^r$ , are paid to landowners who are living in neither of the two regions and spend their income in markets not included in our model economy. Since the wages described in equations (7) to (10) are the only source of income to agents, we know that the optimal allocation between consumption  $G$  and housing services  $T$  for each agent of education type

$I$ , living in region  $r$  is as follows:

$$G_{I,r} = \theta \frac{w_I^r}{p_G} \quad T_{I,r} = (1 - \theta) \frac{w_I^r}{p_T^r} \quad (12)$$

where  $I = H, L$  and  $r = C, P$ .

The equilibrium price for the tradable good  $p_G$  adjusts such that market-clearing on the global goods market is achieved. In equilibrium the total expenditures on the tradable good have to equal the total supply of the tradable good. Since all workers are rewarded in compliance with a constant returns to scale production technology in both regions (ignoring their impact on  $A_r$ ), the sum of all wages paid in region  $r$  equals the overall output of the tradable good in the respective region. Therefore the condition for market-clearing on the goods market becomes

$$\underbrace{\theta \frac{Y^c}{p_G} + \theta \frac{Y^p}{p_G}}_{\text{demand for the tradable good}} = \underbrace{Y^c + Y^p}_{\text{supply of the tradable good}} \quad (13)$$

This equilibrium condition for the goods market indicates that the price of the tradable good has to equal  $p_G = \theta$  in both locations.

The price  $p_T^r$  adjusts in each region such that market-clearing on the regional housing market is achieved. In equilibrium, the total expenditures on housing services in region  $r$  have to equal the total supply of housing services in the respective region. Therefore the condition for market-clearing on the regional housing market becomes

$$\underbrace{(1 - \theta) \frac{Y^r}{p_T^r}}_{\text{demand for housing in region } r} = \underbrace{1}_{\text{supply of housing in region } r} \quad (14)$$

Solving for the price of housing services  $p_T^r$  yields  $p_T^r = (1 - \theta)Y^r$ . We clearly see that the crowding effect in our model economy is mainly captured by a higher price of housing services. Since the supply of housing in region  $r$  is fixed, an increase in the worker's expenditures in this region directly drives up the prices on the regional housing market.

The optimal allocation between consumption  $G$  and housing services  $T$  can thus be written as

$$G_{I,r} = w_I^r \quad T_{I,r} = \frac{w_I^r}{Y^r} \quad (15)$$

### 3.4. Optimal Choice of Knowledge Spread

The highly educated located in the city have a direct impact on their wage by choosing their knowledge spread  $\delta_i$ . This knowledge spread  $\delta_i$  determines the range of individuals they are able to communicate with and additionally has an impact on the potential outcome of learning. From equation (15) we can see that maximizing wages in turn leads to maximization of utility. Therefore the highly educated workers in the city face the following maximization problem:

$$\begin{aligned} \max_{\delta_i \in [0,1]} w_H^c &= \alpha A_c \left( \sum_{H_r} e_{i,c} \right)^{\alpha-1} e_{i,r} L_r^{1-\alpha} = \\ &= \alpha A_c \left( \sum_{H_c} \left[ 1 + \delta_i \left( 1 - b_1 \frac{\delta_i}{4} \right) \right] \right)^{\alpha-1} \left[ 1 + \delta_i \left( 1 - b_1 \frac{\delta_i}{4} \right) \right] L_c^{1-\alpha} \end{aligned} \quad (16)$$

In section 3.1. we explained in detail why the impact on the total factor productivity  $A_c$  is not taken into account by the highly educated when choosing their spread of knowledge  $\delta_i$ . Innovative output is considered as a by-product that is generated in the process of learning through face-to-face meetings. Highly educated agents contribute to innovative output, but they do not consider this impact when choosing their knowledge horizon. Therefore, they only take into account the impact of their knowledge spread  $\delta_i$  on the personal effectiveness  $e_{i,c}$ . Maximizing wages is thus equivalent to maximizing the personal effectiveness  $e_{i,c}$ , which yields a solution of  $\delta_i = \delta^* = \frac{2}{b_1}$ . This value of  $\delta^*$  is chosen by all highly educated workers located in the city. By construction (we set  $b_1 > 2$ ) highly educated agents are selective in their choice of partners they are willing to accept. They only agree to be matched with individuals yielding a positive learning outcome  $e_{i,c} > 1$  for them. In all other cases highly educated workers in the city prefer solitary learning over the transfer of knowledge.

### 3.5. Spatial Equilibrium

The previously established framework includes two centrifugal forces that induce agglomeration, namely the two forms of knowledge accumulation resulting from face-to-face contacts. As the highly educated cannot benefit from face-to-face interactions in the periphery, this mechanism provides the incentive for them to move to the city. On the other hand, our model also considers two centripetal forces limiting the extent of agglomeration. The first centripetal force are the crowding effects in the housing markets, which lead to increasing house prices

in more densely populated regions. The second centripetal effect stems from the decreasing returns in highly educated labor embodied in the Cobb-Douglas production function.

In order to achieve the spatial equilibrium in our model economy, utility levels given by (11) of the highly educated have to be equal across both regions. The condition for a spatial equilibrium is given by the following equation:

$$\underbrace{\left(G_H^c\right)^\theta \left(T_H^c\right)^{1-\theta}}_{\text{level of utility in the city}} = \underbrace{\left(G_H^p\right)^\theta \left(T_H^p\right)^{1-\theta}}_{\text{level of utility in the periphery}} \quad (17)$$

Evaluating the wage equations (7) and (8) for highly qualified workers in both regions at the chosen knowledge spread  $\delta_i^* = \frac{2}{b_1}$  and using the optimal allocations given by (15) yields the utility levels in both regions. As highly educated workers are freely mobile, they move between the two regions until the utility levels are equalized. The equilibrium condition can thus be written as

$$\left(\alpha\left(1 + \frac{a_1}{b_1^2}\right)\left(1 + \frac{1}{b_1}\right)^\alpha H_c^\alpha\right)^\theta \left(\frac{\alpha}{H_c}\right)^{1-\theta} = \left(\alpha H_p^\alpha\right)^\theta \left(\frac{\alpha}{H_p}\right)^{1-\theta} \quad (18)$$

where the left-hand side represents the level of utility achieved in the city and the right-hand side represents the level of utility that is achieved in the periphery.

Solving for the equilibrium allocation of highly qualified workers gives us

$$\frac{H_c}{H_p} = \left[ \left(1 + \frac{a_1}{b_1^2}\right)\left(1 + \frac{1}{b_1}\right)^\alpha \right]^{\frac{\theta}{1-\theta-\alpha\theta}} \quad (19)$$

Due to the immobility of less educated workers, an increase in  $\frac{H_c}{H_p}$  also signifies an increase in the city's population. This result clearly reflects the agglomeration and dispersion forces at work. The expression in square brackets captures the impact of the face-to-face meetings on the accumulation of ideas and skills in the city. The interplay of these effects manifests our agglomeration force. The centripetal forces, i.e. the crowding effect in house prices and decreasing returns to highly educated work, are represented in the exponent by  $\theta$  and  $\alpha$  respectively.

### 3.6. Social Inefficiencies

Social inefficiency arises in our model due to the uncompensated innovative output. When choosing their knowledge spread, agents only maximize their

expected learning output and do not consider the impact of their choice of  $\delta$  on innovation. As the quality of innovation increases in the diversity of knowledge types while the quality of learning decreases in diversity, this leads to an equilibrium knowledge spread that is smaller than socially optimal. To show this inefficiency formally, we consider the impact of  $\delta$  on output of the tradable good in the city. For a given distribution of the highly educated, overall utility in the city depends only and is increasing in output of the tradeable good. It is straightforward to show that the socially optimal choice of  $\delta$  is the same for all agents, so we focus on these symmetric cases. This allows us to write output in the city as

$$Y_c = \left(1 + \frac{a_1\delta^2}{4}\right)H_c \left[\left(1 + \delta - \frac{b_1\delta^2}{4}\right)H_c\right]^\alpha L_c^{1-\alpha} \quad (20)$$

Differentiating with respect to  $\delta$  yields

$$\begin{aligned} \frac{\partial Y_c}{\partial \delta} = & a_1 \frac{\delta}{2} H_c \left[\left(1 + \delta - \frac{b_1\delta^2}{4}\right)H_c\right]^\alpha L_c^{1-\alpha} + \\ & \alpha \left(1 + \frac{a_1\delta^2}{4}\right)H_c \left[\left(1 + \delta - \frac{b_1\delta^2}{4}\right)H_c\right]^{\alpha-1} \left[\left(1 - \frac{b_1\delta}{2}\right)H_c\right] L_c^{1-\alpha} \end{aligned} \quad (21)$$

We evaluate this derivative at  $\delta^* = \frac{2}{b_1}$  in order to determine the social efficiency of the equilibrium choice. Using the envelope theorem and considering that  $\delta^*$  maximizes effectiveness, we know that the marginal effect of changing  $\delta$  on effectiveness is equal to 0. Therefore, the derivative evaluated at  $\delta^*$  simplifies to

$$\left. \frac{\partial Y_c}{\partial \delta} \right|_{\delta=\delta^*} = \frac{a_1}{b_1} H_c \left[\left(1 + \frac{1}{b_1}\right)H_c\right]^\alpha L_c^{1-\alpha} > 0 \quad (22)$$

This result confirms the intuition provided above, the marginal effect of increasing  $\delta$  at  $\delta^*$  on innovative output is positive while the marginal effect on learning is 0. This implies that  $\delta^*$  cannot be the socially efficient choice as increasing  $\delta$  marginally increases overall utility in the city.

As previously shown, the equilibrium allocation of the highly qualified depends on the magnitude of knowledge accumulation and therefore on  $\delta$ . The inefficient choice of  $\delta^*$  leads to less than optimal knowledge accumulation and therefore smaller agglomeration forces than socially optimal. Increasing  $\delta$  beyond  $\delta^*$  towards the socially optimal level thus also increases the share of highly qualified living in the city. Formally, the equilibrium allocation depending on  $\delta$  is

$$\frac{H_c}{H_p} = \left[\left(1 + \frac{a_1\delta^2}{4}\right)\left(1 + \delta - \frac{b_1\delta^2}{4}\right)^\alpha\right]^{\frac{\theta}{1-\theta-\alpha\theta}} \quad (23)$$

Differentiating with respect to  $\delta$  yields

$$\frac{\partial \frac{H_c}{H_p}}{\partial \delta} = \frac{\theta}{1 - \theta - \alpha\theta} \left[ \left(1 + \frac{a_1\delta^2}{4}\right) \left(1 + \delta - \frac{b_1\delta^2}{4}\right)^\alpha \right]^{\frac{2\theta + \alpha\theta - 1}{1 - \theta - \alpha\theta}} \left[ a_1 \frac{\delta}{2} \left(1 + \delta - \frac{b_1\delta^2}{4}\right)^\alpha + \left(1 + \frac{a_1\delta^2}{4}\right) \left(\alpha \left(1 + \delta - \frac{b_1\delta^2}{4}\right)^{\alpha-1} \left(1 - \frac{b_1\delta}{2}\right)\right) \right] \quad (24)$$

Again, we evaluate the derivative at  $\delta^*$  and find that the marginal effect driven by effectiveness is 0, while the marginal effect from innovation is positive. The overall effect from increasing  $\delta$  on the share of highly educated living in the city is therefore

$$\left. \frac{\partial \frac{H_c}{H_p}}{\partial \delta} \right|_{\delta=\delta^*} = \frac{\theta}{1 - \theta - \alpha\theta} \left[ \left(1 + \frac{a_1\delta^2}{4}\right) \left(1 + \delta - \frac{b_1\delta^2}{4}\right)^\alpha \right]^{\frac{2\theta + \alpha\theta - 1}{1 - \theta - \alpha\theta}} \frac{a_1}{b_1} \left[ \left(1 + \frac{1}{b_1}\right) \right]^\alpha > 0 \quad (25)$$

We can conclude from this preliminary analysis, that the agents' ignorance of innovative effects leads to knowledge spreads that are narrower than socially optimal. This inefficiency means that agglomeration forces do not reach their optimal extent and therefore leads to cities that are smaller than socially desirable.

#### 4. Conclusion

If it is true that distance still plays a crucial role in the transmission of codifiable knowledge between people, then cities, offering a dense urban environment, continue to keep their role as a meeting point for people to exchange their information. There is strong empirical evidence showing the continuing importance of urban knowledge spillovers. Audretsch and Feldmann (1996) show that more than 96 percent of product innovations stem from metropolitan areas and Glaeser and Maré (2001) find that the urban wage premium is most accurately explained by learning opportunities in cities.

The aim of this paper is to develop a spatial model that explicitly incorporates the different types of knowledge spillovers taking place in cities and to show how they affect the migration decision of highly educated individuals. We use a static general equilibrium framework with two types of labor (highly and less educated workers) and two asymmetric locations: The city and the periphery, where only the city provides highly educated workers with the opportunity to exchange knowledge via face-to-face interactions. Our model incorporated two forms of knowledge spillovers happening in these meetings whose intensities were dependent on the similarity of knowledge background of the interacting

individuals: First, the individual build-up of skills through the process of learning increases in the similarity of knowledge backgrounds. And second, innovative output generated in a meeting decreases in the similarity of knowledge backgrounds. This reflects the general sentiment that diversity stimulates the emergence of new ideas. We also assumed that highly educated workers only focus on the build-up of their personal skills when deciding about the range of individuals in the city they accept to be matched with, whereas innovative output is a by-product of the process of learning.

The interplay of agglomeration and dispersion forces determines the allocation of highly educated workers in the spatial equilibrium. Moving to the city gives them the chance to increase their personal effectiveness through the process of learning in face-to-face meetings. On the other hand there are two dispersion forces at work: First, the crowding effect in the regional housing market drives up prices for housing services and secondly, highly educated workers face decreasing returns to scale to their supplied work. However, the equilibrium allocation of highly educated workers across the two regions is socially inefficient. As mentioned above highly educated workers only focus on the build-up of their personal skills since the increase in personal effectiveness is directly compensated by firms. Therefore highly educated workers in the city choose a relatively narrow range of individuals, they accept to be matched with. A Social Planner however, would recognize that meetings between more diverse individuals in the city would have a positive impact on the innovative output. This inefficient decision of the highly educated additionally implicates that agglomeration forces do not reach their optimal extent and therefore create cities that are smaller than socially optimal. We are the first to explicitly model the impact of different forms of knowledge spillovers on agglomeration forces. However, there are still weak points in our model environment. So far, we take the asymmetry between the two locations as exogenous, meaning that we assumed the extreme case that knowledge spillovers are only possible in the city and not in the periphery. A more refined approach would be to set up a model with two symmetric locations, where the asymmetry emerges endogenously with the quality of interactions depending on the density of highly educated workers. Additionally we made the assumption that the highly educated fail to anticipate their impact on the innovative output. We are quite convinced that people underestimate or do not get sufficiently compensated for their contribution to the emergence of innovations. However, the assumption of people not taking into account their impact on the innovative output at all is not very satisfactory. Another promising extension to our analysis is the introduction of a dynamic framework that could provide valuable insights on the development of productivity in cities and wage premia on the individual level.

In spite of these weak points, we believe that our model's insights on the micro-foundations of different types of knowledge spillovers provide a valuable contribution to the understanding of agglomeration forces.



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