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## Optimal Number of Firms in the Wireless Markets<sup>\*</sup>

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#### Abstract

In this paper, we design a theoretical model to analyze the impact of the number of firms on investment in the wireless communications industry. Our model extends the Salop's framework by introducing investment in quality that either reduces the marginal cost of production or shifts the consumers' valuation upward. We find that an increase in the number of firms reduces their incentives to invest in quality. The impact on the aggregate industry investment can be non-monotone. These theoretical findings are supported by empirical evidence from the mobile telecommunications industry. More specifically, we find that mobile operators' investment in network infrastructure is not affected when going from two to three firms; but decreases above three firms. In addition, there is an inverted-U relationship between the industry investment and the number of mobile operators; the maximum being reached at three or four mobile operators.

Keywords: Market structure, Investment, Mobile Telecommunications. JEL Classification: D21, D22, L13, L40.

<sup>\*</sup>The usual disclaimer applies.

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## 1 Introduction

The impact of the market structure on investment is an important question in the telecommunication industry, owing to the role of the diffusion of new technologies on the long run growth rate (See Comin and Mestieri (2013)). However, the economic literature does not help answering this question as its predictions crucially depend on several parameters (Schmutzler, 2013). The impact of the market structure on investment thus depends on the characteristics of each industry.

In this paper, we propose a theoretical model tailored to the wireless market in order to analyze how the number of wireless operators affects investment in this market. Our model extends the Salop's framework by introducing investment that either reduces the marginal cost of production or shifts the consumers' valuation upward. This model predicts that, everything else equal, an operator's investment falls as the number of wireless operators increases. In addition there may be an inverted-U relationship between the aggregate industry investment and the number of wireless operators.

We empirically test these predictions using firm level panel data from 38 national markets worldwide. The dataset provides information about firms' investment in network infrastructure in each of these markets, as well as the number of operators and a number of characteristics of the markets. The identification strategy relies on the exogeneity of the number of operators and their date of entry into the market. The date of entry into the market is used as an instrument for the relative size, in terms of subscriber market share, of an operator.

It turns out that operators' investment in network infrastructure is not affected when going from two to three firms; but decreases above three firms. This result accords well with the theoretical prediction, though the flatness of the investment between two and three operators may be explained by the coverage of the market. The magnitude of this negative effect of the number of firms is smaller for the larger operators. In addition, we find an inverted-U relationship between the industry investment and the number of mobile operators; the maximum being reached at three or four mobile operators.

The results of this paper fall into the broad literature on the impact of competition on innovation pioneered by Schumpeter (1942) and Arrow (1962). Dasgupta and Stiglitz (1980) finds a negative correlation between market concentration and innovation but did not deal with any causal effect. More recent papers such as Boone (2008), Vives (2008) and Belleflamme and Vergari (2011) deal with the causal impact of competition on investment but only from a theoretical perspective. Given the lack of robustness of their theoretical predictions, Schmutzler (2013) proposes a general and unified theoretical framework showing that the impact of competition on investment depends on the characteristic of each industry. Some papers such as Aghion, Bloom, Blundell, Griffith, and Howitt (2005) provide some empirical evidence but at the macroeconomic level or using experimental evidence at the microeconomic level (Sacco and Schmutzler, 2011). In addition, these empirical papers do not focus more specifically on the impact of the number of firms. This paper is among the firsts to provide a theoretical framework backed by empirical and causal evidence on this issue at the microeconomic level, thanks to the availability of an original dataset and the regulatory framework of the wireless industry which provides exogenous variables as instruments.

The rest of the paper is organized as follows. Section 2 presents the theoretical model, and section 3 presents a numerical application of theoretical results. Section 4 presents the dataset, the empirical model, identification strategies and the results. Section 5 concludes.

### 2 The Theoretical Model

The model of Salop (1979) provides a useful framework to estimate the number of firms in a market. This model assumes that all the firms are regularly spread on a circle where consumers are uniformly distributed. It is assumed that the space of product is totally homogeneous, thus the location of firms does not matter.

The perimeter of the circle and the density of consumers are equal to unity. Consumers are moving along the circle and the transportation cost is equal to t. They want to purchase one unit of the good of one of the firms. The surplus generated by purchasing to firm i is  $s_i$ .

We consider the following two steps game. First firms decide whether they enter the market with a level of quality represented by s and cost of production c. For firm i, we denote  $d_i = s_i - c_i$ , with  $c_i \leq s_i$ . d represents somehow the efficacy of the firm, both from the quality side, s and from the cost side, c. In order to choose the parameter  $d_i$ , firm i invests an amount  $F_i(d_i)$  increasing with  $d_i$ . This suggests that investment in the one hand increases quality and in the other hand decreases costs. Second, firms compete in price and firm i set price  $p_i$ . The game is solved by backward induction. We assume that N firms enter the market, therefore the distance between two firms is 1/N.



Figure 1: Circular Market

In a first time, we assume that levels of quality s are high enough such that the market is fully covered. In that case, the utility for a customer located at a distance x from firm i to purchase firm i's offer is:  $U_i = s_i - tx - p_i$ . The utility for this consumer to purchase firm i + 1's offer is:  $U_{i+1} = s_{i+1} - t(\frac{1}{N} - x) - p_{i+1}$ . This consumer is indifferent between firm i and i + 1 if  $U_i = U_{i+1}$ . This occurs for  $x = \frac{s_i - s_{i+1} + p_{i+1} - p_i + \frac{t}{N}}{2t}$ . Same manner, the indifferent consumer between firm i and firm i - 1 is located at  $-y = \frac{s_i - s_{i-1} + p_{i-1} - p_i + \frac{t}{N}}{2t}$ . (See Figure.1)

The market share of firm *i* is  $\sigma_i = x - y = \frac{1}{N} + \frac{1}{t} \left( s_i - \frac{s_{i-1} + s_{i+1}}{2} + \frac{p_{i-1} + p_{i+1}}{2} - p_i \right)$ . Profit of firm *i* writes  $\pi_i = (p_i - c_i) \sigma_i - F_i(d_i)$ .

The first order condition leads to Nash equilibrium prices:  $p_i^* = c_i + \sigma_i t$ .

Hence firm *i*'s equilibrium profit is  $\pi_i^* = \sigma_i^2 t - F_i(d_i)$ .

The sum of the profits in the whole industry is:  $\sum_i \pi_i^* = HHI * t - \sum_i F_i(d_i)$  where HHI is the Herfindahl-Hirschmann index. This relation allows to calculate the value of t in the market when profits, market shares and investments are known in the market. t represents the parameter of differentiation between firms on the market. If t = 0 firms are perfect substitutes and  $p_i^* = c_i$ .

The number of firms that decide to enter the market derives from the zero profit condition. This condition applies for all firms, otherwise, the weakest one would not enter. As the market is not necessarily symmetric, the zero profit condition for the weakest firm which is somehow the weak link determines the number of firms in the market.

The asymmetry among firms is caused by the differences in efficiency and by the differences in the preferences of consumers. In the model, the surplus for consumer, s, may be different among firms. Even for the same investment, the surplus for consumers minus cost of production, d, has no reason to be equal.  $F_i(d_i) = F_j(d_j)$  does not necessarily implies that  $d_i = d_j$ .

We denote  $\lambda_i$ , the difference between *i*'s market share and the average market share:  $\lambda_i = \sigma_i - \frac{1}{N}$  assuming that firm *k* is the weakest firm, profit of firm *k* can be rewritten:  $\pi_k = \left(\frac{1}{N} + \lambda_k\right)^2 t - F_k(d_k)$ . The equation applied to the weakest firm provides the zero profit condition that allows to determine the number of firms that can operate in the market. Notice that in this case, for the weakest firm,  $\lambda_k \leq 0$ .  $\lambda_k = 0$  means that the market is symmetrical.

The zero profit condition for firm k writes:  $\left(\frac{1}{N} + \lambda_k\right)^2 t - F_k(d_k) \ge 0$  which leads to the condition:  $N \le \frac{1}{\sqrt{\frac{F_k}{t} - \lambda_k}}$ 

When all firms are identical, market is symmetrical and  $N \leq \sqrt{\frac{t}{F}}$ , we find again the result of Salop highlighted by (Tirole 1988).

#### 2.1 Market shares

The market shares of firm *i* writes:  $\sigma_i = \frac{1}{2N} + \frac{1}{2t} \left( d_i - \frac{d_{i+1} + d_{i-1}}{2} \right) + \frac{\sigma_{i-1} + \sigma_{i+1}}{4}$ 

Firm *i*'s market share,  $\sigma_i$  can be expressed as a linear combination of the quality of each firms  $d_i, d_{i+1}, ..., d_N$ . The impact of each competitor on firm *i*'s market share depends on its distance from firm *i*. As a result, the coefficient for firm i + j and firm i - j are the same and we can write:

$$\sigma_i = \frac{1}{N} + \frac{\beta_0 d_i}{t} + \sum_{j=1}^{(N-1)/2} \frac{\beta_j (d_{i+j} + d_{i-j})}{t}$$
(1)

if N is odd

And

$$\sigma_i = \frac{1}{N} + \frac{\beta_0 d_i}{t} + \sum_{j=1}^{N/2} \frac{\beta_j (d_{i+j} + d_{i-j})}{t}$$
(2)

if N is even, because in that case firm i + N/2 is also the firm i - N/2. j represents the distance from firm i.

Similarly, when N is odd firm i + (N+1)/2 is also the firm i - (N-1)/2 and firm i - (N+1)/2 is also the firm i + (N-1)/2.

The coefficients  $\beta_i$  can be written (see proof in the annexes):

For N = 2:  $\beta_0 = \frac{1}{3}$ ;  $\beta_1 = -\frac{1}{6}$ For N = 3:  $\beta_0 = \frac{2}{5}$  and  $\beta_1 = -\frac{1}{5}$ For N = 4:  $\beta_0 = \frac{5}{12}$ ;  $\beta_1 = -\frac{1}{6}$  and  $\beta_2 = -\frac{1}{24}$  $\beta_{(N-1)/2} = \frac{-1}{(5\lambda(N)+14\mu(N))}$ ;  $\beta_0 = \frac{2\lambda(N)+6\mu(N)}{(5\lambda(N)+14\mu(N))}$ ;  $\beta_1 = -\frac{\lambda(N)+2\mu(N)}{(5\lambda(N)+14\mu(N))}$  and  $\beta_{2+j} = \frac{-\lambda(N-2j)}{(5\lambda(N)+14\mu(N))}$  if N is odd and N > 3 and

 $\beta_{N/2} = \frac{-1}{(5\gamma(N) + 14\delta(N))}; \beta_0 = \frac{2\gamma(N) + 6\delta(N)}{(5\gamma(N) + 14\delta(N))}; \beta_1 = -\frac{\gamma(N) + 2\delta(N)}{(5\gamma(N) + 14\delta(N))} \text{ and } \beta_{2+j} = \frac{-\gamma(N-2j)}{(5\gamma(N) + 14\delta(N))}$  if N is even and N > 4.

With  $\lambda(N+2) = 4\lambda(N) - \lambda(N-2)$  and  $\lambda(N) = 0$  if N is even.  $\lambda(5) = 1; \lambda(7) = 3;$  $\mu(N) = \sum_{k=5}^{N} \lambda(k)$  and  $\gamma(8) = 4\gamma(6) - 2\gamma(4)$  and  $\gamma(N+2) = 4\gamma(N) - \gamma(N-2)$  if N > 6 and  $\gamma(N) = 0$  if N is odd.  $\gamma(4) = 1; \gamma(6) = 4; \delta(N) = \sum_{k=4}^{N} \gamma(k)$ 

For simplification, we denote, if N is odd and N > 3:  $\alpha_0(N) = 2\lambda(N) + 6\mu(N)$  and  $\alpha_1(N) = \lambda(N) + 2\mu(N)$ , then  $2\alpha_0(N) + \alpha_1(N) = 5\lambda(N) + 14\mu(N)$ . The coefficients can be written:

$$\beta_0 = \frac{\alpha_0}{(2\alpha_0 + \alpha_1)} ; \ \beta_1 = -\frac{\alpha_1}{(2\alpha_0 + \alpha_1)}; \ \beta_{2+j} = \frac{-\lambda(N-2j)}{(2\alpha_0 + \alpha_1)} \text{ and } \beta_{(N-1)/2} = \frac{-1}{(2\alpha_0 + \alpha_1)}.$$
  
For  $N = 2 : \alpha_0 = 1 ; \ \alpha_1 = 1$ 

For N = 4:  $\alpha_0 = 5$ ;  $\alpha_1 = 2$  and  $\alpha_2 = 1$ 

As a result: 
$$\sigma_i = \frac{1}{N} + \frac{\alpha_0 d_i - \sum_{j=1}^{N-1} \alpha_j (d_{i+j} + d_{i-j})}{(2\alpha_0 + \alpha_1)t}$$

And similarly if N is even and N > 4:  $\alpha_0 = \frac{1}{2}(2\gamma(N) + 6\delta(N))$  and  $\alpha_1 = \frac{1}{2}(\gamma(N) + 2\delta(N))$ , then  $2\alpha_0 + \alpha_1 = \frac{1}{2}(5\gamma(N) + 14\delta(N))$ . The coefficients can be written:

$$\beta_0 = \frac{\alpha_0}{(2\alpha_0 + \alpha_1)}$$
;  $\beta_1 = -\frac{\alpha_1}{(2\alpha_0 + \alpha_1)}$ ;  $\beta_{2+j} = \frac{-\gamma(N-2j)}{(2\alpha_0 + \alpha_1)}$  and  $\beta_{N/2} = \frac{-2}{(2\alpha_0 + \alpha_1)}$ .

Notice that  $d_{i+N/2} = d_{i-N/2}$  thus  $d_{i+N/2} + d_{i-N/2} = 2d_{i+N/2}$ . This explains the coefficient 2 in  $\beta_{N/2}$ .

For N = 3:  $\alpha_0 = 2$  and  $\beta_1 = 1$ 

As a result: 
$$\sigma_i = \frac{1}{N} + \frac{\alpha_0 d_i - \sum_{j=1}^{N/2} \alpha_j (d_{i+j} + d_{i-j})}{(2\alpha_0 + \alpha_1)t}$$

Market share can be written:

$$N = 2; \sigma_i = \frac{1}{2} + \frac{(d_i - d_{i+1})}{3t}$$

$$N = 3; \sigma_i = \frac{1}{3} + \frac{(2d_i - d_{i+1} - d_{i-1})}{5t}$$

$$N = 4; \sigma_i = \frac{1}{4} + \frac{(5d_i - 2d_{i+1} - 2d_{i-1} - d_{i+2})}{12t}$$

$$N = 5; \sigma_i = \frac{1}{5} + \frac{(8d_i - 3d_{1+1} - 3d_{i-1} - d_{i+2} - d_{i-2})}{19t}$$

$$N = 6; \sigma_i = \frac{1}{6} + \frac{(19d_i - 7d_{i+1} - 7d_{i-1} - 2d_{i+2} - 2d_{i-2} - d_{i+3})}{45t}$$

The impact of the other firms on firm *i*'s market share depends on their distance to firm *i*. The coefficient affected to each firm increases with its proximity with firm *i*. The coefficients affected to  $d_{i+2}$  or  $d_{i-2}$  are weaker than the coefficient affected to  $d_{i+1}$  or  $d_{i-1}$ .

The table below shows the evolution of coefficients up to N = 10.

N	2	3	4	5	6	7	8	9	10
α.0	1	2	5	8	19	30	71	112	265
α1	1	1	2	3	7	11	26	41	97
α.2			1	1	2	3	7	11	26
α.3					1	1	2	3	7
α.4							1	1	2
α.5									1

Table 1: Coefficients of the market shares

#### 2.2 Incentives to invest

What is the level of quality  $d_i$  and the fixed costs  $F_i$  a firm has to spend to maximize its equilibrium profit:  $\pi_i = \sigma_i^2 t - F_i(d_i)$ 

The first order condition leads to:

 $\frac{\partial F_i}{\partial d_i} = 2\sigma_i \frac{\partial \sigma_i}{\partial d_i} t$ 

The calculation of market share  $\sigma_i$  allows to calculate  $\frac{\partial \sigma_i}{\partial d_i}$ . However the result depends on the number of firms.

$$\begin{aligned} \frac{\partial \sigma_i}{\partial d_i} &= \beta_0 / t = \frac{\alpha_0(N)}{(2\alpha_0(N) + \alpha_1(N))t}; \frac{\partial F_i}{\partial d_i} = 2\beta_0 \sigma_i = \frac{2\alpha_0(N)\sigma_i}{2\alpha_0(N) + \alpha_1(N)} \\ N &= 2; \ \frac{\partial \sigma_i}{\partial d_i} &= \frac{1}{3t}; \frac{\partial F_i}{\partial d_i} = \frac{2\sigma_i}{3} \\ N &= 3; \ \frac{\partial \sigma_i}{\partial d_i} &= \frac{2}{5t}; \ \frac{i}{\partial d_i} = \frac{4\sigma_i}{5} \\ N &= 4; \ \frac{\partial \sigma_i}{\partial d_i} &= \frac{5}{12t}; \ \frac{\partial F_i}{\partial d_i} = \frac{10\sigma_i}{12} \\ N &= 5; \ \frac{\partial \sigma_i}{\partial d_i} &= \frac{8}{19t}; \ \frac{\partial F_i}{\partial d_i} = \frac{16\sigma_i}{19} \\ N &= 6; \ \frac{\partial \sigma_i}{\partial d_i} &= \frac{19}{45t}; \ \frac{\partial F_i}{\partial d_i} = \frac{38\sigma_i}{45} \end{aligned}$$

 $\frac{\partial \sigma_i}{\partial d_i}$  is increasing in N.  $\frac{\partial \sigma_i}{\partial d_i}(N+1) - \frac{\partial \sigma_i}{\partial d_i}(N) = \beta_0(N+1) - \beta_0(N).$ 

 $\forall N \in \mathbb{N}$ , even or odd,  $\beta_0(N+1) - \beta_0(N) > 0$ , therefore  $\frac{\partial \sigma_i}{\partial d_i}$  is increasing in N (see proof in annexes).

The impact of quality  $d_i$  on market share  $\sigma_i$  increases with the number of firms but the growth decreases sharply and converge toward an asymptote  $\overline{\beta_0} = \frac{14+8\sqrt{3}}{33+19\sqrt{3}}$  (See proof in annexes). The asymptote is represented in the following graph (figure 2) in red line.



Figure 2: Impact of quality on market share

The impact of an increase in quality  $d_i$  is higher when the number of firm is high because the distance between firms is lower and therefore the influence of the quality is stronger. However, the impact of the marginal increase in the number of firms is decreasing because the gap between 2 and 3 firms is higher than the gap between 5 and 6 firms. Although the impact of quality on market share increases, the number of firm reduces the average market share, as a result, the incentives to invest tend to decrease with the number of firms  $\frac{\partial F_i}{\partial d_i} = 2\beta_0\sigma_i$ .

If the market is symmetrical,  $\frac{\partial F_i}{\partial d_i} = \frac{2\beta_0}{N}$ , the incentive to invest, i.e the impact of the quality on the incentives to invest decreases with the number of firms N.

The following graph (Figure 3) illustrates the decrease in incentives to invest  $\frac{\partial F_i}{\partial d_i}$  according to the number of firms.



Figure 3: Incentives to invest at firm level

It is noticeable that the difference of impact between 2 and 3 firms is rather large, while the difference of impact between 5 and 6 firms is very weak. It is assumed that  $F_i(d_i)$  is increasing and convex. Increasing because a higher investment entails a better quality, and convex because the marginal increase in quality becomes more and more costly. Because of the convexity of  $F_i$ , an increase in  $\frac{\partial F_i}{\partial d_i}$  corresponds to an increase in  $F_i$ . The investment does not depend only on the impact of the quality on market share,  $\frac{\partial \sigma_i}{\partial d_i}$ , but also on the market share itself. The higher the market share, the higher the investment. This is the reason why the asymmetry between firms impacts the investment on the market.

Since  $F_i(d_i)$  is concave and  $\frac{\partial F_i}{\partial d_i}$  is decreasing in N, investment  $F_i(d_i)$  and  $d_i$  also tend to decrease with N.

#### 2.3 Consumer surplus and Social welfare

Consumers' surplus is the difference between the surplus of purchasing and the price. For a consumer purchasing *i*'s offer, the surplus is  $s_i - p_i = d_i - \sigma_i t$ . Consumers that purchase firm *i*'s offer are located between -y and x. At equilibrium, Consumers' surplus of consumers purchasing *i*'s offer writes  $CS_i = \int_{-y}^{x} (d_i - \sigma_i t - tz) dz = \sigma_i d_i - \sigma_i^2 t - \frac{(x^2+y^2)}{2}$ . At the industry level  $CS =_{i=1}^{N} CS_i$ .

 $CS = E(d) + \left(\frac{(N-1)}{\beta_0} - \frac{3N}{2}\right)t \ var(\sigma) - \frac{5t}{4N} + Nt \ cov(x, y)$  where E(d) is the mean of quality level  $d_i$ .  $E(d) = \frac{\sum_{i=1}^{N} d_i}{N}$ ,  $var(\sigma)$  is the variance of the market share and cov(x, y) the covariance between x and y. (See proof in annexes)

Same manner, Welfare generated by consumers purchasing *i*'s offer is  $W_i = CS_i + \pi_i$ and at the industry level:  $W =_{i=1}^{N} W_i = E(d) + \left(\frac{(N-1)}{\beta_0} - \frac{N}{2}\right)t \ var(\sigma) - \frac{t}{4N} + Nt \ cov(x, y) - \sum_{i=1}^{N} F_i$  (see proof in annexes)

E(d) tends to decrease with the number of firms as the investment decreases. The term  $\left(\frac{(N-1)}{\beta_0} - \frac{3N}{2}\right) \ge 0$  whatever the number of firms, N. Therefore  $\left(\frac{(N-1)}{\beta_0} - \frac{N}{2}\right) > \left(\frac{(N-1)}{\beta_0} - \frac{5N}{4}\right)$  is also positive.  $\frac{5t}{4N}$  as well as  $\frac{t}{4N}$  decrease with N, thus  $-\frac{5t}{4N}$ , as well as  $-\frac{t}{4N}$  are increasing.  $F_i$  tends to decrease with N, but it is not possible to conclude for  $\sum_{i=1}^{N} F_i$ . As a result, the evolution of Consumer surplus and Welfare according to the number of firms is ambiguous. The heterogeneity of market shares increases both consumer surplus and welfare.

#### $\blacksquare$ *Thesymmetriccase*

If the market is symmetrical,  $var(\sigma) = 0$ ;  $\forall i \in \mathbb{N}, d_i = d$  and  $F_i = F$ . Therefore  $CS = d - \frac{5t}{4N}$  and  $W = d - \frac{t}{4N} - NF$ 

The terms  $-\frac{5t}{4N}$  and  $-\frac{t}{4N}$  represent the losses due to the transportation cost. These losses tend to decrease with the number of firms as the distance form consumers to their preferred choice tend to decrease. The quality d tend to decrease with the number of firms as the term  $\frac{\partial F}{\partial d}$  decreases and the convexity of F(d) entails a decrease both in F and in d. The evolution of the investment of the industry NFis ambiguous and depends of F(d).

However, it should be noted that the impact of investments on quality is decisive. If this impact is high, the decrease in investment with the number of firm has also a high negative impact on the quality d. If this impact is sufficiently high, Consumer surplus and welfare are decreasing with the number of firms.

In information technologies and in particular in wireless technologies, this impact may be significant as the technological progress is high.

The evolution of consumer surplus according to the number of firms is written:

 $CS(N+1)-CS(N) = d(N+1)-d(N) + \frac{5t}{4N(N+1)}$ . We know that d is decreasing in N and  $\frac{5t}{4N(N+1)}$  is positive. Consumer surplus is increasing if  $\frac{5t}{4N(N+1)} > d(N) - d(N+1)$ , this means that as long as the growth of surplus caused by the lower of the average distance between consumer and the firm they purchase compensates for the decline in d, consumer surplus increases, otherwise it decreases.

Same manner, the evolution of Welfare is written:  $W(N+1) - W(N) = d(N+1) - d(N) + \frac{t}{4N(N+1)} + NF(N) - (N+1)F(N+1)$ 

The sign of W(N+1)-W(N), in addition, depends on the variation of the investment at the industry level. Those parameters depends on the relation between investment and quality F(d).

## **3** Numerical Application

The impact of the number of firms on Investment, consumer surplus and welfare depend on the function F(d). In this section, we will make a hypothesis on this function and compare the theoretical results with empirical observations.

We assume that a unit investment allows to build a capacity  $\rho$  in the network. An amount of investment F allows to build a capacity  $C = \rho F$ . According to the Weber-Fechner law, a capacity C represents a gross utility proportional to the logarithm of C. Without any capacity, gross utility equals 0 thus gross utility is written  $s = \omega \ln(C + 1)$ .  $\omega$  represents the consumer's sensitivity to the network capacity and  $\rho$  represents the impact of investment on capacity. This leads to the following relationship between F and d:

$$F(d) = \frac{e^{\frac{d+c}{\omega}} - 1}{\rho}$$

#### 3.1 Investment at the firm and the industry level

This relation allows to calculate the incentive to invest  $\frac{\partial F_i}{\partial d_i} = \frac{e^{\frac{d_i+c_i}{\omega}}}{\omega\rho} = 2\beta_0\sigma_i$ . This leads to  $d_i = \omega \ln(2\omega\rho\beta_0\sigma_i) - c_i$ ;

 $F_i(d_i) = 2\omega\beta_0\sigma_i - 1/\rho$  and  $\sum_{i=1}^N F_i = 2\omega\beta_0 - N/\rho$ . It is noticeable that, in this case, whatever the differences in market share, the investment at the industry level

is unchanged.

F must be positive, this induces a lower bound to the market share.  $\sigma_i > 1/2\omega\rho\beta_0$ and an upper bound to the number of firms  $\sum_{i=1}^N \sigma_i = 1 > N/2\omega\rho\beta_0$ , thus  $N < 2\omega\rho\beta_0$ .

 $F_i(d_i)$  tends to decrease with N because  $\beta_0$  increases slowly and  $\sigma_i$  tends to decrease in average like 1/N and  $\beta_0/N$  is decreasing for  $N \ge 2$ .

At the industry level,  $\sum_{i=1}^{N} F_i$  is increasing if  $\beta_0(N+1) - \beta_0(N) > \frac{1}{2\omega\rho}$ . The left term is decreasing and tends toward 0 while the right term is constant (see annexes). This means that there is a number of firm that maximizes the investment of the industry. This number depends on the parameter  $\omega$  and  $\rho$  which may differ from one market to another. An empirical estimation is required to estimate the number of firms that maximizes investment.

The graph below represents the number of firms maximizing investment according to  $1/2\omega\rho$ .



Figure 4: Number of firms maximizing investment

In this graph, the higher  $1/2\omega\rho$ , the lower the number of firms maximizing investment.

#### 3.2 Consumer Surplus and Welfare in symmetrical market

In symmetrical market, consumer surplus is written:  $CS(N) = \omega \left[ \ln(2\omega\rho\beta_0(N)) - \ln(N) \right] - c - \frac{5t}{4N}$ 

Variation of consumer surplus from N firms to N + 1 firms is written:

$$CS(N+1) - CS(N) = \omega \ln \left[ \ln \left( \frac{\beta_0(N+1)}{\beta_0(N)} \right) - \ln \left( \frac{N+1}{N} \right) \right] + \frac{5t}{4N(N+1)}$$
$$CS(N+1) - CS(N) \text{ is positive if } \frac{t}{\omega} > \frac{4N(N+1)}{5} \ln \left( \frac{(N+1)\beta_0(N)}{N\beta_0(N+1)} \right)$$

The right term is increasing in N (see annexes), therefore, there is a finite number of firms  $N_{CS} \geq 2$  that maximizes consumer surplus. t represents consumer sensitivity to horizontal differentiation and  $\omega$  the impact of consumer sensitivity to quality or somehow, to vertical differentiation. the ratio  $t/\omega$  represents the ratio between horizontal and vertical differentiation. A high ratio indicates that consumer are relatively more sensitive to horizontal than vertical differentiation. The higher is the ratio, the higher the number of firm required to maximize consumer surplus.

Similarly, welfare is written:  $W(N) = \omega \left[ \ln(2\omega\rho\beta_0(N)) - \ln(N) \right] - c - \frac{t}{4N} - NF(N)$ Variation of welfare from N firms to N + 1 firms is written:  $W(N + 1) - W(N) = \omega \ln \left[ \ln \left( \frac{\beta_0(N+1)}{\beta_0(N)} \right) - \ln \left( \frac{N+1}{N} \right) \right] + \frac{t}{4N(N+1)} + NF(N) - (N+1)F(N+1)$ W(N+1) - W(N) is positive if  $\frac{t}{\omega} > 4N(N+1) \left[ \ln \left( \frac{(N+1)\beta_0(N)}{N\beta_0(N+1)} \right) + 2(\beta_0(N+1) - \beta_0(N)) - \frac{1}{\omega\rho} \right]$ 

The upper bound for N implies that  $1/\omega\rho < 2\beta_0(N)/N$  this means that the right term is increasing in N (see annexes). As a result, there is a finite number of firms  $N_W \ge 2$  that maximizes the welfare. The ratio  $t/\omega$ , in the same way as consumer surplus determines the optimal number of firms  $N_W$ . Furthermore, in the case of welfare, the parameter  $\rho$ , which represents the impact of investment on capacity, acts in the same direction as  $\omega$ , it reduces  $N_W$ .

In the case of wireless industry, the impact of investment on quality perceived by consumers seems quite crucial.

## 4 Empirical Evidence

#### 4.1 Dataset

The empirical estimation relies on a new dataset built by aggregating information from four databases. The first is the Wireless Intelligence database managed by the GSMA. It provides information about the list of firms within each market along with their dates of entry. The second database provides information about their capital expenditures, market shares as well as the aggregate capital expenditures at the market level. This database is managed by Yankee Group. For some markets, the Yankee Group database provides information about the aggregate investment without the investment data of all the firms in the markets, particularly the smallest ones.<sup>1</sup>

As we do not directly observe the capital expenditures without the license fees, we retrieve information about the date of purchase of a license from the World Cellular Information Services database managed by Ovum. This information will be useful to control for the license fees included in the capital expenditures. We also extract the penetration rate of mobile telephony from this database as a proxy for the market coverage. Finally, we also extract the size of working age population from the World Development Indicator database of the World Bank, as a proxy for the market size.

The final dataset is a panel of 91 mobile network operators from 38 countries worldwide, observed over 10 years, from 2004 to 2013. The sample does not include China due to an extreme value issue. It also does not include Canada, the United States of America and India, as these markets are made of several regional markets which makes it difficult to assess the impact of the number of firms on the aggregate investment. The firm level sample is made of 818 observations, whereas the market level sample is made of 378 observations.

#### 4.2 The Empirical Model

To test the prediction of the theoretical model, we estimate a set of reduced form econometric models. These models rely on firm level data for both the symmetric and the asymmetric cases, and market level data for the impact of the number of firms on their aggregate investment. They will be derived from the generic empirical model presented below.

Following the theoretical model, the investment of a firm depends on the number of firms in the market and its relative efficiency with respect to its rivals. Thus, the econometric model writes<sup>2</sup>:

$$y_{ijt} = \alpha + f(N_{jt}) + \delta R E_{ijt} + \gamma X_{jt} + \mu_j + \nu_t + \epsilon_{ijt}$$
(3)

<sup>&</sup>lt;sup>1</sup>This may be explained by the fact that aggregate data are readily provided by the national regulators, while the firm level data requires some costly researches to be gathered. Meanwhile, investors are more interested in the performance of the largest firms. Thus, the benefit of gathering the data is not worth the cost of research for the smallest firms.

<sup>&</sup>lt;sup>2</sup>Note that econometric model does not include any firm fixed effect as we will need to rely on an instrument perfectly collinear with the firms' fixed effects, namely their dates of entry into the market

Where  $y_{ijt}$  is the investment of firm *i* in market *j* at time *t*, measured by the capital expenditures.  $N_{jt}$  is the number of firms, which may change over time within some markets. *f* is a function of the number of firms, flexibly specified in order to capture a non-monotone relationship. Basically, *f* is a sum of dummies indexed by the number of firms. It takes a market with a specified number of firms as reference and compares the investment under a given number of firms with respect to this reference market. It formally writes:

$$f(N_{jt}) = \sum_{k \neq 3} \beta_k \mathbb{1}(k = N_{jt})$$

k is an index characterizing the number of firms within a market. By setting  $k \neq 3$ , we are taking markets with three firms as the reference. The estimation is not sensitive to this choice. Thus  $\beta_k$  represents the difference in investment of firm in a market with k firms with respect to a similar firm in a market with three firms. This is the parameter that characterizes the impact of the number of firms on investment. Typically, if  $\beta_2$  were positive and significant; while  $\beta_k$  (k > 3) were negative and significant, thus we would conclude that the relationship between the number of firms and investment is downward sloping. On the contrary, if  $\beta_2$  were negative and significant, thus we would conclude that there is an inverted-U relationship between investment and the number of firms, with a maximum in markets with three firms.

 $RE_{ijt}$  represents the relative efficiency of a firm *i* with respect to the other firms within the market *j* at time *t*. Theoretically, the relative efficiency refers to the gap in terms of marginal cost between a firm and its rivals. As this gap should translate into difference in the market shares of the firms, we define the relative efficiency variable as the difference between a firm's market share and that of its rivals. More specifically, the relative efficiency writes:

$$RE_{ijt} = \sigma_{ijt} - (1 - \sigma_{ijt}) = 2\sigma_{ijt} - 1$$

 $\sigma_{ijt}$  is the subscribers market share of firm *i*.

By construction, the relative efficiency variable is strictly superior to -1 and inferior to 1. It depends on the number of firms in a market, reflecting the fact that a change in the number of firms also affects the relative efficiency of all the firms. Its coefficient  $\delta$  captures the effect of the relative efficiency on investment. We expect this coefficient to be positive, as more efficient firms will investment more than the less efficient ones.

 $X_{jt}$  is a set of control variables at the market level, including the market size, measured by the working age population between 15 and 64 years old, and the market coverage measured by the one year lagged penetration of mobile telephony. The lagged of the penetration rate is chosen to ensure its exogeneity with respect to the investment.

 $\mu_j$  and  $\nu_t$  are respectively market and year fixed effects deemed to capture unobservable market or year specific determinants of investment.  $\epsilon_{ijt}$  represents the unobserved idiosyncratic shocks affecting firms' investment. In the following section, we present the properties of these idiosyncratic shocks and discuss how they affect our identification strategy of assessing the impact of the number of firms on investment.

#### 4.3 The Identification Strategy

The identification of the impact of the number of firms on investment relies on two sources of variation. The first is the change in the number of firms within a market from one year to another, due to an entry or a merger. Of the 38 national markets in our sample, 29 have experienced at least an entry or a merger between 2004 and 2013 (see tables 2 and 3). The timing of these changes in the market structure differs across markets. Actually, national regulatory authorities can be thought of doing an experiment about the number of mobile network operators that would raises investment and lowers prices. This experiment is probably not random, as its may depend on the investment of firms. However, given two markets with similar characteristics in terms of size, and coverage, we may consider that the difference in the timing of the change in the market structure, whether an entry or a merger, is random with respect to the firms' investment. Therefore, the cross-year variation in the number of firms within a market can be used as an exogenous variation to identify the impact of the number of firms on investment.

The second source of variation is cross-markets; that is the difference in terms of the number of firms within two markets with similar characteristics in terms of size and coverage. Indeed, under free entry, two markets with similar characteristics should have the same number of firms. However, given that entry into the mobile telecommunications markets is regulated, some pro-competitive regulators may accommodate more firms through the process of spectrum licensing; while those which are less pro-competitive would accommodate less firms either through the spectrum licensing process of through mergers. As a result, any difference in terms of the number of firms between two similar markets would reflect the preference of the regulator which is independent from the investment of the firms. Therefore, the cross-market variation in the number of firms can also be used a way to identify the impact of the number of firms on investment.

The first source of variation requires more statistical power as it focuses on the change in the number of firms within each market. More specifically, it requires, a change in the number of firms from every possible initial number of firms. For instance, if we restrict the estimation to those markets with two to six firms, as it is typically the case in the mobile telecommunications industry, this strategy requires to have sufficient markets with 2, 3, 4, 5 and 6 initial firms in order to be able to identify the impact of a change in the number of firms on their investment. Otherwise, we would not be able to identify significant effects when they actually exist. Meanwhile, this is source of variation that yield the least unbiased estimates of the impact of the number of firms on investment.

Under the assumption of symmetric markets, we will estimate the following equation which is derived from equation (3):

$$y_{ijt} = \alpha + f(N_{jt}) + \gamma X_{jt} + \mu_j + \nu_t + \epsilon_{ijt}$$

$$\tag{4}$$

By accounting for the market specific effects, the OLS estimation of this equation will rely on the first source of variation in order to identify the impact of the number of firms on investment. We could remove the market specific effects, assuming that they are not correlated with the number of firms, given the characteristics X. In that case, the identification will rely on the cross market variation.

In order to account for the asymmetry between firms, as predicted by the theory, we need to include the relative efficiency variable into this equation. However, this variable is typically endogenous as more efficient firms may be less affected by the idiosyncratic shocks and conversely. To overcome this endogeneity issue, we implement an instrumental variable strategy. The instrument for the relative efficiency of a firm is its year of entry into the market. We expect that mobile network operators that enter earlier into the market shall be on average more efficient that the late entrants. This efficiency gap between early and late entrants may be explained by economies of scale which are significant in the telecommunications industry. Meanwhile, the date of entry into the market is a regulatory decision, and thus exogenous with respect to firms' relative efficiency. As a consequence, the year of entry can be used a valid instrument for firms' relative efficiency. One limitation of this instrument is the fact that it only exploits firm specific variation in efficiency.

The instrumental variable estimation is implemented on the following equation:

$$y_{ijt} = \alpha + f(N_{jt}) + \delta R E_{ijt} + \gamma X_{jt} + \mu_j + \nu_t + \epsilon_{ijt}$$

Theoretically, the aggregate investment does no longer depends on the asymmetry between firms. Thus we estimate the following equation:

$$Y_{jt} = \alpha + g(N_{jt}) + \gamma X_{jt} + \mu_j + \nu_t + \varepsilon_{jt}$$
(5)

All estimations correct for arbitrary heteroskedasticity and autocorrelation using the Bartet Kernel of bandwidth 2. We implement the weak instruments test using the instrumental variables Stata routine proposed by Baum, Schaffer, and Stillman (2007). The Kelibergen-Paap Wald rank F-statistics are compared to the critical values tabulated by Stock and Yogo (2002) to test the weakness of the instruments.

#### 4.4 Results

Table 4 in appendix presents the summary statistics of the main variables used in the estimation. The following table 5 in appendix presents the econometric estimation results. The first four variables of this table are dummies that characterize the number of MNOs in a given market.

#### ■ The impact of the number of firms on investment in a symmetric framework

Specifications (1), (2) and (3) test the first theoretical prediction of the paper; that is a rise in the number of firms decreases their investment in a symmetric framework. They presents the OLS estimates of the equation 4, controlling for market size and coverage in specification (2) and for country fixed effects in specification (3). The results of specification (1) shows an inverted-U relationship between the number of firms and investment, when we do not control for the market size and coverage. More specifically, the negative and significant coefficient associated with markets with 2 firms (-0.56) implies that the average investment is lower in markets with two firms than in those markets with three firms. Likewise, the negative and significant coefficients associated with the other market dummies means that the average investment is also lower in these markets than in a market with three firms. Therefore, the maximal average investment is reached in those markets with three firms.

However, once we control for the market size and coverage in specification (2), the coefficient of the markets with two MNOs turns out to be positive and no longer significant. In addition, the magnitude of the fall in investment above three MNOs increases, as suggested by the rise in the absolute value of the point estimates associated with dummies for markets with more than 3 MNOs. These two changes correspond to a rise in the stepness of the negative relationship between the number of firms and investment. This rise can be explained by the differences in the size of the markets. Actually, The point estimate of this variable is positive and significant, reflecting the fact that firms in larger market invest more on average. But in addition, larger markets also host more firms. This is particularly the case between markets with two MNOs and markets with three MNOs. The changes in the point estimates of the 2 MNOs dummies from negative and significant to positive and non significant after controlling for the market size means that MNOs invest less in 2 MNOs markets because these markets have smaller size. As shown in tables 2 and 3, national markets such as Croatia, New Zealand, Norway and the United Arab Emirates are the ones which host two firms.

# ■ The impact of the number of firms on investment in an asymmetric framework

As these estimates rely on cross-markets variations, they do not account for a potential correlation between the unobserved market specific effects and the unobservables residuals in equation (3). For instance, entry or merger may happen to occur in those markets where investment is more sensitive to the number of firms, due to unobserved and time independant preferences for quality. In that case, we would overestimate the impact of the number of firms on investment. Specification (3) deals with this issue by controlling for market fixed effects. As expected, there is a fall in the absolute values of the impact of the number of firms on investment, as suggested by the fall in the absolute values of the point estimates of the market dummies between specifications (2) and (3). Still, the downward sloping relationship between the number of firms and the average investment remains. Therefore, the first prediction of the theoretical model regarding the impact of the number of firms on investment in a symmetric market is validated. As we control for market specific effects, the market size is no longer a significant predictor due to a loss of power. The standard error of its point estimates rises as there are less variation in the market size within a given country. On the other hand, the point estimate of the market coverage, measured by the lagged penetration rate of mobile telephony, becomes significant due to a rise in its magnitude. This point estimate was downward biased in the previous specification as countries with structurally low investment have low penetration and conversely.

Specifications (4) and (5) accounts for the asymmetry between the MNOs, as predicted by the theoretical model. This asymmetry is measured as the difference between the market share of a given MNO and that of its rivals. Specifications (4) and (5) can be viewed respectively as the first and the second stages of the two-stages least squares strategy used for the instrumental variable estimation. The instrument of the asymmetry index is the year of entry. Specification (4) shows that this instrument has a negative and significant impact on a firm's relative efficiency. In other words, firms that enter later in the market are on average lagging behind in terms of market shares.

Specification (5) exploits this variation in the relative efficiency of the firms to estimate the causal impact of the number of firms on their investment, controlling for their relative efficiency, the market size and coverage, as well as year, regions and country fixed effects. This specification represent the main relationship between investment and the number of firms derived from the theoretical model. Not surprisingly, it turns out that the more efficient firms invest more than the lagging ones. In addition the downward relationship between the investment and the number of firms remains. However, the impact of the number of firms is attenuated compared to the point estimates of specification (3) which assumes symmetry. Therefore, a rise in the number of firms decreases their indidvidual investment, but this effect is smaller for the more efficient firms. Conversely a merger would raise all firms investment and particularly for the most efficient ones. Besides, the market size and coverage turns out to be good determinants of firms investment. Each firm invest more in larger market and as the market is more and more covered.

#### ■ The impact of the number of firms on the aggregate investment

While these results hold at the firm level, a regulator may be more interested in raising the aggregate investment at the market level. Specification (6) presents the OLS estimates of the impact of the number of firms on the aggregate investment.

We find that the aggregate investment is lower in a market with two MNOs than in a market with three. This is also true for markets with five or six MNOs. However, there is no statistical difference between the aggregate investment in a market with three MNOs and the one with four, even though the aggregate investment in the latter is slightly smaller. Therefore, the relationship between the aggregate investment and the number of MNOs is an inverted-U with a maximum at three or four MNOs.

## 5 Conclusion

This paper proposes a theoretical model to analyze the impact of the number of firms on investment in the wireless industry. This model extend the classical Salop framework by introducing a first stage of investment in quality. It shows that a rise in the number of firms decreases the operators' investment; but the relationship between the aggregate investment and the number of operators can be inverted-U. Using firm level data and an instrumental variable estimation, it finds that these theoretical predictions are consistent with the data.

These results suggest a tradeoff between the firm and the industry investments. The industry investment obtained with four operators is not statistically different from the one obtained with three operators. The advantage of having four operators is a fall in the price of the offers. In the meantime, the firm's level investment is significantly lower with four operators than with three. It may therefore be the case that the unit price paid by consumer is larger with four firms than with three if the dynamic efficiency effect is significant.

A future work would assess the magnitude of the dynamic efficiency effect in the wireless industry. Besides, this paper presents a static model of the impact of the number of firms on investment. A way forward could be to assess the effect of the specific market dynamics such as entry, merger and exit on the investment and pricing of the mobile network operators.

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## A Appendix

#### A.1 Theory

1)  $\sigma_i$  can be expressed as a linear combination of the quality of each firms  $d_i, d_{i+1}, ..., d_N$ . The impact of each firm on the market share of firm *i* depends on its distance from firm *i*. As a result, the coefficient for firm i + j and firm i - j are the same and we can write:

 $\sigma_i = 1/N + \beta_0 d_i/t + \sum_{j=1}^{(N-1)/2} \beta_j (d_{i+j} + d_{i-j})/t \text{ if } N \text{ is odd and } \sigma_i = 1/N + \beta_0 d_i/t + \sum_{j=1}^{N/2} \beta_j (d_{i+j} + d_{i-j})/t \text{ if } N \text{ is even, because in that case firm } i + N/2 \text{ is also the firm } i - N/2.$  Similarly, when N is odd firm i + (N+1)/2 is also the firm i - (N-1)/2 and firm i - (N+1)/2 is also the firm i + (N-1)/2.

We know that  $\sigma_i = \frac{1}{2N} + \frac{1}{2t} \left( d_i - \frac{d_{i+1} + d_{i-1}}{2} \right) + \frac{\sigma_{i-1} + \sigma_{i+1}}{4}$ . When N is odd, this expression yields:  $1/N + \beta_0 d_i/t + \sum_{j=1}^{(N-1)/2} \beta_j (d_{i+j} + d_{i-j})/t = \frac{1}{2N} + \frac{1}{2t} \left( d_i - \frac{d_{i+1} + d_{i-1}}{2} \right) + \frac{1}{2N} + \frac{1}{4t} \left[ \beta_0 \left( d_{i+1} + d_{i-1} \right) + \sum_{j=1}^{(N-1)/2} \beta_j (d_{i+j+1} + d_{i-j+1} + d_{i+j-1} + d_{i-j-1}) \right]$ 

This provides a system of equations that can be solved:

If N = 3 then the expression is  $1/3 + \beta_0 d_i/t + \beta_1 (d_{i+1} + d_{i-1})/t = \frac{1}{6} + \frac{1}{2t} \left( d_i - \frac{d_{i+1} + d_{i-1}}{2} \right) + \frac{1}{6} + \frac{1}{4t} \left[ \beta_0 \left( d_{i+1} + d_{i-1} \right) + \beta_1 (2d_i + d_{i+2} + d_{i-2}) \right]$  and for  $N = 3 \ d_{i+2} = d_{i-1}$  and  $d_{i-2} = d_{i+1}$  therefore  $\beta_0 d_i/t + \beta_1 (d_{i+1} + d_{i-1})/t = \frac{1}{2t} \left( d_i - \frac{d_{i+1} + d_{i-1}}{2} \right) + \frac{1}{4t} \left[ (\beta_0 + \beta_1) \left( d_{i+1} + d_{i-1} \right) + 2\beta_1 d_i \right]$ As a result  $\beta_0 = \frac{1}{2} + \frac{\beta_1}{2}$  and  $\beta_1 = -\frac{1}{4} + \frac{\beta_0 + \beta_1}{4}$ . This yields  $\beta_0 = \frac{2}{5}$  and  $\beta_1 = -\frac{1}{5}$ 

If N > 3 then

 $\begin{array}{l} \beta_{0} \ = \ \frac{1}{2} + \frac{\beta_{1}}{2}; \beta_{1} \ = \ -\frac{1}{4} + \frac{\beta_{0} + \beta_{2}}{4}; \beta_{2} \ = \ \frac{\beta_{1} + \beta_{3}}{4}; ...; \beta_{(N-1)/2} \ = \ \frac{\beta_{(N-3)/2} + \beta_{(N+1)/2}}{4}. \ \text{As} \\ \beta_{(N+1)/2} \ = \ \beta_{(N-1)/2} \ \text{this last equation becomes } \beta_{(N-3)/2} \ = \ 3\beta_{(N-1)/2}. \ \text{We can write} \\ \beta_{(N-5)/2} \ = \ 4\beta_{(N-3)/2} \ - \ \beta_{(N-1)/2}; \beta_{(N-7)/2} \ = \ 4\beta_{(N-5)/2} \ - \ \beta_{(N-3)/2} \ \text{and more generative} \\ \text{ally } \ \forall k \ \in \ \mathbb{N}; \ k \ \leq \ N \ - \ 4; \ \beta_{(N-k)/2} \ = \ 4\beta_{(N-k+2)/2} \ - \ \beta_{(N-k+4)/2}. \ \text{If } \ N \ = \ 5, \text{then } \beta_{2} \ = \\ \beta_{(N-1)/2}; \ \text{if } \ N \ = \ 7 \ \text{then } \ \beta_{2} \ = \ \beta_{(N-3)/2} \ = \ 3\beta_{(N-1)/2}; \ \text{if } \ N \ = \ 9; \text{then } \ \beta_{2} \ = \ \beta_{(N-5)/2} \ = \\ 4\beta_{(N-3)/2} \ - \ \beta_{(N-1)/2} \ = \ 11\beta_{(N-1)/2}. \ \text{Let } \ \lambda(N) \ \in \ \mathbb{N}, \ \text{we can write} \ \beta_{2} \ = \ \lambda(N)\beta_{(N-1)/2} \ \\ \text{with } \ \lambda(N+2) \ = \ 4\lambda(N) \ - \ \lambda(N-2) \ \text{and } \ \lambda(N) \ = \ 0 \ \text{if } \ N \ \text{is even. } \ \lambda(5) \ = \ 1; \ \lambda(7) \ = \ 3. \ \\ \text{Generally we can write } \ \forall j \ \in \ \mathbb{N}, \ \beta_{2+j} \ = \ \lambda(N-2j)\beta_{(N-1)/2} \ \end{array}$ 

The sum of market shares  $\sum_{i=1}^{N} \sigma_i = 1$ ;  $\sum_{i=1}^{N} \sigma_i = 1 + \sum_{i=1}^{N} \left[ \beta_0 d_i + \sum_{j=1}^{(N-1)/2} \beta_j (d_{i+j} + d_{i-j}) \right] / t$ , therefore  $\beta_0 + 2 \sum_{j=1}^{(N-1)/2} \beta_j = 0$  or,  $\beta_0 + 2\beta_1 = 0$  for N = 3 and  $\beta_0 + 2\beta_1 + 2 \sum_{j=2}^{(N-1)/2} \beta_j = 0$  when N > 3. In this case, using the following change of variable  $\begin{aligned} k &= N - 2j + 4 \text{,the equation becomes } \beta_0 + 2\beta_1 + 2\sum_{k=5}^N \beta_{2+(N-k)/2} = 0 \text{ and finally} \\ \beta_0 + 2\beta_1 + 2\beta_{(N-1)/2} \sum_{k=5}^N \lambda(k) = 0. \text{ For simplification, we denote } \mu(N) = \sum_{k=5}^N \lambda(k). \end{aligned}$ The system of equations leads to:  $\beta_{(N-1)/2} = \frac{-1}{(5\lambda(N)+14\mu(N))}; \beta_0 = \frac{2\lambda(N)+6\mu(N)}{(5\lambda(N)+14\mu(N))}; \beta_1 = -\frac{\lambda(N)+2\mu(N)}{(5\lambda(N)+14\mu(N))} \text{ and } \beta_{2+j} = \frac{-\lambda(N-2j)}{(5\lambda(N)+14\mu(N))}. \end{aligned}$ 

Let us denote  $\alpha_0 = 2\lambda(N) + 6\mu(N)$  and  $\alpha_1 = \lambda(N) + 2\mu(N)$ , then  $2\alpha_0 + \alpha_1 = 5\lambda(N) + 14\mu(N)$ . The coefficients can be written:

$$\beta_0 = \frac{\alpha_0}{(2\alpha_0 + \alpha_1)}; \ \beta_1 = -\frac{\alpha_1}{(2\alpha_0 + \alpha_1)}; \ \beta_{2+j} = \frac{-\lambda(N-2j)}{(2\alpha_0 + \alpha_1)} \text{ and } \beta_{(N-1)/2} = \frac{-1}{(2\alpha_0 + \alpha_1)}$$

As a result:  $\sigma_i = \frac{1}{N} + \frac{\alpha_0 d_i - \sum_{j=1}^{N-1} \alpha_j (d_{i+j} + d_{i-j})}{(2\alpha_0 + \alpha_1)t}$ 

Similarly, when N is even, We know that  $\sigma_i = \frac{1}{2N} + \frac{1}{2t} \left( d_i - \frac{d_{i+1} + d_{i-1}}{2} \right) + \frac{\sigma_{i-1} + \sigma_{i+1}}{4}$ . When N is even, this expression yields:  $1/N + \beta_0 d_i/t + \sum_{j=1}^{N/2} \beta_j (d_{i+j} + d_{i-j})/t = \frac{1}{2N} + \frac{1}{2t} \left( d_i - \frac{d_{i+1} + d_{i-1}}{2} \right) + \frac{1}{2N} + \frac{1}{4t} \left[ \beta_0 \left( d_{i+1} + d_{i-1} \right) + \sum_{j=1}^{N/2} \beta_j (d_{i+j+1} + d_{i-j+1} + d_{i+j-1} + d_{i-j-1}) \right]$ 

This provides a system of equations that can be solved:

If N = 2, then the expression is  $\frac{1}{2} + \beta_0 d_i / t + 2\beta_1 (d_{i+1}) / t = \frac{1}{4} + \frac{1}{2t} (d_i - d_{i+1}) + \frac{1}{4} + \frac{1}{4} [2\beta_0 d_{i+1} + 4\beta_1 d_i]$ 

As a result  $\beta_0 = \frac{1}{2} + \beta_1$  and  $2\beta_1 = -\frac{1}{2} + \frac{\beta_0}{2}$ . This yields  $\beta_0 = \frac{1}{3}$  and  $\beta_1 = \frac{-1}{6}$ 

If N = 4, then the expression is  $1/4 + \beta_0 d_i/t + \beta_1 (d_{i+1} + d_{i-1})/t + 2\beta_2 d_{i+2} = \frac{1}{8} + \frac{1}{2t} \left( d_i - \frac{d_{i+1} + d_{i-1}}{2} \right) + \frac{1}{8} + \frac{1}{4t} \left[ (\beta_0 + 2\beta_2)(d_{i+1} + d_{i-1}) + 2\beta_1 d_i + 2\beta_1 d_{i+2} \right]$ As a result  $\beta_0 = \frac{1}{2} + \frac{2\beta_1}{4}$ ;  $\beta_1 = -\frac{1}{4} + \frac{\beta_0 + 2\beta_2}{4}$  and  $2\beta_2 = \frac{2\beta_1}{4}$  This yields  $\beta_0 = \frac{5}{12}$ ;  $\beta_1 = \frac{-1}{6}$  and  $\beta_2 = \frac{-1}{24}$ 

If N > 4 then

 $\begin{array}{l} \beta_0 = \frac{1}{2} + \frac{\beta_1}{2}; \beta_1 = -\frac{1}{4} + \frac{\beta_0 + \beta_2}{4}; \forall j \in \left] 1, N/2 - 1\right[; \beta_{N/2-j} = \frac{\beta_{N/2-j+1} + \beta_{N/2-j-1}}{4}; \beta_{N/2-1} = \frac{\beta_{N/2-2} + 2\beta_{N/2}}{4}; \text{ and because firm } N/2 \text{ is also firm } -N/2, \text{ we have } 2\beta_{N/2} = \frac{\beta_{N/2-1}}{2}. \text{ We can write } \beta_{N/2-j-1} = 4\beta_{N/2-j} - \beta_{N/2-j+1}. \text{ Let } \gamma(N) \in \mathbb{N}, N > 4 \text{ we can write } \beta_2 = \gamma(N)\beta_{N/2} \text{ with } \gamma(N+2) = 4\gamma(N) - 2\gamma(N-2) \text{ for } N > 6 \text{ and } \gamma(8) = 4\gamma(6) - 2\gamma(4) \text{ for } N = 6 \text{ and } \gamma(N) = 0 \text{ if } N \text{ is odd. We know that } \gamma(4) = 1; \gamma(6) = 4; \text{ Therefore } \gamma(8) = 14. \text{Generally we can write } \forall j \in \mathbb{N}, \beta_{2+j} = \gamma(N-2j)\beta_{N/2} \end{array}$ 

The sum of market shares  $\sum_{i=1}^{N} \sigma_i = 1$ ;  $\sum_{i=1}^{N} \sigma_i = 1 + \sum_{i=1}^{N} \left[ \beta_0 d_i + \sum_{j=1}^{N/2} \beta_j (d_{i+j} + d_{i-j}) \right] / t$ , therefore  $\beta_0 + 2 \sum_{j=1}^{N/2} \beta_j = 0$  or,  $\beta_0 + 2\beta_1 = 0$  for N = 2 and  $\beta_0 + 2\beta_1 + 2 \sum_{j=2}^{N/2} \beta_j = 0$ when N > 2. In this case, using the following change of variable k = N - 2j + 2 4,the equation becomes  $\beta_0 + 2\beta_1 + 2\sum_{k=4}^N \beta_{2+(N-k)/2} = 0$  and finally  $\beta_0 + 2\beta_1 + 2\beta_{N/2}\sum_{k=4}^N \gamma(k) = 0.$ 

For simplification, we denote  $\delta(N) = \sum_{k=4}^{N} \gamma(k)$ . The system of equations leads to:  $\beta_{N/2} = \frac{-1}{(5\gamma(N)+14\delta(N))}; \beta_0 = \frac{2\gamma(N)+6\delta(N)}{(5\gamma(N)+14\delta(N))}; \beta_1 = -\frac{\gamma(N)+2\delta(N)}{(5\gamma(N)+14\delta(N))}$  and  $\beta_{2+j} = \frac{-\gamma(N-2j)}{(5\gamma(N)+14\delta(N))}$ .

Let us denote  $\alpha_0 = \frac{1}{2} (2\gamma(N) + 6\delta(N))$  and  $\alpha_1 = \frac{1}{2} (\gamma(N) + 2\delta(N))$ , then  $2\alpha_0 + \alpha_1 = \frac{1}{2} (5\gamma(N) + 14\delta(N))$ .

The coefficients can be written:

 $\beta_0 = \frac{\alpha_0}{(2\alpha_0 + \alpha_1)}$ ;  $\beta_1 = -\frac{\alpha_1}{(2\alpha_0 + \alpha_1)}$ ;  $\beta_{2+j} = \frac{-\gamma(N-2j)}{(2\alpha_0 + \alpha_1)}$  and  $\beta_{N/2} = \frac{-2}{(2\alpha_0 + \alpha_1)}$ .

Notice that  $d_{i+N/2} = d_{i-N/2}$  thus  $d_{i+N/2} + d_{i-N/2} = 2d_{i+N/2}$ . This explains the coefficient 2 in  $\beta_{N/2}$ .

As a result:  $\sigma_i = \frac{1}{N} + \frac{\alpha_0 d_i - \sum_{j=1}^{N/2} \alpha_j (d_{i+j} + d_{i-j})}{(2\alpha_0 + \alpha_1)t}$ 2) If N is odd,  $\beta_0(N+2) - \beta_0(N) = \frac{\lambda(N)\mu(N+2) - \lambda(N+2)\mu(N)}{[5\lambda(N+2) + 14\mu(N+2)][5\lambda(N) + 14\mu(N)]}$ We know that for N > 3;  $4\lambda(N) - \lambda(N-2) = \lambda(N+2)$ . Similarly,  $4\mu(N) - \mu(N-2) = \mu(N+2)$  because  $4\mu(N) - \mu(N-2) = 4\lambda(N) - \lambda(N-2) + 4\mu(N-2) - \mu(N-4) = \lambda(N+2) + 4\mu(N-4)$   $2) - \mu(N-4)$  and thus,  $4\mu(N-2) - \mu(N-4) = \lambda(N) + 4\mu(N-4) - \mu(N-6)$ , therefore,  $4\mu(N) - \mu(N-2) = \lambda(N+2) + \lambda(N) + 4\mu(N-4) - \mu(N-6)$  and same manner:  $4\mu(N-4) - \mu(N-6) = \lambda(N-2) + 4\mu(N-6) - \mu(N-8)$  and so on... As a result:  $4\mu(N) - \mu(N-2) = \lambda(N+2) + \lambda(N) + \lambda(N-2) + ...\lambda(11) + 4\mu(7) - \mu(5) = \lambda(N+2) + \lambda(N) + \lambda(N-2) + ...\lambda(11) + 15$ 

And yet  $15 = \lambda(9) + \lambda(7) + \lambda(5)$ .

Finally  $4\mu(N)-\mu(N-2)=\sum_{k=5}^{N+2}\lambda(k)=\mu(N+2)$ 

now,  $\lambda(N)\mu(N+2) - \lambda(N+2)\mu(N) = 4\lambda(N)\mu(N) - \lambda(N)\mu(N-2) - 4\lambda(N)\mu(N) + \lambda(N-2)\mu(N) = \lambda(N-2)\mu(N) - \lambda(N)\mu(N-2)$ 

$$\lambda(N-2)\mu(N) - \lambda(N)\mu(N-2) = 4\lambda(N-2)\mu(N-2) - \lambda(N-2)\mu(N-4) - 4\lambda(N-2)\mu(N-2) + \lambda(N-4)\mu(N-2) = \lambda(N-4)\mu(N-2) - \lambda(N-2)\mu(N-4)$$

By iteration of the calculation, we can write  $\lambda(N)\mu(N+2) - \lambda(N+2)\mu(N) = \lambda(5)\mu(7) - \lambda(7)\mu(5) = 1$ 

Therefore  $\beta_0(N+2) - \beta_0(N) = \frac{1}{[5\lambda(N+2) + 14\mu(N+2)][5\lambda(N) + 14\mu(N)]}$ 

If N is even, for N > 4 a similar calculation yields  $\gamma(N)\delta(N+2) - \gamma(N+2)\delta(N) = \gamma(6)\delta(8) - \gamma(8)\delta(6) = 6$ 

Therefore  $\beta_0(N+2) - \beta_0(N) = \frac{6}{[5\gamma(N+2)+14\delta(N+2)][5\gamma(N)+14\delta(N)]}$ 

 $\begin{aligned} 3) \text{ If N is even, } &\beta_0(N+1) - \beta_0(N) = \frac{30(\gamma(N)\mu(N+1) - \delta(N)\lambda(N+1)) - 28(\delta(N)\lambda(N+1) - \gamma(N)\mu(N+1))}{[5\lambda(N+1) + 14\mu(N+1)][5\gamma(N) + 14\delta(N)]} = \\ &\frac{2(\gamma(N)\mu(N+1) - \delta(N)\lambda(N+1))}{[5\lambda(N+1) + 14\mu(N+1)][5\gamma(N) + 14\delta(N)]} \end{aligned}$ We have  $\gamma(N) = \lambda(N-1) + \lambda(N+1)$  and  $\delta(N) = \mu(N-1) + \mu(N+1)$ Indeed,  $\gamma(6) = \lambda(5) + \lambda(7) = 4$  and  $\delta(6) = \mu(5) + \mu(7) = 5$ . If both relations are fulfilled at rank N then they are also fulfilled at rank N + 2  $\gamma(N+2) = 4\gamma(N) - \gamma(N-2) = 4(\lambda(N+1) + \lambda(N-1)) - (\lambda(N-1) + \lambda(N-3)))$   $\gamma(N+2) = 4\lambda(N+1) - \lambda(N-1) + 4\lambda(N-1) - \lambda(N-3) = \lambda(N+3) + \lambda(N+1)$ The sign of  $\beta_0(N+1) - \beta_0(N)$  is the sign of  $\gamma(N)\mu(N+1) - \delta(N)\lambda(N+1)$ and  $\gamma(N)\mu(N+1) - \delta(N)\lambda(N+1) = [\lambda(N-1) + \lambda(N+1)]\mu(N+1) - [\mu(N-1) + \mu(N+1)]\lambda(N+1) + \lambda(N-1) + \lambda(N-1)]$ 

If N is even, N + 1 and N - 1 are odd and thus  $\lambda(N - 1)\mu(N + 1) - \mu(N - 1) = 1$ .

As a result  $\beta_0(N+1) - \beta_0(N) = \frac{2}{[5\lambda(N+1)+14\mu(N+1)][5\gamma(N)+14\delta(N)]} > 0$ If N is odd,  $\beta_0(N+1) - \beta_0(N) = \frac{2(\lambda(N)\delta(N+1)-\mu(N)\gamma(N+1))}{[5\gamma(N+1)+14\delta(N+1)][5\lambda(N)+14\mu(N)]} = \frac{2}{[5\gamma(N+1)+14\delta(N+1)][5\lambda(N)+14\mu(N)]}$ because  $\lambda(N)\delta(N+1) - \mu(N)\gamma(N+1) = \lambda(N)\mu(N+2) - \mu(N)\lambda(N+2) = 1$ 

4) If N is odd, We know that  $\lambda(N+2) = \mu(N+2) - \mu(N)$  thus  $\lambda(N) = \mu(N) - \mu(N-2)$ 

Replacing this result in  $\beta_0(N) = \frac{2\lambda(N)+6\mu(N)}{5\lambda(N)+14\mu(N)}$  yields  $\beta_0(N) = \frac{8\mu(N)-2\mu(N-2)}{19\mu(N)-5\mu(N-2)}$ Same manner, if N is even, $\beta_0(N) = \frac{8\delta(N)-2\delta(N-2)}{19\delta(N)-5\delta(N-2)}$ 

 $\mu$  and  $\delta$  are Lucas sequences. Indeed  $\mu(N+2)=P\mu(N)-Q\mu(N-2)$  and  $\delta(N+2)=P\delta(N)-Q\delta(N-2)$ 

where P = 4 and Q = 1. The characteristic equation of the recurrence relation is  $x^2 - 4x + 1 = 0$ 

The discriminant is  $\Delta = 12$ . We denote  $a = \frac{4+\sqrt{12}}{2}$  and  $b = \frac{4-\sqrt{12}}{2}$ 

Therefore, we can write  $\lambda, \mu, \gamma$  and  $\delta$  as a function of  $N : \lambda(N) = \frac{(1+a)a^{(N-5)/2} + (1+b)b^{(N-5)/2}}{6}; \gamma(N) = a^{(N-4)/2} + b^{(N-4)/2}; \mu(N) = \frac{a^{(N-3)/2} - b^{(N-3)/2}}{\sqrt{12}}$  and  $\delta(N) = \frac{a^{(N-4)/2}(1+a) - b^{(N-4)/2}(1+b)}{\sqrt{12}}$ 

As a result  $\forall N \in \mathbb{N}; N \ge 2; \beta_0(N) = \frac{a^{(N-5)}(8a-2)-(8b-2)}{a^{(N-5)}(19a-5)-(19b-5)}$ 

$$\lim_{N \to +\infty} \beta_0(N) = \overline{\beta_0} = \frac{8a-2}{19a-5} = \frac{14+8\sqrt{3}}{33+19\sqrt{3}}$$

$$\beta_0(N+1) - \beta_0(N) = \frac{2(a-b)(a-1)a^{(N-5)}}{(19a-5)^2a^{(2N-9)} - 6(a+1)a^{(N-5)} + (19b-5)^2} = \frac{2(a-b)(a-1)a^{(N-5)}}{(a^{2N+1} - (1+a)a^N + 1)(19b-5)^2} \text{ and } \\ \frac{\beta_0(N+1) - \beta_0(N)}{\beta_0(N+2) - \beta_0(N+1)} = \frac{(19a-5)^2a^{(2N-7)} - 6(a+1)a^{(N-4)} + (19b-5)^2}{(19a-5)^2a^{(2N-8)} - 6(a+1)a^{(N-4)} + (19b-5)^2a} \text{ and } \lim_{N \to +\infty} \left[ \frac{\beta_0(N+1) - \beta_0(N)}{\beta_0(N+2) - \beta_0(N+1)} \right] = a$$

5) Consumer Surplus is written:  $CS_i = \int_{-y}^{x} (d_i - \sigma_i t - tz) dz = \sigma_i d_i - \sigma_i^2 t - \frac{(x^2 + y^2)t}{2}$   $CS =_{i=1}^{N} CS_i = \sum \sigma_i d_i - \sum \sigma_i^2 t - \frac{(x^2 + y^2)t}{2} = \sum \sigma_i d_i - HHIt - \left(\frac{1}{4N} + Nvar(x)\right) t$  $var(\sigma) = var(x) + var(y) + 2cov(x, y) \text{ and } var(x) = var(y) \text{ thus, } var(x) = var(\sigma)/2 - cov(x, y)$ 

$$HHI = Nvar(\sigma) + 1/N$$

Market share depends on the position among firms. However, for the markets operated by N firms, in average:

 $\begin{aligned} \sigma_i &= \frac{1}{N} + \beta_0 \frac{d_i}{t} + (N-1)\beta_0 \sum_{j \neq i} \frac{d_j}{t}. \text{ Therefore quality of firm } j \neq i \text{ is written:} \\ d_j &= \frac{(N-1)t}{N\beta_0} + (\sigma_j - \sigma_i) + d_i \text{ and this leads to } var(d) = \left(\frac{(N-1)t}{N\beta_0}\right)^2 var(\sigma). \end{aligned}$ 

As a result,  $\sum \sigma_i d_i = E(d) + \frac{(N-1)t}{\beta_0} var(\sigma).$ 

this yields:  $CS = E(d) + \frac{(N-1)t}{\beta_0} var(\sigma) - HHIt - \frac{t}{4N} - Nvar(x)t = E(d) + \frac{(N-1)t}{\beta_0} var(\sigma) - \frac{5t}{4N} - \frac{3Nvar(\sigma)t}{2} + Ncov(x,y)t$ Thus  $CS = E(d) - \frac{5t}{4N} + \left(\frac{(N-1)}{\beta_0} - \frac{3N}{2}\right)t \ var(\sigma) + Nt \ cov(x,y)$ 

## A.2 Empirics

	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Belgium	3	3	3	3	3	3	3	3	3	3
	2	2	2	2	2	2	2	2	2	2
Ecuador	3	3	3	3	3	3	3	3	3	3
	2	2	2	2	2	2	2	2	2	2
Germany	4	4	4	4	4	4	4	4	4	4
	4	4	4	4	4	4	4	4	4	4
Hungary	3	3	3	3	3	3	3	3	3	3
	2	2	2	2	2	2	2	2	2	2
Indonesia	6	6								
	3	3								
Saudi Arabia						4	4	4	4	4
						1	1	1	1	1
South Korea	3	3	3	3	3	3	3	3	3	3
	3	3	3	3	3	3	3	3	3	3
Turkey	3	3	3	3	3	3	3	3	3	3
	2	2	3	3	3	3	3	3	3	3
UAE				2	2	2	2	2	2	2
				2	2	2	2	2	2	2
Argentina	5	4	4	4	4	4	4	4	4	4
	4	4	4	4	4	4	4	4	4	4
Australia	4	4	4	4	4	3	4	4	3	3
	3	3	3	3	3	3	3	3	3	3
Austria	5	5	4	4	4	4	4	4	4	3
	2	2	2	2	2	2	2	2	2	2
Chile	4	3	4	4	4	4	5	5	6	5
	2	3	3	3	3	3	3	3	3	3
Croatia	2	2	4	4	4	4	4	4	4	4
	2	2	2	2	2	2	2	2	2	2
Czech Republic	3	3	3	4	4	4	4	4	4	4
	1	1	1	1	1	1	1	1	1	1
Denmark	4	4	4	4	5	5	5	5	5	5
	3	3	3	3	3	3	3	3	3	3
Finland	3	3	3	4	4	4	4	4	4	4
	1	1	1	1	1	1	1	1	1	1
France	3	3	3	3	3	3	3	3	4	4
	3	3	3	3	3	3	3	3	4	4
Greece	4	4	4	3	3	3	3	3	3	3
	1	1	1	1	1	2	2	2	2	2
The markets above the line are those that did not experience any change in their market structure. The actual number of mobile operators is in bold. The observed number of operators is under the actual figure.										

#### A.2.1 Evolution of the number of firms in each market

Table 2: Evolution of the number of firms across markets

	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Hong Kong	5	5	5	5	6	6	6	6	6	6
	2	2	2	2	2	2	2	2	2	2
Ireland	3	4	4	4	4	5	5	5	5	5
	1	1	1	1	1	1	1	1	1	1
Lithuania	3	3	3	3	3	4	4	4	4	4
	1	1	1	1	1	1	1	1	1	1
Malaysia	3	3	3	5						
	3	3	3	3						
Mexico	5	5	5	4	4	5	5	5	5	5
	3	3	3	3	3	3	3	3	3	3
Netherlands	4	4	4	3	3	3	3	3	4	4
	3	3	3	3	3	3	3	3	3	3
New Zealand	2	2	2	2	2	3	3	3	3	3
	2	2	2	2	2	2	2	2	2	2
Norway	2	2	4	4	4	4	5	4	4	4
	2	2	2	2	2	2	2	2	2	2
Philippines	5	5	5	5	5	4	5	4	4	4
	2	2	2	2	2	2	2	2	2	2
Poland	3	3	4	4	5	6	6	5	6	6
	2	3	3	3	3	3	3	3	3	3
Portugal	3	3	3	3	3	3	3	3	3	4
	3	3	3	3	3	3	3	3	3	3
Romania			4	5	5	4	4	4	4	4
			1	1	1	1	1	1	1	1
Singapore	4	4	4	4	5	5	5	4	4	4
	3	3	3	3	3	3	3	3	3	3
Spain	3	3	4	4	4	4	6	6	6	6
	3	3	4	4	4	4	4	4	4	4
Sweden	5	5	5	6	6	6	5	5	5	5
	3	3	3	3	3	3	3	3	3	3
Switzerland	3	4	4	5	4	4	4	4	3	3
	1	1	1	1	1	1	1	1	1	1
Thailand	5	5	5	6	6	6	6	5	5	5
	2	2	2	2	2	2	2	2	2	2
United Kingdom	5	5	5	5	5	5	4	4	4	4
-	5	5	5	5	5	5	4	4	4	4
Venezuela		5	3	3	3	3	3	3	3	3
		1	1	1	1	1	1	1	1	1
The actual number of	mobile oper	rators is i	n bold. T	he obse	rved nur	nber of o	operators	s is unde	er the ac	tual

Table 3: Distribution of the number of firms across markets and years

#### A.2.2 Summary Statistics

	Obs.	Mean	Std. Dev.	Min	Max
Capital expenditures (millions US dollars, constant 2013)	818	415.47	486.82	2.11	3164.69
Market share (%)	818	33.00	14.84	0.21	95.57
Relative efficiency (2*market share -1)	818	-0.34	0.29	-0.99	0.91
Purchase a license (dummy)	818	0.27		0	1
Year of entry	818	1994.2	5.40	1981	2012
Year of observation	818	2008.5	2.86	2004	2013

 Table 4: Summary Statistics

#### A.2.3 Econometric estimation results

	1			Relative	Logorithm of investment		
_	Logaritr	im of investme	ent	efficiency	Logarithm of	Investment	
	(1)	(2)	(3)	(4)	(5)	(6)	
2 MNOs (dummy)	-0.56***	0.07	0.02	0.04	-0.01	-0.36**	
	(0.16)	(0.18)	(0.15)	(0.06)	(0.13)	(0.17)	
4 MNOs (dummy)	-0.25**	-0.43***	-0.35***	-0.03	-0.25***	-0.17	
	(0.10)	(0.09)	(0.10)	(0.03)	(0.07)	(0.12)	
5 MNOs (dummy)	-0.42***	-0.69***	-0.39***	-0.05	-0.26***	-0.44***	
	(0.14)	(0.10)	(0.12)	(0.04)	(0.09)	(0.12)	
6 MNOs (dummy)	-0.48**	-1.00***	-0.48**	-0.05	-0.34**	-0.56**	
	(0.19)	(0.17)	(0.19)	(0.04)	(0.14)	(0.23)	
Year of entry (1981-2012)				-0.03***			
				(0.00)			
Relative efficiency (-1 < RE < 7	1)				1.80***		
					(0.16)		
Market size (in millions)		0.03***	0.06	-0.00	0.06***	0.03***	
		(0.00)	(0.04)	(0.02)	(0.02)	(0.00)	
Coverage (lagged rate of pene	etration)	0.28	0.61***	0.02	0.56***	0.64**	
		(0.23)	(0.24)	(0.10)	(0.17)	(0.30)	
Spectrum license (dummy)	0.04	-0.01	-0.00	0.00	-0.01	0.03	
	(0.08)	(0.06)	(0.04)	(0.01)	(0.04)	(0.08)	
Year fixed effects	v	٧	V	v	v	٧	
Region fixed effects	v	٧	V	v	v	٧	
Country fixed effects			v	v	v		
Constant	5.79***	5.10***	4.46***	68.99***	4.95***	5.71***	
	(0.13)	(0.25)	(0.30)	(4.23)	(0.24)	(0.32)	
Observations	818	818	818	818	818	378	
R-squared	0.136	0.430	0.691	0.576	0.823	0.575	

Column (1) corresponds to the OLS estimation of the impact of the number of firms on the investment of each firm, assuming symmetric market structure. Column (2) controls for market size and coverage. Column (3) accounts for the market fixed effects. This latter specification uses the cross year source of identification. Column (4) is the first stage of the IV estimation of the impact of the number of firms on investment, controlling for the asymmetry in terms of marginal cost in column (5). This asymmetry is measured by the relative efficiency variable as the difference between a firm's market share and that of its rivals  $(m_i - (1 - m_i))$ . The year of entry into the market is used an instrument for the relative efficiency variable. The Cragg-Donald Wald F statistic for weak instrument is 477.4, larger than the critical value at 10% (16.4). Column (6) presents the OLS estimation results at the aggregate market level. The market size is measured as the working age population (15 - 64 years old).In all the specifications, the markets with three firms have been used as reference.

Significant at 1%<sup>(\*\*\*)</sup>, 5%<sup>(\*\*)</sup> and 10%<sup>(\*)</sup>. Robust standard errors corrected for arbitrary heteroskedasticity and autocorrelation are in parentheses.

Table 5: The impact of the number of firms on investment in new technologies