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Second-Best Renewable Subsidies to De-Carbonize the Economy: Commitment and the Green Paradox

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Abstract

Climate change must deal with two market failures: global warming and learning by doing in renewable use. The first-best policy consists of an aggressive renewables subsidy in the near term and a gradually rising and falling carbon tax. Given that global carbon taxes remain elusive, policy makers have to use a second-best subsidy. In case of credible commitment, the second-best subsidy is set higher than the social benefit of learning. It allows the transition time and peak warming close to first-best levels at the cost of higher fossil fuel use (weak Green Paradox). If policy makers cannot commit, the second-best subsidy is set to the social benefit of learning. It generates smaller weak Green Paradox effects, but the transition to the carbon-free takes longer and cumulative carbon emissions are higher. Under first-best and second best with pre-commitment peak warming is 2.1 - 2.3 °C, under second best without commitment 3.5°C, and without any policy temperature 5.1°C above pre-industrial levels. Not being able to commit yields a welfare loss of 95% of initial GDP compared to first best. Being able to commit brings this figure down to 7%.

JEL-codes: H210, Q510, Q540.

Keywords: first-best and second-best policy, commitment, Markov-perfect, Ramsey growth, carbon tax, renewables subsidy, learning by doing, directed technical change.

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1. Introduction

Climate policy has to deal with two crucial market failures: the failure for markets to price carbon to fully internalize all future damages arising from burning another unit of carbon and the failure of markets to internalize the full benefits of learning by doing in using renewable energy (Goulder and Mathai, 2000; de Zwaan et al., 2002; Popp, 2004; Edenhofer et al., 2005). To correct for these market failures the first-best policy has to be two-pronged: a carbon tax that must be set to the social cost of carbon (SCC), i.e., the present value of all future marginal global warming damages resulting from burning one extra unit carbon today,[1] and a renewable subsidy that must be set to the social benefit of learning by doing (SBL), i.e., the present value of all future reductions in the cost of renewables from using one unit of renewable energy today. Politicians are, however, keener on the carrot than the stick and thus prefer subsidies to taxes. Thirty years of international climate negotiations have failed miserably and national renewable policies may be called for when agreements on international carbon taxation fail to materialize. This brings us in the realm of second-best economics. Our objective is, therefore, to investigate how well a second-best Markov-perfect optimal renewable subsidy in the absence of a carbon does in the decentralized market economy compared with the first-best climate policy and business as usual.

Second-best issues are omnipresent in public economics but rarely discussed in climate change economics.[2] Grimaud et al. (2011) analyse optimal first-best and second-best climate policies in a decentralized market economy with directed technical change and endogenous growth. Kalkuhl et al. (2013) use a sophisticated IAM of growth and climate change with stock-dependent fossil fuel extraction costs to investigate the impact of optimal second-best renewable energy subsidies when carbon taxation is infeasible in a decentralized market economy.[3] These

[1] The optimal carbon price can be found on an efficient emissions market or as the shadow price of direct control legislation. Fischer et al. (2003) discusses effects of endogenous technical change on instrument choice.

[2] Apart from the voluminous literature on the double dividend hypothesis surveyed by Bovenberg and Goulder (2002) which deals with static second-best issues, the exceptions are Barrage (2014) and Schmitt (2013) who discuss optimal climate policy with distortionary labour and capital income taxation, respectively, with and without commitment. Van der Ploeg (2015) discusses the theory of second-best optimal carbon taxation in a two-period, three-country framework, highlights the rent grabbing component of the unilateral carbon tax and shows that the second-best optimal future carbon tax given a first-best carbon tax that is set below the optimal social cost of carbon is set too low as well.

[3] Kalkuhl et al. (2013) maximize welfare under the additional constraint of a peak warming of 2°C and the associated cumulative carbon budget for the optimal first-best or second-best climate policies. In our framework policy has to trade off small reductions in future global warming against small reductions in consumption now. The resulting maximum degree of global warming depends on the rate of pure time preference, intergenerational inequality aversion and trend growth, and is thus not necessarily equal to 2°C. In contrast to Kalkuhl et al. (2013), we find that the optimal second-best renewable subsidy is able to

studies find that a second-best subsidy is an apt measure for compensating the missing carbon price but assume that policy makers can commit to announcements about future policies even though given the forward-looking nature of scarcity rents on fossil fuel there is an incentive to re-optimize and deviate from announcements about future policies. We study the time-consistent Markov perfect second-best policy and find that the loss of commitment has significant costs in terms of welfare and environmental damage.

We characterize the equilibrium conditions for the first-best and second-best policy in an integrated assessment model (IAM) of growth and climate change with stock-dependent extraction costs, ongoing technical progress, and structural change and renewable energy a perfect substitute for fossil fuel.[4] This implies that fossil fuel is exhaustible and that the price of fossil fuel contains two forward-looking elements: the scarcity rent (the present discounted value of all future increases in extraction costs resulting from an extracting an extra unit of fossil fuel) and the carbon tax. The endogenous scarcity rent responds to policy and is lowered by subsidies for renewable energy. In the absence of a carbon tax, market prices for fossil fuel fall in the second-best setting, leading to increased carbon emissions relative to business as usual and a *weak* Green Paradox (Sinn, 2008; Gerlagh, 2011). The second-best Markov-perfect

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lock up a large fraction of fossil fuel reserves and thus despite some short-run adverse weak Green Paradox effects boosts welfare and gets close to the first-best optimum.

[4] We suppose that the cost of renewable energy falls as experience increases (Arrow, 1962). Tiang and Popp (2014) provide recent econometric evidence for significant learning by doing effects in renewable energy generation, suggesting that each new wind power project in China (with 60 GW capacity) leads to a unit cost reduction of 0.25%. De Zwaan et al. (2002) are the first to address optimal climate policy in the face of learning by doing in renewables in integrated assessment models. Popp (2004) studies endogenous technical progress in a fully calibrated IAM; Popp et al. (2010) review the implications of technical innovation and diffusion for the environment; and Goulder and Mathai (2000) study the implications in a stylised model of climate change. Manne and Richels (2004) argue that endogenous technical progress does not alter climate policy recommendations, specifically the transition timing. Jouvet and Schumacher (2012) find the opposite and so does Popp (2004) who studies optimal policies in an adapted version of DICE of Nordhaus (2008). Hübler et al. (2012) present a multi-region IAM with endogenous growth and study region-specific welfare effects. Fischer and Newell (2008) study optimal interaction of policy instruments in a calibrated model of heterogeneous energy producers limited to the US energy sector. None of these studies considers the "laissez faire" decentralized economy. Grimaud et al. (2001) characterize the "laissez faire" equilibrium in a stylized model. Various studies examine second-best carbon tax policy in which not enough instruments are available to the government (Hart, 2008; Graeker and Pade, 2009). Second-best subsidies have been studied using large, numerical global energy models (Edenhofer et al. 2005; Bosetti et al., 2006). Fossil fuel stocks are assumed abundant. Thus these studies cannot address how much fossil fuel to lock up in the crust of the earth and the time of phasing in renewables does not depend on expectations about future policies and energy prices. These are crucial features of our analysis. Tsur and Zemel (2005) analyze growth and R&D in a model with scarce resources, but do not offer a calibration and estimates of optimal climate policy.

climate policy assumes that policy makers cannot commit to announced future renewable subsidies and is equal to the social benefit of learning. If commitment is credible, policy makers can improve on the Markov-perfect second-best optimal renewable subsidy by pushing the subsidy above the SBL and, thereby, compensating for the lack of a carbon tax. It brings forward extraction of fossil fuel at the cost of accelerated global warming in the short run as fossil fuel owners fear that their resources will be worth less in the future. However, compared with business as usual, the second-best policy locks up more fossil fuel in the ground and curbs global warming in the long run but less so than in the first best. Our second-best Markov-perfect framework investigates whether the extra fossil fuel that is locked up forever is big enough to avoid a *strong* Green Paradox (Gerlagh, 2011).

Our calibrated IAM suggests that the first-best climate policy requires an aggressive and temporary renewable subsidy and a gradually rising carbon tax to price out fossil fuel with the required carbon tax to keep out fossil fuel in the carbon-free era falling with time. The first-best climate policy enforces a carbon budget of 320 GtC and brings down the maximum global mean temperature to 2.1°C. With commitment, the second-best subsidy for renewable energy fully compensates for the missing carbon tax such that the transition to carbon-free energy coincides with first-best. The lacking carbon tax, however, induces higher fossil fuel of 60 GtC during the fossil era and peak warming of 2.3°C. The second-best Markov-perfect renewable subsidy is relevant if commitment is infeasible and uses a significantly higher carbon budget of 1080 GtC, implying peak warming of 3.5 °C. This compares to a business as usual outcome of 2500 GtC carbon burnt and pre-industrial temperature increases of 5.1°C. There is no strong Green Paradox as the Markov Perfect second-best renewable subsidy without commitment reduces welfare relative to under first best by 95 percent of initial GDP compared to a welfare loss of six times initial GDP under business as usual. Being able to commit brings this figure down to 7%. However, policy makers have an incentive to renege after some time has lapsed by dropping the renewable subsidy and postponing the carbon-free era.

Section 2 discusses a simple two-stock model of carbon accumulation in the atmosphere and global mean temperature due to Golosov et al. (2014) and discusses our benchmark specification of climate damages which are bigger at higher temperatures than Nordhaus (2008, 2014) following recent suggestions by Stern (2013) and Dietz and Stern (2014). Section 3 formulates the command optimum for our general equilibrium IAM of climate change and Ramsey growth. Section 4 derives the market outcome of our IAM and shows how to derive the optimal first-best and second-best Markov-perfect climate policies. Section 5 offers policy simulations and highlights the effects of first-best and second-best Markov-perfect climate policies on untapped fossil fuel, the time it takes to phase in renewable energy and to reach the

carbon-free era, and welfare. There is also a discussion of the second-best optimal policy if pre-commitment is feasible. Section 6 concludes.

2. The carbon cycle, temperature and global warming damages

We use an annual version of the decadal model of the carbon cycle put forward by Golosov et al. (2014) and based on Archer (2005) and Archer et al. (2009):

$$(1) \qquad E_{t+1}^P = E_t^P + \varphi_L F_t, \quad \varphi_L = 0.2, \quad E_0^P = 103 \text{ GtC},$$

$$(2) \qquad E_{t+1}^T = (1-\varphi)E_t^T + \varphi_0(1-\varphi_L)F_t, \quad \varphi = 0.002304, \quad \varphi_0 = 0.393, \quad E_0^T = 699 \text{ GtC},$$

where E_t^P is the part of the stock of carbon (GtC) that stays thousands of years in the atmosphere, E_t^T the remaining part of the stock of atmospheric carbon (GtC) that decays at rate φ, and F_t the rate of fossil fuel use (GtC/decade).[5] About 20% of carbon emissions stay up 'forever' and the remainder has a mean life of about 300 years, so $\varphi = 1-(1-0.0228)^{1/10} = 0.002304$, where 0.0228 is the parameter proposed for the decadal model in Golosov et al. (2014). The parameter $\varphi_0 = 0.393$, is calibrated so that about half the carbon impulse is removed after 30 years.

The equilibrium climate sensitivity (ECS = ω) is the rise in global mean temperature after a doubling of the total carbon stock in the atmosphere, E_t. A typical estimate for the ECS is 3 (IPCC, 2007). Following Golosov et al. (2014), we ignore lags between atmospheric carbon and temperature:

$$(3) \qquad T_t = \omega \ln\left(E_t / 596.4\right) / \ln(2), \quad \omega = 3, \quad E_t \equiv E_t^P + E_t^T,$$

where 596.4 GtC is the IPCC figure for the pre-industrial stock of carbon.[6] The evolution of the stock of fossil fuel reserves S_t (measured at the start of period *t*) follows from the depletion equation:

$$(4) \qquad S_{t+1} = S_t - F_t, \quad S_0 = 4000 \text{ GtC}.$$

[5] The three reservoirs used by Nordhaus (2008) highlight the exchange of carbon with the deep oceans arising from the acidification of oceans limiting the capacity to absorb carbon. Our carbon cycle ignores time-varying coefficients as in Bolin and Erikkson (1958). It also abstracts from diffusive rather than advective transfers of heat to the oceans (Allen et al., 2009) which leads to longer and greater warming (Bronselaer et al., 2013; Baldwin, 2014).

[6] This temperature lag lowers the SCC (van der Ploeg and Rezai, 2014).

Nordhaus (2008) has combined detailed micro estimates of the costs of global warming to get aggregate macro costs of global warming of 1.7% of world GDP at 2.5 ^{o}C. This figure is used to calibrate the fraction of production that is left after global warming damages:

$$(5) \qquad \tilde{Z}(T_t) = \frac{1}{1+\zeta_1 T_t^{\zeta_2} + \zeta_3 T_t^{\zeta_4}}, \quad \text{so} \quad Z(E_t) \equiv \tilde{Z}\left(\omega \ln\left(E_t / 596.4\right) / \ln(2)\right),$$

with $\zeta_1 = 0.00284$, $\zeta_2 = 2$, and $\zeta_3 = \zeta_4 = 0$.[7] Weitzman (2010) and Dietz and Stern (2014) argue that damages rise more rapidly at higher levels of temperature than suggested by (5). Assuming that damages are 50% of world GDP at 6^{o} C and 99% at 12.5 ^{o}C, Ackerman and Stanton (2012) recalibrate (5) with $\zeta_1 = 0.00245$, $\zeta_2 = 2$, $\zeta_3 = 5.021 \times 10^{-6}$, and $\zeta_4 = 6.76$. The extra term in the denominator is included to capture potentially catastrophic losses at high temperatures.[8]

3. Ramsey growth and climate change: the command optimum

The social planner maximizes utilitarian social welfare

$$(6) \qquad \sum_{t=0}^{\infty} (1+\rho)^{-t} L_t U_t(C_t / L_t) = \sum_{t=0}^{\infty} (1+\rho)^{-t} L_t \left[\frac{(C_t / L_t)^{1-1/\eta} - 1}{1 - 1/\eta} \right],$$

where L_t is the size of the exogenous world population at time t, C_t aggregate consumption at time t, U the instantaneous CES utility function, $\rho > 0$ the rate of pure time preference and $\eta > 0$ the elasticity of intertemporal substitution. The ethics of climate policy depend on how much weight is given to future generations and how small intergenerational inequality aversion (IIA = $1/\eta$) is or how easy it is to substitute current for future consumption per head. The most ambitious climate policies result if society has a low rate of time preference and a low IIA (low ρ, high η).

Output is produced with three inputs: capital K_t, labour, L_t, and energy. Energy is either renewable R_t (e.g., solar or wind energy) or fossil fuel (oil, natural gas and coal), F_t. The production function $H(.)$ has constant returns to scale, is concave, and satisfies the Inada conditions. Renewables are subject to learning, so their unit production cost $b(B_t)$ falls with cumulated past production B_t and thus $b' < 0$. Fossil fuel extraction cost is $G(S_t)F_t$ with S_t

[7] The damage function resulting from the DICE-07 model is almost distinguishable (up to 7 ^{o}C) from that of the DICE-2013R model (see http://www.econ.yale.edu/~nordhaus/homepage/Web-DICE-2013-April.htm).

[8] We abstract from positive feedback and uncertain climate catastrophes that occur once temperature exceeds certain thresholds (e.g., Lemoine and Traeger, 2014; Lontzek et al., 2015; van der Ploeg and de Zeeuw, 2015).

remaining reserves, and rise as less accessible fields have to be explored, $G' < 0$. What is left of production after covering the cost of energy is allocated to consumption C_t, investments $K_{t+1} - K_t$, and depreciation δK_t where δ is the depreciation rate:

$$(7) \qquad K_{t+1} = (1-\delta)K_t + Z(E_t)H(K_t, L_t, F_t + R_t) - G(S_t)F_t - b(B_t)R_t - C_t.$$

The initial capital stock K_0 is given. Renewable knowledge accumulates according to

$$(8) \qquad B_{t+1} = B_t + R_t, \quad B_0 = 0.$$

Current technological options favour fossil energy; complete decarbonization requires substantial reductions in the cost of renewables versus that of fossil fuel. Apart from carbon taxes, technological progress is an important factor in determining the optimal combination of fossil and renewable energy sources (Acemoglu et al., 2012; Mattauch et al., 2012). We thus capture learning and lock-in effects by making the cost of renewables a decreasing function of past cumulated renewable energy production, $b' < 0$ with $B_t = \sum_{s=0}^{t} R_s$. We assume instantaneous and perfect spill-over of learning.[9]

Proposition 1: *The social optimum maximizes (6) subject to (1)-(8). It must satisfy the Euler equation for consumption growth*

$$(9) \qquad \frac{C_{t+1}/L_{t+1}}{C_t/L_t} = \left(\frac{1+r_{t+1}}{1+\rho}\right)^{\eta}, \quad r_{t+1} \equiv Z_{t+1}H_{K_{t+1}} - \delta,$$

and the efficiency conditions for energy use

$$(10a) \quad Z(E_t)H_{F_t+R_t}(K_t, L_t, F_t + R_t) \le G(S_t) + \theta_t^S + \theta_t^E, \quad F_t \ge 0, \quad \text{c.s.},$$

$$(10b) \quad Z(E_t)H_{F_t+R_t}(K_t, L_t, F_t + R_t) \le b(B_t) - \theta_t^B, \quad R_t \ge 0, \quad \text{c.s.},$$

where the scarcity rent , the SCC and the SBL are, respectively, given by

$$(11) \quad \theta_t^S = -\sum_{s=0}^{\infty}\left[G'(S_{t+1+s})F_{t+1+s}\Delta_{t+s}\right],$$

$$(12) \quad \theta_t^B = -\sum_{s=0}^{\infty}\left[b'(B_{t+1+s})R_{t+1+s}\Delta_{t+s}\right] \text{ and}$$

[9] We prefer learning-by-doing over other specifications of endogenous technical change, such as investment in R&D in Bovenberg and Smulders (1996) and Acemoglu et al. (2012), due to the better, albeit limited, availability of empirically validated learning curves. See also footnote 3.

(13) $$\theta_t^E = -\sum_{s=0}^{\infty}\left[\left\{\varphi_L + \varphi_0(1-\varphi_L)(1-\varphi)^s\right\}\Delta_{t+s}Z'(E_{t+1+s})H(K_{t+1+s}, L_{t+1+s}, F_{t+1+s}+R_{t+1+s})\right],$$

with the compound discount factors given by $\Delta_{t+s} \equiv \prod_{s'=0}^{s}(1+r_{t+1+s'})^{-1}, s \geq 0.$

Proof: see appendix A.

The Euler equation (9) states that growth in consumption per capita rises with the social return on capital (r_{t+1}) and falls with the rate of time preference, especially if IIA = $1/\eta$ is small. Equation (10a) states that, if fossil fuel is used, its marginal product should equal the sum of current extraction cost, $G(S_t)$, the scarcity rent, θ_t^S, and the SCC, θ_t^E. If fossil fuel is not used, its marginal product is below marginal cost. Equation (10b) indicates that, if renewable energy is used, its marginal product must equal its current cost $b(B_t)$ *minus* the SBL, θ_t^B.

Equation (11) states that the scarcity rent of keeping an extra unit of fossil fuel unexploited is the present discounted value of all future reductions in fossil fuel extraction costs. It stems from the Hotelling rule which states that the return on extracting an extra unit of fossil, i.e., the rate of interest ($r_t\theta_t^S$) minus the increase in future extraction cost $\left(-G'(S_{t+1})F_{t+1}\right)$, must equal the expected capital gain from keeping an extra unit of fossil fuel in the earth $(\theta_{t+1}^S - \theta_t^S)$. Due to the availability of renewable energy as a backstop, increasing extraction costs imply that fossil fuel will be eventually phased out and typically part of fossil fuel reserves will be abandoned and locked up. The Hotelling scarcity rent then captures the increase in all future extraction costs resulting from extracting an extra unit of fossil fuel today. Equation (12) indicates that the SBL equals the present discounted value of all future learning-by-doing reductions in the cost of renewable energy.

Equation (13) states that the SCC equals the present discounted value of all future marginal global warming damages from burning one unit of carbon today, taking due account of that part stays in the atmosphere for ever and the rest gradually decays at a rate corresponding to roughly 1/300 per year.

Proposition 2: *If the utility function is logarithmic, the production function is Cobb-Douglas, global warming damages are* $Z(E_t) \cong \exp\left[-\tilde{\zeta}(E_t - 581)\right]$, *depreciation of physical capital is 100% every period and energy production does not require capital input, the SCC becomes*

(13′) $$\theta_t^E = \tilde{\zeta}\left[\left(\frac{1+\rho}{\rho}\right)\varphi_L + \left(\frac{1+\rho}{\rho+\varphi}\right)\varphi_0(1-\varphi_L)\right]Z(E_t)H(K_t, L_t, F_t+R_t).$$

Proof: see Golosov et al. (2014).

The optimal SCC is proportional to world GDP. The factor of proportionality The factor of proportionality is independent of the factor production shares; it is big if the social rate of discount ρ is small, the permanent fraction of the atmospheric stock of carbon φ_L is large, and the lifetime of the transient component of the atmospheric stock of carbon $1/\varphi$ is large. This result supposes unit IIA.[10]

4. Ramsey growth and climate change: the decentralized market outcome

In a decentralized market economy one needs to consider the behaviour of producers of final goods, fossil fuel and renewable energy and of households. Final goods producers operate under perfect competition. They take the output price (the numeraire), the wage w_t, the market interest rate r_{t+1}, the market price for fossil fuel p_t, the specific carbon tax τ_t, the market price for renewable energy q_t, the renewable subsidy υ_t and the carbon stock E_t as given. They choose labour, capital and energy to maximize profits, $Z(E_t)H(K_t, L_t, F_t + R_t) - w_t L_t - (r_{t+1} + \delta)K_t - (p_t + \tau_t)F_t - (q_t - \upsilon_t)R_t$,

where $r_{t+1} + \delta$ is the user cost of capital. This leads to the following efficiency conditions:

$$(14)\quad Z(E_t)H_{K_t} = r_{t+1} + \delta,\ Z(E_t)H_{L_t} = w_t,\ \left.\begin{matrix} Z(E_t)H_{F_t} \le p_t + \tau_t \\ F_t \ge 0 \end{matrix}\right\} c.s.,\ \left.\begin{matrix} Z(E_t)H_{R_t} \le q_t - \upsilon_t \\ R_t \ge 0 \end{matrix}\right\} c.s.$$

Making use of (14), we obtain the net output function

$$(15)\quad Y_t \equiv Z(E_t)H(K_t, L_t, F_t + R_t) - \delta K_t - (p_t + \tau_t)F_t - (q_t - \upsilon_t)R_t = Y(E_t, K_t, L_t, p_t + \tau_t, q_t - \upsilon_t),$$

where $Y_{E_t} = Z(E_t)'H_t < 0, Y_{K_t} = r_{t+1}, Y_{L_t} = w_t, Y_{p_t+\tau_t} = -F_t \le 0$ and $Y_{q_t-\upsilon_t} = -R_t \le 0.$

Fossil fuel owners also operate under perfect competition and maximize the present discounted value of their profits, $\sum_{t=0}^{\infty} \tilde{\Delta}_t \left[p_t F_t - G(S_t)F_t \right]$ with $\tilde{\Delta}_t \equiv \prod_{s=0}^{t} (1 + r_{1+s})^{-1}, t \ge 0,$ subject to the depletion equation (4), taking the market price of fossil fuel p_t as given and internalising the adverse effect of current depletion on future extraction costs. They thus set the price of fossil fuel equal to extraction cost plus the scarcity rent (11) which stems from the Hotelling rule:

[10] Nordhaus (1991) and Golosov et al. (2014) also give approximate rules for the optimal SCC, which depend on IIA and the trend rate of growth of the economy. Rezai and van der Ploeg (2014) derive an approximate rule for the optimal SCC which also allows for population growth, climate damages not proportional to GDP, and mean reversion in damages to TFP growth.

(16) $p_t = G(S_t) + \theta_t^S$ where $\theta_{t+1}^S = (1+r_{t+1})\theta_t^S + G'(S_{t+1})F_{t+1}$.

Producers of renewable energy also operate under perfect competition and maximize the present value of their profits, $\sum_{t=0}^{\infty} \tilde{\Delta}_t \left[\{q_t - b(B_t)\} R_t \right]$, taking the market price of renewable energy q_t and the stock of accumulated knowledge about using renewable energy B_t as given. They thus set the price of renewable energy equal to the marginal cost of producing it: $q_t = b(B_t)$.

Households maximize utility (6) subject to the budget constraint $A_{t+1}^H = (1+r_{t+1})A_t^H + w_t L_t + \Theta_t - C_t$, where A_t^H denotes household assets and Θ_t lump-sum transfers from the government. This gives rise to the same Euler equation for optimal consumption growth as in the command economy, (9).

The government balances its books, $\tau_t F_t = \upsilon_t R_t + \Theta_t$, so that it hands net revenue from taxes and subsidies as lump-sum transfers. Asset and final goods market equilibrium require $A_t^H = K_t$ and $Z(E_t)H(K_t, L_t, F_t + R_t) = C_t + K_{t+1} - (1-\delta)K_t + G(S_t)F_t + b(B_t)R_t$. Using (15) and the pricing conditions for energy producers, the latter becomes $K_{t+1} = K_t + Y_t - C_t + (\theta_t^S + \tau_t)F_t - \upsilon_t R_t$.

4.1. Replicating the first-best optimum in the market economy

The first fundamental theorem of welfare economics indicates that the first-best optimum for the command economy can with suitable taxes and subsidies be replicated in the market economy.

Proposition 3: *The social optimum is replicated in the decentralized market economy if* $\tau_t = \theta_t^E$ *and* $\nu_t = \theta_t^B, \forall t \geq 0,$ *where these follow from (12) and (13).*

Proof: Comparing conditions of proposition 2 with the efficiency conditions and market equilibrium conditions of the decentralized market economy, we can demonstrate that these are identical if the specific carbon tax is set to the first-best SCC and the renewable subsidy is set to the optimal SBL. □

The first best thus emerges in the market economy if the specific carbon tax is set to the optimal SCC, the renewable subsidy is set to the optimal SBL, and net revenue is rebated in lump sums.

4.2. Second-best climate policies in the market economy: with and without commitment

As shown in Grimaud et al. (2011) and Kalkuhl et al. (2013), calculating second-best climate policies is more cumbersome. The reason is that the first fundamental theorem of welfare economic no longer holds if the full set of instruments is no longer available as would be the

case when the government can optimally choose the renewable subsidy when the carbon tax is absent (or constrained to a sub-optimal value). The government chooses the renewable subsidy to maximize welfare subject to the behavioural, market equilibrium and budget constraints of the market economy as described in section 4.1. Making use of the net output function (15), the government's second-best problem can thus be stated as:

$$(17) \qquad \underset{\{\upsilon_t, t\geq 0\}}{\text{Max}} \sum_{t=0}^{\infty} (1+\rho)^{-t} L_t \left[\frac{(C_t / L_t)^{1-1/\eta} - 1}{1 - 1/\eta} \right],$$

subject to

$$(18a) \qquad E_{t+1}^P = E_t^P + \varphi_L \text{Max}\left[0, -Y_{p_t}(E_t, K_t, L_t, p_t, q_t - \upsilon_t)\right], \quad E_0^P \text{ given},$$

$$(18b) \qquad E_{t+1}^T = (1-\varphi) E_t^T + \varphi_0 (1-\varphi_L) \text{Max}\left[0, -Y_{p_t}(E_t, K_t, L_t, p_t, q_t - \upsilon_t)\right], \quad E_0^T \text{ given},$$

$$(18c) \qquad S_{t+1} = S_t - \text{Max}\left[0, -Y_{p_t}(E_t, K_t, L_t, p_t, q_t - \upsilon_t)\right], \quad S_0 \text{ given},$$

$$(18d) \qquad B_{t+1} = B_t + \text{Max}\left[0, -Y_{q_t - \upsilon_t}(E_t, K_t, L_t, p_t, q_t - \upsilon_t)\right], \quad B_0 = 0.$$

$$(18e) \qquad \begin{aligned} K_{t+1} = K_t + Y(E_t, K_t, L_t, p_t, q_t - \upsilon_t) - C_t + \theta_t^S \text{Max}\left[0, -Y_{p_t}(E_t, K_t, L_t, p_t, q_t - \upsilon_t)\right] \\ - \upsilon_t \text{Max}\left[0, -Y_{q_t - \upsilon_t}(E_t, K_t, L_t, p_t, q_t - \upsilon_t)\right], \end{aligned}$$

$$(19a) \qquad C_{t+1} = \left(\frac{1 + Y_{K_t}(E_t, K_t, L_t, p_t, q_t - \upsilon_t)}{1+\rho} \right)^{\eta} \left(\frac{L_{t+1}}{L_t} \right) C_t,$$

$$(19b) \qquad \begin{aligned} \theta_{t+1}^S = \left[1 + Y_{K_t}(E_t, K_t, L_t, p_t, q_t - \upsilon_t)\right] \theta_t^S + \\ G'(S_{t+1}) \text{Max}\left[0, -Y_{p_{t+1}}(E_{t+1}, K_{t+1}, L_{t+1}, p_{t+1}, q_{t+1} - \upsilon_{t+1})\right]. \end{aligned}$$

where $p_t = G(S_t) + \theta_t^S$ and $q_t = b(B_t)$. Equation (17) is the same objective as in (6) but with a different choice set. Equations (18a), (18b) and (18c) restate equations (1), (2) and (4) with fossil fuel use substituted from the net output function (15). Equation (18d) describes the evolution of knowledge in producing renewable energy and stems from (6) and (15). Equation (18e) is the goods market equilibrium condition using (15). Equations (18a-e) give the dynamics for the predetermined state variables of our IAM. The dynamics for the non-predetermined states are given by (19a) the Euler equation for consumption from (9) and (19b) the Hotelling rule for the scarcity rent from (16), where the interest rate and fossil fuel use come from the net output function.

Given that empirically the cost of renewable energy is above that of fossil fuels, the second-best optimal outcome, i.e. with pre-commitment, for the market economy that results from the optimal control problem (17)-(19) consists of an initial phase where only fossil fuel is used, possibly an intermediate phase where fossil fuel and renewable energy use are alongside each other,[11] and a final carbon-free phase. The renewable subsidy is only defined and effective during the intermediate and renewable phase. The policy maker can bring forward the transition time to the carbon-free era by setting higher subsidy levels than the SBL, and thereby get closer to the first best.

Such strategic considerations are not feasible for the policy maker without commitment: the Markov-perfect second-best policy equals the SBL and does not attempt to manipulate the optimal time of transition to the carbon-free era. To see this, one has to solve the problem (17)-(19) using the principle of dynamic programming. Starting with the final phase, we note that the in-situ stock of fossil fuel remains unchanged whilst the carbon in the atmosphere gradually decays leaving ultimately only the permanent component. Since we have $Z(E_t)H_{R_t} = b(B_t) - \upsilon_t$, renewable use increases in capital, the stock of renewable knowledge and the renewable subsidy but falls with global warming. Working backwards, we obtain the following proposition.

Proposition 4: *During the final carbon-free phase and the phase where fossil fuel and renewable energy are used together, the Markov-perfect second-best optimal renewable subsidy equals the SBL:*

$$(20) \qquad \upsilon_t = \theta_t^B = -\sum_{s=0}^{\infty}\left[b'(B_{t+1+s})R_{t+1+s}\Delta_{t+s}\right].$$

Proof: see appendix B. The Markov-perfect second-best optimal renewable subsidy equals the SBL, but this does not necessarily coincide with the first-best optimal SBL and renewable subsidy. To see this, note that the first phase where only fossil fuel is used has no policies and can be solved as if it were business as usual. Still, the outcomes during this first phase are not business as usual for two reasons. First, the renewable subsidy with and without commitment ensures that more fossil fuel is locked up forever. This follows from the arbitrage condition that at the end of phase one (supposing that the intermediate phase is degenerate for the time being) the economy must be indifferent between fossil fuel and renewable energy and a vanishing scarcity rent at that time:

[11] Since fossil fuel and renewable energy are perfect substitutes, simultaneous use is infeasible without learning by doing, renewable subsidy or carbon tax (except possibly for a single period of time). Learning by doing introduces convexity in the renewable production cost so an intermediate phase with simultaneous use might emerge. In fact, such a phase often does not occur and if it occurs it is at most for one or a few years.

$$(21) \qquad G(S_t) < b(0) - \upsilon_t, \quad 0 \le t < t_{CF}, \qquad G(S_t) \ge b(B_t) - \upsilon_t, \quad S_t = S_{t_{CF}}, \quad \forall t \ge t_{CF},$$

where t_{CF} is the time when the economy for the first time uses only renewable energy. From (21) we see that a renewable subsidy increases the stock of untapped fossil fuel and thus curbs the length of the first phase. Second, the renewable subsidy lowers fossil fuel prices in the first phase and thus induces a weak Green Paradox as at any point of time emissions are higher than under business as usual. A renewable subsidy thus curbs cumulative emissions but boosts emissions in the short run.

At the time of the switch to the final carbon-free phase, the price of energy must be continuous to rule out arbitrage opportunities. Hence, renewable energy use immediately after time t_{CF} must equal fossil fuel use immediately before time t_{CF}, and thus must be higher due to the weak Green Paradox effects in the initial phase. The second-best optimal social benefit of learning by doing (20) must thus at time t_{CF} and thereafter be higher than the first-best optimal SBL. In this sense the second-best optimal subsidy over-compensates for the lack of a carbon tax. The extent to which it is higher depends on the trade-off between adverse short-run weak Green Paradox effects and long-run benefits of locking up carbon. Hence, the upward adjustment of the SBL is less if fossil fuel demand is relatively elastic and fossil fuel supply is relatively inelastic.[12]

4.3. Announcement of future second-best optimal climate policies

As already mentioned, if policy makers can commit to announcements about the future renewable subsidy, they can boost welfare by pushing the renewable subsidy above the SBL and thereby bringing forward the carbon-free era, locking up more fossil fuel, and curbing cumulative carbon emissions. However, such a policy is time inconsistent and not credible.[13] As after some time there is less fossil fuel in situ, and consequently the supply of fossil fuel is more elastic and therefore weak Green Paradox effects are less after some time. Re-optimization would then lead to a downward adjustment of the renewable subsidy. As a result, the phasing out of fossil fuel will be postponed and more fossil fuel reserves will be burnt leading to higher cumulative carbon emissions and higher peak global warming. In our simulations we contrast the second-best renewable policy with and without commitment and highlight the cost of not

[12] One can also calculate the second-best carbon tax in the absence of a renewable subsidy. The derivation is similar and the global second-best carbon tax will be set to the SCC, where the SCC will differ due to the initial phase being longer and the in-situ stock of fossil fuel at the end of the initial phase being lower. With commitment, the second-best carbon tax compensates for the lack of a renewable subsidy. Due to its political irrelevance, we do not study the second-best carbon tax further.

[13] The co-states μ_t^C and μ_t^S for the non-predetermined variables driven by (19) are predetermined. Optimality requires that $\mu_0^C = 0$ and $\mu_0^S = 0$. The second-best optimal subsidy is time consistent if these co-states remain zero forever. If not, it is time inconsistent as it pays to renege and re-optimize.

being able to commit. We also show that welfare rises if policy makers renege on the former outcome just before the fossil fuel was meant to be phased out.

5. Policy simulation and optimization

Here we compare the scenarios for the market economy summarized in figure 1 and tables 1-2:

I. the first-best outcome where the carbon tax is set to the optimal SCC, $\tau_t = \theta_t^E$, and the renewable subsidy to the optimal SBL, $\upsilon_t = \theta_t^B, \forall t \geq 0$ (solid green lines);
II. the second-best renewable subsidy without commitment (long-dashed red lines);
III. the second-best optimal renewable subsidy with pre-commitment (dotted blue lines);
IV. BAU with no carbon tax or renewable subsidy (dot-dashed brown lines).

In our simulations time runs from 2010 till 2600 and is measured in years.[14] The functional forms and calibration of the carbon cycle, temperature module and global warming damages have been discussed in section 2. The functional forms and benchmark parameter values for the economic part of our model put forward in sections 3 and 4 are discussed in appendix C. We choose standard macroeconomic parameter values for capital depreciation and intertemporal preferences and adopt assumptions on near-term productivity and population growth from Nordhaus (2014). Current production possibilities imply low fossil fuel extraction costs and an initially high cost for renewable energy generation due to past biases in innovation towards fossil energy production. The calibration of our benchmark scenario reflects this cost structure. We use a CES production function and elasticity of substitution between energy and the capital-labour aggregate of ϑ = 0.5). We refer the reader to appendix C for more details.

5.1. First best: how to quickly de-carbonize and leave more fossil fuel untapped

Under the first-best scenario I (solid green) consumption, GDP and the capital stock monotonically increase. The transition to renewable energy takes place smoothly as soon as 2038; fossil energy is phased out completely by 2041 (see Table 1). Over this period 320 GtC are burnt, so most of the 4000 GtC of fossil fuel reserves are abandoned. Table 2 shows that leads to a maximum increase in temperature of 2.1°C or a maximum atmospheric carbon stock of 970 GtC (from (3)), which is close to the maximum of a trillion tons of carbon argued for in

[14] The problem is solved numerically with the optimization solver CONOPT3 in GAMS. We solve the model in finite time. The turnpike property ensures that all equilibrium paths approach the steady state quickly such that it renders terminal conditions essentially unimportant. In contrast to the first best, the second best needs additional constraints for market equilibrium and private sector behaviour.

Allen et al. (2009).[15] This rapid and unambiguous first-best transformation towards a carbon-free economy is achieved through the implementation of a carbon tax and a renewable subsidy policy. Both follow an inverted U-shaped time profile. The global carbon tax starts at 109 $/tC or 30 $/tCO2 and reaches a maximum of 175 $/tC or 48 $/tCO2 at the end of the fossil era, after which the tax falls and becomes obsolete as learning in renewables reduces their cost. The renewable subsidy starts at 350 $/tC or 95$/tCO2 in the first period of renewable use and rapidly falls to zero as all learning has occurred by the end of this century. The optimal policy mix, therefore, combines a quick and aggressive subsidy to phase in renewable energy quite early on and a carbon tax which gradually rises and falls to depress fossil energy use until renewable energy sources are competitive.

Table 1: Transition times and carbon budget

	Fossil fuel only	Renewable Only	Carbon used
I First best	2010-2038	2041 –	320 GtC
II Second-best subsidy: no commitment	2010-2076	2083 –	1080 GtC
III Second-best subsidy: commitment	2010-2039	2040 –	380 GtC
IV Business as usual	2010-2176	2177 –	2500 GtC

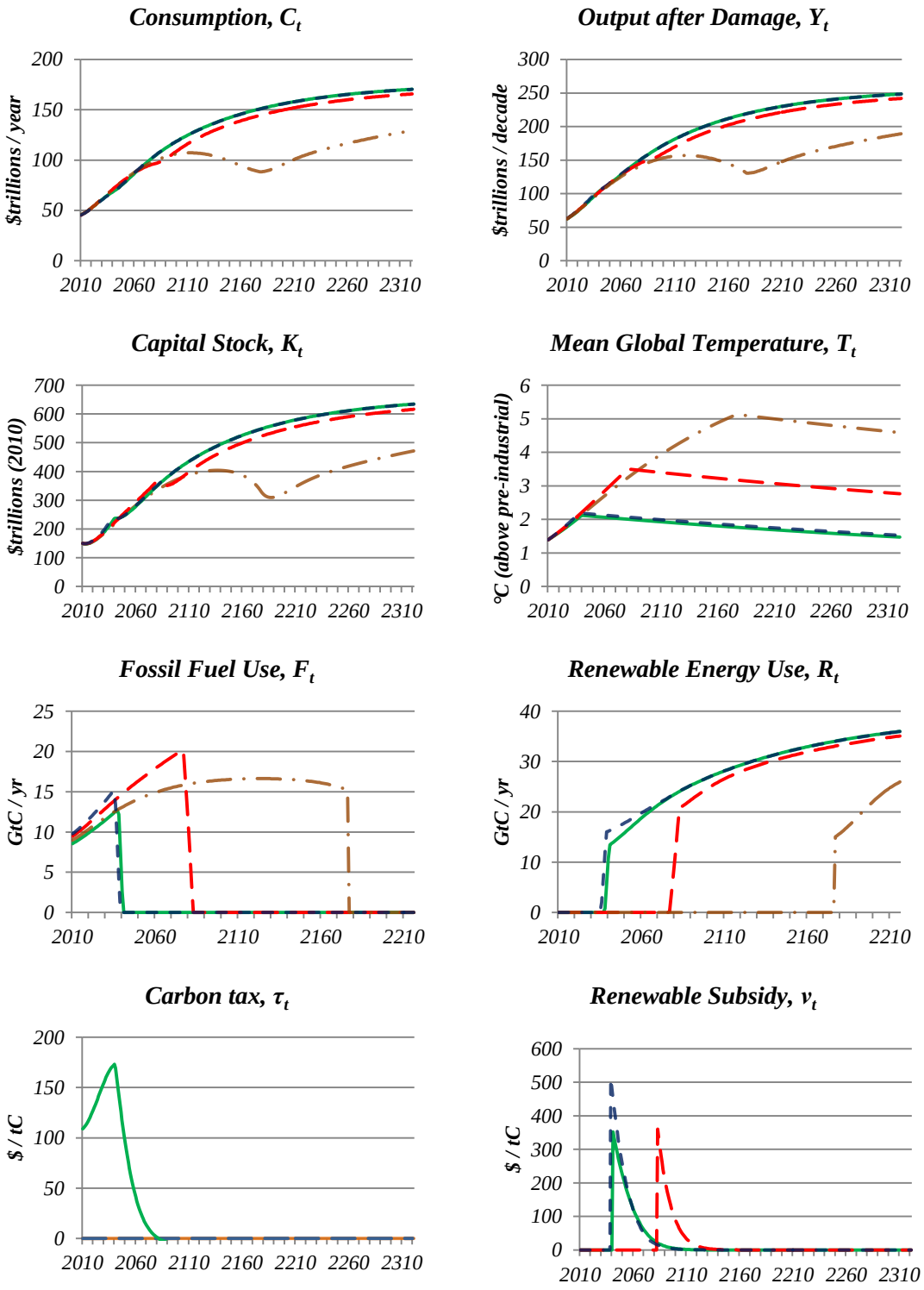
5.2. Business as usual and Markov-perfect second-best policies

In the business as usual scenario IV (dot-dashed brown) both externalities remain uncorrected. As a result the economy uses much more fossil fuel, 2500 GtC in total, so much less fossil fuel is left in the crust of the earth. Global mean temperature increases by a maximum of 5.1°C matching recent IPCC and IEA estimates for business as usual. The transition to renewable energy occurs much later, in 2175, and abruptly. The reason is that climate benefits of renewable energy and learning go unnoticed. The impacts of the climate and learning externalities are large enough to drastically change accumulation paths as temperatures rise. This is can be seen in the "kinks" in the business-as-usual paths in figure 1 and is also reflected in the substantial welfare loss of about 6 times initial GDP.[16] Failure to cooperate induces excessive fossil fuel extraction and capital accumulation leading to high global warming damages. Damages under business as usual are large enough to lower factor returns sufficiently

[15] Recent estimates by the IPCC (2014) state that cumulative emissions have to be limited to 790 GtC (with an uncertainty range of 700-860 GtC) if global warming is to remain below 2°C. By 2011 520 GtC had been emitted, giving a remaining carbon budget of only 270 GtC.

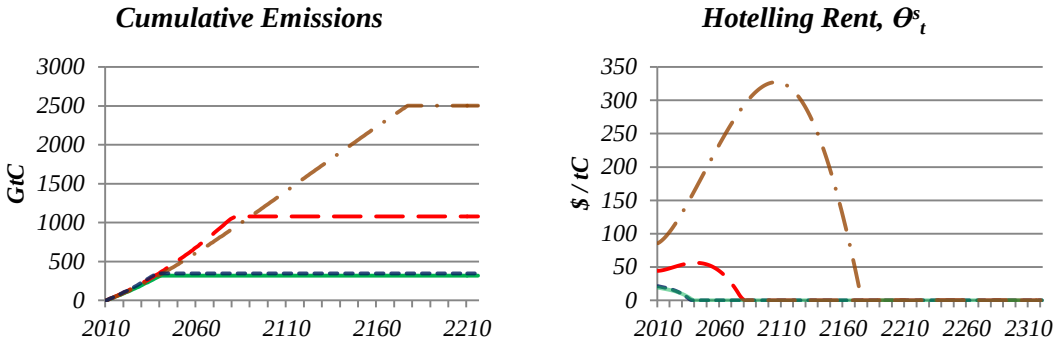
[16] Stern (2007) expresses cost of inaction in annuity terms; Nordhaus (2008) in terms of today's consumption. We calculate the difference in the total welfare, evaluated at initial prices, and express it as a share of initial GDP. Our welfare measure equals the loss due to inaction as a share of initial GDP.

Figure 1: Policy simulations



Key: first best (——), second-best subsidy (— —), BAU (— · —), second-best subsidy pre-commitment (- - -)

Figure 1: Policy simulations (continued)



Key: first best (——), second-best subsidy (— —), BAU (– · –), second-best subsidy pre-commitment (- - -)

to induce decumulation of capital and a fall in consumption. From 2140-2190 the capital stock falls by 25% from a peak of $410 trillion to a trough $310 trillion, consumption drops by 17% from a peak of $107 trillion to a trough of $88 trillion. Once extraction costs rise above the cost of renewable energy, the climate crisis ends. As the economy switches to renewable energy and stocks of atmospheric carbon recede, the return to capital, the interest rate and investment increase.

Table 2: Global carbon taxes, renewable subsidies, and welfare losses

	Welfare Loss (% of GDP)	Maximum carbon tax τ ($/tC)	Maximum renewable subsidy ($/tC)	max T (°C)
I First best	0%	175 $/GtC	350 $/GtC	2.1 °C
II Second-best subsidy: no commitment	-95%	N/A	360 $/GtC	3.5 °C
III Second-best subsidy: commitment	-6%	N/A	510 $/GtC	2.2 °C
IV Business as usual	-598%	N/A	N/A	5.1 °C

Failure to reach an international climate agreement or the political infeasibility of carbon taxes might lead to the implementation of a second-best renewable subsidy. Without commitment such a subsidy delays the transition by about 40 years (long-dashed red). The subsidy starts at a similar level as under first-best but the delayed transition increases total carbon use to 1080 GtC (less than half of BAU but still 3 to 4 times the optimal carbon budget) and increases peak temperature significantly to 3.5°C. The reduction of the carbon budget relative to BAU leads to a weak Green Paradox effect (Sinn, 2008) in the absence of a correcting carbon tax as fossil fuel use increases above BAU levels but for a shorter period (see *Fossil Fuel Use* panel in figure 1).[17] In the long run more carbon is locked up in the crust of the earth than under BAU. As a

[17] Under Leontief production technology, there are no Green Paradox effects as the energy demand is a fixed proportion of output.

result of this, welfare is higher than under BAU and there is no strong Green Paradox effect. The cost of second-best is significant as welfare falls by 95% compared to the first best, but this fall is a lot of less than the fall of almost 6 times initial GDP under BAU.

5.3. Second-best renewable subsidy with pre-commitment and time inconsistency

To facilitate comparison with Kalkuhl et al. (2013), we also indicated in figure 1 with the dotted blue lines the optimal announcement of second-best optimal renewable subsidies when pre-commitment is feasible. It is clear from the simulations that with pre-commitment the renewable subsidy is pushed above the SBL as this brings forward the carbon-free era by more than four decades to before the first-best timing (from 2083 to 2039) and locks up more fossil fuel in the crust of the earth. There is some acceleration of global warming in the short run arising from the weak Green Paradox effect, but more fossil fuel is locked up in the long run and therefore cumulative emissions and maximum global warming are cut down (from 1080 GtC to 345 GtC and from 3.5°C to 2.2°C). Although carbon emissions are higher by 30 GtC and global mean temperature by 0.1°C relative to the first-best outcome, this is exclusively due to the weak Green Paradox effect. In the absence of carbon taxation, fossil fuel prices are depressed relative to BAU under the second-best subsidy for renewable energy as global warming is forced down to a figure that is very close to the first-best outcome. To mimic the first-best outcome while having to accept somewhat higher fossil fuel use during the fossil era, renewable energy needs to be phased in earlier than first-best. These inefficiencies are relatively small and welfare falls by less than 6% relative to the first-best outcome, which is a lot less than the 95% of initial GDP when pre-commitment to future climate policies is not possible.

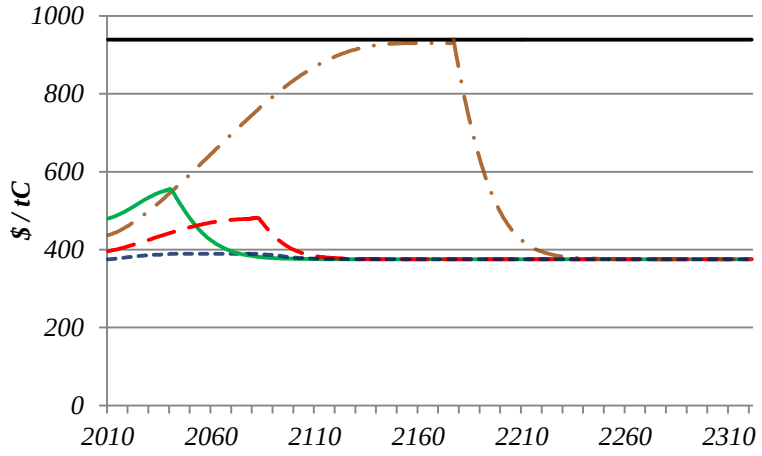
Alas, the pledges of policy makers are not credible and there is an incentive to deviate from the initial policy announcement later on. To illustrate this point, we give policy makers the option to re-optimize after 25 years. This leaves only 5 years of fossil fuel use and 55 GtC to be burnt. Policy makers renege on their announcements by subscribing to more ambitious climate targets: the subsidy for renewable energy is increased by almost 10% to 50 $/tC and as the result cumulative carbon emissions are depressed by nearly 15 GtC as the linkage to the weak Green Paradox effect with higher fossil fuel use in the first 25 years is severed. In choosing to surprise private agents by pushing up subsidies for renewable energy, expectations are falsified and welfare is increased by 0.1% and peak temperature is lowered by a tiny amount (0.03°C) relative to the second-best outcomes. This occurs at the expense of a very small additional weak Green Paradox effect. This illustrates that, in the absence of a credible and effective commitment mechanism, the second-best renewable subsidies calculated under the assumption are pre-commitment are likely to be reneged on and are thus time inconsistent. If pre-

commitment cannot be guaranteed, the second-best renewable energy subsidies calculated under the assumption of no pre-commitment and discussed in section 5.2 will be relevant as they are credible and time consistent albeit at the expense of lower welfare and higher peak warming. Our results demonstrate the importance of commitment devices in climate policy.

5.4. Time paths for the market price of fossil fuel and renewable energy

The weak Green Paradox effects are best seen when plotting the market prices of energy, depicted in figure 2. The price of fossil energy consists of the sum of marginal extraction cost and the Hotelling rent plus any carbon tax (see equation (10a)). The market price of renewable energy is set to its production cost minus any learning subsidy (see equation (10b)). Initially prices are rising in all scenarios and only on these rising sections are fossil fuels used. The solid black line gives the initial cost of renewable energy.

Figure 2: The market prices of energy during the transition ($/tC)



Key: first best (——), second-best subsidy (— —), BAU (– · –), second-best subsidy pre-commitment (- - -)

First, consider the business as usual scenario IV (dot-dashed brown) where the carbon tax and the subsidy are set to zero. The market cost of renewable energy is above the market price of fossil energy and constant in the absence of any production and/or subsidy. Fossil fuel is in use initially and its price rises due to increasing extraction costs and the increasing Hotelling rents. As extraction costs approach the back-stop price, the fossil fuel era draws to an end and the Hotelling rent falls. This fall in the Hotelling rent mitigates rising extraction costs and prolongs the era of fossil fuel use. In its final period, the scarcity value reaches zero and extraction costs equal renewable unit costs. After this switch point, the cost of fossil energy, consisting now only of the extraction cost component, remains constant. The cost of producing renewable energy falls quickly and approaches its lower floor due to learning by doing.

With a second-best subsidy without commitment, scenario II (dashed red), the market price of energy falls below its business-as-usual level as fossil fuel owners anticipate that their resources will be worth less in the future. Lower market prices temporarily stimulate higher fossil fuel use (of up to 28%), faster extraction, and acceleration of global warming. Once the SBL is sufficiently high to make fossil fuel uncompetitive, renewable energy is produced and energy prices start to fall as past learning lowers production costs.

First best policy, scenario I (solid green), precludes the weak Green Paradox effect by setting a carbon tax which equals the SCC. This lifts energy prices above BAU levels initially. The tax allows enables an earlier transition to renewable energy.

The second-best subsidy with pre-commitment, scenario III (dotted blue), compensates for the missing carbon tax by increasing the renewable energy subsidy beyond the SBL. Energy prices are lower initially and kept lower for longer as the subsidy prices fossil energy out of the market to ensure learning makes renewable energy competitive\e even as the subsidy recedes.

6. Conclusion

Our integrated assessment of climate change and Ramsey growth highlights the costs associated with second-best policy in the absence of a carbon tax. While the first-best policy prices carbon and subsidizes renewable use to curb fossil fuel use and promote substitution away from fossil fuel towards renewables, increase untapped fossil fuel, and bring forward the carbon-free era, we show that second-best policy has significant costs in terms of welfare and peak warming. The first-best policy mix limits the total amount of carbon burnt is 320 GtC and maximum warming to 2.1°C, whereas under the Markov-perfect second-best policy 1080 GtC are burnt and temperature rises by as much as 3.5°C. The associated welfare loss amounts to nearly today's world GDP compared to a welfare loss of almost 6 times today's world GDP under business as usual which sees global warming rise to 5.1°C as the total amount of carbon burnt is much higher (2,500 GtC). A subsidy to renewable energy without taxing fossil fuel encourages higher fossil fuel use in the short run (up to 30% above fossil fuel use under business as usual), but locks up more carbon and curbs cumulative carbon emissions.

Previous studies on second-best climate policy have assumed commitment by policy makers is possible, finding that the absence of a carbon tax does not add significant welfare losses. Our results show that these findings are due to the assumption of commitment which allows a first-best transition timing. Due the weak Green Paradox effect, however, total carbon use increases by 120 GtC to 380 GtC and peak warming to 2.3°C. This slight increase in temperature lowers

welfare by 7% of initial GDP. Welfare is higher than under business as usual (no strong Green Paradox effect), since the second-best optimal subsidy locks up more carbon in the earth and limits peak global warming. However, the second-best policy is not credible as it pays policy makers to renege and drop the renewable subsidy after some time has lapsed.

Under business as usual, inducing maximum warming of 5.1 °C. The welfare loss without policy is almost 6 times today's world GDP. Second-best renewable subsidies are, therefore, better than doing nothing, but are insufficient to combat climate change. It is important that renewable subsidies are complemented by a carbon tax to avoid excessive extraction in the short run associated with the weak Green Paradox effect. If policy makers can pre-commit to announced renewable subsidies, they can do better but they would have an incentive to renege and therefore such announcements are not credible.

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Appendix A: Proof of proposition 1

The adjoined Lagrangian for our model reads:

$$L \equiv \sum_{t=0}^{\infty}(1+\rho)^{-t}\left[L_t U_t(C_t/L_t) - \mu_t^S(S_{t+1}-S_t+F_t) - \mu_t^B(B_{t+1}-B_t-R_t)\right]$$

$$+\sum_{t=0}^{\infty}(1+\rho)^{-t}\left[\mu_t^P(E_{t+1}^P - E_t^P - \varphi_L F_t) + \mu_t^T\left\{E_{t+1}^T - (1-\varphi)E_t^T - \varphi_0(1-\varphi_L)F_t\right\}\right]$$

$$-\sum_{t=0}^{\infty}(1+\rho)^{-t}\lambda_t\left[K_{t+1} - (1-\delta)K_t - Z(E_t^P+E_t^T)H(K_t,L_t,F_t+R_t) + G(S_t)F_t + b(B_t)R_t + C_t\right],$$

where μ_t^S denotes the shadow value of *in-situ* fossil fuel, μ_t^B the shadow value of learning by doing, μ_t^P and μ_t^T the shadow disvalue of the permanent and transient stocks of atmospheric carbon, and λ_t the shadow value of manmade capital. Necessary conditions for a social optimum are:

(A2a) $U'(C_t/L_t) = (C_t/L_t)^{-1/\eta} = \lambda_t$,

(A2b) $Z_t H_{F_t+R_t} \le G(S_t) + \left[\mu_t^S + \varphi_L\mu_t^P + \varphi_0(1-\varphi_L)\mu_t^T\right]/\lambda_t, \quad F_t \ge 0, \quad$ c.s.,

(A2c) $Z_t H_{F_t+R_t} \le b(B_t) - \mu_t^B/\lambda_t, \quad R_t \ge 0, \quad$ c.s.,

(A2d) $(1-\delta+Z_{t+1}H_{K_{t+1}})\lambda_{t+1} = (1+\rho)\lambda_t$,

(A2e) $\mu_{t+1}^S = (1+\rho)\mu_t^S + G'(S_{t+1})F_{t+1}\lambda_{t+1}$,

(A2f) $\mu^B_{t+1} = (1+\rho)\mu^B_t - b'(B_{t+1})R_{t+1}\lambda_{t+1}$,

(A2g) $\mu^P_{t+1} = (1+\rho)\mu^P_t + Z'(E^P_{t+1} + E^T_{t+1})H_{t+1}\lambda_{t+1}$,

(A2h) $(1-\varphi)\mu^T_{t+1} = (1+\rho)\mu^T_t + Z'(E^P_{t+1} + E^T_{t+1})H_{t+1}\lambda_{t+1}$.

Equations (A2a) and (A2d) yield the Euler equation (9). The Kuhn-Tucker conditions (A2b) and (A2c) give (10a) and (10b) after defining $\theta^S_t \equiv \mu^S_t / \lambda_t$, $\theta^B_t \equiv \mu^B_t / \lambda_t$ and $\theta^E_t \equiv \left[\varphi_L \mu^P_t + \varphi_0(1-\varphi_L)\mu^T_t\right] / \lambda_t$ in final good units. (A2e) and (A2d) yield the Hotelling rule

(A3) $\theta^S_{t+1} = (1+r_{t+1})\theta^S_t + G'(S_{t+1})F_{t+1}$.

Forward summation over time of (A3) and using the transversality conditions gives (11). Equations (A2f) and (A2d) yield

(A4) $\theta^B_{t+1} = (1+r_{t+1})\theta^B_t + b'(B_{t+1})R_{t+1}$.

Forward summation over time of (A4) and using the transversality conditions yields (12). Defining $\theta^P_t \equiv \mu^P_t / \lambda_t$ and $\theta^T_t \equiv \mu^T_t / \lambda_t$ in final good units we use (A2g), (A2h) and (A2a) to get:

(A5a) $\theta^P_{t+1} = (1+r_{t+1})\theta^P_t + Z'(E^P_{t+1} + E^T_{t+1})H_{t+1}$,

(A5b) $(1-\varphi)\theta^T_{t+1} = (1+r_{t+1})\theta^T_t + Z'(E^P_{t+1} + E^T_{t+1})H_{t+1}$.

Solving (A5a) and (A5b), using the transversality conditions and $\theta^E_t \equiv \varphi_L\theta^P_t + \varphi_0(1-\varphi_L)\theta^T_t$, we obtain equation (13).

Appendix B: Proof of proposition 4

The Lagrangian for the final phase of the second-best outcome is defined as

$$\Lambda \equiv \sum_{t=t_R}^{\infty}(1+\rho)^{-t} L_t\left[\frac{(C_t/L_t)^{1-1/\eta}-1}{1-1/\eta}\right] + \lambda^B_t\left[B_{t+1} - B_t + Y_{b(B_t)-\upsilon_t}(E_t, K_t, L_t, ., b(B_t)-\upsilon_t)\right]$$

(B1) $$+\lambda^K_t\left[K_{t+1} - K_t - Y(E_t, K_t, L_t, ., b(B_t)-\upsilon_t) + C_t - \upsilon_t Y_{b(B_t)-\upsilon_t}(E_t, K_t, L_t, ., b(B_t)-\upsilon_t)\right]$$

$$+\mu^C_t\left[C_{t+1} - \left(\frac{1+Y_{K_{t+1}}(E_{t+1}, K_{t+1}, L_{t+1}, ., b(B_{t+1})-\upsilon_{t+1})}{1+\rho}\right)^{\eta}\left(\frac{L_{t+1}}{L_t}\right)C_t\right],$$

where t_{CF} denotes the start of the final carbon-free phase. The optimality conditions for this phase are

(B2a) $$\frac{d\Lambda}{d\upsilon_t} = \left(\lambda_t^K \upsilon_t - \lambda_t^B\right) Y_{q_t,q_t} + \eta \mu_{t-1}^C Y_{K_t,q_t} \left(\frac{1+r_t}{1+\rho}\right)^{\eta-1} \left(\frac{L_t}{L_{t-1}}\right) C_{t-1} = 0,$$

(B2b)

$$\frac{d\Lambda}{dB_t} = (1+\rho)\lambda_{t-1}^B - \lambda_t^B + \left[\lambda_t^B Y_{q_t,q_t} + \lambda_t^K (R_t - \upsilon_t Y_{q_t,q_t}) - \eta \mu_{t-1}^C Y_{K_t,q_t} \left(\frac{1+r_t}{1+\rho}\right)^{\eta-1} \left(\frac{L_t}{L_t - 1}\right) C_{t-1}\right] b'(B_t) = 0,$$

(B2c) $$\frac{d\Lambda}{dK_t} = \lambda_t^B Y_{K_t,q_t} + (1+\rho)\lambda_{t-1}^K - \lambda_t^K (1 + r_t + \upsilon_t Y_{K_t,q_t}) - \eta \mu_{t-1}^C Y_{K_t,K_t} \left(\frac{1+r_t}{1+\rho}\right)^{\eta-1} \left(\frac{L_t}{L_{t-1}}\right) C_{t-1} = 0.$$

(B2d) $$\frac{d\Lambda}{dC_t} = (C_t / L_t)^{-1/\eta} + \lambda_t^K + (1+\rho)\mu_{t-1}^C - \mu_t^C \left(\frac{1+r_{t+1}}{1+\rho}\right)^{\eta} \left(\frac{L_{t+1}}{L_t}\right) = 0.$$

One can verify that the optimal Markov-perfect second-best renewable subsidy is time consistent in the final phase by verifying that a solution to (18), (19) and (21) exists if $\mu_t^C = 0, t \geq 0.$ Let us suppose this is the case, We then get from (B2a) that $\upsilon_t = \lambda_t^B / \lambda_t^K$, so that (B2b) gives $(1+\rho)\lambda_{t-1}^B = \lambda_t^B + \lambda_t^K b'(B_t) R_t$ and (B2c) gives $(1+\rho)\lambda_{t-1}^K = \lambda_t^K (1 + r_{t+1})$. Equation (B2d) yields $L_t (C_t / L_t)^{1-1/\eta} + \lambda_t^K = 0,$ which together with the last condition yields the Euler equation (19a). We also have $(1+\rho)\upsilon_{t-1}\lambda_{t-1}^K = \upsilon_t \lambda_t^K + \lambda_t^K b'(B_t) R_t$ or $\upsilon_t = (1 + r_{t+1})\upsilon_{t-1} - + b'(B_t) R_t,$ which indeed yields (20) and is the same as (12).

Using a similar procedure, one show that the Markov-perfect renewable phase during the phase were fossil fuel and renewable energy are used alongside each other is given by the same expression for the SBL.

Appendix C: Functional forms and calibration

Our model runs on an *annual* instead of decadal time grid as earlier integrated assessment models or semi-decadal as in Nordhaus (2014). This allows us to pinpoint much more accurately the transition times towards the carbon-free era and a coarse time grid has been shown to introduce significant numerical biases (Cai et. al, 2012).

Preferences

We have CES utility and set the elasticity of intertemporal substitution to η = ½ and thus intergenerational inequality aversion to 2. The rate of pure time preference ρ is 1% per year.

Cost of energy

We employ an extraction technology of the form $G(S) = \gamma_1 (S_0 / S)^{\gamma_2}$, where γ_1 and γ_2 are positive constants. This specification implies that reserves will not be fully be extracted; some fossil fuel

remains untapped in the crust of the earth. Extraction costs are calibrated to give an initial share of energy in GDP between 6%-7% depending on the policy scenario. This translates to fossil production costs of $350/tC ($35/barrel of oil), where we take one barrel of oil to be equivalent to 1/10 ton of carbon, giving $G(S_0) = \gamma_1 = 0.35$. The IEA (2008) long-term cost curve for oil extraction gives a doubling to quadrupling of the extraction cost of oil if another 1000 GtC are extracted. Since we are considering all carbon-based energy sources (not only oil) which are more abundant and cheaper to extract, we assume only a doubling of production costs if a total 2000 GtC is extracted. With $S_0 = 4000\,\text{GtC}$,[18] this gives $\gamma_2 = 1$.[19] This implies that we assume very low extraction costs and a high initial stock of reserves which biases our findings toward using more fossil fuel longer.

Initial capital stock and depreciation rate

The initial capital stock is set to 200 (US$ trillion), which is taken from Rezai et al. (2012). We set the depreciation rate δ to be 0.1 per year.

Global production and global warming damages

Output before damages is

$$H_t = \left[(1-\beta)\left(AK_t^{\alpha}(A_t^L L_t)^{1-\alpha} \right)^{1-1/\vartheta} + \beta\left(\frac{F_t + R_t}{\sigma}\right)^{1-1/\vartheta} \right]^{\frac{1}{1-1/\vartheta}}, \quad \vartheta \geq 0,\ 0 < \alpha < 1 \text{ and } 0 < \beta < 1.$$ This is a constant-returns-to scale CES production function in energy and a capital-labour composite with θ the elasticity of substitution and β the share the parameter for energy. The capital-labour composite is defined by a constant-returns-to-scale Cobb-Douglas function with α the share of capital, A total factor productivity and A_t^L the efficiency of labour. The two types of energy are perfect substitutes in production. Damages are calibrated so that they give the same level of global warming damages for the initial levels of output and mean temperature. It is convenient to rewrite production before damages as

$$H_t = H_0\left[(1-\beta)\left(\frac{AK_t^{\alpha}(A_t^L L_t)^{1-\alpha}}{H_0} \right)^{1-1/\vartheta} + \beta\left(\frac{F_t + R_t}{\sigma H_0} \right)^{1-1/\vartheta} \right]^{\frac{1}{1-1/\vartheta}}.$$ We set the share of capital to α =

[18] Stocks of carbon-based energy sources are notoriously hard to estimate. IPCC (2007) assumes in its A2- scenario that 7000 $GtCO_2$ (with 3.66 tCO2 per tC this equals 1912 GtC) will be burnt with a rising trend this century alone. We roughly double this number to get our estimate of 4000 GtC for initial fossil fuel reserves. Nordhaus (2008) assumes an upper limit for carbon-based fuel of 6000 GtC in the DICE-07.

[19] Since $G(2000)/G(4000) = (4000/2000)^{\gamma_2} = 2^{\gamma_2}$ and $2^{\gamma_2} = 2.$

0.35, the energy share parameter to $\beta = 0.06$, and the elasticity of factor substitution $\vartheta = 0.5$ (as in the WITCH 2008 model) which we will refer to as the CES run. Initial world GDP in 2010 is \$63 trillion. Given $A_1^L = 1$, we calibrate $A = 3.78$ to yield initial output under "laissez faire". The energy intensity of output σ is calibrated to an initial energy use of 9 GtC under "laissez faire", $\sigma = 0.15$.

Population growth and labour-augmenting technical progress

Population in 2010 (L_1) is 6.5 billion people. Following Nordhaus (2008) and UN projections population growth is given by $L_t = 8.6 - 2.1e^{-0.35t}$. Population growth starts at 1% per year and falls below 0.1% percent within six decades and flattens out at 8.6 billion people. In the sensitivity analysis in section 4.3 we assume faster growth and a higher plateau to reflect more recent forecasts. Without loss of generality the efficiency of labour $A_t^L = 3 - 2e^{-0.2t}$ starts out with $A_1^L = 1$ and an initial Harrod-neutral rate of technical progress of 2% per year. The efficiency of labour stabilizes at 3 times its current level.

Cost of the renewable and learning by doing

We model learning by doing with initial cost reductions and a lower limit for the cost of the renewable, i.e., $b(B_t) = \chi_1 + \chi_2 e^{-\chi_3 B_t}$, $\chi_1, \chi_2, \chi_3 \geq 0$. This formulation differs from the usual power law definition of learning curves (Manne and Richels, 2004) but allows us to better calibrate initial learning rates (which can reach infinity for power law) and specify a lower limit for unit cost. We calibrate unit cost of renewable energy to the percentage of GDP necessary to generate all energy demand from renewables. Under a Leontief technology, with $\vartheta \to 0$, energy demand is $\sigma Z_t H_t$. The cost of generating all energy carbon free is $\sigma Z_t H_t b / Z_t H_t = \sigma b_t$. Nordhaus (2008) states that it costs 5.6% of GDP to decarbonize today's economy in a model of back-stop mitigation. We increase this cost to 7-8% of GDP in order to reach the learning potential given below. To calibrate the production cost of renewable energy, this cost estimate needs to be combined with the cost of producing conventional energy which ranges between 6%-7%. This gives $\sigma b_1 = 0.14$ or with $\sigma = 0.15$ we get $b_1 = b(0) = \chi_1 + \chi_2 = 0.94$ or \$940/tC. Through learning by doing this cost can be reduced by 60% to a lower limit of 9% of GDP, so that $b(\infty) = \chi_1 = 0.563$ and thus $\chi_2 = 0.375$. In our simulations with $\vartheta > 0$, this lower limit falls to about 6% of GDP due to substitution of energy through capital, i.e. in the long run the share of energy falls back to its current level. We assume that experience in renewable production lowers unit cost at a falling rate and the parameter χ_3 measures this speed of learning. We calibrate learning such that costs would decrease slowly. We suppose a mere

1.25% reduction in cost if all of world energy use would be supplied by renewable sources in the initial period, so that χ_3 = 0.00375. These assumptions imply at most a 14% reduction for a doubling of experience at the peak of learning-by-doing and fall within the broad range reported for learning rates in renewable generation (McDonald and Schrattenholzer, 2001). Manne and Richels (2004) use a 20% cost reduction for a doubling of experience (and thus need to impose unrealistic constraints on renewable use due to the strong curvature of the power-law learning curve). Alberth and Hope (2007) assume a cost reduction of 5%-25% for a doubling of experience so our calibrated 14% falls in this range. Goulder and Mathai (2000) calibrate their learning-by-doing technology to a 0.5%-4% of GDP cost of a 25% emission reduction in 2020. This translates into a cost of renewable energy which is a factor 3-8 times current fossil energy prices. In our calibration current renewable production prices are little less than 3 times those of fossil energy. These numbers illustrate the large scientific uncertainties surrounding the possible trajectories of renewable energy prices. Our calibration falls within the ballpark figures presented above, providing higher costs relative to Nordhaus (2008) who also considers potential CCS technologies in his assessment.