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# University of Kent School of Economics Discussion Papers

# Tax Reforms in Search-and-Matching Models with Heterogeneous Agents

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# Tax Reforms in Search-and-Matching Models with Heterogeneous Agents

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#### Abstract

Using a Mortensen-Pissarides search-and-matching framework, this paper investigates the importance of search frictions in determining the welfare and distributional effects of tax reforms that re-allocate the tax burden from capital to labour income. Calibrating the model to the UK economy, we find that the tax reforms are Pareto improving but increase inequality in the long run, despite welfare losses for at least one segment of the population in the short run. The results are robust to the variations in the relative bargaining power of workers and different specifications of unemployment benefit. But the welfare gains are higher for all agents if the relative bargaining power of workers is reduced or we assume that unemployment benefit depends on past wages.

JEL codes: E21, E24, E62

Key words: search frictions, agent heterogeneity, unemployment

benefits, tax reforms

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# 1 Introduction

Over the past two decades, the Mortensen-Pissarides search-and-matching framework has become a powerful tool for the analysis of unemployment, job vacancies and labour market behavior. The theory focuses on the interaction between unemployment and job creation. It builds on the idea that matching in the labour markets takes time and is costly. Frictions originate from lack of coordination between unemployed workers and vacant jobs that disrupts the ability to form employment relationships. A significant body of literature has applied the search-and-matching models to explain the cyclical fluctuations in unemployment (see e.g. Yashiv (2007) for a review of this literature). For example, Rogerson et al. (2005) discuss the usefulness of a range of search-theoretic models for analyzing the unemployment dynamics, job turnover and wages. There is also a debate on whether or not a calibrated matching model can quantitatively account for observed aggregate fluctuations in the labour markets (see e.g. Shimer (2005), Hall and Milgrom (2008) and Pissarides (2009)).

Another strand of the literature has studied the effects of factor income taxation on welfare and improving labour market efficiency. For instance, Shi and Wen (1999) show that labour income taxation is more costly than capital income taxation with realistic parameter values by computing the marginal deadweight losses associated with capital and labour income tax. Boone and Bovenberg (2002) explore the optimal role of the tax system in alleviating labour market imperfections and raising revenue. They suggest that the optimal tax system should not distort the labour market tightness and only the ad valorem component of the wage tax should be employed to raise revenue. Domeij (2005) examines the condition under which a long-run optimal zero capital tax can be obtained by assuming that the government has access to a commitment technology and optimally chooses capital and labour income taxation. He finds that when the Hosios parameter restriction<sup>2</sup> is satisfied or

<sup>&</sup>lt;sup>1</sup>See Mortensen and Pissarides (1994) and Pissarides (2001).

<sup>&</sup>lt;sup>2</sup>This refers to the condition that the elasticity of search in job matches coincides with the relative bargaining power of workers in the wage bargaining. We will discuss this condition in more detail in sub-section 2.8.

the government can use subsidies to vacancies or unemployment, the optimal capital tax in the long-run is always zero.<sup>3</sup> If these conditions do not hold, a non-zero capital tax can then work to correct the distortions arising from search externalities.

This paper contributes to the growing literature on tax policy study in search-and-matching models. It aims to shed some light on the welfare and distributional issues of re-allocating the tax burden from capital to labour income. We study the importance of search frictions in determining the effects of such tax reforms in both the long- and short-run. To the best of our knowledge, this question is not answered yet in the search-and-matching literature. In our setup, the government taxes capital income, including interest on savings and profits, and labour income by using two different tax rates to finance its spending requirements. We stay as close as possible to the standard search-and-matching model with capital accumulation (see e.g. Merz (1995) and Andolfatto (1996)), but incorporate household heterogeneity. Following Judd (1985), Lansing (1999) and Ardagna (2007), the households are divided into capitalists and workers in terms of economic roles. Only capitalists have access to the asset markets and only workers can work. The setup of household heterogeneity allows us to examine the distributional effects of income taxation and inequality issues.

Pissarides (1998) suggests that unemployment benefits specification is one of the key influences on the effects of employment tax reforms on unemployment and wages. In this paper, we further examine an alternative specification of unemployment benefits by assuming that they are proportional to past wages but are constant in the long run. As Chéron and Langot (2010) point out, this setup for unemployment benefits can introduce a feedback effect of distribution of wages on the distribution of unemployment benefits and is important for the predictions of models with search frictions. Therefore, we intend to analyse the importance of the specification of unemployment benefits in determining the effects of tax reforms and explore the different mechanisms of tax reforms driving the results.

 $<sup>^3</sup>$ In contast, Klein and Rios-Rull (2003) show that the optimal capital tax rate tends to be high when the government cannot commit to future tax rates.

We calibrate the model so that its steady-state solution can reflect the main features of the current UK economy, in particular, its tax structure and labour market characteristics. The UK is chosen to illustrate the quantitative analysis since its tax structure stands in stark contrast with other European countries, by having a very high effective capital to labour income tax ratio. Since the effects of tax reforms that re-allocate the tax burden from capital to labour income are monotonic in our model, we focus on the reform of eliminating the capital income tax which is widely investigated in optimal taxation literature.

Our main findings are summarized as follows. First, in a model with search and matching frictions, the tax reform considered is Pareto improving in the long run although it increases inequality between agents. In other words, all the agents are better off, despite higher welfare gains for the capitalists compared to the workers. This happens because, in the new, postreform economy, the increase in labour income tax and labour productivity due to higher capital accumulation leads to an increase in the bargained wage rate. Unemployment benefit increases as it is assumed to be proportional to the wage rate. As a result, the search-unemployment of workers increases. The higher wage rate reduces the expected profits of firms even if the higher labour productivity has a positive profits effect. Thus, the firms open less job vacancies. The net wage rate rises in the new, post-reform economy. This is because the productivity gains outweigh the increase in labour income tax which is beneficial to the workers. Overall, the income, consumption and welfare of workers increase in the long run. Capitalists can directly benefit from the zero capital tax as the interest income from capital increases. The capital income effect is bigger than the labour income effect. Capitalists benefit more from this tax reform and therefore inequality increases. However, the capital tax cut met with the labour tax increase hurts the workers and also worsens the aggregate welfare over the transition. This is because the positive effects resulting from higher capital accumulation take time to be realized. As a result, the combination of an initially lower net wage rate and higher search-unemployment creates short-run losses for the workers and aggregate economy which are reversed in the long run. We also show that our

results are robust to variations in the relative bargaining power of workers in the Nash bargain. Increasing the workers' bargaining power makes the tax reform less efficient in terms of welfare improving.

Second, when we assume that unemployment benefits depend on past wages, the model can generate similar welfare results in both the long- and short-run although the mechanism driving results is different. The tax reform is still Pareto improving but generates higher welfare gains for all agents in the long run. This happens because, there are larger positive effects resulting from higher capital accumulation. On one hand, unemployment benefits remain the same at the steady-state of post-reform economy so that searchunemployment goes down when the net wage rate rises. On the other hand, the firms open more job vacancies as higher capital accumulation increases firms' expected profits from a successful match. In turn, the tightness of labour market is reduced which can lead to the increase in employment. Thus, the long-run welfare gains for the workers are higher in the post-reform economy since working can generate higher labour income for them. Similar to the long-run welfare implications, the tax reform has higher welfare effects for all agents in the transition period. In other words, the lifetime welfare gains for capitalists are higher and welfare losses for workers are lower. The lifetime aggregate welfare improves. The net wage increases over time as the capital tax cuts have continuous effects on boosting capital accumulation. The unemployment benefits depend on past wages and their path follows the path of wage rate which causes the inertia in the increases in unemployment benefits during the transition. This tends to weaken the increase in searchunemployment which is beneficial to the workers. The welfare of workers is raised more quickly in the new, post-reform economy. To summarise, these imply that the long- and short-run welfare gains of tax reforms are higher for all agents by assuming unemployment benefits depending on past wages.

The rest of this paper is organised as follows: Section 2 describes the model structure. Section 3 discusses the calibration of the model to the current UK economy and gives the old, pre-reform steady-state. The results of tax reforms are analysed in Section 4. Section 5 studies an alternative specification of unemployment benefits and investigates the effects of tax

# 2 The model

#### 2.1 Economic environment

The features of the economy are summarized as follows. Infinitely lived households, homogenous firms, and a government populate the economy. Households are divided into capitalists and workers in terms of whether they can save or work. Only capitalists can save and own firms. Their income is comprised of interest income from physical capital and dividends of firms. Workers cannot save and consume all their disposable income in each period. The workers can engage in one of three activities: working to obtain wage income, searching for a job or enjoying leisure. If employed, they sell one unit labour endowment to only one firm at a time. The labour supply is thus indivisible. At any given period of time, the workers searching for jobs are randomly matched with job vacancies open by firms through a matching function. The search frictions can generate unemployment. Unemployment arises as the job seekers are not successful in their search for new employment. The unemployed workers receive unemployment benefits from the government. When there is a successful match, the matched worker and firm bargain over the wage rate to maximise a weighted average of worker's and firm's surpluses. Furthermore, we assume that two workers who are hired at different times must be paid the same wage at any given time. If the bargaining is successful, the worker will be employed by the firm in the following period. In this sense, employment at a given period of time is predetermined. It changes as unemployed workers get new jobs and employed workers separate from old jobs at an exogenous rate of separation. Following Andolfatto (1996), Merz (1995) and Pissarides (1998), we assume that workers are identical in the labour market. Individual employment risk is completely smoothed by using employment lotteries<sup>4</sup> and all workers have

<sup>&</sup>lt;sup>4</sup>The technique is introduced in models with indivisible labour, e.g. Hansen (1985) and Rogerson (1988).

equal employment and income.<sup>5</sup> Hence, one worker can be thought of as being endowed with one unit of time in each period, which can be split into: working, search and leisure. The firms produce final goods by employing capital and labour. As discussed above, since employment is predetermined, the firms open job vacancies at a constant resource cost to target their next-period employment. The government taxes interest income from physical capital, profits and labour income to finance its spending requirements.

# 2.2 Population composition

Total population is given by N which is exogenous and constant over time with the population of capitalists and workers respectively being denoted by  $N^k$  and  $N^w$ . The population shares of capitalists and workers are assumed to be:  $N^k/N \equiv n^k$ , and  $N^w/N \equiv n^w = 1 - n^k$ , respectively. We further assume that each capitalist owns one single firm, so that  $N^f = N^k$ .

# 2.3 Matching technology

As in the standard search-and-matching literature,<sup>6</sup> the matching technology is represented by the Cobb-Douglas function:<sup>7</sup>

$$M_t = mS_t^{\eta} V_t^{1-\eta} \tag{1}$$

where  $M_t$  is the new matches at t;  $S_t = N^w s_t$  denotes the aggregate number of workers who are looking for a job;  $V_t = N^f v_t$  denotes the aggregate number of job vacancies created by firms in the labour market; m > 0 represents the constant efficiency of matching;  $0 < \eta < 1$  denotes the elasticity of searches in the matching. In addition, we define the ratio:  $z_t = V_t/S_t$ , as the tightness of the labour market. The smaller the ratio of  $z_t$ , the tighter the labour market and therefore the harder for an unemployed worker to match with a

<sup>&</sup>lt;sup>5</sup>For a model with price taking firms and uninsured idiosyncratic employment risk due to incomplete financial markets and search frictions (see e.g. Krusell *et al.* (2010)).

<sup>&</sup>lt;sup>6</sup>See Pissarides (1986) and Blanchard and Diamond (1989).

<sup>&</sup>lt;sup>7</sup>Mortensen and Wright (2002) discuss results for increasing-returns-to-scale matching function and find that equilibria are unlikely to be efficient.

job vacancy.

The probability at which aggregate job searches lead to a new job match is given by:

$$p_t = \frac{M_t}{S_t} = mS_t^{\eta - 1}V_t^{1 - \eta} = mz_t^{1 - \eta}$$
 (2)

and its inverse,  $1/p_t$ , measures the duration of a search.

The probability at which a job vacancy can be matched with an unemployed worker is calculated by:

$$q_t = \frac{M_t}{V_t} = mS_t^{\eta} V_t^{-\eta} = mz_t^{-\eta}$$
 (3)

and its inverse,  $1/q_t$ , measures the duration of a job vacancy.

# 2.4 Utility function

The objective of household  $i \equiv k, w$ , is to maximise his discounted lifetime utility. A representative capitalist has preferences represented by the following lifetime utility function:

$$U_t^k = \sum_{t=0}^{\infty} \beta^t u_t^k \tag{4}$$

where  $0 < \beta < 1$  stands for the constant rate of time preference. The instantaneous utility function of the capitalist is given by:

$$u_t^k = \frac{\left(C_t^k\right)^{1-\sigma}}{1-\sigma} \tag{5}$$

where  $C_t^k$  is the capitalist's private consumption; and  $\sigma > 1$  is the coefficient of relative risk aversion.

The lifetime utility of a representative worker is as follows:

$$U_t^w = \sum_{t=0}^{\infty} \beta^t u_t^w \tag{6}$$

with the instantaneous utility function given by:

$$u_t^w = \frac{(C_t^w)^{1-\sigma}}{1-\sigma} - \xi \frac{(e_t + s_t)^{\mu}}{\mu} \tag{7}$$

where  $C_t^w$  is the worker's private consumption;  $e_t$  is the employment;  $s_t$  is the search activities of worker which effectively belongs to unemployment;  $e_t + s_t$  is therefore the labour force participation rate and  $s_t/(e_t + s_t)$  gives the unemployment rate;  $\xi > 0$  is a disutility parameter attached to the non-leisure activities;  $\frac{1}{\mu-1} > 0$  measures the wage elasticity of labour force participation. Different from the capitalist, exerting work or search effort in the labour market generates disutility for the worker.

# 2.5 Capitalists

The within-period budget constraint of each capitalist is described as:

$$C_{t}^{k} + I_{t}^{k} = r_{t} K_{t}^{k} - \tau_{t}^{k} (r_{t} - \delta) K_{t}^{k} + (1 - \tau_{t}^{k}) \pi_{t}^{k} + \overline{G}_{t}^{t}$$
(8)

where  $I_t^k$  is the capitalist's private investment;  $K_t^k$  is the physical capital held by the capitalist at the beginning of t;  $r_t$  is the gross return to physical capital;  $\pi_t^k$  is profits received from firms which are taxed at the same rate as interest income from savings;  $\overline{G}_t^t$  is per capita transfer from the government;  $0 \le \tau_t^k < 1$  is the tax rate on capital income; and  $0 < \delta < 1$  is the constant depreciation rate of physical capital.

The capital stock evolves according to:

$$K_{t+1}^{k} = (1 - \delta) K_{t}^{k} + I_{t}^{k}. \tag{9}$$

We then rewrite the budget constraint of the capitalist by making use of equation (9). This yields:

$$C_t^k + K_{t+1}^k = R_t K_t^k + (1 - \tau_t^k) \pi_t^k + \overline{G}_t^t.$$
 (10)

Here, we have defined a new variable,  $R_t = 1 - \delta + r_t - \tau_t^k (r_t - \delta)$ , as the

net return to physical capital after taxation and depreciation.

The capitalist's optimization problem is to choose  $\left\{C_t^k, K_{t+1}^k\right\}_{t=0}^{\infty}$  to maximise (4) subject to the constraint (10) taking the return to capital  $\left\{r_t\right\}_{t=0}^{\infty}$ , profits  $\left\{\pi_t^k\right\}_{t=0}^{\infty}$ , policy variables  $\left\{\tau_t^k, \overline{G}_t^t\right\}_{t=0}^{\infty}$  and the initial condition for  $K_0^k$  as given.<sup>8</sup>

#### 2.6 Workers

Each worker's within-period budget constraint is given by:

$$C_t^w = (1 - \tau_t^w) w_t e_t + \overline{G}_t^u s_t + \overline{G}_t^t$$
(11)

where  $w_t$  is the gross wage rate;  $0 \le \tau_t^w < 1$  is the tax rate on labour income; and  $\overline{G}_t^u$  denotes per capita unemployment benefits offered by the government. Following Pissarides (1998), Shi and Wen (1999) and Ardagna (2007), we assume that unemployment benefits are proportional to the wage rate, i.e.  $\overline{G}_t^u = \overline{r}_t w_t$ , where  $\overline{r}_t$  is defined as the replacement ratio. Unemployment benefits are less than the net wage rate, i.e.  $\overline{r}_t w_t < (1 - \tau_t^w) w_t$ , searching is costly to the worker.

Employment evolves according to:

$$e_{t+1} = p_t s_t + (1 - \gamma) e_t \tag{12}$$

where  $0 < \gamma < 1$  is the constant exogenous rate of job separation. The worker chooses  $\{C_t^w, s_t, e_{t+1}\}_{t=0}^{\infty}$  to maximise (6) subject to the constraints (11) and (12), taking the gross wage rate  $\{w_t\}_{t=0}^{\infty}$ , the matching probability  $\{p_t\}_{t=0}^{\infty}$ , policy variables  $\{\tau_t^w, \overline{\tau}_t, \overline{G}_t^t\}_{t=0}^{\infty}$  and the initial condition for  $e_0$  as given.

#### 2.7 Firms

A representative firm produces the final goods with a constant-returns-to-scale technology in two productive inputs: capital,  $K_t^f$ , and labour,  $L_t^f$ . The

<sup>&</sup>lt;sup>8</sup>The utility-maximization of the capitalist is given in the Appendix 7.1.

<sup>&</sup>lt;sup>9</sup>The utility-maximization of the worker is given in the Appendix 7.2.

production function is given by:

$$Y_t^f = A \left( K_t^f \right)^{\alpha} \left( L_t^f \right)^{1-\alpha} \tag{13}$$

where  $0 < \alpha, 1 - \alpha < 1$  denote the constant output elasticities of capital and labour, respectively.

Two remarks follow. First, employment is pre-determined at any given period of time prior to the production taking place. The firm takes the number of workers currently employed as given. It opens new vacancies in order to employ the desired number of workers next period. There is an exogenous resource cost from creating a new vacancy. The firm also needs to decide on the size of the capital stock that it needs for production. Second, the firm can earn positive profits. This is because the firm can influence its future employment by controlling the currently created job vacancies. This forward-looking decision making results in a marginal product of labour which is higher than the marginal cost of labour, in other words, the gross wage, so that the search frictions result in positive profits in the product markets.

The job transition function which links the future number of filled jobs to the net hiring plus the current stock of filled jobs is given by:

$$L_{t+1}^{f} = q_t v_t + (1 - \gamma) L_t^f \tag{14}$$

where the old jobs dissolve at a constant rate of  $0 < \gamma < 1$ .

The profits function of the firm is given by:

$$\pi_t^f = Y_t^f - r_t K_t^f - w_t L_t^f - \nu v_t \tag{15}$$

where  $\nu > 0$  stands for the constant resource cost of opening a new vacancy.

It is worth noting that the profit-maximisation problem of th firm is intertemporal, in the sense that the firm can influence its future employment by choosing the number of vacancies created in contemporaneous period. The firm takes the factor prices  $\{r_t, w_t\}_{t=0}^{\infty}$ , the matching probability  $\{q_t\}_{t=0}^{\infty}$  and an initial condition for  $L_0^f$  as given, and chooses  $\left\{K_t^f, v_t, L_{t+1}^f\right\}_{t=0}^{\infty}$  to

maximise the present value of a stream of profits:

$$\sum_{t=0}^{\infty} \prod_{i=0}^{t} R_i^{-1} \pi_t^f \tag{16}$$

subject to the constraints (13), (14), and (15).<sup>10</sup>

The profit-maximising conditions of the firm can be summarized by two equations in the following:

$$r_t = \alpha \frac{Y_t^f}{K_t^f} \tag{17}$$

which states that the rate of return on capital is equal to the marginal product of capital.

The second condition is given by:

$$\frac{\nu}{q_t} = R_{t+1}^{-1} \left[ (1 - \alpha) \frac{Y_{t+1}^f}{L_{t+1}^f} - w_{t+1} + \frac{\nu(1 - \gamma)}{q_{t+1}} \right]$$
(18)

which states that the average vacancy creation costs of a successful match at time t are equal to the discounted expected value of profits brought about at time t + 1.

# 2.8 Wage determination

The wage rate is determined by a Nash bargaining between a pair of matched worker and firm with the aim of maximising the weighted product of their surpluses resulting from a successful match. The worker's utility can be increased by  $(1 - \tau_t^w) w_t u_{1,t}^w - u_{2,t}^w$  if he is employed.<sup>11</sup> We rewrite the worker's surplus as  $w_t - \frac{u_{2,t}^w}{(1-\tau_t^w)u_{1,t}^w}$  by separating out the wage rate,  $w_t$ . The quantity,  $\frac{u_{2,t}^w}{(1-\tau_t^w)u_{1,t}^w}$ , is then interpreted as the worker's reservation wage. By employing one additional unit of labour, the firm can increase its profits by,  $Y_{2,t}^f - w_t$ ,

<sup>&</sup>lt;sup>10</sup>The profit-maximization of the firm and the derivation of its expected profits are provided in the Appendix 7.3 and 7.4, respectively.

<sup>&</sup>lt;sup>11</sup>We assume that an unemployed worker is not entitled to the unemployment benefits if he does not accept a potential job. If this is the case, the worker leaves the labour force and he is then counted as part of  $(1 - e_t - s_t)$ .

which is the firm's marginal profitability from hiring. To summarise, the worker and the firm bargain over the wage rate to maximise the following weighted average of surpluses:

$$\left[w_t - \frac{u_{2,t}^w}{(1 - \tau_t^w) u_{1,t}^w}\right]^{\phi} \left[Y_{2,t}^f - w_t\right]^{1-\phi}$$
(19)

where  $0 \le \phi \le 1$  represents the constant relative bargaining power of the worker.

The wage rate after a successful bargaining is given by: 12

$$w_t = (1 - \phi) \frac{u_{2,t}^w}{(1 - \tau_t^w) u_{1,t}^w} + \phi Y_{2,t}^f.$$
 (20)

It shows that the wage rate is a weighted average of the reservation wage (the outside factor) and labour productivity (the inside factor). Since we do not study heterogeneity of labour, all the workers who are hired will receive the same wage from the firms at any given time. It further implies that we work with a symmetric equilibrium.

As in Domeij (2005) and Arseneau and Chugh (2008), we assume that the households cannot affect the reservation wage via the marginal rate of substitution between consumption and leisure. This assumption is standard in the literature. As such, only the labour tax changes affect the reservation wage and therefore the equilibrium wage rate.

There exist externalities in the matching models which arise from the fact that one additional job seeker can increase the probability that a firm is matched with a worker, i.e. a positive externality, but decrease the probability of a job seeker already existing in the markets to be matched with a firm, i.e. a negative externality. As pointed out by Mortensen (1982) and Hosios (1990), the workers and firms ignore the externalities created by their choices and therefore search inefficiency arises. But in a model with no distortionary taxes, Hosios (1990) shows that two opposite externalities will be balanced and the search inefficiency vanishes when the elasticity of searches

<sup>&</sup>lt;sup>12</sup>The derivation of the wage rate in Nash bargain is given in the Appendix 7.5.

in the matching is equal to the relative bargaining power of the workers, i.e.  $\eta = \phi$ .

# 2.9 Government and market clearing conditions

The per-capita government budget constraint equating spending and revenues is given by:

$$\overline{G}_t^t + n^w \overline{G}_t^u s_t = n^k \tau_t^k (r_t - \delta) K_t^k + n^k \tau_t^k \pi_t^k + n^w \tau_t^w w_t e_t.$$
(21)

To ensure that the government budget is balanced in each period, we allow the labour income tax,  $\tau_t^w$ , to be residually determined.

The capital markets clear when the supply is equal to the demand for capital per capita:

$$K_t^k = K_t^f. (22)$$

All the profits of firms are equally distributed to the capitalists which gives the following per capita market clearing condition for the dividends:

$$\pi_t^k = \pi_t^f. (23)$$

In the labour markets, the equality of per capita labour supply and demand is given by:

$$n^w e_t = n^k L_t^f. (24)$$

Finally, in the goods markets, the economy's per capita resource constraint is satisfied:

$$n^{k}Y_{t}^{f} = n^{k}C_{t}^{k} + n^{w}C_{t}^{w} + n^{k}\left(K_{t+1}^{k} - (1 - \delta)K_{t}^{k}\right) + n^{k}\nu\nu_{t}.$$
 (25)

# 2.10 Decentralized equilibrium (given policy)

We summarize the decentralized equilibrium conditions in real terms in the following. Given the paths of prices  $\{w_t, r_t\}_{t=0}^{\infty}$ , the paths of policy instruments  $\{\tau_t^k, \overline{r}_t, \overline{G}_t^t\}_{t=0}^{\infty}$  and initial conditions for  $K_0^k$  and  $e_0$ , a decentralized

equilibrium (DE) is defined to be an allocation  $\{C_t^k, K_{t+1}^k, C_t^w, s_t, e_{t+1}, v_t\}_{t=0}^{\infty}$  and one residually determined policy instrument,  $\tau_t^w$ , such that (i) capitalists, workers, and firms undertake their respective optimization problems outlined above; (ii) wage rate is determined by a Nash bargaining between a pair of matched worker and firm; (iii) all budget constraints are satisfied; and (iv) all markets clear.

Thus, the DE consists of the capitalist's optimality conditions for  $C_t^k$  and  $K_{t+1}^k$ ; the worker's optimality conditions for  $C_t^w$ ,  $s_t$  and  $e_{t+1}$ ; the firm's first-order conditions for  $K_t^f$ ,  $v_t$  and  $L_{t+1}^f$ ; the optimality condition for the wage rate in the Nash-bargain,  $w_t$ ; the evolution of employment,  $e_t$ ; the budget constraints of worker and government, i.e.  $BC^w$  and  $BC^g$ ; the aggregate resource constraint, RC; and the market clearing conditions in the capital, dividends and labour markets, i.e.  $MC_K$ ,  $MC_{\pi}$  and  $MC_L$ .<sup>13</sup>

# 3 Calibration and steady-state solution

The structural parameters of the model is next calibrated so that the model's steady-state solution reflects the main empirical characteristics of the UK economy, particularly the features of its labour market. Table 1 below reports the structural parameters in the model.

Preferences: Time is measured in quarters. The rate of time preference is set to 0.99 which is standard in the real business cycle literature for quarterly calibration. In the utility function, we use a value for  $\sigma$  that is common in DGE literature, i.e.  $\sigma=2$ . The value of  $\mu$  is set at 5 to obtain the labour participation elasticity of  $\frac{1}{\mu-1}=0.25$ . This elasticity value falls in the range in Killingsworth (1983). The value of  $\xi$  is calibrated to get the labour participation rate of 63%.<sup>14</sup>

Production: We use a standard value for the capital productivity parameter, i.e.  $\alpha = 0.35$ . The quarterly depreciation rate of physical capital is 2.5%. These two parameters imply a realistic steady-state capital-to-output

<sup>&</sup>lt;sup>13</sup>Note that relying on Walras's law, we drop the budget constraint of the capitalist from the DE.

<sup>&</sup>lt;sup>14</sup>See Schweitzer and Tinsley (2004) for the empirical evidence.

ratio of 8.12 on a quarterly basis. The unit cost of opening a vacancy of  $\nu$  is calibrated to get the steady-state unemployment rate of 7%. The unemployment rate corresponds to the data average from 1970 to 2010 from the UK Office for National Statistics (ONS). The productivity parameter, A, is normalized to 1.

Table 1: Calibration (pre-reform)

	(F)	
Parameter	Definition	Value
$0 < \beta < 1$	rate of time preference	0.990
$\sigma > 1$	relative risk aversion	2.000
$\mu > 1$	elasticity parameter in utility	5.000
$0 < \delta < 1$	depreciation rate on capital	0.025
$\xi > 0$	utility parameter	5.900
$0 < \gamma < 1$	job separation rate	0.050
$0 < \alpha < 1$	capital's share	0.350
$\nu > 0$	job posting cost	0.628
$0 \le \phi \le 1$	worker's bargaining power	0.500
m > 0	efficiency of matching	0.699
$0 < \eta < 1$	elasticity of unemployment	0.500
$0 < n^k < 1$	population share of capitalists	0.115
A > 0	TFP level	1.000
$0 \leq \tau^k < 1$	tax rate on capital income	0.442
$0 < \overline{r} < 1$	replacement ratio	0.500
$\overline{G}^t > 0$	per capita government transfer	0.342

Labour market: We assume that the capitalists do not work in the model. But they can save in the form of capital, own firms and receive profits. Following Ardagna (2007), we treat the self-employed as capitalists to calibrate the population share of capitalists. The data of self-employment becomes available from 1992 for the UK economy in the Labour Force Statistics (LFS) database. The population shares of capitalists is set to the data averages of 0.115. With regards to the bargaining process, we set the worker's bargaining power to 0.5 which features a symmetric Nash bargaining solution. In later

tax policy analysis, we allow this parameter to take a range of different values as a robustness test, i.e.  $\phi \in [0.25, 0.375, 0.5, 0.625, 0.75]$ . The elasticity of unemployment in the matching function is set to 0.5. The exogenous job separation rate  $\gamma$  is set to 0.05 as in e.g. Shi and Wen (1997 and 1999) and Domeij (2005). The inverse of  $\gamma$  gives the average duration of a job. Our calibration implies that the average duration of a job is five years which is consistent with the data average from 1992 to 2010 from OECD. Stat database. The matching technology is represented by a homogenous of degree one function and characterized by the efficiency parameter, m. We calibrate, m, to obtain the duration of unemployment of 4.5 months at the steady-state when the tightness of labour market is 0.9. The unit cost of creating a vacancy yields the duration of a job vacancy of 4 months, similar to Pissarides (2006).

Policy instruments: Effective average tax rates for capital and labour income from 1970 to 2005 are constructed by following the approach in Conesa et.al. (2007), i.e.  $\tau^k = 0.442$ , and  $\tau^w = 0.27$ . We then calibrate the per capita government transfers,  $\overline{G}_t^t$ , to obtain the steady-state  $\tau^w$  of 0.27. The replacement ratio is set to 0.5 which is comparable with the data for industrialised countries (see e.g. Nickell and Nunziata (2001)) and between the values used in previous studies, ranging from 0.45 (Shi and Wen (1999)) to 0.6 (Pissarides (1998)).

The parameters imply the steady-state solution which is reported in Column (1) of Table 2 below. The net returns to labour and capital,  $\widetilde{w} = (1 - \tau^w) w$  and  $\widetilde{r} = (1 - \tau^k) (r - \delta)$  are useful for the policy analysis which follows. The steady-state disposable income of capitalists and workers is given by  $Y^k$  and  $Y^w$ , respectively. The lifetime welfare of agent is obtained using the formula  $U^i_{ss} = \frac{(1-\beta^T)}{1-\beta}u^i$ , for i = k, w, where  $u^i$  is the welfare of i calculated at the steady-state using (5) and (7) and T = 1000. The aggregate or social welfare,  $U_{ss}$ , is defined in the Benthamite fashion as the average welfare of all agents in the economy.<sup>15</sup>

The steady-state solution for the above parameterisation implies the following shares of public spending in GDP:  $\frac{G^t}{Y} = 0.213$  and  $\frac{G^u}{Y} = 0.024$ , which

The aggregate welfare is given by:  $U_{ss} = n^k U_{ss}^k + n^w U_{ss}^w$ .

further implies that the government spending in transfers is about 23.7% of GDP consistent with UK data from the OECD.Stat database.

# 4 Distributional effects of tax reforms

# 4.1 The long-run effects

We first examine the long-run effects of reducing the tax burden on capital income in the model. In all cases, we find that the effects of capital tax reductions are monotonic and increase with the magnitude of the capital tax cut. Hence, we particularly analyse the effects of abolishing capital taxation which is associated with a concurrent increase in labour income tax rate,  $\tau^w$ , to generate the required tax revenues to finance public spending. The zero capital tax has been intensively examined in the optimal taxation literature. We evaluate the effects of the tax reform by comparing the post- with the pre-reform steady-states with main focuses on the labour market and the distribution of welfare. The steady-state allocations together with welfare of agents after the tax reform are shown in Column (2) of Table 2.

As can been seen in Table 2, the implementation of a zero capital income tax will be Pareto improving in the long-run (see Us), even if it increases inequality (see  $\frac{Y^k}{Y^w}$ ). In other words, all the agents are better off after the tax reform, although the gains for the capitalists compared to the workers are higher. This is consistent with Judd's (1985) results that it is optimal for both capitalists and workers to choose a zero capital tax in the long-run.

The trade-off for the workers after implementing the zero capital tax can be seen by noting that, the labour tax,  $\tau^w$ , increases (i.e. from 0.27 to 0.343) to make up for the tax revenue losses, due to the elimination of the capital tax. In turn, the workers' reservation wage,  $\frac{u_{2,t}^w}{(1-\tau_t^w)u_{1,t}^w}$ , increases. Meanwhile, the labour productivity,  $Y_{2,t}^f$ , increases which induced by higher capital accumulation. Both changes intend to increase the wage rate which can be seen in the optimality condition in the bargaining (20). Therefore, the bargained wage rate, w, increases at new, post-reform steady-state (i.e. from 1.956 to 2.195). The net return to labour increases as well (i.e. from

1.428 to 1.441), since the increase in the wage rate outweighs the increase in the labour tax.

Table 2: Long-run effects of tax reform  $(\overline{G}_t^u = \overline{r}_t w_t)$ 

	Pre-reform	Post-reform
	(1)	(2)
$\frac{C^k}{Y}$	0.108	0.125
$\frac{C^w}{Y}$	0.675	0.614
$\frac{C}{Y}$	0.783	0.739
$\frac{Y^k}{Y^w}$	3.547	4.691
$\frac{C^w}{Y}$ $\frac{C}{Y}$ $\frac{Y^k}{Y^w}$ $\frac{I^k}{Y}$ $\frac{K^k}{Y}$ $\frac{G^t}{Y}$	0.203	0.249
$\frac{K^k}{Y}$	8.120	9.971
$\frac{G^t}{Y}$	0.213	0.192
$\frac{G^u}{Y}$	0.024	0.026
e	0.586	0.582
s	0.044	0.048
v	0.306	0.279
$\tau^w$	0.270	0.343
w	1.956	2.195
$\widetilde{w}$	1.428	1.441
r	0.043	0.035
$\widetilde{r}$	0.010	0.010
z	0.900	0.759
p	0.663	0.609
q	0.737	0.802
$U^k$	-66.392	-51.725
$U^w$	-93.563	-92.779
U	-90.439	-88.058

The increase in the wage rate leads to an increase in unemployment benefits. This is because unemployment benefits are proportional to the wage rate, i.e.  $\overline{G}^u = \overline{r}w$ . One one hand, the number of workers looking for jobs at the steady-state, s, increases (i.e. from 0.044 to 0.048). On the other

hand, the increase in the wage rate reduces firms' expected profits from a successful match, despite the higher labour productivity due to higher capital accumulation. Thus, the firms reduce the number of vacancies open for unemployed workers (i.e. from 0.306 to 0.279). These changes, in turn, imply a tighter labour market which can be seen that z falls from 0.9 to 0.759. A tighter labour market implies a lower probability for an unemployed worker to match with a job vacancy (i.e. from 0.663 to 0.609), and a higher probability at which a job vacancy can be matched with an unemployed worker (i.e. from 0.7371 to 0.802). According to the employment evolution equation (12), steady-state employment falls (i.e. from 0.586 to 0.582). The income, consumption and welfare of workers rise resulting from two positive effects. On one hand, unemployment benefits are higher resulting from the increasing wage rate so that income from search is higher. On the other hand, the increased net wage rate raises the income from working. The tax reform can also benefit the capitalists since the elimination of capital tax boosts investment and capital. The pre- and post- reform investment-to- and capital-tooutput ratios are (0.203, 8.120) and (0.249, 9.971), respectively. As a result, the income, consumption and welfare of capitalists increase. Thus, all agents benefit from the reform that implements a zero capital tax in the long-run. Capitalists directly benefit from the zero capital tax and also the increased capital. The capital income effect is bigger than the labour income effect. Hence, capitalists benefit more from this tax reform and inequality increases despite the Pareto superiority of the reform.

#### 4.2 The transitional effects

In contrast to the above steady-state analysis, we now investigate the welfare effects of the tax reform in the transition period. The literature suggests that during the transition period, capital tax cuts met by labour tax increases will hurt the agents whose income reply on labour income, even if there are benefits to them in the long-run (see e.g. Garcia-Milà et al. (2010) and Angelopoulos et al. (2011)). To assess the implications of the transition period in our model, in Table 3 as follows, we present compensating

consumption supplement,  $\zeta^i$  to measure relevant welfare gains/costs in the post-reform economy for each type of agent.<sup>16</sup> It can capture the importance of the timing of the benefits and costs of eliminating the capital tax.

In contrast to our findings in Table 2, the results in Table 3 suggest that there are welfare losses for the workers and at the aggregate level over the lifetime, although this tax reform is Pareto improving in the long run. It predicts that the elimination of capital tax will hurt the workers and also worsen social welfare during the transition.

Table 3: Welfare effects of tax reform  $(\overline{G}_t^u = \overline{r}_t w_t)$ 

$\zeta_{ss}^k$	0.221
$\zeta^w_{ss}$	0.008
$\zeta_{ss}$	0.032
$\zeta^k_{lt}$	0.174
$\zeta^w_{lt}$	-0.028
$\zeta_{lt}$	-0.004

To understand the underlying transmission mechanism driving these results, we first plot the transitional paths of variables to the new steady state. We assume that, at initial period, the capital tax unexpectedly and permanently shifts from 0.442 to 0. In response to the permanent policy change, the responses of variables are illustrated in Figure 1 as follows. In other words, Figures 1 shows how the economy gradually coverages to the new steady-state. These paths are generated by simulating the model as it converges to the new, post-reform steady-state, starting from the pre-reform steady-state (see e.g. Giannitsarou (2006)).

#### [Figure 1 about here]

Table 4 further presents the effects of the zero capital tax on the key variables in the short-run, i.e. 1, 10, 20, 50 and 100 periods after the zero

<sup>&</sup>lt;sup>16</sup>In particular, this has been obtained using the formula  $\left(\frac{U_A^i}{U_B^i}\right)^{\frac{1}{1-\sigma_1}} - 1$ , where  $U_A^i$  and  $U_B^i$  is welfare post- and pre-reform, respectively.

capital tax has been implemented, as well as the long-run, i.e. 200 periods after the reform and also at the new steady-state.

Table 4: Transitional effects of zero  $\tau^k$   $(\overline{G}_t^u = \overline{r}_t w_t)$ 

	Pre-reform			P	ost_reform	n		
	Old SS	Period 1	10	20	50	100	200	New SS
$C^k$	1.506	1.633	1.679	1.723	1.815	1.889	1.927	1.933
$C^w$	1.222	1.106	1.126	1.144	1.184	1.215	1.231	1.233
$K^k$	113.124	113.877	119.711	125.315	137.554	147.764	153.180	154.044
$ au^w$	0.270	0.384	0.378	0.371	0.358	0.349	0.344	0.343
w	1.956	1.973	2.011	2.043	2.110	2.164	2.191	2.195
$\widetilde{w}$	1.428	1.223	1.256	1.288	1.355	1.409	1.437	1.441
e	0.586	0.583	0.582	0.582	0.582	0.582	0.582	0.582
s	0.044	0.056	0.055	0.054	0.051	0.049	0.048	0.048
v	0.306	0.228	0.240	0.247	0.261	0.272	0.278	0.279
z	0.900	0.532	0.563	0.594	0.664	0.722	0.754	0.759
p	0.663	0.510	0.524	0.539	0.569	0.594	0.607	0.609
q	0.737	0.959	0.931	0.907	0.858	0.822	0.805	0.802

First, we note that the labour tax initially goes above its new, post-reform steady-state (i.e. 0.384 versus 0.343). This intends to increase the bargained wage rate via its positive effect on the reservation wage as discussed above. However, the labour productivity does not increase much initially, as the higher capital accumulation due to elimination of capital tax is not yet realized at first which weakens the increase in wage rate. These two effects, on the whole, result in slightly rising wage rate for the first period (i.e. from 1.956 to 1.973). This, in turn, leads to higher unemployment benefits. The net wage rate falls short of its old, pre-reform steady-state (i.e. from 1.428 to 1.223), as the initial increase in labour tax exceeds the increase in wage rate. The search-unemployment overshoots its post-reform steady-state (i.e. 0.056 versus 0.048). This result is driven by the higher unemployment benefits and lower net wage rate in the post-shock economy. The firms cut job vacancies in the short-run, since prior to the new steady-state, the positive higher capital

accumulation effect on labour productivity is not strong enough to outweigh the negative profits effect induced by rising wage rate. As capital accumulates and this is transformed into higher labour productivity, the firms begin to open more job vacancies over time, although the number is less than the old, pre-reform steady-state. Labour market tightness is increasing as search-unemployment falls and the number of vacancies increases over time. As can be seen in Figure 1, employment at the first period falls, but remains almost unchanged over time. This is because the increase in p and decrease in s effectively net out over time, which leaves no effect on employment. The combination of lower net wage rates and higher search-unemployment creates short-run losses for the workers and also aggregate welfare worsens, which are reversed in the long run, similar to Domeij and Heathcote (2004).<sup>17</sup>

# 4.3 Changes in bargaining power of workers

As have discussed earlier, the choice of worker's bargaining power,  $\phi$ , is crucial in the models with search frictions due to the existence of externalities. We now illustrate the degree to which our results are robust to variations in this parameter and examine the importance of worker's bargaining power on the welfare effects of elimination of capital income tax. Our calibration above is based on the Hosios condition,  $\eta = \phi$ . In what follows, we examine changes in  $\phi$  that encompass the entire range used in the literature, see e.g. Domeij (2005). In Table 5, for each value of  $\phi$ , we report the differences in the long run between the pre- and post-reform steady-state for the key economic variables.<sup>18</sup> We also report the compensating consumption supplement for

<sup>&</sup>lt;sup>17</sup>They find that in the heterogeneous agent economy capital tax cuts are supported only by a minority of households during the transition.

<sup>&</sup>lt;sup>18</sup>Note that for all cases considered, the parameters,  $\nu$  and  $\overline{G}^t$  are re-calibrated so that the base in all cases is an economy with 7% unemployment and 27% labour tax rate. The remaining parameters used are as in Table 1.

each agent and the aggregate economy at the steady-state.

Table 5: Changes in worker's bargaining power for tax reforms (difference from pre-reform policy)

			*		
	0.250	0.375	0.500	0.625	0.750
$ \%\Delta C^k$	0.2926	0.2868	0.2836	0.2814	0.2800
$\%\Delta C^w$	0.0099	0.0092	0.0090	0.0089	0.0089
$\%\Delta\left(\frac{C^k}{Y}\right)$	0.1665	0.1607	0.1575	0.1553	0.1539
$\%\Delta\left(\frac{C^w}{Y}\right)$	-0.0886	-0.0897	-0.0901	-0.0903	-0.0905
$\%\Delta\left(\frac{Y^k}{Y^w}\right)$	0.3236	0.3232	0.3227	0.3222	0.3219
$\%\Delta K^k$	0.3606	0.3614	0.3617	0.3620	0.3621
$\Delta  au^w$	0.0791	0.0753	0.0734	0.0721	0.0713
$\Delta w$	0.1332	0.1257	0.1221	0.1200	0.1186
$\Delta\widetilde{w}$	0.0103	0.0095	0.0093	0.0093	0.0093
$\zeta^k_{ss}$	0.2264	0.2229	0.2209	0.2196	0.2187
$\zeta^w_{ss}$	0.0092	0.0086	0.0084	0.0083	0.0083
$\zeta_{ss}$	0.0341	0.0332	0.0328	0.0326	0.0325

As can be seen in Table 5, the welfare gains for all agents from elimination of the capital tax are decreasing in  $\phi$ . In other words, for fixed  $\eta$ , increasing the workers' bargaining power makes the tax reform less efficient in terms of welfare improving. As discussed above, the wage rate is rising after the tax reform via the higher labour productivity and concurrent increase in labour tax channel. The results in Table 5 show that, as  $\phi$  increases, the tax reform has a smaller effect on labour tax but bigger effect on labour productivity due to higher capital accumulation. As the relative bargaining power of worker increases, the importance of the increase in labour tax has been improved relative to the labour productivity. Hence, the tax reform exerts a smaller effect on wage rate and in turn on unemployment benefits. The effect on labour tax and wage rate. Therefore, given that the effects on net wage rate and unemployment benefits have been reduced, the income, consumption and welfare for the workers increase by less as  $\phi$  goes up.

As  $\phi$  increases, the job posting cost needs to fall in the pre-reform economy such that the new calibration can yield the same unemployment rate as in the base case. This implies that the costs of posting vacancies have been reduced. As a result, the expected profits of firms are getting bigger and the firms increase production. It implies that the tax reform has a bigger effect on boosting investment of capitalists and therefore a smaller effect on consumption increase. The welfare gains for the capitalists become smaller as  $\phi$  goes up. Finally, it is worth noting that inequality improves as can be seen from decreasing relative income of capitalists and workers,  $\frac{Y^k}{Y^w}$ . It indicates that increasing the workers' bargaining power can help to reduce the income gap between capitalists and workers.

# 5 Alternative specification of unemployment benefits

In this section, we employ an alternative specification of unemployment benefits. Pissarides (1998) and Koskela and von Thadden (2008) have discussed the importance of the specification of unemployment benefits in the wage bargaining. We now assume that unemployment benefits depend on past wages due to some institutional features in the labour market, see e.g. Blanchard and Katz (1999) and Chéron and Langot (2010). Thus, unemployment benefit,  $\overline{G}_t^u$ , is specified as follows:

$$\overline{G}_t^u = \left(\frac{z}{w}\right) w_{t-1} \tag{26}$$

where w is the steady-state wage rate. As can be seen, unemployment benefits are proportional to past wages by the constant  $\frac{z}{w} > 0$  in the transition period. However, in the steady-state, they are constant and equal to z > 0. When the wage rate rises after the tax reform, unemployment benefits remain the same. Hence, this new specification of unemployment benefits is important in determining both the long- and short-run results of the tax reforms.

The parameter, z, is re-calibrated to obtain the steady-state  $\tau^w$  of 0.27. All the other parameters are used as in Table 1.

# 5.1 The long-run effects of tax reforms

We first examine the importance of this new specification of unemployment benefits in determining the long-run effects of the tax reforms. Column (1) and Column (2) of Table 6 present the pre- and post-reform steady-states, respectively. We report the steady-state allocations and welfare of agents.

The results in Table 6 show that there are welfare gains for all agents in the long run if the government chooses a zero capital tax and increases labour tax to make up for the tax revenue losses although this tax reform increases inequity. It implies that the tax reform is still Pareto improving despite increasing income difference (see  $\frac{Y^k}{Y^w}$ ). In the new unemployment benefits setup, the tax reform has different effects on labour market which can be seen from the changes in labour market variables. The unemployment benefits remain the same in the post-reform economy. In this case, the searchunemployment only depends on net wage rate. As discussed before, the net wage increases after the tax reform and therefore search-unemployment falls (i.e. from 0.044 to 0.041). The firms create more vacancies at the post-reform steady-state (i.e. from 0.306 to 0.334). This is because, on one hand, the increase in wage rate is relatively smaller (i.e. from 1.956 to 2.195 versus from 1.956 to 2.187), which implies smaller negative revenue effect. On the other hand, the higher in labour productivity due to higher capital accumulation increases the firms' expected profits from a successful match. The production is more profitable at the post-reform steady-state which can be seen in equation (18) since the average creation costs are lower. The labour market tightness is rising (i.e. from 0.9 to 1.064) when v increases and s decreases. Therefore, the probability at which unemployed workers can be matched with job vacancies increases (i.e. from 0.663 to 0.721), and the probability at which job vacancies can be matched with job seekers decreases (i.e. from 0.737 to 0.677). In turn, employment goes up at the post-reform steady-state (i.e. from 0.586 to 0.588). This creates one additional channel for the increases in income, consumption, and welfare of workers as working can generate higher income relative to searching. The steady-state welfare gains for workers are therefore when unemployment benefits are assumed to depend on past wages.

Table 6: Long-run effects of tax reform  $(\overline{G}_t^u = \left(\frac{z}{w}\right) w_{t-1})$ 

	Pre-reform	Post-reform
	(1)	(2)
$\frac{C^k}{Y}$	0.108	0.125
	0.675	0.612
$\frac{C}{Y}$	0.783	0.737
$\frac{Y^k}{Y^w}$	3.547	4.711
$\frac{I^k}{Y}$	0.203	0.249
$\frac{C^w}{Y}$ $\frac{C}{Y}$ $\frac{Y^k}{Y^w}$ $\frac{I^k}{Y}$ $\frac{K^k}{Y}$ $\frac{G^t}{Y}$	8.120	9.971
$\frac{G^t}{Y}$	0.213	0.190
$\frac{G^u}{Y}$	0.024	0.020
e	0.586	0.588
s	0.044	0.041
v	0.306	0.334
$\tau^w$	0.270	0.331
w	1.956	2.187
$\widetilde{w}$	1.428	1.462
r	0.043	0.035
$\widetilde{r}$	0.010	0.101
z	0.900	1.064
p	0.663	0.721
q	0.737	0.677
$U^k$	-66.392	-51.093
$U^w$	-93.563	-92.142
U	-90.439	-87.421

As explained before, a zero capital tax boosts investment and capital.

The income, consumption and welfare of capitalists increase at the post-reform steady-state. Furthermore, the capitalists can gain more from the tax reform due to the increase in firms' profits in the production. To summarise, if unemployment benefits depend on past wages, the tax cuts met by the labour tax increases can result in higher welfare gains for all agents and the tax reform is still Pareto improving in the long run. Hence, the formation of unemployment benefits only influences the magnitude of steady-state welfare effects of tax reforms. But as discussed above, the tax reforms have different effects on labour market variables so that the mechanism driving the results is different.

#### 5.2 The transitional effects of tax reforms

We then analyse how the results will change during the transition period. We report the same variables in Table 7 in order to compare with those in Table 3.

Table 7: Welfare effects of zero  $\tau^k$   $(\overline{G}_t^u = \left(\frac{z}{w}\right) w_{t-1})$ 

$\zeta_{ss}^k$	0.230
$\zeta^w_{ss}$	0.015
$\zeta_{ss}$	0.040
$\zeta^k_{lt}$	0.186
$\zeta^w_{lt}$	-0.019
$\zeta_{lt}$	0.005

As can be seen, our main result that the capital tax cuts will hurt the agents whose income reply on labour income during the transition period stands in the model with new specification of unemployment benefits. As in the long run, the tax reform has higher welfare effects for all agents in transition period. It is worth noting that the aggregate welfare losses turn into the welfare gains over the lifetime. Our results show that the tax reforms imply short-run welfare losses only for the workers, similar to Ardagna (2007).<sup>19</sup>

 $<sup>^{19}</sup>$ She employs a model with unionised labour market to examine exogenous changes in

To understand what drives these results, we evaluate the transitional dynamics between steady-states. Table 8 reports the effects of tax reform in the short-run and Figure 2 plots the transitional paths of variables.

Table 8: Transitional effects of zero  $\tau^k$   $(\overline{G}_t^u = (\frac{z}{w}) w_{t-1})$ 

	D C					(w)	-	
	Pre-reform	Post_reform						
	Old SS	Period 1	10	20	50	100	200	New SS
$C^k$	1.506	1.644	1.693	1.739	1.835	1.912	1.951	1.957
$C^w$	1.222	1.110	1.132	1.151	1.192	1.224	1.240	1.242
$K^k$	113.124	113.809	119.927	125.829	138.688	149.328	154.870	155.723
$ au^w$	0.270	0.372	0.364	0.358	0.345	0.336	0.332	0.331
w	1.956	1.958	1.994	2.028	2.099	2.154	2.182	2.187
$\widetilde{w}$	1.428	1.237	1.272	1.305	1.375	1.430	1.458	1.462
e	0.586	0.590	0.590	0.590	0.589	0.589	0.588	0.588
s	0.044	0.049	0.047	0.046	0.043	0.042	0.041	0.041
v	0.306	0.288	0.291	0.299	0.314	0.327	0.333	0.334
z	0.900	0.761	0.805	0.847	0.940	1.017	1.058	1.064
p	0.663	0.609	0.627	0.643	0.678	0.705	0.719	0.721
q	0.737	0.801	0.779	0.759	0.721	0.693	0.679	0.677

The tax reform has an immediate effect on boosting capital accumulation in this model for the first period (from 113.124 to 113.809). This is transformed into higher labour productivity and thus higher wage rate as can be seen in the wage condition (20). The wage rate is rising over time since capital accumulation is increasing which results in increasing labour productivity. The net wage,  $\tilde{w}$ , initially falls short of the old, pre-reform steady-state, due to the large increase in labour tax as discussed before. Thus, the search-unemployment at the first period overshoots its old steady-state. The net wage is rising over time which causes the decrease in search-unemployment. Besides, unemployment benefits depend on past wages and their path follows the path of wage rate which causes the inertia in the increases in un-

fiscal instruments accommodated by changes in government debt and finds that workers' welfare goes down after the increase in labour tax.

employment benefits in the transition. This tends to weaken the increase in search-unemployment. As a result, the increase in net wage dominates and search-unemployment falls during the transition. The firms cut job vacancies at the first period (i.e. from 0.306 to 0.288) as the positive profits effect due to the higher capital accumulation is not yet realized and the increase in wage rate makes the production less profitable. As more capital is built up, the firms open more job vacancies in the labour market. The labour market tightness is reduced for the first period (i.e. from 0.9 to 0.761) due to less available vacancies and more search-unemployment. In turn, employment increases for the first period (i.e. from 0.586 to 0.590), but there are small fluctuations of employment over time, which can be seen in Figure 2.

#### [Figure 2 about here]

We see in Figure 2, that the tax cuts have larger effects on increasing capital accumulation and raising the net wage rate during the transition. As a result, the income, consumption and welfare of capitalists increase by more over time, and the income, consumption, and welfare of workers is raised more quickly relative to the model with old specification of unemployment benefits. Thus, the lifetime welfare gains for capitalists are higher and the lifetime welfare losses for workers are lower. The lifetime social welfare improves in the aggregate economy.

# 6 Conclusions

This paper investigated the effects of tax policy on unemployment, distribution of income and the welfare of agents assuming household heterogeneity. The households were divided into capitalists and workers. Only workers worked and only capitalists had access to the asset market. The analysis was conducted in a search and matching model. Unemployed workers sought potential job opportunities and firms opened new job vacancies to employ the desired number of workers in the following period. The wage rate was determined in a Nash bargaining between a pair of worker and firm once

they were matched through a Cobb-Douglas matching function. If the bargaining was successful, the worker was employed by the firm in the following period and the firm produced employing capital and labour. In this sense, employment was pre-determined at any given period of time. The government taxed interest income from physical capital, profits and labour income to finance its spending. The model was calibrated to match the main characteristics of the UK economy, with particular focuses on its labour market. In the tax reform experiments, we analysed the effects of capital tax cuts associated with concurrent labour tax increases. This allowed us to examine the productivity-tax burden trade-off and different impacts on heterogeneous households. Our main findings are summarized as follows.

First, in a model with search and matching frictions, the tax reform considered is Pareto improving in the long run although it increases inequality between agents. In other words, all the agents are better off, despite higher welfare gains for the capitalists compared to the workers. However, the capital tax cut met with the labour tax increase hurts the workers and also worsens the aggregate welfare in the transition period. This is because the positive effects resulting from higher capital accumulation take time to be realized. As a result, the combination of an initially lower net wage rate and higher search-unemployment creates short-run losses for the workers and aggregate economy, which are reversed in the long run. We also show that our results are robust to variations in the relative bargaining power of workers in the Nash bargain. Increasing the workers' bargaining power makes the tax reform less efficient in terms of welfare improving.

Second, when we assume that unemployment benefits depend on past wages, the model can generate similar welfare results in both the long- and short-run although the mechanism driving results is different. The tax reform is still Pareto improving in the long run but generates higher welfare gains for all agents. Similar to the long run results, the tax reform has higher welfare effects for all agents in the transition period. In other words, the lifetime welfare gains for capitalists are higher and welfare losses for workers are lower. As a result, the lifetime aggregate welfare improves. This is mainly because the capital tax cuts have larger and quicker effects on increasing capital

accumulation and raising the net wage during the transition. Therefore, the welfare of workers is raised more quickly in the new, post-reform economy. To summarise, the long- and short-run welfare gains of tax reforms are higher for all agents by assuming unemployment benefits depending on past wages.

Our analysis makes clear that the tax reform of reducing capital tax and a concurrent labour tax increase can increase the welfare of all agents but with the sacrifice of inequality under different specifications of unemployment benefits. We further see that, in the short run, the tax reform will hurt the agents who rely on labour income. Thus, our analysis adds to the tax policy studies in the search-and-matching literature and offers new results about the redistributional effects of the tax policy.

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# 7 Appendix

## 7.1 Optimization of capitalists

The Lagrangian function of the capitalist is written as:

$$L^{k} = \sum_{t=0}^{\infty} \beta^{t} \left\{ \frac{\left(C_{t}^{k}\right)^{1-\sigma}}{1-\sigma} + \lambda_{t}^{1} \left[ R_{t} K_{t}^{k} + \left(1-\tau_{t}^{k}\right) \pi_{t}^{k} + \overline{G}_{t}^{t} - C_{t}^{k} - K_{t+1}^{k} \right] \right\}$$

where  $\lambda_t^1$  is the Lagrangian multiplier on the capitalist's budget constraint.

The first-order condition (FOC) for  $\mathcal{C}_t^k$  is:

$$(1 - \sigma) \frac{\left(C_t^k\right)^{-\sigma}}{1 - \sigma} - \lambda_t^1 = 0$$
$$\frac{1}{\left(C_t^k\right)^{\sigma}} = \lambda_t^1.$$

The FOC for  $K_{t+1}^k$  is:

$$\beta \lambda_{t+1}^1 R_{t+1} - \lambda_t^1 = 0$$

$$\beta \lambda_{t+1}^1 R_{t+1} = \lambda_t^1.$$

If we combine these two FOCs, we can get:

$$\frac{1}{\left(C_t^k\right)^{\sigma}} = \beta \frac{R_{t+1}}{\left(C_{t+1}^k\right)^{\sigma}}.$$

#### 7.2 Optimization of workers

The Lagrangian function of the worker is written as:

$$L^{w} = \sum_{t=0}^{\infty} \beta^{t} \left\{ \frac{\left(C_{t}^{w}\right)^{1-\sigma}}{1-\sigma} - \xi \frac{\left(e_{t}+s_{t}\right)^{\mu}}{\mu} + \lambda_{t}^{2} \left[p_{t}s_{t}+\left(1-\gamma\right)e_{t}-e_{t+1}\right] + \lambda_{t}^{3} \left[\left(1-\tau_{t}^{w}\right)w_{t}e_{t} + \overline{G}_{t}^{u}s_{t} + \overline{G}_{t}^{t} - C_{t}^{w}\right] \right\}$$

where  $\lambda_t^2$  is the Lagrangian multiplier on the evolution equation of employment and  $\lambda_t^3$  is the Lagrangian multiplier on the worker's budget constraint.

The FOC for  $C_t^k$  is:

$$(1 - \sigma) \frac{\left(C_t^w\right)^{-\sigma}}{1 - \sigma} = \lambda_t^3$$
$$\frac{1}{\left(C_t^w\right)^{\sigma}} = \lambda_t^3.$$

The FOC for  $s_t$  is:

$$-\xi \mu \frac{(e_t + s_t)^{\mu - 1}}{\mu} + \lambda_t^2 p_t + \lambda_t^3 \overline{G}_t^u = 0$$
$$\xi (e_t + s_t)^{\mu - 1} = p_t \lambda_t^2 + \lambda_t^3 \overline{G}_t^u$$

We substitute out  $\lambda_t^3$  using the FOC for  $C_t^k$  and solve for  $\lambda_t^2$ :

$$\xi \left(e_t + s_t\right)^{\mu - 1} = p_t \lambda_t^2 + \frac{\overline{G}_t^u}{\left(C_t^w\right)^{\sigma}}$$

$$\frac{\xi \left(e_t + s_t\right)^{\mu - 1}}{p_t} - \frac{\overline{G}_t^u}{p_t \left(C_t^w\right)^{\sigma}} = \lambda_t^2$$

The FOC for  $e_{t+1}$  is:

$$-\beta \xi \mu \frac{(e_{t+1} + s_{t+1})^{\mu - 1}}{\mu} - \lambda_t^2 + \beta \lambda_{t+1}^2 (1 - \gamma) + \lambda_{t+1}^3 (1 - \tau_{t+1}^w) w_{t+1} = 0$$

$$\beta \xi \left( e_{t+1} + s_{t+1} \right)^{\mu - 1} = -\lambda_t^2 + \beta \left[ \lambda_{t+1}^2 \left( 1 - \gamma \right) + \lambda_{t+1}^3 \left( 1 - \tau_{t+1}^w \right) w_{t+1} \right]$$

We can rewrite it by substituting out  $\lambda_t^2$ ,  $\lambda_{t+1}^2$ , and  $\lambda_{t+1}^3$  by using conditions above:

$$\beta \xi \left( e_{t+1} + s_{t+1} \right)^{\mu - 1} = -\frac{\xi \left( e_t + s_t \right)^{\mu - 1}}{p_t} + \frac{\overline{G}_t^u}{p_t \left( C_t^w \right)^{\sigma}} +$$

$$+ \beta E_t \left[ \left( \frac{\xi \left( e_{t+1} + s_{t+1} \right)^{\mu - 1}}{p_{t+1}} - \frac{\overline{G}_{t+1}^u}{p_t \left( C_{t+1}^w \right)^{\sigma}} \right) (1 - \gamma) +$$

$$+ \frac{1}{\left( C_{t+1}^w \right)^{\sigma}} \left( 1 - \tau_{t+1}^w \right) w_{t+1} \right].$$

#### 7.3 Optimization of firms

The Lagrangian function of the firm is written as:

$$L^{f} = \sum_{t=0}^{\infty} \prod_{i=0}^{t} R_{i}^{-1} \left\{ A \left( K_{t}^{f} \right)^{\alpha} \left( L_{t}^{f} \right)^{1-\alpha} - r_{t} K_{t}^{f} - w_{t} L_{t}^{f} - \nu v_{t} + \lambda_{t}^{4} \left[ q_{t} v_{t} + (1 - \gamma) L_{t}^{f} - L_{t+1}^{f} \right] \right\}$$

where  $\lambda_t^4$  is the Lagrangian multiplier on the evaluation of firm's labour input. The FOC for  $K_t^f$  is:

$$\prod_{i=0}^{t} R_i^{-1} \alpha A \left( K_t^f \right)^{\alpha} \left( L_t^f \right)^{1-\alpha} - \prod_{i=0}^{t} R_i^{-1} r_t = 0$$

$$r_t = \alpha A \left( K_t^f \right)^{\alpha} \left( L_t^f \right)^{1-\alpha}$$

$$r_t = \alpha \frac{Y_t^f}{K_t^f}.$$

The FOC for  $v_t$  is:

$$-\prod_{i=0}^{t} R_i^{-1} \nu + \prod_{i=0}^{t} R_i^{-1} \lambda_t^4 q_t = 0$$

$$\nu = \lambda_t^4 q_t.$$
(27)

The FOC for  $L_{t+1}$  is:

$$\prod_{i=0}^{t+1} R_i^{-1} \left[ (1-\alpha) A \left( K_{t+1}^f \right)^{\alpha} \left( L_{t+1}^f \right)^{-\alpha} - w_{t+1} \right] -$$

$$- \prod_{i=0}^{t} R_i^{-1} \lambda_t^4 + \prod_{i=0}^{t+1} R_i^{-1} \lambda_{t+1}^4 \left( 1 - \gamma \right) = 0$$

$$R_{t+1}^{-1} \left[ (1-\alpha) A \left( K_{t+1}^f \right)^{\alpha} \left( L_{t+1}^f \right)^{-\alpha} - w_{t+1} \right] = \lambda_t^4 - R_{t+1}^{-1} \lambda_{t+1}^4 \left( 1 - \gamma \right)$$

$$R_{t+1}^{-1} \left[ (1 - \alpha) \frac{Y_{t+1}^f}{L_{t+1}^f} - w_{t+1} \right] = \lambda_t^4 - R_{t+1}^{-1} \lambda_{t+1}^4 (1 - \gamma).$$
 (28)

We then solve for  $\lambda_t^4$  in condition (27) and substitute the expression into condition (28):

$$R_{t+1}^{-1} \left[ (1-\alpha) \frac{Y_{t+1}^f}{L_{t+1}^f} - w_{t+1} \right] = \frac{\nu}{q_t} - R_{t+1}^{-1} \frac{\nu}{q_{t+1}} (1-\gamma)$$

which can be simplified to:

$$\frac{\nu}{q_t} = R_{t+1}^{-1} \left[ (1 - \alpha) \frac{Y_{t+1}^f}{L_{t+1}^f} - w_{t+1} + \frac{\nu(1 - \gamma)}{q_{t+1}} \right].$$

#### 7.4 Derivation of firm's expected profits

We first rewrite the present value of the stream of firm's profits in (16) starting from time 1 by making use of two first-order conditions of firms set out above, the profits equation (15), the law of motion for the firm's employment (14), and the properties of the production function:

$$\begin{split} \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \pi_{t}^{f} &= \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \left( Y_{t}^{f} - r_{t} K_{t}^{f} - w_{t} L_{t}^{f} - \nu v_{t} \right) \\ &= \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \left( Y_{t}^{f} - \alpha Y_{t}^{f} - w_{t} L_{t}^{f} - \nu v_{t} \right) \\ &= \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \left[ (1 - \alpha) Y_{t}^{f} - w_{t} L_{t}^{f} \right] - \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \nu v_{t} \\ &= \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \left( f_{L_{t}^{f}} L_{t}^{f} - w_{t} L_{t}^{f} \right) - \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \nu v_{t} \\ &= \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \left( f_{L_{t}^{f}} - w_{t} \right) L_{t}^{f} - \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \nu v_{t} \\ &= \sum_{t=0}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \left( f_{L_{t+1}^{f}} - w_{t+1} \right) L_{t+1}^{f} - \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \nu v_{t} \\ &= \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \left[ (1 - \alpha) Y_{t}^{f} - w_{t} L_{t}^{f} - \nu v_{t} \right] \\ &= \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \left[ (1 - \alpha) \frac{Y_{t+1}^{f}}{L_{t+1}^{f}} - w_{t} L_{t}^{f} \right] - \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \nu v_{t} \\ &= \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \left[ (1 - \alpha) \frac{Y_{t+1}^{f}}{L_{t+1}^{f}} - w_{t} L_{t}^{f} - \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \nu v_{t} \right] \\ &= \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \left[ (1 - \alpha) \frac{Y_{t+1}^{f}}{L_{t+1}^{f}} - w_{t+1} \right] L_{t+1}^{f} - \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \nu v_{t} \end{split}$$

Then, we substitute out  $\left( (1-\alpha) \frac{Y_{t+1}^f}{L_{t+1}^f} - w_{t+1} \right)$  by making use of the con-

dition (18). The r.h.s. of above equation can be rewritten as:

$$\begin{split} &\sum_{t=0}^{\infty} \prod_{i=0}^{t} R_{i+1}^{-1} L_{t+1}^{f} \left[ \frac{\nu}{q_{t}} R_{t+1} - \frac{\nu(1-\gamma)}{q_{t+1}} \right] - \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \nu v_{t} \\ &= \sum_{t=0}^{\infty} \prod_{i=0}^{t} R_{i+1}^{-1} L_{t+1}^{f} \frac{\nu}{q_{t}} R_{t+1} - \sum_{t=0}^{\infty} \prod_{i=0}^{t} R_{i+1}^{-1} L_{t+1}^{f} \frac{\nu(1-\gamma)}{q_{t+1}} - \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \nu v_{t} \\ &= \sum_{t=0}^{\infty} \prod_{i=0}^{t} R_{i+1}^{-1} \left( q_{t} v_{t} + (1-\gamma) L_{t}^{f} \right) \frac{\nu}{q_{t}} R_{t+1} \\ &- \sum_{t=0}^{\infty} \prod_{i=0}^{t} R_{i+1}^{-1} L_{t+1}^{f} \frac{\nu(1-\gamma)}{q_{t+1}} - \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \nu v_{t} \\ &= \left( \sum_{t=0}^{\infty} \prod_{i=0}^{t} R_{i+1}^{-1} R_{t+1} \nu v_{t} - \sum_{t=1}^{\infty} \prod_{i=1}^{t} R_{i}^{-1} \nu v_{t} \right) \\ &+ \left( \sum_{t=0}^{\infty} \prod_{i=0}^{t} R_{i+1}^{-1} (1-\gamma) L_{t}^{f} \frac{\nu}{q_{t}} R_{t+1} - \sum_{t=0}^{\infty} \prod_{i=0}^{t} R_{i+1}^{-1} L_{t+1}^{f} \frac{\nu(1-\gamma)}{q_{t+1}} \right) \\ &= \nu v_{0} + (1-\gamma) L_{0}^{f} \frac{\nu}{q_{0}}. \end{split}$$

Making use of the evolution equation of employment,  $L_1^f = q_0 v_0 + (1 - \gamma) L_0^f$ , we can rewrite the final expression above as follows:

$$\nu v_0 + (1 - \gamma) L_0^f \frac{\nu}{q_0} = L_1^f \frac{\nu}{q_0}$$

so that

$$\sum_{t=1}^{\infty} \prod_{i=1}^{t} R_i^{-1} \pi_t^f = L_1^f \frac{\nu}{q_0}$$

or

$$\nu = q_0 \frac{\sum_{t=1}^{\infty} \prod_{i=1}^{t} R_i^{-1} \pi_t^f}{L_1^f}.$$
 (29)

It states that, in equilibrium, the costs of posting a vacancy today should equalize the discounted value of stream of profits brought about by each filled vacancy tomorrow. This implies that the marginal cost of a vacancy is equal to the marginal benefit of filling it in the next period.

### 7.5 Solution to the Nash bargain

The FOC with respect to  $w_t$  is given by:

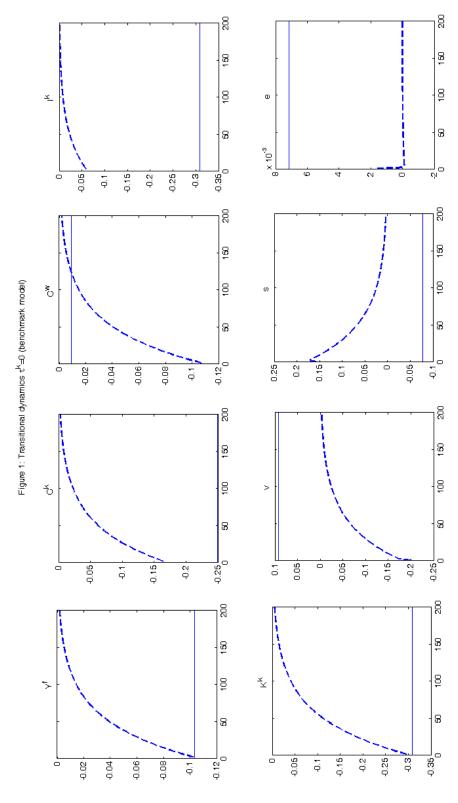
$$\phi \left[ w_t - \frac{u_{2,t}^w}{(1 - \tau_t^w) u_{1,t}^w} \right]^{\phi - 1} \left[ Y_{2,t}^f - w_t \right]^{1 - \phi} -$$

$$- (1 - \phi) \left[ w_t - \frac{u_{2,t}^w}{(1 - \tau_t^w) u_{1,t}^w} \right]^{\phi} \left[ Y_{2,t}^f - w_t \right]^{-\phi} = 0$$

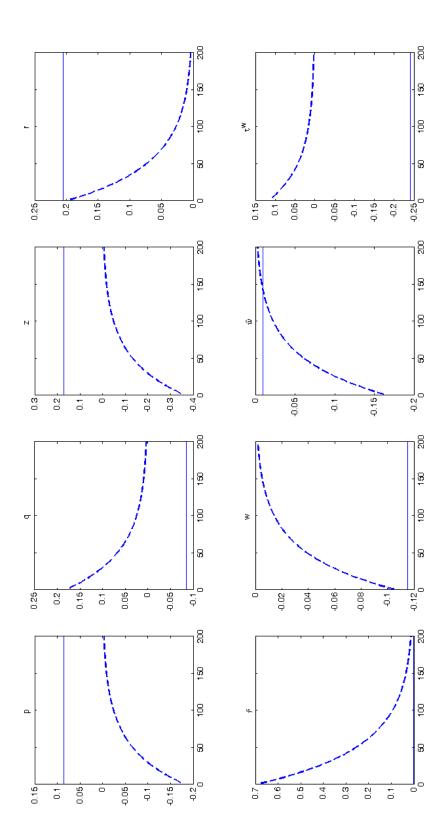
$$\phi \left[ Y_{2,t}^f - w_t \right] = (1 - \phi) \left[ w_t - \frac{u_{2,t}^w}{(1 - \tau_t^w) u_{1,t}^w} \right]$$

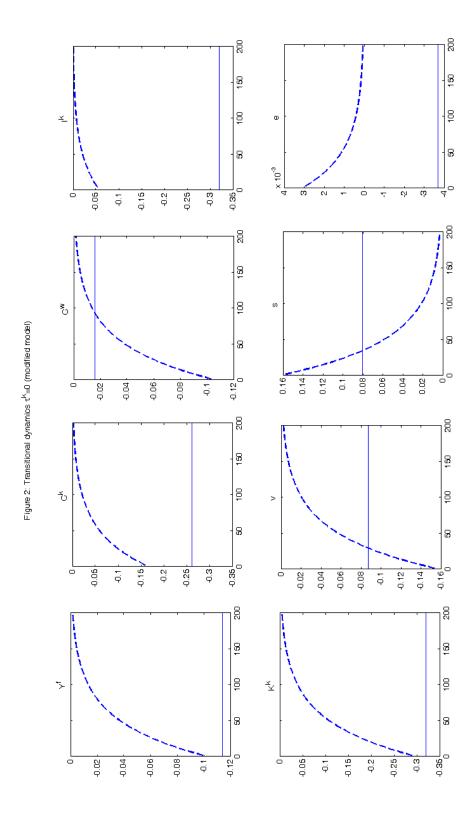
$$\phi Y_{2,t}^f - \phi w_t = (1 - \phi) w_t - (1 - \phi) \frac{u_{2,t}^w}{(1 - \tau_t^w) u_{1,t}^w}$$

$$w_t = (1 - \phi) \frac{u_{2,t}^w}{(1 - \tau_t^w) u_{1,t}^w} + \phi Y_{2,t}^f.$$



% dev. from post-reform steady-state





% dev. from post-reform steady-state

