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# Divided Majority and Information Aggregation: Theory and Experiment\*

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## Abstract

We propose a theory-based experimental approach to compare the properties of *approval voting* (AV) with those of *plurality*. This comparison is motivated by the theoretical predictions that, in our aggregate uncertainty setup, AV should produce close to first-best outcomes, while plurality will not. The experiment shows, first, that welfare gains are substantial. Second, both aggregate and individual responses are in line with theoretical predictions, and thus with strategic voting. Finally, subjects' behavior under AV highlights the need to study equilibria in asymmetric strategies.

**JEL Classification:** C72, C92, D70

**Keywords:** Multicandidate Elections, Information Aggregation, Plurality, Approval Voting, Laboratory Experiments

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# 1 Introduction

Election procedures are a defining feature of democracies. They also determine how decisions are made in smaller groups, from parliaments to board committees. Designing a voting system that produces the best possible outcomes given voter preferences and information is thus essential.<sup>1</sup> The literature proposes a variety of voting systems, each of them with potential strengths and weaknesses, and only a few are used in practice (Bormann and Golder 2013). Not that there is either an empirical proof or even a common belief that those in use are actually better. To the contrary, their flaws have been repeatedly emphasized. But we lack evidence that the alternatives would perform sufficiently better, and this stalls reform.<sup>2</sup>

Enhancing our knowledge about the capacity of alternative voting systems to outperform the ones currently in use requires a combination of theory and empirics: we need theory to identify which electoral systems are potentially best performing. We need empirics to test whether theoretical predictions are met in practice. The empirical question is twofold: first, do voters actually behave as theory predicts (in particular, are they strategic or sincere)? Second, can a change in the voting system actually deliver the predicted welfare gains? A strong limitation to empirical work is that observational data are scant, since only a handful of electoral systems are used in practice.

In this paper, we adopt a theory-based experimental approach to achieve a dual objective: (i) comparing the welfare properties of two voting systems, and (ii) shedding new light on the debate about whether voters behave strategically. Theory shows that, in large electorates, *approval voting* (AV) should produce close to first-best welfare results if voters face aggregate uncertainty and behave strategically (Bouton and Castanheira 2012).<sup>3</sup> With the objective of testing our results in the laboratory, we develop a model to compare AV with *plurality voting* (the system used e.g. in the U.S. and the U.K.) for any electorate size.

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<sup>1</sup>The voting system must be able to aggregate heterogeneous preferences (see e.g. Borda 1781, Arrow 1951, Brams and Fishburn 1978, Myerson and Weber 1993, Myerson 1999, 2002, Castanheira 2003, Myatt 2007, Bouton 2013, Felsenthal and Machover 2012, and the references therein) and/or heterogeneous information (see e.g. Condorcet 1785, Austen-Smith and Banks 1996, Feddersen and Pesendorfer 1996, 1997, 1998; Myerson 1998, Piketty 2000, Bhattacharya 2012, Mandler 2012, and the references therein).

<sup>2</sup>These limitations resonate in civil society, where there is growing frustration with existing electoral systems. A large number of activists lobby in favor of reforming the electoral system (e.g. the Electoral Reform Society ([www.electoral-reform.org.uk](http://www.electoral-reform.org.uk)) and the Fair Vote Reforms initiative ([www.fairvote.org](http://www.fairvote.org))), and many official proposals have been introduced. A recent example comes from the UK, which held a national referendum in 2011 on whether to replace plurality voting with alternative voting.

<sup>3</sup>Under AV, voters can “approve” of as many candidates as they want. Each approval counts as one point and the candidate obtaining the largest number of points wins.

We show that AV should produce significant welfare gains.

Our laboratory experiments confirm these predictions. The welfare-maximizing alternative wins with much higher probability under AV, and welfare gains are actually higher than theory predicts. This is for three reasons: first, the subjects' behavior is closer to the theoretical prediction in AV than in plurality. The multiplicity of equilibria in plurality produces coordination failures, which reduces welfare compared to theoretical predictions. Second, AV appears more robust to collective mistakes than plurality. Third, the experiment shows that voters specialize in AV (i.e. adopt asymmetric strategies) in ways that produce higher welfare than predicted by the symmetric strategy equilibrium.

In our setup, majority voters have common value preferences but face aggregate uncertainty: they are divided by opposing information as to which of two majority alternatives is the best. The minority supports another candidate, who is a Condorcet loser.<sup>4</sup> In plurality, we find that aggregate uncertainty produces a novel “informative equilibrium” in which all three alternatives receive a strictly positive vote share. This equilibrium coexists with “Duverger’s Law equilibria” in which majority voters coordinate all their ballots on a single candidate.<sup>5</sup> The experiment identifies when each equilibrium gets selected: subjects select a Duverger’s Law equilibrium when the size of the minority is large – the informative equilibrium would then result in much lower welfare. When the minority is small, subjects select the informative equilibrium, even when it results in moderately lower welfare than a Duverger’s Law equilibrium. In contrast, AV typically features a unique (symmetric) equilibrium, which produces strictly higher welfare than any equilibrium in plurality.

While our setup differs from the traditional one (no aggregate uncertainty, and voters have full information about the relative value of each alternative), we perceive that an aggregate uncertainty setup is both more realistic and necessary to capture empirically relevant voting behavior. The voters’ imperfect information captures “rational ignorance” and actually explains why inferior equilibria disappear in AV (see Bouton and Castanheira 2012 for more detail). The common value component also provides majority voters with two conflicting incentives: on the one hand, they benefit from aggregating the information dispersed in the electorate – this requires dividing their ballots across the two majority

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<sup>4</sup>In a large Poisson game setup, Bouton and Castanheira (2012) show that the theoretical properties of AV remain the same when majority voters are also divided by heterogeneous preferences. We can thus focus on the simpler case of pure common values without losing the insights from the more general setup.

<sup>5</sup>In a different setup, the pioneering work by Myatt (2007) already identified aggregate uncertainty as key to understand the properties of plurality. Before his work, the literature typically assumed that voters know the distribution of preferences in the electorate. In this case, only Duverger’s Law equilibria should be stable (Riker 1982, Palfrey 1989, Bouton *et al.* 2015).

alternatives. On the other hand, they want to defeat the Condorcet loser – this requires coordinating their ballots on a single majority alternative. Varying the size of the minority in the lab alters the relative value of these two incentives, and allows us to test novel implications of the model on the subjects’ strategic responses.

Our results have several implications for future research. Regarding plurality, our finding that aggregate uncertainty produces an empirically-relevant “informative” equilibrium shows that one cannot systematically associate “sincere voting” with “non-strategic” or non “short term instrumentally rational” voting (Cox 1997). Which ballot is a voter’s best response actually depends on which equilibrium is selected by the rest of the electorate. This modifies the way in which we typically measure strategic voting (see *a.o.* Guarnascheli et al. 2000, Feddersen 2004, Hortala-Vallve and Llorente-Saguer 2010, Kawai and Watanabe 2013, Spenkuch 2013, and Esponda and Vespa 2014). In our experiment, and taking account of equilibrium selection, the fraction of “strategic” subjects is found to be comprised between a lower bound of 27.78% and an upper bound of 72.23% across treatments.

Regarding approval voting, the experiment also reveals that subjects coordinated on an *asymmetric* equilibrium. In that equilibrium, some subjects (almost) always double vote for the two majority alternatives, and other subjects (almost) always single vote for their preferred alternative, given their signal. We verify that such asymmetric equilibria in pure strategy exist theoretically, and produce higher welfare than the symmetric equilibrium. This pattern points to the need to consider equilibria in asymmetric strategies in future theoretical research (see also Ladha et al. 1996). It also suggests that the subjects can actually much better exploit the favorable properties of AV than what is typically perceived (see *e.g.* the debates between Brams and Fishburn 1983, versus Niemi 1984, Saari and Newenhizen 1988, and Nagel 2007).

There are obviously other voting systems that could and should be considered (*a.o.* runoff voting and Borda count). Our focus on plurality voting and approval voting is arbitrary to some extent but several reasons justify it. First, plurality voting is one of the most widely used electoral systems around the world (see *e.g.* Bormann and Golder 2013). The other main contender is runoff voting, which also suffers from significant weaknesses, some similar to those of plurality (see Bouton 2012 and Bouton and Gratton 2015). Second, the voting literature highlights that approval voting has desirable properties (see *e.g.* Brams and Fishburn 1978, 1983, Myerson 1999, 2002, Weber 1995, Forsythe et al. 1996, Laslier 2010, Nuñez 2010, and Bouton and Castanheira 2012). By contrast, a voting system like Borda features significant weaknesses, in particular a lack of decisiveness (see Myerson and

Weber 1993, Forsythe *et al.* 1996, Myerson 2002).

Forsythe *et al.* (1993, 1996) are closest to our paper. There are few other papers comparing plurality to AV: for instance, Rapoport *et al.* (1991), Van der Straeten *et al.* (2010), Dellis *et al.* (2011) or Bassi (2015) study whether voters behave more or less strategically under plurality and/or whether Duverger’s Law applies in each system. These papers focus on the case of private values and perfect information (see also Rietz 2008 and Palfrey 2013 for detailed reviews of that literature). By contrast, our majority voters have common values and they are uncertain about their preferred alternative. Our paper also relates to the experimental literature on the Condorcet jury theorem, with the difference that we consider three alternatives; see *e.g.* Guarnaschelli *et al.* (2000), Battaglini *et al.* (2008, 2010), Goeree and Yariv (2011), and Bhattacharya *et al.* (2014).

## 2 The Model

While the literature typically focuses on results valid for arbitrarily large electorates, we want to identify theoretical results that are valid for any population size. This is necessary to test the results in the laboratory. We thus consider a voting game with an electorate of fixed and finite size who must elect one alternative  $P$  out of three:  $A$ ,  $B$  and  $C$ .

The electorate is split in two groups:  $n$  active voters who constitute a majority, and  $n_C$  passive voters who constitute a minority.<sup>6</sup> There are two states of nature:  $\omega = \{a, b\}$ , which materialize with probabilities  $q(\omega) > 0$ . The actual state of nature is not observed before the election.

A majority voter’s utility depends on the policy outcome and on the state of nature. Utility is highest if the winning alternative matches the state, intermediate if there is a mismatch between the two, and lowest if alternative  $C$  is elected:

$$\begin{aligned} U(P, \omega) &= V > 0 \text{ if } (P, \omega) = (A, a) \text{ or } (B, b) \\ &= v \in (0, V) \text{ if } (P, \omega) = (A, b) \text{ or } (B, a) \\ &= 0 \text{ if } P = C, \forall \omega. \end{aligned} \tag{1}$$

Minority voters prefer  $C$  to either  $A$  or  $B$ , and are indifferent between the latter options. Hence, their dominant strategy is to vote for  $C$ . We focus on the interesting case in which  $C$ -voters represent a large minority:  $n - 1 > n_C > n/2$ . Thus,  $C$  is a Condorcet loser but

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<sup>6</sup>The results directly extend to  $n$  and  $n_C$  being drawn from either a binomial or a Poisson distribution.

it can win the election if active voters split their votes between  $A$  and  $B$ .<sup>7</sup>

**Timing.** Before the election (at **time 0**), nature chooses whether the state is  $a$  or  $b$ . At **time 1**, each voter receives a signal  $s \in S \equiv \{s_A, s_B\}$ , with conditional probabilities  $r(s|\omega) \in (0, 1)$  and  $r(s_A|\omega) + r(s_B|\omega) = 1$ . These signals are informative,  $r(s_A|a) > r(s_A|b)$ , but private and voter cannot communicate (communication is studied *a.o.* by Coughlan 2000, Persico 2004, and Austen-Smith and Feddersen 2006). We say that the distribution of signals is *unbiased* if  $r(s_A|a) = r(s_B|b)$  and *biased* if  $r(s_A|a) \neq r(s_B|b)$ . By convention, we focus on the case in which the “more abundant” signal is  $s_A$ :  $r(s_A|a) + r(s_A|b) \geq 1$ .

Having received her signal, the voter updates her beliefs through Bayes’ rule:  $q(\omega|s) = q(\omega)r(s|\omega) / (q(a)r(s|a) + q(b)r(s|b))$ . Like Bouton and Castanheira (2012), we assume that signals are sufficiently strong to create a *divided majority*:

$$q(a|s_A) > 1/2 > q(a|s_B). \quad (2)$$

That is, conditional on receiving signal  $s_A$ , alternative  $A$  yields strictly higher expected utility than alternative  $B$ , and conversely for a voter who receives signal  $s_B$ .

The election is held at **time 2**, when the actual state of nature is still unobserved. Payoffs realize at **time 3**: the winner of the election and the actual state of nature are revealed, and each voter receives utility  $U(P, \omega)$ .

**Strategy space and equilibrium concept.** How voters can allocate their votes depends on whether the electoral rule is plurality or approval voting. In *plurality (PL)*, each voter can vote for exactly one alternative or abstain. Their action set is:

$$\Psi_{PL} = \{A, B, C, \emptyset\},$$

where, by an abuse of notation, action  $A$  (respectively  $B, C$ ) denotes a ballot in favor of  $A$  (resp.  $B, C$ ), and  $\emptyset$  denotes abstention – which will prove to be a dominated action.

In *approval voting (AV)*, each voter can vote for one, two or three alternatives – or abstain:

$$\Psi_{AV} = \{A, B, C, AB, AC, BC, ABC, \emptyset\},$$

where, by an abuse of notation, action  $A$  denotes a ballot in favor of  $A$  only, action  $BC$

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<sup>7</sup>A possible alternative interpretation of our setup is that voters vote on whether to reform a status quo policy ( $C$ ). Two policies could replace this status quo ( $A$  and  $B$ ), and a qualified majority of  $\frac{n_C}{n+n_C}$  is required for passing a reform (see e.g. Dewan and Myatt 2007).

denotes a joint approval of  $B$  and  $C$ , etc. Each approval counts as one vote: when a voter only approves of  $A$ , then only alternative  $A$  is credited with a vote. If the voter approves of both  $A$  and  $B$ , then  $A$  and  $B$  are credited with one vote each, and so on.

Let an *action profile*  $x$  be the vector that lists the *realized* number of ballots  $\psi \in \Psi_R$ ,  $R \in \{PL, AV\}$  at time 2. The total number of votes received by an alternative  $\psi$  is denoted by  $X_\psi$ . Under plurality the total number of votes received by alternative  $A$ , for instance, is simply:  $X_A = x_A$ . Under AV, it is:  $X_A = x_A + x_{AB} + x_{AC} + x_{ABC}$ . The winner of the election is the alternative receiving the largest number of votes – ties are broken by a fair dice.

A symmetric strategy is a mapping  $\sigma : S \rightarrow \Delta(\Psi_R)$ , and we denote by  $\sigma_s(\psi)$  the probability that some randomly sampled active voter who received signal  $s$  plays  $\psi$ . Given a strategy  $\sigma$ , the *expected share of active voters* playing action  $\psi$  in state  $\omega$  is  $\tau_\psi^\omega(\sigma) = \sum_s \sigma_s(\psi) r(s|\omega)$ , whereas the *expected number of ballots*  $\psi$  is  $\mathbf{E}[x(\psi) | \omega, \sigma] = \tau_\psi^\omega(\sigma) n$ .

For now, we focus on the properties of symmetric Bayesian Nash equilibria that satisfy what we call *sincere stability*. That is, the equilibrium must be robust to the case in which voters may tremble by voting sincerely (that is:  $\sigma_{s_A}(A), \sigma_{s_B}(B) \geq \varepsilon > 0$ , and we look for sequences of equilibria with  $\varepsilon \rightarrow 0$ ). Sincere stability, by imposing that a small fraction of the voters votes for their preferred alternative, implies that at least some pivot probabilities remain strictly positive. This eliminates equilibria in weakly dominated strategies such as the ones in which all voters play the same action only because this strategy implies that all pivot probabilities are zero.

The advantage of our sincere stability refinement is twofold: it captures the essence of properness in a tractable way,<sup>8</sup> and it is behaviorally relevant. Indeed, experimental data (both in our experiments and others) suggest that some voters vote for their ex ante most preferred alternative no matter what.

### 3 Plurality

This section analyzes key equilibrium properties of plurality voting (see Appendix A1 for technical details and proofs). Below, we show that two types of equilibria coexist: in one, all majority voters play the same (pure) strategy independently of their signal: they all

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<sup>8</sup>We do not use more traditional refinement concepts such as *perfection* or *properness* because, in the voting context, the former does not have much bite, and the latter is quite intractable since it requires a comparison of all pivot probabilities for totally mixed strategies.



vote for either  $A$  or  $B$ . This type of equilibrium is known as a *Duverger’s Law equilibrium*, in which only two alternatives receive a strictly positive vote share. In the second type of equilibrium, a majority voter’s strategy does depend on her signal. Depending on parameter values, this equilibrium either features *sincere voting*, that is, voters with signal  $s_A$  (resp.  $s_B$ ) vote  $A$  (resp.  $B$ ) or a strictly mixed strategy in which voters with the most abundant signal ( $s_A$  by convention) mix between  $A$  and  $B$ . We find that these three-party equilibria exist for any population size, are robust to signal biases, and do not feature any tie. They are thus not “knife edge” in the sense of Palfrey (1989).

### 3.1 Duverger’s Law Equilibria

The game theoretic version of Duverger’s Law (Duverger 1963, Riker 1982, Palfrey 1989, Myerson and Weber 1993, Cox 1997) states that, when voters behave strategically, only two alternatives should obtain a strictly positive fraction of the votes in plurality elections. In our setup, these equilibria are as follows:

**Definition 1** *A Duverger’s Law equilibrium is such that either all majority voters vote for  $A$  or all vote for  $B$ .*

These Duverger’s Law equilibria ensure that  $C$  cannot win the election, since either  $A$  or  $B$  receive  $n(> n_C)$  votes. However, they also prevent information aggregation: the winner of the election is fully determined by *ex ante* voter coordination, which amounts to throwing away all the voters’ private signals. Our first proposition is that:

**Proposition 1** *Duverger’s Law equilibria always exist under plurality.*

The intuition for the proof (see Appendix A1) is straightforward: if an alternative, say  $A$ , collects the ballots from almost all the other voters, then a ballot for  $A$  is much more likely to be pivotal against  $C$  than a ballot for  $B$ . This ensures that all majority voters value a vote for  $A$  strictly more than a vote for  $B$ : they do not want to waste their ballot on an alternative that is very unlikely to win.

### 3.2 Informative Equilibria

In Duverger’s Law equilibria, voters discard the information in their possession. Yet, this type of equilibrium is typically considered the only reasonable one if voters are *short-term*

*instrumentally rational*, in Cox’s (1997) terminology. Indeed, in a world without aggregate uncertainty, equilibria with more than two alternatives obtaining votes are typically “knife edge” and “expectationally unstable” (Palfrey 1989, Fey 1997).<sup>9</sup> Therefore, empirical research typically associates strategic voting with the voters’ propensity to abandon their preferred but non-viable candidates, and vote for more serious contenders (see Cox 1997, Alvarez and Nagler 2000, Blais et al 2005, Fujiwara 2011, Spenkuch 2013).<sup>10</sup>

Instead, Propositions 2 and 3 below show that even if all voters are *short-term instrumentally rational*, there exists an equilibrium in which no candidate is abandoned by her supporters. This breaks the link between non-instrumental voting and the observation that only relatively low fractions of the electorate switch to their second-best alternative. This discussion revolves around the existence and stability of what we call an *informative equilibrium*:

**Definition 2** *An informative equilibrium is such that (i) all alternatives receive a strictly positive vote share, (ii) these vote shares are different across alternatives (no knife-edge equilibrium), and (iii) A is the strongest majority contender in state a, and B in state b.*

To prove the existence of such an equilibrium, we first focus on the case in which information is close to being symmetric across states. Then, voters vote *sincerely* in an informative equilibrium: a voter who receives signal  $s_A$  votes for  $A$ , whereas a voter who receives signal  $s_B$  votes for  $B$ . That is, abandoning one’s preferred candidate is *not* a best response when one expects other voters to vote sincerely:

**Proposition 2** *In the unbiased case  $r(s_A|a) = r(s_B|b)$ , the sincere voting equilibrium exists  $\forall n, n_C$ . Moreover, there exists a value  $\delta(n, n_C) > 0$  such that sincere voting is an equilibrium for any distribution satisfying  $r(s_A|a) - r(s_B|b) < \delta(n, n_C)$ .*

The intuition is that, in the unbiased case, sincere voting implies that the likelihood of being pivotal against  $C$  is the same with an  $A$ -ballot in state  $a$  as with a  $B$ -ballot in state  $b$ . Therefore,  $s_A$ -voters strictly prefer to vote for  $A$  and  $s_B$ -voters strictly prefer to vote for  $B$ . Importantly, this equilibrium exists even if signals are slightly biased.

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<sup>9</sup>In contrast, Dewan and Myatt (2007), Myatt (2007) and Bouton *et al.* (2015) emphasize the existence of three-candidate equilibria when there is aggregate uncertainty. In our setup as well, informative equilibria would still exist if  $s_A$ -voters always preferred  $A$  and  $s_B$ -voters always preferred  $B$ , *i.e.* if they had private value preferences.

<sup>10</sup>Kawai and Watanabe (2013) acknowledge the weakness of the private-value pivotal voter model in organizing observational data.

The pros and cons of sincere voting are the exact flip-side of the ones identified for Duverger’s Law equilibria: as illustrated by the following example, it produces information, but does not guarantee a defeat of the Condorcet loser.

**Example 1** Consider a case in which  $n = 12$ ,  $n_C = 7$ , and  $r(s_A|a) = r(s_B|b) = 2/3$ .<sup>11</sup> Sincere voting implies that the best alternative ( $A$  in state  $a$ ;  $B$  in state  $b$ ) wins with a probability of 73%.  $C$  has the second largest expected vote share and wins with a probability of 23% in either state. The alternative with the lowest –but strictly positive– vote share is  $B$  in state  $a$  and  $A$  in state  $b$ .

When  $n_C$  is 9, the alternative with the largest expected vote share is  $C$ , who then wins with a probability larger than 71%, whereas the best alternative wins with a probability below 29%.

This example illustrates the voters’ main trade-off in plurality: voting sincerely aggregates information, but at the cost of letting  $C$  win some times. Coordinating on a Duverger’s Law equilibrium reverses the costs and benefits. Thus, coordinating on the informative equilibrium becomes costly when the size of the minority becomes large. In Example 1,  $C$ ’s probability of winning jumps from 23% to above 71% when  $n_C$  increases from 7 to 9. Expected payoffs drop accordingly, from 152 to 70. While our theory is silent about equilibrium selection, this suggests that the informative equilibrium should be empirically more relevant when minority size is small and conversely when it is large. We return to this when discussing the experimental results (see Section 6).

Based on this example and on Proposition 2, one may be misled into thinking that informative equilibria require signals to be (almost) unbiased. Instead, the fact that the signal structure becomes too biased to sustain sincere voting produces an informative equilibrium in which voters increase their support for the *weakest* candidate:

**Proposition 3** Let  $r(s_A|a) - r(s_B|b) > \delta(n, n_C)$ . Then, there exists an informative equilibrium in which voters with signal  $s_A$  play a non-degenerate mixed strategy:  $\sigma_{s_A}(A) \in (0, 1)$  and  $\sigma_{s_B}(B) = 1$ .

The intuition for the proof is best conveyed through a second numerical example, which illustrates that the existence of this equilibrium does not rely on some form of symmetry between vote shares (see also Bouton and Castanheira 2009 for the case of large electorates):

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<sup>11</sup>Each numerical example reproduces the parameters used in one of the treatments of our laboratory experiments (see Section 5). In all examples, the two states of nature are equally likely, and the payoffs are:  $V = 200$ ;  $v = 110$  and the value of  $C$  is 20.

**Example 2** Let electorate size still be  $n = 12$  and  $n_C = 7$ , but the signal structure be  $r(s_A|a) = 8/9 > 2/3 = r(s_B|b)$ . For these parameter values, an  $s_A$ -voter would strictly prefer to vote for  $B$  if all the other voters were to vote sincerely. Indeed, under sincere voting, a larger  $r(s_A|a)$  substantially decreases the probability of being pivotal in favour of  $A$  in state  $a$ . In contrast, the probability of being pivotal for  $B$  in state  $b$  remains high, and now dominates all the other pivot probabilities. Hence,  $G(A|s_A) - G(B|s_A) < 0$ .

The informative equilibrium is reached when  $\sigma_{s_A}(A) = 0.915$  and  $\sigma_{s_B}(B) = 1$ : by reducing the expected vote share of  $A$  and increasing that of  $B$ , the relative probability of being pivotal in favor of  $A$  in state  $a$  increases to the point in which  $s_A$ -voters are indifferent between voting  $A$  and  $B$ , whereas  $s_B$ -voters still strictly prefer to vote  $B$ . Importantly, all vote shares are strictly positive and the full information Condorcet winner is the most likely winner in both states of nature (their winning probabilities are respectively 96% and 79% in states  $a$  and  $b$ ):

$$\tau_A^a = 0.81 > \tau_B^b = 0.69 > \frac{n_C}{n} = 0.58 > \tau_A^b = 0.31 > \tau_B^a = 0.19.$$

This informative equilibrium gives  $C$  a strictly positive probability of victory (3% in state  $a$  and 18% in state  $b$ ) but expected utility is higher in this equilibrium than in a Duverger's Law equilibrium.

## 4 Approval Voting

This section analyzes key equilibrium properties of AV (see Appendix A2 for technical details and proofs). First, note that the action set under AV is an extension of the action set under plurality. As shown by Ahn and Oliveros (2013, Proposition 1), this implies that, in a pure common-value game, there exists an equilibrium in AV for which welfare is weakly higher than for any equilibrium in plurality.<sup>12</sup> We prove an even stronger result (which also extends the analysis of Bouton and Castanheira 2012): with a minority that has preferences opposite to the majority's, the welfare dominance of AV over plurality is *strict*.

**Theorem 1** *There always exists an equilibrium in AV for which expected welfare is strictly higher than for any equilibrium in plurality. In that equilibrium, some voters must double-vote, and  $\sigma_{s_A}(A), \sigma_{s_B}(B) > 0$ .*

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<sup>12</sup>Ahn and Oliveros (2013) exploit McLennan (1998)'s argument to show that, in a common value setup, one can rank equilibrium outcomes under AV as opposed to plurality and negative voting. By revealed preferences, since the action set in the two other rules is a strict subset of the action set under AV, "the maximal equilibrium utility under approval voting is greater than or equal to the maximal equilibrium utility under plurality voting or under negative voting." (p. 3).

The intuition for the proof (see Appendix A2) is as follows: the only difference between the set of undominated actions in plurality and in AV is the possibility to double-vote  $AB$ . Following Ahn and Oliveros (2013), if one voter wants to double-vote, the other voters' expected utility must also increase. We find that a voter's strict best response is precisely to double-vote when the other voters single-vote "excessively" (Lemma 2 in Appendix A2), which implies that all equilibria in plurality are strictly payoff dominated.

Another question is to assess the magnitude of the potential welfare gains from moving from plurality to AV. In a large electorate setting, AV selects the best alternative with a probability that converges to 1. Welfare may thus increase up to  $V$ . Achievable welfare in plurality depends on which equilibrium is selected. In a Duverger's Law equilibrium,  $C$  cannot win. But, depending on the probability of each state of nature, welfare may lie anywhere between  $v$  and  $V$ . In an informative equilibrium,  $C$  wins with strictly positive probability and, as illustrated in Example 1, may even become the most likely winner. Expected welfare may thus fall below  $v$ . We come back to this in Section 6.5.

A shortcoming of Theorem 1 is that it does not establish equilibrium uniqueness, which would require arbitrarily large population sizes (see Bouton and Castanheira, 2012), while our focus is on finite – possibly small – population sizes. Yet, our next theorem identifies unique voting patterns for any *interior* equilibrium:

**Theorem 2** *Whenever both  $s_A$ - and  $s_B$ -voters adopt a nondegenerate mixed strategy in equilibrium, voters with signal  $s_A$  only mix between  $A$  and  $AB$ , and voters with signal  $s_B$  only mix between  $B$  and  $AB$ .*

This theorem builds on the comparison between the preferences of  $s_A$  and  $s_B$  voters: conjecture for instance a case in which the former play  $B$  with strictly positive probability. Since a voter with signal  $s_B$  values  $B$  even more, she must only play  $B$ , which contradicts the nature of an interior equilibrium. As explained, finite population sizes imply that we cannot extend this result beyond interior equilibria. Yet, numerical simulations showed that, for all the parameter values we checked, the equilibrium was unique and such that voters with signal  $s_A$  only mix between  $A$  and  $AB$ , while voters with signal  $s_B$  never play  $A$ . This held both for interior equilibria and for equilibria in which (one of the two groups of) voters play a degenerate strategy.

Two additional examples are useful to illustrate these results and better understand the features and comparative statics of voting equilibria in AV:

**Example 3** Consider the same set of parameters as in Example 1:  $n = 12$ ,  $n_C = 7$  or  $9$ , and  $r(s_A|a) = r(s_B|b) = 2/3$ . The unique equilibrium is such that:

$$\begin{aligned} \sigma_{s_A}(A) = \sigma_{s_B}(B) = 0.64 \text{ and } \sigma_{s_A}(AB) = \sigma_{s_B}(AB) = 0.36 \text{ when } n_C = 7, \\ \sigma_{s_A}(A) = \sigma_{s_B}(B) = 0.30 \text{ and } \sigma_{s_A}(AB) = \sigma_{s_B}(AB) = 0.70 \text{ when } n_C = 9. \end{aligned}$$

When  $n_C = 7$ , these equilibrium strategies imply that  $A$  wins with a probability of 82% in state  $a$  (as does  $B$  in state  $b$ ), whereas  $C$ 's probability of winning is below 1%. When  $n_C = 9$ ,  $A$  wins with a probability of 73% in state  $a$  (as does  $B$  in state  $b$ ), whereas  $C$ 's probability of winning remains as low as 1.5%. These values should be contrasted with the sincere voting equilibrium in plurality (see Example 1), in which the probability of selecting the best outcome was substantially lower, and the risk that  $C$  wins was substantially larger.

Comparing equilibrium behavior with  $n_C = 7$  and  $n_C = 9$  in Example 3 shows that the larger  $n_C$ , the more double-voting in equilibrium. This pattern was found to be monotonic and consistent across numerical examples for any value of  $n$  and signal structures.

**Example 4** Consider the same set of parameters as in Example 3, except for  $r(s_A|a) = 8/9$ . This reproduces the biased signal setup of Example 2. As in Example 3, the equilibrium is unique:  $\sigma_{s_A}(A) = 0.26 < \sigma_{s_B}(B) = 0.52$  and  $\sigma_{s_A}(AB) = 0.74 > \sigma_{s_B}(AB) = 0.48$ . This equilibrium implies that  $A$  wins with a probability of 87% in state  $a$ , whereas  $B$  wins with a probability of 90% in state  $b$ .  $C$ 's winning probabilities are 0.5% in state  $a$  and 2.8% in state  $b$ .

This example highlights a second feature of AV: when the signal structure is biased (here towards  $s_A$ ),  $s_A$ -voters double-vote more than  $s_B$ -voters. In other words, in equilibrium, voters who support the strongest alternative should lend support to the underdog, by double-voting more. Such predictions (predicted behavior is different between plurality and approval voting, and across types) will allow us to disentangle rational behavior from sincere voting in the experiment.

## 5 Experimental Design and Procedures

We test our theoretical predictions through controlled laboratory experiments. To this end, we introduced subjects to a game that had the very same structure as the one presented in the model. All participants were given the role of an active voter, whereas passive voters

were simulated by the computer.<sup>13</sup> In all treatments, the number of active voters was  $n = 12$  and subjects were told that the computer casts  $n_C$  votes for *gray* in each election ( $n_C$  was set equal to 7 or 9 depending on the treatments).

Following the experimental literature on the Condorcet Jury Theorem, the two states of the world were called *blue jar* and *red jar*, whereas the signals were called *blue ball* and *red ball*. One of the jars was selected randomly by the computer, with equal probability. The subjects were not told which jar had been selected, but were told how the probability of receiving a ball of each color depended on the selected jar. After seeing their ball, each subject could vote from a set of three candidates: *blue*, *red* or *gray*.<sup>14</sup> *Blue* and *red* were the two majority candidates and *gray* was the Condorcet loser. As in Guarnaschelli *et al.* (2000), abstention was not allowed (remember that abstention is a strictly dominated action in our setup).<sup>15</sup>

The subjects' payoff depended on the color of the selected jar and on that of the election winner. If the color of the winner matched that of the jar, the payoff to all members of the group was 200 euro cents. If the winner was blue and the jar red or the other way around, their payoff was 110 cents. Finally, if gray won, their payoff was 20 cents.

We consider three treatment variables: the *voting rule* (plurality or approval voting), the *size of the minority* (small or large), and the *signal structure* (biased or unbiased). Each treatment corresponds to one of the examples in Sections 3 and 4. Table 1 summarizes the six different treatments.

Experiments were conducted at the BonnEconLab of the University of Bonn between July 2011 and January 2012. We ran a total of 18 sessions with 2 groups of 12 subjects in each of them. No subject participated in more than one session. Students were recruited through the online recruitment system ORSEE (Greiner 2004) and the experiment was programmed and conducted with the software z-Tree (Fischbacher 2007).

All experimental sessions were organized along the same procedure: subjects received detailed written instructions, which an instructor read aloud.<sup>16</sup> Each session proceeded in two parts: in the first part, subjects played one of the treatments in fixed groups for 100

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<sup>13</sup>Morton and Tyran (2012) show that preferences in one group are not affected by preferences of an opposite group. Therefore, having computerized rather than human subjects should not alter the behavior of majority voters in a significant way. Partisans (the equivalent to our passive voters) simulated by the computer has been used in previous studies – see Battaglini *et al.* (2008, 2010).

<sup>14</sup>The colors that we used in the experiments were *blau*, *rot* and *schwarz*.

<sup>15</sup>In a setting related to ours, Forsythe *et al.* (1993) allowed for abstention and found that the abstention rate was as low as 0.65%. Note also that, under AV, abstention is strategically equivalent to approving of all candidates. Subjects chose that option only 0.08% of the time.

<sup>16</sup>The instructions can be found in the supplementary appendix.

Treatment	Voting rule	Minority size ( $n_C$ )	Blue balls in blue jar	Red balls in red jar	Sessions / Ind. Obs.	Group #
PL7	Plurality	7	2/3	2/3	3 / 6	1-6
PL9	Plurality	9	2/3	2/3	3 / 6	7-12
AV7	Approval	7	2/3	2/3	3 / 6	13-18
AV9	Approval	9	2/3	2/3	3 / 6	19-24
PL7B	Plurality	7	8/9	2/3	3 / 6	25-30
AV7B	Approval	7	8/9	2/3	3 / 6	31-36

Table 1: Treatment overview. Note: "Blue balls in blue jar" stands for the "fraction of blue balls in the blue jar" and "ind. obs." stands for "individual observations".

periods. Before starting, subjects were asked to answer a questionnaire to check their full understanding of the experimental design. At the end of the experiment, subjects received new instructions, and made 10 choices in simple lotteries, as in Holt and Laury (2002). We ran this second part to elicit the subjects' risk preferences.

To determine payment, the computer randomly selected four periods from the first part and one lottery from the second part.<sup>17</sup> In total, subjects earned an average of €13.47, including a show-up fee of €3. Each experimental session lasted approximately one hour.

Our choice of fixed matching is not innocuous. The main advantage of fixed matching is that, for given costs, it delivers more independent units of observation, hence more power for non-parametric tests. A typical drawback of fixed matching is that it favours repeated game effects. One might thus fear that outcomes based on fixed matching could display more cooperative behavior than in the theory. However, this is not an issue in our setup: since voters have common values, there is no gap between a potential "cooperative equilibrium" and the equilibrium in a one-shot game.<sup>18</sup> Hence, the only relevant effect of fixed matching is that it may facilitate the subjects' learning about the equilibrium selected by the group. For instance, Forsythe et al. (1993, 1996) observe that Duverger's Law equilibria emerge more easily among voters with a common history. Since we are more interested in the equilibrium properties of the voting systems under consideration than out-of-equilibrium coordination failures, fixed matching emerges as a natural choice. Finally, given equilibrium multiplicity under plurality and equilibrium uniqueness under AV, fixed matching essentially stacks the

<sup>17</sup>In the first round of experiments (the seven sessions with the groups 1, 2, 7, 8, 9, 10, 13, 14, 15, 16, 19, 20, 21 and 22), we selected seven periods to determine payment. We reduced this to four periods after realizing that the experiment had taken much less time than expected. We find no difference in behavior between these two sets of sessions.

<sup>18</sup>In the setup of the Condorcet Jury Theorem, Ali et al. (2008) find no significant difference between random matching (or ad hoc committees) and fixed matching (or standing committees).



deck against the latter in welfare comparisons.

Several reasons justify our choice of a large number of repetitions. First, repetitions allow subjects to get familiar with the trade-offs of the environment. Second, despite fixed matching, we anticipated that subjects would need time to (i) solve the coordination problems under plurality (due to the co-existence of informative and Duverger’s Law equilibria), and (ii) fine-tune their mixed strategies under AV. A large number of repetitions allows us to (i) identify possible learning effects on those dimensions, and (ii) study how the systems perform once they had time to adjust their strategy. Last but not least, we can still capture possible coordination failures or departures from the right equilibrium mix, and their effects on systems’ performances, by focusing on the first periods of the experiment. Accordingly, we distinguish between the results in the first and the last 50 periods.

## 5.1 Hypotheses

In this section, we distill our theoretical results into testable hypotheses. Hypotheses H1-H3 refer to voting behavior while H4 refers to welfare comparisons.

**Hypothesis 1** *The frequency of sincere voting will be (weakly) higher in treatment PL7 than in treatment PL9.*

**Hypothesis 2** *The frequency of double voting will be higher in treatment AV7 than in treatment AV9.*

**Hypothesis 3** *Voters will compensate for biases in the signal structure, that is:*

**H3a:** *Conditional on converging to an informative equilibrium, the frequency of sincere voting under PL7B will be lower for blue than for red voters.*

**H3b:** *The frequency of double voting under AV7B will be higher for blue than for red voters.*

**Hypothesis 4** *For each combination of  $n_C$  and signals, average payoffs will be higher under AV than under PL.*

## 6 Experimental Results

### 6.1 Unbiased Treatments

#### 6.1.1 Plurality

As shown in Section 3, two types of equilibria coexist under plurality when information is unbiased: in Duverger’s Law equilibria, participants should disregard their signal and coordinate on always voting blue or always voting red. In sincere voting equilibria, participants should vote their signal. Table 2 shows the average frequencies with which subjects voted sincerely (we call this *voting the signal*), for the color opposite to their signal (we will call this *voting opposite*) or for gray.

In the presence of a small minority (PL7), the subjects’ voting behavior is consistent with sincere voting: taking an average across all groups and periods, 91.38% of the ballots were sincere in PL7, with a lowest value of 86.42% in one independent group. This behavior is quite stable over time: regressing the frequency of “voting the signal” on the period number, we find that the coefficient is not significantly different from zero ( $p = 0.648$ ). This evidence of sincere voting stands in stark contrast with the findings of Forsythe *et al.* (1996), which only found convergence to Duverger’s Law Equilibria.

Voting behavior is substantially different in the presence of a large minority (PL9). First, only 63.86% of the observations are consistent with sincere voting. This percentage is significantly lower than the one observed in PL7 (Mann-Whitney,  $z = 2.882$ ,  $p < 0.01$ ) and, hence, Hypothesis H1 is validated.<sup>19</sup> Second, the frequency of voting the signal significantly decreases over time: the predicted frequency of sincere voting drops from 73.90% in the first periods to 53.70% in the last period. That is, participants begin the experiment by voting sincerely (94.44% of them voted their signal in the first period), and then they adjust their behavior by increasingly voting against their signal. This pattern actually reveals a progressive shift from a sincere voting equilibrium to a Duverger’s Law equilibrium, with sometimes significant coordination failures in the interim periods. Figure 1 illustrates this shift. As one can see, all six groups eventually converged to a Duverger’s Law equilibrium.

This raises two empirical questions regarding equilibrium selection. The first one is *why* all groups selected a Duverger’s Law equilibrium in the PL9 treatment, and the informative equilibrium in the PL7 treatment. The second question is *how* each PL9 group selected its

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<sup>19</sup>In all nonparametric tests we used a group as an independent observation, because from period 2 onwards, individual choices were affected by observing other group members. Unless otherwise noted, we aggregated the data across all 100 periods in a matching group.

Treatment	Minority Size		Periods	Periods	Equilibrium	
			1-50	51-100	Sincere Voting	Duverger's Law
PL7	Small	Signal	91.80	90.94	100.00	50.00
		Opposite	7.78	8.89	-	50.00
		Gray	0.42	0.17	-	-
PL9	Large	Signal	68.47	59.25	100.00	48.92*
		Opposite	31.11	40.67	-	51.08*
		Gray	0.41	0.08	-	-

Table 2: Aggregate voting behavior in plurality treatments with unbiased information, separated by first and second half, and equilibrium predictions. \*In the case of Duverger's Law in PL9, the prediction is adjusted to the color that each group converged to.

Duverger's Law equilibrium.

We can identify at least two reasons why Duverger's Law equilibria are the most natural focal point in PL9: first, according to Example 1, the expected utility in the informative equilibrium is 69.76 in PL9, instead of 152.76 in PL7. This compares with an expected utility of 155 in a Duverger's Law equilibrium. The incentive to get away from sincere voting is thus more important in PL9. Second, the range of strategy profiles for which sincere voting is a best response is quite narrow in the case of PL9. The phase diagrams in Figure 2 illustrate this graphically. One can readily see that the attraction zone of the sincere voting equilibrium is much larger in PL7 than in PL9.<sup>20</sup> Therefore, even relatively small departures from sincere voting make it optimal to vote for the leading majority candidate in PL9.

Turning to the second question, most groups coordinated on the first color that obtained strictly more than six of the majority votes.<sup>21</sup> This is in line with the findings of Forsythe et al. (1993, p235): "*a majority candidate who was ahead of the other in early elections tended to win the later elections, while the other majority candidate was driven out of subsequent races*". Yet, the transition from sincere voting to the selected Duverger's Law equilibrium can take a substantial amount of time: the first period from which either blue or red consistently obtained enough votes to win was 50, 59, 83, 63, 21 and 26 for groups 7-12 respectively. This shows that experiments using shorter horizons may fail to capture equilibrium convergence (the welfare consequences of these coordination failures are analyzed in Section 6.5).

<sup>20</sup>Confronting the strategies actually played by the subjects to these theoretical predictions, we found that, even in early periods, the typical voting realization falls outside the sincere voting attraction zone in PL9, and inside that zone in PL7.

<sup>21</sup>It happened in period 1 for four groups and in period 2 for one group. The only exception is group 11, where blue got 7 votes in the first period and then red received more votes from period 2 onwards.

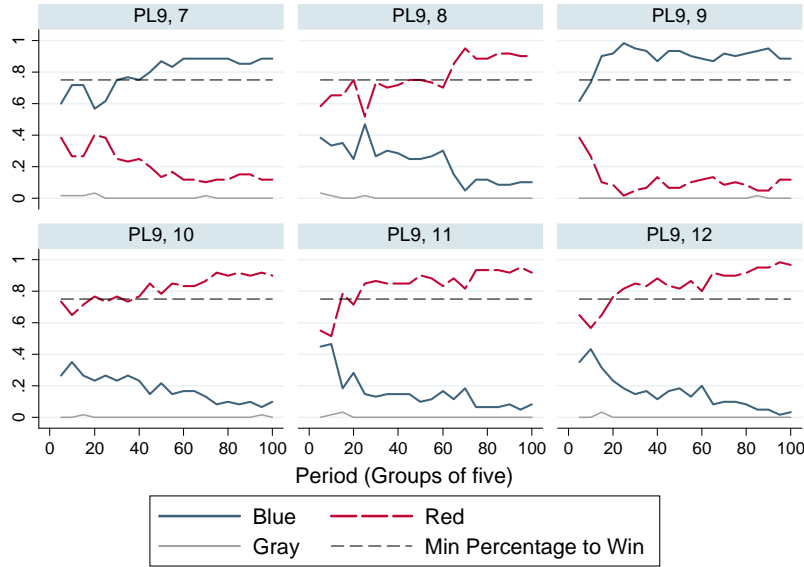


Figure 1: Frequency of voting blue, red and gray, irrespective of the signal in groups of treatment PL9. The dashed line indicates the minimum frequency of vote share required to defeat gray (in case nobody from the majority votes for the Condorcet loser).

### 6.1.2 Approval Voting

Table 3 summarizes the subjects' behavior in AV treatments. These two treatments reproduce the parametric cases covered in Example 3, which we found to display a unique symmetric equilibrium. In that equilibrium a voter should only single-vote her signal or double-vote blue and red. More than 94% of actions were in line with this prediction.

The second prediction drawn from Example 3 refers to the effect of minority size: it should increase the frequency of double-voting (Hypothesis H2). Table 3 shows that this is indeed the way in which the subjects adapted their behavior: the percentage of double-voting was multiplied by more than two, from 23.29% in treatment AV7 to 48.66% in treatment AV9. This difference is significant at 1% (Mann-Whitney,  $z = 2.722$ ,  $p < 0.01$ ). We observe an increase of the frequency of double voting over time. However, the average frequency of double voting in AV9 is higher than in AV7 in every single period (and statistically significant at 10% in all but periods 1, 2, 3, 4 and 6).

Although the comparative statics go in the direction predicted by theory, the amount of double-voting was well below theoretical predictions. These differences are significant at 5% in both AV7 and AV9 (Mann-Whitney,  $z = 2.201$ ,  $p < 0.05$ ). One might think that

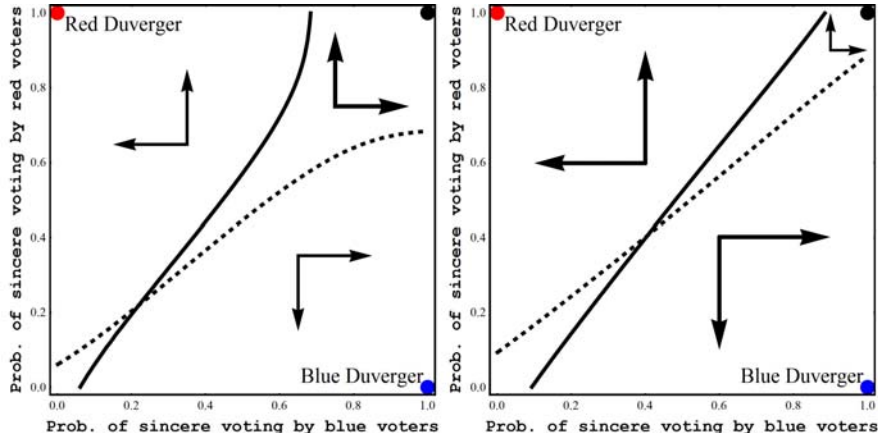


Figure 2: Phase diagram of treatments PL7 and PL9. The horizontal axis displays the probability of sincere voting by blue voters while the vertical axis displays the probability of sincere voting by red voters. The solid line indicates the indifference curve for the blue voters, while the dashed line indicates the indifference curve for the red voters. The arrows help identify the attraction zones of each of the three equilibria mentioned.

risk aversion (or, more precisely, the lack of risk aversion) helps explain this discrepancy. However, we do not find any significant relation between a subject's level of risk aversion and her propensity to double-vote. Another possibility for this discrepancy is that it is more costly for subjects to double-vote than to single-vote (they have to click twice instead of once). A third, and perhaps more subtle, possibility is that subjects did not play mixed strategies and coordinated on an asymmetric equilibrium. We discuss this in Section 6.3.

Finally, we note that aggregate behavior is homogeneous across independent groups in AV7. In contrast, there is one group in AV9 (group 22) for which behavior differs substantially from the other five groups: in group 22, 17.33% of the votes were single-votes against the signal. According to our theoretical predictions, this should not happen in equilibrium. Looking closer at individual voting data, we observe that four subjects consistently single-voted red irrespective of their signal. This implies that red was consistently winning the elections, which prevented information aggregation.<sup>22</sup>

<sup>22</sup>Aggregate measures displayed in Table 3 are robust to the exclusion of group 22. In the last 50 periods, for example, the level of double voting would switch from 51.64 to 52.80 and the level single voting would switch from 43.33 to 44.63. In the same vein, the tests provided throughout the section are also robust to the exclusion of group 22.

Treatment	Minority Size		Periods	Periods	Equilibrium
			1-50	51-100	
AV7	Small	Signal	70.92	71.94	64.00
		Double Vote	22.22	24.36	36.00
		Opposite	6.50	3.69	-
		Gray	0.36	0.00	-
AV9	Large	Signal	47.08	43.33	30.00
		Double Vote	45.67	51.64	70.00
		Opposite	6.86	4.97	-
		Gray	0.39	0.06	-

Table 3: Aggregate voting behavior in approval voting treatments with unbiased information. Gray refers to voting for gray or a combination of gray and others.

## 6.2 The Effects of Biased Information

In PL7, we observed that all independent groups coordinated on the sincere voting equilibrium. One reason might be the symmetry between the blue and red signals, which made coordination challenging for the subjects. To test whether this is the case, treatment PL7B instead makes the signal structure strongly biased in favor of the blue signal by setting  $r(\text{blue ball} \mid \text{blue jar}) = 8/9$ . So, if the voters were to keep playing sincere, blue would win disproportionately more often than red. Propositions 1 and 3 show that voters may still coordinate on either the Duverger’s Law equilibrium or on the informative equilibrium in which blue voters should mix between voting blue and voting red (see Example 2).

In the experiment, we observe that one independent group (group 28) coordinated on the “blue” Duverger’s Law equilibrium. The other five adopted a strategy coherent with the informative equilibrium of Example 2. Let us analyze each in turn: in group 28, almost all subjects cast a blue ballot as of period 31. From that period onwards, blue consistently obtained enough votes to win. Table 4 summarizes the behavior of the other five independent groups. Looking at the last 50 periods, we observe that, in line with theoretical predictions, red subjects voted sincerely with a higher probability than blue subjects. The difference between these two behaviors is statistically significant (Wilcoxon,  $z = 2.023, p < 0.043$ ). This result is in line with Hypothesis H3a. Interestingly, this pattern only appears after subjects had enough time to solve the coordination problem and fine-tune their strategies. Indeed, the difference is not statistically significant in the first 30, 40, or 50 periods. Actually, despite some heterogeneity across groups, there is no statistically significant difference between the theoretical prediction and the observed frequency of voting blue when getting a blue ball (Mann-Whitney,  $z = 0.405, p = 0.686$ ). Note however that,

	Periods	Periods	Equilibrium
	1-50	51-100	
Signal if blue	92.99	90.75	91.53
Opposite if blue	6.89	8.38	8.47
Signal if red	96.39	97.48	100
Opposite if red	3.13	1.74	0
Gray	0.27	0.83	0

Table 4: Aggregate voting behavior in treatment PL7B. Group 28 was excluded given that it converged to a Duverger’s Law equilibrium.

as pointed out in Brown and Rosenthal (1990) and subsequent papers (see, e.g. Cason and Friedman 1993, and Cason et al. 2010), the fact that average play converges to the one predicted in equilibrium need not imply that behavior does. We explore this issue further in Section 6.3.

The model helps identify two reasons why the informative equilibrium is more likely to be selected. First, it yields a higher expected payoff than Duverger’s Law equilibria (178.37 instead of 155). Second, as identified by the phase diagram in Figure 3, when starting from sincere voting (the top-right corner), the local dynamics of individual best responses point towards the informative mixed strategy equilibrium (the black dot on the graph) rather than towards either Duverger’s Law equilibria. This provides additional evidence that informative equilibria are empirically relevant when voters face aggregate uncertainty.

Turning to approval voting, the only difference between treatments AV7B and PL7B is that subjects can exploit the possibility of double-voting instead of having to vote for or against their signal. According to Example 4, blue voters should double-vote more often than red voters, and no subject should single-vote against his or her signal. Table 5 shows that the subjects’ behavior was in line with this prediction. The difference between the blue and red voters is significant not only for the second half of the sample but also for the whole experiment (Mann-Whitney,  $z = 2.201$ ,  $p = 0.028$ ). This confirms Hypothesis H3b.

### 6.3 Asymmetric Equilibria with Approval Voting

Let us now focus on individual voting behavior. We saw in Section 6.1.2 that the fraction of subjects who double-vote is lower than predicted by the theory in a symmetric equilibrium. To obtain a better understanding of this gap between the theory and the observations, we can delve deeper into individual behavior. Figure 4 disaggregates voting behavior at the individual level in the last 50 periods of treatments AV7 (left panel) and AV9 (right

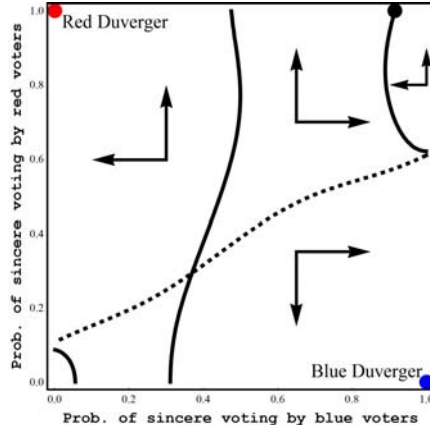


Figure 3: Phase diagram of treatment PL7B. The horizontal axis displays the probability of sincere voting by blue voters, while the vertical axis displays the probability of sincere voting by red voters. The solid line indicates the indifference curve for the blue voters; the dashed line indicates the indifference curve for the red voters.

	Periods 1-50	Periods 51-100	Equilibrium
Signal if blue	66.95	61.16	50.1
Double-vote if blue	29.87	37.16	49.9
Signal if red	74.56	80.52	92.6
Double-vote if red	20.52	17.98	7.4
Opposite	2.94	1.56	0
Gray	0.89	0.06	0

Table 5: Aggregate Voting Behavior in treatment AV7B.

panel).<sup>23</sup>

According to Theorem 2, in a symmetric equilibrium, subjects should adopt the same strategy of mixing between voting their signal and double-voting. If all subjects voted in this way, all the circles in Figure 4 should be located at the same point on the negative diagonal between (0,1) and (1,0). While most circles are indeed on this diagonal, we observe that very few subjects are in the vicinity of the orange triangle, which describes the predicted symmetric strategy.<sup>24</sup> Instead, we observe two opposite subject clusters: one that plays the

<sup>23</sup>See supplementary appendix for similar figures in all other treatments. Interestingly, we do not observe such asymmetric behavior in plurality treatments.

<sup>24</sup>Fewer observations are located along this negative diagonal in the first 50 periods. This shows again that, when subjects are given more time, they do fine-tune their strategy, which becomes progressively closer to theoretical predictions (see Section 6.4).



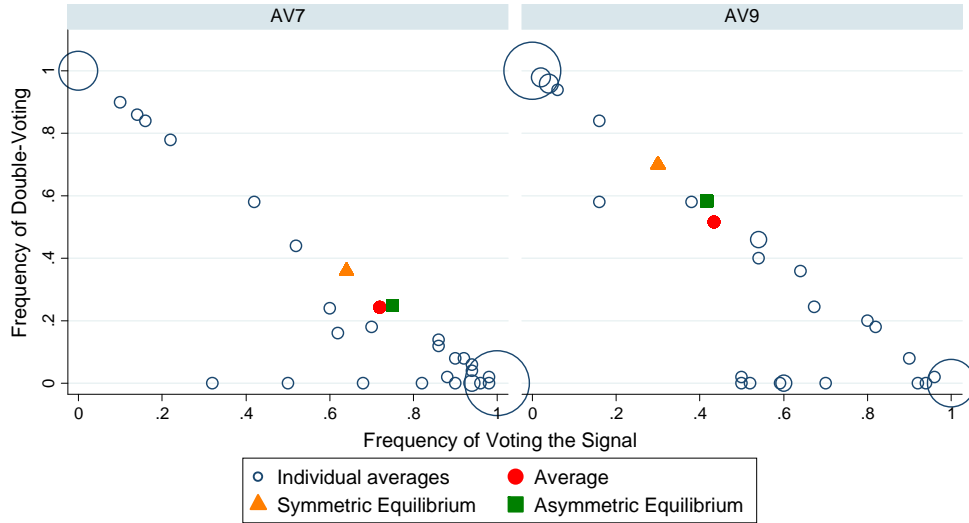


Figure 4: Individual behavior in AV treatments with unbiased information. Each hollow circle in the graph corresponds to the observed frequency of play: its size represents the number of subjects who actually adopted that frequency of play. The red circle represents the average frequency of play observed, the orange triangle represents the symmetric equilibrium prediction and the green square represents the asymmetric equilibrium prediction.

pure strategy of (almost) always double-voting and another one with subjects who (almost) always single-vote their signal – 84.72% of the subjects either voted their signal or double-voted at least 75% of the times in the last 50 periods of the experiment, and 73.77% played a pure strategy of casting the same ballot 100% of the times. The treatment effect between AV7 and AV9 observed in Section 6.1.2 is mainly driven by a switch in the relative number of subjects in each cluster: in treatment AV7, 20.83% double vote at least 75% of the time, while this number increases to 45.83% in the case of AV9 (Mann-Whitney,  $z = 2.934$ ,  $p = 0.003$ ).<sup>25</sup>

This pattern points at the need to consider asymmetric strategies. Pushing the line of reasoning of McLennan (1998) and Ahn and Oliveros (2013) further, allowing for asymmetric strategies can be interpreted as another extension of the group’s choice set, which can increase expected welfare. Allowing some voters to specialize in double or single-voting may produce significant advantages. The challenge is to identify potential equilibria by relaxing the assumption of symmetric strategies, ubiquitous as it is in the voting literature.<sup>26</sup>

<sup>25</sup>The number of players who double vote is quite homogeneous across independent groups: 2, 2, 2, 3, 3 and 3 in AV7, versus 6, 6, 5, 4, 7 and 5 in AV9.

<sup>26</sup>There are noticeable exceptions such as McLennan (1998), and Ladha, Miller and Oppenheimer (1996).

Relaxing the constraint that voters who receive the same signal must play the same strategy, the following proposition proves, for a broad set of parameter values (including the ones used in the experiment), the existence of at least one equilibrium in which voters play asymmetric strategies. It also characterizes this *asymmetric equilibrium*: voters specialize independently of their signal in either single-voting or double-voting. That is, some voters always single-vote and others always double-vote. If the signal structure is sufficiently unbiased, all “single-voters” vote their signal, i.e.  $A$  if signal  $s_A$  and  $B$  if signal  $s_B$ . If the bias in the signal structure is stronger, then the voters receiving the less abundant signal vote sincerely whereas those who receive the more abundant signal mix between  $A$  and  $B$ .

**Proposition 4** *Suppose that  $q(a) = q(b)$ ,  $r(s_A|a) \geq r(s_B|b)$  and  $V \leq 2v$ . Any strategy profile satisfying the following conditions is an asymmetric equilibrium:*

1.  $2n_C - n + 1$  voters always double-vote;
2. The rest of the voters single-vote informatively with  $\sigma_{s_B}^{1v}(B) = 1$  and

$$\sigma_{s_A}^{1v}(A) = \begin{cases} \frac{\rho^{\frac{n-n_C}{n-n_C-1}} - 1}{\rho^{\frac{n-n_C}{n-n_C-1}} r(s_A|a) - r(s_A|b)} & \text{if } \rho^{\frac{n-n_C}{n-n_C-1}} > \frac{r(s_B|b)}{r(s_B|a)}, \text{ where } \rho = \frac{r(s_A|a)}{r(s_A|b)}, \\ 1 & \text{if } \rho^{\frac{n-n_C}{n-n_C-1}} \leq \frac{r(s_B|b)}{r(s_B|a)} \end{cases},$$

where  $\sigma_s^{1v}(\psi)$  is the probability that a single-voter of type  $s$  plays action  $\psi$ .

**Proof.** See supplementary appendix. ■

Such an asymmetric behavior is an equilibrium because the voters who specialize in single-voting perceive the expected payoff of each ballot differently from voters specializing in double-voting. In particular, “single-voters” are pivotal only when  $A$ ,  $B$ , and  $C$  receive exactly the same number of votes, whereas “double-voters” are pivotal if either  $A$  is trailing behind by one vote or if it is leading by one vote. The best responses of these two groups of voters are thus different. The following example illustrates this result in more detail.

**Example 5** *Assume (as in Example 3)  $n = 12$ ,  $n_C = 7$ , and  $q(s_A|a) = q(s_B|b) = 2/3$ . In the asymmetric equilibrium,  $2n_C - n + 1 = 3$  voters double-vote, and the other 9 single-vote their signal.*

*Compared with the symmetric equilibrium, the aggregate level of double-voting decreases from 36% to 25%, but this is enough to ensure that the Condorcet loser never wins the*

*election. Indeed, with 3 double-votes and 9 single-votes, one of the two majority alternatives must receive at least 8 votes, i.e. strictly more than the Condorcet loser. Finally, the likelihood of choosing the best candidate increases from 82% in the symmetric equilibrium to 85.5%. Information aggregation is improved because the (expected) number of voters who reveal their information, i.e. the expected number of single-voters, is larger in this asymmetric equilibrium than in the symmetric one (9 vs. 7.68).*

Such asymmetric equilibria under AV appear to organize laboratory data better than the symmetric equilibrium. In treatment AV7, the predicted level of double-voting in the asymmetric equilibrium is 25%, to be compared with the observed 24.46% in the laboratory. This difference is not significant (Wilcoxon,  $z = -0.524$ ,  $p = 0.60$ ). In the case of AV9, the predicted level of double-voting is 58.33% compared to the observed 51.64%. The difference is still significant (Wilcoxon,  $z = 2.201$ ,  $p = 0.028$ ), although the gap is much smaller than with the symmetric equilibrium.

Three remarks are in order. First, the equilibrium in Proposition 4 makes an interesting prediction for the biased treatment AV7B: the level of double-voting should be independent of the signal structure. This is not what we observe in the data (see Table 5). Second, the existence of such an asymmetric equilibrium does not invalidate the welfare results in Theorem 1: indeed, the logic of the proof is the same. Finally, it is possible that fixed matching facilitated the emergence of asymmetric equilibria.

## 6.4 On Voter Rationality

A heated debate in political science is whether one may take seriously the assumption that voters behave strategically (see Section 3.2 for a short discussion). We explore this question in three steps. First, we analyze whether subject behavior follows some minimal criteria of self-interest. Second, we study the groups' behavior across treatments to confront the sincere voting and strategic voting hypotheses. Third, we assess the fraction of subjects who adopted a rational voting behavior.

A minimal criterion of self-interest is to check whether subjects played dominated strategies. We find that less than 2.1% of the subjects chose a dominated action more than 5% of the times – this pattern is constant across treatments. A stricter test is whether subjects play actions that are not in their best-response set. Concretely, this means not voting sincerely in PL7, and not voting for same color as the rest of the group in PL9 (*i.e.* one action out of three is in each voter's best-response set). In AV, the best response set con-

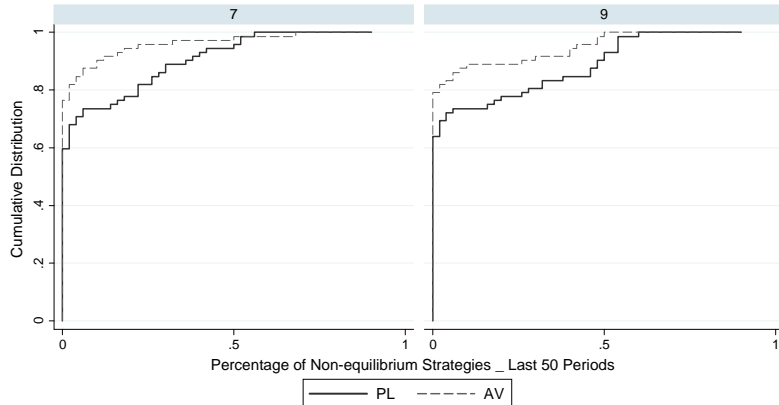


Figure 5: Cumulative distribution of percentage of each subject’s propensity to cast ballots not in their best response set in the last 50 periods. The left (right) panel describes behavior under a minority of 7 (9).

tains sincere single- and double-voting (*i.e.* two actions out of seven). Figure 4 illustrates the outcome. Two clear conclusions emerge: first, more than 70% of the subjects played a non-equilibrium strategy less than 5% of the time. Second, subjects made more mistakes under plurality than under AV (see also Section 6.5).

We need other metrics to discriminate between strategic and sincere behavior. For instance, if 100% of the subjects single voted their signal in AV9, then 100% of the data are both in the undominated and the best-response sets, despite the fact that this cannot be an equilibrium behavior. Exploiting the testable implications of our model, we can reject the hypothesis that all subjects behave sincerely. In particular, (1) 40.67% of the votes in PL9 were incompatible with sincere voting (see Table 2). (2) In line with Hypothesis 2, the voters’ propensity to double vote is significantly higher in AV9 than in AV7. (3) In line with Hypothesis 3, subjects compensated for the biased signals in AV7B and PL7B. (4) In line with Theorem 2, the switch from PL7 to AV7 produced a clear drop in “single-vote the signal” (from 90.94% to 71.94%) and the switch from PL9 to AV9 induced subjects to essentially stop “single-voting opposite” (from 40.67% to 4.97% – these differences are statistically significant: Mann-Whitney,  $z = 2.882$ ,  $p < 0.01$  in both cases).

Note still that similar arguments can be made to exclude the reverse hypothesis that all subjects behaved as predicted by the model: in PL9, for instance, 15.28% of the subjects voted sincerely more than 90% of the times. An obvious question is thus to assess which fraction of subjects behaved “rationally” according to the model. To do so, we identify

a lower and an upper bound on the fraction of subjects displaying a behavioral response that is only compatible with strict criteria of rational behavior. First observe that, in the last 50 periods of PL9, 72.23% of the subjects voted for the majority candidate more than 90% of the times (and 64% of the subjects voted so all the time). Second, observe that the propensity to double vote increased by 27.78% between AV7 and AV9. In summary, a lower bound is 27.78% and an upper bound 72.23%. This figure is in line with recent findings on observational data in the literature (Kawai and Watanabe 2013, Spenkuch 2013).

## 6.5 Welfare

Previous laboratory experiments about multicandidate elections were based on theories that are inconclusive when it comes to comparing welfare across voting systems (this is the case, for instance, with the theoretical predictions of Myerson and Weber 1993 used in Forsythe et al. 1996). A valuable feature of our common value setup is that it allows for such direct comparisons: by Theorem 1, in equilibrium, the active voters' payoff should be strictly higher with AV than with plurality.

Table 6 (in columns 2 and 3) displays the average payment obtained by the subjects in each treatment, respectively for the first and second 50 periods. Comparing PL and AV treatments two by two, one can see that realized payoffs are systematically higher in AV treatments. All these differences are significant at the 1% confidence level.<sup>27</sup> This validates Hypothesis H4. This comparison is robust to various time windows. If we consider groups of 10 periods, the average payoff is always higher under AV than under PL (and significantly higher in 20 out of 30 cases). Interestingly, realized payoffs under AV are relatively close to the payoffs that a benevolent dictator would achieve if informed about all the signals in the group (191.00, 188.90 and 197.60 in the last 50 periods for AV7, AV9 and AV7B, respectively).

These results lead to several conclusions. First, plurality performs relatively poorly both when coordination problems are salient and when they have been resolved. Why AV performs better even in early periods can be due to several reasons. For instance, (i) subjects fine-tune their strategies much faster under AV than they can solve coordination problems under plurality. (ii) AV does not require full fine-tuning to perform better than plurality. (iii) Plurality performs very poorly unless coordination problems have been fully addressed.

Regarding point (i), we observed in Section 6.4 that subjects make fewer “mistakes” in

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<sup>27</sup>Mann-Whitney tests are:  $z = 2.882$  and  $p\text{-value} = 0.0039$  for AV7-PL7,  $z = 2.722$  and  $p\text{-value} = 0.0065$  for AV9-PL9, and  $z = 2.913$  and  $p\text{-value} = 0.0036$  for AV7B-PL7B.

Treatment	Periods	Periods	Equilibrium	
	1-50	51-100	Symmetric*	Asymmetric
PL7	136.70	138.50	154.87	-
PL9	101.15	147.80	156.20	-
PL7B	169.85	171.95	178.76	-
AV7	167.00	183.95	179.65	189.80
AV9	146.75	168.95	164.70	181.10
AV7B	188.90	192.50	193.58	194.68

Table 6: Average payoff and theoretical predictions. \* In the case of plurality, equilibrium predictions refer to the equilibrium where experimental groups converged to.

AV than in plurality. This happens despite the fact that voters have to choose from a larger choice set in AV than in plurality. This suggests that the “individual complexity” of having to choose from a larger choice set is actually much simpler to address than the “strategic complexity” of having to adapt behavior to which equilibrium was selected by the other voters.

Regarding point (ii), one should realize that AV can produce quite higher welfare than PL even if subjects do not perfectly fine-tune their strategy to the equilibrium predicted by the theory. To show this, we calculate the predicted level of expected welfare under AV when voters adopt any given mixture between single and double-voting, and compare it with the case in which voters coordinate perfectly on the Duverger’s Law equilibrium under plurality, *i.e.* the payoff-maximizing equilibrium both in PL7 and PL9. This clearly stacks the deck against AV. The results shown in Figure 6 are striking: with symmetric strategies (remember from Section 6.3 that asymmetric strategies perform even better), the welfare dominance of AV over plurality is robust to large mistakes in the mix between single and double-voting. For a small minority, virtually any positive amount of double-voting ( $> 0.174\%$ ) makes AV welfare-superior to plurality. For a large minority, AV dominates plurality as long as voters double-vote with a probability higher than 45.1%. In contrast, in plurality treatments PL7 and PL9, respectively, at least 2/3 and 83.3% of the subjects must coordinate on a same color to achieve the maximum achievable payoff of 155. This shows that (a) AV does not require perfect fine-tuning to perform better than plurality, and (b) plurality performs poorly unless a very large fraction of the voters have coordinated on the same equilibrium.

Regarding point (iii), it is interesting to look at the effect of the size of the minority on welfare. In plurality, the expected payoff should be strictly decreasing in  $n_C$  in an informative equilibrium. In a Duverger’s Law equilibrium instead, it should remain independent

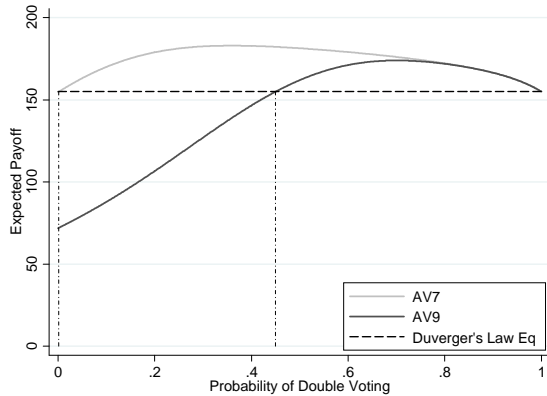


Figure 6: Expected payoff under AV for different symmetric strategies. The light (dark) gray line represents the expected payoff under AV as a function of the frequency of double voting, when the minority is small (large). The dashed line is a benchmark that represents the expected payoff of the Duverger’s Law equilibrium strategy.

of  $n_C$ , at 155. Table 6 shows an interesting reversal: in the first 50 periods, the average payoff is higher in treatment PL7 than in treatment PL9, while the opposite is true for the second half. This is explained by the substantial costs of the coordination failures that we observe in the first 50 periods in PL9. Across the entire experiment session, payoff is lower under PL9 than under PL7 (Mann-Whitney test,  $z = 1.922$ ,  $p\text{-value} = 0.0547$ ), while the opposite should hold if coordination was perfect.

## 7 Conclusions

In this paper, we adopted a theory-based experimental approach to achieve a dual objective: (i) comparing the welfare properties of plurality and approval voting, and (ii) shedding new light on the debate about whether voters behave strategically.

Our experiments confirmed that the theoretical promises of AV are largely met in practice, both at the aggregate level – welfare increases substantially – and in terms of individual responses – subjects’ strategic behavior moves in line with theoretical predictions. This gives weight to “rational-strategic” theories of voting, at least in a setup with a small electorate and a large number of repetitions that allows voters to learn how their actions map into outcomes.

Clearly, this paper can only produce part of the answers needed to form definite proposals

about how to reform our collective decision-making processes. In the future, we will also need to consider other systems (e.g. Instant-Runoff), pre-election polls, preference settings (e.g. majority voters have a mix of private and common values), and larger electorates.

## References

- Ahn, D. and S. Oliveros** (2013), Approval Voting and Scoring Rules with Common Values, *mimeo*.
- Ali, N., Goeree, J., Kartik, N. and T. Palfrey** (2008), Information Aggregation in Ad Hoc and Standing Committees, *American Economic Review*, Papers & Proceedings, 98(2), 181-186.
- Alvarez, M. and J. Nagler** (2000), A New Approach for Modelling Strategic Voting in Multiparty Elections, *British Journal of Political Science*, 30(1), 57-75
- Arrow, K.** (1951), *Social Choice and Individual Values*, New York: Wiley.
- Austen-Smith, D. and J. S. Banks** (1996), Information Aggregation, Rationality, and the Condorcet Jury Theorem, *American Political Science Review*, 90, 34–45.
- Austen-Smith, D. and T. Feddersen** (2006). Deliberation, Preference Uncertainty and Voting Rules, *American Political Science Review*, 100(2): 209-217.
- Bassi, A.** (2015). Voting systems and strategic manipulation: An experimental study, *Journal of Theoretical Politics*, 27(1): 58-85.
- Battaglini, M., Morton, R. and T. Palfrey** (2008), Information Aggregation and Strategic Abstention in Large Laboratory Elections. *American Economic Review*, Papers & Proceedings, 98(2): 194-200.
- Battaglini, M., Morton, R. and T. Palfrey** (2010), The Swing Voter’s Curse in the Laboratory. *Review of Economic Studies*, 77, 61-89.
- Bhattacharya, S.** (2012), Preference Monotonicity and Information Aggregation in Elections. *Econometrica*, 81(3), 1229-1247.
- Bhattacharya, S., J. Duffy and S. T. Kim** (2014). Compulsory versus Voluntary Voting: An Experimental Study. *Games and Economic Behavior*, 84, 11-131.
- Blais, A., R. Young and M. Turcotte** (2005), Direct or indirect? Assessing two approaches to the measurement of strategic voting, *Electoral Studies*, 24 (2), 163-176.
- Bormann, N.-C., and M. Golder** (2013). Democratic Electoral Systems around the world, 1946–2011, *Electoral Studies*, 32(2), 360–369.
- Bouton, L.** (2012), A Theory of Strategic Voting in Runoff Elections. *American Economic Review*, 103(4), 1248-1288.
- Bouton, L., and M. Castanheira** (2009), The Condorcet-Duverger Trade-Off: Swing



- Voters and Voting Equilibria, in Aragonés, E., C. Beviá, H. Llavador and N. Schofield, eds.: *The Political Economy of Democracy*. Fundacion BBVA, 121-142.
- Bouton, L., and M. Castanheira** (2012), One Person, Many Votes: Divided Majority and Information Aggregation. *Econometrica*, 80(1), 43-87.
- Bouton, L., M. Castanheira and A. Llorente-Saguer** (2015), Multicandidate Elections: Aggregate Uncertainty in the Laboratory, CEPR Discussion Paper No. DP10481.
- Bouton, L., and G. Gratton** (2015), Majority Runoff Elections: Strategic Voting and Duverger's Hypothesis. *Theoretical Economics*, 10(2), 283-314.
- Borda, J.-C.** (1781), *Mémoires sur les élections au Scrutin*, Paris: Histoire de l'Académie Royale des Sciences.
- Brams, S. and P. Fishburn** (1978), Approval Voting, *American Political Science Review*, 72, 113-134.
- Brams, S. and P. Fishburn** (1983). *Approval Voting*. 1st ed., Boston: Birkhauser.
- Brown, J. and R. Rosenthal** (1990), Testing the Minimax Hypothesis: A Re-examination of O'neill's Game Experiment. *Econometrica*, 58(5), 1065-1081.
- Cason, T. and D. Friedman** (2003), Buyer Search and Price Dispersion: A Laboratory Study. *Journal of Economic Theory*, 112, 232-260.
- Cason, T., D. Friedman and E. Hopkins** (2010), Testing the TASP: An Experimental Investigation of Learning in Games with Unstable Equilibria. *Journal of Economic Theory*, 145, 2309-2331.
- Castanheira, M.** (2003), Why Vote for Losers?, *Journal of the European Economic Association*, 1, 1207-1238.
- de Condorcet, N.C.** (1785). *Essai sur l'Application de l'Analyse à la Probabilité des Décisions Rendues à la Pluralité des Voix*. Paris.
- Coughlan, P.** (2000), In Defense of Unanimous Jury Verdicts: Mistrials, Communication, and Strategic Voting, *American Political Science Review*, 94, 375-393.
- Cox, G.** (1997). *Making Votes Count*. Cambridge, UK: Cambridge University Press.
- Dellis, A., S. D'Evelyn and K. Sherstyuk** (2011), Multiple Votes, Ballot Truncation and the Two-party System: An Experiment, *Social Choice and Welfare*, 37, 171-200.
- Dewan, T. and D. Myatt** (2007), Leading the Party: Coordination, Direction, and Communication, *American Political Science Review*, 101, 825-843.
- Duverger, M.** (1963), *Political Parties: their Organization and Activity in the Modern State*, North, B. and North R., tr. New York: Wiley, Science Ed.
- Esponda, I. and E. Vespa** (2014), Hypothetical Thinking and Information Extraction: Pivotal Voting in the Laboratory, *American Economic Journal: Microeconomics*, 6(4), 180-202.

- Feddersen, T.** (2004), Rational Choice Theory and the Paradox of Not Voting, *Journal of Economic Perspectives*, 18, 99-112.
- Feddersen, T. and Pesendorfer, W.** (1996), The Swing Voter's Curse, *American Economic Review*, 86, 408-424.
- Feddersen, T. and Pesendorfer, W.** (1997), Voting Behavior and Information Aggregation in Elections with Private Information, *Econometrica*, 65, 1029-1058.
- Feddersen, T. and Pesendorfer, W.** (1998), Convicting the Innocent: the Inferiority of Unanimous Jury Verdicts, *American Political Science Review*, 92, 23-35.
- Felsenthal, D. and M. Machover** (2012), *Electoral Systems. Paradoxes, Assumptions, and Procedures*, Editors, Berlin Heidelberg: Springer-Verlag
- Fey, M.** (1997), Stability and Coordination in Duverger's Law: A Formal Model of Pre-election Polls and Strategic Voting. *American Political Science Review*, 91(1), 135-47.
- Fischbacher, U.** (2007), z-Tree - Zurich Toolbox for Readymade Economic Experiments, *Experimental Economics*, 10, 171-178.
- Forsythe, R., R.B. Myerson, T.A. Rietz and R.J. Weber** (1993), An Experiment on Coordination in Multi-Candidate Elections: The Importance of Polls and Election Histories, *Social Choice and Welfare*, 10: 223-247.
- Forsythe, R., R.B. Myerson, T.A. Rietz and R.J. Weber** (1996), An Experimental Study of Voting Rules and Polls in Three-Way Elections. *International Journal of Game Theory*, 25, 355-83.
- Fujiwara, T.** (2011), A Regression Discontinuity Test of Strategic Voting and Duverger's Law, *Quarterly Journal of Political Science*, 6, 197-233.
- Goeree, J. and L. Yariv** (2011). An Experimental Study of Collective Deliberation, *Econometrica*, 79(3): 893-921.
- Greiner, B.** (2004), An Online Recruitment System for Economic Experiments, in *Forschung und Wissenschaftliches Rechnen 2003*, ed. by K. Kremer and V. Macho. GWDG Bericht 63. Gesellschaft für Wissenschaftliche Datenverarbeitung Göttingen: Datenverarbeitung, 79-93.
- Guarnaschelli, S., McKelvey, R., and T. Palfrey** (2000), An Experimental Study of Jury Decision Rules, *American Political Science Review*, 94(2): 407-423.
- Holt, C. and S. Laury** (2002), Risk Aversion and Incentive Effects, *American Economic Review*, 92, 1644-1655.
- Hortala-Vallve, R and A. Llorente-Saguer** (2010). "A Simple Mechanism for Resolving Conflict", *Games and Economic Behavior*, 70, 375-391.
- Kawai, K. and Y. Watanabe** (2013), Inferring Strategic Voting, *American Economic Review*, 103(2), 624-662.

- Ladha, K., G. Miller, and J. Oppenheimer** (1996), Information Aggregation by Majority Rule: Theory and Experiments. Washington University Typescript.
- Laslier, J.-F.** (2010), The Leader Rule: A Model of Strategic Approval Voting in a Large Electorate, *Journal of Theoretical Politics*, 22(1): 64-84.
- Mandler, M.** (2012). The fragility of information aggregation in large elections, *Games and Economic Behavior* 74: 257–268.
- McLennan, A.** (1998), Consequences of the Condorcet Jury Theorem for Beneficial Information Aggregation by Rational Agents, *American Political Science Review*, 92, 413-418.
- Morton, R. and J.-R. Tyran,** (2012), Ethical vs. Selfish Motivations and Turnout in Small and Large Elections, *mimeo*.
- Myatt, D.** (2007), On the Theory of Strategic Voting, *Review of Economic Studies*, 74: 255-281.
- Myerson, R.** (1998), Extended Poisson Games and the Condorcet Jury Theorem, *Games and Economic Behavior*, 25, 111-131.
- Myerson, R.** (1999), Theoretical Comparison of Electoral Systems, *European Economic Review*, 43, 671-697.
- Myerson, R.** (2002), Comparison of Scoring Rules in Poisson Voting Games, *Journal of Economic Theory*, 103, 219-251.
- Myerson, R. and R. Weber,** (1993), A Theory of Voting Equilibria, *American Political Science Review*, 77, 102–14.
- Nagel, J.** (2007). "The Burr Dilemma in Approval Voting." *Journal of Politics*, 69, 43-58.
- Niemi, R.G.** (1984), "The Problem of Strategic Behavior under Approval Voting", *American Political Science Review*, 78(4), 952-958
- Nuñez, M.** (2010), "Condorcet Consistency of Approval Voting: A Counter Example in Large Poisson Games", *Journal of Theoretical Politics*, 22(1), 64-84.
- Palfrey, T.** (1989), "A Mathematical Proof of Duverger's Law", *In Models of Strategic Choice in Politics*, edited by P. C. Ordeshook. Ann Arbor, Michigan: University of Michigan Press.
- Palfrey, T.** (2013), "Experiments in Political Economy", forthcoming in *Handbook of Experimental Economics* Vol. 2 (J. Kagel and A. Roth, eds.), Princeton University Press.
- Persico, N.** (2004). Committee Design with Endogenous Information, *Review of Economic Studies*, 71: 165-191.
- Piketty, T.** (2000), Voting as Communicating, *Review of Economic Studies*, 67, 169-191.
- Rapoport, A., D. Felsenthal, Z. Maoz** (1991), Sincere versus strategic voting behavior in small groups. In: Palfrey T (ed) Laboratory research in political economy. The University of Michigan Press, AnnArbor, 201–235.

- Rietz, T.** (2008), “Three-Way Experimental Election Results: Strategic Voting, Coordinated Outcomes and Duverger’s Law”, *Handbook of Experimental Economics*.
- Riker, W.** (1982), The Two-Party System and Duverger’s Law: An Essay on the History of Political Science, *American Political Science Review*, 84, 1077-1101.
- Saari, D. and J. Van Newenhizen** (1988), “The problem of indeterminacy in approval, multiple, and truncated voting systems”, *Public Choice* 59, 101-120
- Spenkuch, J.** (2012), On the Extent of Strategic Voting, *mimeo*, University of Chicago.
- Van der Straeten, K., Laslier, J.-F., Sauger, N., and Blais, A.** (2010), Strategic, Sincere, and Heuristic Voting under Four Election Rules: An Experimental Study, *Social Choice and Welfare*, 35, 435-472.
- Weber, R.** (1995), Approval Voting, *Journal of Economic Perspectives*, 9, 29-49.

# Appendices

## Appendix A1: Plurality

### Pivot Events, Probabilities, and Expected Gains

When deciding for which alternative to vote, a voter must assess the expected value of each feasible action, which depends on *pivot events*. We denote by  $piv_{QP}$  the pivot event that one voter's ballot changes the outcome from a victory of  $P$  towards a victory of  $Q$ . The probability of  $piv_{QP}$  in state  $\omega$  is denoted  $p_{QP}^\omega$ .

The fact that  $n_C > n/2$  implies that the probability of being pivotal between  $A$  and  $B$  is zero:  $\Pr(piv_{AB}) = 0$ . Only  $\Pr(piv_{AC})$  and  $\Pr(piv_{BC})$  can be strictly positive. A direct corollary is that voting for  $C$  and abstaining are dominated actions. It remains to check which of actions  $A$  and  $B$  have the highest expected payoff.

A ballot, say  $A$ , can only be pivotal if  $x_A \in \{n_C - 1, n_C\}$ . To assess the probability of such an event, a voter must identify the distribution of the other  $n - 1$  votes, given the strategy  $\sigma$ . Dropping  $\sigma$  from the notation for the sake of readability, the pivot probabilities in favor of  $A$  and  $B$  are:

$$p_{AC}^\omega \equiv \Pr(piv_{AC}|\omega, \text{Plurality}) = \frac{(n-1)!}{2} \frac{(\tau_A^\omega)^{n_C-1} (\tau_B^\omega)^{n-n_C-1}}{(n_C-1)!(n-n_C-1)!} \left[ \frac{\tau_A^\omega}{n_C} + \frac{\tau_B^\omega}{n-n_C} \right], \quad (3)$$

$$p_{BC}^\omega \equiv \Pr(piv_{BC}|\omega, \text{Plurality}) = \frac{(n-1)!}{2} \frac{(\tau_B^\omega)^{n_C-1} (\tau_A^\omega)^{n-n_C-1}}{(n_C-1)!(n-n_C-1)!} \left[ \frac{\tau_B^\omega}{n_C} + \frac{\tau_A^\omega}{n-n_C} \right], \quad (4)$$

where the two terms between brackets represent the cases in which one vote respectively breaks and makes a tie. Note that pivot probabilities are continuous in  $\tau_A^\omega$  and  $\tau_B^\omega$ . The expected gains of action  $A$  and  $B$  over abstention are thus:

$$G(A|s) = q(a|s) p_{AC}^a V + q(b|s) p_{AC}^b v (> 0), \quad (5)$$

$$G(B|s) = q(a|s) p_{BC}^a v + q(b|s) p_{BC}^b V (> 0), \quad (6)$$

and the payoff difference between the two is:

$$G(A|s) - G(B|s) = q(a|s) [V p_{AC}^a - v p_{BC}^a] + q(b|s) [v p_{AC}^b - V p_{BC}^b], \quad (7)$$

#### 7.0.1 Proofs of Section 3

**Proof of Proposition 1..** Consider *e.g.*  $\sigma_{s_A}(A) = \varepsilon$  and  $\sigma_{s_B}(B) = 1$ . From (3) and (4), we have:

$$\frac{p_{AC}^\omega}{p_{BC}^\omega} = \left( \frac{\tau_A^\omega}{\tau_B^\omega} \right)^{2n_C-n} \frac{\tau_A^\omega (n-n_C) + \tau_B^\omega n_C}{\tau_A^\omega n_C + \tau_B^\omega (n-n_C)} \xrightarrow{\varepsilon \rightarrow 0} 0.$$

Hence, from (7), we have that  $G(A|s) - G(B|s) < 0$  for any  $\varepsilon$  in the neighborhood of 0. ■

**Proof of Proposition 2.** We start with the unbiased case, *i.e.*  $r(s_A|a) = r(s_B|b)$ . Under sincere

voting,  $\sigma_{s_A}(A) = 1 = \sigma_{s_B}(B)$ , (3) and (4) imply  $p_{AC}^a = p_{BC}^b > p_{AC}^b = p_{BC}^a$ . Then, from (7):

$$G(A|s) - G(B|s) = [Vp_{AC}^a - vp_{BC}^a] [q(a|s) - q(b|s)].$$

Hence  $G(A|s_A) - G(B|s_A) > 0 > G(A|s_B) - G(B|s_B)$ , since  $q(a|s_A) - q(b|s_A) > 0 > q(a|s_B) - q(b|s_B)$ . Sincere voting is thus an equilibrium strategy. By the continuity of pivot probabilities with respect to  $\tau_A^\omega$  and  $\tau_B^\omega$ , it immediately follows that there must exist a value  $\delta(n, n_C) > 0$  such that sincere voting is an equilibrium for any  $|r(s_A|a) - r(s_B|b)| < \delta(n, n_C)$ . ■

**Proof of Proposition 3.** Consider a distribution of signals such that  $r(s_A|a) - r(s_B|b) > \delta(n, n_C)$ , in which case sincere voting is not an equilibrium. That is, there exists a signal  $\bar{s} \in \{s_A, s_B\}$  such that all the voters who received signal  $\bar{s}$  strictly prefer to deviate from a strategy profile  $\sigma^{\text{sincere}} \equiv \{\sigma_{s_A}(A), \sigma_{s_B}(B)\} = \{1, 1\}$ .

**Case 1:**  $\bar{s} = s_A$ . In this case,  $\sigma^{\text{sincere}} \Rightarrow G(A|s_A) - G(B|s_A) < 0$ . Now, consider a second strategy profile  $\sigma' \equiv \left\{ [r(s_A|a) + r(s_A|b)]^{-1}, 1 \right\}$ . With this profile, we have:  $\tau_A^a = \tau_B^b$  and  $\tau_A^b = \tau_B^a$ , and thus  $p_{BC}^b = p_{AC}^a > 0$  and  $p_{AC}^b = p_{BC}^a > 0$  and, from (7):

$$G(A|s) - G(B|s) = [Vp_{AC}^a - vp_{BC}^a] [q(a|s) - q(b|s)], \quad (8)$$

where (i)  $[Vp_{AC}^a - vp_{BC}^a]$  is positive, and (ii)  $[q(a|s) - q(b|s)]$  is positive for  $s_A$  and negative for  $s_B$ . In other words, all voters would strictly prefer to deviate from  $\sigma'$  by voting sincerely. This means that the value of  $G(A|s_A) - G(B|s_A)$  changes sign when  $\sigma_{s_A}(A)$  is increased from  $[r(s_A|a) + r(s_A|b)]^{-1}$  to 1. Since all pivot probabilities are continuous in  $\sigma_{s_A}$ , there must exist a value  $\sigma_{s_A}^*(A) \in \left( [r(s_A|a) + r(s_A|b)]^{-1}, 1 \right)$  such that voters with signal  $s_A$  are indifferent between playing  $A$  and  $B$ . It is easy to check that, for this strategy, a voter who received signal  $s_B$  strictly prefers to play  $B$ , and hence that  $\{\sigma_{s_A}(A), \sigma_{s_B}(B)\} = \{\sigma_{s_A}^*(A), 1\}$  is an equilibrium.

**Case 2:**  $\bar{s} = s_B$ . In this case,  $\sigma^{\text{sincere}} \Rightarrow G(A|s) - G(B|s) > 0$  for both signals. Now, consider another strategy profile  $\sigma'' \equiv \{\varepsilon, 1\}$ , with  $\varepsilon \rightarrow 0$  (and hence  $\sigma_{s_A}(B) \rightarrow 1$ ). From Proposition 1, this strategy profile implies  $G(A|s) - G(B|s) < 0$  for both signals. By the continuity of the payoffs with respect to  $\sigma_{s_A}(A)$ , there must therefore exist a value  $\sigma_{s_A}^{**}(A) \in (0, 1)$  such that  $G(A|s_A) - G(B|s_A) = 0$  and, by the same argument as above,  $G(A|s_B) - G(B|s_B) < 0$ . Hence, the strategy profile  $\{\sigma_{s_A}(A), \sigma_{s_B}(B)\} = \{\sigma_{s_A}^{**}(A), 1\}$  is an equilibrium.

Note that sincere stability is not a binding restriction, since all voters vote for their preferred alternative with a probability strictly larger than 0. ■

## Appendix A2: Approval Voting

### Preliminaries

Under AV, voters have a larger choice set. Single approvals ( $A, B, C$ ) have the same effect as in plurality. Double or triple approvals instead allow voters to abstain selectively. The following lemma identifies the set of dominated and undominated actions:

**Lemma 1** *Independently of a voter's signal, the actions  $\psi \in \{C, AC, BC, ABC, \emptyset\}$  are weakly dominated by some action in  $\psi \in \{A, B, AB\}$ . Hence, in equilibrium:*

$$\sigma_s(A) + \sigma_s(B) + \sigma_s(AB) = 1, \quad \forall s \in \{s_A, s_B\}. \quad (9)$$

*Proof.* Straightforward. ■

### Pivotal Events, Probabilities, and Expected Gains

Let  $\pi_{QP}^\omega$  denote the probability that a single- $Q$  ballot is pivotal in favor of  $Q$  against  $P$  in state  $\omega \in \{a, b\}$  when the voting rule is AV. There are two important differences with the probabilities  $p_{QP}^\omega$  in plurality. First,  $\pi_{AB}^\omega$  can be strictly positive since double-voting can increase the score of both  $A$  and  $B$  above that of  $C$ . Second, we need to determine the value of the double ballot  $AB$ .

The pivotal event  $piv_{AC}^{AV}$  is defined as follows:

$$\begin{aligned} x_A &> x_B - 1 \text{ and } x_A + x_{AB} \in \{n_C - 1, n_C\}, \text{ or} \\ x_A &= x_B - 1 \text{ and } x_B + x_{AB} = n_C. \end{aligned}$$

Given the multinomial distribution,  $\Pr(x|\omega) = n! \prod_{\psi \in \Psi_{AV}} \frac{\tau_\psi^\omega(\sigma)^{x(\psi)}}{x(\psi)!}$ , the probability of event  $piv_{AC}^{AV}$  in state  $\omega$  is:

$$\begin{aligned} \pi_{AC}^\omega &\equiv \Pr(piv_{AC}^{AV}|\omega, AV) = (n-1)! \sum_{i=0}^1 \sum_{x_{AB}=0}^{2(n_C-i)-n} \frac{(\tau_A^\omega)^{n_C-i-x_{AB}} (\tau_{AB}^\omega)^{x_{AB}} (\tau_B^\omega)^{(n-1)-(n_C-i)}}{2(n_C-i-x_{AB})! x_{AB}! (n-1-n_C+i)!} \\ &\quad + \frac{(n-1)! [\tau_A^\omega \tau_B^\omega]^{n-1-n_C} (\tau_{AB}^\omega)^{2n_C+1-n}}{3 [(n-1-n_C)!]^2 (2n_C-n+1)!} + \frac{(n-1)! (\tau_A^\omega)^{n-1-n_C} (\tau_{AB}^\omega)^{2n_C-n} (\tau_B^\omega)^{n-n_C}}{6 (n-1-n_C)! (n-n_C)! (2n_C-n)!}. \end{aligned}$$

The probability of event  $piv_{BC}^{AV}$  in state  $\omega$ ,  $\pi_{BC}^\omega$ , can be computed similarly. The pivot probability of  $piv_{AB}^{AV}$  in state  $\omega$  is given by:

$$\begin{aligned} \pi_{AB}^\omega &\equiv \Pr(piv_{AB}^{AV}|\omega, AV) = (n-1)! \sum_{i=0}^1 \sum_{k=i}^{n+2(i-1)-n_C} \frac{(\tau_A^\omega)^{k-i} (\tau_{AB}^\omega)^{n+(i-1)-2k} (\tau_B^\omega)^k}{2k!(k-i)!(n+i-1-2k)!} \\ &\quad + \frac{(n-1)! [\tau_A^\omega \tau_B^\omega]^{n-1-n_C} (\tau_{AB}^\omega)^{2n_C+1-n}}{3 [(n-1-n_C)!]^2 (2n_C-n+1)!} + \frac{(n-1)! (\tau_A^\omega)^{n-1-n_C} (\tau_{AB}^\omega)^{2n_C-n} (\tau_B^\omega)^{n-n_C}}{6 (n-1-n_C)! (n-n_C)! (2n_C-n)!}, \end{aligned}$$

and  $\pi_{BA}^\omega$  is defined in the same way. The expected value  $G^{AV}$  of single- $A$  and single- $B$  ballots under AV are thus:

$$G^{AV}(A|s) = q(a|s) [\pi_{AC}^a V + \pi_{AB}^a (V - v)] + q(b|s) [\pi_{AC}^b v + \pi_{AB}^b (v - V)], \quad (10)$$

$$G^{AV}(B|s) = q(a|s) [\pi_{BC}^a v + \pi_{BA}^a (v - V)] + q(b|s) [\pi_{BC}^b V + \pi_{BA}^b (V - v)]. \quad (11)$$

The value of a double ballot follows almost immediately from (10) and (11):

$$G^{AV}(AB|s) = q(a|s) [\pi_{AC}^a V + \pi_{BC}^a v - \phi^a] + q(b|s) [\pi_{AC}^b v + \pi_{BC}^b V - \phi^b], \quad (12)$$

where  $\phi^a$  and  $\phi^b$  are correcting terms for three-way ties. These correcting terms are:<sup>28</sup>

$$\begin{aligned}\phi^a &= [\Pr(X_A = X_B = n_C - 1|a)(V + v) + \Pr(X_A = X_B + 1 = n_C|a)v + \dots \\ &\quad \dots + \Pr(X_A + 1 = X_B = n_C|a)V + \Pr(X_A = X_B = n_C|a)(V + v)]/6, \text{ and} \\ \phi^b &= [\Pr(X_A = X_B = n_C - 1|b)(V + v) + \Pr(X_A = X_B + 1 = n_C|b)V + \dots \\ &\quad \dots + \Pr(X_A + 1 = X_B = n_C|b)v + \Pr(X_A = X_B = n_C|b)(V + v)]/6.\end{aligned}$$

To understand what these correcting terms represent, consider the case in which, without voter  $i$ 's ballot, both alternatives  $A$  and  $B$  lose to  $C$  by one vote. A single- $A$  ballot creates a tie between  $A$  and  $C$ . Thus, the ballot is pivotal in favor of  $A$  with probability  $1/2$ . Likewise, a single- $B$  ballot is pivotal in favor of  $B$  with probability  $1/2$ . Instead, a double-vote  $AB$  creates a three-way tie, the winning probabilities of  $A$  and  $B$  are  $1/3$  instead of  $1/2$ . Summing up the probabilities  $\pi_{AC}^a$  and  $\pi_{BC}^a$  overestimates the value of the double ballot by  $(V + v)/6$ .  $\phi^a$  and  $\phi^b$  correct for such overestimations in that and three other cases: when  $A$  trails behind  $B$  and  $C$  by one vote, when  $B$  trails behind  $A$  and  $C$  by one vote, and when  $A$ ,  $B$  and  $C$  have the same number of votes. We directly see that  $\phi^\omega = 0$  when  $\tau_{AB}^\omega \in \{0, 1\}$ , or  $\tau_A^\omega = 0$ , or  $\tau_B^\omega = 0$ . These correcting terms become vanishingly small and can be omitted when the population size increases towards infinity.

It follows that the payoff differential between actions  $A$  and  $AB$  is:

$$\begin{aligned}G^{AV}(A|s) - G^{AV}(AB|s) &= q(a|s)[\pi_{AB}^a(V - v) - \pi_{BC}^a v + \phi^a] \\ &\quad + q(b|s)[\pi_{AB}^b(v - V) - \pi_{BC}^b V + \phi^b].\end{aligned}\tag{13}$$

The first term in (13) may either be positive or negative. The second is strictly negative. Similarly, the first term in (14) is strictly negative:

$$\begin{aligned}G^{AV}(B|s) - G^{AV}(AB|s) &= q(a|s)[\pi_{BA}^a(v - V) - \pi_{AC}^a V + \phi^a] \\ &\quad + q(b|s)[\pi_{BA}^b(V - v) - \pi_{AC}^b v + \phi^b].\end{aligned}\tag{14}$$

These payoffs differentials highlight the main trade-off faced by voters under AV: on the one hand, they want to double-vote more if  $C$  is a threat. On the other hand, they want to double-vote less if  $C$  is *not* a threat. In particular, if no other voter double-votes, a vote can never be pivotal between  $A$  and  $B$ :  $\pi_{AB}^\omega = \pi_{BA}^\omega = \phi^\omega = 0$ , and both payoff differentials must be negative. Hence:

**Lemma 2** *The strategies that are an equilibrium in plurality cannot be an equilibrium in AV.*

Conversely, imagine that all the other voters double-vote. In that case:  $\pi_{AC}^\omega = \pi_{BC}^\omega = \phi^\omega = 0$ , and either (13) or (14) must be strictly positive. Hence:

**Lemma 3** *Pure double-voting is never an equilibrium in AV.*

Next, we show that different voter types cannot mix over the same set of actions in equilibrium:

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<sup>28</sup>Proof available upon request.



**Lemma 4** *If there exists a signal  $s$  such that*

$$\begin{aligned} G^{AV}(A|s) - G^{AV}(AB|s) = 0 & \text{ then } G^{AV}(A|s_A) - G^{AV}(AB|s_A) > G^{AV}(A|s_B) - G^{AV}(AB|s_B) \quad (15) \\ G^{AV}(B|s) - G^{AV}(AB|s) = 0 & \text{ then } G^{AV}(B|s_B) - G^{AV}(AB|s_B) > G^{AV}(B|s_A) - G^{AV}(AB|s_A), \text{ and} \\ G^{AV}(A|s) - G^{AV}(B|s) = 0 & \text{ then } G^{AV}(A|s_A) - G^{AV}(B|s_A) > G^{AV}(A|s_B) - G^{AV}(B|s_B). \end{aligned}$$

**Proof.** We detail the proof for (15). It is similar for the other two implications. Remember that the second term in (13) is necessarily negative. Thus  $G^{AV}(A|s) - G^{AV}(AB|s) = 0$  implies that the first term must be strictly positive. It follows immediately that:

$$G^{AV}(A|s) - G^{AV}(AB|s) \geq 0 \text{ iff } \frac{q(a|s)}{q(b|s)} \geq \frac{\pi_{AB}^b(V-v) + \pi_{BC}^bV - \phi^b}{\pi_{AB}^a(V-v) - \pi_{BC}^av + \phi^a}.$$

Thus, (15) follows from  $\frac{q(a|s_A)}{q(b|s_A)} > \frac{q(a|s_B)}{q(b|s_B)}$ . ■

Finally, neither  $A$  nor  $B$  can have  $n$  votes with probability 1:

**Lemma 5** *In any voting equilibrium under AV, neither  $A$  nor  $B$  can be approved by all voters.*

**Proof.** We use a proof by contradiction, for the limit case  $\varepsilon \rightarrow 0$ . By definition the results hold when  $\varepsilon > 0$ . Policy  $A$  is approved by all voters iff  $\sigma_{s_A}(A) + \sigma_{s_A}(AB) = 1 = \sigma_{s_B}(A) + \sigma_{s_B}(AB)$ . In this case, we have:  $x_A + x_{AB} = n$  and hence  $\pi_{AC}^\omega = 0 = \pi_{BC}^\omega$  and  $\phi^\omega = 0$ . The only possible pivot events are when  $x_{AB} = n - 1$  or  $n - 2$ . Hence:

$$\begin{aligned} G(A|s) - G(AB|s) &= [q(a|s)\pi_{AB}^a - q(b|s)\pi_{AB}^b](V-v) \geq 0 \\ G(B|s) - G(AB|s) &= [q(b|s)\pi_{BA}^b - q(a|s)\pi_{BA}^a](V-v) \geq 0. \end{aligned}$$

with:  $\pi_{AB}^\omega = \frac{(\tau_{AB}^\omega)^{n-1}}{2}$ , and  $\pi_{BA}^\omega = \frac{(\tau_{AB}^\omega)^{n-2}}{2} [(n-1) + (2-n)\tau_{AB}^\omega]$ . Therefore,

$$\frac{\pi_{BA}^b}{\pi_{BA}^a} = \left( \frac{\tau_{AB}^b}{\tau_{AB}^a} \right)^{n-2} \frac{(n-1) + (2-n)\tau_{AB}^b}{(n-1) + (2-n)\tau_{AB}^a}, \quad (16)$$

$$\frac{\pi_{AB}^a}{\pi_{AB}^b} = \left( \frac{\tau_{AB}^a}{\tau_{AB}^b} \right)^{n-1}. \quad (17)$$

Now, we show that  $\frac{\pi_{BA}^b}{\pi_{BA}^a}$  is increasing in  $\frac{\tau_{AB}^b}{\tau_{AB}^a}$  (from (16)), it is straightforward that  $\frac{\pi_{AB}^a}{\pi_{AB}^b}$  is also increasing in  $\frac{\tau_{AB}^a}{\tau_{AB}^b}$ . Taking logs, we have that the RHS of (16) is:

$$(n-2) [\log \tau_{AB}^b - \log \tau_{AB}^a] + \log [(n-1) + (2-n)\tau_{AB}^b] - \log [(n-1) + (2-n)\tau_{AB}^a].$$

Differentiating with respect to  $\tau_{AB}^b$  yields:  $\frac{n-2}{\tau_{AB}^b} - \frac{n-2}{(n-1)+(2-n)\tau_{AB}^b}$ , which is non-negative iff  $\tau_{AB}^b \leq 1$ . Therefore, we have  $\pi_{AB}^b > \pi_{AB}^a$  and  $\pi_{BA}^b > \pi_{BA}^a$  when  $\tau_{AB}^b > \tau_{AB}^a$ , and conversely.

We now use this result to prove that  $A$  cannot be approved by all voters. From Theorem 1, Lemma 3, and Lemma 4, there are 2 cases to check: (i)  $\sigma_{s_A}(A) = 1$  and  $\sigma_{s_B}(A) \in [0, 1)$ , and (ii)  $\sigma_{s_B}(A) = 0$  and  $\sigma_{s_A}(A) \in (0, 1]$ . If  $\sigma_{s_A}(A) = 1$  and  $\sigma_{s_B}(A) \in [0, 1)$ , then  $\tau_{AB}^b > \tau_{AB}^a$ . Hence,

we have that  $\pi_{BA}^b > \pi_{BA}^a$ , which implies  $G(B|s_B) - G(AB|s_B) > 0$ . Thus, there cannot be any equilibrium in which  $\sigma_{s_A}(A) = 1$  and  $\sigma_{s_B}(A) \in [0, 1)$ . If  $\sigma_{s_A}(A) \in (0, 1]$  and  $\sigma_{s_B}(A) = 0$ , then either  $\tau_{AB}^a > \tau_{AB}^b$  or  $\tau_{AB}^a < \tau_{AB}^b$ . If  $\tau_{AB}^a > \tau_{AB}^b$ , then  $\pi_{AB}^a > \pi_{AB}^b$ , and thus  $G(A|s_A) - G(AB|s_A) > 0$ . If  $\tau_{AB}^a < \tau_{AB}^b$ , then  $\pi_{BA}^b > \pi_{BA}^a$ , and thus  $G(B|s_B) - G(AB|s_B) > 0$ . Therefore, there cannot be any equilibrium in which  $\sigma_{s_B}(A) = 0$  and  $\sigma_{s_A}(A) \in (0, 1]$ . ■

## Proofs of Section 4

**Proof of Theorem 1.** From McLennan (1998), a strategy that maximizes expected utility must be an equilibrium of such a common value game (and any finite Bayesian game like ours must have an equilibrium). Now, conjecture some strategy profile  $\sigma$  that can be played under plurality. That is,  $\sigma_s(AB) = 0$  for  $s = s_A, s_B$ . By Lemma 2,  $G^{AV}(A|s) - G^{AV}(AB|s) < 0$  and  $G^{AV}(B|s) - G^{AV}(AB|s) < 0, \forall s$ , which implies that  $\tau_{AB}^\omega = 0$  cannot be part of an equilibrium, and that the welfare-maximizing equilibrium under AV must produce strictly higher expected utility than plurality.

It remains to show that this equilibrium is sincerely stable. We actually show the stronger statement that, to maximize expected welfare, a strategy must satisfy  $\sigma_{s_A}(A), \sigma_{s_B}(B) > 0$ . We show this by contradiction: suppose that  $\hat{\sigma}$  maximizes expected welfare and is such that  $\hat{\sigma}_{s_A}(A) = 0$ . By Lemma 5, we have  $\tau_A^\omega, \tau_B^\omega, \tau_{AB}^\omega > 0$  and hence  $\hat{\sigma}_{s_B}(A) > 0$ . Then, compare  $\hat{\sigma}$  with some other strategy  $\sigma'$  in which  $s_A$ -voters transfer some of their votes from  $B$  towards  $AB$ , whereas  $s_B$ -voters adapt their voting strategy so as to maintain all vote shares unchanged in state  $b$ .<sup>29</sup>

The total vote share of  $A$  in state  $a$  must then increase (i.e.  $\tau_A^a(\sigma') + \tau_{AB}^a(\sigma') > \tau_A^a(\hat{\sigma}) + \tau_{AB}^a(\hat{\sigma})$ ), together with the expected fraction of double-votes. As a result, in state  $a$ , the probability that  $A$  wins must increase, whereas the probability that  $C$  wins decreases weakly. In state  $b$ , winning probabilities are unchanged. Hence,  $\hat{\sigma}$  cannot maximize expected welfare. ■

**Proof of Theorem 2.** We prove the Theorem in two steps. First, we show that there is no interior equilibrium in which a voter strictly mixes across the three actions  $A, B$ , and  $AB$ . Second, we show that  $s_A$ -voters never play  $B$ , nor  $s_B$ -voters play  $A$  in an interior equilibrium. It follows that the only possible interior equilibrium is such that voters with signal  $s_A$  mix between  $A$  and  $AB$ , and voters with signal  $s_B$  mix between  $B$  and  $AB$ .

First, conjecture an equilibrium in which  $\sigma_{s_A}(A), \sigma_{s_A}(B), \sigma_{s_A}(AB) > 0$ . This requires  $G(A|s_A) = G(B|s_A) = G(AB|s_A)$ . In this case, by Lemma 4,  $s_B$ -voters must be playing  $B$  with probability 1, i.e.  $\sigma_{s_B}(B) = 1$ . The equilibrium is therefore not interior, a contradiction. Similarly,  $s_A$ -voters must play  $A$  with probability 1 if  $s_B$ -voters strictly mix between  $A, B$ , and  $AB$ .

Second, imagine that  $s_B$ -voters play  $A$  with strictly positive probability in equilibrium:  $\sigma_{s_B}(A) \in (0, 1)$ . This requires either (i)  $G(A|s_B) = G(AB|s_B) \geq G(B|s_B)$  or (ii)  $G(A|s_B) = G(B|s_B) \geq G(AB|s_B)$ . By Lemma 4, both (i) and (ii) imply that  $G(A|s_A) > G(AB|s_A), G(B|s_A)$ , and hence that  $A$ 's strategy cannot be interior. By symmetry,  $\sigma_{s_A}(B) \in (0, 1)$  cannot be part of an interior equilibrium either. ■

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<sup>29</sup>If  $\hat{\sigma}_{s_A}(AB) = 1$ , then one must consider a transfer of  $s_A$ -votes from  $AB$  towards  $A$ , and  $s_B$ -voters adapt their strategy to maintain all  $\tau_\psi^a$  unchanged.

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