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**Working Paper**

## Tuition Fees, as User Prices, and Private Incentives

CESifo Working Paper, No. 5991

**Provided in Cooperation with:**

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*Suggested Citation:* Economides, George; Philippopoulos, Apostolis; Sakkas, Stelios (2016) : Tuition Fees, as User Prices, and Private Incentives, CESifo Working Paper, No. 5991, Center for Economic Studies and Ifo Institute (CESifo), Munich

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## Tuition Fees, as User Prices, and Private Incentives

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CESIFO WORKING PAPER NO. 5991  
CATEGORY 1: PUBLIC FINANCE  
JULY 2016

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ISSN 2364-1428

# Tuition Fees, as User Prices, and Private Incentives

## Abstract

This paper studies the aggregate and distributional implications of introducing tuition fees for public education services into a tax system with income and consumption taxes. The setup is a neoclassical growth model where agents differ in capital holdings. We show that the introduction of tuition fees (a) improves individual incentives to work and/or save and (b) can be both efficient and equitable. The focus is on the role of tuition fees as an extra price and how this affects private incentives.

JEL-Codes: H400, H200, D600.

Keywords: user prices, tax mix, efficiency, equity.

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June 30, 2016

We thank seminar participants, in particular Arye Hillman, Christos Kotsogiannis, Assaf Razin and Heinrich Ursprung, at the conference on “Public policy and public sector reforms” held at Delphi, Greece, on 5-6 June 2015. We have also benefited from discussions with Kostas Angelopoulos, Harris Dellas, Saqib Jafarey, Jim Malley, Hyun Park and Peter Birch Sørensen. This research has been co-financed by the European Union (European Social Fund - ESF) and Greek national funds through the Operational Program “Education and Lifelong Learning” of the National Strategic Reference Framework (NSRF) - Research Funding Program ARISTEIA II\_Public Sector Reform\_5328; we are grateful for financial support. Any views and errors are entirely our own.

## 1. Introduction

The 2008 world financial and economic crisis has brought into the spotlight a number of issues related to the reform of the public sector. In addition to the first obvious task, which is public debt sustainability, there are two other interrelated issues: how to improve the provision of public goods and services and how to reduce the social burden for this provision.<sup>1</sup> The latter, namely, the reduction of the social burden for the provision of public goods and services, has to do not only with the classic quest for the least distorting tax mix but also with the search for new sources of public revenue. User prices for excludable public goods and services can possibly play this role and, among a wide variety of such user prices, the most debated example is tuition fees for publicly provided education services.<sup>2</sup> A common objection to tuition fees is the view that they are “unjust”, in the sense that they tend to increase income inequality. But, is it really the case?

This paper studies the aggregate and distributional implications of tuition fees for publicly provided education services. Within a rather standard general equilibrium framework, we show that the introduction of tuition fees, modeled as user prices for publicly provided private education services, allows for the creation of a new market and this new market enhances individual incentives and opportunities. As a result, such tuition fees can, not only make everybody better off (i.e. they are Pareto efficient) but also reduce income inequality.

Our model is as follows. We use a rather standard neoclassical growth model with human capital. Individual human capital can be augmented by (among other things) publicly provided education services. These services are modeled as an excludable public good. Their provision requires public funds. We compare two different systems of public financing. According to the first system, public education services are provided “free” of charge,

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<sup>1</sup> See Sørensen (2016) for a recent survey of the economics of the public sector and the need for reforms.

<sup>2</sup> For examples and reviews of user prices for excludable public goods and services, like education, health, child care, elderly care, etc, see Cullis and Jones (1998, chapter 12) and Hillman (2009, chapter 3). Tuition fees for public education services are a popular example of user prices; see also Atkinson and Stiglitz (1980, chapter 16). See chapter 6 in the book of Milton and Rose Friedman (1980) for the history of private and public education system in the US. See Jacobs and van der Ploeg (2006), for European practice. See Barr (2012) for reforms of higher education finance, including tuition fees, in England. For the political economy of tuition fees and public education spending policy, see e.g. Merzlyn and Ursprung (2005), Soares (2006), Haupt (2012), Kauder and Potrafke (2013) and Epple and Romano (1996, 2014).

meaning that they are paid by general (income and consumption) taxes. According to the second system, there are tuition fees, in the form of user prices for publicly provided private education services. In the latter case, private agents' utility maximization problem gives the individual demand for those services as a negative function of the user price; this allows individuals to voluntarily choose the amount of public education services they want to use and hence the amount they wish to pay. Then, the user price, or the tuition fee, emerges as a consequence of this voluntary demand and the quantity made available by the government. As an extension, we also allow for a minimum uniform provision of the same education services, which is determined exogenously by the government, so, in this case, individuals choose how much to top up by paying fees.<sup>3</sup> To study distributional issues, we obviously need a model with heterogeneous agents. Following a long tradition in the literature on tax policy and social conflict that dates back to e.g. Judd (1985), we assume that households are divided into two distinct social groups, called capitalists and workers: while both groups can accumulate some type of human capital and provide labor services, only capitalists own the physical stock.<sup>4</sup> The model is solved numerically using common parameter values.

Our first result is about efficiency. When, other things equal, we compare an economy without user prices for publicly provided private education services to the same economy with such prices, the latter is always Pareto improving. In other words, with tuition fees, modeled as user prices, both social groups, capitalists and workers, gain in terms of income and welfare. This holds even when the government also makes available a minimum uniform provision financed by general taxes, so that, in this case, individuals have the choice to top up or opt out of using marketed education services; our solutions imply that they find it optimal to top up, and this applies especially to workers (see below for income distribution). Intuitively, the introduction of user prices for publicly provided education services, and in particular the creation of an extra market for this type of public service (see below for a further discussion), helps individuals to realize that, in order to afford its provision, they need higher income and hence they need to work more hours and/or save more. To put it differently, with user prices and the associated education choice, the cost of public education

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<sup>3</sup> The minimum or compulsory provision can be thought of as primary and secondary education, while the top up can be thought of as tertiary or higher education.

services is internalized at individual, as opposed to social level, and this, *ceteris paribus*, strengthens the individual incentive to work and/or save. It should be noticed that this is the opposite from an increase in taxes which, *ceteris paribus*, distorts incentives. Not surprisingly, the more private are the benefits from the provision of public education services, the higher are the efficiency gains from tuition fees.

Our second result is about distribution. Our results show that the income of workers rises by more than that of capitalists when we introduce user prices for publicly provided private education services and move to a more efficient economy. In other words, in equilibrium, the introduction of such prices proves to be a progressive policy. Loosely speaking, workers, or the poor, view tuition charges, and the associated work effort to pay for them, as an opportunity to climb the income ladder, while capitalists enjoy anyway an extra source of income coming from physical capital so that their incentive to invest in human capital is weaker. The rise in the gross income of workers more than offsets their higher user payments, so that their net income rises by more than the capitalists' and eventually long-term inequality falls in terms of net incomes. Therefore, in equilibrium, and in the case of user prices for publicly provided education services that augment private human capital, not only everybody gets better off (relative to the case without user prices), but also inequality, as measured by changes in net incomes, is reduced in equilibrium. When the government also makes available a minimum uniform provision financed by general taxes, other things equal, net income inequality falls, but this comes at the cost of making everybody worse off. This problem becomes more acute when there is only a minimum uniform provision and nobody has the choice to top up; in this case, inequality falls but at the cost of immiserating everybody.

Before we carry on, it should be stressed that here we use a stylized general equilibrium model that allows us to make our main point boldly. Namely, to show that, to the extent that agents can afford to pay user charges, so that education choices are feasible, the introduction of user prices for publicly provided private education services can improve both efficiency and equity. Of course, we realize that when some agents cannot afford the payment of user charges, government intervention is needed to supplement the market mechanism and

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<sup>4</sup> See Lansing (2011) for a review of general equilibrium models with concentrated ownership of capital as a type of agent heterogeneity. According to empirical evidence by e.g. Krueger et al. (2010), concentration in capital

give everybody the opportunity of education. Actually, our results are consistent with this; as said above, if, for some reason, workers do not pay user charges, inequality rises without a minimum uniform provision financed by general taxes. Thus, in such cases, the government should intervene to ensure the provision of a minimal education system to everybody, especially to those with poor initial background (see e.g. Hillman and Jenker, 2004, and Cunha and Heckman, 2009). Nevertheless, we also showed that, in this case, there is a tradeoff between efficiency and equality, so social judgments have to be made. But our main argument for tuition fees (seen as a user price), still holds, even if some households are exempted from paying for schooling directly.

The rest of the paper is organized as follows. Section 2 compares our work to the literature. Section 3 presents the model and its main results. Extensions and robustness checks are in section 4. Section 5 closes the paper. An Appendix provides algebraic details.

## **2. Related literature and how our work differs**

As said in the opening paragraph of section 1, the public provision of private goods/services has always been a debated issue. There is a rich and still open literature on the possible merits of such provision (see e.g. Ireland (1990), Blomquist and Christiansen (1995, 1998), Broadway and Marchand (1995), Epple and Romano (1996), Bergstrom and Blomquist (1996), Broadway et al. (1998), Pirttilä and Tuomala (2002), Blomquist et al. (2010), Fang and Norman (2014)). Most of these papers focus on informational asymmetries and how the public provision of private goods, jointly with taxation and subsidies, can alleviate informational problems and the associated inefficiencies. Also, most of these papers, contrary to ours, use partial equilibrium models, where the (user) price of the publicly provided private good is exogenous or is set equal to the endogenously determined price of the private good. Among the above papers, the one closer to ours is Pirttilä and Tuomala (2002), who also use a general equilibrium model, in the sense that public provision of private goods affects, among other things, productivity, wages and the possibility of redistribution. The same applies to Ott and Turnovsky (2006), who, however, use a single agent model so there are no distribution effects. There is also a rich literature focusing on tuition fees. For instance, Heckman et al.

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ownership is one of the key determinants of income inequality.

(1999), Blundell et al. (2003) and Malchow-Møller et al. (2011), among many others, have studied the implications of fees vis-à-vis other taxes and how these implications differ between a general- and a partial-equilibrium approach.<sup>5</sup>

However, our work takes a new theoretical perspective and thereby provides a new argument for tuition fees. It treats tuition fees as a user price (the price of publicly provided private education services) rather than as a different tax instrument. And a new price naturally implies a new market. To put it differently, as the word “price” itself indicates, user prices in general, and tuition fees in particular, open up a new market, specifically, the market for publicly provided education services. Then, the expansion of the set of prices/markets available strengthens individual incentives and gives individuals more power to affect their income distribution.<sup>6</sup> The new market makes everybody better off and especially those with the stronger incentives who are the relatively poor. In addition, in a general equilibrium setup, there is a double dividend in the sense that a more efficient economy implies larger tax bases which, in turn, allow the reduction of distorting taxes and this reduction further stimulates incentives and the macro economy.

### **3. A general equilibrium model with tuition fees as user prices**

#### *3.1 Informal description of the model and discussion of key assumptions*

We construct a dynamic general equilibrium model with tuition fees, modeled as user prices, for publicly provided education services. The model was described in the Introduction above but here we further elaborate two things: the role of public education services and the type of agent heterogeneity assumed.

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<sup>5</sup> Other related papers include Gertler et al. (1987), Fraser (1996), Swope and Janeba (2006), Fuest and Kolmar (2007), Huber and Runkel (2009) and Ellingsen and Paltseva (2012). In particular, Gertler et al. (1987) study the impact of introducing user fees in the health care system in Peru. Fraser (1996) focuses on the provision of public goods under different public financing schemes including user fees. Swope and Janeba (2006) analyze how populations with different preferences choose different public financing schemes. Fuest and Kolmar (2007) focus on the use of user fees under cross-border externalities. Huber and Runkel (2009) also focus on the use of user fees under tax competition; they also provide useful empirical evidence for the use of user fees in the US. Ellingsen and Paltseva (2012) focus on the possible inefficiencies of Lindahl prices.

<sup>6</sup> Milton and Rose Friedman (1980, chapter 1) discuss the three interrelated functions of prices (to help agents to act correctly, to determine the income distribution and to transmit information). Here, by assumption, we do not have information issues.



Public education services can augment the accumulation of individual human capital. In particular, individual human capital increases over time thanks to individual time devoted to education and publicly provided education services (in the robustness section below we also allow households to supplement such services from the private market). Agents choose optimally the quantity of those publicly provided education services (which work like an excludable public good) by paying the user price (or tuition fee) to the government. These services can be fully private or can create positive externalities to other individuals. Our modeling of human capital is as in the related growth literature (see e.g. Aghion and Howitt, 2009, for a review chapter) with the exception of user prices.

To study distributional issues, we simply assume that households are permanently allocated to two social groups<sup>7</sup> and that these groups differ in capital ownership. In particular, capital is in the hands of a group of agents called capitalists, while workers do not save or borrow. As said in the Introduction, this is one of most common types of agent heterogeneity used by the literature on tax policy.<sup>8</sup>

The model is deterministic. Individuals within each social group are identical. Time is discrete and infinite.

### 3.2 *Households as capitalists*

There are  $k = 1, 2, \dots, N^k$  identical capitalists. Each  $k$  maximizes discounted lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t u(c_{k,t}, z_{k,t}) \tag{1}$$

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<sup>7</sup> Thus, we assume away issues of occupational choice and social mobility. See e.g. Acemoglu (2009, chapter 23) who adds fixed costs associated with social mobility.

<sup>8</sup> Although capital ownership is the only type of heterogeneity assumed in the paper, we report that our results are robust to adding more types of heterogeneity, like assuming that the two social groups accumulate different types of human capital and thus supply different types of labour services earning different wages.

where  $c_{k,t}$  is  $k$ 's consumption;  $z_{k,t}$  is  $k$ 's leisure time; and  $0 < \beta < 1$  is the subjective discount factor.<sup>9</sup> In each period, one unit of time can be allocated to leisure,  $z_{k,t}$ , work,  $l_{k,t}$ , and education,  $e_{k,t}$ , so that the time constraint is  $z_{k,t} + l_{k,t} + e_{k,t} = 1$ .

The period utility function,  $u(\cdot)$ , has the usual properties. For our numerical solutions, we will use the simple additive form:

$$u(c_{k,t}, z_{k,t}) = \mu_1 \log(c_{k,t}) + \mu_2 \log(1 - l_{k,t} - e_{k,t}) \quad (2)$$

where  $\mu_1, \mu_2 > 0$  are preference parameters.

The within period budget constraint of each  $k$  is:

$$(1 + \tau_t^c)c_{k,t} + k_{k,t+1} - (1 - \delta^k)k_{k,t} + p_t^e g_{k,t}^e = (1 - \tau_t^y)(r_t k_{k,t} + w_t l_{k,t} h_{k,t}) \quad (3)$$

where  $k_{k,t+1}$  is  $k$ 's end-of-period physical capital stock;  $r_t$  is the market return to the beginning-of-period capital stock,  $k_{k,t}$ ;  $w_t$  is the wage rate;  $h_{k,t}$  is  $k$ 's beginning-of-period human capital stock so that  $l_{k,t} h_{k,t}$  denotes  $k$ 's effective labor;  $p_t^e$  is the price of publicly provided education services relative to the price of the private good, which means that each  $k$  voluntarily pays  $p_t^e g_{k,t}^e$  for the public education services provided to him/her personally,  $g_{k,t}^e$ ;  $0 < \tau_t^c, \tau_t^y < 1$  are consumption and income tax rates respectively; and  $0 \leq \delta^k \leq 1$  is the physical capital depreciation rate. Dividends received from firms are omitted since profits will be zero in equilibrium.

The motion of human capital of each  $k$  is (see e.g. Arcalean and Schiopu, 2010, for a review of the related literature):

$$h_{k,t+1} = (1 - \delta^h)h_{k,t} + B(e_{k,t})^{\theta_1} (h_{k,t})^{\theta_2} (\tilde{g}_{k,t}^e)^{\theta_3} \quad (4)$$

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<sup>9</sup> For simplicity, we abstract from utility-enhancing public goods. We report that this is not important to our results.

where  $0 \leq \delta^h \leq 1$  is the human capital depreciation rate;  $B > 0$  is a scale parameter;  $\theta_1 + \theta_2 + \theta_3 = 1$  are technology parameters; and  $\tilde{g}_{k,t}^e$  denotes public education services enjoyed by each  $k$ . Following e.g. Atkinson and Stiglitz (1980, equation 16-56) and Alesina and Wacziarg (1999), in order to allow for possible education externalities, we define  $\tilde{g}_{k,t}^e$  as:

$$\tilde{g}_{k,t}^e \equiv g_{k,t}^e + \gamma \sum_{j \neq k}^N g_{j,t}^e \quad (5)$$

where the parameter  $0 \leq \gamma \leq 1$  measures the strength of external effects from all other agents' public education services. Thus, when  $\gamma = 0$  in (5), the publicly provided education service is fully private, meaning that there are no external effects. Notice that each individual  $k$  chooses  $g_{k,t}^e$ , and thus pays  $p_t^e g_{k,t}^e$  voluntarily and optimally, by taking  $g_{j,t}^e$  as given (however, the latter, namely  $g_{j,t}^e$ , is also chosen by other individuals).

Each capitalist acts competitively choosing  $\{c_{k,t}, l_{k,t}, e_{k,t}, k_{k,t+1}, h_{k,t+1}, g_{k,t}^e\}_{t=0}^{\infty}$  to solve the above problem. The first-order conditions are written and discussed in the Appendix.

It is useful to stress two things at this point. First, the first-order condition for  $g_{k,t}^e$  gives a private demand function for the publicly provided private good which is similar to those in e.g. Atkinson and Stiglitz (1980, equation 16-57) and Ott and Turnovsky (2006). In other words, each individual chooses his expenditure on this good so as his own marginal rate of substitution equals the marginal economic rate of transformation (see Appendix for this demand function). Second, we have started with the case in which all publicly provided education services are voluntary, namely, each individual optimally chooses the amount he/she wishes to pay. In section 4 below, we will add a uniform provision which is exogenously set by the government.

### 3.3 *Households as workers*

There are  $w = 1, 2, \dots, N^w$  identical workers, where  $N^w = N - N^k$ . Their problem is similar to that of the capitalists above except that workers receive labor income only. Thus, we can omit details. Each  $w$  maximizes:

$$\sum_{t=0}^{\infty} \beta^t u(c_{w,t}, z_{w,t}) \quad (6)$$

where we will again use the functional form:

$$u(c_{w,t}, z_{w,t}) = \mu_1 \log(c_{w,t}) + \mu_2 \log(1 - l_{w,t} - e_{w,t}) \quad (7)$$

and, as above with the capitalist, the maximization is subject to:

$$(1 + \tau_t^c) c_{w,t} + p_t^e g_{w,t}^e = (1 - \tau_t^y) w_t l_{w,t} h_{w,t} \quad (8)$$

$$h_{w,t+1} = (1 - \delta^h) h_{w,t} + B(e_{w,t})^{\theta_1} (h_{w,t})^{\theta_2} (\tilde{g}_{w,t}^e)^{\theta_3} \quad (9)$$

$$\tilde{g}_{w,t}^e \equiv g_{w,t}^e + \gamma \sum_{j \neq w}^N g_{j,t}^e \quad (10)$$

Each worker acts competitively choosing  $\{c_{w,t}, l_{w,t}, e_{w,t}, h_{w,t+1}, g_{w,t}^e\}_{t=0}^{\infty}$  to solve the above problem. The first-order conditions are written in the Appendix.

### 3.4 Firms

There are  $f = 1, 2, \dots, N^f$  identical firms owned by capitalists. Thus,  $N^f = N^k$ . In each period, each  $f$  maximizes profits:

$$\pi_{f,t} = y_{f,t} - r_t k_{f,t} - w_t l_{f,t} \quad (11)$$

subject to the production function:

$$y_{f,t} = A(k_{f,t})^\alpha (l_{f,t})^{1-\alpha} \quad (12)$$

where  $k_{f,t}$  and  $l_{f,t}$  denote capital and labor inputs used by each firm, while  $A > 0$  and  $0 < \alpha < 1$  are usual parameters.

Firms act competitively. The standard first-order conditions are written in the Appendix.

### 3.5 Government budget constraint

The period budget constraint of the government is (expressed here in aggregate terms):<sup>10</sup>

$$G_t^e = N^k [\tau_t^y (r_t k_{k,t} + w_t h_{k,t} l_{k,t}) + \tau_t^c c_{k,t} + p_t^e g_{k,t}^e] + N^w [\tau_t^y w_t h_{w,t} l_{w,t} + \tau_t^c c_{w,t} + p_t^e g_{w,t}^e] \quad (13)$$

where  $G_t^e$  denotes total government spending on public education services (all other variables have been defined above).

The market for publicly provided education services will clear when the quantity demanded by private agents equals the quantity provided by the government, that is when  $G_t^e = N^k g_{k,t}^e + N^w g_{w,t}^e$  or equivalently  $g_t^e = \nu^k g_{k,t}^e + \nu^w g_{w,t}^e$ , where  $g_t^e \equiv G_t^e / N$  denotes per capita public spending on education services and where  $\nu^k \equiv N^k / N$  and  $\nu^w \equiv N^w / N = 1 - \nu^k$  are the exogenous population shares. This market-clearing condition will determine the relevant price,  $p_t^e$ .

Hence, inspection of the government budget constraint implies that, in each period, there are three policy instruments  $(\tau_t^y, \tau_t^c, g_t^e)$  out of which one is residually determined to close the government budget (see the next subsection).

### 3.6 Decentralized competitive equilibrium (DCE) for any feasible policy

In the decentralized competitive equilibrium (DCE), households maximize utility, firms maximize profits, all constraints are satisfied and all markets clear (see Appendix for market-clearing conditions). The resulting DCE is summarized by a dynamic system of 15 equations in 15 variables, which are  $\{c_{k,t}, l_{k,t}, e_{k,t}, k_{k,t+1}, h_{k,t+1}, \rho_{k,t}, g_{k,t}^e, c_{w,t}, l_{w,t}, e_{w,t}, h_{w,t+1}, \rho_{w,t}, g_{w,t}^e, p_t^e\}_{t=0}^\infty$  and one of the three fiscal policy instruments,  $\{\tau_t^y, \tau_t^c, g_t^e\}_{t=0}^\infty$ , that follows residually (again, see Appendix for the DCE system). As said above, the user price,  $\{p_t^e\}_{t=0}^\infty$ , is endogenously

<sup>10</sup> For simplicity, we use a single income tax rather than separate taxes on capital income and labour income. Also, for simplicity, we do not include public debt. These assumptions are not important to our main results.

determined to equate demand and supply in the market for publicly provided education services.

Without loss of generality, we will treat  $\{g_t^e\}_{t=0}^\infty$  as the residually determined policy instrument at DCE level, which means that, in this regime, the two tax rates,  $\{\tau_t^y, \tau_t^c\}_{t=0}^\infty$ , are set exogenously. We choose  $\{g_t^e\}_{t=0}^\infty$  as the residually determined policy instrument in order to capture the idea that the government accommodates the quantity of publicly provided education services demanded voluntarily by private agents. However, the classification of policy instruments into endogenous and exogenous (namely, whether  $\{g_t^e\}_{t=0}^\infty$  is the residually determined instrument as we assume here, or whether one of the tax rates plays this role so that  $\{g_t^e\}_{t=0}^\infty$  is set exogenously or chosen optimally by the government) is not important to our qualitative results (see subsection 4.1 below for details).

We can now study the implications of tuition fees as user prices.

### 3.7 *Implications of tuition fees, as user prices, on publicly provided education services*

Our main goal is to compare the above economy, in which each individual voluntarily and optimally determines his own expenditure on publicly provided education services and where tuition fees play the role of the price in the market for those services, to the same economy when this market is closed down, other things equal. The former, more general, case is captured by the DCE system defined above. The latter can follow as a special case if we simply set  $p_t^e \equiv 0$  in this system.

Due to its nonlinearity, the model is solved numerically. In so doing, we use commonly employed values for parameters and exogenous policy instruments (see notes in Table 1 for baseline parameter and policy values). Table 1 reports the steady state solution of the DCE system with, and without, user prices. As said above, in the case with user prices, we set the income tax rate and the consumption tax rate exogenously by allowing public spending on education services to be the residually determined policy instrument. On the other hand, in the case without user prices, and in order to make the comparison of alternative public systems meaningful, we exogenously set public spending on education services at the same levels as found in the case with tuition fees where private agents were free to choose these levels, and allow one of the tax rates, in particular the consumption tax rate, to play the role of

the residually determined policy instrument (see below for other cases and robustness). In other words, in the case without user prices, although each private agent enjoys the same amount as before in the case with user prices (which was voluntarily and optimally chosen by him), this amount is now provided free of charge by the government (meaning it is paid by general taxes). As we shall see below, this means fewer choices and this affects individual incentives.

In Table 1, in each regime, with and without fees, we present the solution for various degrees of social externalities, where recall that  $\gamma = 0$  is the special case without external effects, namely the case in which the publicly provided good is fully private (we report that our results are robust to any value of  $\gamma$  in the whole range  $0 \leq \gamma \leq 1$ ). Numbers in bold letters indicate those variables whose values are set exogenously as explained in the previous paragraph.

Table 1 around here

Regarding efficiency, the steady state solutions in Table 1 imply that the introduction of user prices improves individual incentives and leads to a more efficient economy. In particular, individual and aggregate hours of work ( $l_k, l_w, l$ ), individual and aggregate hours of education ( $e_k, e_w, e$ ), capitalists' physical capital ( $k_k$ ), individual and aggregate human capital ( $h_k, h_w, h$ ), individual gross incomes ( $y_k^g, y_w^g$ ), individual net-of-tax incomes ( $y_k^n, y_w^n$ ),<sup>11</sup> as well as aggregate output ( $y$ ), they all rise as we switch to a tuition fee system, other things equal. The same applies to individual and aggregate utility ( $u_k, u_w, u$ ). In other words, the introduction of user prices (here in the form of tuition fees) is Pareto-improving in the sense that both social groups become better off. As the degree of social externalities increases (namely, as  $\gamma$  rises), user payments, public good provision and average labor and education time all fall, as a result of common free-riding incentives. On the other hand, as  $\gamma$  rises, average private consumption rises.

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<sup>11</sup> Net income is defined as gross income minus all types of taxes and user charges. Thus,  $y_k^n \equiv (1-\tau^y)(rk_k + wh_k l_k - \tau^c c_k - pg_k)$  and  $y_w^n \equiv (1-\tau^y)wh_w l_w - \tau^c c_w - pg_w$ .

Regarding income distribution, the introduction of user prices decreases the ratio of capitalist's net income to the worker's net income, namely,  $y_k^n / y_w^n$  falls. This is for any value of the externality parameter. Recall that this particular ratio, namely, the distribution between income going to those who hold capital and those who do not, has always been an important measure of (in)equality in the analysis of the incidence of changes in tax policy.<sup>12</sup> Table 1 also reports results for the ratio of the gross income of capitalists to the gross income of workers,  $y_k^g / y_w^g$ , which falls too when we introduce tuitions fees. The intuition behind our results is as follows. When we introduce tuition fees modeled as user prices, we move to a more efficient economy with more markets, more choices and hence better incentives at individual level. In a more efficient economy, the gross income of both agents rises. Actually, as Table 1 shows, the gross income of workers rises by more than the gross income of capitalists so that  $y_k^g / y_w^g$  falls. This happens because workers find it optimal to pay more on tuition charges than capitalists,  $g_k^e < g_w^e$ , and hence to accumulate more human capital,  $h_k < h_w$ , in all cases with tuition fees. To put it differently, income from labor, which is affected by human capital accumulation, is the only source of income for workers, whereas capitalists can enjoy also income from physical capital; hence, workers are more willing to invest in human capital relative to capitalists. The gross income effect more than outweighs any payments for tuition fees and taxes,<sup>13</sup> meaning that the rise in gross income is so strong for the worker so that his/her net income rises despite having to pay higher tuition charges; hence,  $y_k^n / y_w^n$  falls. Therefore, the introduction of tuition fees becomes a progressive policy in equilibrium. Generally speaking, even when a policy change looks regressive at first sight, it may eventually be progressive once effects of individual incentives and general equilibrium effects on prices and returns are taken into account. This is like a double dividend result: more efficiency and more equality and this works via better incentives.

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<sup>12</sup> See e.g. the discussion by Turnovsky (1995, p. 340).

<sup>13</sup> The effective average tax rate on the capitalist relative to the effective tax on the worker, defined as  $t_k / t_w$ , where  $t_k \equiv \frac{\tau^y (rk_k + wh_k l_k) + \tau^c c_k + pg_k}{rk_k + wh_k l_k}$  and  $t_w \equiv \frac{\tau^y wh_w l_w + \tau^c c_w + pg_w}{wh_w l_w}$ , falls when we introduce user prices. Solutions for these variables are available upon request.



## 4. Extensions and sensitivity analysis

The above results are robust to a number of extensions and checks. In what follows, we present those that we believe are more important.

### 4.1 *Robustness to the way of modeling policy*

We find it useful to start with a clarification related to the way of modeling policy. Our results do not depend on which fiscal policy instrument is residually determined to satisfy the government budget constraint. This holds both in the case with, and without, tuition fees. In particular, as said above, in the case without tuition fees, we exogenously set public spending as found in the case with fees, and allowed one of the tax rates to play the role of the residually determined policy instrument. We did so (namely, we allowed the government to replicate the same quantity of the public good) not only because this looks more natural to us but also because our purpose is to investigate the aggregate and distributional implications of different combinations of public financing policy instruments when the amount of publicly provided education services remains the same. However, we report that this model specification is not important to our results. For example, we could alternatively assume, in the case without fees, that the tax rates remain exogenously set and that it is the amount of the publicly provided good that plays the role of the residually determined instrument.

Our results also do not depend on the number of policy instruments used. In particular, the introduction of fees is efficiency enhancing even if they replace an existing tax. In other words, the efficiency gains do not arise simply because the government has an additional policy instrument (namely, fees) at its disposal.

Finally, we report that our results are robust to changes in the values of the exogenously set policy instruments.

### 4.2 *Adding a minimum uniform amount of publicly provided education services*

We now add a minimum uniform public provision of education. In other words, the government provides an exogenously determined minimum amount of public education services (say, a fraction of GDP as in the data) received by everybody and, at the same time, as in the baseline model of the previous section individuals can voluntarily and optimally top

up by paying for publicly provided education services beyond the minimum uniform provision. As said above, one could think of minimum provision of education as representing the compulsory part of schooling, i.e. primary and secondary education.

In terms of modeling, the uniform provision, denoted as  $\bar{g}_t$  in per capita terms, enters agents' human capital accumulation functions. Namely, equations (4) and (9) change respectively to:

$$h_{k,t+1} = (1 - \delta^h)h_{k,t} + B(e_{k,t})^{\theta_1} (h_{k,t})^{\theta_2} (\bar{g}_t + \tilde{g}_{k,t}^e)^{\theta_3} \quad (14)$$

$$h_{w,t+1} = (1 - \delta^h)h_{w,t} + B(e_{w,t})^{\theta_1} (h_{w,t})^{\theta_2} (\bar{g}_t + \tilde{g}_{w,t}^e)^{\theta_3} \quad (15)$$

The minimum uniform amount  $\bar{g}_t$  is provided “freely” (meaning it is financed by general taxes) as a regular public good to everybody. Thus, it enters on the spending side of the government budget constraint which changes from (13) to:

$$\bar{g}_t + g_t^e = n^k [\tau_t^y (r_t k_{k,t} + w_t l_{k,t} h_{k,t}) + \tau_t^c c_{k,t} + p_t^e g_{k,t}^e] + n^w [\tau_t^y w_t l_{w,t} h_{w,t} + \tau_t^c c_{w,t} + p_t^e g_{w,t}^e] \quad (16)$$

The new equilibrium system is presented in the Appendix. Numerical steady state solutions are reported in Table 2. In these solutions, we set  $\bar{g}_t$  to be 5% of GDP which is close to the data in most OECD countries (but recall that, since the solution for output is endogenous,  $\bar{g}_t$  is also endogenous). To save on space, we focus on one value of social externalities only, for instance  $\gamma = 0.15$ . On the other hand, Table 2 includes several other subcases, all of them within the spirit of the current policy experiment, meaning that now there are deviations from the polar case of the previous section where all individual purchases of publicly provided education services were voluntarily and optimally chosen. Again, as in Table 1, numbers in bold indicate values exogenously set.

In particular, in Table 2, the first column starts with the case with voluntary top ups by all individuals, namely, individually chosen expenditures which are in addition to the minimum uniform provision. The second column reports the case where all components of public education services are exogenously set as in the first column but now this is provided

without user charges (in other words, the first two columns repeat the same experiment as in Table 1 above, except that now there is also a minimum uniform provision). The third column reports the case without fees again, as in the second column, but now the two social groups are forced to receive the same amount of public services (namely, we exogenously set  $g_e^k = g_e^w = g_e$ , where  $g_e$  is exogenously set as found in the first column). The fourth column reports the case in which only capitalists are free to top up by paying fees, while workers make use of the minimum uniform amount only. The fifth column (second from the end) reports the case with uniform provision only so that nobody is allowed to top up and the residually determined policy instrument is the consumption tax rate. The sixth and last column repeats the experiment of the fifth column but now we keep both tax rates (income and consumption tax rates) at the exogenously set level they had at the first column and the residually policy instrument is  $\bar{g}$ , namely, the uniform provision of education services.

Table 2 around here

Inspection of the new solutions in Table 2 reveals that the main result remains as in the previous section. Namely, the first two columns in Table 2 confirm that allowing for voluntary tuition fees, modeled as user prices, can make everybody better off and can also reduce net, as well as gross, income inequality, even when an exogenously set minimum uniform provision is also made available by the government. In particular, in Table 2, as it was the case in Table 1, both  $y_k^n$  and  $y_w^n$  fall, and  $y_k^n / y_w^n$  rises, as we move from the first column with user prices to the second column without user prices.

But there are new results too. A comparison of the results in the first two columns in Table 2 (with a minimum uniform provision) to the corresponding results in Table 1 (without a minimum uniform provision) reveals that, other things equal, the addition of a minimum provision by the government leads to lower net incomes for all agents (both  $y_k^n$  and  $y_w^n$  are lower in Table 2 relative to Table 1) but, on the other hand, net income inequality, as measured by  $y_k^n / y_w^n$ , is lower in Table 2, as one would probably expect.

We now continue with the new cases studied in Table 2 (column 3 and after). Comparison of the results in the third column to those in the first column implies that net

incomes fall, and also inequality rises, when we move from the first column to the third column in which all agents are forced to use the same uniform amount. That is, a one-size-fits-all forced education policy hurts everybody and also worsens inequality. In the fourth column, workers get worse off when their choices are reduced and have to make use of the minimum uniform amount only. In particular,  $y_w^n$  falls and  $y_k^n / y_w^n$  rises relative to the first column. Actually, we report that comparative static exercises, within this regime, show that as the exogenously set minimum uniform provision rises (resp. falls), net income inequality falls (resp. rises) but this makes everybody worse off (resp. better off) in terms of individual net income level. In the fifth column (second from the end), where there is only a uniform provision and nobody has the choice to top up, inequality falls, namely  $y_k^n / y_w^n$  rises, but this is at the cost of immiserating everybody (for instance, per capita output falls by more than 30% relative to per capita output of the first column). In the last column, in which we repeat the same experiment as in the fifth column but now we keep the tax rates exogenously set as in the first column and treat  $\bar{g}$  as the residually adjusting fiscal instrument, both  $y_k^n$  and  $y_w^n$  fall, and also the ratio  $y_k^n / y_w^n$  decreases, meaning that the net income of capitalists falls by more. However, again, greater equality comes at the cost of important efficiency losses (for instance, per capita output falls by more than 15% relative to per capita output in the first column).

### 4.3 Adding private education goods/services

Our results are also robust to a more general motion for human capital. In particular, in the model above, and specifically in equations (4) and (9), we did not allow individuals to supplement education services from the private market and we also used a Cobb-Douglas function for the accumulation of private human capital. Now we use (see also Stokey, 1996, for a similar functional form although in a different context):

$$h_{i,t+1} = (1 - \delta^h)h_{i,t} + B e_{i,t}^{\theta_1} h_{i,t}^{\theta_2} [\varphi s_{i,t}^v + (1 - \varphi) \tilde{g}_{i,t}^v]^{\theta_3/v} \quad (17)$$

where  $s_{i,t}$  is private goods/services used in education by each agent  $i \equiv k, w$ . That is, now agents can use both private time and private goods/services for their education. Also,

$0 < \varphi < 1$  and  $0 \leq \nu \leq 1$  are new parameters, where the elasticity parameter,  $\nu$ , measures the degree of substitutability between the two inputs; if  $\nu = 1$ , private education services and public education services are perfect substitutes; if  $\nu = 0$ , the function becomes Cobb-Douglas in all arguments. Results for the new model in the long run are reported in Table 3, which, for comparison reasons, is the exact analogue of Table 1. All main results remain as in Table 1.

Table 3 around here

#### 4.4 *Robustness to parameterization*

We finally report that our results are robust to changes in parameter values. It is worth emphasizing that our results are also robust to the specification of the utility function. For instance, our results continue to hold when we assign a higher degree of disutility to effort made for work than to effort made for education, since studying can be thought of, not only as an investment in future human capital, but also as a utility-enhancing activity in the current period. (in the letter only we add: but we do need to have a net cost associated with current education time in order to get a standard well-defined optimality condition with respect to this control variable).

### 5. **Closing remarks and possible extensions**

We studied the aggregate and distributional implications of introducing tuition fees, modeled as user prices for publicly provided education services. Employing a rather standard general equilibrium setup, we showed that the introduction of such user prices to a system with general taxes can, not only improve efficiency, but also reduce inequality at least in most cases. Thus, it is possible to find public financing policies that are both efficient and equitable. Heckman and Jacobs (2010) have argued similarly in their study for various education policies in Europe.

Since the main results have already been listed in the Introduction above, we close with possible extensions. In addition to the caveats discussed in the Introduction, the paper can be enriched in several other ways. For instance, we can also study transition effects. That

is, we can study the aggregate and distributional implications when we depart from the steady state of the economy without tuition fees and travel over time towards the steady state of the same economy with tuition fees. In addition, instead of studying the implication of an exogenous switch (reform) to a tuition fee system, we could study optimally chosen tax and fee policy, both in a Benthamite and a political economy setup. Regarding political economy issues, here we abstracted from them in order to focus on the role of tuition fees as an extra price. However, such issues can be added to our framework. We leave these extensions for future work.

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## APPENDIX

In the equations presented below, to save on space, we include from the very beginning the uniform per capita provision of public education services,  $\bar{g}_t \geq 0$ , added in section 4. Thus, to get the baseline model of section 3, we just set  $\bar{g}_t = 0$ .

### Households as capitalists

The optimality conditions of each capitalist include his/her constraints and also:

$$\frac{1}{(1 + \tau_t^c)c_{k,t}} = \frac{\beta[1 - \delta^k + (1 - \tau_{t+1}^y)r_{t+1}]}{(1 + \tau_{t+1}^c)c_{k,t+1}} \quad (\text{A.1})$$

$$\frac{\mu_2}{1 - l_{k,t} - e_{k,t}} = \frac{\mu_1(1 - \tau_t^y)w_t h_{k,t}}{(1 + \tau_t^c)c_{k,t}} \quad (\text{A.2})$$

$$\frac{\mu_2}{1 - l_{k,t} - e_{k,t}} = \rho_{k,t} B \theta_1 (e_{k,t})^{\theta_1 - 1} (h_{k,t})^{\theta_2} (g_{k,t}^e + \gamma \sum_{j \neq k}^N g_{j,t}^e)^{\theta_3} \quad (\text{A.3})$$

$$\rho_{k,t} = \frac{\beta \mu_1 (1 - \tau_{t+1}^y) w_{t+1} l_{k,t+1}}{(1 + \tau_{t+1}^c) c_{k,t+1}} + \beta \rho_{k,t+1} \left[ 1 - \delta^h + B \theta_2 (e_{k,t+1})^{\theta_1} (h_{k,t+1})^{\theta_2 - 1} (g_{k,t+1}^e + \gamma \sum_{j \neq k}^N g_{j,t+1}^e)^{\theta_3} \right] \quad (\text{A.4})$$

$$\frac{\mu_1 p_t^e}{(1 + \tau_t^c) c_{k,t}} = \rho_{k,t} B \theta_3 (e_{k,t})^{\theta_1} (h_{k,t})^{\theta_2} (g_{k,t}^e + \gamma \sum_{j \neq k}^N g_{j,t}^e)^{\theta_3 - 1} \quad (\text{A.5})$$

where  $\rho_{k,t}$  is the dynamic multiplier associated with the motion of human capital as written in the text.

Note that by using (A.2) and (A.3) into the optimality condition for the excludable public good, (A.5), we can rewrite (A.5) as:

$$g_{k,t}^e = \frac{\theta_3 (1 - \tau_t^y) w_t h_{k,t} e_{k,t} - \bar{g}_t - \gamma \sum_{j \neq k}^N g_{j,t}^e}{\theta_1 p_t^e}$$

which is the private agent's demand function for public education services. It states that the agent's demand for such services increases with their relative productivity (as measured by  $\theta_3$  vis-à-vis  $\theta_1$ ) as well as with the wage earned by the human capital co-generated by these services, while it decreases with the tuition fee,  $p_t^e$ . For related demand functions for

excludable public goods and services, see e.g. Atkinson and Stiglitz (1980, equation 16-57), Ott and Turnovsky (2006) and Economides and Philippopoulos (2015).

### Households as workers

The worker's optimality conditions are as those of the capitalist without the Euler condition for physical capital. Thus, we have the constraints and also:

$$\frac{\mu_2}{1 - l_{w,t} - e_{w,t}} = \frac{\mu_1(1 - \tau_t^y)w_t h_{w,t}}{(1 + \tau_t^c)c_{w,t}} \quad (\text{B.1})$$

$$\rho_{w,t} = \frac{\mu_1(1 - \tau_t^y)w_t h_{w,t}}{(1 + \tau_t^c)c_{w,t}} \frac{1}{B\theta_1(e_{w,t})^{\theta_1-1}(h_{w,t})^{\theta_2}(g_{w,t}^e + \gamma \sum_{j \neq w}^N g_{j,t}^e)^{\theta_3}} \quad (\text{B.2})$$

$$\rho_{w,t} = \frac{\beta\mu_1(1 - \tau_{t+1}^y)w_{t+1}l_{w,t+1}}{(1 + \tau_{t+1}^c)c_{w,t+1}} + \beta\rho_{w,t+1} \left[ 1 - \delta^h + B\theta_2(e_{w,t+1})^{\theta_1}(h_{w,t+1})^{\theta_2-1}(g_{w,t+1}^e + \gamma \sum_{j \neq w}^N g_{j,t+1}^e)^{\theta_3} \right] \quad (\text{B.3})$$

$$g_{w,t}^e = \frac{\theta_3(1 - \tau_t^y)w_t h_{w,t} e_{w,t}}{\theta_1 p_t^e} - \bar{g}_t - \gamma \sum_{j \neq w}^N g_{j,t}^e \quad (\text{B.4})$$

### Firms

Firms act competitively. The standard first-order conditions are:

$$r_t = \frac{\alpha y_{f,t}}{k_{f,t}} \quad (\text{C.1})$$

$$w_t = \frac{(1 - \alpha)y_{f,t}}{l_{f,t}} \quad (\text{C.2})$$

so that profits are zero in equilibrium.

### Market-clearing conditions

The market-clearing conditions in the labor market, the capital market, the private good market and the market for publicly provided education services are respectively:

$$\sum_{k=1}^{N^k} l_{k,t} h_{k,t} + \sum_{w=1}^{N^w} l_{w,t} h_{w,t} = \sum_{f=1}^{N^k} l_{f,t} \quad (\text{D.1})$$

$$\sum_{k=1}^{N^k} k_{k,t} = \sum_{f=1}^{N^k} k_{f,t} \quad (\text{D.2})$$

$$N^k [c_{k,t} + k_{k,t+1} - (1 - \delta^k)k_{k,t}] + N^w c_{w,t} + G_t^e + N\bar{g}_t = N^k y_{f,t} \quad (\text{D.3})$$

$$G_t^e = N^k g_{k,t}^e + N^w g_{w,t}^e \quad (\text{D.4})$$

Notice that using the optimality conditions (A.5) and (B.4) for  $g_{k,t}^e$  and  $g_{w,t}^e$  respectively into (D.4), or equivalently into  $g_t^e = \nu^k g_{k,t}^e + \nu^w g_{w,t}^e$  (where  $g_t^e \equiv G_t^e / N$ ,  $\nu^k \equiv N^k / N$  and  $\nu^w \equiv N^w / N = 1 - \nu^k$ ), we get an expression for the user price,  $p_t^e$ :

$$p_t^e = \frac{\theta_3(1 - \tau_t^y)w_t[\nu^k h_{k,t} e_{k,t} + \nu^w h_{w,t} e_{w,t}]}{\theta_1 g_t^e}$$

where, as said in the text,  $g_t^e \equiv G_t^e / N$ .

### Decentralized competitive equilibrium (for any feasible policy)

Collecting the above equations, the DCE can be summarized by the following system of equilibrium conditions (as said above, to get the baseline model of section 3 without uniform provision, we just set  $\bar{g}_t = 0$ ).

#### Capitalist

$$\frac{1}{(1 + \tau_t^c)c_{k,t}} = \frac{\beta[1 - \delta^k + (1 - \tau_{t+1}^y)r_{t+1}]}{(1 + \tau_{t+1}^c)c_{k,t+1}} \quad (\text{E.1})$$

$$\frac{\mu_2}{1 - l_{k,t} - e_{k,t}} = \frac{\mu_1(1 - \tau_t^y)w_t h_{k,t}}{(1 + \tau_t^c)c_{k,t}} \quad (\text{E.2})$$

$$\rho_{k,t} = \frac{\mu_1(1 - \tau_t^y)w_t h_{k,t}}{(1 + \tau_t^c)c_{k,t}} \frac{1}{B\theta_1(e_{k,t})^{\theta_1-1}(h_{k,t})^{\theta_2}(\bar{g}_t + g_{k,t}^e + \gamma[(N^k - 1)g_{k,t}^e + N^w g_{w,t}^e])^{\theta_3}} \quad (\text{E.3})$$

$$\rho_{k,t} = \frac{\beta\mu_1(1 - \tau_{t+1}^y)w_{t+1}l_{k,t+1}}{(1 + \tau_{t+1}^c)c_{k,t+1}} + \quad (\text{E.4})$$

$$+ \beta\rho_{k,t+1} \left[ 1 - \delta^h + B\theta_2(e_{k,t+1})^{\theta_1}(h_{k,t+1})^{\theta_2-1}(\bar{g}_{t+1} + g_{k,t+1}^e + \gamma[(N^k - 1)g_{k,t+1}^e + N^w g_{w,t+1}^e])^{\theta_3} \right]$$

$$g_{k,t}^e = \frac{\theta_3(1 - \tau_t^y)w_t h_{k,t} e_{k,t}}{\theta_1 p_t^e} - \bar{g}_t - \gamma[(N^k - 1)g_{k,t}^e + N^w g_{w,t}^e] \quad (\text{E.5})$$

$$h_{k,t+1} = (1 - \delta^h)h_{k,t} + B(e_{k,t})^{\theta_1}(h_{k,t})^{\theta_2}(\bar{g}_t + g_{k,t}^e + \gamma[(N^k - 1)g_{k,t}^e + N^w g_{w,t}^e])^{\theta_3} \quad (\text{E.6})$$

*Worker*

$$\frac{\mu_2}{1-l_{w,t}-e_{w,t}} = \frac{\mu_1(1-\tau_t^y)w_t h_{w,t}}{(1+\tau_t^c)c_{w,t}} \quad (\text{E.7})$$

$$\rho_{w,t} = \frac{\mu_1(1-\tau_t^y)w_t h_{w,t}}{(1+\tau_t^c)c_{w,t}} \frac{1}{B\theta_1(e_{w,t})^{\theta_1-1}(h_{w,t})^{\theta_2}(\bar{g}_t + g_{w,t}^e + \gamma[(N^w-1)g_{w,t}^e + N^k g_{k,t}^e])^{\theta_3}} \quad (\text{E.8})$$

$$\begin{aligned} \rho_{w,t} = & \frac{\beta\mu_1(1-\tau_{t+1}^y)w_{t+1}l_{w,t+1}}{(1+\tau_{t+1}^c)c_{w,t+1}} + \\ & + \beta\rho_{w,t+1} \left[ 1 - \delta^h + B\theta_2(e_{w,t+1})^{\theta_1}(h_{w,t+1})^{\theta_2-1}(\bar{g}_{t+1} + g_{w,t+1}^e + \gamma[(N^w-1)g_{w,t+1}^e + N^k g_{k,t+1}^e])^{\theta_3} \right] \end{aligned} \quad (\text{E.9})$$

$$g_{w,t}^e = \frac{\theta_3(1-\tau_t^y)w_t h_{w,t} e_{w,t}}{\theta_1 p_t^e} - \bar{g}_t - \gamma[(N^w-1)g_{w,t}^e + N^k g_{k,t}^e] \quad (\text{E.10})$$

$$h_{w,t+1} = (1-\delta^h)h_{w,t} + B(e_{w,t})^{\theta_1}(h_{w,t})^{\theta_2}(\bar{g}_t + g_{w,t}^e + \gamma[(N^w-1)g_{w,t}^e + N^k g_{k,t}^e])^{\theta_3} \quad (\text{E.11})$$

$$(1+\tau_t^c)c_{w,t} + p_t^e g_{w,t}^e = (1-\tau_t^y)w_t h_{w,t} l_{w,t} \quad (\text{E.12})$$

*Resource constraint*

$$n^k [c_{k,t} + k_{k,t+1} - (1-\delta^k)k_{k,t}] + n^w c_{w,t} + g_t^e + \bar{g}_t = n^k y_{f,t} \quad (\text{E.13})$$

*Government budget constraint*

$$\bar{g}_t + g_t^e = n^k [\tau_t^y (r_t^k k_{k,t} + w_t l_{k,t} h_{k,t}) + \tau_t^c c_{k,t} + p_t^e g_{k,t}^e] + n^w [\tau_t^y w_t l_{w,t} h_{w,t} + \tau_t^c c_{w,t} + p_t^e g_{w,t}^e] \quad (\text{E.14})$$

*Market-clearing condition for publicly provided education services*

$$g_t^e = n^k g_{k,t}^e + n^w g_{w,t}^e \quad (\text{E.15})$$

In the above equations, we use for output and factor returns:

$$n^k y_{f,t} = A(n^k k_{k,t})^\alpha (n^k l_{k,t} h_{k,t} + n^w l_{w,t} h_{w,t})^{1-\alpha} \quad (\text{E.16a})$$

$$r_t = \frac{\alpha y_{f,t}}{k_{k,t}} \quad (\text{E.16b})$$

$$w_t = \frac{(1-\alpha)n^k y_{f,t}}{(n^k l_{k,t} h_{k,t} + n^w l_{w,t} h_{w,t})} \quad (\text{E.16c})$$

We thus have a dynamic system of 15 equations, (E.1-E.15), in 15 endogenous variables, which are  $\{c_{k,t}, l_{k,t}, e_{k,t}, k_{k,t+1}, h_{k,t+1}, \rho_{k,t}, g_{k,t}^e, c_{w,t}, l_{w,t}, e_{w,t}, h_{w,t+1}, \rho_{w,t}, g_{w,t}^e, p_t^e\}_{t=0}^{\infty}$  and one of the fiscal policy instruments,  $\{\tau_t^y, \tau_t^c, g_t^e, \bar{g}_t\}_{t=0}^{\infty}$ , which follows residually to satisfy the within-period government budget constraint; in the case with tuition fees, this role is played by  $g_t^e$ , while the other fiscal instruments are set exogenously.

**Table 1: Steady state solutions with and without tuition fees**

variables	with fees					without fees				
	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.15$	$\gamma = 0.2$	$\gamma = 0.25$	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.15$	$\gamma = 0.2$	$\gamma = 0.25$
$c_k$	0.0478	0.0519	0.0535	0.0548	0.0560	0.0406	0.0479	0.0502	0.0520	0.0535
$l_k$	0.2959	0.2870	0.2850	0.2835	0.2822	0.2823	0.2823	0.2823	0.2822	0.2822
$e_k$	0.1968	0.1909	0.1896	0.1886	0.1877	0.1878	0.1878	0.1877	0.1877	0.1877
$k_k$	1.1557	1.1961	1.2201	1.2430	1.2644	1.0489	1.1374	1.1719	1.2020	1.2283
$h_k$	0.1437	0.1517	0.1552	0.1584	0.1612	0.1381	0.1494	0.1539	0.1578	0.1611
$g_e^k$	0.0218	0.0151	0.0120	0.0085	0.0043	<b>0.0218</b>	<b>0.0151</b>	<b>0.0120</b>	<b>0.0085</b>	<b>0.0043</b>
$y_k^g$	0.1766	0.1822	0.1856	0.1889	0.1919	0.1608	0.1743	0.1795	0.1841	0.1881
$T_k$	0.0364	0.0346	0.0346	0.0346	0.0348	0.0363	0.0354	0.0356	0.0359	0.0363
$y_k^n$	0.1403	0.1476	0.1511	0.1543	0.1572	0.1245	0.1389	0.1440	0.1482	0.1518
$c_w$	0.0482	0.0527	0.0544	0.0559	0.0572	0.0414	0.0490	0.0515	0.0533	0.0549
$l_w$	0.3335	0.3257	0.3241	0.3231	0.3225	0.3159	0.3159	0.3159	0.3159	0.3159
$e_w$	0.2218	0.2166	0.2156	0.2149	0.2145	0.2102	0.2102	0.2102	0.2102	0.2102
$h_w$	0.1652	0.1757	0.1803	0.1845	0.1883	0.1577	0.1712	0.1764	0.1810	0.1850
$g_e^w$	0.0282	0.0284	0.0296	0.0312	0.0331	<b>0.0282</b>	<b>0.0284</b>	<b>0.0296</b>	<b>0.0312</b>	<b>0.0331</b>
$y_w^g$	0.0701	0.0729	0.0744	0.0759	0.0773	0.0634	0.0688	0.0709	0.0728	0.0744
$T_w$	0.0219	0.0201	0.0200	0.0200	0.0201	0.0220	0.0198	0.0195	0.0195	0.0194
$y_w^n$	0.0482	0.0527	0.0544	0.0559	0.0572	0.0414	0.0490	0.0515	0.0533	0.0549
$g_e$	0.0263	0.0245	0.0243	0.0244	0.0245	<b>0.0263</b>	<b>0.0245</b>	<b>0.0243</b>	<b>0.0244</b>	<b>0.0245</b>
$p$	0.2343	0.1372	0.1136	0.0970	0.0846	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
$y$	0.1021	0.1057	0.1078	0.1098	0.1117	0.0927	0.1005	0.1035	0.1062	0.1085
$\tau^c$	<b>0.10</b>	<b>0.10</b>	<b>0.10</b>	<b>0.10</b>	<b>0.10</b>	0.3012	0.1937	0.1717	0.1601	0.1508
$u_k$	-1.6234	-1.5732	-1.5575	-1.5446	-1.5337	-1.6630	-1.5967	-1.5775	-1.5635	-1.5518
$u_w$	-1.6992	-1.6458	-1.6299	-1.6171	-1.6066	-1.7214	-1.6542	-1.6348	-1.6206	-1.6086
$u$	-1.6775	-1.6240	-1.6082	-1.5954	-1.5847	-1.7039	-1.6370	-1.6176	-1.6034	-1.5916
$y_k^g / y_w^g$	2.5187	2.5011	2.4950	2.4894	2.4832	2.5347	2.5313	2.5305	2.5295	2.5282
$y_k^n / y_w^n$	2.9109	2.7984	2.7756	2.7603	2.7489	3.0033	2.8325	2.7974	2.7783	2.7626

**Notes:**  $\alpha = 0.36$ ,  $\delta^k = 0.08$ ,  $\delta^h = 0.05$ ,  $\beta = 0.99$ ,  $B = 0.05$ ,  $\mu = 0.4$ ,  $A = 1$ ,  $\tau^y = 0.15$ ,  $\theta_1 = 0.6$ ,  $\theta_2 = 0.3$ ,  $\theta_3 = 0.1$ ,  $N^k = 3$ ,  $N^w = 7$ ,  $n^k = 0.3$ ,  $n^w = 0.7$ . Numbers in bold denote exogenously set policy variables.

**Table 2: Steady state solutions with and without tuition fees when there is also a minimum uniform provision of public education**

variables	with fees	without fees	without fees ( $g_e^k = g_e^w = g_e$ )	only capitalists pay ( $g_e^w = 0$ )	only $\widehat{g}_e$ ( $g_e^k = g_e^w = g_e = \mathbf{0}$ )	only $\widehat{g}_e$ ( $g_e^k = g_e^w = g_e = \mathbf{0}$ )
$c_k$	0.0521	0.0495	0.0509	0.0572	0.0476	0.0466
$l_k$	0.2842	0.2822	0.2833	0.2981	0.2833	0.2833
$e_k$	0.1890	0.1877	0.1884	0.1983	0.1884	0.1885
$k_k$	1.1839	1.1420	1.1428	1.1410	0.7862	0.9928
$h_k$	0.1509	0.1499	0.1548	0.1732	0.1065	0.1345
$g_e^k$	0.0078	<b>0.0078</b>	<b>0.0181</b>	0.0553	<b>0</b>	<b>0</b>
$y_k^g$	0.1801	0.1749	0.1770	0.1867	0.1218	0.1537
$T_k$	0.0332	0.0341	0.0346	0.0382	0.0112	0.0277
$y_k^n$	0.1468	0.1408	0.1423	0.1485	0.1105	0.1260
$c_w$	0.0531	0.0507	0.0501	0.0494	0.0469	0.0459
$l_w$	0.3234	0.3159	0.3159	0.3159	0.3159	0.3159
$e_w$	0.2151	0.2102	0.2102	0.2102	0.2102	0.2102
$h_w$	0.1754	0.1719	0.1700	0.1591	0.1169	0.1477
$g_e^w$	0.0226	<b>0.0226</b>	<b>0.0181</b>	<b>0</b>	<b>0</b>	<b>0</b>
$y_w^g$	0.0722	0.0691	0.0684	0.0640	0.0470	0.0594
$T_w$	0.0191	0.0184	0.0182	0.0145	0.0001	0.0135
$y_w^n$	0.0531	0.0507	0.0501	0.0494	0.0469	0.0459
$g_e$	0.0181	<b>0.0181</b>	<b>0.0181</b>	0.0166	<b>0</b>	<b>0</b>
$\overline{g}$	0.0052	0.0050	0.0050	0.0050	0.0035	0.0178
$p$	0.1318	<b>0</b>	<b>0</b>	0.0805	<b>0</b>	<b>0</b>
$y$	0.1046	0.1009	0.1009	0.1008	0.0695	0.0877
$\tau^c$	<b>0.10</b>	0.1592	0.1589	<b>0.10</b>	-0.1474	<b>0.10</b>
$u_k$	-1.5662	-1.5836	-1.5740	-1.5559	-1.6008	-1.6094
$u_w$	-1.6382	-1.6408	-1.6452	-1.6508	-1.6720	-1.6806
$u$	-1.6166	-1.6236	-1.6238	-1.6223	-1.6506	-1.6592
$y_k^g / y_w^g$	2.4937	2.5301	2.5884	2.9172	2.5884	2.5885
$y_k^n / y_w^n$	2.7654	2.7775	2.8384	3.0032	2.3565	2.7459

**Notes:**  $\gamma = 0.15$ ,  $\alpha = 0.36$ ,  $\delta^k = 0.08$ ,  $\delta^h = 0.05$ ,  $\beta = 0.99$ ,  $B = 0.05$ ,  $\mu = 0.4$ ,  $A = 1$ ,  $\tau^y = 0.15$ ,  $\theta_1 = 0.6$ ,  $\theta_2 = 0.3$ ,  $\theta_3 = 0.1$ ,  $N^k = 3$ ,  $N^w = 7$ ,  $n^k = 0.3$ ,  $n^w = 0.7$ ,  $s_e^g = 0.05$ .

Numbers in bold denote exogenously set policy variables.



**Table 3: Steady state solutions with and without tuition fees (private education and CES human capital function)**

variables	with fees					without fees				
	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.15$	$\gamma = 0.2$	$\gamma = 0.25$	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.15$	$\gamma = 0.2$	$\gamma = 0.25$
$c_k$	0.0382	0.0397	0.0403	0.0409	0.0414	0.0347	0.0376	0.0385	0.0393	0.0400
$l_k$	0.2959	0.2900	0.2883	0.2870	0.2858	0.2881	0.2868	0.2863	0.2860	0.2857
$e_k$	0.1968	0.1929	0.1918	0.1909	0.1901	0.1916	0.1908	0.1905	0.1902	0.1900
$k_k$	0.9230	0.9295	0.9361	0.9433	0.9506	0.8684	0.8960	0.9080	0.9188	0.9290
$h_k$	0.1148	0.1171	0.1183	0.1194	0.1205	0.1116	0.1158	0.1175	0.1190	0.1204
$s_k$	0.0018	0.0014	0.0013	0.0012	0.0011	0.0017	0.0013	0.0012	0.0011	0.0011
$g_e^k$	0.0156	0.0113	0.0090	0.0065	0.0036	<b>0.0156</b>	<b>0.0113</b>	<b>0.0090</b>	<b>0.0065</b>	<b>0.0036</b>
$y_k^g$	0.1411	0.1418	0.1426	0.1436	0.1446	0.1330	0.1372	0.1391	0.1407	0.1423
$T_k$	0.0273	0.0263	0.0262	0.0261	0.0261	0.0272	0.0266	0.0267	0.0268	0.0269
$y_k^n$	0.1138	0.1154	0.1165	0.1175	0.1185	0.1058	0.1106	0.1124	0.1140	0.1154
$c_w$	0.0385	0.0402	0.0409	0.0415	0.0421	0.0352	0.0382	0.0393	0.0401	0.0408
$l_w$	0.3335	0.3283	0.3269	0.3259	0.3252	0.3233	0.3216	0.3210	0.3206	0.3202
$e_w$	0.2218	0.2184	0.2175	0.2168	0.2163	0.2150	0.2139	0.2135	0.2133	0.2130
$h_w$	0.1319	0.1353	0.1369	0.1385	0.1401	0.1278	0.1325	0.1345	0.1363	0.1380
$s_w$	0.0023	0.0018	0.0017	0.0015	0.0014	0.0021	0.0017	0.0016	0.0014	0.0014
$g_e^w$	0.0202	0.0209	0.0218	0.0228	0.0241	<b>0.0202</b>	<b>0.0209</b>	<b>0.0218</b>	<b>0.0228</b>	<b>0.0241</b>
$y_w^g$	0.0560	0.0565	0.0570	0.0575	0.0580	0.0526	0.0543	0.0550	0.0556	0.0563
$T_w$	0.0152	0.0145	0.0144	0.0144	0.0145	0.0152	0.0143	0.0141	0.0141	0.0141
$y_w^n$	0.0408	0.0420	0.0426	0.0431	0.0435	0.0374	0.0400	0.0408	0.0415	0.0422
$g_e$	0.0188	0.0180	0.0179	0.0179	0.0179	<b>0.0188</b>	<b>0.0180</b>	<b>0.0179</b>	<b>0.0179</b>	<b>0.0179</b>
$p$	0.1457	0.0952	0.0818	0.0720	0.0643	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
$y$	0.0815	0.0821	0.0827	0.0833	0.0840	0.0767	0.0791	0.0802	0.0812	0.0821
$\tau^c$	<b>0.10</b>	<b>0.10</b>	<b>0.10</b>	<b>0.10</b>	<b>0.10</b>	0.2080	0.1611	0.1503	0.1437	0.1377
$u_k$	-1.7134	-1.6883	-1.6768	-1.6686	-1.6613	-1.7365	-1.7020	-1.6911	-1.6825	-1.6746
$u_w$	-1.7892	-1.7599	-1.7501	-1.7419	-1.7349	-1.8020	-1.7656	-1.7539	-1.7447	-1.7364
$u$	-1.7664	-1.7378	-1.7281	-1.7199	-1.7128	-1.7824	-1.7465	-1.7350	-1.7260	-1.7178
$y_k^g / y_w^g$	2.5187	2.5073	2.5027	2.4980	2.4932	2.5291	2.5298	2.5296	2.5296	2.5293
$y_k^n / y_w^n$	2.7885	2.7460	2.7358	2.7284	2.7227	2.8319	2.7683	2.7534	2.7443	2.7358

**Notes:** See notes in Table 1 and also  $\nu = 0.5$ , and  $\phi = 0.7$ .