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# Minimum Wages and Relational Contracts

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#### Abstract

The need to give incentives is usually absent in the literature on minimum wages. However, especially in the service sector it is important *how well* a job is done, and employees must be incentivized to perform accordingly. Furthermore, many aspects regarding service quality cannot be verified, which implies that relational contracts have to be used to provide incentives. The present article shows that in this case, a minimum wage increases implemented effort, i.e., realized service quality, as well as the efficiency of an employment relationship. Hence, it can be explained why productivity and service quality went up after the introduction of the British National Minimum Wage, and that this might actually have caused a more efficient labor market. Furthermore, if workers have low bargaining power, a higher minimum wage also increases firm profits and consequently employment. Therefore, the present article presents a new perspective on reasons for why minimum wages often have no or only negligible employment effects.

Keywords: Minimum Wages, Relational Contracts, Bargaining. JEL Classification: C73, D21, J24, J31.

### 1 Introduction

Minimum wage laws and its positive or negative effects are one of the most controversially debated issues in economics. When trying to understand its consequences, though, only limited attention has been paid to how a minimum wage affects the provision of incentives.

In this article, I show that a minimum wage has a crucial impact on a firm's optimal choice of incentives and consequently on efficiency, profits and employment.

In a partial-equilibrium setting where wages are determined by a bargaining process between firms and workers, firms will require their workers to do a better job in return for high wage payments induced by a binding minimum wage. Given performance is not verifiable, an appropriate minimum wage can then increase the surplus of an employment relationship. If a worker's bargaining power is low, a binding minimum wage will furthermore increase profits and consequently also employment – because the minimum wage serves as a commitment device to pay higher wages in the future. I therefore show that via the channel of increasing a worker's effective bargaining power and implementable effort, a higher minimum wage can induce a Pareto improvement yielding higher profits, worker rents and employment.

Minimum wages are especially relevant in the service sector.<sup>1</sup> There in particular what matters is *how well* - and not only *that* - a job is done. The degree of service quality provided by employees is important for customer satisfaction and will have an impact on a firm's profits. Take employees of a fast food restaurant, who are supposed to be friendly to customers and careful when preparing the food. A cleaner can do a superficial job or clean everything thoroughly, and a nightwatchman might be more or less attentive. Then, it is necessary to incentivize workers, and the question arises how this should and can be done. As many aspects of service quality are highly subjective and cannot be precisely measured, it will generally be difficult to capture all relevant dimensions in an explicit, i.e., court enforceable, contract. Hence, relational contracts are needed, which generally are used in settings where employees need to be given incentives to perform a desired task, but where it is impossible or at least very difficult to verify effort and output.

The present article analyzes the impact of a minimum wage on the optimal choice of incentives within a relational contracting framework. A labor market with many homogeneous firms and workers exists, where entry is costly for firms. In every period of an infinite horizon game, the terms of an employment relationship for a match consisting of one firm and one worker are determined by a bargaining process between the two. As a result, each gets a fixed share of the resulting relationship surplus. In order to create a surplus, though, workers must exert effort, which they will only do if they believe to be sufficiently compensated. Since no formal contracts are feasible, their willingness to exert effort depends on the future rents they expect to achieve within the relationship, and these rents are increasing in their bargaining power. Low bargaining power of workers is hence associated with low effort and a low

<sup>&</sup>lt;sup>1</sup>As an example, in an overview on minimum wages in Canada, Sussman and Tabi (2004, p. 9) state that "[a]lmost all minimum wage workers were employed in the service sector".

relationship surplus. Then, an (exogenous) increase of workers' bargaining power can even yield higher profits for firms – namely if the associated surplus increase more than offsets the higher rents workers collect. Firms are not able to imitate this effect and implement higher effort by promising workers a larger share of the future surplus – because they cannot credibly commit to pay workers more in the future than given by their bargaining power.

A binding minimum wage is basically equivalent to an increase in a worker's bargaining power: Knowing they will earn higher rents in the future, workers are willing to increase effort today in order to keep their job. A binding minimum wage hence increases effort and generally the surplus of an employment relationship. If workers' bargaining power has initially been rather low (which seems to be true for many jobs where a minimum wage is relevant), a binding minimum wage can further increase profits and consequently – as the number of firms and hence jobs is determined by a zero-profit condition – also total employment.

There is evidence that a minimum wage increases productivity, and that this is driven by higher effort levels of employees. Galinda-Rueda and Pereira (2004) and Rizov and Croucher (2011) analyze the effects of the introduction of a National Minimum Wage in Britain in 1999 on labor productivity. Both find a positive and significant effect - in particular in the service sector. In addition, several surveys attempt to provide a better understanding of the specific channels that induced the observed increase in productivity. These surveys find that a substantial amount of firms responded to the minimum wage by inducing higher effort of workers or by providing higher service quality (Low Pay Commission, 2001, or Heyes and Gray, 2003).

Hirsch et.al (2011) show that these effects could also be observed in the US. They state that managers responded to a minimum wage increase with – inter alia – higher prices and higher performance standards. Furthermore, the workforce in their sample was supposed to improve service quality in order to become more productive, where managers in particular used approaches to boost the morale of employees.

Concerning the relationship between a minimum wage and profits (which in my setting is positive for high and negative for low values of worker bargaining power), Harasztosi and Lindner (2015) observe a non-negative effect. They show that this has been driven by a substantial increase in sales revenues, which indicates that their results are consistent with my story: An increased service quality might have allowed firms to charge higher prices and generate higher sales revenues. Moreover, the positive effect of a minimum wage on (net) sales was mainly observed in the service industry.

Finally, their empirical outcomes support my prediction that employment effects of a minimum wage should be driven by its impact on profits if relational contracts are relevant: Harasztosi and Lindner (2015) not only find that the minimum wage increase did not affect profits, but also observe no negative employment effects in the service industry.

### **Related Literature**

An important and considerable amount of research deals with employment effects of minimum wages. The hypothesis derived from the standard textbook model of a labor market - that a binding minimum wage leads to job losses - is now seriously questioned. Empirical studies like Katz and Krueger (1992), Card and Krueger (1994), Machin and Manning (1994) and more recently Dube et al. (2010) or Harasztosi and Lindner (2015) suggest that the employment effect of a minimum wage is not necessarily negative and might even be slightly positive. Other articles (for overviews see Brown, 1999, or Neumark and Wascher, 2007) still claim that a minimum wage destroys jobs.

Several theoretical models have been developed to explain the observed patterns. Bhashkar and To (1999), for example, develop a model of monopsonistic competition where a minimum wage raises employment per firm but causes firms to exit the market. Generally rent-creating search frictions are used as an explanation for the seemingly counterintuitive outcome that a minimum wage does not necessarily destroy jobs (based on Burdett and Mortensen, 1998, see also Card and Krueger, 1995, Flinn, 2006, or Dube et al., forthcoming). These approaches, though, would generally predict a negative effect of a minimum wage on profits, which is not found by Harasztosi and Lindner (2015). There, a higher minimum wage neither has a negative effect on employment nor on profits in the service industry, a result my model can generate.

Furthermore, these articles abstract from incentives, which have been given almost no attention in the relevant literature. Exceptions are Kadan and Swinkels (2010, 2013) and Rebitzer and Taylor (1995). Kadan and Swinkels (2010, 2013) analyze the effect of a wage floor in a standard moral hazard setting. They show that a minimum wage generally has a negative impact on induced effort levels. Different from my setting, they assume that workers are risk averse, effort cannot be observed, and an explicit contract is feasible. Then, a higher wage floor (i.e. payments that have to be made for the lowest output realization) generally increases the marginal costs of inducing effort, reducing total incentives given to employees. However, the non-verfiability of certain activities will often render explicit contracts infeasible, especially in the service sector where minimum wage laws are particularly important. Rebitzer and Taylor (1995) develop an efficiency wage model where a minimum wage makes it easier for firms to prevent a given number of employees from shirking. Thereby, the authors can explain positive employment effects of a minimum wage, which however is also associated with a profit reduction<sup>2</sup>. Furthermore, they do not take the impact on a worker's productivity into account, which in my model is the driving factor of potential positive employment effects of a minimum wage.

Concluding, I derive a new potential driving force for positive employment effects of a minimum wage: It increases workers' effort and potentially has a positive impact on profits, namely when a firm's commitment to compensate workers is insufficient. The consequence that higher profits then imply more demand for workers naturally follows.

While I focus on the impact of a minimum wage on the quality of provided work, quantity aspects have been analyzed as well. Strobl and Walsh (2011), for example, use a competitive model of the labor market to show that a minimum wage can either increase or decrease the hours worked by an employee. I abstract from the amount of hours worked - which are verifiable - and instead focus on the usually non-verifiable aspect of service quality.

Finally, this article relates to the literature on relational contracts. MacLeod and Malcomson (1989) and Levin (2003) are arguably the most prominent contributions to relational contracting in a setting with just one principal and one agent, and show that optimal contracts can take a rather simple form. I analyze a setting with many principals and agents and potential rematching (comparable to MacLeod and Malcomson, 1998), where players bargain over the relationship surplus (following Miller and Watson, 2013).

<sup>&</sup>lt;sup>2</sup>Georgiadis (2013) empirically analyzes how efficiency wages respond to a minimum wage. Although he finds evidence in favor of an efficiency wage model, he does not observe a negative effect on profitability, which a standard efficiency wage model would predict.

### 2 Model Setup

### 2.1 Environment and Production

The market I consider consists of a mass 1 of identical workers ("he") and a potentially infinite mass of identical firms ("it"). Principals and workers are risk neutral. The time horizon is infinite, time is discrete (with periods t = 1, 2, ...), and all players share a common discount factor  $\delta$ . To become active in the market, though, a firm has to pay one-time entry costs k > 0 (after which it can stay in the market forever). The decision whether to enter or not is made by each unactive firm at the beginning of every period. The mass of active firms in period t is denoted by  $M_t$ .

In every period, all workers and all active firms either are part of a firm-worker match or not. The matching process for unmatched parties is random and frictionless and takes place after  $M_t$  has been determined: If  $M_t < 1$  (i.e., there are more workers than firms), every unmatched firm is randomly matched with exactly one unmatched worker. Then, all firms are part of a match, while  $1-M_t$  workers remain unmatched. If  $M_t > 1$ , every unmatched worker is randomly matched with exactly one unmatched firm. In the following I focus on symmetric matches, hence omit indices.

Afterwards, every matched pair (which can either be a new match or one with longer tenure) starts a bargaining process, which is further described below and – if successful – determines a wage payment  $w_t$  from firm to worker, an effort level  $e_t \ge 0$  the worker is supposed to choose, and intended continuation play. Workers then consume  $w_t$  and actually exert effort  $e_t$ , generating output  $y_t = e_t \theta$ . The size of  $\theta$  can be a function of  $M_t$ , the number of firms active in the market, where I naturally assume  $d\theta/dM \le 0$ , reflecting a negative effect of competition on an individual firm's revenues. While output is directly consumed by the firm, a worker faces effort costs  $c(e_t)$ , with c(0) = 0 and c', c'' > 0. Hence, first-best effort  $e^{FB}$  – maximizing the total per-period surplus in a match – is characterized by

$$\theta - c'(e^{FB}) = 0.$$

Note that I exclude the possibility to pay an additional bonus after effort has been exerted. It turns out that this assumption is without loss of generality.

All unmatched players and those who are part of a match but where bargaining has failed consume their exogenous outside utilities in the respective period, and potentially re-enter the matching market in the subsequent period. For simplicity, I set all players's *exogenous* outside utility levels to zero (note that an unmatched player's *endogenous* reservation utility - which reflects the possibility of finding a job with a positive rent and is further described below - can actually be positive).

At the end of every period, an exogenous shock is realized which makes some workers leave the market (for example because the partner found a job somewhere else). With probability  $(1 - \gamma)$ , each worker - no matter whether part of a match or not - leaves the market, and remains for another period with probability  $\gamma$ . If a worker exits the market for exogenous reasons, he leaves for good and receives a payoff normalized to zero from then on. Furthermore, the number of employees remains fixed over time, hence  $(1 - \gamma)$  new workers enter the market in every period.

Finally, all matches where workers have not left the market decide whether they want to remain matched for another period, in which case they again start the bargaining process in period t + 1. If any player decides to leave the match, both players re-enter the matching market in the subsequent period.

The timing within a period t is summarized in the following graph:

	Random matching		W gets $w_t$ supplies $e_t$	-	ration sions
Unactive F		Bargaining	W le	eaves	$\gamma \ {f new}$
decides about	,		with prob. $\gamma$		${f W}$ enter
market entry					

Note that effort in my setting refers to non-verifiable measures such as a worker's motivation and not aspects like working hours, which clearly would be verifiable (in the conclusion, I sketch the implications of also having a verifiable effort dimension in my model). Therefore, my model arguably mainly applies to the service sector where it seems very difficult to pin down aspects like provided service quality in formal, court-enforceable contracts. I do not claim, though, that relational contracts generally are more relevant in the service sector than in other industries. On the contrary, relational contracts seem to be important in many sectors, governing many relationships within and between firms (see Gibbons and Henderson, 2013, or Malcomson, 2013, for surveys). What I argue is that for low-paying jobs where a minimum wage is potentially relevant, formal performance measures exist to a lesser degree in the service sector than, for example, in manufacturing (in addition, the number of low-end jobs has become rather small in developed countries<sup>3</sup>). Exam-

<sup>&</sup>lt;sup>3</sup>I thank an anonymous referee for bringing up this point.

ples for the importance of relational contracts in the service sector are provided by Gibbons and Henderson (2013), who discuss the example of Nordstrom employees who are expected to exercise "good judgement in all situations", which arguably is very difficulty to precisely define in formal contracts. Furthermore, Shemwell et.al (1994) find that issues such as trust and commitment are very important in customer service-provider relationships.

### 2.2 Payoffs

Using  $d_t^P \in \{0, 1\}$  to describe whether a firm is in a relationship in period t and whether bargaining has led to an agreement (i.e.,  $d_t^P = 0$  if either the firm is not part of a match in period t, or if bargaining has not been successful), the payoff stream of an arbitrary firm at the beginning of a period t equals

$$\Pi_t = \mathbf{E}\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} d_{\tau}^P \left(e_{\tau}\theta - w_{\tau}\right)\right].$$

Using  $d_t^A \in \{0,1\}$  to describe whether a worker is in a relationship (by construction, this implies that  $d_t^A = 0$  once a worker has left the market for exogenous reasons) and whether bargaining has led been successful, an arbitrary worker receives

$$U_t = \mathbb{E}\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} d_{\tau}^A \left(w_{\tau} - c(e_{\tau})\right)\right].$$

### 2.3 Observability of Effort and Output

Effort as well as output can be observed by both, firm and worker, but not by anyone outside the respective match, hence is not verifiable. This implies that effort in my setting should not be mistaken for working hours – those are verifiable and can hence be enforced with the use of formal contracts. Effort rather reflects issues like an employee's motivation or provided service quality.

### 2.4 Bargaining Process

Wages and intended effort levels are determined by a bargaining process within a match. Concerning the specifications of this bargaining process, I mainly follow Miller and Watson (2013) who adopt the literature on alternating-offer bargaining (Rubinstein, 1982, or Shaked and Sutton, 1984) to a repeated setting when workers

must be incentivized but formal contracts are not feasible. I slightly adjust their approach when necessary, in order to account for differences between my and their setting.

Formally, Miller and Watson (2013) assume a generalized alternating-offer bargaining protocol. Offers not only specify wage and effort levels for a given period, but also (desired) continuation play (potentially contingent on the full history of the game). For a full formal description, I refer to Miller and Watson (2013), for my purpose the following components are sufficient:

The number of bargaining rounds in every period is arbitrary and may be finite or infinite. What matters is that all rounds occur instantaneously, hence bargaining is not costly in terms of time discounting.

In every bargaining round, either the firm or the worker is randomly selected to make an offer. If this offer is accepted by the counterpart, bargaining ends and the agreement is sealed by a formal short-term contract. This contract states that firm and worker have formed an employment relationship in the respective period (which is important if a minimum wage is present) and determines a wage payment  $w_t$ . Although intended effort  $e_t$  is part of a bargaining agreement as well, it cannot be specified by the formal contract because of its assumed non-verifiability.

The random process determining who is allowed to make an offer in each bargaining round is not further specified. It has to be designed in a way, though, that if an agreement is reached, the net surplus of the relationship (created in the current as well as all future periods) is distributed according to fixed shares, where a worker gets  $\alpha \in [0, 1]$  of this surplus and the firm  $1 - \alpha$ . The share  $\alpha$  reflects respects respective bargaining powers and is determined by the number of firms active in the market, with  $d\alpha/dM \geq 0$  (hence, the more firms are on the market, the lower is the individual firm's bargaining power). The net surplus is defined as the difference between payoffs after an agreement and payoffs under disagreement, i.e. after bargaining has failed:

Bargaining can fail if an offer is rejected. More precisely, if an offer is rejected, bargaining may break down (which is determined by some random breakdown process) or continue for another round. If bargaining breaks down, play in the respective period is denoted to be under disagreement. Then, no contract is signed for the respective period, no payment is made and no effort exerted (I assume that the worker can only work for the firm if an employment contract has been signed). In this case, players consume their exogenous outside options in the respective periods and re-enter the matching market in the subsequent period (in Appendix B, I relax this assumption and allow players to remain matched in the subsequent period after disagreement in the current one). Therefore, disagreement payoffs are unique and cannot be affected by players. This is different from Miller and Watson (2013), where players remain matched throughout with their initial counterpart, which allows them to also bargain over disagreement play in future periods (which can be arbitrarily history-dependent as long as it constitutes a subgame perfect equilibrium).

Finally, I exclude the possibility of bargaining at later points in time within a period. Otherwise, the player on the short side of the market could induce the other one to make a side-payment in order to avoid a separation.

### 2.5 Strategies and Equilibrium Concept

I impose a couple of restrictions on feasible strategies. First, strategies are assumed to be *contract-specific* in the sense of Board and Meyer-ter-Vehn (2015). This implies that actions of firms and workers, as well as bargaining and disagreement outcomes, do not depend on the identity of the worker, calendar time, or history outside the current relationship (the latter, though, is already implied by my assumption that what happens within a match can only be observed by the involved players). However, a player's strategy will still depend on the strategies of all market participant, since the possibility of a re-match determines everyone's endogenous outside option. Hence, the market as a whole will be in a social equilibrium, as for example described by Ghosh and Ray (1996), Kranton (1996), or MacLeod and Malcomson (1998). More precisely, the equilibrium concept I introduce is a so-called *contractual social equilibrium*. Following Milller and Watson (2013) and extending it to a setting with many players, this concept describes a subgame perfect equilibrium (SPE), where the following assumptions on bargaining play restrict the set of SPE under consideration.

First, internal consistency of agreements is required in a sense that given players agree on continuation payoffs that are SPE payoffs, then continuation play yields the respective payoffs.

Second, under disagreement continuation play is independent of the reason for *why* bargaining has failed (for example whether an unexpected offer has been made). Note that in my benchmark setting where under disagreement, players automatically re-enter the matching market in the subsequent period, this assumption is already implied by my restriction to contract-specific strategies. However, it becomes relevant in the extension analyzed in Appendix B, where players can remain matched

in the subsequent periods following disagreement in the previous one.<sup>4</sup>

Third, players never agree on an outcome that is Pareto dominated by another one, hence strategies are renegotiation-proof.

Concluding, I follow Miller and Watson (2013) and adapt it to my setting where workers can be replaced. A *contractual social equilibrium* describes a subgame perfect equilibrium with a restriction to contract-specific strategies, furthermore assuming that bargaining yields consistent outcomes, disagreement triggers a separation, and that agreement does not involve Pareto dominated outcomes.

Finally, note that all this implies that for a given  $M_t$ , the game is stationary and I can omit time subscripts. This is implied by payoffs under agreement being constant over time – because disagreement outcomes are unique and payoffs under agreement are given by a fixed share of the net surplus which players aim to maximize in every period.

### 3 Results Without a Minimum Wage

#### 3.1 Steady-State Payoffs

In the following, I describe payoffs in the steady state of the game absent a minimum wage. Because there are no matching frictions, this steady state is immediately reached. Furthermore, I assume that no one expects a minimum wage to be introduced in the future. Hence, the number of firms M – and consequently values of  $\alpha$ and  $\theta$  – are (expected to be) constant over time.

Given that workers do not get fired and always accept employment offers in equilibrium, an employed worker's discounted payoff stream is denoted by U and equals

$$U = w - c(e^*) + \delta \gamma U, \tag{1}$$

where  $e^*$  is equilibrium effort in any period t. Future payoffs only enter with probability  $\gamma$ , since the worker might leave the market for exogenous reasons (with probability  $(1 - \gamma)$ ), then receiving a payoff of zero.

<sup>&</sup>lt;sup>4</sup>Miller and Watson (2013) also assume that under disagreement, no transfer is made (formally, continuation play under disagreement is independent of the transfer made in the period where disagreement occurs - hence there is no incentive to make any transfer given bargaining has failed). Such a restriction is not needed in my setting, though, where successful bargaining is sealed by a formal short-term contract, and where effort and wage payments are only possible within such an agreement.

A firm's payoff in any period it is in an employment relationship is denoted by  $\Pi$  and equals

$$\Pi = e^*\theta - w + \delta \left[\gamma \Pi + (1 - \gamma)\overline{\Pi}\right] \ .$$

Again, the current relationship is only continued with probability  $\gamma$ . If it breaks up because the employed worker leaves the market, the firm's payoff at the beginning of next period – then being unmatched – is  $\overline{\Pi}$ . Equivalently, an unmatched worker's utility at the beginning of a period is  $\overline{U}$ .  $\overline{U}$  and  $\overline{\Pi}$  hence describe players' endogenous reservation utilities  $\overline{U}$ , in contrast to their exogenous outside utilities which are set to zero.

The values  $\overline{U}$  and  $\overline{\Pi}$  depend on the number of firms that are active in the market. If firms are on the short side, i.e. if M < 1, they always find a match with probability 1, and  $\overline{\Pi} = \Pi$ . Unemployed workers, on the other hand, are only matched with probability  $\mu \equiv \frac{(1-\gamma)M}{(1-\gamma)+\gamma(1-M)} = \frac{(1-\gamma)M}{1-\gamma M}$ , where the nominator gives the number of available jobs at the beginning of a period (consisting of matches that broke up at the end of the previous period because the respective worker left the market), and the denominator the number of workers looking for a job (consisting of those who newly entered the market as well as those who have been unemployed in the previous period and remained in the market). Therefore,  $\overline{U} = \mu U + \delta(1-\mu)\gamma \overline{U}$  if M < 1.

If M > 1, on the other hand, firms with free vacancies are only matched with probability  $\nu \equiv \frac{1-\gamma}{M-\gamma}$ , whereas workers always find a job. Hence,  $\overline{U} = U$  and  $\overline{\Pi} = \nu \Pi + \delta(1-\nu)\overline{\Pi}$  if M > 1.

The payment w is determined by the outcome of the bargaining process that takes place at the beginning of every period and splits the surplus of an employment relationship accordingly. There, note that the gross surplus generated within a period of a given match is  $e^*\theta - c(e^*)$ , where  $e^*$  is equilibrium effort. Given a match can terminate for exogenous reasons, the total gross surplus of a given relationship is  $S^G = \frac{e^*\theta - c(e^*)}{1 - \delta\gamma}$ . This implies that the net surplus generated within a given match at the beginning of a period is  $S = S^G - \Pi^D - U^D$ , where  $\Pi^D$  and  $U^D$  are a firm's and a worker's disagreement payoffs and will be formally defined below. Since the bargaining process allocates the share  $\alpha$  of the net surplus to the worker, and  $1 - \alpha$ to the firm of a match, utilities are

$$U = U^{D} + \alpha S = (1 - \alpha)U^{D} + \alpha \left(\frac{e^{*}\theta - c(e^{*})}{1 - \delta\gamma} - \Pi^{D}\right)$$
$$\Pi = \Pi^{D} + (1 - \alpha)S = \alpha\Pi^{D} + (1 - \alpha)\left(\frac{e^{*}\theta - c(e^{*})}{1 - \delta\gamma} - U^{D}\right).$$

Disagreement implies that the involved parties do not sign an employment contract in the respective period. Both consume their exogenous outside utilities which are set to zero and re-enter the matching market in the subsequent period where random re-matching is assumed<sup>5</sup>. This confirms that disagreement payoffs are unique and – given the mass of firms M and hence  $\alpha$  are constant – consequently equilibrium payoffs are unique and constant in every period as well, independent of the history of the game.

Hence,

and

$$\Pi^D = \delta \overline{\Pi}.$$

 $U^D = \delta \gamma \overline{U}$ 

Having unique disagreement payoffs is different from Miller and Watson (2013), who focus on a game with two players that also remain matched under disagreement. Then, positive effort can be part of the disagreement outcome, which therefore is not unique and can be a function of the full history of the game. I exclude the possibility of effort under disagreement and more generally that a firm and a worker have a relationship where the worker potentially exerts effort without signing an employment contract (which seals an agreement). This is done for the following reason: Reaching an agreement is necessary to induce a wage payment (as in Miller and Watson, 2013). If a relationship including effort were also feasible without signing a contract, i.e. without a wage payment, this could be a way to circumvent a minimum wage. For the real-world settings I am interested in, a scenario where a worker works for a firm without signing an employment contract and without receiving a wage does not seem too realistic, and in particular not legal in most jurisdictions.

### 3.2 Equilibrium Outcomes for a Given Number of Firms

Since no external enforcement mechanism exists, it must be optimal for an employed worker to choose equilibrium effort  $e^*$ . Implementable effort levels are determined by the difference between a worker's continuation utilities after choosing equilibrium effort  $e^*$  and his continuation utilities after choosing any other effort level. Since continuation utilities are unique in equilibrium and independent of the history of

<sup>&</sup>lt;sup>5</sup>In Appendix B, I allow players to remain matched in the subsequent period if play is under disagreement in the current one.

the game, only a termination of the match can punish workers for a deviation.<sup>6</sup> Furthermore, a termination threat can be used to provide incentives for a worker because – following a deviation – a separation is indeed weakly optimal for one player: If M < 1, a firm will induce the separation, because then  $\overline{\Pi} = \Pi$  and it is indifferent between starting a new or sticking to its old relationship. If M > 1, a separation is weakly optimal for the worker.

Taking this into account, implementable effort levels are determined by a worker's incentive compatibility (IC) constraint, which is

$$-c(e^*) + \delta\gamma \left(U - \overline{U}\right) \ge 0. \tag{IC}$$

As a first result, the (IC) constraint implies that positive effort is only feasible if firms are on the short side of the market:

**Lemma 1:** No effort can be implemented for  $M \ge 1$ .

Proof:  $M \ge 1$  implies  $\overline{U} = U$ , giving the (IC) constraint  $-c(e^*) \ge 0$ , which has  $e^* = 0$  as a unique solution.

Slightly anticipating the analysis of the equilibrium number of firms in section 3.3, Lemma 1 also implies that  $M \ge 1$  cannot determine an equilibrium:  $e^* = 0$  means that firms make no profits; given strictly positive entry costs k, no firm would enter if it expected the number of firms to be larger than 1. In the following, I therefore stick to M < 1 in which case the (IC) constraint becomes

$$-c(e^*) + \delta\gamma \left(\frac{\alpha \left(1-\delta\right) \left(e^*\theta - c(e^*)\right) \left(1-\mu\right)}{\left(1-\alpha\delta\right) \left(1-\delta\gamma\right) + \alpha\delta\gamma\mu \left(1-\delta\right)}\right) \ge 0.$$
(IC)

There, note that although the situation after leaving a given match, i.e., the value of  $\overline{U}$ , is taken as given by an individual worker, symmetry of the game implies that effort and bargaining levels are the same also in future matches and hence affect the (IC) constraint equivalently.

The (IC) constraint is the only constraint that has to be considered explicitly. Other potential constraints automatically hold: Firms and workers are generally

<sup>&</sup>lt;sup>6</sup>However, note that even if continuation utilities could be made contingent on the history, it would still be optimal to use a termination threat in order to provide incentives: It is optimal to use the harshest feasible punishment for any observable deviation (Abreu, 1988); furthermore – since he is always free to leave at the end of a period – a worker cannot be made worse off within than outside a match.

willing to enter a match, because a zero outside-payoff implies that a positive net surplus (which is realized for equilibrium effort levels  $e^* \ge 0$  which are below firstbest effort  $e^{FB}$ ) together with any sharing rule  $(\alpha, 1 - \alpha), \alpha \in [0, 1]$ , makes players better-off within a match than outside (this holds unless a minimum wage – introduced below – is so high that a profitable labor relationship is not feasible for the firm). Furthermore, neither firm nor worker ever have an incentive to terminate a match at the end of a period and go for a new partner. The firm would be rematched with probability 1, however its payoff from starting a new relationship is the same as when remaining with the current partner. The agent would be rematched with a probability smaller than 1, hence his payoff from starting a new relationship is strictly lower than when remaining with the current partner.

Concluding, the problem here is to maximize gross relationship surplus,  $S^G = \frac{e^*\theta - c(e^*)}{1 - \delta\gamma}$ , subject to the (IC) constraint. If the constraint is violated for first-best effort  $e^{FB}$ , implemented effort will be inefficiently low and  $e^* < e^{FB}$ . If the constraint holds for first-best effort, then  $e^* = e^{FB}$ . Hence, no higher effort level than  $e^{FB}$  will ever be implemented, at least as long a minimum wage wage is not too high. In the following, I analyze how the allocation of bargaining power affects implementable effort and players' payoffs.

**Comparative Statics** Fixing the number of firms M, I now explore the impact of an exogenous change of  $\alpha$  on implementable effort and utilities. First, an increase in a worker's bargaining power  $\alpha$  always increases implementable effort and a worker's utility U. Interestingly, firms could also benefit from workers having higher bargaining power:

**Proposition 1** Implementable effort and workers' utilities increase in  $\alpha$ . Furthermore, there exists an  $\overline{\alpha} \in (0, 1)$  such that profits  $\Pi$  increase in  $\alpha$  for  $\alpha \leq \overline{\alpha}$  and decrease in  $\alpha$  for  $\alpha > \overline{\alpha}$ .

The proof can be found in Appendix A.

Implementable effort increases in the difference between a worker's continuation payoff U and his outside option  $\overline{U}$ , and a higher  $\alpha$  increases U by more than  $\overline{U}$ (because M < 1). Furthermore, for small values of  $\alpha$ , (IC) is more likely to bind, restricting effort and hence efficiency. This relationship between bargaining powers and implementable effort and hence efficiency is different from standard relational contracting models<sup>7</sup>, where those generally are functions of the continuation surplus of a relationship (in Levin, 2003, for example, rent distribution can be completely separated from surplus maximization). This difference is driven by the assumptions impose I impose on considered outcomes, in particular that bargaining takes place in every period and that players never agree on an outcome that is Pareto dominated by another one. Hence, if a firm tried to implement higher effort by promising the worker a larger continuation payoff, such a promise would not be credible. Both would correctly anticipate that the next bargaining round would again yield a sharing rule determined by the original bargaining shares (otherwise, the firm also had an incentive to replace its current worker), and that firm and worker would accept this outcome independent of any promises made in the past.

The inability to promise workers a larger share of the surplus can harm firms, namely when  $\alpha$  and hence implementable effort are rather small. To grasp the intuition for this result, first note that in the extreme case of  $\alpha = 0$ , actually no effort at all can be implemented: Plugging  $\alpha = 0$  into the (IC) constraint gives  $-c(e^*) \geq 0$ , which only holds for  $e^* = 0$ . But zero effort also implies zero surplus and hence zero profits. Therefore, a strictly positive  $\alpha(<1)$  which is associated with strictly positive effort leads to strictly positive profits and is hence preferred by a firm. More generally, a higher  $\alpha$  has a direct and an indirect effect on profits II. Whereas the direct effect is always negative (for a given surplus, a worker receives a larger share), the indirect effect is positive if the (IC) constraint binds. Then,  $\alpha$  increases the total surplus which increases profits as well. If  $\alpha$  and hence  $e^*$  are rather small, the latter effect is likely to dominate. On the other hand, for larger values of effort and  $\alpha$ , the negative direct effect will dominate. Then, inducing additional effort becomes more costly (since effort costs are convex), and the increasing surplus does not make up for the smaller share the firm can keep.

Concluding, if  $\alpha$  is rather small, firms would like to voluntary leave a larger share of the surplus to workers, but cannot commit to do so. Since  $\alpha$  is assumed to depend on the number of active firms, though, it cannot be treated in isolation from M and hence  $\mu$ . This will be explored in the following section.

<sup>&</sup>lt;sup>7</sup>As in standard in relational contracting models, though, implementable effort also increases in  $\gamma$  and  $\delta$ . Furthermore, higher values of these parameters benefit both players, since they only affect the total surplus without having any redistributional consequences.

### 3.3 Equilibrium Number of Firms

Now, M is not taken as given anymore and I take into account that firms can freely enter (and leave) the market. Entry is associated with one-time costs k > 0 (exit is costless). Furthermore, I assume  $\frac{d\alpha}{dM} \ge 0$ , i.e. more firms being active in the market (weakly) increases each worker's bargaining power. I simplify the analysis, though, by assuming  $d\theta/dM = 0$ . Otherwise, first-best effort  $e^{FB}$  would also be a function of M. In general, one would expect a negative impact of M on  $\theta$ , since more firms should be associated with a more intense competition on (not further modelled) product markets. Having  $d\theta/dM < 0$  would not change my results (in particular the below proclaimed positive impact of a minimum wage on efficiency would remain as long as negative impact of M on  $\theta$  is not too large) and not generate any additional insights.

Furthermore, I focus on equilibria where a strictly positive number of firms is active and where this number is unique. This is implied by the three following properties: First, k is not too large compared to potential profits. Second, I abstract from coordination issues that might initially be present: An equilibrium with M = 0could exist even in the presence of high potential profits. A single firm would probably not have an incentive to enter if it did not expect others to do so as well, because this firm would have a very high bargaining power ( $\alpha$  would likely be close to zero). Then, hardly any effort could be enforced with the resulting negative consequences on profits. Third, I assume that II is concave in M (not further pursuing what this requires for  $\alpha(M)$ ) and that at least one M > 0 exists that satisfies the zero-profit condition

$$-k + \delta \Pi = 0. \tag{2}$$

Then, the number of firms in equilibrium,  $M^*$ , is unique; it is given by the zero-profit condition (2), and in addition  $d\Pi/dM < 0$  holds.<sup>8</sup>

To better understand the interaction between M and profits (which determines the zero-profit condition (2)), I take a closer look at

$$\frac{d\Pi}{dM} = \frac{\partial\Pi}{\partial M} + \frac{\partial\Pi}{\partial\alpha}\frac{\partial\alpha}{\partial M} + \frac{\partial\Pi}{\partial e}\frac{de}{dM}$$

<sup>&</sup>lt;sup>8</sup>There might exist another value of M that also satisfies the zero-profit condition, but where  $d\Pi/dM > 0$  holds; this cannot constitute an equilibrium, though, because in this case additional entry would yield higher, i.e., strictly positive, profits.

The first term,  $\partial \Pi / \partial M$ , is negative: A higher M increases  $\mu$ , i.e. an unmatched worker's prospects of finding a job, and consequently an employed worker's outside option  $\overline{U}$ . Ceteris paribus, the net surplus of an employment relationship is hence reduced and thereby a firm's profits  $\Pi$ .

The second term,  $(\partial \Pi/\partial \alpha) (\partial \alpha/\partial M)$  is negative as well and captures the negative direct effect of  $\alpha$  on profits, which is driven by workers being able to assume a larger share of a given surplus. The third term,  $(\partial \Pi/\partial e) (de/dM)$ , can be positive or negative, due to the ambiguous effect of M on implementable effort. Again, there is a negative direct effect: Because a worker's outside option  $\overline{U}$  increases in M, a worker's (IC) constraint is tightened. However, there is also the already workedout positive effect of  $\alpha$  (which increases in M) on implementable effort. If  $\alpha$  and hence implementable effort is rather small, this last effect can dominate and lead to a positive total effect of M on  $\Pi$ . I do not assume a specific relationship between M and  $\alpha$ , but having an  $\alpha$  close to zero if only a very small number of firms is present seems sensible, indicating that there should be a range where additional entry actually increases the profits of individual firms, i.e. where entering firms exert a *positive* externality on other firms. This non-monotonic relationship between firm size and profits is not at the core of this paper, but might be interesting on its own because textbook models usually assume a negative relationship.

### 4 Minimum Wage

Now, assume there is a minimum wage, which I define as a lower bound on per-period wages,  $\overline{w} \geq 0$ . I first explore the effect of changes of  $\overline{w}$  on implementable effort and on payoffs for a given market structure, i.e. for fixed values of  $\alpha$  and M. Then, I analyze the effect of a minimum wage on the entry and exit of firms and consequently on equilibrium employment. Note that I consider the effects of unanticipated changes on a minimum wage. Even if changes were anticipated, though, the qualitative effects would remain and only magnitudes would change.

In the previous analysis, I have been silent on wages and described payoffs solely as functions of relationship surplus and bargaining shares. For example, a worker's utility is  $U = \frac{\alpha(1-\delta)\frac{e^*\theta-c(e^*)}{1-\delta\gamma}(1-\delta\gamma(1-\mu))}{(1-\alpha\delta)(1-\delta\gamma)+\alpha\delta\gamma\mu(1-\delta)}$ , where however wage payments from firm to worker are needed in order to give a worker his share of the surplus (the output  $e^*\theta$  is directly consumed by the firm and not verifiable, hence cannot be shared physically). Since payoffs and equilibrium effort are the same in every period, the same holds for wages, and I denote the equilibrium per-period wage  $w^*$ . Then, I can also write a worker's utility stream as  $U = \frac{w^* - c(e^*)}{1 - \delta\gamma}$ , which gives a per-period wage  $w^* = c(e^*) + \alpha \frac{(1-\delta)(e^*\theta - c(e^*))(1-\delta\gamma(1-\mu))}{(1-\alpha\delta)(1-\delta\gamma) + \alpha\delta\gamma\mu(1-\delta)}$ . This indicates that a minimum wage does only affect outcomes if it binds, i.e. if  $\overline{w} > w^*$ . What is also needed for this to hold, though, is that a non-binding minimum wage does not affect disagreement outcomes. This is true in my setting, where an agreement has to be sealed by a contract specifying a wage for the respective period. Under disagreement, no employment contract is signed (otherwise, the employment contract signed under disagreement would also have to include a wage that is not below a minimum wage, and the latter would affect outcomes even if it were not binding under agreement) and no payments are made. Hence, a minimum wage solely affects bargaining in a sense that the contract sealing the bargaining phase cannot specify a wage below  $\overline{w}$ .

Before presenting my results, note that having a binding minimum wage implies that it might (and generally will) be impossible for the firm to get a share  $1 - \alpha$ of the surplus of an employment relationship, but that its maximum feasible payoff is equivalent to a strictly lower share of the surplus. I assume that in this case, the firm's payoff is as high as feasible and that the firm is still willing to enter an employment relationship as long as this is profitable.

#### 4.1 Minimum Wage and Effort

My first result establishes a positive relationship between a minimum wage and equilibrium effort:

**Proposition 2** Assume a binding minimum wage, i.e.,  $\overline{w} > w^*$ . Then, implemented effort is strictly larger than  $e^*$  and further increasing in  $\overline{w}$ .

The proof can be found in Appendix A.

A binding minimum wage always increases implemented effort, which implies that it also increases the efficiency of an employment relationship as long as effort is inefficiently low. This is driven by two implications of a mandatory wage raise. First, a worker's future benefit of keeping the job (i.e.,  $U - \overline{U}$ ) goes up, which increases the effort costs he is willing to bear today in order to not get fired, and consequently increases implementable effort. Second, higher effort is not only feasible but will also be implemented and agreed upon in the bargaining process. This is because a (binding) minimum wage ceteris paribus increases a worker's payoff above the level indicated by his bargaining power  $\alpha$ . Then, a higher effort level is the only way to have an effective sharing rule that comes closer to "fair" levels.

However, the higher implementable effort cannot completely make up for the additional transfer, hence additional rents are shifted from firms to workers. Consequently, maximum feasible effort will be implemented, which is characterized by  $c(\overline{e}) = \overline{w}\delta\gamma(1-\mu)$ , with  $d\overline{e}/d\overline{w} = \delta\gamma(1-\mu)/c' > 0$ . Only if efficient effort can already be implemented if a minimum wage is not present, effective bargaining shares might not be affected by the presence of a binding minimum wage. Then, implemented effort will actually be inefficiently high. This case seems less interesting, though, because a rather high effort level without a minimum wage (for example because  $\alpha$  is large) implies that high wages are paid anyway, making it less likely that a minimum wage actually binds. Furthermore, markets where a minimum wage is relevant supposedly are characterized by low levels of workers' bargaining power. This conjecture is supported by Manning (2003), who presents evidence that employers actually set wages in markets where a minimum wage is relevant. He states that "for the average worker in a non-union setting, this does seem to be the appropriate assumption" (p. 4). Concluding, a binding minimum wage is expected to have a positive effect on the efficiency of employment relationships if those are characterized by low wages and consequently low employee effort, and where effort is hard to verify as for example in the service industry.

Indeed, there is evidence that the productivity of firms has gone up after the introduction of a minimum wage, and that these productivity gains were particularly significant for firms in the service industry. Galindo-Rueda and Pereira (2004) analyze how British firms responded to the introduction of a National Minimum Wage in 1999. They find a positive one-off effect on labor productivity (measured as gross output relative to employment), which in addition is only observed in the service sector and not in manufacturing.

Rizov and Croucher (2011) conduct a further study on the effect of the British National Minimum Wage. They compute a structural estimation of production functions within disaggregate 4-digit industries, controlling for supply and demand factors that affect firms. They find that productivity substantially went up after the introduction - and subsequent increases - of the minimum wage, again with a substantially higher impact in service industries than in manufacturing.

Both studies can only speculate on the factors that cause the observed productivity increases, though. In general, productivity might go up because of reductions in employment or working hours (which however was not observed in both studies), the adjustment of prices, or issues like training, changes in the organizational structure of firms, or - as is the point of this paper - the provision of more effort and hence a higher service quality. Several studies attempt to fill this gap, conducting extensive surveys in which managers were asked how they responded to the introduction of the British National Minimum Wage. Manning et. al (2003) focus on workers in the residential care homes industry. They find that the effect of the minimum wage on worker effort is positive, however not significantly different from zero. The British Low Pay Commission (Low Pay Commission, 2001) - which is supposed to analyze the impact of the British National Minimum Wage and make recommendation concerning potential increases - initiated many research projects to study the exact impact of the National Minimum Wage. They find that 30~% of all firms in the surveys responded by improving the quality of provided services. In one of the involved projects, Heyes and Gray (2003) conduct a survey of small-scale enterprises in the Yorkshire and Humberside region, with a special focus on service industries (motor services, retail, care homes, hairdressing and hospitality). There, 61~% of the firms state that "Increasing workers' level of effort" was an important or very important response to the minimum wage. The point "Improving quality of products and/or services" is regarded as important or very important by 63% of the respondents.

Hirsch et.al (2011) show that these effects could also be observed in the US. They analyze increases of the US federal minimum wage between 2007 and 2009, using a sample of 81 quick-service restaurants in Georgia and Alabama. Their data includes a written survey of restaurant managers and qualitative data collected in interviews with restaurant managers. They observe that managers responded to the minimum wage increase with – amongst others – higher prices and higher performance standards. Furthermore, the workforce was supposed to become more productive and increase sales through improved service, where managers in particular used approaches to boost the morale of employees to increase their productivity.

### 4.2 Minimum Wage and Payoffs

As described in the previous section, a minimum wage has two implications in my model: it shifts rents from firms to workers, but also increases the efficiency of an employment relationship. Unless effort is inefficiently high, workers hence always benefit from a higher (binding) minimum wage. The case is less straightforward for firms. Although they are directly harmed by the rent-shifting effect, they also benefit from the increased efficiency of an employment relationship. If effort is rather low, the latter effect dominates, and firms' profits can actually go up following a minimum wage increase.

**Corollary 1**: An employed worker's payoff U increases in a binding minimum wage as long as effort is inefficiently low, and might or might not increase otherwise; a firm's payoff  $\Pi$  increases in a binding minimum wage as long as effort is below the level characterized by  $\delta\gamma (1-\mu)\theta - c' = 0$ , and decreases otherwise.

*Proof*: If a minimum wage binds and effort is inefficiently low, then  $U = \frac{\overline{w} - c(\overline{e})}{1 - \delta \gamma}$  and  $\Pi = \frac{\overline{e}\theta - \overline{w}}{1 - \delta}$ , where  $\overline{e}$  is characterized by  $\overline{w} - \frac{c(\overline{e})}{\delta \gamma(1-\mu)} = 0$ , hence  $\frac{d\overline{e}}{d\overline{w}} = \frac{\delta \gamma(1-\mu)}{c'}$ . Therefore,  $\frac{dU}{d\overline{w}} = \frac{1 - \delta \gamma(1-\mu)}{1 - \delta \gamma} > 0$  and  $\frac{d\Pi}{d\overline{w}} = \frac{\delta \gamma(1-\mu)\theta - c'}{(1-\delta)c'}$ , which is positive for  $\delta \gamma (1-\mu)\theta - c' \geq 0$  and negative otherwise. If effort is inefficiently high, then either  $U = \frac{\overline{w} - c(\overline{e})}{1 - \delta \gamma}$  or  $U = \frac{\overline{w} - c(\widehat{e})}{1 - \delta \gamma}$ , where  $\hat{e}$  is characterized by  $\overline{w} = c(\hat{e}) + \alpha \frac{(1-\delta)(\hat{e}\theta - c(\hat{e}))(1-\delta \gamma(1-\mu))}{(1-\alpha\delta)(1-\delta \gamma(1-\mu))(\theta - c')}$  (see above). In the first case,  $dU/d\overline{w} > 0$ ; in the second case,  $\frac{dU}{d\overline{w}} = \frac{\alpha(1-\delta)(1-\delta\gamma(1-\mu))(\theta-c')}{[c'(1-\delta\gamma)(1-\alpha)+\alpha(1-\delta)(1-\delta\gamma(1-\mu))\theta](1-\delta\gamma)}$ , which is negative for inefficiently high effort levels.

Whereas the intuition for the mostly positive effect of a minimum wage on Ufor a given market structure is straightforward (note that this also implies that  $d\overline{U}/d\overline{w} > 0$ ), the driver of a potentially positive value of  $d\Pi/d\overline{w}$  is less obvious. It is the same, though, as the one of a potentially positive impact of a worker's bargaining power  $\alpha$  on  $\Pi$ : Larger (future) rents for workers who keep their jobs increase their willingness to exert effort today, because the non-verifiability of effort requires effort costs to be offset by future rents. If effort has been rather low, the associated efficiency increase can more than offset the pure rent-shifting effect. I cannot state which effect is supposed to dominate in the real world. However, since markets where a minimum wage binds are supposedly associated with workers having rather small bargaining power (see Manning, 2003), the impact of a minimum wage on profits should at least be not too negative.

Whereas there is vast evidence that employees benefit from a higher minimum wage (at least those that keep their jobs; potential employment effects are analyzed in the next section, see Holzer et al., 1991, or Harasztosi and Lindner, 2015), only little research exists that assesses the interaction between minimum wages and profits (theoretically, search-and-matching models that try to explain non-negative employment effects such as Dube et. al, forthcoming, but also Manning, 2003, or Flinn, 2006, would generally predict a negative effect on profits). Exceptions for

empirical research are Draca et. al (2011) and Harasztosi and Lindner (2015):

Draca et.al (2011) find a negative effect of the British National Minimum Wage on profits. However, due to a lack of more detailed data, they use the profit-tosales ratio as a measure for profits, although both are not identical. In my setting, for example, the profit-to-sales would correspond to  $(e\theta - w)/e\theta$ . This measure would *always* decrease in a higher (binding) minimum wage, even if profits  $\Pi$  were increasing in  $\overline{w}^9$ .

Harasztosi and Lindner (2015) analyze the impact of a large and persistent increase in the minimum wage in Hungary in 2001, utilizing detailed information on firms' balance-sheets and income statements. They find that the higher minimum wage had large positive effects on labor costs and earnings, however that profits did not decline. Instead, sales substantially went up. Harasztosi and Lindner (2015) claim that higher costs must have been passed on to (final) customers via higher prices. I present a complementary story, claiming that in order to being able to increase prices, firms must also have increased service quality. This claim is further supported by their result that the positive effect of the minimum wage on (net) sales is almost entirely driven by the service industry.

### 4.3 Minimum Wage and Employment

Finally, I assess the impact of a minimum wage on total employment, i.e. the number of firms M in the economy. Because M is determined by a zero-profit condition, there is a strong link between the relationship of  $\overline{w}$  with profits and the effect it has on employment.

**Proposition 3** Equilibrium employment  $M^*$  is increasing in a binding minimum wage as long as effort is below the level characterized by  $\delta \gamma (1 - \mu) \theta - c'$ . For higher effort levels, there exists a threshold  $\hat{e}$  such that employment  $dM^*/d\overline{w} = 0$  for  $e \leq \hat{e}$  and  $dM^*/d\overline{w} < 0$  for  $e > \hat{e}$ .

The proof can be found in Appendix A.

<sup>&</sup>lt;sup>9</sup>To see that, take  $\frac{d(\overline{e}\theta - \overline{w})/\overline{e}\theta}{d\overline{w}} = -\frac{(\overline{e}c' - c(\overline{e}))/c'}{\overline{e}^2\theta}$ , where I took into account that in the case of inefficiently low effort  $\overline{e}$  is defined by  $\overline{w}\delta\gamma(1-\mu) - c(\overline{e}) = 0$ . Because  $c(\cdot)$  is a strictly convex function with c(0) = 0,  $\overline{e}c' - c(\overline{e}) > 0$ , and  $\frac{d(\overline{e}\theta - \overline{w})/\overline{e}\theta}{d\overline{w}} < 0$ .

If a relatively low minimum wage binds in a given industry (indicating that without the minimum wage, wages and hence implemented effort would be rather low), a moderate increase is likely to have a positive employment effect. The intuition for this result is straighforward, given the previously derived effects of a minimum wage on effort and profits. If workers' bargaining power  $\alpha$  is low, a higher minimum wage effectively not only increases effort, but also a firm's profits.<sup>10</sup> Starting from a steady state where  $M^*$ , the total employment level in the industry under consideration, has been generated by a zero-profit condition, higher profits naturally trigger an entry of additional firms.

At intermediate levels, the minimum wage will not have direct negative effects on employment, even though firms' profits go down. This is due to entry costs k: Firms that have entered the market make positive profits, and entry costs k are sunk. So a reduction in profits does not automatically make them give up and leave the market. In the long run, though, employment effects should also be negative: I do not model the possibility of an exogenous exit of firms. If this was possible, then in a steady state without a minimum wage, exiting firms would be replaced by new entries, not affecting equilibrium employment. If a minimum wage reduced profits, though, some exiting firms would not be replaced by new entries, triggering a negative employment effect in the long run. Finally, if the profit reduction is so large that firms even make ex-post losses, some of them will give up, triggering an immediate negative effect on employment.

There exists a large empirical literature on (often non-existent) employment effects of a minimum wage (as examples, take Card and Krueger (1994), Katz and Krueger (1992), Machin and Manning (1994), or more recently Dube et al. (2010) and Harasztosi and Lindner (2015)). Several theoretical explanations have been offered to explain this apparent puzzle, where the predominant approach involves a labor market with matching frictions. Based on the Burdett and Mortensen (1998) search-and-matching model frame, Card and Krueger (1995) or Dube et al. (forthcoming), among others, show that a minimum wage can reduce matching frictions by reducing separations: Because a higher minimum wage induces less wage dispersion, the likelihood of finding a better job is lower. This results in fewer job-to-job transitions and less occupancies firms must fill. In a similar vein, Flinn (2006) uses the positive effect of a minimum wage on workers' bargaining power. Abstracting from on-the-job search, a higher bargaining power increases workers' participation

<sup>&</sup>lt;sup>10</sup>However, note that I neglect a potential negative effect of additional entry on  $\theta$ . Taking this into account would only have a quantitive and not a qualitative effect on my results.

in a given market and induces a larger search intensity, which might eventually increase employment. Generally, if the reduction of matching frictions is sufficiently large to overcome the negative effects of a minimum wage (lower profits make some low-productivity jobs disappear, low-productivity workers have lower chances to find a job), the total employment effects do not have to be negative.

Whereas these models can explain the (partial) absence of negative employment effects of a minimum wage, strong assumptions are needed – especially on the matching process – to generate the desired outcomes. I offer a complementary explanation, where the labor market friction is grounded in an agency problem between a firm and its workers: The latter must be motivated to exert effort, and effort includes dimensions such as motivation or service quality. Formal, court-enforceable contracts hence cannot be used to provide incentives and relational contracts serve as substitutes.

Furthermore, in addition to delivering a new perspective on the impact of a minimum wage on labor markets, my approach also links employment outcomes to the observed empirical connection between a minimum wage and effort of workers, in particular in the service sector. Besides delivering a theoretical explanation for this link, I state that it might further be a driving force behind employment effects of a minimum wage – via its impact on a firm's profits. Support for this view is provided by Harasztosi and Lindner (2015). In their sample, they do not observe a decline in profits following a minimum wage increase. Furthermore, they only detect small (if any) disemployment effect, and then mostly in manufacturing and exporting industries, and not in the service industry, where my model mainly applies to. These connections cannot be made by search-and-matching models. First, those do not consider the importance of how well a job is done, and the efficiency only depends on firm and worker characteristics. Second, given an occupation is filled, a minimum wage there has a negative effect on profits.

### 5 Discussion and Conclusion

Incentives should not be neglected when analyzing the impact of a minimum wage. If relevant aspects of performance like the friendliness towards customers cannot be verified, relational contracts must be used to give incentives. As firms cannot commit to pay workers more in the future than given by their bargaining power, they enforce inefficiently low service quality. If forced to pay a higher wage than actually intended, they also require higher levels of effort. Thus, a minimum wage can increase service quality and even the efficiency of many occupations. If workers' bargaining power has initially been low, a minimum wage might even increase profits and consequently employment.

The mechanism proposed in this article requires a (potentially) infinite duration of an employment relationship, at least there may be no final period after which it ends for sure. Otherwise – because workers are only willing to exert effort if their future utility (conditional on keeping their job) is sufficiently high -a standard unravelling argument would imply zero effort in all periods for tempory workers. But temporary workers – like students having a holiday job – certainly assume a non-negligible share of minimum wage jobs (for example in fast food outlets). To conclude this article, I want to show that by slightly extending the model, I can also reasonably deal with temporary workers: Assume there is a second effort dimension which is verifiable, like hours worked or exercising standard tasks. The first effort dimension still is not verifiable and captures difficult-to-measure aspects such as motivation and provided service quality. Whereas incentive provision for the nonverifiable component would still require the use of relational contracts, formal, courtenforceable short-term contracts could be used for the verifiable effort component (then, the first-best could and would be implemented in this dimension). First of all, note that now, even within a finite employment relationship, some non-verifiable effort might be implementable: If workers have non-zero bargaining power and can hence secure a rent in the last period of an employment relationship (where only the verifiable effort component can be enforced), they will be willing to exert nonverifiable effort in the second-to-last period (and before) in order to keep their job<sup>11</sup>. Implementable non-verifiable effort, though, will always be lower than within a potentially infinite employment relationship – because a longer horizon implies that total expected future rents are higher in the latter case.

Now, as long as a worker's bargaining power is low, the non-verifiable effort component is of negligible importance, and whether a worker is temporary or not hardly makes a difference. This changes with a (higher) minimum wage. Then, the non-verifiable effort component becomes more relevant, and temporary workers are less and less valuable to a firm compared to permanent ones.

<sup>&</sup>lt;sup>11</sup>Some additional assumptions would have to be imposed, though, in order to generate this outcome, because it would have to be optimal for a firm to only fire the worker after not exerting equilibrium effort in the non-verifiable dimension.

There is at least indirect evidence that, when facing a higher minimum wage, firms become more inclined to employ permanent workers. This evidence relates minimum wages to the average tenure of employees, which should go up if less temporary workers are hired. Portugal and Cardoso (2006) find that separations of teenage workers in Portugal decreased after a minimum wage increase, while Dube et al. (2010) observe that average tenure rose substantially in restaurants in San Francisco (while employment remained unchanged). Finally, Dube et al. (forthcoming) find strong evidence that turnover rates for teenagers and restaurant workers fall after a minimum wage increase. Moreover, the majority of the reduction in separations in their sample is observed for workers with less than one quarter tenure, which should include most students with holiday jobs.

### Appendix A – Omitted Proofs

**Proof of Proposition 1.** I consider exogenous changes in  $\alpha$  that affect the economy as a whole, hence also a worker's outside option  $\overline{U}$ . Changes in match-specific bargaining powers would yield very similar results.

First, I show that implementabe effort increases in  $\alpha$ . If the (IC) constraint does not bind, effort is at its first-best and not affected by marginal changes in  $\alpha$ . Now, assume that (IC) binds. Then,  $\frac{de^*}{d\alpha} = -\frac{\delta\gamma \frac{(1-\delta)(e^*\theta-c(e^*))(1-\mu)}{(1-\alpha\delta)(1-\delta\gamma)+\alpha\delta\gamma\mu(1-\delta))^2}}{-c'+\delta\gamma \left(\frac{\alpha(1-\delta)(\theta-c')(1-\mu)}{(1-\alpha\delta)(1-\delta\gamma)+\alpha\delta\gamma\mu(1-\delta)}\right)}$ , which is positive. This is because the denominator – which reflects the partial derivative of the left-hand side of the (IC) constraint with respect to effort – must be negative. If it were not negative, higher effort would relax the (IC) constraint, contradicting that it binds. To show that U is increasing in  $\alpha$ , note that  $\frac{dU}{d\alpha} = \frac{\partial U}{\partial \alpha} + \frac{\partial U}{\partial e} \frac{de^*}{d\alpha}$ , and  $U = \frac{\alpha(1-\delta)\frac{e^*\theta-c(e^*)}{1-\delta\gamma} + (1-\alpha)\delta\gamma\overline{U}}{1-\alpha\delta} = \frac{\alpha(1-\delta)\frac{e^*\theta-c(e^*)}{1-\delta\gamma}(1-\delta\gamma(1-\mu))}{(1-\alpha\delta)(1-\delta\gamma)+\alpha\delta\gamma\mu(1-\delta)}$  (since  $\overline{U} = \frac{\mu}{1-\delta\gamma(1-\mu)}U$ ). The direct effect of  $\alpha$  on U,  $\frac{\partial U}{\partial \alpha} = \frac{(1-\delta)(1-\delta\gamma(1-\mu))(e^*\theta-c(e^*))}{[(1-\alpha\delta)(1-\delta\gamma)+\alpha\delta\gamma\mu(1-\delta)]^2} > 0$ . Naturally,  $\partial U/\partial e \ge 0$  as long as effort is not at its first-best. Finally,  $de^*/d\alpha > 0$  for  $e^* < e^{FB}$  and  $de^*/d\alpha = 0$ for  $e^* = e^{FB}$ , establishing  $dU/d\alpha > 0$ .

Regarding the impact of  $\alpha$  on profits  $\Pi$ , I have  $\frac{d\Pi}{d\alpha} = \frac{\partial\Pi}{\partial\alpha} + \frac{\partial\Pi}{\partial e} \frac{de^*}{d\alpha}$  and  $\Pi = \frac{(1-\alpha)(e^*\theta-c(e^*))}{(1-\alpha\delta)(1-\delta\gamma)+\alpha\delta\gamma\mu(1-\delta)}$ . There  $\partial\Pi/\partial\alpha < 0$ , whereas  $\frac{\partial\Pi}{\partial e} \frac{de^*}{d\alpha} \ge 0$ . Hence,  $d\Pi/d\alpha < 0$  if the (IC) constraint does not bind and  $e^* = e^{FB}$ . For a binding (IC) constraint and  $e^* < e^{FB}$ , I have  $\frac{d\Pi}{d\alpha} = -\frac{(e^*\theta-c(e^*))(1-\delta)}{[(1-\delta\gamma)(1-\alpha\delta)+\mu\alpha\delta\gamma(1-\delta)]^2} \left\{ \frac{\delta\gamma(1-\mu)\theta-c'}{(e^*-c'(1-\mu))(1-\alpha\delta)-\mu(1-\alpha)\delta\gamma} \right\}$ . This is positive for  $\delta\gamma (1-\mu)\theta - c' > 0$  and negative for  $\delta\gamma (1-\mu)\theta - c' < 0$  (the denominator of the second term again represents the partial derivative of the lefthand side of the (IC) constraint with respect to effort and must hence be negative). To establish the existence of  $\overline{\alpha} \in (0, 1)$ , with  $d\Pi/d\alpha > 0$  for  $\alpha < \overline{\alpha}$  and  $d\Pi/d\alpha < 0$  for  $\alpha > \overline{\alpha}$ , note that for  $\alpha = 0$ , the (IC) constraint becomes  $-c(e^*) \ge 0$ , which only holds for  $e^* = 0$ . Hence,  $\Pi = 0$  for  $\alpha = 0$ , whereas  $\Pi > 0$  for  $0 < \alpha < 1$  (because of  $de^*/d\alpha \ge 0$ , with a strict inequality for  $e^* < e^{FB}$ ). Furthermore, for  $\alpha = 1$  the (IC) constraint becomes  $-c(e^*) + e^*\theta\delta\gamma (1-\mu) \ge 0$ . If it binds, then  $\delta\gamma (1-\mu)\theta - c' < 0$  because otherwise, a larger effort level would relax the (IC) constraint. If it does bind, this inequality holds as well because then,  $\theta - c' = 0$ . Again using  $de^*/d\alpha \ge 0$ , this establishes the existence of  $\overline{\alpha} \in (0, 1)$ , with  $d\Pi/d\alpha > 0$  for  $\alpha < \overline{\alpha}$  and  $d\Pi/d\alpha < 0$ 

**Proof of Proposition 2.** Note that absent a minimum wage, equilibrium wages are  $w^* = c(e^*) + \alpha \frac{(1-\delta)(e^*\theta - c(e^*))(1-\delta(1-\mu)\gamma)}{(1-\alpha\delta)(1-\delta\gamma) + \alpha\delta\gamma\mu(1-\delta)}$ , where  $e^*$  is equilibrium effort absent a

minimum wage. If  $\overline{w} \leq w^*$ , the minimum wage is not binding and irrelevant, in particular since it does not (directly) impact disagreement outcomes. Therefore, assume that  $\overline{w} > w^*$ . This has two effects. First, it represents a redistribution from firm to worker, hence  $\Pi$  goes down and U increases. Second, the increase in U also increases implementable effort. In the following, denote by  $\overline{e}$  the maximum implementable effort given the minimum wage is paid to the worker. For  $\overline{w}$  and  $\overline{e}$ ,  $U = \frac{\overline{w} - c(\overline{e})}{1 - \delta \gamma}$ , and  $\overline{e}$  is characterized by  $-c(\overline{e}) + \delta \gamma \frac{\overline{w} - c(\overline{e})}{1 - \delta \gamma} \left(1 - \frac{\mu}{1 - \delta(1 - \mu)\gamma}\right) = 0$ , i.e., by

$$c(\overline{e}) = \overline{w}\delta\gamma \left(1 - \mu\right),\tag{3}$$

with

$$\frac{d\overline{e}}{d\overline{w}} = \frac{\delta\gamma\left(1-\mu\right)}{c'} > 0. \tag{4}$$

Next, I show that  $\overline{e}$  will actually be implemented – at least as long as  $e^* < e^{FB}$ .  $\overline{e}$  would not be implemented if it resulted in workers getting a share of the surplus that is below  $\alpha$ . Then, bargaining would yield a different (lower) effort level. Only if the worker's effective surplus is at least  $\alpha$ , maximum effort  $\overline{e}$  will be implemented (note that it is not feasible to reduce the worker's effective surplus share – payments cannot be reduced below  $\overline{w}$ , and effort cannot be increased above  $\overline{e}$ ).

Put differently, I have to compare  $\frac{\overline{w}-c(\overline{e})}{1-\delta\gamma}$ , the worker's payoff given a binding minimum wage and maximum implementable effort in that case, to  $\frac{\alpha(1-\delta)\frac{\overline{e}\theta-c(\overline{e})}{1-\delta\gamma}(1-\delta(1-\mu)\gamma)}{(1-\alpha\delta)(1-\delta\gamma)+\alpha\delta\gamma\mu(1-\delta)}$ , which represents the worker's "fair" payoff given  $\overline{e}$  is implemented, i.e. given he receives a share  $\alpha$  of the total net surplus when effort is  $\overline{e}$ .

If  $\frac{\overline{w}-c(\overline{e})}{1-\delta\gamma} \geq \frac{\alpha(1-\delta)\frac{\overline{e}\theta-c(\overline{e})}{1-\delta\gamma}(1-\delta(1-\mu)\gamma)}{(1-\alpha\delta)(1-\delta\gamma)+\alpha\delta\gamma\mu(1-\delta)}$ , then  $\overline{e}$  will be implemented, and the worker is paid  $\overline{w}$  in every period. Otherwise, implemented effort is set to a level such that this condition holds as an equality. This condition can be rewritten to  $\overline{w} \geq c(\overline{e}) + \frac{\alpha(1-\delta)(\overline{e}\theta-c(\overline{e}))(1-\delta(1-\mu)\gamma)}{(1-\alpha\delta)(1-\delta\gamma)+\alpha\delta\gamma\mu(1-\delta)}$ . Using (3) which gives  $\overline{w} = \frac{c(\overline{e})}{\delta\gamma(1-\mu)}$ , and plugging it into the condition gives

$$-c(\overline{e}) + \frac{\alpha\delta\gamma\left(1-\delta\right)\left(1-\mu\right)\left(\overline{e}\theta - c(\overline{e})\right)}{\left(1-\alpha\delta\right)\left(1-\delta\gamma\right) + \alpha\delta\gamma\mu\left(1-\delta\right)} \le 0.$$
(5)

Now, recall that the (IC) constraint without a minimum wage is

$$-c(e^*) + \frac{\alpha\delta\gamma\left(1-\delta\right)\left(1-\mu\right)\left(e^*\theta - c(e^*)\right)}{\left(1-\alpha\delta\right)\left(1-\delta\gamma\right) + \alpha\delta\gamma\mu\left(1-\delta\right)} \ge 0.$$

The left-hand-side of the (IC) constraint is concave in effort, and furthermore decreasing in effort for levels above  $e^*$  in case it binds. In addition  $\overline{e} > e^*$  for a binding minimum wage, hence (5) holds if (IC) binds absent a minimum wage, and  $\overline{e}$  is actually implemented. Since  $\overline{e} > e^*$  and  $d\overline{e}/d\overline{w} > 0$ , the proposition is proven in case  $e^* < e^{FB}$ . Now, assume that  $e^* = e^{FB}$ . Then, condition (5) might or might not hold for a given value of  $\overline{w}$ . Assume it does not hold (otherwise, I am done). Then, the worker would get less than his "fair" share when exerting maximum feasible effort  $\overline{e}$ , hence effort is set such that  $U = \frac{\alpha(1-\delta)\frac{e\theta-c(e)}{1-\delta\gamma}(1-\delta\gamma(1-\mu))}{(1-\alpha\delta)(1-\delta\gamma)+\alpha\delta\gamma\mu(1-\delta)}$  and  $\overline{w} = c(e) + \alpha \frac{(1-\delta)(e\theta-c(e))(1-\delta\gamma(1-\mu))}{(1-\alpha\delta)(1-\delta\gamma)+\alpha\delta\gamma\mu(1-\delta)}$ , giving  $\frac{de}{d\overline{w}} = \frac{1}{c'+\alpha \frac{(1-\delta)(\theta-c')(1-\delta\gamma(1-\mu))}{(1-\alpha\delta)(1-\delta\gamma)+\alpha\delta\gamma\mu(1-\delta)}} = \frac{[(1-\alpha\delta)(1-\delta\gamma)+\alpha\delta\gamma\mu(1-\delta)]}{c'(1-\delta\gamma)(1-\alpha)+\alpha(1-\delta)(1-\delta\gamma(1-\mu))\theta} > 0.$ 

**Proof of Proposition 3.** Assume we are in a steady state equilibrium where equilibrium employment is characterized by  $-k + \delta \Pi = 0$ . The unexpected introduction/increase of a binding minimum wage  $\overline{w}$  has the following effect: If  $-k + \delta \Pi$  is increased, employment goes up. Otherwise,  $M^*$  is reduced if and only if the minimum wage increase leads to  $\Pi < 0$ , and remains unaffected as long as  $\Pi \ge 0$ .

In the first case, I compute  $\frac{dM^*}{d\overline{w}} = -\frac{d\Pi/\overline{w}}{d\Pi/dM}$ . Since the denominator has to be negative (otherwise, the situation before would not have constituted an equilibrium), the sign of  $dM^*/d\overline{w}$  is the same as the sign of  $d\Pi/\overline{w}$ . In Corollary 1, I established  $\frac{d\Pi}{d\overline{w}} = \frac{\delta\gamma(1-\mu)\theta-c'}{(1-\delta)c'}$ , giving the first part of the proposition. Concerning the remainder of the proposition, note that if the minimum wage binds,  $\Pi$  is concave in  $\overline{w}$ . This follows from effort  $\overline{e}$  being characterized by  $\overline{w} - \frac{c(\overline{e})}{\delta\gamma(1-\mu)} = 0$ , and the convexity of  $c(\cdot)$ .

## Appendix B – Worker can be Retained Under Disagreement

Assume that under disagreement, the firm can promise the worker to also bargain with him in the next period (i.e., both players do not enter the matching pool then). Everything else remains as before, in particular disagreement still implies that players do not sign an employment contract in the respective period. Note that since M < 1, the firm can unilaterally make the decision whether to remain matched with the worker and does sot in a profit-maximizing way (since  $\Pi = \overline{\Pi}$ , ex-post the firm is indifferent whether to retain the current or hire a new worker). The worker would always be willing to remain matched, since this increases his disagreement and hence also his agreement utility.

Denote the probability with which the firm retains the worker by  $\rho$ . As long

as no (binding) minimum wage is present, this gives a worker the disagreement utility  $U^D = \delta \gamma \left(\rho U + (1-\rho)\overline{U}\right)$ . Furthermore,  $\Pi^D = \delta \left(\rho \Pi + (1-\rho)\overline{\Pi}\right) = \delta \Pi$ since  $\Pi = \overline{\Pi}$ .

The effect of having a higher  $\rho$  on profits is twofold and consists of a direct and an indirect effect (comparable to a higher worker bargaining power  $\alpha$ ). Because it increases  $U^D$  and therefore also  $U(=U^D + \alpha S)$ , the direct effect is negative: For a given effort level, a higher  $\rho$  shifts surplus from firm to worker. The indirect effect is positive, though. Because a higher  $\rho$  increases U, it also increases implementable effort. If the latter has been rather low (for example due to a low value  $\alpha$ ), the indirect effect dominates and the firm would rather retain the worker.

This is summarized in the following Lemma:

**Lemma A1**: If maximum enforceable effort is characterized by  $\delta\gamma\theta - c' > 0$  for  $\alpha = 1$ , then  $\rho = 1$ . Otherwise, there is a  $\hat{\alpha} \in (0, 1)$  such that  $\rho = 1$  for  $\alpha < \hat{\alpha}$  and  $\rho = 0$  for  $\alpha > \hat{\alpha}$ . For  $\alpha = \hat{\alpha}$ ,  $\rho \in (0, 1)$  and set in a way such that  $\delta\gamma\theta - c' = 0$ .

#### Proof:

Note that when setting  $\rho$  in a given relationship, the firm treats  $\overline{U}$  as a constant. The same holds for the effect on effort in the worker's (IC) constraint.

Furthermore,  $\frac{d\Pi}{d\rho} = \frac{\partial\Pi}{\partial\rho} + \frac{\partial\Pi}{\partial e} \frac{de^*}{d\rho}$ , where  $\frac{\partial\Pi}{\partial\rho} = (1-\alpha)\delta\gamma \frac{-\alpha(1-\delta)\frac{e^*\theta-c(e^*)}{1-\delta\gamma} + \overline{U}(1-\alpha\delta-(1-\alpha)\delta\gamma)}{(1-\alpha\delta-(1-\alpha)\rho\delta\gamma)^2}$ . Because of the (IC) constraint  $-c(e^*) + \delta\gamma \left(\frac{\alpha(1-\delta)\frac{(e^*\theta-c(e^*))}{1-\delta\gamma} - \overline{U}(1-\alpha\delta-(1-\alpha)\delta\gamma)}{1-\alpha\delta-(1-\alpha)\rho\delta\gamma}\right) \ge 0$ , the nominator must be negative, hence  $\partial\Pi/\partial\rho < 0$ . Furthermore, since a higher  $\rho$  increases U, the (IC) constraint is relaxed and more effort can be implemented:

$$\frac{de^*}{d\rho} = -\frac{\delta\gamma\left(\frac{(1-\alpha)\delta\gamma\left[\alpha(1-\delta)\frac{\left(e^*\theta-c(e^*)\right)}{1-\delta\gamma} - \overline{U}(1-\alpha\delta-(1-\alpha)\delta\gamma)\right]}{(1-\alpha\delta-(1-\alpha)\rho\delta\gamma)^2}\right)}{-c' + \delta\gamma\left(\frac{\alpha(1-\delta)\frac{\left(\theta-c'\right)}{1-\delta\gamma}}{1-\alpha\delta-(1-\alpha)\rho\delta\gamma}\right)} > 0$$

Since profits increase in effort  $\left(\frac{\partial \Pi}{\partial e} = \frac{(1-\rho\delta\gamma)(1-\alpha)\frac{\theta-c'}{1-\delta\gamma}}{1-\alpha\delta-(1-\alpha)\rho\delta\gamma} \ge 0\right)$ , a higher effort level has positive effect on profits.

Combining these results gives

$$\frac{d\Pi}{d\rho} = -\frac{(1-\alpha)\delta\gamma\left[\alpha(1-\delta)\frac{e^*\theta-c(e^*)}{1-\delta\gamma} - (1-\alpha\delta-(1-\alpha)\delta\gamma)\overline{U}\right]}{(1-\alpha\delta-(1-\alpha)\rho\delta\gamma)(1-\delta\gamma)} \left\{\frac{\delta\gamma\theta-c'}{-c'(1-\alpha\delta-(1-\alpha)\rho\delta\gamma) + \delta\gamma\alpha(1-\delta)\frac{(\theta-c')}{1-\delta\gamma}}\right\}, \text{ and}$$
  
the sign is determined by the sign of  $\delta\gamma\theta-c'$ .

Therefore, I have to distinguish between the following cases:

- $\delta\gamma\theta c' > 0$  for  $\rho = 1$  (in which case it will also be positive for  $\rho = 0$ ) implies  $\rho^* = 1$
- $\delta\gamma\theta c' < 0$  for  $\rho = 0$  implies  $\rho^* = 0$
- $\delta\gamma\theta c' > 0$  for  $\rho = 0$  but  $\delta\gamma\theta c' < 0$  for  $\rho = 1$  implies that  $\rho^*$  is chosen such that  $\delta\gamma\theta c' = 0$ .

Concerning the threshold  $\hat{\alpha}$  note that enforceable effort is increasing in  $\alpha$ . For  $\alpha = 0$ , no effort can be enforced. For  $\alpha = 1$ , the (IC) constraint becomes  $-c(e^*) + \delta\gamma (1-\mu) e^*\theta \ge 0$ . If it does not bind,  $e^*$  is at its first-best level, hence characterized by  $\theta - c' = 0$ , implying  $\delta\gamma\theta - c' < 0$ . If it binds, then  $-c(e^*) + \delta\gamma (1-\mu) e^*\theta = 0$  and  $\delta\gamma (1-\mu)\theta - c' < 0$  (otherwise, higher effort would relax the constraint, contradicting that it binds). Hence, both cases  $-\delta\gamma\theta - c' > 0$  and  $\delta\gamma\theta - c' < 0 - \alpha$  are possible for  $\alpha = 1$  and a binding (IC) constraint.

### Binding Minimum Wage

If players face a minimum wage, the situation resembles the one in the main part – because  $\rho$  only has a direct effect on  $U^D$  which is irrelevant in case of a binding minimum wage (the worker gets more than what is specified by his bargaining power). Only the threshold above where the minimum wage becomes binding is larger than before if effort without a minimum wage is rather low (i.e., lower than the level characterized by  $\delta \gamma \theta - c' = 0$ ): Recall that absent a minimum wage, the wage in the main part was  $w^* = c(e^*) + \alpha \frac{(1-\delta)(e^*\theta - c(e^*))(1-\delta\gamma(1-\mu))}{(1-\alpha\delta)(1-\delta\gamma) + \alpha\delta\gamma\mu(1-\delta)}$ , which for  $\rho = 0$  also holds in this section. Hence, if effort is already relatively high without a minimum wage, the impact as well as the threshold were it becomes binding are unaffected (in this case, since  $\frac{d\Pi}{d\overline{w}} = \frac{\delta\gamma(1-\mu)\theta-c'}{(1-\delta)c'}$ , the effect of a higher minimum wage on profits is unambigously negative). For a relatively low  $\alpha$  such that effort is below the level characterized by  $\delta\gamma\theta - c' = 0$ , hence where  $\rho = 1$ , wages absent a minimum wage are higher than before (implemented effort as well as the worker's utility for a given effort level are higher). Therefore, the threshold above where the minimum wage binds is higher than in the main part. As soon as it becomes binding, though, nothing changes.

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