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by

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Education, lifetime labor supply, and longevity improvements*

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Abstract

This paper presents an analysis of the differential role of mortality for the optimal schooling and retirement age when the accumulation of human capital follows the so-called "Ben-Porath mechanism". We set up a life-cycle model of consumption and labor supply at the extensive margin that allows for endogenous human capital formation. This paper makes two important contributions. First, we provide the conditions under which a decrease in mortality leads to a longer education period and an earlier retirement age. Second, those conditions are decomposed into a Ben-Porath mechanism and a lifetime-human wealth effect vs. the years-to-consume effect. Finally, using US and Swedish data for cohorts born between 1890 and 2000, we show that our model can match the empirical evidence.

JEL: I25, J10, J24, J26.

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1 Introduction

In many countries, economic development has been accompanied with significant increases in life expectancy and reductions in labor supply. Over the nineteenth and twentieth century record life expectancy (at birth) has increased by 40 years at a rate of 3 months per year (Oeppen and Vaupel, 2002; Lee, 2003), while labor supply has decreased at two extensive margins: later entrance in the labor market and earlier retirement.[1] These historical trends are shown in Figure 1 for males born in the United States from 1850 to 1930. Between the 1850 and 1930 cohorts, average years of schooling increased from 8 to 13.28 (an increase of 5 years), average retirement age decreased from 69 to 63.8 (a declined of 5 years), and the life expectancy at age 5 increased from 52.5 to 66.7 (an increase of 14 years).

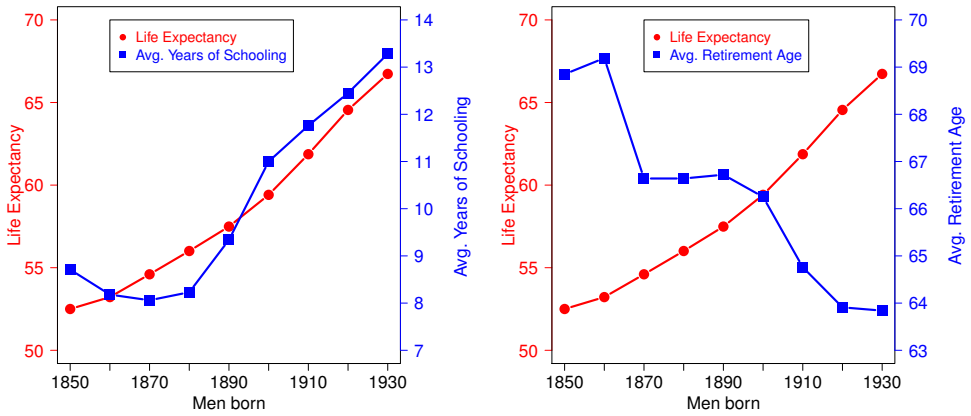


Figure 1: Life expectancy at age 5, average years of schooling, and average retirement age for US men by birth cohort

Source: Hazan (2009). The average retirement age were calculated using the labor force participation rates from age 45 to 80.

These trends are not exclusively of the United States but are, instead, common to most advanced countries. Prior to industrialization, male literacy rates started to increase in the most advanced countries (Cippola, 1969; Cervellati and Sunde, 2005; Boucekkine et al., 2007). This process continued with an expansion of primary education enrollment rates at the end of the nineteenth century and first half of the twentieth century (Be-

[1]Before 1950, most of the gain in life expectancy was due to large reductions in death rates at younger ages (Oeppen and Vaupel, 2002). Still, although at a lower rate, the life expectancy at ages 6 and 16 have also risen for the last 160 years in developed countries like Sweden at a steady pace of 1¾ and 1½ months per year, respectively.

navot and Riddle, 1988). By 1950, the average length of schooling for males was around six years in the most advanced countries and has increased up to twelve years in 2010 (Barro and Lee, 2013). Over the same period, labor force participation rates for old workers started to fall until recently, even before the introduction of pension systems (Costa, 1998; Schieber and Shoven, 1999). In 1970, the average retirement age was 68 in OECD countries and has declined to age 63 in 2010 – see OECD (2009).

Existing theoretical models that analyze the effect of mortality on education and retirement implicitly assume a positive causal relationship between the length of schooling, retirement, and life expectancy; for example Boucekkine et al. (2002), Echevarria (2004), Echevarria and Iza (2006), Ferreira and Pessoa (2007), and Zhang and Zhang (2009), among others. Thus, according to the existing literature, a decline in mortality leads to an increase in education and a postponement of the retirement age, which contradicts the historical empirical evidence. The motivation of our study is therefore to provide possible explanations under which a decrease in mortality leads to an increase in schooling and early retirement.

There are two important dimensions to be considered when modeling the effect of mortality on schooling and retirement. First, the observed positive link between human capital investment and life expectancy that is theoretically replicated through the well-known Ben-Porath (1967)'s mechanism (de la Croix and Licandro, 1999; Kalemli-Ozcan et al., 2000; Zhang et. al., 2001, 2003; Cervellati and Sunde, 2005; Soares, 2005; Zhang and Zhang, 2005; Jayachandran and Lleras-Muney, 2009; Oster et al., 2013), except for Hazan and Zoabi (2006). Second, the recent findings showing that the link between the life expectancy and labor supply depends on the age pattern of mortality improvements. In particular, on the one side, for a given retirement age it has been shown that only improvements in survival during prime-working ages –and not longevity per se– increase human capital investment (Cervellati and Sunde, 2013; de la Croix, 2015). On the other, for a given educational attainment mortality declines during adulthood may cause early retirement, while reductions in mortality at older ages delay retirement (d'Albis et al., 2012; Strulik and Werner, 2012).[2]

In this paper, we set up a life-cycle model of consumption and labor supply at the extensive margin that allows for endogenous human capital formation through the Ben-Porath's mechanism. First, we explain the differential role of mortality on the optimal schooling choice and retirement choice. Second, we use the model to study whether the observed decline

[2]Empirical investigations of the mortality decline over the last two centuries show that mortality does not improve uniformly across age groups (Lee, 1994; Wilmoth and Horiuchi, 1999; Cutler et al., 2006). Early stages of the mortality transition are mainly characterized by reductions of mortality for infants and children, while recent mortality declines occur at older ages.

in mortality across cohorts born in US and Sweden can produce a monotonic increase in schooling followed by a decline in retirement. Our model has one key feature. Following the literature on human capital formation, individuals have a relative disutility from attending school, or aversion to schooling time, like in Heckman et al. (1998), Bils and Klenow (2000), Card (2001), Oreopoulos (2007), Restuccia and Vandenbroucke (2013). This feature has been shown to be important to account for the substantial difference between the returns to schooling and the marginal cost of schooling (Oreopoulos, 2007).[3] Our model differs from the previous literature in two aspects. First, we model the labor supply decision at the extensive margin (retirement) rather than at the intensive margin (hours worked). Thus, we complement the recent work by Restuccia and Vandenbroucke (2013), who have shown that the increase in life expectancy during the last century only counts for 3% of the decline in hours worked in the US, by empirically showing the effect of life expectancy on retirement. Second, since the decline in mortality does not occur uniformly across age-groups, following d'Albis et al. (2012) we model the age-specific mortality rates non-parametrically. Thus, using the derivative of a functional (Ryder and Heal, 1973; d'Albis et al., 2012), we analyze the impact of a mortality decline at any arbitrary age on human capital investment and retirement.

This paper provides two important contributions. First, we find that when there exists an aversion to schooling time, an increase in the length of schooling might lead to a decline in the retirement age. An increase in productivity raises the marginal benefit of continue working, inducing individuals to retire later, whereas an increase in productivity also implies that individuals use the additional income to raise consumption and leisure (substitution effect). The net effect is ambiguous and depends on the strength of the income effect versus the substitution effect. We show that the critical parameter driving the strength of the income effect is the aversion to schooling time. Hence, only when we assume that there exists aversion to schooling time we can replicate that an increase in life expectancy may cause an increase in the length schooling and early retirement, reconciling the empirical facts with the economic theory. Second, using a general utility function we provide the economic intuition of our results by decomposing the differential effect of mortality on schooling and retirement into a *Ben-Porath mechanism* and a *lifetime human wealth* effect vs. the *years-to-consume* effect. The *Ben-Porath mechanism* is the positive effect on the

[3]The empirical work on the returns to education suggests that the aversion to schooling time captures sizable nonpecuniary effects of schooling. Examples of negative nonpecuniary effects of schooling are high psychic costs of school and higher risk and uncertainty (Heckman, 2003; Carneiro et al., 2003; Cunha et al., 2005; Heckman et al., 2006), while suggested positive nonpecuniary effects of compulsory schooling are fostering trust and the reduction of teen fertility, criminal activity, or smoking (Oreopoulos and Salvanes, 2011), among others.

marginal benefit of schooling caused by gains in life expectancy. *Lifetime human wealth* effect stands for the positive impact that a mortality decline has on consumption because it raises the likelihood of receiving a future labor income stream. On the contrary, the *years-to-consume* effect, which is always negative, reflects the overall reduction in consumption due to a longer lifespan.

We also perform a quantitative exercise of our model using US and Swedish mortality data. We restrict the parameters of the model so that it reproduces the years of schooling and retirement age observed in the data for the cohort born in 1890.[4] Then, we compute cohort-specific sequences of years of schooling and retirement ages assuming that each cohort faces a different survival probability. Using a stylized model, we find that improvements in the survival probability may account for a decline in the retirement age around 1.2 years (or 40% of the total decline) and an increase in years of education around 1.2 year (or 30%) between the 1890 and 1930 cohorts. Thereby, our results suggest that the effect of increases in life expectancy on the labor supply is stronger than previously suggested. In addition, since in the earlier stage of mortality transition, a decline in mortality mainly occurred to younger people, whereas in the later stage, a decline in mortality has mainly occurred at older ages, we show that the optimal retirement age bottomed out for cohorts born in the 1920s and it is expected to increase from now on.

The paper is organized as follows: Section 2 introduces the model setup and presents the first-order conditions for optimal consumption, length of schooling, and retirement. Furthermore, the relationship between the optimal length of schooling and retirement is explained. In Section 3, we study –using the Volterra derivative of a functional– the differential role of mortality on the optimal length of schooling and retirement. For a better understanding on the role of mortality on each variable, we distinguish between the partial and total impact of mortality on education and retirement, separately. In Section 4, we solve the model numerically and demonstrate, using a simple quantitative exercise, how the mortality transition may increase the length of schooling and reduce the retirement age. Concluding remarks are made in Section 5.

[4]Starting from this cohort guarantees that in both countries individuals received pension benefits upon retirement from the PAYG pension system. In the United States the Social Security Act was signed in 1935 by Franklin D. Roosevelt. In Sweden the old-age pension system switched from a fully funded system that dates back to 1913 to a pay-as-you-go system in 1935 (Palme, 2005). The universal coverage was achieved in 1946 in Sweden, while more than 60% of workers were covered by the Social Security in 1940 in the US (Schieber and Shoven, 1999).

2 The model

We setup a consumer's problem that consists in choosing the optimal number of years of schooling (S), optimal retirement age (R), and the optimal consumption path ($c(x)$) in order to maximize the expected lifetime utility ($V(S,R,c)$). We assume time is continuous. Agents face lifetime uncertainty, which is represented by the survival function

$$p(x) = e^{-\int_0^x \mu(q)dq}, \tag{1}$$

where $p(x)$ is the (unconditional) probability of surviving to age x, $p(0) = 1$, $p(\omega) = 0$, $\omega \in (0,\infty)$ denotes the maximum age, and $\mu(q) \geq 0$ is the mortality hazard rate at age q.

Schooling and labor supply are indivisible and the transitions from schooling to working and from working to retirement are irreversible, as in Boucekkine et al. (2002), Echevarria (2004), Echevarria and Iza (2006), and Cai and Lau (2012). We also assume that agents do not save with a bequest motive in mind and there exists a perfect annuity market, which grants that agents borrow and lend freely at a fixed interest rate. Thereby, consumers optimally choose to purchase annuities (Yaari, 1965). The instantaneous expected utility depends positively on current consumption and negatively on current non-leisure time. The utility of consumption $U(c)$ is an increasing and concave function (i.e. $U_c(\cdot) > 0$, $U_{cc}(\cdot) < 0$).[5] Let $\tilde{\phi}(S,x)$ denote the disutility of non-leisure time at age x of an individual who has completed S years of schooling. Assume $\tilde{\phi}(S,x)$ is a positive and increasing function with respect to age (i.e. $\tilde{\phi}(S,x) > 0$, $\tilde{\phi}_x(S,x) > 0$), which reflects the fact that the disutility of not enjoying leisure is increasing with age (Hazan, 2009; Kalemli-Ozcan and Weil, 2010; d'Albis et al., 2012; Cai and Lau, 2012). After retirement, $\tilde{\phi}(S,x)$ equals zero. Then, assuming that agents discount future utility flows at a subjective discount rate ρ, the expected lifetime utility, conditional on the years of schooling (S), retirement (R), and consumption path (c) is

$$V(S,R,c) = \int_0^{\omega} e^{-\rho x} p(x) U\big(c(x)\big)dx - \int_0^{R} e^{-\rho x} p(x)\tilde{\phi}(S,x)dx. \tag{2}$$

Following Heckman et al. (1998), Bils and Klenow (2000), Card (2001), Oreopoulos (2007), Restuccia and Vandenbroucke (2013), we assume agents may have different preferences between schooling time and working time

$$\tilde{\phi}(S,x) = \begin{cases} \phi(x) + \psi(x) & \text{if } x \leq S, \\ \phi(x) & \text{if } x > S, \end{cases} \tag{3}$$

[5] We use subscripts to denote the derivative with respect to the variable in the subscript, and apply the same notation for partial derivatives.

where $\phi(x) > 0$ (with $\phi_x(x) \geq 0$) is the underlying disutility of non-leisure time and $\psi(x)$ is the relative disutility from attending school or aversion to schooling time. Factor $\psi(x)$ is positive when the agent prefers work to schooling or negative when schooling is preferred to work. We assume that if $\psi(x)$ is positive, the aversion to schooling time increases with age (i.e. $\psi_x(x) \geq 0$), whereas if $\psi(x)$ is negative, our agent has a decreasing preference for schooling, or $\psi_x(x) \leq 0$. As a particular case, notice $\psi(x) = 0$ for all $x \in (0, \omega)$ is implicitly assumed in Boucekkine et al. (2002), Echevarria (2004), Echevarria and Iza (2006), Ferreira and Pessoa (2007), Zhang and Zhang (2009), Kalemli-Ozcan and Weil (2010), and Cai and Lau (2012), among many others.

Labor income, denoted by y, is assumed to be proportional to years of schooling (S) and years of post-schooling experience (E), which is equal to $x - S$ (Mincer, 1974). Thus, we write labor income at age $x = S + E$ conditional on S years of schooling as $y(S, S + E) = w(E)h(S + E)$, where $w(E) > 0$ represents the wage rate per unit of human capital with $E = x - S$ years of post-schooling experience and $h(S + E)$ is the stock of human capital of an individual at age x with S years of schooling. We assume there exists an $\bar{E} \in [0, R - S]$, for $R > S$, such that

$$\begin{cases} w_E(E) \geq 0 & \text{if } E \leq \bar{E}, \\ w_E(E) \leq 0 & \text{if } E \geq \bar{E}, \end{cases} \tag{4}$$

which represents the usual hump-shape pattern of the labor income found in empirical data (Heckman et al., 2006).[6] This assumption although realistically capture the observed shape of lifetime income profiles, it is not necessary for the results presented in the paper.[7] Assume the law of motion of human capital of an individual at age x with S years of schooling accumulates according to a Ben-Porath (1967) technology

$$h_x(x) = \begin{cases} q(h(x)) - \delta h(x) & \text{if } x \leq S \\ -\delta h(x) & \text{otherwise,} \end{cases} \tag{5}$$

where $q(\cdot)$ is the human capital production function (with $q_h(\cdot) > 0$ and $q_{hh}(\cdot) < 0$), and $\delta > 0$ is the human capital depreciation rate, which is assumed constant across age.[8] As a result, the law of motion of financial

[6] Using data from the US Decennial Censuses, the estimated coefficients from Mincer log earnings regressions for white males during the period 1940-90 report an average $\bar{E}$ value of 30.1 years (Heckman et al., 2006, Table 2, p. 326).

[7] A sufficient condition for our results to hold is that the wage rate is a non-increasing function upon retirement. See Appendix A.

[8] The functional form $h(S) = e^{\theta(S)}$, used by Hazan (2009), p. 1834, can be obtained assuming either that $\delta = 0$ or that $q(h(x))$ is equal to $(\theta_x(x) + \delta)h(x)$.

wealth at age x ($a(x)$) is

$$a_x(x) = \begin{cases} [r + \mu(x)]a(x) + y(S, x) - c(x) & \text{if } S < x < R, \\ [r + \mu(x)]a(x) - c(x) & \text{otherwise,} \end{cases} \tag{6}$$

with boundary conditions $a(0) = 0$ and $a(\omega) = 0$, where r is the real interest rate. Integrating (6) with respect to age, subject to the boundary conditions, we obtain the standard lifecycle budget constraint faced by our individual:

$$\int_0^{\omega} e^{-rx} p(x)c(x)dx = \int_S^R e^{-rx} p(x)y(S, x)dx \equiv W(S, R), \tag{7}$$

where $W(S, R)$ is the lifecycle earnings (measured at age 0) conditional on S years of schooling and retirement age R. For the sake of comparison with the literature on the impact of mortality on retirement and education, notice that we implicitly assume that the only pecuniary cost of schooling is foregone labor income (Kalemli-Ozcan et al., 2000; Hazan, 2009; Cai and Lau, 2012; Cervellati and Sunde, 2013). Tuition costs, earnings while in school, and taxes are also modeled in the returns to education literature (Willis, 1986; Card, 2001; Heckman et al., 2006).

2.1 Optimal consumption, length of schooling, and retirement age

Following d'Albis and Augeraud-Véron (2008), Heijdra and Romp (2009), and d'Albis et al. (2012) we obtain our agent's optimal consumption path, length of schooling, and retirement in two steps. First, we derive the optimal consumption path. We define the optimal consumption at age x, conditional on the length of schooling (S) and retirement age (R), as $c(x, S, R)$. Second, based on the conditional consumption path derived in the first step, we obtain the optimal length of schooling and retirement age. Let us define $\hat{V}(S, R)$ as the expected lifetime utility conditional on the optimal consumption path.

In Proposition 1, we characterize the optimal consumption path, the optimal length of schooling, and the optimal retirement age. The proof is given in Appendix A.

Proposition 1 *For the life-cycle model given by (1)-(6), the optimal consumption path, conditional on a length of schooling S and a retirement age R, is characterized by*

$$U_c\big(c(x, S, R)\big) = e^{(\rho - r)x} U_c\big(c(0, S, R)\big). \tag{8}$$

Moreover, an interior optimal length of schooling (S^*) satisfies

$$\int_{S^*}^{R} e^{-r(x-S^*)} \frac{p(x)}{p(S^*)} y_S(S^*, x)dx = y(S^*, S^*) + \frac{e^{(r-\rho)S^*}\psi(S^*)}{U_c\big(c(0, S^*, R)\big)}, \tag{9}$$

and an interior optimal retirement age (R^*) is given by

$$U_c\big(c(0, S, R^*)\big)e^{-rR^*}y(S, R^*) = e^{-\rho R^*}\phi(R^*). \tag{10}$$

Eq. (8) is the standard Euler condition characterizing the consumption path. The left-hand side of Eq. (9) is the marginal benefit of the S^*-th year of schooling, whereas the right-hand side represents the marginal cost of the S^*-th year of schooling. The first term is the foregone earnings, or pecuniary cost of schooling, and the second term represents the nonpecuniary cost (if $\psi(S^*) > 0$) or benefit (if $\psi(S^*) < 0$) from attending schooling. Let us define $f(S, R)$ as the marginal effect of an additional unit of schooling (measured at age S) on lifecycle earnings:

$$f(S, R) \equiv \frac{W_S(S, R)}{e^{-rS}p(S)} = \int_S^R e^{-r(x-S)} \frac{p(x)}{p(S)} y_S(S, x)dx - y(S, S), \tag{11}$$

or, equivalently, the marginal benefit of the S-th year of schooling minus the foregone labor income at age S (measured at age S). Provided the labor income is separable in education and experience, from (5) Eq. (11) can be rewritten, after rearranging, as

$$f(S, R) = \frac{W(S, R)}{e^{-rS}p(S)} \left(\frac{q\big(h(S)\big)}{h(S)} - \delta \right.$$
$$\left. -r - \frac{\int_S^R e^{-(r+\delta)x}\mu(x)p(x)w(x-S)dx}{\int_S^R e^{-(r+\delta)x}p(x)w(x-S)dx} - \frac{e^{-rR}p(R)y(S, R)}{W(S, R)} \right), \tag{12}$$

where $q(h(S))/h(S) - \delta$ is the rate of return to education at the S-th unit of schooling (henceforth $r^h(S)$). The third and fourth terms inside the parenthesis represent the average return lost in the capital market from postponing the entrance in the labor market. Specifically, the fourth term is the average risk premium lost from the S-th unit of schooling, which hereinafter we denote by $\bar{\mu}_{[S,R]}$. The last term is the income lost at retirement relative to the lifetime wealth. For notational convenience, let us denote the sum of the negative terms in (12) as $\bar{r}(S, R)$; that is

$$\bar{r}(S, R) = r + \bar{\mu}_{[S,R]} + \frac{e^{-rR}p(R)y(S, R)}{W(S, R)}. \tag{13}$$

Eq. (13) represents the hurdle rate or annuitized marginal cost of the S-th unit of schooling, expressed in terms of foregone earnings, conditional on

the retirement age R.[9] Assuming there is no mortality risk and considering that R tends to infinity, Eq. (13) reduces to the real interest rate. Thus, if $\psi(S^*)$ is zero, we obtain the result that individuals invest in schooling until the marginal return to education equals the return to capital, see Willis (1986).

Substituting (11)-(13) in (9), and rearranging, gives

$$r^h(S^*) = \bar{r}(S^*, R) + \frac{e^{-\rho S^*} p(S^*)\psi(S^*)}{W(S^*, R)U_c(c(0, S^*, R))}. \tag{14}$$

Eq. (14) implies that the return to education at the S^*-th unit of schooling is equal to the sum of the marginal cost of the S^*-th unit of schooling expressed in terms of foregone earnings and the nonpecuniary cost/benefit of schooling. Eq. (14) implies that when working is preferred to schooling ($\psi(S^*) > 0$), individuals underinvest in education since $r^h(S^*) > \bar{r}(S^*, R)$. In contrast, when schooling is preferred to work ($\psi(S^*) < 0$), individuals over-investment in education since $r^h(S^*) < \bar{r}(S^*, R)$. As a consequence, if education were considered a pure investment good ($\psi(S^*) = 0$), $r^h(S^*) = \bar{r}(S^*, R^*)$.

Empirically, the econometric estimations of returns to education report values of $r^h(S^*)$ exceeding those of $\bar{r}(S^*, R^*)$. For example, Card (1999) finds a wide range of rates of returns to education in the US centered around 8% per year, while Heckman et al. (2008) estimate also for the US that the returns to education range between 10 to 15% per year. In contrast, when education is considered a pure investment good, the rate of return to education for an individual with 10 years of education does not exceed 3% per year for a wide range of feasible retirement ages.[10] Several explanations are suggested in the literature for the positive difference between $r^h(S^*)$ and $\bar{r}(S^*, R^*)$. The most common ones are high "psychic cost" of school, uncertainty, and heterogeneity among individuals (Carneiro et al., 2003; Cunha et al., 2005; Heckman et al., 2006), the myopic behavior of adolescents (Oreopoulos, 2007), while credit constraints might be important for going to college decisions (Belley and Lochner, 2007), but not for most students (Carneiro and Heckman, 2002; Heckman et al., 2006). Henceforth, following the literature on returns to education, we assume hereinafter that $\psi(x) > 0$ for all $x \in (0, S)$. As a consequence, $r^h(S^*) > \bar{r}(S^*, R)$.

Eq. (10) is the optimal retirement age condition. Eq. (10) implies that the marginal benefit of continued working at age R^*, which is equivalent to

[9] The hurdle rate is: the minimum return required to make an individual financially better off from taking one year of school instead of one year of work (Oreopoulos, 2007).

[10] A value of 3% has been calculated based on the wage rate per unit of human capital $\log w(x-S) = \log w(0) + 0.094(x-S) - 0.0013(x-S)^2$ withdrawn from Table 2 (Heckman et al., 2006, p. 326), US death rates of males from the cohort born in year 1900 (Bell et al., 1992), an interest rate of 3%, and no human capital depreciation rate.

the additional labor income at age R^* measured in utility terms, equals the marginal cost of working at age R^*, or the disutility of continued working at age R^*. This optimal retirement age condition was first derived by Sheshinski (1978).

The first important results one can obtain from Proposition 1 are the effects of an increase in the optimal length of schooling and retirement age on the optimal consumption path. Differentiating (7) and (8) with respect to S, substituting, and using (11) gives

$$\frac{c_S(0,S^*,R)}{c(0,S^*,R)} = \frac{e^{-rS^*}p(S^*)\sigma(c(0,S^*,R))f(S^*,R)}{\int_0^\omega e^{-rx}p(x)\sigma(c(x,S^*,R))c(x,S^*,R)dx}. \tag{15}$$

where

$$\sigma(c) = -\frac{U_c(c)}{c\cdot U_{cc}(c)} > 0, \tag{16}$$

is the intertemporal elasticity of substitution (IES) for consumption c. Using (7) and (12)-(14), Eq. (15) becomes[11]

$$\frac{c_S(0,S^*,R)}{c(0,S^*,R)} = \frac{\sigma(c(0,S^*,R))}{\sigma(c(\bar{x},S^*,R))}(r^h(S^*) - \bar{r}(S^*,R)), \tag{18}$$

Assuming a constant IES, Eq. (18) implies that the relative increase in the initial consumption due to an additional unit of schooling is equal to the difference between the return to education and the marginal cost of the S^*-th unit of schooling expressed in terms of foregone earnings. As a consequence, an additional investment in schooling is efficient when $r^h(S^*) > \bar{r}(S^*,R)$, and inefficient when $r^h(S^*) < \bar{r}(S^*,R)$.

To analyze the impact of retirement on the optimal consumption path we differentiate (7) and (8) with respect to R. Substituting and using (10), we have

$$\frac{c_R(0,S,R^*)}{c(0,S,R^*)} = \frac{e^{-rR^*}p(R^*)\sigma(c(0,S,R^*))y(S,R^*)}{\int_0^\omega e^{-rx}p(x)\sigma(c(x,S,R^*))c(x,S,R^*)dx}, \tag{19}$$

which is equivalent to

$$\frac{c_R(0,S,R^*)}{c(0,S,R^*)} = \frac{\sigma(c(0,S,R^*))}{\sigma(c(\bar{x},S,R^*))}\frac{e^{-rR^*}p(R^*)y(S,R^*)}{W(S,R^*)}. \tag{20}$$

[11] Applying the mean value theorem for integration, there exists an $\bar{x} \in (0,\omega)$ such that

$$\int_0^\omega e^{-rx}p(x)\sigma(c(x,S^*,R))c(x,S^*,R)dx = \sigma(c(\bar{x},S^*,R))\int_0^\omega e^{-rx}p(x)c(x,S^*,R)dx = \sigma(c(\bar{x},S^*,R))W(S^*,R). \tag{17}$$

For a constant IES, Eq. (20) states that the relative impact of delaying retirement on the initial consumption is equal to the weight of labor income at age R^* in lifecycle earnings. Thereby, contrary to an increase in the length of schooling, an increase in the retirement age always raises the optimal consumption path because the agent receives an additional labor income at age R^*.

2.2 Relationship between years of schooling and retirement

In the previous subsection we have shown the first-order conditions for an optimum of S^* and R^*, separately, and how they impact on the optimal consumption path. In this subsection, we turn to a detailed study about the relationship between the optimal years of schooling and the optimal retirement age.

Let us denote $c(0, S^*, R^*)$ as c^* and $c(\bar{x}, S^*, R^*)$ as $\bar{c}$. Applying the implicit-function theorem to the first order condition for S^* around the point (S^*, R^*), we can examine the impact on the optimal length of schooling of a change in the retirement age. Totally differentiating (9) with respect to R and S, taking $\psi(S^*)$ as common factor, and rearranging, we obtain

$$\left.\frac{dS^*}{dR}\right|_{R=R^*} = \frac{\frac{f_R(S^*,R^*)}{f(S^*,R^*)} - \frac{1}{\sigma(c^*)}\frac{c_R^*}{c^*}}{\frac{\psi_S(S^*)}{\psi(S^*)} + \frac{1}{\sigma(c^*)}\frac{c_S^*}{c^*} - \frac{f_S(S^*,R^*)}{f(S^*,R^*)}}. \tag{21}$$

Similarly, applying the implicit-function theorem, we totally differentiate (10) with respect to S and R to examine the impact on the optimal retirement age of a change in the length of schooling

$$\left.\frac{dR^*}{dS}\right|_{S=S^*} = \frac{-\frac{1}{\sigma(c^*)}\frac{c_S^*}{c^*} + r^h(S^*) + \delta - \frac{w_{R-S}(R^*-S^*)}{w(R^*-S^*)}}{\frac{1}{\sigma(c^*)}\frac{c_R^*}{c^*} - \frac{w_{R-S}(R^*-S^*)}{w(R^*-S^*)} + \delta + \frac{\phi_R(R^*)}{\phi(R^*)}}. \tag{22}$$

Provided (S^*, R^*) is an interior solution of our problem, substituting (11) and (20) in (21)-(22), and using (13), we have[12]

$$\text{sign}\left[\frac{dS^*}{dR^*}\right] = \text{sign}\left[\frac{\bar{r}(S^*, R^*) + \sigma(\bar{c})\left[\delta - \frac{w_{R-S}(R^*-S^*)}{w(R^*-S^*)}\right]}{1-\sigma(\bar{c})} - r^h(S^*)\right], \tag{23}$$

Eq. (23) implies that the length of schooling S^* and the retirement age R^* may be either positively or negatively related. On the one hand, looking at Eq. (22), we have that an additional year of schooling after age S^*

[12] From now on, we use the term dS^*/dR^* as a shorthand notation for $\left.\frac{dS^*}{dR}\right|_{R=R^*} \Big/ \left.\frac{dR^*}{dS}\right|_{S=S^*}$.

increases the labor income at age R^* by $r^h(S^*) + \delta - \frac{w_{R-S}(R^*-S^*)}{w(R^*-S^*)}$, which increases the marginal benefit of working. As a consequence, our individual optimally postpones the retirement age in order to reap the benefits of schooling. On the other hand, the increase in education may also change the marginal utility of consumption, and hence the marginal benefit of working, by $-\frac{1}{\sigma(c^*)}\frac{c_S^*}{c^*}$.[13] Thus, the net effect of a change in education on retirement depends upon the strength of the income effect, reflected by the IES (Imrohoroglu and Kitao, 2009; Keane, 2011), and the difference between $r^h(S^*)$ and $\bar{r}(S^*, R^*)$. At the extreme cases, when $\sigma(\bar{c})$ tends to one or $\bar{r}(S^*, R^*) = r^h(S^*)$, we have that the sign of $\frac{dS^*}{dR}\big|_{R=R^*}$ and $\frac{dR^*}{dS}\big|_{S=S^*}$ depend on the sign of $\bar{r}(S^*, R^*) + \delta - \frac{w_{R-S}(R^*-S^*)}{w(R^*-S^*)}$, which is always positive whenever $R^* - S^* \geq \bar{E}$.

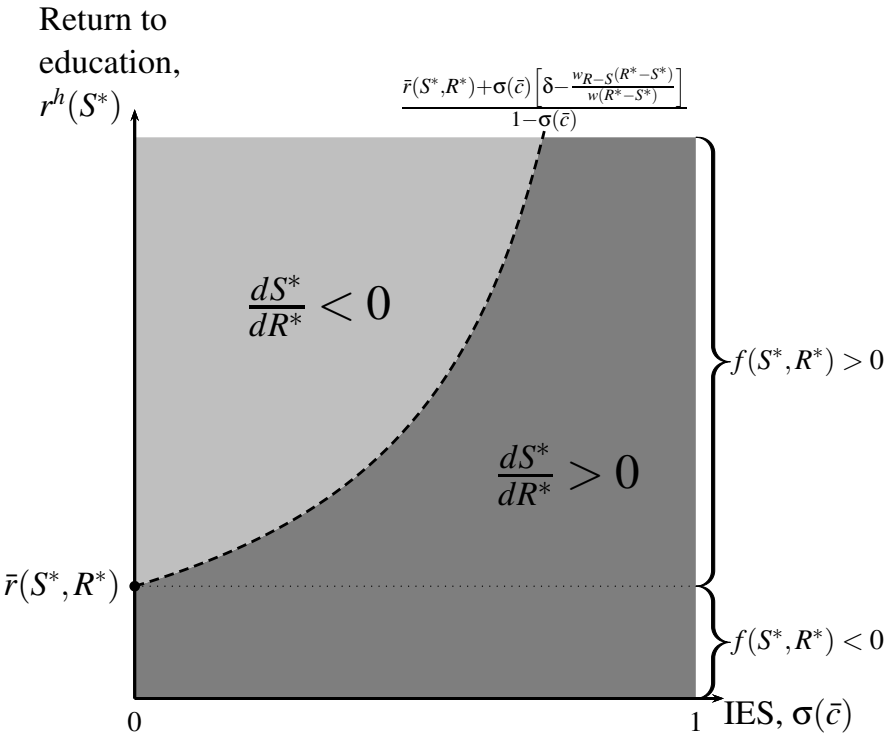


Figure 2: Relationship between S^* and R^* by return to education and intertemporal elasticity of substitution

Figure 2 summarizes the result obtained in Eq. (23). For any given wage

[13] Differentiating (8) with respect to S, we have

$$\frac{1}{\sigma(c^*)}\frac{c_S^*}{c^*} = \frac{1}{\sigma(c(x, S^*, R^*))}\frac{c_S(x, S^*, R^*)}{c(x, S^*, R^*)} \text{ for all } x \in [0, \omega).$$

rate per unit of human capital, Figure 2 is divided into two shaded areas. A dark gray area that contains the combination of $(r^h(S^*), \sigma(\bar{c}))$ values for which S^* and R^* are positively related, and a light gray area with the combination of $(r^h(S^*), \sigma(\bar{c}))$ values for which S^* and R^* are negatively related. It is clear looking at Figure 2 that S^* and R^* are positively related whenever the return to education is equal to, or lower than, $\bar{r}(S^*, R^*)$ (dark gray area below the horizontal dotted line in Figure 2). In this region, an increase in the retirement age leads to an increase in the optimal length of schooling (Ben-Porath, 1967), as well as an increase in schooling yields an increase in the retirement age (Boucekkine et al., 2002; Echevarria and Iza, 2006). However, when the return to education is higher than $\bar{r}(S^*, R^*)$, S^* and R^* can either be positively or negatively related. The black dashed line in Figure 2 delimits the combination of $(r^h(S^*), \sigma(\bar{c}))$ values at which S^* and R^* are not related to each other; i.e. $\frac{dS^*}{dR^*} = 0$. The light gray area, located at the upper-left corner, is characterized by low IES and high return to education. In this area, the income effect dominates. Thus, for a sufficiently high return to education and low IES, when a positive income shock increases the optimal years of schooling, the optimal retirement age decreases, since individuals purchase more leisure time, and the positive effect on years of schooling gets reinforced. The same effect would take place if the positive income shock initially reduces the retirement age. Notice, however, the negative relation between S^* and R^* vanishes as the return to education approaches the dashed line, which eventually occurs when the length of schooling is sufficiently large. On the contrary, in the dark gray area, where the strength of the income effect diminishes –as a consequence of a positive income shock that raises the retirement age– the optimal years of schooling increases and the rise in the retirement age gets also reinforced.

3 Differential impact of mortality decline on optimal schooling years and retirement age

In this Section, we study the impact of a mortality decline at an arbitrary age (x_0) on the optimal length of schooling (S^*) and the optimal retirement age (R^*). For clarity of exposition, we make explicit the dependence of the optimal schooling and retirement age on each other and on the underlying mortality schedule; i.e. $S^* \equiv S^*(R^*; \mu)$ and $R^* \equiv R^*(S^*; \mu)$.[14]

Eqs. (24a)-(24b) below show how the effect of a mortality decline at an

[14]Let the continuous function $\mu : [0, \omega) \to \mathbf{R}^+, x_0 \mapsto \mu(x_0)$ represents the mortality hazard rate at any age $x_0 \in [0, \omega)$.

arbitrary age x_0 is characterized by the sum of two partial effects:

$$\operatorname{sign}\left[\frac{-dS^*}{d\mu(x_0)}\right] = -\operatorname{sign}\left[S^*_{\mu(x_0)}(R^*;\mu) + \frac{dS^*}{dR^*}R^*_{\mu(x_0)}(S^*;\mu)\right], \tag{24a}$$

$$\operatorname{sign}\left[\frac{-dR^*}{d\mu(x_0)}\right] = -\operatorname{sign}\left[R^*_{\mu(x_0)}(S^*;\mu) + \frac{dR^*}{dS^*}S^*_{\mu(x_0)}(R^*;\mu)\right]. \tag{24b}$$

See the proof in Appendix D. The first partial effect is the impact of a mortality decline at x_0 on S^* and R^* –respectively– holding all other variables unchanged, while the second partial effect is the impact of retirement (resp. schooling) on schooling (resp. retirement) that is mediated by a change in mortality. Thus, as shown in (24a) and (24b), the effect of mortality on S^* and R^* are intertwined.

For exposition clarity, in Section 3.1 we first focus on the analysis of the partial effects and, in Section 3.2, we study the total impact of a mortality decline on both variables.

3.1 Partial effect

Following the same order as the derivation of the first-order conditions, we first study the partial impact that a mortality decline has on the optimal consumption path and, second, we continue with the analysis of the partial effect of a mortality decline on the optimal length of schooling and retirement age.

To study the effect of mortality on our variables of interest, we make use of the derivative of a functional (Ryder and Heal, 1973; d'Albis et al., 2012) to obtain, through (1), that

$$-\frac{\partial p(x)}{\partial \mu(x_0)} = \begin{cases} p(x) & \text{if } x_0 \leq x, \\ 0 & \text{if } x_0 > x. \end{cases} \tag{25}$$

Eq. (25) means that a mortality decline at age x_0 has no effect on the survival probability before age x_0, but it increases the survival probability at ages above or equal to x_0. From (25) we derive the impact that a mortality decline at an arbitrary age x_0 has on the optimal consumption path and, in particular, on the initial optimal consumption (c^*). Differentiating (7) and (8) with respect to $-\mu(x_0)$, substituting, and rearranging gives

$$\frac{1}{c^*}\frac{-\partial c^*}{\partial \mu(x_0)} = -\frac{\sigma(c^*)}{\sigma(\bar{c})}\frac{e^{-rx_0}p(x_0)a(x_0)}{W(S^*,R^*)}. \tag{26}$$

Notice in Eq. (26) that if the IES is constant across the lifecycle, the relative impact of a mortality decline at age x_0 on the initial consumption is minus the ratio between the financial wealth position at age x_0 and

lifecycle earnings. Thereby, the sign of the impact of a decline in mortality at age x_0 on the optimal consumption path is equal to minus the sign of the financial wealth at age x_0. Moreover, according to Eq. (26) a decline in mortality at two different ages does not necessarily have the same impact on consumption.[15]

Eq. (26) is the extension of Eq. (B.5) in d'Albis et al. (2012) to a model with endogenous human capital investment. Like d'Albis et al. (2012) we show that the optimal consumption path increases with a decline in mortality at age x_0 when $a(x_0) < 0$, while the optimal consumption declines when $a(x_0) > 0$, for all $x_0 \in [0, \omega)$. The intuition is simple. On the one hand, a decline in mortality increases the number of years the agent is expected to live. As a consequence, agents compensate a longer lifespan with an overall reduction in consumption. This effect, which is always negative, is named the "years-to-consume" effect. On the other hand, a mortality decline during the working period raises the likelihood of receiving a future labor income stream, which leads to an overall increase in the consumption path. This other effect, which is always positive, is named the "lifetime human wealth" effect. For a better understanding, Proposition 2 gives the net result of these two opposite effects using a CIES utility function. The proof is given in Appendix B.

Proposition 2 *For the life-cycle model given by (1)-(6), if $U_c(c)$ is a power function, the overall result of $\frac{1}{c^*}\frac{-\partial c^*}{\partial \mu(x_0)}$ is the same as that of*

$$g(x_0) = \frac{\int_{S^*}^{R^*} e^{-rx}\left[\frac{-\partial p(x)}{\partial \mu(x_0)}\right] y(S^*, x)dx}{\int_{S^*}^{R^*} e^{-rx}p(x)y(S^*, x)dx} - \frac{\int_{x_0}^{\omega} e^{-[(1-\sigma)r+\sigma\rho]x}p(x)dx}{\int_0^{\omega} e^{-[(1-\sigma)r+\sigma\rho]x}p(x)dx}, \tag{28}$$

where $\sigma \in [0, 1]$ is the intertemporal elasticity of substitution. Moreover, there exists a critical point x_c within the open interval (S^*, R^*) such that

$$\begin{cases} g(x_0) > 0 & \text{for all } x_0 < x_c, \\ g(x_0) = 0 & \text{for all } x_0 = x_c, \\ g(x_0) < 0 & \text{for all } x_0 > x_c. \end{cases} \tag{29}$$

The first component of (28) is the "lifetime human wealth" effect, while the second component represents the "years-to-consume" effect. An illustration of the shape of both effects across the life-cycle is given in Figure 3. Notice the lifetime human wealth effect dominates the years-to-consume

[15] The partial impact of a mortality decline at all ages on the initial consumption is

$$\frac{1}{c^*}\frac{-\partial c^*}{\partial \mu} = \int_0^{\omega} \frac{1}{c^*}\frac{-\partial c^*}{\partial \mu(x)}d\mu(x) = -\frac{\sigma(c^*)}{\sigma(\bar{c})}\frac{\int_0^{\omega} e^{-rx}p(x)a(x)d\mu(x)}{W(S^*, R^*)}. \tag{27}$$

effect up to age $x_c \in (S^*, R^*)$, the year at which the financial wealth is zero, $a(x_c) = 0$. Therefore, a mortality decline early in life leads to an overall increase in consumption. In contrast, a decline in mortality at ages above x_c leads to an overall decline in consumption because the years-to-consume effect dominates the lifetime human wealth effect. Though for simplicity we have not modeled any retirement pension system, our results are robust to the introduction of a more general and realistic framework. Indeed, the introduction of an income during the retirement period will extend the lifetime human wealth effect up to age ω, shifting the age x_c toward older ages.

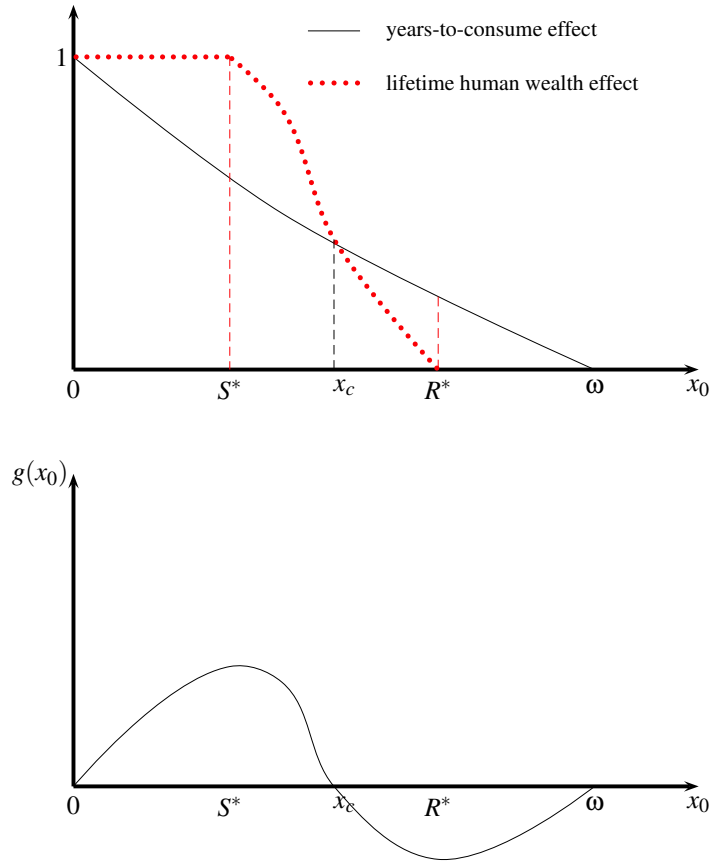


Figure 3: The lifetime human wealth and years-to-consume effect

The partial impact of a decline in mortality on the length of schooling and retirement age is given in Proposition 3. The proof is given in Appendix C.

Proposition 3 *For the life-cycle model given by (1)-(6),*

(a) the sign of $-S^*_{\mu(x_0)}$ *is the same as that of*

$$\frac{r^h(S^*) - \bar{r}(S^*, R^*)}{\sigma(\bar{c})} a(x_0), \tag{30}$$

when $x_0 \leq S^*$ *and* $x_0 \geq R^*$*, and*

$$\frac{r^h(S^*) - \bar{r}(S^*, R^*)}{\sigma(\bar{c})} a(x_0) + \int_{x_0}^{R^*} e^{-r(x-x_0)} \frac{p(x)}{p(x_0)} y_S(S^*, x) dx, \tag{31}$$

when $x_0 \in (S^*, R^*)$*, and*

(b) the sign of $-R^*_{\mu(x_0)}$ *is the same as that of* $a(x_0)$*.*

In Proposition 3(b) we obtain the same "consumption-leisure" relationship as in d'Albis et al. (2012). That is, given that consumption and leisure are normal goods, Proposition 3(b) implies that if a mortality decline yields an increase in consumption because the lifetime human wealth effect dominates the years-to-consume effect, agents anticipate their optimal retirement age in order to enjoy more leisure time. Similarly, when the decline in mortality implies that the years-to-consume effect dominates the lifetime human wealth effect, agents diminish their consumption and postpone their optimal retirement age.

Proposition 3(a) extends the years-to-consume effect and lifetime human wealth effect reasoning to the accumulation of human capital. In this regard, we obtain unambiguous results concerning the sign on the optimal length of schooling of a mortality decline at ages before the entrance into the labor market, S^*, and after the optimal retirement age, R^*. Specifically, Proposition 3(a) implies that when $r^h(S^*) > \bar{r}(S^*, R^*)$, if a mortality decline yields an increase in consumption because the lifetime human wealth effect dominates the years-to-consume effect, agents reduce their investment in education. Recall that this happens during the schooling period as Figure 3 shows. In contrast, a decline in mortality after the optimal retirement age leads to more years of schooling. Instead, if $r^h(S^*) = \bar{r}(S^*, R^*)$ –as frequently assumed in the literature– a decline in mortality during the schooling period or during retirement period does not have an impact on the optimal length of schooling.

During the working period, Proposition 3(a) shows that a decline in mortality positively affects education through the second term in Eq. (31), which reflects the effect of a mortality decline at age x_0 on the marginal benefit of schooling (measured at age x_0), also known as the Ben-Porath mechanism. Actually, this is the only component driving the effect of mortality on the length of schooling when $r^h(S^*) = \bar{r}(S^*, R^*)$, but it is not the case whenever $r^h(S^*) \neq \bar{r}(S^*, R^*)$. In other words, under a model of human capital investment with a fixed retirement age, only improvements

in survival during prime-working ages triggers the Ben-Porath mechanism (Cervellati and Sunde, 2013).

Next we use the results obtained in Proposition 3 to derive the total effects.

3.2 Total effects

In this section we complete the analysis studying the total impact of a mortality decline at an arbitrary age on the optimal length of schooling and retirement age. To do so, we combine the partial effects, presented in Section 3.1, according to Eqs. (24a) and (24b).

Proposition 4 gives under the strict concavity of the expected lifetime utility, the sign of a decline in mortality at an arbitrary age x_0 on the optimal length of schooling and retirement age. See Appendix D for the proof.

Proposition 4 *Assuming the strict concavity of* $\hat{V}(S,R)$*, for the life-cycle model given by (1)-(6),*

(a) the sign of $\frac{-dS^*}{d\mu(x_0)}$ *is the same as that of*

$$\begin{cases} a(x_0)\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{dc^*}{dS^*} + \int_{x_0}^{R^*} e^{-r(x-x_0)}\frac{p(x)}{p(x_0)}y_S(S^*,x)dx & \text{if } S^* < x_0 < R^*, \\ a(x_0)\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{dc^*}{dS^*} & \text{otherwise,} \end{cases} \tag{32}$$

(b) and the sign of $\frac{-dR^*}{d\mu(x_0)}$ *is the same as that of*

$$\begin{cases} a(x_0)\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{dc^*}{dR^*} + \frac{dS^*}{dR^*}\int_{x_0}^{R^*} e^{-r(x-x_0)}\frac{p(x)}{p(x_0)}y_S(S^*,x)dx & \text{if } S^* < x_0 < R^*, \\ a(x_0)\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{dc^*}{dR^*} & \text{otherwise.} \end{cases} \tag{33}$$

Eqs. (32) and (33) show that the total impact of a decline in mortality at an arbitrary age x_0 on the optimal length of schooling and retirement age is given by three factors: (i) the "lifetime human wealth" effect versus the "years-to-consume" effect, which is reflected by the financial wealth at age x_0, i.e. $a(x_0)$; (ii) the total impact of years of schooling and retirement on the initial consumption; and (iii) the effect of mortality on the marginal benefit of schooling or Ben-Porath mechanism. Factors (i) and (ii) only have an impact when $r^h(S^*) \neq \bar{r}(S^*,R^*)$. The first thing to notice is that the increase in the marginal benefit of schooling due to a decline in mortality always has a positive impact on education, but it is not necessarily true on the optimal retirement age since it depends on the relationship between education and retirement. If S^* and R^* are negatively related,

agents anticipate their retirement age and enjoy more leisure time when a decline in mortality causes an increase in the marginal benefit of schooling. In contrast, if S^* and R^* are positively related, agents postpone their retirement age in order to reap the benefits of schooling. This is because in the former alternative the income effect dominates over the substitution effect, whereas in the latter the substitution effect dominates over the income effect.

During the schooling period and the retirement period, the effect of mortality on the marginal benefit of schooling is null. Thereby, in these two periods, the sign of the total impact of a decline in mortality on the optimal length of schooling and retirement age solely depend on the lifetime human wealth effect versus the years-to-consume effect and the total impact of years of schooling and retirement on the initial consumption:

$$\begin{aligned}\text{sign}\left[\frac{-dS^*}{d\mu(x_0)}\right] &= \text{sign}\left[a(x_0)\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{dc^*}{dS^*}\right],\\ \text{sign}\left[\frac{-dR^*}{d\mu(x_0)}\right] &= \text{sign}\left[a(x_0)\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{dc^*}{dR^*}\right],\end{aligned}$$

for all $x_0 \in [0, S^*] \cap [R^*, \omega]$. On the one side, from (6) we know that $a(x_0) < 0$ during the schooling period, while $a(x_0) > 0$ during the retirement period. On the other side, combining (18), (20)-(22) we have

$$\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{dc^*}{dS^*} = (1-\lambda^R)\frac{r^h(S^*)-\bar{r}(S^*,R^*)}{\sigma(\bar{c})}+\lambda^R\left(r^h(S^*)+\delta-\frac{w_{R-S}(R^*-S^*)}{w(R^*-S^*)}\right), \tag{34}$$

where

$$\lambda^R = \frac{1}{\sigma(c^*)}\frac{c_R^*}{c^*}\Bigg/\left(\frac{1}{\sigma(c^*)}\frac{c_R^*}{c^*}-\frac{w_{R-S}(R^*-S^*)}{w(R^*-S^*)}+\delta+\frac{\phi_R(R^*)}{\phi(R^*)}\right). \tag{35}$$

Since λ^R takes values between zero and one, assuming $r^h(S^*)-\bar{r}(S^*,R^*) > 0$ and $R^*-S^* > \bar{E}$, it is straightforward to show that (34) is always positive. Therefore, like Cai and Lau (2012) a decline in mortality during the schooling period has a negative impact on education, while a decline in mortality during the retirement period has a positive impact on education. On the contrary, the total impact on the initial consumption of an increase in the optimal retirement age is *a priori* ambiguous. In particular, it can be shown

$$\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{dc^*}{dR^*}\begin{cases}\geq 0 \Leftrightarrow \frac{dS^*}{dR^*} \geq -\frac{c_R^*}{c_S^*},\\ \leq 0 \Leftrightarrow \frac{dS^*}{dR^*} \leq -\frac{c_R^*}{c_S^*}.\end{cases} \tag{36}$$

Consequently, according to (23), the total impact of a decline in mortality –during the schooling and retirement periods– on the optimal retirement age coincides with that on the length of schooling if, and only if, the relationship

between the optimal years of education and retirement age is positive; otherwise, when $\frac{dS^*}{dR^*} < 0$, both alternatives are possible.

The direct consequence of this ambiguity is that the impact of a reduction of $\mu(x_0)$ on $R^* - S^*$ is in general ambiguous. In the next section we perform a simple numerical analysis using observed mortality data in order to show the impact of $\mu(x_0)$ on $R^* - S^*$ under different scenarios.

4 Quantitative exercise

In this Section we study numerically the impact of the epidemiological transition on the optimal years of schooling and retirement age when both variables are endogenous. We thus abstract from the effect that a pension system or any policy reform may have on our decision variables. This is a strong simplification but it allows us to focus on the main point of the article: the effect of mortality declines on education and retirement.

For comparability with the existing literature, our quantitative exercise exploits the data used by Hazan (2009). Moreover, we extend the analysis by using death rates of Swedish males born between 1890 and 2000 in order to see the effect that an alternative observed mortality pattern may have on labor supply. Our analysis delivers two important results. First, when the life expectancy rises, our model is capable of producing a decline in the optimal retirement age and an increase in years of schooling. Second, since in the earlier stage of mortality transition, a decline in mortality belongs mainly to younger people, whereas in the later stage, a decline in mortality decline has mainly occurred at older ages, we show that the optimal retirement age stops declining after the cohort born in year 1920 and increases thereafter.

4.1 Data

To estimate the marginal effect of the decline in mortality on the length of schooling and retirement age, we just need data on mortality rates. Nevertheless, to realistically match the effective retirement age and years of schooling for the cohort born in 1890 (our baseline cohort), we also collect data on labor force participation rates and years of schooling for Swedish males and US males. The effective retirement age for Swedish males is based on employment rates, taken from census data, for the period 1910-1985 and labor force surveys for the period 1975-2004 published by Statistics Sweden.[16] Years of schooling for birth cohorts born in Sweden

[16]Employed and working age population in Sweden during the period 1910-1985 is available at http://www.scb.se/tidsseriehafteforvarvsarbetandefob1910-1985. Before the 1980s, unemployment rates above age 45 were roughly constant and lower than 2 percent (Ljungqvist and Sargent, 1995; Holmlund, 2009). Therefore, we do not

is calculated based on the number of students by educational attainment reported by de la Croix et al. (2008).[17] US data on labor force participation rates and on years of schooling for cohorts born between 1890 and 1930 are taken from Hazan (2009).

As widely shown in the literature, our data suggest a clear downward trend in the effective retirement age across cohorts.[18] Specifically, US males born in 1890 retired on average at age 66.7, while US males born in 1930 retired on average at age 63.8. Similarly, in Sweden, the expected retirement age of males born in 1890 was 65.9, while it declined to age 63.17 for males born in 1930. In this paper, however, for comparability reasons we only make use of the retirement age and length of schooling data points for US males born in year 1890. Similar results, though, are obtained using the retirement age and length of schooling for Swedish males born in 1890. These results are shown in a supplementary appendix upon request.

4.1.1 Mortality data

Like Hazan (2009), we combine the data reported by Haines (1998) and Bell et al. (1992) to produce the probability of dying at each age for US males born between 1890 and 2000. Notice, though, that US mortality data for cohorts born before 1933 are not based on complete death registrations and census data.[19] The probability of dying at each age for Swedish males born between 1890 and 1911 is taken from Human Mortality Database (2013). Deaths rates for cohorts born after year 1911 are constructed applying the Lee-Carter model (Lee and Carter, 1992).[20]

expect a significant error by combining both datasets.

[17] Information on number of students and enrollment rates by educational attainment in Sweden from year 1768 to 2002 are withdrawn from http://perso.uclouvain.de/david.delacroix/data/swedish-educ-data.pdf.

[18] We assume individuals might retire from age 45 and that no withdrawal from the labor market occur after age 80. The effective retirement age for the birth cohort i (R^i) is calculated according to

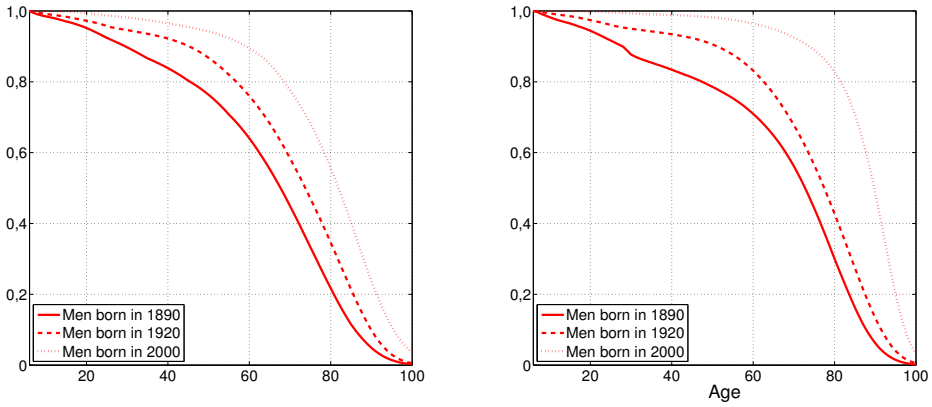
$$R^i = \sum_{x=45}^{80} x \left(\text{lfp}_x^i - \text{lfp}_{x+1}^i\right) \Big/ \sum_{x=45}^{80} \left(\text{lfp}_x^i - \text{lfp}_{x+1}^i\right) ,$$

where lfp_x^i denotes the labor force participation at age x for the cohort born in year i.

[19] Death rates based on complete death registrations and census data for cohorts born before year 1900 are only available for a small set of countries: Belgium, Denmark, Finland, France, Iceland, Italy, Netherlands, Sweden and Switzerland. The same standards of quality for the US demographic data start in 1933. This information can be drawn from the Human Mortality Database.

[20] Future age-specific death rates from year 2011 are projected applying the Lee-Carter method to actual Swedish period-death rates from year 1945 up to 2011. Thus, it is assumed that the log of the death rate is explained by the following multiplicative process:

$$\log m_{\tau,x} = a_x + k_\tau b_x + \epsilon_{t,x}, \text{where } \epsilon_{\tau,x} \text{ is } i.i.d.(0, \sigma_\epsilon),$$



(a) US males (b) Swedish males

Figure 4: Conditional survival probability at age 6 across cohorts by country: Cohorts 1890-2000.

Note: Survival probabilities for cohorts born after 1920 is based on forecasted data.

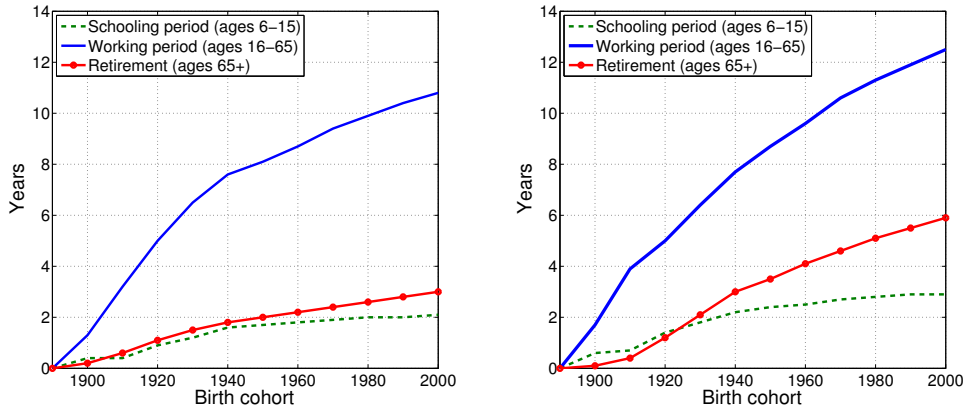
Figure 4 presents the cohort survival probabilities conditional on reaching age 6 for males born in 1890, 1920, and 2000 in (a) US and (b) Sweden. Both panels show the progressive increase in the conditional survival probability for younger cohorts in US and Sweden. This pattern implies that the life expectancy from age 6 of US males (i.e. the area below the survival probability curve) is expected to increase from 56.9 years for the cohort born in 1890 to 72.8 years for the cohort born in 2000.[21] In Sweden, the expected increase in life expectancy at age 6 is greater than that of US, rising from 59.7 years for Swedish males born in year 1890 to 80.9 for those born in 2000. Therefore, the life expectancy gap between both countries is expected to increase from 2.8 for cohorts born in 1890 to 8.1 years for cohorts born in 2000.

Figure 5 shows the contribution of the mortality decline by stage of life on the increase in life expectancy across males born in US and Sweden from 1890 to 2000. We distinguish three periods: schooling from ages 6 to 15 (dashed green line), working period from ages 16 to 65 (solid blue line), and retirement for ages above 65 (circle red line).

According to Figure 5, most years-gained occur during (prime) working ages for the oldest cohorts. Specifically, five additional years were gained,

where $m_{\tau,x}$ is the death rate at age x in year τ, a_x, b_x are age-specific constants, and k_τ is a time-varying index obtained through the singular-value decomposition of a matrix of death rates.

[21] Life expectancy at age 6 is calculated as: $e_t(6) = \int_6^\omega \frac{p_t(s)}{p_t(6)} ds$, where t denotes the birth cohort.



(a) US males (b) Swedish males

Figure 5: Stage of life decomposition of differences in life expectancy from age 6 across cohorts by country (baseline cohort 1890): Cohorts 1890-2000.

Note: Authors' calculations based on the decomposition of life expectancy proposed by Arriaga (1984).

on average, in both countries at (prime) working ages between the cohort born in 1890 and that in 1920, while the total contribution of the mortality reduction during the schooling period and the retirement period was slightly above two years of age. Between the cohort born in 1920 and that in year 2000 a further increase of 5.8 and 7.5 years at (prime) working ages is expected in the US and Sweden, respectively. Hence, as it is shown in Figure 5, the increasing life expectancy gap between both countries for younger cohorts is mainly due to the faster increase in Sweden compared to that in the US in the life expectancy at age 65. In particular, Swedish males born in 2000 are expected to gain at least three more years of age than their counterparts in US.

In the next section we perform a numerical simulation to show the consequences of the different mortality transition on the optimal length of schooling and retirement age.

4.2 Numerical simulation

To solve the model, we follow Cervellati and Sunde (2013) and consider a constant intertemporal elasticity of substitution (CIES) utility function,

$$U(c) = \frac{c^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}, \text{ with } \sigma \in (0, 1]. \tag{37}$$

The underlying disutility of non-leisure time is assumed constant $\phi(x) = \phi$. For simplicity, we assume a constant aversion to schooling time, $\psi(x) = \psi$, for all $x \in (0, S)$. Since our individual devotes her full time to education while she is in school, we use the following simplified version of the Ben-Porath human capital production function

$$q\big(h(x)\big) = \xi h(x)^{\gamma}, \text{ with } \xi > 0 \text{ and } \gamma \in (0, 1), \tag{38}$$

where ξ is a scaling factor and γ is the returns to scale in human capital investment. Similar to Hazan (2009) and Cervellati and Sunde (2013) the wage rate is assumed to be constant.

To shed light on the effects of mortality on S^* and R^*, we introduce further simplifying assumptions. We assume zero discounting, $r = \delta = \rho = 0$, so that the inverse of $\bar{r}(S^*, R^*)$ coincides with the expected lifetime labor supply (ELW)[22]

$$\bar{r}(S^*, R^*) = \left[\int_{S^*}^{R^*} \frac{p(x)}{p(S^*)} dx\right]^{-1}. \tag{40}$$

Integrating (5) and using (38) the return to education at age S becomes

$$r^h(S^*) = \xi / \left(h(0) + (1-\gamma)\xi S^*\right). \tag{41}$$

As in Cervellati and Sunde (2013), we set $\gamma = 0.65$, $\sigma = 0.5$, the wage rate per unit of human capital to one, and fix $h(0) = 1$. In order to show the importance on the results of the relationship between S^* and R^*, we run three alternative simulations combining three different returns to education function. In particular, we set ξ to 1, .25, and 0.075 so as to have a return to education after 13 years in school around 18%, 11.5% (Heckman et al., 2008), and 5.5%.[23] Finally, to isolate the effect of the decline in mortality, we restrict the parameters of the model so that it reproduces the years of schooling and retirement age observed in the data for the cohort born in 1890 in the United States. Table 1 reports the values of ϕ and ψ for US and Sweden, which are calculated so as to have an initial optimal length of schooling of 9.3 years and an optimal retirement age of 66.7 years for males born in 1890 (Hazan, 2009).

[22] Under the assumptions zero discounting and constant wage rate, the annuitized marginal cost of the S-th unit of schooling becomes

$$\bar{r}(S^*, R^*) = \frac{\int_{S^*}^{R^*} \mu(x)p(x)dx}{\int_{S^*}^{R^*} p(x)dx} + \frac{p(R^*)}{\int_{S^*}^{R^*} p(x)dx}. \tag{39}$$

Since it follows from (1) that $-dp(x) = \mu(x)p(x)dx$, substituting it in (39), and rearranging, gives Eq. (40).

[23] Notice that when $h(0) = 1$, ξ is equal to the initial return to education $r^h(0)$.

Table 1: Values of ϕ and ψ

CIES	Human capital scaling factor	Underlying disutility of leisure		Aversion to schooling time		Ratio	
σ	ξ	ϕ		ψ		ψ/ϕ	
		US	SWE	US	SWE	US	SWE
0.50	1.000	0.03	0.03	0.27	0.29	9.0	9.3
0.50	0.250	0.34	0.36	1.67	1.82	4.9	5.0
0.50	0.075	1.01	1.07	1.59	1.75	1.6	1.6

Note: US and SWE stand for United States of America and Sweden, respectively.

4.3 Results

We divide this Section in three parts. First, we study the effect of the aversion to schooling time assumption for modeling the impact of mortality on total years worked. Second, we analyze the differential effect of mortality declines at different stages of life on the optimal years of schooling and optimal retirement age. Finally, we estimate quantitatively the importance of the aversion to schooling time for explaining the evolution of the years worked.

4.3.1 The importance of the aversion to schooling time assumption

To show the impact of mortality on the labor supply for different relationship between S^* and R^*, we compute for each country three simulations that results from three different human capital scaling factors. The parameters of the model are held constant across cohorts and fixed at the values that reproduce the years of schooling and retirement age observed in the data for the cohort born in 1890 in the United States. Hence, in each controlled experiment, any variation in S^* and R^* across cohorts is solely due to changes in mortality.

Table 2 reports the absolute change in S^* and R^* for cohorts born between year 1890 and 1930 using US and Swedish survival probabilities. The first important result from Table 2 is that the model is capable of predicting that a decline in mortality leads to an increase in schooling followed by a decline in the retirement age. The extent to which the optimal retirement age decreases and the optimal length of schooling increases depends on the ratio ψ/ϕ, or equivalently ξ. Higher (lower) values of ψ/ϕ yield a stronger (weaker) decline in the optimal retirement age and a smaller

Table 2: Absolute change in years of schooling and retirement age between 1890 and 1930

Human capital scaling factor	ξ	1.000	0.250	0.075
Aversion to schooling time ratio	ψ/ϕ	9.0	4.9	1.6
		I	**II**	**III**
US Survival				
Retirement	$R^*_{1930} - R^*_{1890}$	-1.80	-1.17	0.92
Schooling	$S^*_{1930} - S^*_{1890}$	0.68	1.17	2.72
Years worked†		-2.48	-2.34	-1.80
Swedish Survival				
Retirement	$R^*_{1930} - R^*_{1890}$	-1.68	-1.09	0.83
Schooling	$S^*_{1930} - S^*_{1890}$	0.69	1.19	2.76
Years worked†		-2.38	-2.28	-1.93

† Note: Years worked is calculated as the difference between the optimal retirement age and the optimal length of schooling.

(higher) increase in the optimal years of schooling. For instance, if $\xi = 1$ or $\psi/\phi = 9.0$ (column I), the decline in mortality would account for 62.6% (-1.80/-2.88) and 61.6% (-1.68/-2.73) of the decline in the retirement age, while it would account for 17.5% (0.68/3.93) and 17.5% (0.69/3.93) of the increase in years of schooling in the US and Sweden, respectively. Instead, if we assume $\xi = 0.25$ or $\psi/\phi = 4.9$ (column II), the model predicts that the decline in mortality would account for 40.7% (-1.17/-2.88) and 40.0% (-1.09/-2.73) of the decline in the retirement age, whereas it would account for 29.7% (1.17/3.93) and 30.2% (1.19/3.93) of the increase in the years of schooling in the US and Sweden, respectively. The second result is that the optimal years worked, or the difference between the retirement age and the years of schooling, are negatively related to the value of ψ/ϕ. According to Proposition 4, this is because the decline in mortality only affects positively on the optimal years of schooling, while it can have a negative or a positive effect on the optimal retirement age. As explained in Section 3.2, this result is due to the negative relationship between S^* and R^* or, equivalently, because the income effect dominates over the substitution effect. Under the assumptions of zero discounting and a constant wage rate, the sign of the relation between S^* and R^*, or Eq. (23), is given by

$$\text{sign}\left[\frac{dS^*}{dR^*}\right] = \text{sign}\left[\frac{1}{(1-\sigma)(1+\psi/\phi)} - 1\right] \begin{cases} < 0 & \text{if } \frac{\psi}{\phi} > \frac{\sigma}{1-\sigma}, \\ > 0 & \text{otherwise .} \end{cases} \tag{42}$$

Hence, for $\sigma = 0.50$, a value of ψ/ϕ higher (lower) than one implies a

negative (positive) relationship between S^* and R^*. Note therefore that the condition $dS^*/dR^* < 0$, although necessary, is not sufficient to guarantee that a decline in mortality yields earlier retirement. For instance, according to (42), in all simulations the sign of dS^*/dR^* is negative, while the model only predicts an early retirement age in columns I and II. The increase in the retirement age shown in column III is therefore explained by the weak negative relationship between S^* and R^* that results from an aversion to schooling time value of 1.6. In the next subsection, Figures 6 and 7 give further intuitions about the effect of the weak negative relationship between S^* and R^* for the impact of mortality on the years worked.

4.3.2 Contribution of mortality at different stages of life

We complement the empirical analysis by showing the contribution of mortality improvements at different stages of life on the optimal years of schooling and optimal retirement age for our two alternative mortality schedules: males born in US and Sweden. In these simulations, we divide the lifespan in three periods: childhood (ages 6-15), adulthood (ages 16-65), and retirement (ages 65+), which are closely related to the stages of life delimited by the observed length of schooling and retirement age. In order to show the contribution of mortality improvements at each stage of life, we calculate the optimal schooling and retirement considering exclusively the gains in survival during either childhood, or adulthood, or retirement across cohorts ceteris paribus the probability of dying in the other stages of life. We recalculate the survival probability to age x based on the new probability of dying in each age interval.

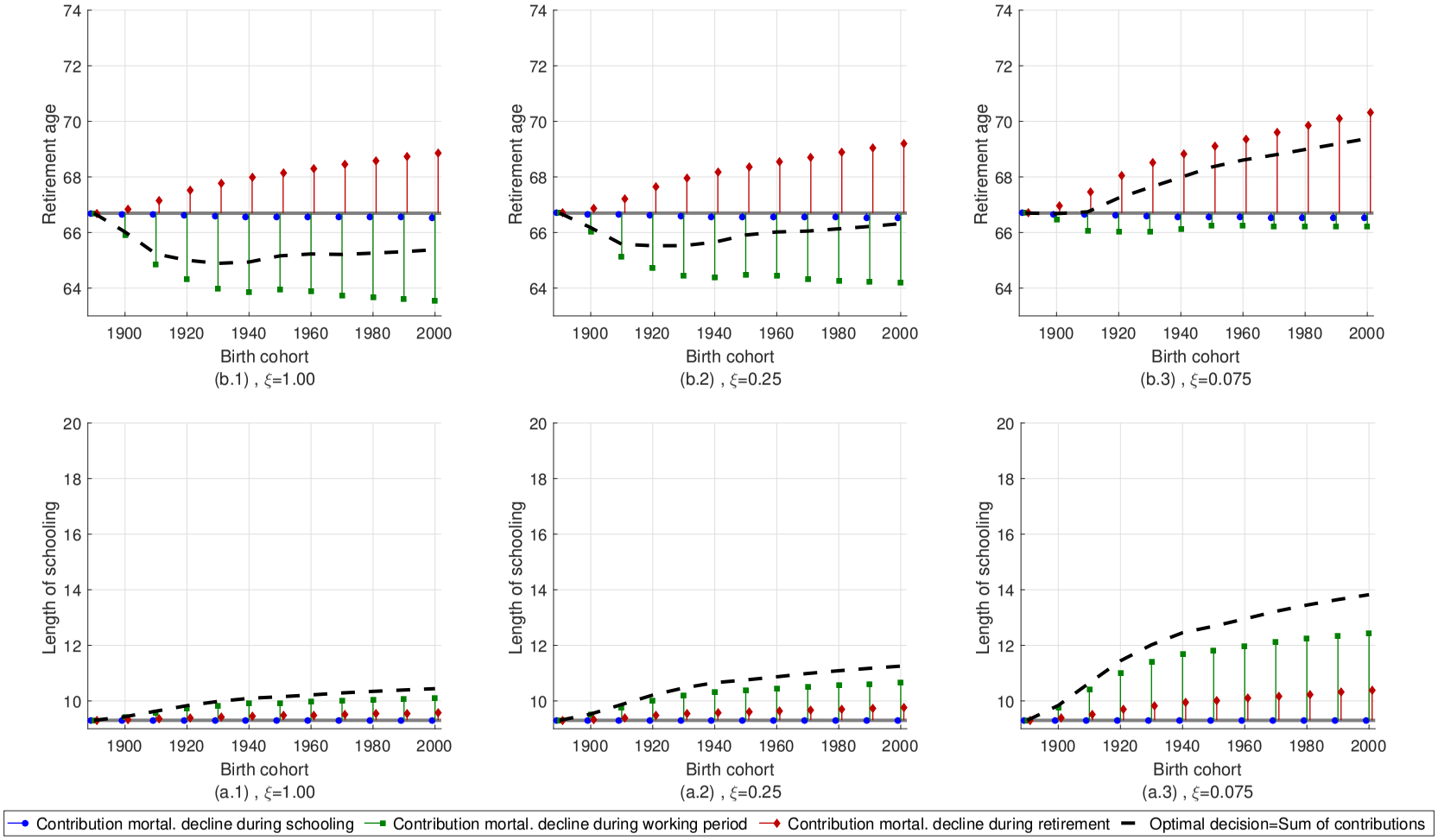


Figure 6: Contribution of mortality decline by stage of life on (a) the optimal length of schooling and (b) the optimal retirement age across cohorts by average return to education, US mortality of males born in 1890-2000

Note: Results for cohorts born after 1930 are based on forecasted mortality rates.

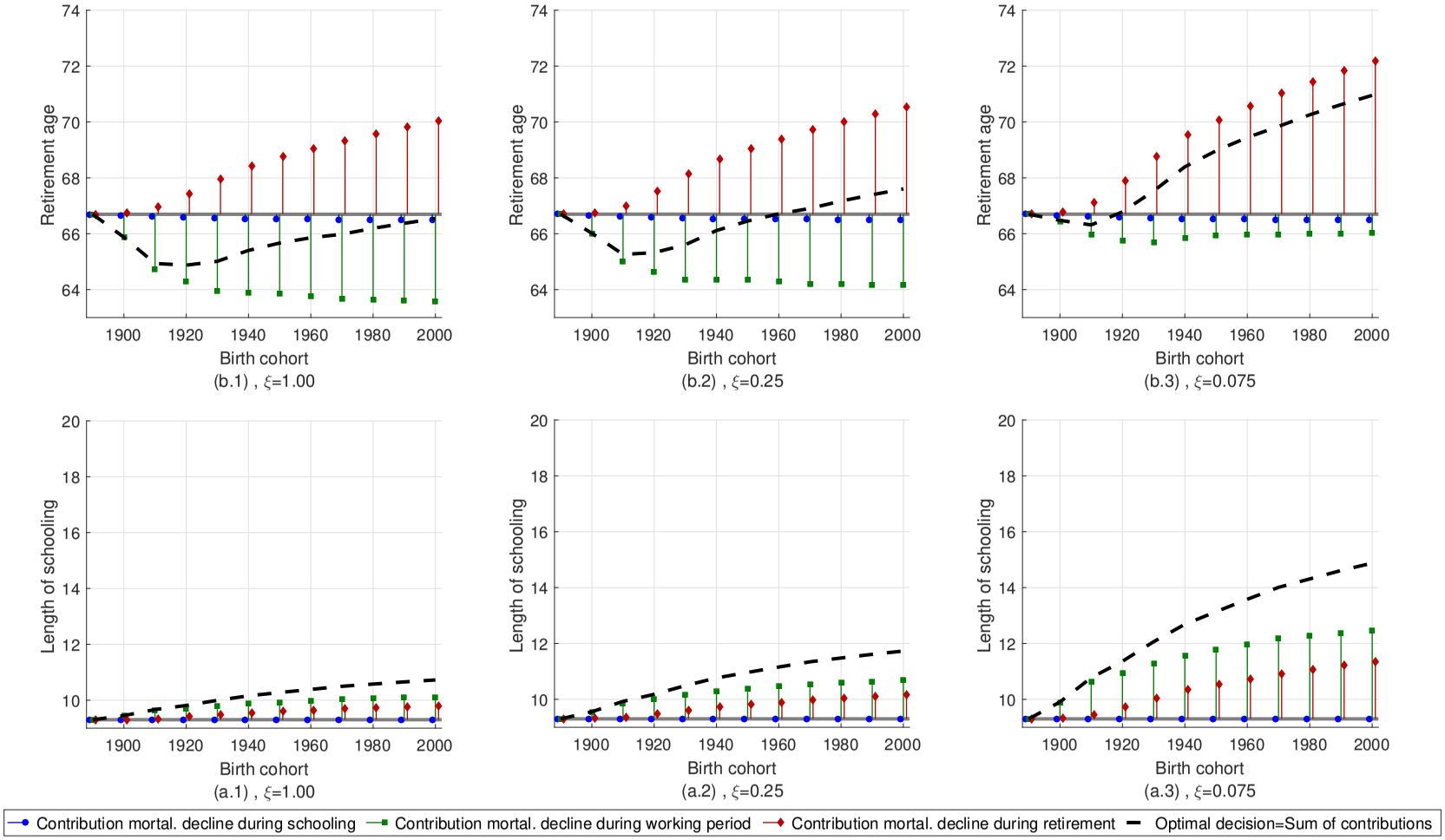


Figure 7: Contribution of mortality decline by stage of life on (a) the optimal length of schooling and (b) the optimal retirement age across cohorts by average return to education, Swedish mortality of males born in 1890-2000

Note: Results for cohorts born after 1930 are based on forecasted mortality rates.

Figures 6 and 7 show how mortality improvements during childhood (circled blue arrows), adulthood (squared green arrows), and retirement (red-diamond arrows) affect on the optimal years of schooling and the retirement age by different functional form of the returns to education. The height of each arrow represents the change from the baseline (solid gray line) in the endogenous variable, while the position of the marker represents the direction of the effect. The evolution of the optimal length of schooling and optimal retirement age –represented by the dashed black line– equals the sum of the heights of the three arrows. Note that the difference between the top and bottom panels for each human capital scaling factor ξ gives the total number of years worked, conditional on reaching the retirement age.

The bottom panels in Figures 6 and 7 show how mortality improvements raise the optimal years of schooling. According to Proposition 4, a decline in mortality during adulthood raises the marginal benefit of schooling and hence the optimal length of schooling, because the likelihood of receiving a future labor earning increases, also known as the Ben-Porath's mechanism. A decline in mortality during retirement also leads to an increase in the length of schooling, because agents continue studying in order to finance the consumption after retirement through an increase in lifetime earnings. That is, the years-to-consume effect dominates over the lifetime human wealth effect. The contribution of mortality improvements during childhood on the optimal length of schooling is negative but negligible. The sum of these three effects of mortality on the optimal length of schooling is represented by the black dashed line.

The panels at the top in Figures 6 and 7 show how mortality improvements change the optimal retirement age. The parameter values assumed in both Figures imply that the income effect dominates over the substitution effect. As a consequence, the relationship between S^* and R^* is negative, or $\frac{dS^*}{dR^*} < 0$. Thus, a decline in mortality during adulthood leads to early retirement (see the second term in Eq. 33). Individuals use the additional income to increase consumption and enjoy more leisure time. Note, however, that the strength of the negative effect decreases (see the length of the green arrows) with lower returns to education. This is because the negative relation between education and retirement vanishes when $r^h(S^*)$ tends to $\bar{r}(S^*, R^*)$ (see Figure 2). Consequently, if the relation between S^* and R^* were positive, a decline in mortality during adulthood would lead to a delay in the retirement age. On the other hand, mortality declines late in life leads to a delay in the optimal retirement age because individuals need more earnings to finance the additional consumption due to a longer retirement period. This positive effect on retirement is indicated by the upward green squared arrow. Similar to the effect on the length of schooling, the contribution of mortality improvements during childhood on

the optimal retirement age is also negative but negligible. The net effect of mortality improvements at different stages of life on the optimal retirement age is represented by the black dashed line. Notice that in all cases we observe a turning point in the evolution of the optimal retirement age after the cohorts born in the 1920s. This is because mortality improvements for cohorts born before the 1920s mainly occurred during childhood and adulthood, whereas the improvements in mortality for more recent cohorts mainly occur at older ages.

4.3.3 Aversion to schooling time values

To assess how reasonable our aversion to schooling timeassumption is, we compute the nonpecuniary cost of schooling that results from our theoretical model and compare it to the values derived in the literature. To do so, we pin down the values of ϕ and ψ that corresponds to the observed length of schooling ($\hat{S}_t$) and retirement age ($\hat{R}_t$) across cohorts and human capital scaling factors.

Assuming a flat wage rate and no discounting, from Eq. (14) the returns to schooling at the optimal schooling decision must satisfy

$$r^h(S^*) = \bar{r}(S^*, R^*)\left[1 + (\psi/\phi)\right], \tag{43}$$

where the ratio (ψ/ϕ) gives information about the importance of the nonpecuniary cost of schooling for the optimal schooling decision. Thus, provided the values of $(\hat{S}_t, \hat{R}_t)$ and $p_t(x)$, from (43) we can calculate the evolution of this ratio as

$$\widehat{(\psi/\phi)}_t = \frac{r^h(\hat{S}_t)}{\bar{r}(\hat{S}_t, \hat{R}_t)} - 1. \tag{44}$$

From Eq. (40) we know that the marginal cost of the $\hat{S}_t$-th unit of schooling (i.e. $\bar{r}(\hat{S}, \hat{R})$) is, by definition, the inverse of the expected lifetime labor supply (ELW). Column III in Table 3 shows that the expected lifetime labor supply has slightly decreased from 42.7 years to 41.3 between cohort 1890 and 1930. Thus, our assumed marginal cost of the $\hat{S}_t$-th unit of schooling, or hardle rate, is close to 2.4% –column IV in Table 3–. A value of 2.4% is between 1.2% and 2.6% estimated for the US; see Table 5 in Oreopoulos (2007).[24]

Using our estimated hardle rate, Figure 8 shows the evolution of the ratio $\widehat{(\psi/\phi)}_t$ that corresponds to the observed length of schooling ($\hat{S}_t$) and retirement age ($\hat{R}_t$) in the US across cohorts for different human capital

[24]The hardle rate values reported by Oreopoulos (2007) are estimated for individuals between age 14 and 16. Thereby, our initial length of schooling of 15.3 years is between the range of ages used by Oreopoulos (2007).

Table 3: Observed average length of schooling ($\hat{S}$), average retirement age ($\hat{R}$), expected lifetime labor supply (ELW), and three hypothetical returns to schooling in the US

Cohort	$\hat{S}+6$	$\hat{R}$	ELW	$\bar{r}(\hat{S},\hat{R})$	$r^h(\hat{S})$		
					ξ =1.00	ξ =.250	ξ =.075
	I	**II**	**III**	**IV**=1/**III**	**V**	**VI**	**VII**
1890	15.3	66.7	42.7	2.3%	23.4%	13.8%	6.0%
1900	17.0	66.3	42.0	2.4%	20.6%	12.7%	5.8%
1910	17.8	64.8	41.7	2.4%	19.5%	12.3%	5.7%
1920	18.5	63.9	41.5	2.4%	18.7%	12.0%	5.7%
1930	19.3	63.8	41.3	2.4%	17.7%	11.6%	5.6%

Note: The expected lifetime labor supply (ELW) is defined as $\int_{\hat{S}_t}^{\hat{R}_t} \frac{p_t(x)}{p_t(\hat{S}_t)} dx$, where $p_t(x)$ is the probability of surviving to age x conditional on reaching age 6 for the cohort born in year t.

scaling factors. We obtain that the ratio $\widehat{(\psi/\phi)}_t$ might range between 1 and 9 according to our different scaling factors. However, since estimated returns to compulsory schooling range between 9.5% and 17.4% (Oreopoulos, 2007), a scaling factor of 0.25 seems to be the most likely case according to column VI in Table 3. In Figure 8, the red dashed curve depicts the evolution of $\widehat{(\psi/\phi)}_t$ that corresponds to the human capital scaling factor of 0.25, which takes values between 4.9 (cohort 1890) and 3.8 (cohort 1930). A human capital scaling factor of 0.25 implies that by staying an additional year in school lifetime wealth would have increased by 11.4% for the cohort born in 1890, while it would have increased by 9.1% for the cohort born in 1930. These values are also within the empirically estimated increase in lifetime wealth from an additional year of compulsory schooling that range between 8.5% and 17.6% (Oreopoulos, 2007).

In sum, we conclude, based on the comparison of our results to the existing empirical literature on the returns to schooling, that the most likely value of ψ/ϕ is 4.9. Therefore, our "aversion to schooling time" is significant and implies that the observed decline in mortality would account for 40.7% of the decline in the retirement age and 29.7% of the increase in the years of schooling in the US.

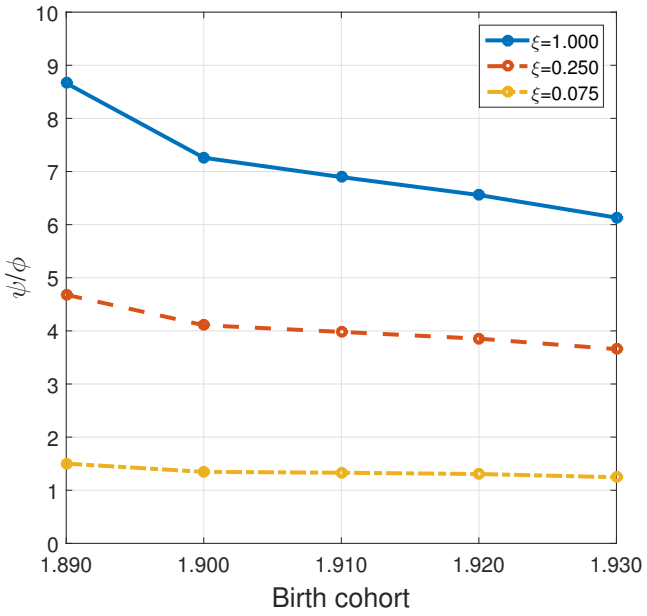


Figure 8: Values of $\widehat{(\psi/\phi)}_t$ that match the observed length of schooling and retirement age by cohort for different ξ values

5 Conclusion

Existing theoretical models predict a causal positive relation between increasing life expectancy and human capital investments and retirement age. However, one salient feature of the economic development during the last two centuries is the negative relation between life expectancy and the extensive labor supply. To reconcile the empirical evidence with economic theory, we develop a lifecycle model with endogenous human capital investment and labor supply in which mortality declines may cause higher schooling and early retirement.

This article makes two important contributions. First, we show that the 'aversion to schooling time' assumption is a necessary, although not sufficient, condition for a decline in mortality to cause higher education and early retirement age. The intuition is as follows. Higher human capital investment, triggered by the Ben-Porath mechanism, causes two opposite effects: on the one side, an increase in earnings at all ages that raises the marginal benefit of working; on the other side, an increase in consumption and hence a decline in the marginal utility of consumption that reduces the marginal benefit of working. Early retirement occurs when the income effect dominates the substitution effect, which in this model depends on the strength of the 'aversion to schooling time'. Second, we derive the dif-

ferential impact of the mortality decline at any arbitrary age on education and retirement. We show that a mortality decline after retirement always results in higher human capital investment and late retirement, whereas a mortality decline at younger ages leads to higher human capital investment and may, or may not, cause early retirement. Using mortality data for cohorts born between year 1890 and 1930 in US and Sweden, we show for reasonable values of the 'aversion to schooling time' and the intertemporal elasticity of substitution that improvements in the survival probability may account for a 40% of the decline in the retirement age and 30% of the increase in the years of education.

For simplicity, our model abstracts from realistic features like the existence of a pension system, the intervention of governments in the access to all levels of education, the introduction of mandatory years of schooling and retirement ages. Still, our results offer an explanation to the empirical evidence, collected during the last centuries in US and Sweden, on the evolution of education and retirement. Moreover, our model is robust to the introduction of the above mentioned features. For instance, if education and retirement are negatively related, positive spillovers from education and publicly provided education will increase the marginal benefit of schooling and will reduce even further the retirement age. Similarly, the existence of pension incentives for early retirement and the overall increase in the labor-augmenting technological progress during the last century would have induced earlier retirement ages and higher increases in the marginal benefit of schooling. Therefore, our results suggest some interesting directions for future research. In particular, first, a logical extension of our framework is the introduction of a pension systems. Second, the implementation of the model in a computable general equilibrium setting in order to analyze the effect of changes in wages and interest rates. The implementation of these issues can provide researchers and policy-makers a better understanding of the effect of changes in the population on modern economic growth.

A Proof of Proposition 1

We first derive the optimal length of schooling condition (S^*) and optimal retirement age (R^*) introduced in Eqs. (9)-(10). Second, we study the conditions for a maximum in S^* and R^*. Substituting the conditional optimal consumption, $c(x, S, R)$, into (2) and differentiating it with respect to S, we obtain

$$V_S(S,R) = \int_0^{\omega} e^{-\rho x} p(x) U_c\big(c(x,S,R)\big) c_S(x,S,R) dx - e^{-\rho S} p(S) \psi(S). \quad (45)$$

Substituting (8) in (45), and rearranging, gives

$$V_S(S,R) = U_c\big(c(0,S,R)\big)\int_0^{\omega} e^{-rx}p(x)c_S(x,S,R)dx - e^{-\rho S}p(S)\psi(S). \quad (46)$$

Differentiating (7) with respect to S, and simplifying, we have

$$\int_0^{\omega} e^{-rx}p(x)c_S(x,S,R)dx = e^{-rS}p(S)\left(\int_S^R e^{-r(x-S)}\frac{p(x)}{p(S)}y_S(S,x)dx - y(S,S)\right). \quad (47)$$

Substituting (47) in (46), and taking $U_c\big(c(0,S,R)\big)e^{-rS}p(S)$ as common factor, we obtain

$$V_S(S,R) = U_c\big(c(0,S,R)\big)e^{-rS}p(S)\times \left(\int_S^R e^{-r(x-S)}\frac{p(x)}{p(S)}y_S(S,x)dx - y(S,S) - \frac{e^{(r-\rho)S}\psi(S)}{U_c\big(c(0,S,R)\big)}\right). \quad (48)$$

Setting $V_S(S,R)$ to zero and simplifying, the first-order condition for an optimal length of schooling is given by (9).

Applying a similar approach, the derivative of (2) with respect to R becomes

$$V_R(S,R) = p(R)\left(U_c\big(c(0,S,R)\big)e^{-rR}y(S,R) - e^{-\rho R}\phi(R)\right). \quad (49)$$

Then, setting $V_R(S,R)$ to zero and simplifying, the first-order condition for an optimal retirement age is given by (10).

Let $\hat{V}(S,R)$ be the expected lifetime utility conditional on the optimal consumption path. Also, let c^* be optimal initial consumption condition on $S = S^*$ and $R = R^*$. In order for $\hat{V}$ to be strictly concave at $S = S^*$ and $R = R^*$ it needs to satisfy

$$\hat{V}_{SS} < 0, \hat{V}_{RR} < 0, \quad (50a)$$

$$\begin{vmatrix} \hat{V}_{SS} & \hat{V}_{SR} \\ \hat{V}_{RS} & \hat{V}_{RR} \end{vmatrix} > 0. \quad (50b)$$

Substituting (11) in (48), differentiating with respect to S around (S^*, R^*), using (16), and simplifying gives

$$\hat{V}_{SS}(S^*,R^*) = U_c\big(c^*\big)e^{-rS^*}p(S^*) \times \left(f_S(S^*,R^*) - f(S^*,R^*)\left(\frac{\psi_S(S^*)}{\psi(S^*)} + \frac{1}{\sigma(c^*)}\frac{c_S^*}{c^*}\right)\right). \quad (51)$$

Since the sign $\left[\frac{\psi_S(S^*)}{\psi(S^*)}\right]$ and sign $[c_S^*]$ is the same as the sign of $f(S^*,R^*)$, a necessary and sufficient condition for $S^* < R^*$ to be a maximum of $\hat{V}(S,R^*)$ is

$$f_S(S^*,R^*) < f(S^*,R^*)\left(\frac{\psi_S(S^*)}{\psi(S^*)} + \frac{1}{\sigma(c^*)}\frac{c_S^*}{c^*}\right). \tag{52}$$

For $f(S^*,R^*) > 0$ notice that $\frac{f_S(S^*,R^*)}{f(S^*,R^*)} - \frac{\psi_{S^*}(S^*)}{\psi(S^*)} < \frac{1}{\sigma(c^*)}\frac{c_S^*}{c^*}$. It is also worth noticing that for $f(S^*,R^*) = 0$, $f_S(S^*,R^*) < 0$.

Differentiating (49) with respect to R, at $S = S^*$ and $R = R^*$, gives

$$\hat{V}_{RR}(S^*,R^*) = U_c(c^*)e^{-rR^*}p(R^*)y(S^*,R^*) \times \left(-\frac{1}{\sigma(c^*)}\frac{c_R^*}{c^*} + \frac{y_R(S^*,R^*)}{y(S^*,R^*)} - \frac{\phi_R(R^*)}{\phi(R^*)}\right). \tag{53}$$

Provided $y_R(S^*,R^*) = \frac{w_{R-S}(S^*-R^*)}{w(R^*-S^*)} - \delta \leq 0$ for all $R^* - S^* \geq \bar{E}$ and since $\phi_x(x) \geq 0$, we conclude that $\hat{V}_{RR}(S^*,R^*) < 0$ is strictly negative. Finally, the second-order conditions of the maximum in $S = S^*$ and $R = R^*$ is satisfied, if (52) holds, $y_R(S^*,R^*) \leq 0$, and

$$\frac{\hat{V}_{SR}(S^*,R^*)}{-\hat{V}_{SS}(S^*,R^*)} \times \frac{\hat{V}_{RS}(S^*,R^*)}{-\hat{V}_{RR}(S^*,R^*)} < 1. \tag{54}$$

Since $\frac{dS^*}{dR}\big|_{R=R^*} = \frac{\hat{V}_{SR}(S^*,R^*)}{-\hat{V}_{SS}(S^*,R^*)}$ and $\frac{dR^*}{dS}\big|_{S=S^*} = \frac{\hat{V}_{RS}(S^*,R^*)}{-\hat{V}_{RR}(S^*,R^*)}$, it follows from (54) that the impact of a change in S^* on R^* differs from the impact of a change in R^* on S^*; i.e. $\frac{dS^*}{dR}\big|_{R=R^*} \times \frac{dR^*}{dS}\big|_{S=S^*} < 1$.

B Proof of Proposition 2

If $U_x(x) = x^{-\frac{1}{\sigma}}$, where σ is the intertemporal elasticity of substitution, from (8) we have $c(x,S,R) = c(0,S,R)e^{\sigma(r-\rho)x}$, for all $x \in (0,\omega)$, substituting it in (7), and simplifying, we obtain

$$c(0,S,R) = \frac{\int_S^R e^{-rx}p(x)y(S,x)dx}{\int_0^\omega e^{-[(1-\sigma)r+\sigma\rho]x}p(x)dx}. \tag{55}$$

Taking logarithms at both sides of (55) and differentiating with respect to $-\mu(x_0)$ gives

$$\frac{1}{c(0,S,R)}\frac{-\partial c(0,S,R)}{\partial \mu(x_0)} = \frac{\int_S^R e^{-rx}\left[\frac{-\partial p(x)}{\partial \mu(x_0)}\right]y(S,x)dx}{\int_S^R e^{-rx}p(x)y(S,x)dx} - \frac{\int_{x_0}^\omega e^{-[(1-\sigma)r+\sigma\rho]x}p(x)dx}{\int_0^\omega e^{-[(1-\sigma)r+\sigma\rho]x}p(x)dx}, \tag{56}$$

Note that the right-hand side of (56) at $S = S^*$ and $R = R^*$ is (28). Now, from (28), we obtain $g(S^*) > 0$, $g(R^*) < 0$,

$$g'(x_0) = -\frac{e^{-rx_0}p(x_0)y(S^*,x_0)}{\int_{S^*}^{R^*} e^{-rx}p(x)y(S^*,x)dx} + \frac{e^{-[(1-\sigma)r+\sigma\rho]x_0}p(x_0)}{\int_0^{\omega} e^{-[(1-\sigma)r+\sigma\rho]x}p(x)dx}, \tag{57}$$

and

$$g''(x_0) = \frac{\left[r + \mu(x_0) - \frac{y_{x_0}(S^*,x_0)}{y(S^*,x_0)}\right] e^{-rx_0}p(x_0)y(S^*,x_0)}{\int_{S^*}^{R^*} e^{-rx}p(x)y(S^*,x)dx} - \frac{\left[(1-\sigma)r + \sigma\rho + \mu(x_0)\right] e^{-[(1-\sigma)r+\sigma\rho]x_0}p(x_0)}{\int_0^{\omega} e^{-[(1-\sigma)r+\sigma\rho]x}p(x)dx}, \tag{58}$$

for any x_0 within the interval (S^*, R^*). Since $g(\cdot)$ is a continuous function in (S^*, R^*), $g(S^*) > 0$, and $g(R^*) < 0$ imply that there exists at least a critical age x_c within the interval (S^*, R^*) such that $g(x_c) = 0$.

In order to prove that x_c is unique, we show that there exists only one local optimum in the interval (S^*, R^*). At a local optimum (denoted by $\tilde{x}_0$, with $g'(\tilde{x}_0) = 0$), from (57) and (58) we obtain

$$g''(\tilde{x}_0) = \frac{\left[\sigma(r-\rho) - \frac{y_{x_0}(S^*,x_0)}{y(S^*,x_0)}\right] e^{-r\tilde{x}_0}p(\tilde{x}_0)y(S^*,\tilde{x}_0)}{\int_{S^*}^{R^*} e^{-rx}p(x)y(S^*,x)dx}. \tag{59}$$

Let $\tilde{x}_0^i$ and $\tilde{x}_0^{ii}$ be two possible candidates, which satisfy that $g'(\tilde{x}_0^i) = 0$ and $g'(\tilde{x}_0^{ii}) = 0$ with $\tilde{x}_0^i < \tilde{x}_0^{ii}$. Provided that $y(S^*, x)$ is strictly concave within the interval (S^*, R^*), $\tilde{x}_0$ is unique ($\tilde{x}_0^i = \tilde{x}_0^{ii}$) either because $\sigma(r-\rho) - \frac{y_{x_0}(S^*,x_0)}{y(S^*,x_0)} < 0$ or $\sigma(r-\rho) - \frac{y_{x_0}(S^*,x_0)}{y(S^*,x_0)} > 0$ for all $\tilde{x}_0 \in (S^*, R^*)$, or $\tilde{x}_0^i$ is a local maximum and $\tilde{x}_0^{ii}$ a local minimum, which proves that x_c is unique.

C Proof of Proposition 3

Given the implicit-function theorem holds, there is one unique function $\Gamma(R;\mu)$ that equals S^* for any $(R;\mu)$ around $(R^*;\mu(x_0))$. Provided the optimal length of schooling condition (9)

$$\hat{V}_S(\Gamma(R;\mu), R;\mu) = \hat{V}_S(S^*, R^*;\mu(x_0)) = 0, \tag{60}$$

and assuming $\hat{V}(S, R;\mu)$ is strictly concave around the point $(S^*, R^*;\mu(x_0))$. Then, we differentiate (60) with respect to a mortality decline at an arbitrary age x_0, $-\mu(x_0)$, to obtain the marginal impact of a mortality decline

on the optimal length of schooling. Applying the Chain rule in (60) we obtain

$$-S^*_{\mu(x_0)} = \frac{\frac{-\partial f(S^*,R^*)}{\partial \mu(x_0)} - \frac{f(S^*,R^*)}{\sigma(c^*)}\frac{1}{c^*}\frac{-\partial c^*}{\partial \mu(x_0)}}{f(S^*,R^*)\left(\frac{\psi_S(S^*)}{\psi(S^*)} + \frac{1}{\sigma(c^*)}\frac{c^*_S}{c^*}\right) - f_S(S^*,R^*)}. \tag{61}$$

Thus, from (52) we have

$$\text{sign}\left[-S^*_{\mu(x_0)}\right] = \text{sign}\left[\frac{-\partial f(S^*,R^*)}{\partial \mu(x_0)} - \frac{f(S^*,R^*)}{\sigma(c^*)}\frac{1}{c^*}\frac{-\partial c^*}{\partial \mu(x_0)}\right]. \tag{62}$$

Note from (25) that the first term on the right-hand side of (62) is zero whenever $x_0 \leq S^*$ or $x_0 \geq R^*$. Substituting (12)-(13) and (26) in (62), and taking $e^{-r(x_0-S^*)}\frac{p(x_0)}{p(S^*)}$ as common factor gives

$$\text{sign}\left[-S^*_{\mu(x_0)}\right] = \text{sign}\left[\frac{r^h(S^*) - \bar{r}(S^*,R^*)}{\sigma(\bar{c})}a(x_0)\right]. \tag{63}$$

when $x_0 \leq S^*$ and $x_0 \geq R^*$, and

$$\text{sign}\left[-S^*_{\mu(x_0)}\right] = \text{sign}\left[\frac{r^h(S^*) - \bar{r}(S^*,R^*)}{\sigma(\bar{c})}a(x_0) + \int_{x_0}^{R^*} e^{-r(x-x_0)}\frac{p(x)}{p(x_0)}y_S(S^*,x)dx\right]. \tag{64}$$

when $x_0 \in (S^*, R^*)$. This proves Proposition 3(a).

Similarly, given that the implicit-function theorem holds, there is one unique function $\Upsilon(S;\mu)$ that equals R^* for any $(S;\mu)$ around $(S^*;\mu(x_0))$. Provided the optimal retirement age condition (10)

$$\hat{V}_R(S, \Upsilon(S;\mu);\mu) = \hat{V}_R(S^*, R^*;\mu(x_0)) = 0. \tag{65}$$

Assuming $\hat{V}(S,R;\mu)$ is strictly concave around the point $(S^*,R^*;\mu(x_0))$. We obtain after differentiating (65) with respect to $-\mu(x_0)$, and applying the Chain rule,

$$-R^*_{\mu(x_0)} = \frac{-\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{-\partial c^*}{\partial \mu(x_0)}}{\frac{1}{\sigma(c^*)}\frac{c^*_R}{c^*} - \frac{w_{R-S}(R^*-S^*)}{w(R^*-S^*)} + \delta + \frac{\phi_R(R^*)}{\phi(R^*)}}. \tag{66}$$

From (53), we have

$$\text{sign}\left[-R^*_{\mu(x_0)}\right] = -\text{sign}\left[\frac{-\partial c^*}{\partial \mu(x_0)}\right]. \tag{67}$$

Now, in order to prove Proposition (3)(b) we show that the sign of $-R^*_{\mu(x_0)}$ is that of $a(x_0)$.

Differentiating (7) at $S = S^*$ and $R = R^*$ with respect to $-\mu(x_0)$, and rearranging, gives

$$\int_{x_0}^{\omega} e^{-rx} p(x) c(x, S^*, R^*) dx + \int_{0}^{\omega} e^{-rx} p(x) \frac{-\partial c(x, S^*, R^*)}{\partial \mu(x_0)} dx = \int_{S^*}^{R^*} e^{-rx} \frac{-\partial p(x)}{\partial \mu(x_0)} w(x - S^*) h(S^*, x) dx. \quad (68)$$

The intertemporal budget constraint at age x_0 can be expressed as

$$e^{-rx_0} p(x_0) a(x_0) = \begin{cases} \int_{x_0}^{\omega} e^{-rx} p(x) c(x, S^*, R^*) dx - \int_{x_0}^{R^*} e^{-rx} p(x) w(x - S^*) h(S^*, x) dx & \text{if } S^* < x_0 < R^*, \\ \int_{x_0}^{\omega} e^{-rx} p(x) c(x, S^*, R^*) dx & \text{otherwise.} \end{cases} \quad (69)$$

Substituting (69) into (68), we obtain

$$\int_{0}^{\omega} e^{-rx} p(x) \frac{-\partial c(x, S^*, R^*)}{\partial \mu(x_0)} dx = -e^{-rx_0} p(x_0) a(x_0). \quad (70)$$

Differentiating (8) at $S = S^*$ and $R = R^*$ with respect to $-\mu(x_0)$, and simplifying, gives

$$\frac{1}{c(x, S^*, R^*)} \frac{-\partial c(x, S^*, R^*)}{\partial \mu(x_0)} = \frac{\sigma(c(x, S^*, R^*))}{\sigma(c^*)} \frac{1}{c^*} \frac{-\partial c^*}{\partial \mu(x_0)}. \quad (71)$$

Substituting (71) into (70), and rearranging, we get (26). Finally, substituting (26) in (67), we have

$$\text{sign}\left[-R^*_{\mu(x_0)}\right] = -\text{sign}\left[\frac{-\partial c^*}{\partial \mu(x_0)}\right] = \text{sign}\left[a(x_0)\right], \quad (72)$$

which proves Proposition 3(b).

D Proof of Proposition 4

Given the implicit-function theorem holds, there are two unique functions $\Gamma(R; \mu)$ and $\Upsilon(S; \mu)$ that are equal to S^* and R^*, respectively, for any $(S, R; \mu)$ around $(S^*, R^*; \mu(x_0))$. Provided the optimal length of schooling and retirement age conditions, we have:

$$\hat{V}_S(\Gamma(R; \mu), \Upsilon(S; \mu); \mu) = \hat{V}_S(S^*, R^*; \mu(x_0)) = 0, \quad (73a)$$

$$\hat{V}_R(\Gamma(R; \mu), \Upsilon(S; \mu); \mu) = \hat{V}_R(S^*, R^*; \mu(x_0)) = 0. \quad (73b)$$

For notational simplicity, hereinafter we skip the arguments. Writing the system of equations (73) in differential form we have

$$\hat{V}_{S\Gamma}d\Gamma + \hat{V}_{S\Upsilon}d\Upsilon + \hat{V}_{S\mu}d\mu = 0, \tag{74a}$$

$$\hat{V}_{R\Gamma}d\Gamma + \hat{V}_{R\Upsilon}d\Upsilon + \hat{V}_{R\mu}d\mu = 0. \tag{74b}$$

If $\hat{V}_S(\cdot)$ and $\hat{V}_R(\cdot)$ are continuously differentiable with respect to $(S, R; \mu)$, (S^*, R^*) is a solution of the system at the mortality value $\mu(x_0)$, and the Jacobian matrix of the system (73) evaluated at $(S^*, R^*; \mu(x_0))$ is not singular, or $|J| \neq 0$, then the system can be locally solved at $(S^*, R^*; \mu(x_0))$.

The solution of (74), by Cramer's rule, is

$$\begin{pmatrix} \frac{dS^*}{d\mu(x_0)} \\ \frac{dR^*}{d\mu(x_0)} \end{pmatrix} = \frac{1}{|J|} \begin{pmatrix} -\hat{V}_{RR}\hat{V}_{S\mu(x_0)} + \hat{V}_{RS}\hat{V}_{R\mu(x_0)} \\ \hat{V}_{SR}\hat{V}_{S\mu(x_0)} - \hat{V}_{SS}\hat{V}_{R\mu(x_0)} \end{pmatrix}. \tag{75}$$

Taking $\hat{V}_{RR}\hat{V}_{SS}$ as common factor in the right-hand side of (75) gives

$$\begin{pmatrix} \frac{dS^*}{d\mu(x_0)} \\ \frac{dR^*}{d\mu(x_0)} \end{pmatrix} = \frac{\hat{V}_{RR}\hat{V}_{SS}}{|J|} \begin{pmatrix} \frac{\hat{V}_{S\mu(x_0)}}{-\hat{V}_{SS}} + \frac{\hat{V}_{RS}}{-\hat{V}_{SS}}\frac{\hat{V}_{R\mu(x_0)}}{-\hat{V}_{RR}} \\ \frac{\hat{V}_{R\mu(x_0)}}{-\hat{V}_{RR}} + \frac{\hat{V}_{SR}}{-\hat{V}_{RR}}\frac{\hat{V}_{S\mu(x_0)}}{-\hat{V}_{SS}} \end{pmatrix}. \tag{76}$$

Provided the strict concavity of $\hat{V}(S, R; \mu)$ at $(S^*, R^*; \mu(x_0))$ and multiplying both sides of (76) by -1, since we are interested in the effect of a decline in mortality rather than an increase in mortality, we obtain

$$\operatorname{sign}\left[\frac{-dS^*}{d\mu(x_0)}\right] = -\operatorname{sign}\left[S^*_{\mu(x_0)}(R^*; \mu) + \frac{dS^*}{dR^*}R^*_{\mu(x_0)}(S^*; \mu)\right], \tag{77}$$

$$\operatorname{sign}\left[\frac{-dR^*}{d\mu(x_0)}\right] = -\operatorname{sign}\left[R^*_{\mu(x_0)}(S^*; \mu) + \frac{dR^*}{dS^*}S^*_{\mu(x_0)}(R^*; \mu)\right]. \tag{78}$$

This completes the proof of Eqs. (24a) and (24b).

We now differentiate (48) and (49) with respect to $-\mu(x_0)$ at (S^*, R^*), respectively,

$$-\hat{V}_{S\mu(x_0)} = U_c\big(c^*\big)e^{-rS^*}p(S^*)\left(\frac{-\partial f(S^*, R^*)}{\partial\mu(x_0)} - \frac{f(S^*, R^*)}{\sigma(c^*)}\frac{1}{c^*}\frac{-\partial c^*}{\partial\mu(x_0)}\right), \tag{79}$$

$$-\hat{V}_{R\mu(x_0)} = U_c\big(c^*\big)e^{-rR^*}p(R^*)y(S^*, R^*)\left(-\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{-\partial c^*}{\partial\mu(x_0)}\right). \tag{80}$$

Substituting (79)-(80) in (77), taking $\frac{U_c(c^*)e^{-rS^*}p(S^*)}{-\hat{V}_{SS}(S^*,R^*)}$ as common factor, and using (12), (18) and (20), we get

$$\operatorname{sign}\left[\frac{-dS^*}{d\mu(x_0)}\right] = \operatorname{sign}\left[\frac{-\partial f(S^*, R^*)}{\partial\mu(x_0)} - \frac{f(S^*, R^*)}{\sigma(c^*)}\frac{1}{c^*}\frac{-\partial c^*}{\partial\mu(x_0)}\left(1 + \frac{dR^*}{dS^*}\frac{c^*_R}{c^*_S}\right)\right]. \tag{81}$$

Using (11), (18), and (26) we obtain, after rearranging,

$$\text{sign}\left[\frac{-dS^*}{d\mu(x_0)}\right] = \text{sign}\left[\int_{S^*}^{R^*} e^{-r(x-S^*)}\frac{-\partial}{\partial\mu(x_0)}\left[\frac{p(x)}{p(S^*)}\right] y_S(S^*,x)dx \right.$$
$$\left. -e^{-r(x_0-S^*)}\frac{p(x)}{p(S^*)}a(x_0)\frac{1}{\sigma(c^*)c^*}\left(c_S^* + c_R^*\frac{dR^*}{dS^*}\right)\right]. \quad (82)$$

Finally, taking $e^{-r(x_0-S^*)}\frac{p(x)}{p(S^*)}$ as common factor and using the fact that $\frac{dc^*}{dS^*} = c_S^* + c_R^*\frac{dR^*}{dS^*}$ we obtain Proposition 4(a), or Eq. (32).

Using the same steps for the sign of $\frac{-dR^*}{d\mu(x_0)}$ but now taking $-\hat{V}_{RR}$ as common factor, it can be shown that

$$\text{sign}\left[\frac{-dR^*}{d\mu(x_0)}\right] = \text{sign}\left[\frac{dS^*}{dR^*}\int_{S^*}^{R^*} e^{-r(x-S^*)}\frac{-\partial}{\partial\mu(x_0)}\left[\frac{p(x)}{p(S^*)}\right] y_S(S^*,x)dx \right.$$
$$\left. -e^{-r(x_0-S^*)}\frac{p(x)}{p(S^*)}a(x_0)\frac{1}{\sigma(c^*)c^*}\left(c_R^* + c_S^*\frac{dS^*}{dR^*}\right)\right]. \quad (83)$$

Similarly, taking $e^{-r(x_0-S^*)}\frac{p(x)}{p(S^*)}$ as common factor and using the fact that $\frac{dc^*}{dR^*} = c_R^* + c_S^*\frac{dS^*}{dR^*}$ we obtain Proposition 4(b), or Eq. (33).

References

Arriaga, E., 1984. Measuring and explaining the change in life expectancies. Demography 21 (1), 83–96.

Barro, R. J., Lee, J. W., 2013. A new data set of educational attainment in the world, 19502010. Journal of Development Economics 104, 184–198.

Bell, F. C., Wade, A. H., Goss, S. C., 1992. Life tables for the United States social security area. Actuarial Study No. 107, SSA Pub. No. 11-11536.

Belley, P., Lochner, L., 2007. The changing role of family income and ability in determining educational achievement. Journal of Human Capital 1 (1), 37–89.

Benavot, A., Riddle, P., 1988. The expansion of primary education, 1870-1940: Trends and issues. Sociology of Education 61 (3), 191–210.

Ben-Porath, Y., 1967. The production of human capital and the life cycle of earnings. The Journal of Political Economy 75 (4), 352–365.

Bils, M., and Klenow, J. P., 2000. Does schooling cause growth? The American Economic Review 90 (5), 1160–1183.

Boucekkine, R., de la Croix, D., Licandro, O., 2002. Vintage human capital, demographic trends, and endogenous growth. Journal of Economic Theory 104, 340–375.

Boucekkine, R., de la Croix, D., Peeters, D., 2007. Early literacy achievements, population density and the transition to modern growth. Journal of the European Economic Association 5:183–226.

Cai, Z., Lau, P., 2012. Does longevity improvement always raise the length of schooling through the longer-horizon mechanism? Society for Economic Dynamics, 2013 Meeting Papers, number 292.

Card, D., 1999. The causal effect of education on earnings. In O. C. Ashenfelter and D. Card (Eds.), Handbook of labor economics (1st ed., Vol. 3, pp. 1801–1863) Amsterdam: Elsevier.

Card, D., 2001. Estimating the return to education: Progress on some persistent economic problems. Econometrica 69 (5), 1127–1160.

Carneiro, P., Heckman, J. J., 2002. The evidence on credit constraints in post-secondary schooling. Economic Journal 112 (482), 705–734.

Carneiro, P., Hansen, K. T., Heckman, J. J., 2003. 2001 Lawrence R. Klein Lecture estimating distributions of treatment effects with an application to the returns to education and measurement of the effects of uncertainty on college choice. International Economic Review 44 (2), 631–422.

Cervellati, M., Sunde, U., 2005. Human capital formation, life expectancy, and the process of development. American Economic Review 95, 1653–1672.

Cervellati, M., Sunde, U., 2013. Life expectancy, schooling, and lifetime labor supply: Theory and evidence revisited. Econometrica 81 (5), 2055–2086.

Cipolla, C., 1969. Literacy and development in the West. Baltimore: Penguin Books.

Costa, D. L., 1998. The evolution of retirement. In The Evolution of Retirement: An American Economic History, 1880-1990. University of Chicago Press, 6–31.

Cunha, F., Heckman, J., Navarro, S., 2005. Separating uncertainty from heterogeneity in life cycle earnings. Oxford Economic Papers 2005 57 (2), 191–261.

Cutler, D., Deaton, A., Lleras-Muney, A., 2006. The determinant of mortality. Journal of Economic Perspectives 20 (3), 97–120.

d'Albis, H., Augeraud-Véron, E., 2008. Endogenous retirement and monetary cycles. Mathematical Population Studies 15, 214–229.

d'Albis, H., Lau, P., Sanchez-Romero, M., 2012. Mortality transition and differential incentives for early retirement. Journal of Economic Theory 147 (1), 261–283.

de la Croix, D., 2015. Did longer lives buy economic growth? From Malthus to Lucas and Ben-Porath. Discussion Papers (IRES - Institut de Recherches Economiques et Sociales).

de la Croix, D., Licandro, O., 1999. Life expectancy and endogenous growth. Economics Letters 65, 255–263.

de la Croix, D., Lindh, T., Malmberg, B., 2008. Swedish economic growth and education since 1800. Canadian Journal of Economics 41 (1), 166–185.

Echevarria, C. A., 2004. Life expectancy, schooling time, retirement, and growth. Economic Inquiry 42 (4), 602–617.

Echevarria, C. A., Iza, A., 2006. Life expectancy, human capital, social security and growth. Journal of Public Economics 90, 2323–2349.

Ferreira, P. C., Pessoa, S. A., 2007. The effects of longevity and distortions on education and retirement. Review of Economic Dynamics 10 (3), 472–493.

Haines, M. R., 1998. Estimated life table for the United States, 18501910. Historical Methods 31, 149–167.

Hazan, M., Zoabi, H., 2006. Does longevity cause growth? A theoretical critique. Journal of Economic Growth, 11 (4), 363–376.

Hazan, M., 2009. Longevity and lifetime labor supply: Evidence and implications. Econometrica 77 (6), 1829–1863.

Heckman, J. J., Lochner, L. J., Taber, C., 1998. Explaining rising wage inequality: Explorations with a dynamic general equilibrium model of labor earnings with heterogeneous agents. Review of Economic Dynamics 1, 1–58.

Heckman, J. J., 2003. Human capital policy. In: Friedman, Benjamin (Ed.), Inequality in America. MIT Press, Cambridge, M. A.

Heckman, J. J., Lochner, L. J., Todd, P. E., 2006. Earnings functions, rates of return and treatment effects: The Mincer equation and beyond. In: E. Hanushek and F. Welch, Editor(s), Handbook of the Economics of Education 1, 307–458.

Heckman, J. J., Lochner, L. J., Todd, P. E., 2008. Earnings functions and rates of return. Journal of Human Capital 2(1), 1–31.

Heijdra, B. J., Romp, W. E., 2009. Retirement, pensions, and ageing. Journal of Public Economics 93, 586–604.

Holmlund, B., 2009. The Swedish unemployment experience. Oxford review of economic policy, 25 (1), 109–125.

Human Mortality Database, 2013. University of California, Berkeley (USA), and Max Planck Institute for Demography Research (Germany). Available at www.mortality. org or www.humanmortality.de (data download on February 2013).

Imrohoroglu, S., Kitao, S., 2009. Labor supply elasticity and social security reform. Journal of Public Economics 93, 867878.

Jayachandran, S., Lleras-Muney, A., 2009. Life expectancy and human capital investments: Evidence from maternal mortality declines. The Quarterly Journal of Economics 124 (1), 349–397.

Kalemli-Ozcan, S., Weil, D., 2010. Mortality change, the uncertainty effect, and retirement. Journal of Economic Growth 15 (1), 65–91.

Kalemli-Ozcan, S., Ryder, H. E., Weil, D. N., 2000. Mortality decline, human capital investment, and economic growth. Journal of Development Economics 62, 1–23.

Keane, M. P., 2011. Labor supply and taxes: A survey. Journal of Economic Literature 49 (4), 961–1075.

Lee, R. D., 1994. Demography of aging. National Academy Press, Washington, D.C., Ch. The formal demography of population aging, transfers, and the economic life cycle, pp. 8–49.

Lee, R., 2003. The demographic transition: Three centuries of fundamental change. The Journal of Economic Perspectives, 17 (4), 167–190.

Lee, R. D., Carter, L. R., 1992. Modeling and forecasting U. S. mortality. Journal of the American Statistical Association 87 (419), 659–671.

Ljungqvist, L., Sargent, T. J., 1995. The Swedish unemployment experience. European Economic Review, 39 (5), 1043–1070.

Mincer, J., 1974. Schooling, experience and earnings. New York: Columbia University Press.

OECD (2009). Society at a glance 2009: OECD Social Indicators. OECD Publishing.

Oeppen, J., Vaupel, J. W., 2002. Broken limits to life expectancy. Science, 296 (5570), 1029–1031.

Oreopoulos, P., 2007. Do dropouts drop too soon? Wealth, health and happiness from compulsory schooling. Journal of Public Economics 91, 2213–2229.

Oreopoulos, P., Salvanes, K. G., 2011. Priceless: The nonpecuniary benefits of schooling. Journal of Economic Perspectives 25 (1), 159–184.

Oster, E., Shoulson, I., Dorsey, E., 2013. Limited life expectancy, human capital and health investments. The American Economic Review 103 (5), 1977–2002.

Palme, J., 2005. Features of the Swedish pension reform. The Japanese Journal of Social Security Policy 4 (1), 42–53.

Restuccia, D., Vandenbroucke, G., 2013. A century of human capital and hours. Economic Inquiry 51 (3), 1849–1866.

Ryder, H.E., Heal, G.M., 1973. Optimal growth with intertemporally dependent preferences. Review of Economic Studies 40, 1–31.

Sheshinski, E., 1978. A model of social security and retirement decisions. Journal of Public Economics 10, 337–360.

Schieber, S. J., Shoven, J. B. 1999. The real deal: The history and future of social security. London: Yale University Press.

Soares, R. R, 2005. Mortality reductions, educational attainment, and fertility choice. American Economic Review 95 (3), 580–601.

Strulik, H., Werner, K., 2012. Life expectancy, labor supply, and long-run growth: Reconciling theory and evidence, university of Goettingen, Cege Discussion Paper 140.

Willis, R., 1986. Wage determinants: A survey and reinterpretation of human capital earnings functions. In Handbook of Labor Economics, ed. by Orley Ashenfelter and Richard Layard. Amsterdam and New York: North Holland.

Wilmoth, J. R., Horiuchi, S., 1999. Rectangularization revisited: Variability of age at death within human populations. Demography 36, 475–495.

Yaari, M. E., 1965. Uncertain lifetime, life insurance, and the theory of the consumer. Review of Economic Studies 32, 137–150.

Zhang, J., Zhang, J., 2005. The effect of life expectancy on fertility, saving, schooling and economic growth: Theory and evidence. Scandinavian Journal of Economics 107, 45–66.

Zhang, J., Zhang, J., 2009. Longevity, retirement, and capital accumulation in a recursive model with an application to mandatory retirement. Macroeconomic Dynamics 13 (3), 327–348.

Zhang, J., Zhang, J., Lee, R., 2001. Mortality decline and long-run economic growth. Journal of Public Economics 80, 485–507.

Zhang, J., Zhang, J., Lee, R., 2003. Rising longevity, education, savings, and growth. Journal of Development of Economics 70, 83–101.

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