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Commodities, financialization, and heterogeneous agents

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Nicole Branger - Patrick Grüning - Christian Schlag

Commodities, Financialization, and Heterogeneous Agents

SAFE Working Paper No. 131

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Non-Technical Summary

Since the seminal work of Keynes (1930) economists and researchers in finance have studied the theory and empirics of commodity futures markets and their fundamentals extensively. Over the past decade these markets have been the focus of an intense public debate and in academia. The reason is that commodity spot and futures prices, especially those for various food commodities as well as oil and gas, had increased to all-time highs. This pronounced price increase had occurred together with an equally sharp increase in long positions in these contracts held by financial speculators, which often trade in index products written on a basket of commodities. The volume of index-linked commodity investing went up dramatically from the beginning of 2006 until the end of 2007. At around the same time futures and spot prices had risen to all-time highs.

Given this coincidence of large index-linked and financially motivated commodity positions on the one hand and increasing commodity prices on the other, the activities by financial speculators on commodity markets were perceived as harmful from a welfare perspective, especially in the context of food commodities. The poorer parts of the population, in particular in emerging countries, have to spend a large share of their income on these basic commodities. Consequently, it was suggested that futures markets should be regulated tightly via, for example, position limits.

Nevertheless, despite the seemingly clear evidence presented above, the debate about a potential causality going from commodity investing to excessive price increases is still ongoing, with evidence both for and against the hypothesis. One reason for this difference of opinion is probably that the empirical analyses are still plagued by a number of problems. First, the financialization phenomenon in the sense of strongly increasing position sizes held by financial investors can only be observed from 2004 onwards, that is, over a relatively short sample. Second, the data do not usually exhibit the quality needed to study potential causal effects reliably. For example, it is often hard to distinguish financial speculators from other types of investors and to obtain their exact trading positions, since one can only obtain aggregate net positions.

Overall, as this brief discussion shows, there is still need for a deeper theoretical analysis of the impact of commodity trading on spot price dynamics and on the welfare of different economic agents. This is exactly what we want to provide with this paper. More precisely, we want to answer the question of whether and under what conditions the basic fact that commodity price risk becomes tradable on financial markets leads not only to a price reaction, but more importantly to a welfare loss for those agents who are prevented from participating in these markets due to their low wealth. In contrast to other papers dealing with financialization from a pure financial markets perspective, our model focuses explicitly on the double role of a commodity as a source of consumption utility and as an input into a production process.

Our general equilibrium model features two types of goods and four types of agents. The two goods are the basic commodity, which represents a basket of basic commodities like energy and food, and a generic 'non-commodity' good, which can be seen as the result of a finishing process with the basic commodity as one of the inputs. A distinction is made between the two types of

goods to model differences in the consumption bundles between people living in emerging countries, who are potentially strongly dependent on the commodity in their daily consumption, and the inhabitants of richer, more developed countries, for whom the consumption of food is of course still necessary, but where there is also a significant share of other, more refined, goods in the consumption basket.

The choice of agents who populate our model economy is based on a very similar motivation. The four agents are a commodity consumer, an agricultural producer, an industrial producer, and a financial speculator. The first two are again meant to represent an emerging economy with agriculture as a key sector and some basic commodity as the key element of the consumption bundle. The commodity consumer receives an exogenously specified stream of the non-commodity good and has to exchange it for the basic commodity, which immediately implies that high commodity prices represent a bad state for her. The agricultural producer, whose endowment consists of the commodity good, derives utility from the consumption of both this basic good and the refined non-commodity good which she has to buy. With respect to the consumption basket she is structurally equal to the industrial producer, who is endowed with capital and has access to a production technology. She can use this capital and the commodity to produce the non-commodity good. Like the commodity consumer, the financial speculator is endowed with a stream of the non-commodity good, the consumption of which is also the only source of utility for her. She does not consume the commodity good, but may trade commodity derivatives for speculative reasons.

To analyze the equilibrium effects of financial markets in this setup we compare different scenarios with respect to the agents' access to them. In all versions of the model we retain the assumption that the commodity consumer cannot trade financial products. With respect to the other types of agents, we consider the case where all three of them or only two of them can trade instruments called 'bond' and 'commodity derivative'. The quantities of interest we compare across the different scenarios are the agents' wealth and consumption levels and volatilities as well as spot and futures prices and their volatilities, and the reaction of all these key quantities to shocks in the sources of risk in the system.

The main findings of our model with respect to the role of financial markets can be briefly summarized as follows. First, access to financial markets is always beneficial for the agents allowed to trade in the sense that it reduces their consumption volatility. From a welfare point of view it is important that commodity risk is tradable, i.e., that the agricultural producer has access to the financial markets. Once she and the financial speculator can trade on the financial markets, not only do their own consumption growth volatilities decrease but so also do those of the industrial producer and the commodity consumer, so that in this case all agents benefit. Furthermore, compared to the benchmark case without financial markets spot price volatility is much lower. This is no longer true when the financial speculator only trades with the industrial producer. In this case only the two financial market participants enjoy a reduction in consumption volatility, while we find the opposite for the agricultural producer and the commodity consumer.

To sum up, while access to financial markets is always beneficial for the participating agents, the effects for those, who face severe hurdles in their access to these markets are not uniformly clear.

Commodities, Financialization, and Heterogeneous Agents

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Abstract

The term ‘financialization’ describes the phenomenon that commodity contracts are traded for purely financial reasons and not for motives rooted in the real economy. Recently, financialization has been made responsible for causing adverse welfare effects especially for low-income and low-wealth agents, who have to spend a large share of their income for commodity consumption and cannot participate in financial markets. In this paper we study the effect of financial speculation on commodity prices in a heterogeneous agent production economy with an agricultural and an industrial producer, a financial speculator, and a commodity consumer. While access to financial markets is always beneficial for the participating agents, since it allows them to reduce their consumption volatility, it has a decisive effect with respect to overall welfare effects who can trade with whom (but not so much what types of instruments can be traded).

Keywords: Commodities, General Equilibrium, Heterogeneous Preferences, Financial Markets

JEL: E23, G12, G13, Q11, I30

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1 Introduction

Since the seminal work of Keynes (1930) economists and researchers in finance have studied the theory and empirics of commodity futures markets and their fundamentals extensively. Over the past decade these markets have been the focus of intense debates both by the public and academia. The reason is that commodity spot and futures prices, especially those for various food commodities as well as oil and gas, had increased to all time highs, and this pronounced price increase had occurred together with an equally sharp increase in long positions in these contracts held by financial speculators, which often trade in index products written on a basket of commodities. According to Sanders and Irwin (2011) the volume of index-linked commodity investing went up from 90 billion USD at the beginning of 2006 to a peak of just under 200 billion USD at the end of 2007. At around basically the same time futures and spot prices had gone up to all-time highs. For example, Singleton (2013) shows that the NYMEX WTI crude oil price peaked (at about 140 USD per barrel) around August 2008. Other studies providing detailed analyses with similar tendencies in commodity trading activities and price dynamics include Casassus and Collin-Dufresne (2005), Gorton and Rouwenhorst (2006), Hong and Yogo (2009), Acharya, Lochstoer, and Ramadorai (2013), and Gorton, Hayashi, and Rouwenhorst (2013).

Given this coincidence of large index-linked and financially motivated commodity positions on the one hand and increasing commodity prices on the other, the activities by financial speculators on commodity markets were perceived as harmful from a welfare perspective, especially in the context of food commodities, since the poorer parts of the population especially in emerging countries have to spend a large share of their income on these basic commodities. Consequently, it was suggested that they should be regulated tightly via, for example, position limits as proposed by Schumann (2011).

Nevertheless, despite the seemingly clear evidence presented above, there are authors who claim that the causality from commodity investing to excessive price increases assumed by the critics of financialization is not really there. Examples for this view are the papers by Stoll and Whaley (2010), Pirrong (2011), Plante and Yücel (2011a,b), Sanders and Irwin (2011), Fattouh, Kilian, and Mahadeva (2013), and Kilian and Murphy (2013).

On the other hand a number of academic papers and testimonials to government and regulatory committees argue empirically that the observed price increases are in fact mainly due to speculation via index futures. Among these are Masters (2008) and Singleton (2013) who show that intermediate-term growth rates of index positions and managed-money spread positions after controlling for other known factors driving futures prices had the largest impacts on oil prices. Other examples for empirical studies claiming that financial speculation is indeed an explanation for changed commodity price dynamics include Gilbert (2010) and Tang and Xiong (2012). Further studies that point towards changes in trading as well as in futures or spot price dynamics following the ‘financialization’ of commodity markets are Silvennoinen and Thorp (2013), Henderson, Pearson, and Wang (2015), and Cheng, Kirilenko, and Xiong (2015). Cheng and Xiong (2014) provide a survey of the literature on how financial speculators impact commodity markets.

One reason for this mixed picture is probably that the empirical analyses are still plagued by a number of problems. First, the financialization phenomenon in the sense of strongly increasing position sizes held by financial investors can only be observed from 2004 onwards, i.e., over a relatively short sample. Second, the data do not usually exhibit the quality needed to study potential causal effects reliably. For example, it is often hard to distinguish financial speculators from other types of investors and to obtain their exact trading positions, since one can only obtain aggregate net positions.¹

Overall, as this brief discussion shows, there is still need for a deeper theoretical analysis of the impact of commodity trading on spot price dynamics and on the welfare of different economic agents, and this is exactly what we want to provide with this paper.

More precisely, we want to answer the question of whether and under what conditions the basic fact that commodity price risk becomes tradable on financial markets leads not only to a price reaction, but more importantly to a welfare loss by agents, who are prevented from participating in these markets due to their low wealth. In contrast to other papers dealing with financialization from a pure financial markets perspective, e.g. Basak and Pavlova (2015), our model focuses explicitly on the characteristics of a commodity as a source of consumption utility and as an input into a production process. In Basak and Pavlova (2015) the key issue is indexation, i.e., if a tradable asset is contained in some sort of index or not, while the physical properties of the asset or, in our special case, of the commodity are irrelevant. In this sense our analysis provides a benchmark for the analysis of the effects of indexation in this other types of models.

Our general equilibrium model features two types of goods and four types of agents. The two goods are the basic commodity, which represents a basket of basic commodities like energy and food, and a generic ‘non-commodity’ good, which can be seen as the result of a finishing process with the basic commodity as one of the inputs. A distinction is made between the two types of goods to model differences in the consumption bundles between people living in emerging countries, who are potentially strongly dependent on the commodity in their daily consumption, and the inhabitants of richer, more developed countries, for whom the consumption of, for example, food is of course still necessary, but there is also a significant share of other, more refined, goods in the consumption basket.

The choice of agents who populate our model economy is based on a very similar motivation. The four agents are a commodity consumer, an agricultural producer, an industrial producer and a financial speculator. The first two are again meant to represent an emerging economy with agriculture as a key sector and some basic commodity as the key element of the consumption bundle. The commodity consumer receives an exogenously specified stream of the non-commodity good and has to exchange it for the basic commodity, which immediately implies that high commodity prices represent a bad state for her. The agricultural producer, whose endowment consists of the commodity good, derives utility from the consumption of both this basic good and the refined non-commodity good. With respect to the consumption basket she is structurally equal to the industrial producer, who is endowed with capital and has access to a production technology. She can

¹A detailed discussion on these issues can be found in Sanders, Irwin, and Merrin (2010).

use this capital and the commodity to produce the non-commodity good. Like the commodity consumer, the financial speculator is endowed with a stream of the non-commodity good, the consumption of which is also the only source of utility for her. All agents in our model are equipped with recursive preferences of the type introduced by Epstein and Zin (1989).

To analyze the equilibrium effects of financial markets in this setup we compare different scenarios with respect to the agents' access to them. In all versions of the model discussed below we will retain the assumption that the commodity consumer cannot trade financial products. With respect to other types of agents, we will consider the case where all three of them or only one of the two producers and the financial speculator can trade instruments called 'bond' and 'commodity derivative'.² The quantities of interest we compare across the different scenarios are the agents' wealth and consumption levels and volatilities as well as spot and futures prices and their volatilities, and the reaction of all these key quantities to shocks in the sources of risk in the system.

The main findings of our model with respect to the role of financial markets can be briefly summarized as follows. First, access to financial markets is always beneficial for the agents allowed to trade in the sense that it reduces their consumption volatility. From a welfare point of view it is important that commodity risk is tradeable, i.e., that the agricultural producer has access to the financial markets. Once she and the financial speculator can trade on the financial markets, not only do their own consumption growth volatilities decrease but so also do those of the industrial producer and the commodity consumer, so that in this case all agents benefit. Furthermore, compared to the benchmark case without financial markets spot price volatility is much lower. This is no longer true when the financial speculator only trades with the industrial producer. In this case only the two financial market participants enjoy a reduction in consumption volatility, while we find the opposite for the agricultural producer and the commodity consumer. We trace these key results back in detail to the agents' consumption and wealth exposures to shocks, and it is at this point that access to financial markets really matters. Financial instruments enable market participants to share risk across states and across time, so that equilibrium consumption exposures depend on the opportunity to trade the bond or the commodity derivative.

With respect to financial quantities the model produces a reasonable level and volatility of the risk-free rate. Furthermore, the volatilities of futures and spot prices are in line with the dynamics of major commodities.

Our paper is of course not the first to theoretically investigate the link between the commodity and the financial market. The work closest to ours is probably Johnson (2011). The major innovations in our model compared to his setup are the explicit introduction of heterogeneous agents and the more general specification of recursive preferences relative to the constant relative risk aversion utility in his model.

²We introduce these two types of assets into the model to allow agents to share risk across time and states. It turns out that the precise type of payoff tradable in the market is not very important for our overall results.

In terms of the multiple agent setup the papers by Liu, Qiu, and Tang (2011) and Fattouh and Mahadeva (2014) are similar to ours. Liu, Qiu, and Tang (2011) consider a three agent economy featuring a hedger, a speculator, and a financial speculator. The hedger earns an exogenous convenience yield on his inventory of the commodity, while the speculator is providing liquidity. The financial speculator in turn is on average futures contracts long to bet on rising prices. The most important difference between their model and ours is that they analyze a partial equilibrium setting with an exogenously specified convenience yield. This means that one of the key quantities in our model, the relation between spot and futures prices, is not determined in equilibrium in their model and thus does not reflect the agents' optimal consumption and investment strategies.

The model in Fattouh and Mahadeva (2014) also features three agents. The physical speculator obtains an exogenous supply of the commodity which she can sell to the consumer right away or store for later. The financial speculator cannot hold physical inventory, but can trade in futures with the physical speculator and obtains her income from a position in a risky stock. Moreover, investment in a money market account is available for these two agents. The authors conclude from their analysis that fundamentals and not financial speculation were most likely the main determinants of futures and spot prices.

Baker and Routledge (2012) and Baker (2015) also study multiple-agent general equilibrium models. Baker and Routledge (2012) rely on an endowment economy with two consumable goods, a refined final consumption good and oil. There are two groups of investors with recursive preferences which differ with respect to the consumption bundle as well as their time and risk aggregators. They show that exogenous shocks to oil consumption generate endogenous shifts in the wealth distribution, which in turn cause persistent fluctuations of the oil price. Hence, they point to the importance of both fundamental shocks and risk sharing mechanisms on commodity markets as factors explaining the dynamics of futures markets. Baker (2015) considers a model with a producer, a dealer, and a household. Households consume both a refined final good and the commodity. The dealer has access to a storage technology. All three agents trade futures. Financial innovation is captured by a reduction in transaction costs for the household, and proxies the increased inflow of financial investment in commodity markets. The author argues that financialization cannot explain the occurrence of high spot prices, but can lead to lower equilibrium excess returns of futures, a more frequently upward sloping term structure of forward prices, and higher volatilities in spot and futures markets.

Other general equilibrium models involving commodities include Pirrong (2008), Casassus, Collin-Dufresne, and Routledge (2009), Hitzemann (2015), Ready (2016), and Arseneau and Leduc (2013). Pirrong (2008) considers a production economy and introduces stochastic volatility into the economy-wide productivity shock, but he does not provide an explicit welfare analysis in a heterogeneous agent economy as we do in this paper. Casassus, Collin-Dufresne, and Routledge (2009) also analyze a representative agent production economy where the commodity in the authors' focus is probably best represented by oil and thus somewhat different from the mixed commodity that we have in mind in our analysis.

In another representative household model Hitzemann (2015) studies an economy

with long-run productivity risks and an oil-producing sector. Oil is only consumed by the household, but not used in the production of the final consumption good. In this setting about half of the risk premium in futures is explained by fundamental factors. The paper also contributes to the debate on ‘financialization’ by showing that fundamental long-run productivity shocks can explain why futures term structures are more likely to be in contango, and why oil futures prices exhibit more momentum in the data from 2003 to 2008. Ready (2016) studies a related long-run risk production economy model, where oil is also an input to production. He finds similar equilibrium effects caused by the presence of financial speculators on commodity (oil) futures markets. Arseneau and Leduc (2013) consider a general equilibrium model with commodity storage and the commodity being used for both production and consumption of the representative household. Relative to the partial equilibrium framework of Deaton and Laroque (1992) they find that storage leads to a higher persistence in commodity prices and to a lower frequency of stockouts.

In the next section we describe our model and the equilibrium in detail. Afterwards, we discuss our results and calibration in Section 3 and conclude in Section 4. The appendix contains all derivations left out in the main text.

2 Model

We consider a model with two types of goods and four types of agents. The two types of goods are the commodity good and the non-commodity good. We assume that there is an exogenous supply of both the commodity good and the non-commodity good. Furthermore, there is a production technology which can be used to convert the commodity good into the non-commodity good.

There are four agents in our model, the industrial producer, the agricultural producer, the commodity consumer, and the financial speculator. The industrial producer and the agricultural producer derive utility from the commodity good and the non-commodity good. The financial speculator only consumes the non-commodity good, while the commodity consumer derives utility from the commodity good only. The production technology is exclusively available to the industrial producer.

In addition to the explanations provided in the following sections the structure of the model is also summarized graphically in Figure 1. The upper panel in the figure highlights the agents’ endowment streams and the input to their respective utility functions, whereas the lower graph focuses on the different markets represented in our model (commodity spot market and financial markets).

2.1 Agents

There are four agents in our economy, the industrial producer (IP), the agricultural producer (AP), the commodity consumer (CC) and the financial speculator (FS). All agents

have Epstein and Zin preferences given by

$$U_{i,t} = \left\{ (1 - \beta_i) \tilde{C}_{i,t}^{\frac{1-\gamma_i}{\theta_i}} + \beta_i (\mathbb{E}_t[U_{i,t+1}^{1-\gamma_i}])^{\frac{1}{\theta_i}} \right\}^{\frac{\theta_i}{1-\gamma_i}}, \quad (1)$$

for $i \in \{AP, IP, CC, FS\}$. The coefficient of relative risk aversion is denoted by γ_i , ψ_i is the intertemporal elasticity of substitution, and β_i is the time discount factor. We furthermore define $\theta_i = \frac{1-\gamma_i}{1-\frac{1}{\psi_i}}$.

Effective consumption $\tilde{C}_{i,t}$ is the constant-elasticity-of-substitution (CES) aggregate of non-commodity good consumption $C_{i,t}$ and commodity good consumption $Q_{i,t}$, i.e.

$$\tilde{C}_{i,t} = (\phi_i C_{i,t}^{\rho_i} + (1 - \phi_i) Q_{i,t}^{\rho_i})^{\frac{1}{\rho_i}}. \quad (2)$$

The parameter ϕ_i determines the weight of non-commodity consumption. The elasticity of substitution between the two consumption goods is given by $\frac{1}{1-\rho_i}$, where $\rho_i < 1$. In the following, we assume that the industrial and the agricultural producer derive utility from both goods (i.e. $0 < \phi_{IP}, \phi_{AP} < 1$). The commodity consumer derives utility from the commodity good only (i.e. $\phi_{CC} = 0$), while the financial speculator derives utility from the non-commodity good only (i.e. $\phi_{FS} = 1$).

Each agent has an exogenously given endowment which she uses to finance her consumption. The industrial producer owns the (exogenously given) capital and has access to a production technology. She can use capital and the commodity good which she buys in the spot market to produce the non-commodity good. The agricultural producer is endowed with the (exogenously given) supply of the commodity good. By selling parts of her endowment, she obtains the non-commodity good. The commodity consumer and the financial speculator are both endowed with an (exogenously given) stream of the non-commodity good, which the commodity consumer uses to buy the commodity in the spot market.

In order to price the financial assets we introduce in Section 2.3 below, the pricing kernels of the industrial producer, the agricultural producer and the financial speculator are needed. The pricing kernel in units of the non-commodity consumption good of agent $i \in \{IP, AP, FS\}$ is denoted by $M_{t,t+1}^{(i)}$ and is the ratio of agent i 's marginal utility with respect to non-commodity consumption at time $t+1$ to the marginal utility at time t . It is given by³

$$M_{t,t+1}^{(i)} = \frac{\partial U_{i,t} / \partial C_{i,t+1}}{\partial U_{i,t} / \partial C_{i,t}} = \beta_i \left(\frac{C_{i,t+1}}{C_{i,t}} \right)^{-\frac{1}{\psi_i}} \left(\frac{x_{i,t+1}}{x_{i,t}} \right)^{-\xi_i} \left(\frac{U_{i,t+1}^{1-\gamma_i}}{\mathbb{E}_t[U_{i,t+1}^{1-\gamma_i}]} \right)^{1-\frac{1}{\theta_i}}, \quad (3)$$

³The derivation of this pricing kernel for the industrial producer can be found in Appendix A.1, for the agricultural producer in Appendix A.2 and for the financial speculator in Appendix A.3.

where

$$x_{i,t} = \frac{\phi_i C_{i,t}^{\rho_i}}{\phi_i C_{i,t}^{\rho_i} + (1 - \phi_i) Q_{i,t}^{\rho_i}}, \quad \xi_i = \frac{\psi_i(1 - \rho_i) - 1}{\psi_i \rho_i}. \quad (4)$$

The percentage of the budget which the investor spends on non-commodity consumption is denoted by $x_{i,t}$.⁴ Note that $x_{FS,t} \equiv 1$ since the financial speculator only consumes non-commodity goods. Hence, the pricing kernel of the financial speculator reduces to the standard Epstein-Zin pricing kernel derived in Epstein and Zin (1989).

2.2 Endowments and Production

The exogenous supply of the commodity good is given by

$$Q_t = e^{\mu_q \cdot t + q_t}, \quad q_t = (1 - \varphi_q) \bar{q} + \varphi_q q_{t-1} + \varepsilon_{q,t}. \quad (5)$$

It grows with rate μ_q . The overall level of the commodity supply is determined by the long-run mean \bar{q} of the process q_t . Innovations $\varepsilon_{q,t}$ to q_t are normally distributed with mean 0 and standard deviation σ_q , and the persistence of these shocks is determined by φ_q .⁵ Note that the commodity good cannot be stored. Commodities have to be consumed or used in production immediately.

For the non-commodity good, there is also some exogenously given supply. The commodity consumer is endowed with the income stream $Z_{CC,t}$ given by

$$Z_{CC,t} = e^{\mu_{CC} \cdot t + z_{CC,t}}, \quad z_{CC,t} = (1 - \varphi_{CC}) \bar{z}_{CC} + \varphi_{CC} z_{CC,t-1} + \varepsilon_{CC,t}, \quad (6)$$

with growth rate μ_{CC} and normally distributed innovations $\varepsilon_{CC,t}$ with mean 0 and standard deviation σ_{CC} . The persistence of these shocks is determined by φ_{CC} . The overall level of the income stream depends on the long-run mean \bar{z}_{CC} . Analogously, the endowment $Z_{FS,t}$ of the financial speculator is given by

$$Z_{FS,t} = e^{\mu_{FS} \cdot t + z_{FS,t}}, \quad z_{FS,t} = (1 - \varphi_{FS}) \bar{z}_{FS} + \varphi_{FS} z_{FS,t-1} + \varepsilon_{FS,t}. \quad (7)$$

The growth rate is μ_{FS} , the overall level is determined by the long-run mean \bar{z}_{FS} of $z_{FS,t}$, the innovations $\varepsilon_{FS,t}$ are normally distributed with mean 0 and standard deviation σ_{FS} , and the persistence of these shocks is determined by φ_{FS} .

Furthermore, there is a production technology for the non-commodity good, which uses capital K_t and commodities $Q_{P,t}$, available to the industrial producer. The CES

⁴For IP and AP, this follows from the spot price given in Equations (11) and (15).

⁵For the endowment processes, we rely on productivity shocks with an exogenous growth rate as assumed in the macroeconomic literature. The productivity shocks are stationary and temporary, but they have a high persistence. See, for example, Kaltenbrunner and Lochstoer (2010) and Aldrich and Kung (2011), for productivity shocks which are modeled in a similar manner as our endowment processes.

production function Y resulting in non-commodity good output Y_t is given by

$$Y_t = Y(K_t, Q_{P,t}) = (\eta K_t^\nu + (1 - \eta) Q_{P,t}^\nu)^{\frac{1}{\nu}} = Q_{P,t} \left(\eta \left(\frac{K_t}{Q_{P,t}} \right)^\nu + (1 - \eta) \right)^{\frac{1}{\nu}}. \quad (8)$$

The elasticity of substitution between capital and the commodity is $\frac{1}{1-\nu}$ where $\nu < 1$. η gives the weight of capital in production. Capital K is given exogenously. It evolves as

$$K_t = e^{\mu_k t + k_t}, \quad k_t = (1 - \varphi_k) \bar{k} + \varphi_k k_{t-1} + \varepsilon_{k,t}, \quad (9)$$

with growth rate μ_k , overall level determined by the long-run mean \bar{k} , normally distributed innovations $\varepsilon_{k,t}$ with mean 0 and standard deviation σ_k , and the persistence of these innovations is determined by φ_k . In contrast to most macro models the capital endowment is exhausted in the production process and cannot be carried over to the next period. Hence, there is no capital depreciation or investment, i.e. capital is exogenous. More broadly, the capital endowment can be interpreted as labor or technology and all other factors contributing to production (see Johnson (2011) for a discussion).

2.3 Spot Market and Financial Market

The commodity good is traded in the spot market. Here, the industrial producer and the commodity consumer can buy the commodity good from the agricultural producer in exchange for non-commodity goods. The price of the commodity good is denoted by S_t .

Depending on the model under consideration, there might also be a financial market in which (at most) two assets are traded: a bond and a commodity derivative. The bond is a locally risk-free asset which pays one unit of the non-commodity good at the next point in time. The interest rate from t to $t + 1$ is denoted by $R_{f,t}$. It allows to transfer consumption over time. The commodity derivative also has a time to maturity of one period. Its payoff at time $t + 1$ is equal to the spot price S_{t+1} of the commodity good, its price at t is denoted by F_t . In addition to transferring consumption over time, it also allows the spot price risk to be traded.

In the benchmark model there are no financial assets available to the agents. In the most general model (later called IP-AP-FS model), the industrial producer can trade the bond, while the agricultural producer can trade the commodity derivative. In both cases, trading takes place with the financial speculator.

With one-period bonds and commodity derivatives only, the financial market is incomplete. This limits the agents' risk-sharing and consumption-smoothing possibilities, but at the same time allows us to study the impact of financial assets being generally available to the agents and the implications of different asset menus. So we can compare the situation when only the bond is available to that when only the commodity derivative

is traded.⁶ In Section 3.6, we also consider a market in which the industrial producer, the agricultural producer and the financial speculator all have access to both the bond and the commodity derivative, i.e. IP and AP are no longer restricted to one type of contract only.

2.4 Equilibrium

In equilibrium, each agent maximizes her utility, and markets clear. We now look at the optimization problems, derive the individual optimality conditions, and finally turn to the market clearing conditions. The formal definition is given at the end of this section and the detailed equilibrium conditions are given in the appendix. To ensure stable growth, we assume $\mu_k = \mu_q = \mu_{FS} = \mu_{CC}$.

2.4.1 Industrial Producer

The industrial producer is endowed with capital K and has access to the production technology Y . At time t , she decides on how much of the commodity good to buy for production ($Q_{P,t}$) and how much to buy for consumption ($Q_{IP,t}$). If the bond is traded, she also decides on the amount of to invest in the bond ($B_{IP,t+1}$). Her optimization problem is

$$\max_{\{C_{IP,t}, Q_{P,t}, Q_{IP,t}, B_{IP,t+1}\}} \left\{ (1 - \beta_{IP}) \tilde{C}_{IP,t}^{\frac{1-\gamma_{IP}}{\theta_{IP}}} + \beta_{IP} (\mathbb{E}_t[U_{IP,t+1}^{1-\gamma_{IP}}])^{\frac{1}{\theta_{IP}}} \right\}^{\frac{\theta_{IP}}{1-\gamma_{IP}}},$$

subject to the budget restriction

$$C_{IP,t} + B_{IP,t+1} + \frac{\nu_1}{e^{\mu_q \cdot t}} B_{IP,t+1}^2 = Y(K_t, Q_{P,t}) - Q_{P,t} S_t - Q_{IP,t} S_t + B_{IP,t} R_{f,t-1}. \quad (10)$$

The effective consumption $\tilde{C}_{IP,t}$ depends on $C_{IP,t}$ and on $Q_{IP,t}$ and is given by (2).

The budget restriction includes transaction costs $\frac{\nu_1}{e^{\mu_q \cdot t}} B_{IP,t+1}^2$ for holding the bond. These costs are a pure technical condition to obtain a well-defined deterministic steady state. In the benchmark calibration, we will set ν_1 to a small, but positive, number. This approach is thoroughly discussed in Judd and Mertens (2013).⁷ Note that the costs per squared number of the absolute portfolio holdings B_{IP} are $\frac{\nu_1}{e^{\mu_q \cdot t}}$ and thus decreasing as the economy grows. Technically, this is needed to have a stationary equilibrium.⁸

⁶On a complete market differences in asset menus would of course still matter for asset positions, but not for the characteristics of consumption and wealth.

⁷In our setup, one could also allow for sizeable costs which represent the costs of entering the bond market to study the impact of transaction costs on the equilibrium.

⁸The economy is growing, which implies that bond and futures holdings also grow over time. We have to scale all variables by $e^{\mu_q \cdot t}$ to obtain a stationary economy. Since transaction costs depend on the squared positions in the assets, dividing them by $e^{\mu_q \cdot t}$ is necessary to obtain stationary normalized bond

The solution to the optimization problem is derived in Appendix A.1. We give the most important equilibrium conditions in the following three equations.⁹ The intratemporal choice between commodity and non-commodity consumption gives

$$S_t = \frac{1 - \phi_{IP}}{\phi_{IP}} \left(\frac{Q_{IP,t}}{C_{IP,t}} \right)^{\rho_{IP}-1}. \quad (11)$$

The Euler equation for the commodity allocation to production is given by

$$S_t = \frac{\partial Y(K_t, Q_{P,t})}{\partial Q_P} = \left(\eta \left(\frac{K_t}{Q_{P,t}} \right)^\nu + (1 - \eta) \right)^{\frac{1}{\nu}-1} (1 - \eta). \quad (12)$$

The Euler equation for the portfolio holdings is given by

$$1 = \mathbb{E}_t \left[M_{t,t+1}^{(IP)} R_{f,t} - 2\nu_1 \frac{B_{IP,t+1}}{e^{\mu_q t}} \right]. \quad (13)$$

If $\rho_{IP} < \nu$ the producer is better able to substitute capital for commodities in production (i.e. K_t vs. $Q_{P,t}$) than she is able to substitute non-commodity goods for commodities in effective consumption (i.e. $C_{IP,t}$ vs. $Q_{IP,t}$). Thus, when faced with a diminished supply of commodities due to, for example, a negative supply shock the producer will use fewer commodities in production and keep the commodity consumption level relatively unchanged. See Johnson (2011) for this line of argument. Furthermore, Johnson (2011) argues that this assumption is justified by the empirical evidence.

2.4.2 Agricultural Producer

The agricultural producer is endowed with the commodity Q . She has to decide on how much of the commodity to consume ($Q_{AP,t}$) and how much to sell to the industrial producer and the commodity consumer. If she has access to the financial market, she also has to decide on how many commodity derivatives to hold over the next period ($n_{AP,t+1}$). Her optimization problem is

$$\max_{\{C_{AP,t}, Q_{AP,t}, n_{AP,t+1}\}} \left\{ (1 - \beta_{AP}) \tilde{C}_{AP,t}^{\frac{1-\gamma_{AP}}{\theta_{AP}}} + \beta_{AP} (\mathbb{E}_t[U_{AP,t+1}^{1-\gamma_{AP}}])^{\frac{1}{\theta_{AP}}} \right\}^{\frac{\theta_{AP}}{1-\gamma_{AP}}},$$

subject to the budget restriction

$$C_{AP,t} + n_{AP,t+1} F_t + \frac{\nu_2}{e^{\mu_q t}} n_{AP,t+1}^2 = (Q_t - Q_{AP,t}) S_t + n_{AP,t} S_t. \quad (14)$$

and futures holdings.

⁹All equilibrium conditions are summarized in Appendix A.6.1.

The effective consumption $\tilde{C}_{AP,t}$ is given by (2).

Analogous to bonds, holding commodity derivatives also entails transaction costs which are given by $\frac{\nu_2}{e^{\mu_q t}} n_{AP,t+1}^2$. They are again a pure technical condition to obtain a well-defined deterministic steady state.

The solution to this problem is derived in Appendix A.2. The most important equilibrium conditions are given in the following two equations.¹⁰ The intratemporal choice between commodity and non-commodity consumption gives

$$S_t = \frac{1 - \phi_{AP}}{\phi_{AP}} \left(\frac{Q_{AP,t}}{C_{AP,t}} \right)^{\rho_{IP}-1}. \quad (15)$$

The Euler equation for the portfolio holdings is given by

$$F_t = \mathbb{E}_t \left[M_{t,t+1}^{(AP)} S_{t+1} - 2\nu_2 \frac{n_{AP,t+1}}{e^{\mu_q t}} \right]. \quad (16)$$

2.4.3 Commodity Consumer

The commodity consumer is endowed with the exogenous wage stream $Z_{CC,t}$. Her budget restriction is given by

$$Z_{CC,t} = Q_{CC,t} S_t. \quad (17)$$

Since the commodity consumer only obtains utility from commodity consumption Q_{CC} and has no access to financial markets, she will use her full endowment to purchase commodities. She has no possibility to actively influence the risk exposure or timing of her consumption. The commodity consumer is thus strongly exposed to spot price risk.

2.4.4 Financial Speculator

The financial speculator is endowed with the exogenous wage stream $Z_{FS,t}$. She has to decide on her consumption of the non-commodity good ($C_{FS,t}$), the amount she invests into the bond ($B_{FS,t+1}$), and the number of commodity derivatives she holds over the next period ($n_{FS,t+1}$). She maximizes her utility

$$\max_{\{C_{FS,t}, B_{FS,t+1}, n_{FS,t+1}\}} \left\{ (1 - \beta_{FS}) C_{FS,t}^{\frac{1-\gamma_{FS}}{\theta_{FS}}} + \beta_{FS} (\mathbb{E}_t[U_{FS,t+1}^{1-\gamma_{FS}}])^{\frac{1}{\theta_{FS}}} \right\}^{\frac{\theta_{FS}}{1-\gamma_{FS}}},$$

subject to the budget restriction

$$C_{FS,t} + B_{FS,t+1} + \frac{\nu_1}{e^{\mu_q t}} B_{FS,t+1}^2 + n_{FS,t+1} F_t + \frac{\nu_2}{e^{\mu_q t}} n_{FS,t+1}^2 = Z_{FS,t} + B_{FS,t} R_{f,t-1} + n_{FS,t} S_t, \quad (18)$$

¹⁰All equilibrium conditions are summarized in Appendix A.6.2.

where she also has to pay transaction costs for bonds ($\frac{\nu_1}{e^{\mu_q t}} B_{FS,t+1}^2$) and for commodity derivatives ($\frac{\nu_2}{e^{\mu_q t}} n_{FS,t+1}^2$).

The solution to this problem is derived in Appendix A.3. The Euler equations for the optimal positions in financial assets are¹¹

$$1 = \mathbb{E}_t \left[M_{t,t+1}^{(FS)} R_{f,t} - 2\nu_1 \frac{B_{FS,t+1}}{e^{\mu_q t}} \right], \quad (19)$$

$$F_t = \mathbb{E}_t \left[M_{t,t+1}^{(FS)} S_{t+1} - 2\nu_2 \frac{n_{FS,t+1}}{e^{\mu_q t}} \right]. \quad (20)$$

The financial speculator derives utility from the non-commodity good only. Without financial markets, she simply consumes her endowment stream. If financial markets exist, she trades the available assets. In particular, she trades commodity derivatives and thus participates in the “futures market” for the commodity to smooth her consumption and share her risks even if she has no interest in physical delivery of the commodity.

2.4.5 Market Clearing

Market clearing in the spot market for the commodity good implies

$$Q_t = Q_{IP,t} + Q_{AP,t} + Q_{P,t} + Q_{CC,t}. \quad (21)$$

Market clearing in the financial market implies

$$B_{IP,t} + B_{FS,t} = 0, \quad (22)$$

$$n_{AP,t} + n_{FS,t} = 0. \quad (23)$$

The traded assets between the two producers and financial speculator are bonds (money market account) on the non-commodity good and the commodity derivative.

The risk-free rate follows from Equations (13) and (19). If transaction costs were zero¹², it would hold that

$$1 = \mathbb{E}_t \left[M_{t,t+1}^{(IP)} R_{f,t} \right] = \mathbb{E}_t \left[M_{t,t+1}^{(FS)} R_{f,t} \right].$$

The price of the commodity derivative follows from Equations (16) and (20). For zero transaction costs, it was

$$F_t = \mathbb{E}_t \left[M_{t,t+1}^{(AP)} S_{t+1} \right] = \mathbb{E}_t \left[M_{t,t+1}^{(FS)} S_{t+1} \right].$$

¹¹All equilibrium conditions are summarized in Appendix A.6.3.

¹²We will set $\nu_1 = \nu_2 = 0.0001$, so that they are very close to zero.

This can be rewritten as

$$F_t = \frac{1}{R_{f,t}} \mathbb{E}_t^{\mathbb{Q}^{(AP)}} [S_{t+1}] = \frac{1}{R_{f,t}} \mathbb{E}_t^{\mathbb{Q}^{(FS)}} [S_{t+1}], \quad (24)$$

where $\mathbb{Q}^{(i)}$ denotes the risk-neutral probability measure of investor i . The price of the commodity derivative is thus proportional to the futures price of a standard one-period futures contract on the commodity.

2.4.6 Formal Definition of the Equilibrium

We now provide a formal definition of the equilibrium in our economy.

Definition 1. *An equilibrium in this economy is a sequence of effective consumption levels $\{\tilde{C}_{i,t}\}_{i=\{IP,AP,FS,CC\}}$, pricing kernels $\{M_{t,t+1}^{(i)}\}_{i=\{IP,AP,FS\}}$, non-commodity consumption levels $\{C_{i,t}\}_{i=\{IP,AP,FS\}}$, commodity quantities $\{Q_{i,t}\}_{i=\{P,IP,AP,CC\}}$, endowments $\{K_t, Q_t, Z_{FS,t}, Z_{CC,t}\}$, non-commodity production output $\{Y_t\}$, prices $\{R_{f,t}, F_t, S_t\}$ and portfolio holdings $\{B_{IP,t}, B_{FS,t}, n_{AP,t}, n_{FS,t}\}$ such that given exogenous shocks $\{\varepsilon_{k,t}, \varepsilon_{q,t}, \varepsilon_{FS,t}, \varepsilon_{CC,t}\}$:*

1. *Each agent chooses effective consumption (2) optimally to maximize lifetime utility (1), and the pricing kernel processes of the industrial producer, agricultural producer and financial speculator are given by (3).*
2. *The (non-commodity good) budget restrictions for the industrial producer (10), the agricultural producer (14), the financial speculator (18) and the commodity consumer (17) apply and non-commodity production output is given by (8).*
3. *The commodity good allocation and the spot price satisfy (11), (12), (15) and (21).*
4. *Bond holdings, commodity derivative holdings, the risk-free rate and the commodity derivative price jointly satisfy (13), (16), (19) and (20).*
5. *The bond and commodity derivative market clearing conditions (22) and (23) are satisfied.*
6. *Agents' endowments follow from (5), (6), (7) and (9).*

3 Results

In this section we explore the quantitative implications of our model. In Section 3.1 we discuss the four different model calibrations considered. In Sections 3.2 to 3.5 these

different models are analyzed in detail. In our discussion we focus on the properties of the spot price, the agents' consumption exposures, and the resulting implications for the volatilities of wealth and consumption growth. The latter two represent our key metrics for the assessment of the welfare effects generated by the opportunity for certain agents to participate in financial markets.

3.1 Calibration

To understand the equilibrium effects of allowing trading in financial markets for heterogeneous agents relative to the case without financial assets, we will analyze four different model specifications. The benchmark or base case model is one without financial markets (see Section 3.2). The second specification is called the IP-AP-FS model. Here IP and FS can trade the bond, while AP and FS can share risk via the commodity derivative (see Section 3.3). Third, in the IP-FS model the bond is available as a financial instrument to the participating agents (see Section 3.4), and finally in the AP-FS model the commodity derivative is traded (see Section 3.5).

The calibration is not targeted to match as many moments as precisely as possible but rather to produce fairly reasonable quantity and asset pricing implications for aggregate quantities, specifically for commodity-related moments, and to allow for the aforementioned analysis of equilibrium effects of financial market trading to be done meaningfully. The model calibration is quarterly and all parameters discussed below are summarized in Table 1, where Panel A reports parameters common to all model specifications and Panel B reports the parameters that are different across these models.

Our model economy intuitively corresponds to a world economy with emerging countries on the one hand and more developed and industrialized countries on the other. In emerging countries agriculture and the production of other commodities is a key sector, whereas developed countries feature a large industrial sector producing refined goods. The commodity in our economy can thus be interpreted as a basket of basic commodities needed for agents in the economy to sustain themselves, i.e. food commodities like rice, soybeans, etc., and energy resources like oil, gas, etc., needed to prepare and store food and also to heat or cool the living premises. The commodity is also used as input into a production process (e.g., energy for manufacturing plants). Most of the population in emerging countries, in our model represented by CC and AP, will depend strongly on a basic commodity (most importantly food), and on the price of the commodity to finance non-commodity consumption in their daily consumption. On the other hand, for people in developed countries, like IP and FS in our model, other more refined goods represent a much larger share of effective consumption.¹³

¹³Note that we do not model exchange rates or any other typical component of a multi-country model. This means that our economy can also be interpreted as representing a single more developed country with a fairly important commodity-producing sector, a larger industrial sector, a large financial sector, and a poor (for example unemployed) fraction of the population which consumes commodity goods and does not buy manufactured products in large amounts. Australia can be seen as an example of such a country.

The exogenous endowment growth rates $\mu_k = \mu_q = \mu_{FS} = \mu_{CC}$ in the economy are set to obtain an annual growth rate of aggregate consumption of 0.019 corresponding to the empirical value for U.S. post-war data. They are identical to ensure balanced growth so that agents do not die out in the long run.

Since the commodity consumer represents a part of the population with very limited resources only able to sustain herself by consuming the commodity, she should account for a rather small fraction of the total economy. Her consumption share is thus targeted to be roughly 2.5%. Since her resources are very limited, she does not have access to any financial market in any model specification considered. Moreover, the financial speculator represents a rather rich part of the population. Therefore, we set the long-run means in the wage endowments to $\bar{z}_{FS} = 0$ and $\bar{z}_{CC} = -2.9$ so that the latter quantity is sufficiently negative to feature a ‘poor’ consumer and a ‘rich’ speculator.

Another very important quantity in our model is the fraction of expenditure spent on buying commodity goods. Seale Jr., Regmi, and Bernstein (2003) report that in high-income countries (among others Canada, France, Germany, the UK, and the U.S.) households spend on average 16.97% of total expenditures on food. For the largest country in this group, i.e. the United States, the value is 9.73%.¹⁴ The commodity good in our model also includes energy. As discussed in Johnson (2011) on page 17, ‘*the expenditure on energy goods and services was 6.4% in the fourth quarter of 1969 [...] and the expenditure on food and energy goods was 20% of consumption*’ in U.S. data. Taken together, we thus target an aggregate fraction of commodity expenditure to total expenditure, \tilde{X}_{aggr} ,¹⁵ of about 15-20% in the model. Hence, the long-run means in the capital and commodity quantity endowments are assumed to be $\bar{k} = 0$ and $\bar{q} = 0.14$, respectively, so that the commodity has the right degree of scarcity in our model.

Finally, the log return volatility of the commodity derivative and the spot price volatility are also very important asset pricing moments in our model. As explained in the context of Equation (24) the price of the commodity derivative is proportional to the futures price and thus the moments for the log return on the commodity derivative in our model can be meaningfully compared to the log futures returns in the data. In Table 2 we compute the averages and volatilities of these log futures returns for various commodities. Moreover, we report the averages and volatilities of spot prices and the ratio between spot price volatility and average spot price for the same commodities. The average annualized log futures return volatility across these commodities is 27.59 percentage points and the average ratio of the annualized spot price volatility of the average spot price is 0.70 using quarterly futures and spot price data for the time period from the first quarter of 1991 until the second quarter of 2008. The remaining parameters, especially the elasticities of intertemporal substitution, the elasticities and weight parameters in the consumption bundles and the production function, to be discussed below also make sure that the model matches these moments well. Moreover, Table 2 reports statistics on the growth rate of

¹⁴In contrast, middle-income countries (among others Argentina, Estonia, Mexico, and Russia) spend 34.69% of total expenditures on food. Low-income countries (among others Albania, Mali, Nepal, and Pakistan) spend on average 52.58% of their expenditures on food.

¹⁵This fraction is defined by $\tilde{X}_{aggr,t} = \frac{S_t(Q_{IP,t} + Q_{AP,t} + Q_{CC,t})}{C_{IP,t} + C_{AP,t} + C_{FS,t} + S_t(Q_{IP,t} + Q_{AP,t} + Q_{CC,t})}$.

commodity producers' sales (specifically, sales of the farming and the oil and gas extraction industry). These statistics will be used to compare our model-implied commodity sales dynamics with the data.

We choose the elasticities $\nu = 0.10$, $\rho_{IP} = -16.5$, and $\rho_{AP} = -16.5$ to be close to the values estimated in Johnson (2011) using a representative agent economy. Next, the weight of capital in the non-commodity production η is set to 0.95 and thus the commodity accounts for 5% of the resources needed to produce the non-commodity good in line with the literature.¹⁶ Furthermore, we set the weight parameters of non-commodity consumption for the industrial producer and the agricultural producer to be $\phi_{IP} = \phi_{AP} = 0.90$. This reflects in our model that the industrial and agricultural producer are consumers who mainly derive their utility from non-commodity goods.

The preference parameters are identical across agents. The value for the discount factor $\beta_i = \sqrt[4]{0.984}$ is standard in magnitude and used by, for example, Kung and Schmid (2015). The risk aversion coefficient γ_i is set to 10 and is thus in the typical range used in the asset pricing literature. The elasticity of intertemporal substitution (EIS) is set to $\psi_i = 0.5$ and is thus in the range of recent empirical studies estimating this elasticity.¹⁷

The correlation between the capital and commodity shock of the producers is chosen to be 0.58, exactly as in Johnson (2011). We choose the correlation of the wage endowments of the financial speculator and the commodity consumer to be far less, but nevertheless positive, i.e. equal to 0.25, to reflect co-movement in wage increases (or decreases) for both the high-income and low-income parts of the population to some extent. All other correlations are set to 0 for parsimony.

Endowment shock volatilities, i.e. $\sigma_k, \sigma_q, \sigma_{FS}, \sigma_{CC}$, are assumed to be identical and set to 0.02. Moreover, the persistence parameters of each shock, i.e. $\varphi_k, \varphi_q, \varphi_{FS}, \varphi_{CC}$, are also assumed to be identical and set to $\sqrt[4]{0.95}$ as in Kung and Schmid (2015). There is thus no heterogeneity in the (exogenous) level of volatility across agents. This allows us to focus exclusively on the equilibrium price and quantity dynamics induced by differences in preferences, in the type of consumption good, in endowments as well as differences in access to financial markets.

Finally, in order to study the different cases of asset availability discussed in the opening paragraph of this section we set the appropriate portfolio holdings $\{B_{IP}, B_{FS}\}$ and/or $\{n_{AP}, n_{FS}\}$ exogenously to 0, when the respective asset is not traded. Implicitly, the transaction costs parameters ν_1 and/or ν_2 are set to ∞ then. If the asset is traded,

¹⁶Casassus, Collin-Dufresne, and Routledge (2009) and Wei (2003) use an oil or energy share of 0.04 in their production functions. Hence, besides matching the 'standard' labor share of about 0.65-0.70 in production this allows their models to match the energy-labor ratio of 0.05 in empirical data.

¹⁷Vissing-Jørgensen (2002) provides an estimation of the EIS using US household data suggesting that the EIS for households holding assets is significantly positive and below 1. Specifically, she estimates that stockholders have an EIS of around 0.3-0.4 and bondholders of around 0.8-1.0 and that the EIS increases in the size of asset holdings. Moreover, we follow most of the macroeconomics literature in assuming that the EIS is below 1. The literature that combines investment-specific shocks and the analysis of (cross-sectional) asset prices also provides empirical and theoretical justification for an EIS below 1 as discussed by, for example, Papanikolaou (2011).

the transaction costs are assumed to be very small. We set $\nu_1 = \nu_2 = 0.0001$, in close proximity to the value used in Judd and Mertens (2013).

The model is solved using third-order perturbations around the stochastic steady state in Dynare++ 4.4.3. The moments are computed using a simulation of 1,000 economies at quarterly frequency for 500 quarters, from which the first 100 quarters are not considered for the calculation of the moments (‘burn in-period’). Furthermore, impulse response functions are depicted for a length of 20 quarters after an exogenously given positive one standard deviation endowment shock.

3.2 Model without Financial Markets

This section analyzes the baseline model. The agents can trade in the spot market for commodities only, but have access to neither bonds nor commodity derivatives. Analyzing this model setup first allows us to discuss the equilibrium impact of adding financial markets for spot price dynamics and welfare to the economy subsequently. Specifically, by doing this comparison we can derive implications how trading certain kinds of risks affects agents’ consumption and wealth growth volatilities (our main measures for welfare) and the spot price volatility. We will first look briefly at important first and second moments of this baseline model before later turning to analyzing the transmission of agents’ endowment shocks through the economy.

Table 3 reports the first and second moments of growth rates. The average aggregate consumption growth rate is 0.019 and coincides with the growth rates of endowment which are set to 0.019 in the calibration. The volatility of aggregate consumption growth is 3.4 percentage points, and is smaller than the volatility of endowments, which has been set to 0.04 (quarterly: 0.02) for all agents. The same holds true for the volatility of production output which amounts to 0.039. Agent-specific consumption growth rates, however, have volatilities between 4 and 11.3 percentage points and can thus significantly exceed the volatilities of endowment.

As reported in Table 4, 16.1% of total consumption expenditure is spent on the commodity good. This is well in line with the empirical evidence for developed economies that around 17% of total expenditure is spent on food and that around 20% is spent on food and energy in the United States. Furthermore, in our model calibration, about 53% of the commodity is consumed by the industrial producer (IP), about 25% by the agricultural producer (AP) and about 13% by the commodity consumer (CC). The remaining 9% is used in the production process by IP.

Table 4 also gives the consumption shares of the agents. As targeted in the calibration, the consumption share of CC is low and amounts to 2.4%. The financial speculator (FS) accounts for the largest fraction of total consumption in the economy, closely followed by IP. These two agents account for almost 80% of total consumption. AP is responsible for about 18% of total consumption.

The first two moments of the spot price are shown in Table 5. The volatility of the spot price is 0.279 and thus significantly exceeds the volatilities of endowment. The

ratio of the spot price volatility to the average spot price is 0.78 and thus in line with the empirical counterpart reported in Table 2. Moreover, the total commodity sales of AP are equal to $S_t(Q_t - Q_{AP})$ and its growth rate as reported in Table 3 displays an annual volatility of almost 0.07, hence only falling slightly short of the empirical value of 0.11 reported in Table 2. To understand the spot price behavior, we look at impulse response functions. Figure 2 depicts the reaction of the spot price to shocks in capital (Panel A), in the commodity supply (Panel B), and in the endowment streams of the financial speculator (Panel C) and the commodity consumer (Panel D). In addition to the reactions of the spot price, each panel also reports the reactions of the commodity derivative price, the risk-free rate and the return on the commodity derivative.¹⁸ Since the persistence parameters for all four endowment shocks are set to be identical (i.e. $\varphi_k = \varphi_q = \varphi_{FS} = \varphi_{CC}$), the time it takes the economy to return to the steady state is very similar across all impulse response functions. In the remainder of this study, we will thus concentrate on discussing the initial first-period impact of a shock which provides sufficient information given our calibration. Since the persistence is quite high (annually: 0.95), it takes the economy a quite large number of quarters to return to the steady state.

In general, the spot price of the commodity increases in the relative scarcity of the commodity, as measured by C_{IP}/Q_{IP} , K/Q_P , and C_{AP}/Q_{AP} in Equations (11), (12) and (15).¹⁹ Figure 3 depicts the impulse response functions of non-commodity consumption C_i and commodity consumption Q_i for $i \in \{AP, IP\}$, Figure 4 depicts the impulse response functions of the amount Q_P used for production. Following Johnson (2011), we assume $\rho_{IP} = \rho_{AP} < 0 < \nu$, i.e. the elasticity in the consumption bundle is lower than the elasticity in the production function. The economy is thus better able to substitute capital for commodities in production than to substitute the non-commodity good for the commodity good in consumption. Shocks to capital and to the endowments of the investors will thus have a large impact on the amount of the commodity used for production. The large changes in K/Q_P and thus also in the spot price result in a spot price volatility of 0.279 (see Table 5).²⁰

In response to a positive capital shock, the investors will use less of the commodity for production (partly offsetting the increase in the non-commodity good from higher capital) and more for consumption (Figure 4, Panel A). The relative scarcity K/Q_P of the commodity increases, which implies that the spot price of the commodity increases as well. A positive shock to CC's endowment has a similar effect. It also leads to a larger supply of the non-commodity good and a larger demand for the commodity good used for consumption. The amount of the commodity used for production decreases. The higher scarcity of the commodity good leads to an increase of the spot price.

¹⁸The other figures containing impulse response functions have a similar structure. In them, each panel also reports four impulse response functions which look at the impact for agent-specific quantities.

¹⁹Note that the relative scarcity is not exogenously given, but follows from the agents' optimal decisions concerning how much of the commodity to buy and how much then to use for production (in case of IP), and concerning how much of the commodity to sell in exchange for the non-commodity (in case of AP).

²⁰The relations C_{IP}/Q_{IP} and C_{AP}/Q_{AP} between the commodity and the non-commodity good used for consumption are much more stable. The low elasticities of substitution, however, imply that even small changes in these ratios induce large changes in the spot price.

A positive shock to the commodity supply increases the amount of the commodity used for production. The relative scarcity K/Q_P of the commodity decreases, and the spot price drops. Note that an increase in the commodity endowment by 2% induces a much larger decrease in the spot price of more than 6%. Finally, shocks to the endowment of the financial speculator have no impact on the spot price in the economy without financial markets. The speculator only derives utility from the non-commodity good and will thus not participate in the commodity spot market.

Next, we analyze the reaction of agents' effective consumption levels to the four endowment shocks. Figure 5 depicts the impulse response functions for effective consumption of all four agents.

First, FS is only affected by her own endowment shocks as she lives in financial and trade autarky in this model specification. Second, in response to capital shocks both producers can increase their consumption. The increased supply of the capital good allows IP to produce more non-commodity consumption goods. Hence IP consumes more of those and can also buy more commodity goods from AP despite the higher spot price. The increased scarcity of the commodity good and the resulting higher spot price benefits AP since she obtains more non-commodity goods from even selling a smaller fraction of her commodity endowment. She can thus increase both non-commodity and commodity good consumption as well. Due to the increased spot price, CC has to reduce her consumption.

Third and very interestingly, the consumption of both the non-commodity and commodity good for AP decreases in response to commodity endowment shocks (see Figure 3, Panel B). Hence, effective consumption also decreases (see Figure 5, Panel B). Although AP now has more of her own endowment, the negative price reaction in the spot market is so severe that she can afford less non-commodity consumption and even has to sell more of her commodity good to finance her non-commodity good consumption. This effect is similar to what is referred to in the literature as 'immiserizing growth'²¹, i.e., a higher endowment and thus higher endowment growth lead to lower consumption and lower utility for AP. Moreover, this effect is absent in the representative agent model of Johnson (2011). In his paper both the capital and commodity endowment are given to the same agent. However, in our model the agent with the commodity endowment (i.e. the agricultural producer) is dependent on the commodity spot market and thus on the drivers of the spot price. Hence, the elasticity spread advertised by Johnson (2011) in our model gives rise to this additional, at first surprising, result.²² Furthermore, the result hinges exclusively on the condition $\rho_{AP}, \rho_{IP} < \nu$, i.e. investors being better able to substitute capital for commodities in production rather than willing to substitute the non-commodity good for the commodity good in consumption. When the spread between these elasticities becomes smaller, AP suffers less from commodity shocks.²³ In contrast to

²¹See, e.g., the studies by Bhagwati (1958, 1968), Brecher and Díaz Alejandro (1977), Khan (1982) and Beladi and Naqvi (1988).

²²This result also relates to the recent evidence on the link between oil price dynamics and the economic prospects of countries with a large oil-producing sector. Due to the over-supply of oil, oil prices are extremely low at the moment. This has led to adverse economic conditions in countries for which the revenues from oil determine a large share of GDP, such as Norway, Russia, Saudi Arabia, and Venezuela.

²³Using unreported impulse response functions we also checked that for $\rho_{AP} = \rho_{IP} = \nu = 0.1$ the

this, both IP and CC benefit greatly from the large drop in the spot price. They can buy more commodity goods for production of the non-commodity good and for consumption. However, the agricultural producer suffers quite severely in the case when both ρ_{AP} and ρ_{IP} are smaller than ν .

Finally, positive shocks to the commodity consumer's endowment allow CC to buy more commodity goods for consumption. Since the demand for the commodity good increases, the spot price increases. This is in turn also beneficial for AP. IP consumes less as she needs to pay more for the commodity and thus can also produce less non-commodity consumption good units.

Note that the allocation of the commodity to consumption and production, the consumption of the agents, and the spot price are only driven by the weights of the commodity and the non-commodity good, and the elasticity of substitution between them. Neither relative risk aversion γ nor EIS ψ have any impact on the equilibrium quantities and the spot price. The reason is that the investors can only trade in the spot market for the commodity. Without financial assets, they can not shift consumption over time or deviate from the risk of their endowment.

At this point it is worth relating our results to the results of Johnson (2011). His model is able to endogenously create high commodity spot prices by assuming that the substitution elasticity in the consumption bundle is much lower than the one in the production function. He studies the welfare effects of (large) commodity price changes and finds that very large commodity spot prices are considerably welfare-reducing for the representative agent, but that moderately higher commodity prices are welfare-enhancing. Moreover, increasing the elasticity in production leads to higher welfare. The changes in welfare are induced by changes in the effective consumption volatility. We are also interested in the welfare effects induced by changes in consumption volatility. However, we compare heterogeneous agent economies with different financial market structures to study the differential effects on welfare in a model where high commodity spot prices can occur endogenously. Specifically, under which circumstances can financial markets be useful for hedging against these high commodity spot prices and thus increasing (decreasing) welfare (consumption volatilities)?

In the next sections we will analyze different versions of the model when certain agents have access to financial markets. As stated above CC will never be able to participate. We first consider the case when all the other three agents can trade financial instruments (denoted as the IP-AP-FS model). The IP-FS model and the AP-FS model follow.

3.3 IP-AP-FS Model

In this version of the model all agents except CC have access to financial instruments. In particular we allow IP and FS to trade the bond, while AP and FS can trade the agricultural producer can profit from her own commodity endowment shock.

commodity derivative.²⁴

In terms of financial quantities, the return volatility of the commodity derivative is about 11.3 percentage points in this model (see Table 5). Hence, the model explains about 40% of the empirically observed futures return volatility as shown in Table 2. The risk-free rate is a little bit on the high side with around 0.045 and very stable over time with a volatility of about 0.76 percentage points.

The welfare implications of financial market trading are our main focus. So as a first step we look at the agents' wealth levels. They are reported in Table 6, stated in units of the respective effective consumption. Since the consumption bundles of the agents differ from each other, these levels of wealth can not be compared across agents, with the exception of IP and AP, who have the same consumption bundle. In line with the findings for consumption expenditures, IP's wealth level is about twice as big as AP's.

Compared to the baseline model trading in the financial market has a rather small impact on the levels of wealth and effective consumption. The more appropriate metric in our view, however, are not levels, but the volatilities of growth rates for both wealth and consumption, and here the effects are indeed much stronger and clearer (see the lower panel of Table 6). The possibility to trade in financial markets is beneficial for all agents in the sense that these volatilities decline. AP benefits most with a reduction in the volatility of effective consumption growth from 0.0676 to 0.0489, which represents a sizable difference. The very important result here is that consumption volatility declines even for the non-participating CC, albeit to a smaller degree, but one can see here that financial markets indeed have the potential to be uniformly welfare improving.

We will mainly focus on the volatilities of consumption and wealth to assess the welfare implications of trading on financial markets. It also holds for the other cases we analyze that the levels of consumption do not change significantly, so that it is mainly volatility which drives the agents' welfare. Furthermore, since the endowment volatilities are given exogenously, our agents mainly care about the sharing of these exogenous risks via the financial and the commodity market. Volatilities of individual consumption are thus central in our model, which again motivates their use to assess the welfare implications of trading.

To put the above result into perspective, note that it is not clear a priori whether indeed *all* investors will benefit from the existence of financial markets. IP, AP, and CC all trade in the spot market. If trading on the financial market has an impact on the spot price, these agents will be affected. Even if they refrain from trading themselves, their utility with financial markets will be different from their utility without financial markets. The latter is thus no longer a lower bound for their utility which they can enforce by simply not participating in the financial market, and so it is an open question whether they are better or worse off with financial markets than without.

The situation is very different for FS. She does not participate in the spot market and thus lives in autarky when access to financial markets is not possible. Should she

²⁴The results of a sensitivity analysis with respect to the set of assets available to the agents are presented below in Section 3.6.

not benefit from using the financial market to share her risks with the other agents in the economy, she would simply choose not to participate, which would leave her utility unchanged relative to the benchmark case.

A more detailed picture of the impact of financial markets on consumption is presented in Figure 5, which shows the impulse response functions for the agents' effective consumption with respect to different shocks. The asset positions of the agents are given in Table 7.

AP uses a short position in commodity derivatives to hedge against a low spot price. She reduces her exposure to capital shocks, commodity shocks, and CC's endowment shocks in absolute terms (Figure 5, Panels A, B and D), but in turn accepts a positive exposure to FS's endowment shocks (Panel C). IP can significantly reduce her exposure to shocks in q (the driver of commodity supply) and shocks in z_{CC} (i.e. CC's endowment) in absolute terms. In turn, she has to accept a positive exposure to FS's endowment shocks. Her exposure to shocks in her own capital, however, increases slightly. FS can reduce her exposure to her own endowment shocks. She takes on a positive exposure to capital shocks (providing insurance to the agricultural producer who can reduce her exposure). Her exposure to shocks in the commodity supply and CC's endowment is rather small. For these two shocks, she basically acts as an intermediary between AP and IP and allows these two agents to share their risks. Since the reductions in AP's and IP's exposures almost offset each other, there is no need for FS to take a significant exposure herself.

Finally, CC is affected indirectly by the financial market via changes in the spot price. She can reduce her very negative exposure to capital shocks significantly, while her exposure to shocks in FS's endowment changes from zero in the benchmark case to a strongly negative number. Her exposure to her own endowment increases slightly, since she benefits from a smaller increase of the spot price. Her exposure to commodity supply shocks basically does not change.

Figure 2 depicts the reaction of the spot price to shocks. As in the benchmark model the spot price increases with the relative scarcity of the commodity. It thus increases after positive shocks to capital and to the endowments of FS and CC, while it decreases in reaction to positive commodity supply shocks.

Compared to the benchmark case, there is now a positive exposure of the commodity spot price to shocks in FS's endowment. The exposure to capital shocks, which are now also shared by FS, is reduced in absolute terms. The exposure to shocks in the commodity supply and CC's endowment, for which FS does not participate in risk sharing, but mainly acts as an intermediary between IP and AP, remains basically the same.

Overall, the volatility of the spot price is the same with and without trading (see Table 5). Although risk sharing between the agents decreases the exposure of the price to capital shocks, it also implies an exposure to FS's endowment shocks. Overall, the two effects offset each other, and together with the fact that the exposure to commodity shocks and CC's endowment shocks basically does not change, this implies that spot price volatility does not change.

Note that the results for the volatilities of consumption and of the spot price are

different from each other. While the volatilities of consumption decrease, the volatility of the spot price basically does not change. There is thus no causal link between spot price volatility and welfare implications, but trading in financial markets can have different effects on these volatilities.

In Table 8, we report the results of a sensitivity analysis with respect to the preference parameters, i.e., we consider the impact of the elasticity of intertemporal substitution (EIS) and relative risk aversion (RRA) on the equilibrium outcome of the IP-AP-FS model.

A higher EIS implies that the agents care less about smoothing (effective) consumption and use financial markets more strongly when investment opportunities are favorable. It is still as important as before, however, for the agents to have a stable ratio between non-commodity consumption C_i and commodity-consumption Q_i . Hence, financial markets can be expected to be used more strongly to obtain (even more) stable C/Q ratios. Thus the spot price volatility should be decreasing in EIS. This is confirmed by the results when we set the EIS to 0.75 for all agents. In turn consumption growth volatilities increase slightly for all agents except CC. Finally, the volatilities of wealth growth rates decrease, since with ψ closer to 1 the volatility of the wealth-consumption ratio and thus also the volatility of wealth decrease (note that the wealth-consumption ratio would be constant for $\psi = 1$).

In terms of risk aversion we consider the alternative case of $\gamma = 15$ for all agents. The results are presented in the last column of Table 8. The spot price volatility decreases slightly, and consumption volatilities remain more or less unchanged. In particular the small impact on consumption volatilities shows that the agents are already close to the maximal amount of risk sharing for $\gamma = 10$, so that increasing risk aversion further does not change their positions much.

3.4 IP-FS Model

In the IP-FS model the traded asset is the bond, i.e., there is no commodity derivative, and AP does not have access to financial markets. We now first look at the impact on the spot price (see Figure 2), before we turn to the agents' consumption exposures.

The key result here is that the volatility of the spot price increases relative to the benchmark case without financial markets, which may seem counterintuitive at first, since compared to the situation without any financial markets at least some agents now have better opportunities to share risk. In terms of the mechanism, the exposure of the spot price to capital shocks is reduced (Figure 2, Panel A), but at the same time the exposure to commodity shocks and CC's endowment shocks increases in absolute terms (Panels B and D). Furthermore, the spot price increases after a positive shock to FS's endowment (Panel C). In total, spot price volatility increases substantially from 0.2791 to 0.3567 (see Table 5).

Capital shocks are shared between IP and FS, which implies that these shocks have a smaller impact on the relative scarcity of the commodity and a lower impact on the

spot price. AP and CC do not participate in the financial markets and are thus not able to actively share their risks. In the spot market for the commodity, they face IP, who benefits from risk-sharing with the FS and is overall in a better position to determine the terms of trade. The impact of commodity shocks and CC's endowment shocks on the relative scarcity of the commodity is therefore larger than in the benchmark case, which implies a larger price reaction to shocks.

Now we turn to study the impact of bond trading between IP and FS on effective consumption exposures in Figure 5. Compared to the benchmark case without financial markets IP reduces her exposure to her own capital shocks and now takes a positive exposure to shocks in FS's endowment. Her exposure to commodity supply shocks and CC's endowment shocks only goes down a little. FS's exposures are in a sense the mirror images of IP's, i.e., she reduces the exposure to her own shocks and takes the opposite position compared to IP with respect to the other shocks.

Since AP and CC do not participate in the financial market, the change in their exposure to shocks as compared to the benchmark case depends exclusively on the changes in the sensitivity of the commodity spot price. The exposure to capital shocks thus decreases in absolute terms for both agents, while the exposure to commodity shocks and FS's endowment shocks increases. The exposure to CC's endowment shocks also increases for AP. For CC, the exposure to her own endowment shock decreases, since she suffers from the higher spot price after a positive shock.

The volatilities of consumption and wealth are again given in Table 6. The numbers show that the two agents who participate in the financial market benefit from trading. For FS the volatility of consumption and of wealth is smallest in this market setup across all versions of the model. For IP the volatilities decrease slightly compared to the benchmark case. However, now the non-participating agents AP and CC no longer profit from the presence of financial markets. In particular CC suffers from the larger spot price volatility, and now faces a consumption volatility of 0.13 as compared to 0.1126 in the benchmark case, i.e., a relative increase of more than 15%.

3.5 AP-FS Model

When only AP and FS are active on the financial market, we assume they trade the commodity derivative, i.e., there is no bond.

Looking first at the impact of the change in setup on the spot price (see Figure 2) relative to the base case, we see that the exposures of the spot price to capital shocks, commodity shocks, and shocks to CC's endowment all decrease in absolute terms. In turn, there is now a positive exposure to shocks in FS's endowment.

AP directly shares the risk of adverse changes in commodity scarcity with FS. This implies that the impact of these shocks decreases, which in turns leads to a reduced exposure of the spot price. This reduces the volatility of the spot price considerably from 0.2791 to 0.2367 relative to the benchmark case (see Table 5). In contrast, trading between

FS and IP, who is only indirectly affected by commodity shocks, results in a much higher volatility (see Section 3.4). When both IP and AP trade with FS in financial markets, IP's and AP's impact offset each other, and the volatility is basically the same as with no trading in financial markets (see Section 3.3).

Comparing the AP-FS to the IP-AP-FS model shows that the exposure of the spot price to capital shocks increases when only two agents trade, while the exposures to all other shocks decrease in absolute terms. In particular, the exposure to shocks in FS's endowment is now less than half compared to the IP-AP-FS model. The intuition is that IP's actions have the largest impact on the spot price, since she is responsible for determining the supply of the non-commodity good via production. So when IP is allowed to trade with FS to determine the conditions for the supply of the non-commodity good via the bond market, shocks to FS's endowment affect this supply significantly via the bond trading channel, and hence they affect the amount of the non-commodity good that IP would like to produce. In this way, FS's endowment shocks add to the spot price volatility to a larger extent when FS can additionally trade with IP and not just with AP. This effect is absent when only the commodity derivative is traded, so that shocks to FS's endowment only play a minor role for the spot price.

The agents' consumption exposures are the driving forces behind consumption growth volatilities (see Figure 5 for these exposures). AP takes a short position in the commodity derivative to hedge against a low spot price. Compared to the base case she reduces her exposure to shocks to capital, to the commodity supply and to CC's endowment in absolute terms, but takes on a positive exposure to FS's endowment shock. The reduction in exposures is even slightly larger than in the IP-AP-FS model, so that the resulting consumption volatility is also a little bit lower. FS's consumption has a slightly lower exposure to her own endowment shocks and takes the other side of AP's position for the other shocks, i.e., her exposure to both capital shocks and CC's endowment shocks is positive, while it is negative to commodity supply shocks.

The reduced exposure of the spot price to capital shocks implies that IP's consumption is now more exposed to capital shocks, but less so to commodity shocks and CC's endowment shocks. The exposure to FS's endowment shocks is negative, which altogether results in slightly lower consumption and wealth volatilities.

For CC as a non-participating agent consumption is now less exposed to capital and commodity shocks (due to the reduced reaction of the spot price to these sources of risk), while her exposure to her own endowment shocks increases (caused by a less pronounced adverse spot price reaction). The exposure to FS's endowment shocks is negative, and in total CC benefits from a decrease in the volatility of the spot price with a reduction in her consumption volatility from 0.1126 to 0.0997.

In terms of welfare implications the trading of the commodity derivative between AP and FS reduces the volatility of consumption growth and wealth for all agents relative to the base case. The main reasons are the opportunity to share risk with respect to the commodity supply and the resulting lower volatility of the spot price. On the other hand, when only IP and FS can trade in the financial market, the volatilities of wealth increase

for the other two agents (see Tables 3 and 6).

3.6 Additional Analyses

This section provides a number of additional analyses to assess the robustness of our results with respect to alternative specifications of the model and to shed further light on the economic mechanisms behind its main implications.

Since, up to now, we had fixed the set of financial instruments available to the agents in the different versions of the model, we now investigate the robustness of our findings with respect to this feature. Table 9 presents the results for the cases in which IP and FS trade the commodity derivative, and AP and FS trade the bond.

The main take-away from the table is that who is trading with whom matters much more than what is being traded. When all agents except CC are active on financial markets the results remain qualitatively unchanged compared to the findings presented in Section 3.3. Most importantly, it is still true that all agents benefit from the existence of financial markets. The reduction in consumption volatility is now considerably larger for AP and CC, while it is smaller for the other two agents.

Similarly, when IP and FS trade, spot price volatility is almost identical to the value reported in Table 5 for the old asset menu. It increases from around 0.28 in the benchmark case without financial markets to around 0.35 (0.36 for the old asset menu). When AP and FS trade, the spot price volatility drops to around 0.23-0.24 (as before). The sign of the reaction of spot price volatility thus does not depend on which assets are traded, but it depends on who trades. The same basically holds true for the size of the reaction: the impact of the kind of asset is much smaller than the impact of the groups of investors participating in financial markets.

When it comes to the welfare implications, the type of assets available to the agents can be more important, as already seen in the IP-AP-FS model. When only AP and FS trade, AP's consumption volatility is much lower when these two agents trade the bond than when they trade the commodity derivative. CC is also slightly better off in this case, while IP and FS are slightly better off when the commodity derivative is traded. In the IP-FS model the trading agents are better off when the bond is available to them, while AP and CC are better off when the commodity derivative is traded.

When IP and FS trade with each other, IP lacks the possibility to share risk with AP. So when IP trades the bond with FS, she tailors the product to AP's endowment exposure, so that the agents trading in the financial market also profit from indirect risk sharing with AP via the spot market.

AP really benefits from trading the bond with FS. In particular, the exposure of AP's effective consumption to commodity shocks and capital shocks is smaller in absolute terms when she trades the bond than when she trades the commodity derivative. AP thus prefers having a certain lower bound on her non-commodity consumption to being less

exposed to (now less extreme) spot price changes. For the other agents, however, the type of financial instrument which AP and FS actually trade is not so relevant.

Next, Table 10 presents the results for the special case of identical substitution elasticities in consumption and production, i.e., for $\rho_{AP} = \rho_{IP} = \nu = 0.1$. Now, in contrast to the cases discussed so far, AP can profit from her own commodity shock. There is generally more flexibility in the economy, which makes consumption and wealth volatilities decrease. The spot price is also less responsive to endowment shocks. In the case of identical elasticities an important friction in the commodity price dynamics is absent in the sense that now quantity and price dynamics can offset each other exactly. This is also the reason why we see a constant aggregate consumption growth volatility across the different versions of the model.

Additionally, Table 11 reports results for a lower persistence of endowment shocks, i.e., for $\varphi_k = \varphi_q = \varphi_{FS} = \varphi_{CC} = 0.85^{1/4}$ instead of $0.95^{1/4}$ in the original calibration. Most importantly, our results are very robust with respect to a decline in the persistence of endowment shocks. Qualitatively, nothing changes when comparing the different asset menu availabilities in this new calibration, i.e. financial market trading between AP and FS is overall welfare-improving, whereas trading between IP and FS is only beneficial to the participating agents. Moreover, the volatilities of consumption and wealth do not change dramatically. The only large difference is in the spot price volatility which drops to around 0.145 in the benchmark case due to reducing the long-run impact of endowment shocks on the spot price.

Furthermore, we study to what degree the exact specification of preferences matters for our results. General EZ preferences contain time-separable CRRA utility as a special case when the degree of relative risk aversion is equal to the inverse of the EIS. Table 12 reports the results for the case when agents have CRRA preferences, where we keep the elasticity of intertemporal substitution unchanged at $\psi_i = 0.5$ and set $\gamma_i = 2$ for all agents. The results basically do not change as compared to the benchmark calibration (see Table 7). The agents mainly trade to smooth consumption over time, so that the EIS matters much more than the RRA. This finding is also in line with the results of our previous sensitivity analysis with respect to preference parameters in the IP-AP-FS model, reported in Table 8.

Finally, we look at an extension of the IP-AP-FS model, in which these three agents can now trade both the bond and the commodity derivative. This extends the asset menu for IP and AP and allows them to trade with each other directly. The results are reported in Table 13, where we use the same persistence of endowment shocks $\varphi_i = 0.85^{1/4}$ as in Table 11. The asset positions in the upper panel show that AP and IP do indeed make use of the opportunity to trade with each other directly. The absolute positions of AP and IP increase, while the absolute position of FS decreases, since there is now less need for FS to act as an intermediary between IP and AP. Risk sharing becomes easier, and the volatility of the spot price drops from 14.89 to 12.01 percentage points. The impact on the consumption and wealth volatilities is more diverse. IP profits from the larger asset menu in that her consumption volatility drops by 0.5 percentage points. The same holds true for the commodity consumer, who profits from the lower volatility of the spot price.

For FS, the volatility of consumption increases slightly. The largest change, however, is observed for AP, for whom the volatility increases from 3.60 to 6.48 percentage points and is only slightly less than in the benchmark case with no financial markets at all. Again, giving more trading possibilities to IP in particular is harmful for AP.

4 Summary and Conclusion

Motivated by the discussion of the potentially harmful implications of commodity trading for financial (instead of fundamental) reasons we have analyzed an economy populated by heterogeneous agents who differ in their access to financial markets. This access is basically always beneficial for the agents participating in the market, i.e., it reduces the volatility of their consumption stream. We assume throughout the analysis that the agent who is only interested in consumption of the commodity does not have access to these markets, since we expect this to be the case for large parts of the population in emerging economies, which depend heavily on basic agricultural commodities.

As we show in the analysis of our model, it is very important for the welfare implications of financial markets which investors trade in these markets and which do not participate, and we analyze a number of special cases with respect to the group of participating agents. For example, without going into the details of the mechanism again, in the case where only the agricultural producer and the financial speculator trade, the other two agents also benefit in the sense that their consumption volatilities decrease. If, on the other hand, the industrial producer and the financial speculator interact on the financial markets, the other two investors are worse off, i.e., their consumption volatilities go up. Participation of the agricultural producer in financial markets is beneficial for all investors who trade in the spot market, while the opposite is true when the industrial producer has access to those markets. Generally, the distinction between agricultural and industrial producers turns out to be important. Even if they have the same preferences, it matters that one is endowed with capital and the other with commodities. Finally, the presence of the financial speculator is beneficial for the investors she trades with. The effect on the non-participating investors depends on whom she trades with.

We also investigate whether it matters which particular assets are available to the agents. We find that it does, but only to a limited degree, in the sense that the size of the various effects varies, but the signs are robust and remain the same across the different specifications.

An important driver of our results are different elasticities of substitution between capital and the commodity in the production process on the one hand and between the commodity and the other good in some agents' consumption bundle on the other hand. Our choice of parameters with the substitution elasticity in the consumption bundle being less than that between capital and the commodity implies that when the commodity is scarce relative to the non-commodity good the spot price goes up (see also Johnson (2011)). This has the very interesting consequence that the agricultural producer's con-

sumption decreases in response to commodity shocks. Although she now has more of her own endowment available, the negative price reaction in the spot market is so severe that she can ultimately afford even less consumption. This effect is known as ‘immiserizing growth’, when a larger endowment or growth rate leads to lower utility. The higher the spread between the substitution elasticities, the more severe is this effect. In the case of equal elasticities, where we obtain the intuitive result that the agricultural producer can profit from her own commodity shock, financial markets have much less of an impact on consumption volatilities.

Overall, our analysis provides new insights on the role of financial markets in a setting where agents have different consumption bundles and where one group of agents in particular does not have the chance to smooth consumption over time and states via trading products like commodity derivatives, stocks, or bonds. In a sense our results show that efforts to allow a larger share of the population to participate in financial markets always have the potential to improve welfare, but that this is not automatically the case, i.e., the details matter.

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Table 1: Parameters

(a) Panel A: Common parameters across models

Preferences											
β_{IP}	β_{AP}	β_{FS}	β_{CC}	γ_{IP}	γ_{AP}	γ_{FS}	γ_{CC}	ψ_{IP}	ψ_{AP}	ψ_{FS}	ψ_{CC}
0.996	0.996	0.996	0.996	10	10	10	10	0.50	0.50	0.50	0.50
Weights and elasticities											
ρ_{IP}	ρ_{AP}	ϕ_{IP}	ϕ_{AP}	ν	η						
-16.5	-16.5	0.90	0.90	0.10	0.95						
Endowment growth rates and long-run means											
μ_k	μ_q	μ_{FS}	μ_{CC}	\bar{k}	\bar{q}	\bar{z}_{FS}	\bar{z}_{CC}				
0.019/4	0.019/4	0.019/4	0.019/4	0	0.14	0	-2.9				
Endowment shock volatilities and persistences											
σ_k	σ_q	σ_{FS}	σ_{CC}	φ_k	φ_q	φ_{FS}	φ_{CC}				
0.020	0.020	0.020	0.020	$0.95^{1/4}$	$0.95^{1/4}$	$0.95^{1/4}$	$0.95^{1/4}$				
Endowment shock correlations											
$\text{corr}(\varepsilon_k, \varepsilon_q)$	$\text{corr}(\varepsilon_k, \varepsilon_{FS})$	$\text{corr}(\varepsilon_k, \varepsilon_{CC})$	$\text{corr}(\varepsilon_q, \varepsilon_{FS})$	$\text{corr}(\varepsilon_q, \varepsilon_{CC})$	$\text{corr}(\varepsilon_{FS}, \varepsilon_{CC})$						
0.58	0	0	0	0	0.25						

(b) Panel B: Parameters differing across models

	Benchmark	IP-AP-FS	IP-FS	AP-FS
ν_1	∞	0.0001	0.0001	∞
ν_2	∞	0.0001	∞	0.0001

This table reports the quarterly benchmark calibration for our model used to compute the moments in the other tables. Four model specifications are considered: the benchmark model without financial markets; the model where the bond is traded between IP and FS and the commodity derivative is traded between AP and FS (denoted by IP-AP-FS); the model where just the bond is traded between IP and FS (denoted by IP-FS); and the model where just the commodity derivative is traded between AP and FS (denoted by AP-FS). Panel A reports the parameters common to all models, while Panel B shows the parameters differing across the model specifications.

Table 2: Futures returns, spot prices, and commodity sales (data)

(a) Panel A: Futures returns and spot prices

	$\mathbf{E}[\mathbf{r}_{\mathbf{F},t}]$	$\mathbf{E}[\mathbf{S}_t]$	
Crude Oil	9.11	59.30	
Wheat	6.71	278.82	
Soybeans	6.02	237.96	
Oats	7.96	2.82	
Rough Rice	6.00	278.82	
Corn	6.52	116.20	
Mean	7.05	—	
	$\sigma_{\mathbf{r}_{\mathbf{F},t}}$	$\sigma_{\mathbf{S}_t}$	$\sigma_{\mathbf{S}_t}/\mathbf{E}[\mathbf{S}_t]$
Crude Oil	28.71	87.03	1.47
Wheat	25.95	119.94	0.73
Soybeans	25.20	139.66	0.59
Oats	25.59	0.39	0.14
Rough Rice	31.19	198.82	0.71
Corn	28.89	70.31	0.61
Mean	27.59	—	0.70

(b) Panel B: Commodity sales growth rates in U.S. data

	$\mathbf{E}[\Delta\Pi_t]$	$\sigma_{\Delta\Pi_t}$
Farming	0.05	0.09
Oil and gas extraction	0.07	0.20
Total sales	0.06	0.11

This table reports the means and volatilities for log futures returns and for spot prices of various commodities using quarterly data between 1Q/1991 and 2Q/2008 in Panel A. Moreover, the ratio of the spot price volatility to the mean of the spot price is calculated for each commodity. Panel B reports statistics on the annual total commodity sales growth rate $\Delta\Pi_t$ of the farming industry, the oil and gas extraction industry, and the growth rate of the sum of these two industries' sales, respectively, in U.S. data for the period 1963–2014. The log return of a futures contract with settlement price P_t between time $t-1$ and t is defined as $r_{F,t} = \log(P_t) - \log(P_{t-1})$. The spot price is denoted by S_t . The futures and spot price data have been downloaded from <http://www.quandl.com/futures>. The data for commodity sales of commodity producers are from the industry input-output account data (make tables) from the Bureau of Economic Analysis (BEA) and available at http://www.bea.gov/industry/io_annual.htm. Moments reported are measured in percentage points for futures returns, but in dollars for spot prices. The returns are annualized by multiplying quarterly means by 4 and quarterly standard deviations by 2. The spot price volatility is multiplied by 2.

Table 3: Growth rates

	Benchmark	IP-AP-FS	IP-FS	AP-FS
First Moments				
$E[\Delta \log \tilde{C}_{Aggr}]$	1.90	1.90	1.90	1.90
Second Moments				
$\sigma_{\Delta \log \tilde{C}_{Aggr}}$	3.43	3.36	3.70	3.22
$\sigma_{\Delta \log \tilde{C}_{IP}}$	4.68	3.77	4.64	4.50
$\sigma_{\Delta \log \tilde{C}_{AP}}$	6.76	4.89	7.15	4.81
$\sigma_{\Delta \log \tilde{C}_{FS}}$	3.99	3.23	3.14	3.79
$\sigma_{\Delta \log \tilde{C}_{CC}}$	11.26	11.02	13.00	9.97
$\sigma_{\Delta \log C_{IP}}$	4.51	3.67	4.47	4.42
$\sigma_{\Delta \log C_{AP}}$	6.84	5.02	7.36	4.93
$\sigma_{\Delta \log Q_{IP}}$	4.88	4.06	5.04	4.71
$\sigma_{\Delta \log Q_{AP}}$	6.27	4.53	6.66	4.47
$\sigma_{\Delta \log Q_P}$	12.63	13.34	15.78	11.36
$\sigma_{\Delta \log (S(Q-Q_{AP}))}$	6.92	6.77	7.36	6.21
$\sigma_{\Delta \log Y}$	3.87	3.97	4.00	3.90

This table reports the mean of the annual aggregate log consumption growth rate $E[\Delta \log \tilde{C}_{Aggr,t}]$, where aggregate consumption in non-commodity good units is defined by $\tilde{C}_{Aggr,t} = P_{IP,t}\tilde{C}_{IP,t} + P_{AP,t}\tilde{C}_{AP,t} + \tilde{C}_{FS,t} + S_t\tilde{C}_{CC,t}$. $P_{j,t} = \frac{1}{\phi_j} \left(\frac{\tilde{C}_{j,t}}{C_{j,t}} \right)^{\rho_j - 1}$ is the price of the consumption bundle \tilde{C}_j in non-commodity good units for $j = \{IP, AP\}$. Moreover, the annual volatilities of various growth rates are reported: the volatility of aggregate log consumption growth $\sigma_{\Delta \log \tilde{C}_{Aggr}}$, the volatility of agent i 's log effective consumption growth $\sigma_{\Delta \log \tilde{C}_i}$ for $i \in \{IP, AP, FS, CC\}$, the volatility of industrial producer's log non-commodity consumption growth $\sigma_{\Delta \log C_{IP}}$, the volatility of agricultural producer's log non-commodity consumption growth $\sigma_{\Delta \log C_{AP}}$, the volatility of industrial producer's log commodity consumption growth $\sigma_{\Delta \log Q_{IP}}$, the volatility of agricultural producer's log commodity consumption growth $\sigma_{\Delta \log Q_{AP}}$, the volatility of log commodity input to production growth $\sigma_{\Delta \log Q_P}$, the volatility of the value of AP's total commodity sales in non-commodity good units $\sigma_{\Delta \log (S(Q-Q_{AP}))}$, and the volatility of log production output growth $\sigma_{\Delta \log Y}$. The moments for four model specifications are considered: the benchmark model without financial markets; the model where the bond is traded between IP and FS and the commodity derivative is traded between AP and FS (denoted by IP-AP-FS); the model where just the bond is traded between IP and FS (denoted by IP-FS); and the model where just the commodity derivative is traded between AP and FS (denoted by AP-FS). Parameters are reported in Table 1. All models are calibrated at quarterly frequency. The growth rates are time-aggregated to an annual frequency for computing the annual moments. All numbers reported are measured in percentage points.

Table 4: Consumption shares and commodity expenditure ratios

	Benchmark	IP-AP-FS	IP-FS	AP-FS
First Moments				
$E[P_{IP}\tilde{C}_{IP}/\tilde{C}_{Aggr}]$	37.12%	37.19%	37.27%	37.23%
$E[P_{AP}\tilde{C}_{AP}/\tilde{C}_{Aggr}]$	17.65%	17.45%	17.88%	17.29%
$E[\tilde{C}_{FS}/\tilde{C}_{Aggr}]$	42.87%	43.00%	42.50%	43.15%
$E[S_t\tilde{C}_{CC}/\tilde{C}_{Aggr}]$	2.36%	2.36%	2.35%	2.36%
$E[Q_{IP}/Q]$	52.75%	52.89%	52.84%	52.98%
$E[Q_{AP}/Q]$	25.09%	24.82%	25.36%	24.60%
$E[Q_{CC}/Q]$	13.36%	13.44%	13.16%	13.53%
$E[Q_P/Q]$	8.83%	8.85%	8.64%	8.90%
$E[\tilde{X}_{Aggr}]$	16.10%	16.01%	16.34%	15.91%

This table reports the means of effective consumption shares in non-commodity good units for all four agents $P_{i,t}\tilde{C}_{i,t}/\tilde{C}_{Aggr,t}$ with $i \in \{IP, AP, FS, CC\}$ and where $P_{j,t} = \frac{1}{\phi_j} \left(\frac{\tilde{C}_{j,t}}{C_{j,t}} \right)^{\rho_j - 1}$ is the price of the consumption bundle \tilde{C}_j in non-commodity good units for $j = \{IP, AP\}$, $P_{FS,t} \equiv 1$ as financial speculator's consumption is already given in non-commodity good units and $P_{CC,t} \equiv S_t$ is the price of a unit of the commodity in non-commodity good units (i.e. the spot price S_t). Aggregate consumption in non-commodity good units is defined as $\tilde{C}_{Aggr,t} = P_{IP,t}\tilde{C}_{IP,t} + P_{AP,t}\tilde{C}_{AP,t} + \tilde{C}_{FS,t} + S_t\tilde{C}_{CC,t}$. Furthermore, this table reports the means of commodity consumption shares for the producers and commodity consumer $Q_{k,t}/Q_t$ where Q_t is the economy's commodity endowment at time t and $k \in \{IP, AP, CC\}$, the mean of the commodity share used for production by the industrial producer $Q_{P,t}/Q_t$, and the means of the fractions of commodity expenditure to total expenditure for the aggregate economy $\tilde{X}_{Aggr,t} = \frac{S_t(Q_{IP,t} + Q_{AP,t} + Q_{CC,t})}{C_{IP,t} + C_{AP,t} + C_{FS,t} + S_t(Q_{IP,t} + Q_{AP,t} + Q_{CC,t})}$. The moments for four model specifications are considered: the benchmark model without financial markets; the model where the bond is traded between IP and FS and the commodity derivative is traded between AP and FS (denoted by IP-AP-FS); the model where just the bond is traded between IP and FS (denoted by IP-FS); and the model where just the commodity derivative is traded between AP and FS (denoted by AP-FS). Parameters are reported in Table 1. All models are calibrated at quarterly frequency, and all the moments shown in the table are quarterly as well.

Table 5: Asset pricing

	Benchmark	IP-AP-FS	IP-FS	AP-FS
First Moments				
$E[F]$	—	0.352	—	0.350
$E[S]$	0.358	0.356	0.364	0.354
$E[r_f]$	—	4.53	4.49	—
$E[r_Q]$	—	4.65	—	4.48
$E[r_F^{ex}]$	—	0.00	—	0.01
Second Moments				
σ_F	—	27.29	—	23.26
σ_S	27.91	27.92	35.67	23.67
$\sigma_S/E[S]$	77.96	78.43	97.99	66.86
σ_{r_f}	—	0.76	0.77	—
σ_{r_Q}	—	11.33	—	10.05
$\sigma_{r_F^{ex}}$	—	11.19	—	9.95

This table contains the annual first and second asset pricing moments from our model. The commodity derivative price is denoted by F_t and the spot price by S_t . The risk-free rate of the bond from time t to time $t + 1$ is denoted by $r_{f,t}$, the log return on the commodity derivative is defined by $r_{Q,t} = \log(S_t/F_{t-1})$, the log “rolling-over” commodity derivative excess return is given by $r_{F,t}^{ex} = \log(F_t/F_{t-1})$. The moments for four model specifications are considered: the benchmark model without financial markets; the model where the bond is traded between IP and FS and the commodity derivative is traded between AP and FS (denoted by IP-AP-FS); the model with just the bond between IP and FS being traded (denoted by IP-FS); and the model with just the commodity derivative between AP and FS being traded (denoted by AP-FS). Parameters are reported in Table 1. All models are calibrated at quarterly frequency. The means of the prices F_t and S_t are quarterly, the volatilities of these prices are annualized by multiplying quarterly values by 2. The returns are time-aggregated to an annual frequency for computing the annual moments. All numbers reported are measured in percentage points.

Table 6: Agents' welfare

	Benchmark	IP-AP-FS	IP-FS	AP-FS
First Moments				
$E[W_{IP}]$	78.13	78.10	78.49	78.15
$E[W_{AP}]$	38.16	37.74	38.49	37.38
$E[W_{FS}]$	118.91	118.87	118.26	119.19
$E[W_{CC}]$	23.82	23.80	23.93	23.73
<hr style="border-top: 1px dashed black;"/>				
$E[\tilde{C}_{IP}]$	0.642	0.643	0.643	0.644
$E[\tilde{C}_{AP}]$	0.305	0.302	0.309	0.299
$E[\tilde{C}_{FS}]$	1.000	1.002	0.994	1.004
$E[\tilde{C}_{CC}]$	0.154	0.155	0.151	0.156
Second Moments				
$\sigma_{\Delta \log W_{IP}}$	7.48	5.68	7.37	7.15
$\sigma_{\Delta \log W_{AP}}$	11.02	7.42	12.12	7.30
$\sigma_{\Delta \log W_{FS}}$	6.36	4.92	4.78	5.99
$\sigma_{\Delta \log W_{CC}}$	17.51	16.94	20.99	14.69
<hr style="border-top: 1px dashed black;"/>				
$\sigma_{\Delta \log \tilde{C}_{IP}}$	4.68	3.77	4.64	4.50
$\sigma_{\Delta \log \tilde{C}_{AP}}$	6.76	4.89	7.15	4.81
$\sigma_{\Delta \log \tilde{C}_{FS}}$	3.99	3.23	3.14	3.79
$\sigma_{\Delta \log \tilde{C}_{CC}}$	11.26	11.02	13.00	9.97

This table reports the means of the quarterly agents' wealth levels $W_{i,t}$ and of the quarterly agents' consumption levels $\tilde{C}_{i,t}$, and the annual volatilities of agents' log wealth growth rates $\Delta \log W_{i,t}$ and of agents' log consumption growth rates $\Delta \log \tilde{C}_{i,t}$ for $i \in \{IP, AP, FS, CC\}$. Wealth of agent $i \in \{IP, AP, FS, CC\}$ is defined as the claim on the consumption bundle of agent i , i.e. on $\tilde{C}_{i,t}$. This implies that wealth in bundle units, $W_{i,t}$, satisfies the Euler equation $W_{i,t} = \tilde{C}_{i,t} + \mathbb{E}_t[M_{t,t+1}^{(i),aggr} W_{i,t+1}]$, where $M_{t,t+1}^{(i),aggr}$ is the pricing kernel denoted in units of the consumption bundle $\tilde{C}_{i,t}$, defined in Equation (A7) in Appendix A.6.1 for IP, in Equation (A8) in Appendix A.6.2 for AP, and in Equation (A9) in Appendix A.6.3 for FS, respectively. The moments for four model specifications are considered: the benchmark model without financial markets; the model where the bond is traded between IP and FS and the commodity derivative is traded between AP and FS (denoted by IP-AP-FS); the model where just the bond is traded between IP and FS (denoted by IP-FS); and the model where just the commodity derivative is traded between AP and FS (denoted by AP-FS). Parameters are reported in Table 1. All models are calibrated at quarterly frequency. The growth rates are time-aggregated to an annual frequency for computing the annual moments. The means reported are measured in decimals and the volatilities in percentage points.

Table 7: Portfolio holdings, commodity spot price and welfare

	Benchmark	IP-AP-FS	IP-FS	AP-FS
Average Asset Positions				
$E[B_{IP}]$	0.00	0.07	0.98	0.00
$E[B_{FS}]$	0.00	-0.07	-0.98	0.00
$E[n_{AP}]$	0.00	-1.11	0.00	-1.83
$E[n_{FS}]$	0.00	1.11	0.00	1.83
Volatilities				
σ_S	27.91	27.92	35.67	23.67
$\sigma_{\Delta \log W_{IP}}$	7.48	5.68	7.37	7.15
$\sigma_{\Delta \log W_{AP}}$	11.02	7.42	12.12	7.30
$\sigma_{\Delta \log W_{FS}}$	6.36	4.92	4.78	5.99
$\sigma_{\Delta \log W_{CC}}$	17.51	16.94	20.99	14.69
$\sigma_{\Delta \log \tilde{C}_{Aggr}}$	3.43	3.36	3.70	3.22
$\sigma_{\Delta \log \tilde{C}_{IP}}$	4.68	3.77	4.64	4.50
$\sigma_{\Delta \log \tilde{C}_{AP}}$	6.76	4.89	7.15	4.81
$\sigma_{\Delta \log \tilde{C}_{FS}}$	3.99	3.23	3.14	3.79
$\sigma_{\Delta \log \tilde{C}_{CC}}$	11.26	11.02	13.00	9.97

The upper panel of this table reports the agents' average asset positions, i.e., the average bond holdings by IP and FS (denoted by B_{IP} and B_{FS} , respectively) and the average positions in the commodity derivative held by AP and FS (denoted by $n_{AP,t}$ and $n_{FS,t}$, respectively). Additionally, the lower panel repeats some volatilities taken from Tables 3, 5 and 6 to facilitate comparison of the results with our robustness checks for which the results are reported in Tables 8–13.

The moments for four model specifications are considered: the benchmark model without financial markets; the model where the bond is traded between IP and FS and the commodity derivative is traded between AP and FS (denoted by IP-AP-FS); the model where just the bond is traded between IP and FS (denoted by IP-FS); and the model where just the commodity derivative is traded between AP and FS (denoted by AP-FS). Parameters are reported in Table 1. All models are calibrated at quarterly frequency. The growth rates are time-aggregated to an annual frequency for computing the annual moments. The means are reported in decimals, and the volatilities in percentage points.

Table 8: Sensitivity analysis: preference parameters

IP-AP-FS			
	$\gamma = 10$ $\psi = 0.50$	$\gamma = 10$ $\psi = 0.75$	$\gamma = 15$ $\psi = 0.50$
Average Asset Positions			
$E[B_{IP}]$	0.07	-0.27	-0.13
$E[B_{FS}]$	-0.07	0.27	0.13
$E[n_{AP}]$	-1.11	-1.41	-1.04
$E[n_{FS}]$	1.11	1.41	1.04
Volatilities			
σ_S	27.92	26.01	27.21
$\sigma_{\Delta \log W_{IP}}$	5.68	4.66	5.68
$\sigma_{\Delta \log W_{AP}}$	7.42	5.91	7.67
$\sigma_{\Delta \log W_{FS}}$	4.92	3.96	4.91
$\sigma_{\Delta \log W_{CC}}$	16.94	12.95	16.66
$\sigma_{\Delta \log \tilde{C}_{Aggr}}$	3.36	3.31	3.37
$\sigma_{\Delta \log \tilde{C}_{IP}}$	3.77	3.84	3.78
$\sigma_{\Delta \log \tilde{C}_{AP}}$	4.89	4.90	4.94
$\sigma_{\Delta \log \tilde{C}_{FS}}$	3.23	3.26	3.22
$\sigma_{\Delta \log \tilde{C}_{CC}}$	11.02	10.68	11.04

The table reports the results of a sensitivity analysis with respect to the preference parameters for the model where the bond is traded between IP and FS and the commodity derivative is traded between AP and FS (denoted by IP-AP-FS). The column headlines show the respective combination of risk aversion γ and elasticity of intertemporal substitution ψ , where the combination of $\gamma = 10$ and $\psi = 0.5$ represents the original choice of preference parameters. All other parameters are the same as reported in Table 1. The model is calibrated at quarterly frequency. The upper panel shows the agents' average asset positions, i.e., the average bond holdings by IP and FS (denoted by B_{IP} and B_{FS} , respectively) and the average positions in the commodity derivative held by AP and FS (denoted by $n_{AP,t}$ and $n_{FS,t}$, respectively). The lower panel reports the annualized volatility of the spot price S_t , as well as the annual volatilities of agents' log wealth growth rates $\Delta \log W_{i,t}$, of aggregate log consumption growth $\Delta \log \tilde{C}_{aggr,t}$, and of agents' log consumption growth rates $\Delta \log \tilde{C}_{i,t}$ for $i \in \{IP, AP, FS, CC\}$. Aggregate consumption is defined as $\tilde{C}_{Aggr,t} = P_{IP,t} \tilde{C}_{IP,t} + P_{AP,t} \tilde{C}_{AP,t} + \tilde{C}_{FS,t} + S_t \tilde{C}_{CC,t}$. $P_{j,t} = \frac{1}{\phi_j} \left(\frac{\tilde{C}_{j,t}}{C_{j,t}} \right)^{\rho_j - 1}$ is the price of the consumption bundle \tilde{C}_j in non-commodity good units for $j = \{IP, AP\}$. The wealth of agent $i \in \{IP, AP, FS, CC\}$ is defined as the claim on the consumption bundle of agent i , i.e. on $\tilde{C}_{i,t}$. This implies that wealth in bundle units, $W_{i,t}$, satisfies the Euler equation $W_{i,t} = \tilde{C}_{i,t} + \mathbb{E}_t[M_{t,t+1}^{(i),aggr} W_{i,t+1}]$, where $M_{t,t+1}^{(i),aggr}$ is the pricing kernel denoted in units of the consumption bundle $\tilde{C}_{i,t}$, defined in Equation (A7) in Appendix A.6.1 for IP, in Equation (A8) in Appendix A.6.2 for AP, and in Equation (A9) in Appendix A.6.3 for FS, respectively. The growth rates are time-aggregated to an annual frequency for computing the annual moments. The means reported are measured in decimals and the volatilities in percentage points.

Table 9: Robustness check: alternative asset menu

	Benchmark	IP-AP-FS 2	IP-FS 2	AP-FS 2
Average Asset Positions				
$E[B_{AP}]$	0.00	2.01	0.00	2.31
$E[B_{FS}]$	0.00	-2.01	0.00	-2.31
$E[n_{IP}]$	0.00	3.30	3.43	0.00
$E[n_{FS}]$	0.00	-3.30	-3.43	0.00
Volatilities				
σ_S	27.91	27.64	34.82	23.40
$\sigma_{\Delta \log W_{IP}}$	7.48	6.43	7.65	7.87
$\sigma_{\Delta \log W_{AP}}$	11.02	5.04	10.45	5.04
$\sigma_{\Delta \log W_{FS}}$	6.36	5.75	5.57	6.48
$\sigma_{\Delta \log W_{CC}}$	17.51	15.47	18.37	15.49
$\sigma_{\Delta \log \tilde{C}_{Aggr}}$	3.43	3.27	3.59	3.14
$\sigma_{\Delta \log \tilde{C}_{IP}}$	4.68	4.15	4.76	4.61
$\sigma_{\Delta \log \tilde{C}_{AP}}$	6.76	3.50	6.53	3.22
$\sigma_{\Delta \log \tilde{C}_{FS}}$	3.99	3.72	3.59	3.96
$\sigma_{\Delta \log \tilde{C}_{CC}}$	11.26	10.20	11.98	9.35

This table reports results when the asset menu available to the agents is varied so that now the commodity derivative can be traded between IP and FS, and a bond can be traded between AP and FS. Four model specifications are considered: the benchmark model without financial markets; the model where the commodity derivative between IP and FS and the bond is traded between AP and FS (denoted by IP-AP-FS 2); the model where just the commodity derivative is traded between IP and FS (denoted by IP-FS 2); and the model where just the bond is traded between AP and FS (denoted by AP-FS 2). The model parameters are shown in Table 1, where just the trading cost parameters ν_1 and ν_2 need to be flipped for the IP-FS and the AP-FS model. All models are calibrated at quarterly frequency.

The upper panel shows the agents' average asset positions, i.e., the average bond holdings by AP and FS (denoted by B_{AP} and B_{FS} , respectively) and the average positions in the commodity derivative held by IP and FS (denoted by $n_{IP,t}$ and $n_{FS,t}$, respectively). The lower panel reports the annualized volatilities of the spot price S_t , as well as the annual volatilities of agents' log wealth growth rates $\Delta \log W_{i,t}$, aggregate log consumption growth $\Delta \log \tilde{C}_{aggr,t}$, and of agents' log consumption growth rates $\Delta \log \tilde{C}_{i,t}$ with $i \in \{IP, AP, FS, CC\}$. Aggregate consumption is defined as $\tilde{C}_{Aggr,t} = P_{IP,t} \tilde{C}_{IP,t} + P_{AP,t} \tilde{C}_{AP,t} + \tilde{C}_{FS,t} + S_t \tilde{C}_{CC,t}$. $P_{j,t} = \frac{1}{\phi_j} \left(\frac{\tilde{C}_{j,t}}{C_{j,t}} \right)^{\rho_j - 1}$ is the price of the consumption bundle \tilde{C}_j in non-commodity good units for $j = \{IP, AP\}$. The wealth of agent $i \in \{IP, AP, FS, CC\}$ is defined as the value of the claim on agent i 's consumption bundle $\tilde{C}_{i,t}$. This implies that wealth in bundle units, $W_{i,t}$, satisfies the Euler equation $W_{i,t} = \tilde{C}_{i,t} + \mathbb{E}_t[M_{t,t+1}^{(i),aggr} W_{i,t+1}]$, where $M_{t,t+1}^{(i),aggr}$ is the pricing kernel denoted in units of the consumption bundle $\tilde{C}_{i,t}$, defined in Equation (A7) in Appendix A.6.1 for IP, in Equation (A8) in Appendix A.6.2 for AP, and in Equation (A9) in Appendix A.6.3 for FS, respectively. The growth rates are time-aggregated to an annual frequency for computing the annual moments. The means reported are measured in decimals and the volatilities in percentage points.

Table 10: Robustness check: elasticity in consumption equal to elasticity in production

	Benchmark	IP-AP-FS	IP-FS	AP-FS
	$\rho_{IP} = \rho_{AP} = 0.1$	$\rho_{IP} = \rho_{AP} = 0.1$	$\rho_{IP} = \rho_{AP} = 0.1$	$\rho_{IP} = \rho_{AP} = 0.1$
Average Asset Positions				
$E[B_{IP}]$	0.00	-0.11	-0.16	0.00
$E[B_{FS}]$	0.00	0.11	0.16	0.00
$E[n_{AP}]$	0.00	-0.22	0.00	-0.67
$E[n_{FS}]$	0.00	0.22	0.00	0.67
Volatilities				
σ_S	28.58	29.15	29.14	28.57
$\sigma_{\Delta \log W_{IP}}$	6.19	4.84	4.86	6.31
$\sigma_{\Delta \log W_{AP}}$	4.66	4.10	4.15	4.03
$\sigma_{\Delta \log W_{FS}}$	6.36	4.88	4.93	6.07
$\sigma_{\Delta \log W_{CC}}$	6.61	6.53	6.51	6.61
$\sigma_{\Delta \log \tilde{C}_{Aggr}}$	2.71	2.71	2.71	2.71
$\sigma_{\Delta \log \tilde{C}_{IP}}$	3.95	3.19	3.19	3.95
$\sigma_{\Delta \log \tilde{C}_{AP}}$	2.92	2.62	2.65	2.57
$\sigma_{\Delta \log \tilde{C}_{FS}}$	3.99	3.21	3.23	3.84
$\sigma_{\Delta \log \tilde{C}_{CC}}$	4.14	4.08	4.08	4.14

This table reports results for all cases of asset availability with $\rho_{IP} = \rho_{AP} = \nu$, i.e. identical substitution elasticity in consumption as in production. All other parameters are the same as reported in Table 1. All models are calibrated at quarterly frequency.

The upper panel shows the agents' average asset positions, i.e., the average bond holdings by IP and FS (denoted by B_{IP} and B_{FS} , respectively) and the average positions in the commodity derivative held by AP and FS (denoted by $n_{AP,t}$ and $n_{FS,t}$, respectively). The lower panel reports the annualized volatility of the spot price S_t , as well as the annual volatilities of agents' log wealth growth rates $\Delta \log W_{i,t}$, of aggregate log consumption growth $\Delta \log \tilde{C}_{aggr,t}$, and of agents' log consumption growth rates $\Delta \log \tilde{C}_{i,t}$ for $i \in \{IP, AP, FS, CC\}$. Aggregate consumption is defined as $\tilde{C}_{Aggr,t} = P_{IP,t} \tilde{C}_{IP,t} + P_{AP,t} \tilde{C}_{AP,t} + \tilde{C}_{FS,t} + S_t \tilde{C}_{CC,t}$. $P_{j,t} = \frac{1}{\phi_j} \left(\frac{\tilde{C}_{j,t}}{\tilde{C}_{j,t}} \right)^{\rho_j - 1}$ is the price of the consumption bundle \tilde{C}_j in non-commodity good units for $j = \{IP, AP\}$. The wealth of agent $i \in \{IP, AP, FS, CC\}$ is defined as the claim on the consumption bundle of agent i , i.e. on $\tilde{C}_{i,t}$. This implies that wealth in bundle units, $W_{i,t}$, satisfies the Euler equation $W_{i,t} = \tilde{C}_{i,t} + \mathbb{E}_t[M_{t,t+1}^{(i),aggr} W_{i,t+1}]$, where $M_{t,t+1}^{(i),aggr}$ is the pricing kernel denoted in units of the consumption bundle $\tilde{C}_{i,t}$, defined in Equation (A7) in Appendix A.6.1 for IP, in Equation (A8) in Appendix A.6.2 for AP, and in Equation (A9) in Appendix A.6.3 for FS, respectively. The moments for the usual four model specifications, which differ in the asset menu available to the agents, are considered (see for example Table 7). The growth rates are time-aggregated to an annual frequency for computing the annual moments. The means reported are measured in decimals and the volatilities in percentage points. The means reported are measured in decimals and the volatilities in percentage points.

Table 11: Robustness check: lower persistence of endowment shocks

	Benchmark	IP-AP-FS	IP-FS	AP-FS
	$\varphi_i = 0.85^{1/4}$	$\varphi_i = 0.85^{1/4}$	$\varphi_i = 0.85^{1/4}$	$\varphi_i = 0.85^{1/4}$
Average Asset Positions				
$E[B_{IP}]$	0.00	-0.21	0.69	0.00
$E[B_{FS}]$	0.00	0.21	-0.69	0.00
$E[n_{AP}]$	0.00	-0.79	0.00	-1.27
$E[n_{FS}]$	0.00	0.79	0.00	1.26
Volatilities				
σ_S	14.43	14.89	19.73	11.99
$\sigma_{\Delta \log W_{IP}}$	8.02	5.49	7.38	7.89
$\sigma_{\Delta \log W_{AP}}$	11.99	6.14	14.37	6.28
$\sigma_{\Delta \log W_{FS}}$	7.16	5.21	5.07	6.60
$\sigma_{\Delta \log W_{CC}}$	19.81	19.35	25.11	15.91
$\sigma_{\Delta \log \tilde{C}_{Aggr}}$	3.25	3.22	3.60	3.04
$\sigma_{\Delta \log \tilde{C}_{IP}}$	4.40	3.12	4.08	4.32
$\sigma_{\Delta \log \tilde{C}_{AP}}$	6.60	3.60	7.67	3.62
$\sigma_{\Delta \log \tilde{C}_{FS}}$	3.94	2.92	2.83	3.64
$\sigma_{\Delta \log \tilde{C}_{CC}}$	10.97	10.65	13.52	8.96

This table reports results for all cases of asset availability with $\varphi_i = 0.85^{1/4}$ for $i \in \{k, q, FS, CC\}$, i.e. smaller persistence for all endowment shocks. All other parameters are the same as reported in Table 1. All models are calibrated at quarterly frequency.

The upper panel shows the agents' average asset positions, i.e., the average bond holdings by IP and FS (denoted by B_{IP} and B_{FS} , respectively) and the average positions in the commodity derivative held by AP and FS (denoted by $n_{AP,t}$ and $n_{FS,t}$, respectively). The lower panel reports the annualized volatility of the spot price S_t , as well as the annual volatilities of agents' log wealth growth rates $\Delta \log W_{i,t}$, of aggregate log consumption growth $\Delta \log \tilde{C}_{aggr,t}$, and of agents' log consumption growth rates $\Delta \log \tilde{C}_{i,t}$ for $i \in \{IP, AP, FS, CC\}$. Aggregate consumption is defined as $\tilde{C}_{Aggr,t} = P_{IP,t} \tilde{C}_{IP,t} + P_{AP,t} \tilde{C}_{AP,t} + \tilde{C}_{FS,t} + S_t \tilde{C}_{CC,t}$. $P_{j,t} = \frac{1}{\phi_j} \left(\frac{\tilde{C}_{j,t}}{\tilde{C}_{j,t}} \right)^{\rho_j - 1}$ is the price of the consumption bundle \tilde{C}_j in non-commodity good units for $j = \{IP, AP\}$. The wealth of agent $i \in \{IP, AP, FS, CC\}$ is defined as the claim on the consumption bundle of agent i , i.e. on $\tilde{C}_{i,t}$. This implies that wealth in bundle units, $W_{i,t}$, satisfies the Euler equation $W_{i,t} = \tilde{C}_{i,t} + \mathbb{E}_t[M_{t,t+1}^{(i),aggr} W_{i,t+1}]$, where $M_{t,t+1}^{(i),aggr}$ is the pricing kernel denoted in units of the consumption bundle $\tilde{C}_{i,t}$, defined in Equation (A7) in Appendix A.6.1 for IP, in Equation (A8) in Appendix A.6.2 for AP, and in Equation (A9) in Appendix A.6.3 for FS, respectively. The moments for the usual four model specifications, which differ in the asset menu available to the agents, are considered (see for example Table 7). The growth rates are time-aggregated to an annual frequency for computing the annual moments. The means reported are measured in decimals and the volatilities in percentage points.

Table 12: Robustness check: CRRA preferences

	Benchmark	IP-AP-FS	IP-FS	AP-FS
	$\gamma_i = 2$	$\gamma_i = 2$	$\gamma_i = 2$	$\gamma_i = 2$
Average Asset Positions				
$E[B_{IP}]$	0.00	0.53	0.76	0.00
$E[B_{FS}]$	0.00	-0.53	-0.76	0.00
$E[n_{AP}]$	0.00	-1.21	0.00	-1.18
$E[n_{FS}]$	0.00	1.21	0.00	1.18
Volatilities				
σ_S	27.91	27.14	35.89	25.18
$\sigma_{\Delta \log W_{IP}}$	7.48	5.68	7.31	7.20
$\sigma_{\Delta \log W_{AP}}$	11.02	7.03	12.28	6.90
$\sigma_{\Delta \log W_{FS}}$	6.36	4.95	4.77	6.01
$\sigma_{\Delta \log W_{CC}}$	17.51	17.42	21.66	15.25
$\sigma_{\Delta \log \tilde{C}_{Aggr}}$	3.43	3.37	3.71	3.22
$\sigma_{\Delta \log \tilde{C}_{IP}}$	4.68	3.77	4.59	4.52
$\sigma_{\Delta \log \tilde{C}_{AP}}$	6.76	4.81	7.32	4.75
$\sigma_{\Delta \log \tilde{C}_{FS}}$	3.99	3.26	3.14	3.81
$\sigma_{\Delta \log \tilde{C}_{CC}}$	11.26	11.02	13.17	9.99

This table reports results for agents having CRRA preferences and all cases of asset availability by assuming $\gamma_i = 2$. All other parameters are the same as reported in Table 1. Hence, $\gamma_i = 1/\psi_i$, implying that Epstein and Zin preferences reduce to CRRA preferences. All models are calibrated at quarterly frequency.

The upper panel shows the agents' average asset positions, i.e., the average bond holdings by IP and FS (denoted by B_{IP} and B_{FS} , respectively) and the average positions in the commodity derivative held by AP and FS (denoted by $n_{AP,t}$ and $n_{FS,t}$, respectively). The lower panel reports the annualized volatility of the spot price S_t , as well as the annual volatilities of agents' log wealth growth rates $\Delta \log W_{i,t}$, of aggregate log consumption growth $\Delta \log \tilde{C}_{aggr,t}$, and of agents' log consumption growth rates $\Delta \log \tilde{C}_{i,t}$ for $i \in \{IP, AP, FS, CC\}$. Aggregate consumption is defined as $\tilde{C}_{Aggr,t} = P_{IP,t}\tilde{C}_{IP,t} + P_{AP,t}\tilde{C}_{AP,t} + \tilde{C}_{FS,t} + S_t\tilde{C}_{CC,t}$. $P_{j,t} = \frac{1}{\phi_j} \left(\frac{\tilde{C}_{j,t}}{\tilde{C}_{j,t}} \right)^{\rho_j - 1}$ is the price of the consumption bundle \tilde{C}_j in non-commodity good units for $j = \{IP, AP\}$. The wealth of agent $i \in \{IP, AP, FS, CC\}$ is defined as the claim on the consumption bundle of agent i , i.e. on $\tilde{C}_{i,t}$. This implies that wealth in bundle units, $W_{i,t}$, satisfies the Euler equation $W_{i,t} = \tilde{C}_{i,t} + \mathbb{E}_t[M_{t,t+1}^{(i),aggr} W_{i,t+1}]$, where $M_{t,t+1}^{(i),aggr}$ is the pricing kernel denoted in units of the consumption bundle $\tilde{C}_{i,t}$, defined in Equation (A7) in Appendix A.6.1 for IP, in Equation (A8) in Appendix A.6.2 for AP, and in Equation (A9) in Appendix A.6.3 for FS, respectively. The moments for the usual four model specifications, which differ in the asset menu available to the agents, are considered (see for example Table 7). The growth rates are time-aggregated to an annual frequency for computing the annual moments. The means reported are measured in decimals and the volatilities in percentage points. The means reported are measured in decimals and the volatilities in percentage points.

Table 13: Robustness check: full agent trade model

IP-AP-FS-Three-Agent-Trade	
$\varphi_i = 0.85^{1/4}$	
Average Asset Positions	
$E[B_{IP}]$	-1.92
$E[B_{AP}]$	2.04
$E[B_{FS}]$	-0.12

$E[n_{IP}]$	2.81
$E[n_{AP}]$	-2.30
$E[n_{FS}]$	-0.51
Volatilities	
σ_S	12.01

$\sigma_{\Delta \log W_{IP}}$	4.79
$\sigma_{\Delta \log W_{AP}}$	8.94
$\sigma_{\Delta \log W_{FS}}$	5.38
$\sigma_{\Delta \log W_{CC}}$	18.38

$\sigma_{\Delta \log \tilde{C}_{Aggr}}$	3.17

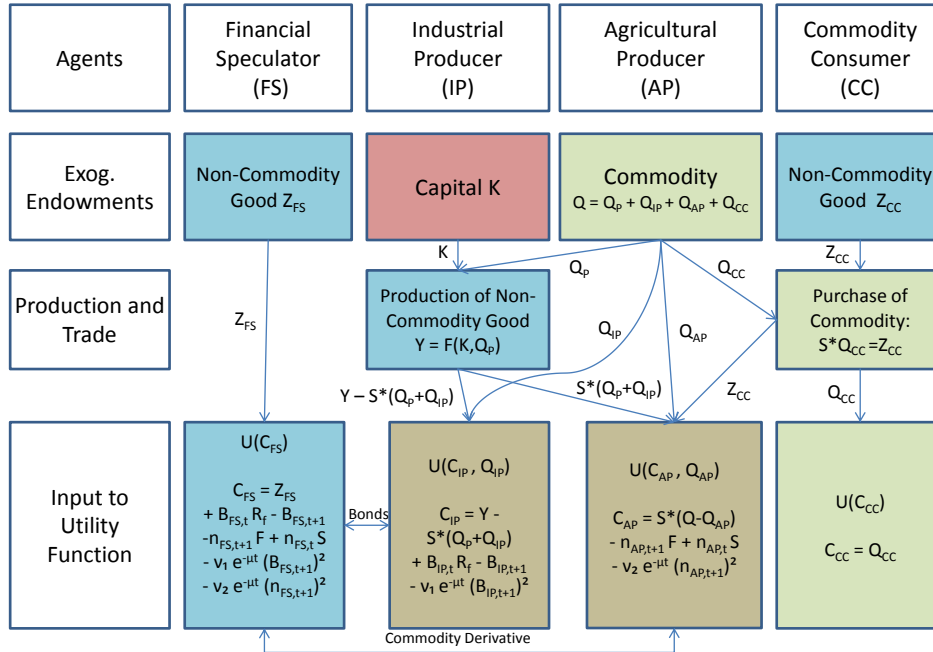
$\sigma_{\Delta \log \tilde{C}_{IP}}$	2.65
$\sigma_{\Delta \log \tilde{C}_{AP}}$	6.48
$\sigma_{\Delta \log \tilde{C}_{FS}}$	3.01
$\sigma_{\Delta \log \tilde{C}_{CC}}$	10.15

The table reports the results of the full participation model by assuming that the two assets in the economy are both traded between three agents, namely IP , AP and FS . The results are produced using the lower persistence in endowment shocks as has also been used in Table 11. Specifically, $\varphi_i = 0.85^{1/4}$. All other parameters are the same as reported in Table 1.

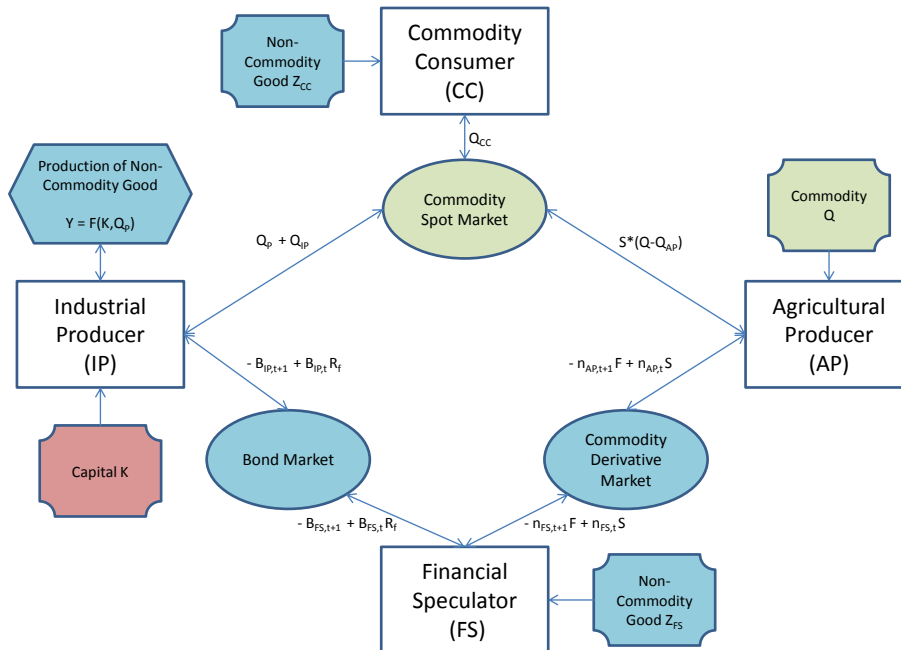
The upper panel shows the agents' average asset positions, i.e., the average bond holdings by IP , AP and FS (denoted by B_{IP} , B_{AP} and B_{FS} , respectively) and the average positions in the commodity derivative held by IP , AP and FS (denoted by $n_{IP,t}$, $n_{AP,t}$ and $n_{FS,t}$, respectively). The lower panel reports the annualized volatility of the spot price S_t , as well as the annual volatilities of agents' log wealth growth rates $\Delta \log W_{i,t}$, of aggregate log consumption growth $\Delta \log \tilde{C}_{aggr,t}$, and of agents' log consumption growth rates $\Delta \log \tilde{C}_{i,t}$ for $i \in \{IP, AP, FS, CC\}$. Aggregate consumption is defined as $\tilde{C}_{Aggr,t} = P_{IP,t}\tilde{C}_{IP,t} + P_{AP,t}\tilde{C}_{AP,t} + \tilde{C}_{FS,t} + S_t\tilde{C}_{CC,t}$. $P_{j,t} = \frac{1}{\phi_j} \left(\frac{\tilde{C}_{j,t}}{C_{j,t}} \right)^{\rho_j - 1}$ is the price of the consumption bundle \tilde{C}_j in non-commodity good units for $j = \{IP, AP\}$. The wealth of agent $i \in \{IP, AP, FS, CC\}$ is defined as the claim to the consumption bundle of agent i , i.e. to $\tilde{C}_{i,t}$. This implies that wealth in bundle units, $W_{i,t}$, satisfies the Euler equation $W_{i,t} = \tilde{C}_{i,t} + \mathbb{E}_t[M_{t,t+1}^{(i),aggr} W_{i,t+1}]$, where $M_{t,t+1}^{(i),aggr}$ is the pricing kernel denoted in units of the consumption bundle $\tilde{C}_{i,t}$, defined in Equation (A7) in Appendix A.6.1 for IP , in Equation (A8) in Appendix A.6.2 for AP , and in Equation (A9) in Appendix A.6.3 for FS , respectively. The growth rates are time-aggregated to an annual frequency for computing the annual moments. The means are reported in decimals, and the volatilities in percentage points.

Figure 1: Visual representation of the model

(a) Panel A: Endowments, production, and utility

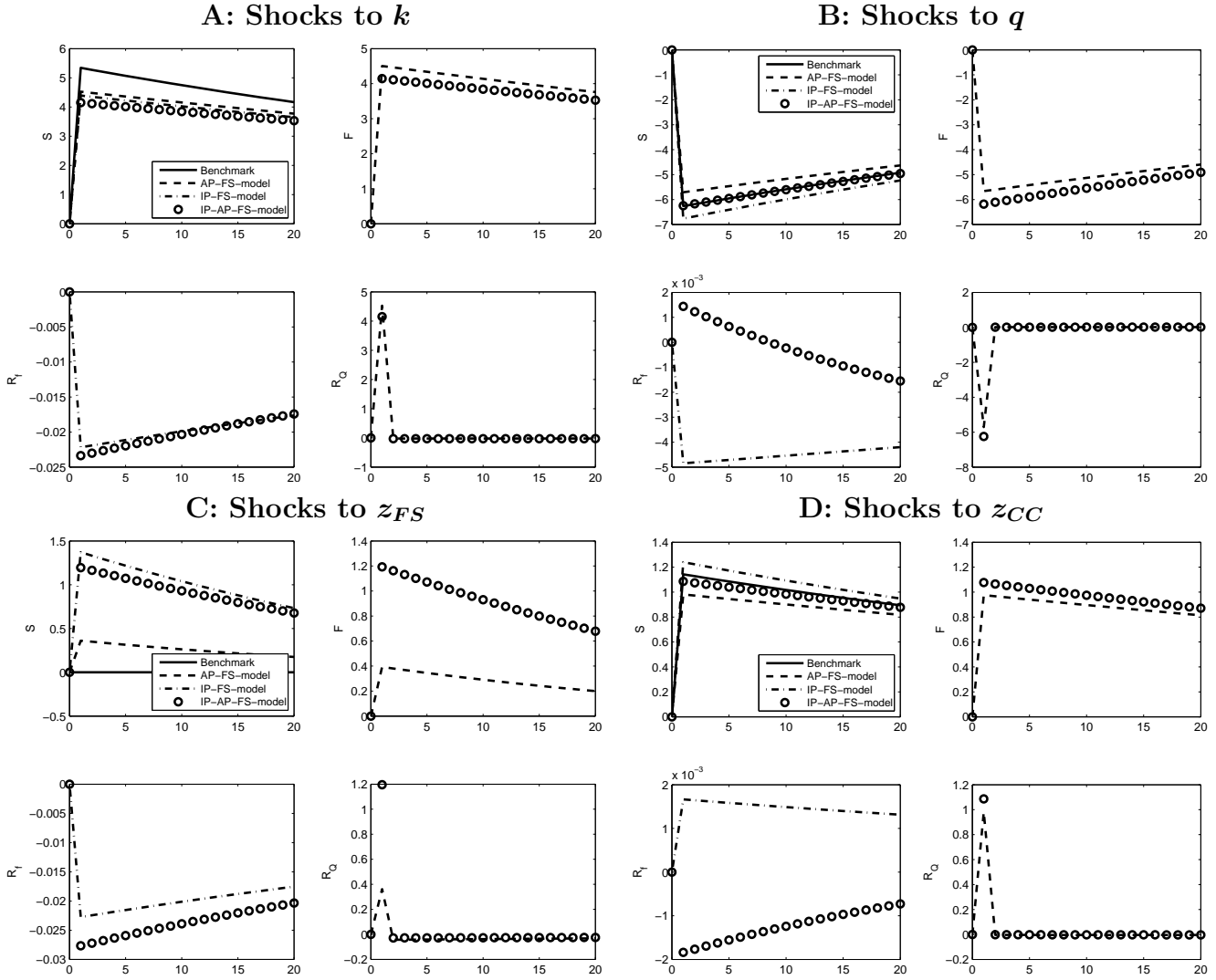


(b) Panel B: Commodity spot market and financial markets



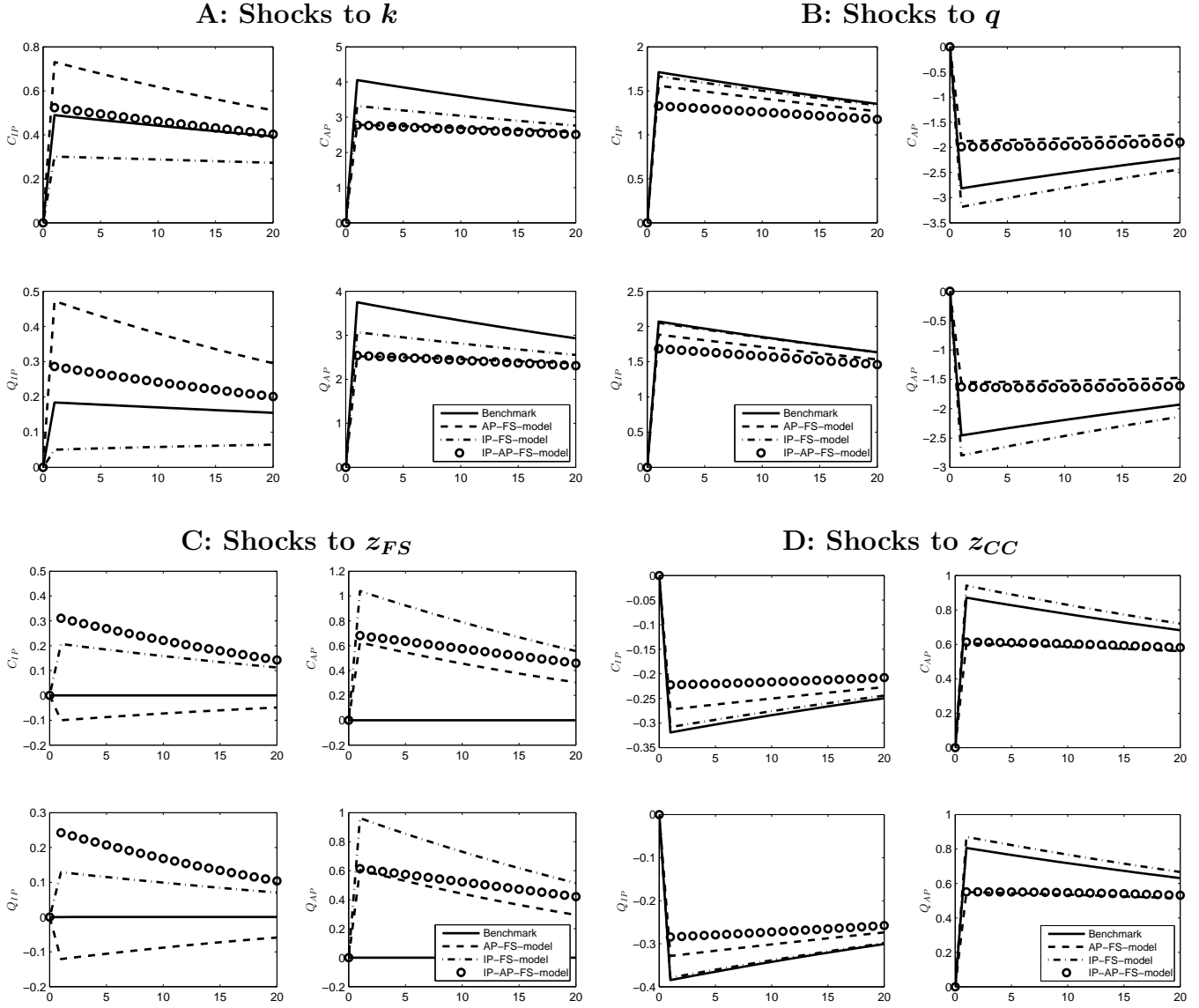
This figure depicts two flow charts of the economy to visualize the agents and their interactions on the commodity and the financial markets. The arrows in the figures visualize the flows of goods, abstracting away from the transaction costs on the financial markets in the lower panel.

Figure 2: Impulse response functions: commodity spot price and asset prices



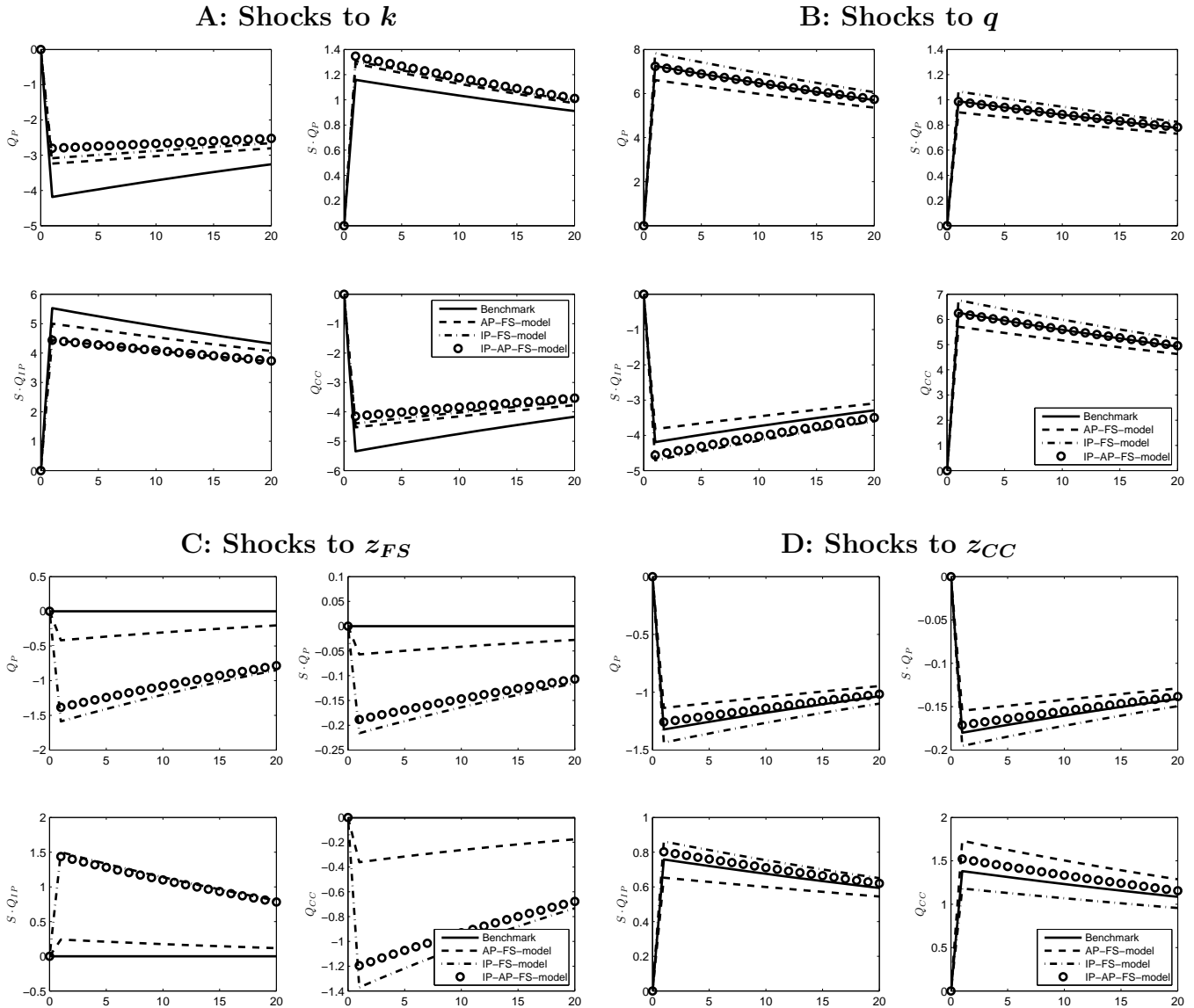
This figure depicts the impulse response functions in log deviations from the steady state in percentage points for the spot price S_t , the commodity derivative price F_t , the gross risk-free interest rate $R_{f,t}$ and the return on the commodity derivative $R_{Q,t} = S_t/F_{t-1}$ for our four model specifications. The graphs in Panel A depict the impulse response function with respect to a one standard deviation shock in the industrial producer's capital endowment k_t . The graphs in Panel B depict the impulse response function with respect to a one standard deviation shock in the agricultural producer's commodity endowment q_t . The graphs in Panel C depict the impulse response function with respect to a one standard deviation shock in the financial speculator's wage endowment $z_{FS,t}$. The graphs in Panel D depict the impulse response function with respect to a one standard deviation shock in the commodity consumer's wage endowment $z_{CC,t}$.

Figure 3: Impulse response functions: consumption (IP and AP)



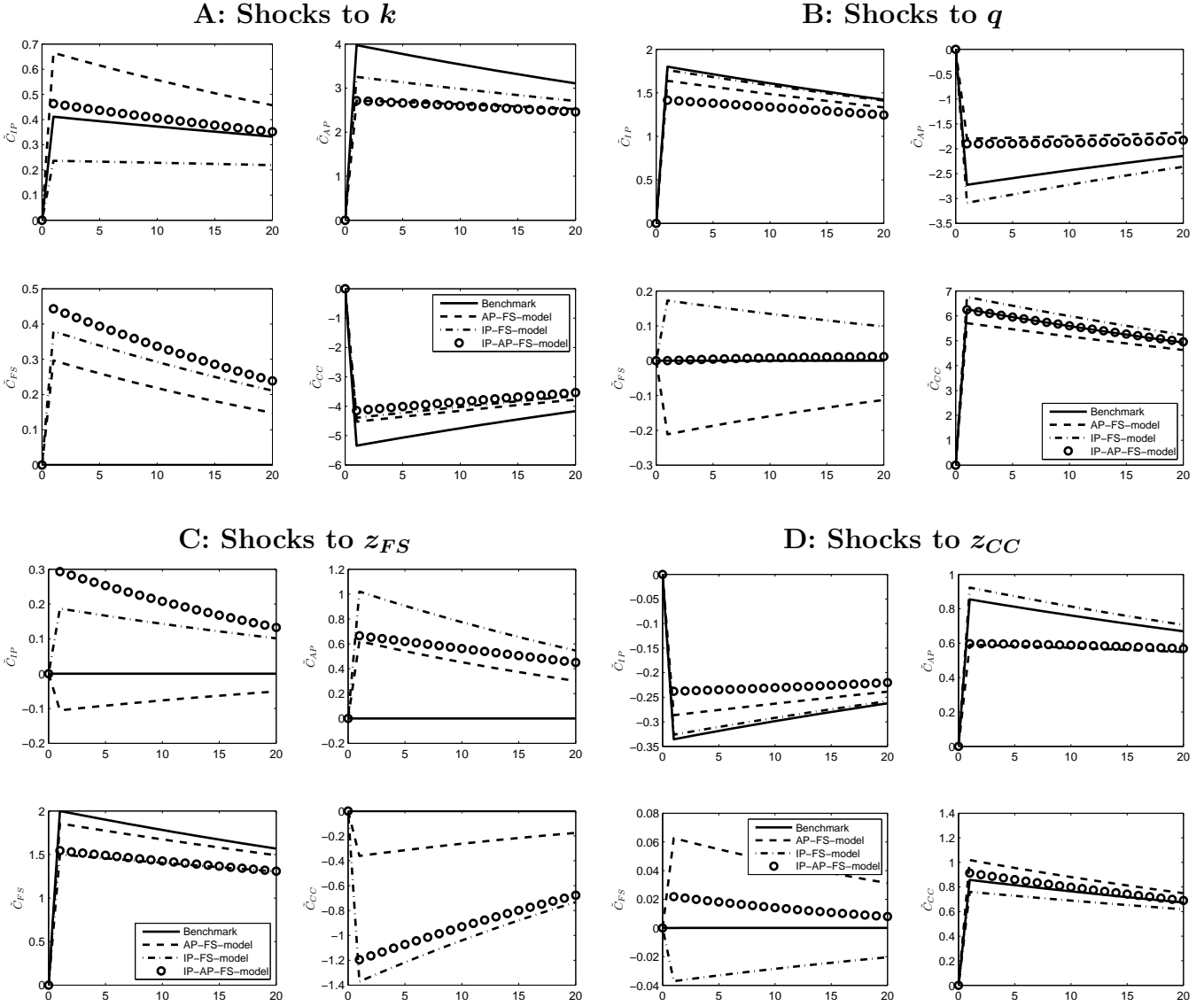
This figure depicts the impulse response functions in log deviations from the steady state in percentage points for the industrial and agricultural producer's non-commodity consumption levels $C_{i,t}$ and commodity consumption levels $Q_{i,t}$, where $i \in \{IP, AP\}$, for our four model specifications. The graphs in Panel A depict the impulse response function with respect to a one standard deviation shock in the industrial producer's capital endowment k_t . The graphs in Panel B depict the impulse response function with respect to a one standard deviation shock in the agricultural producer's commodity endowment q_t . The graphs in Panel C depict the impulse response function with respect to a one standard deviation shock in the financial speculator's wage endowment $z_{FS,t}$. The graphs in Panel D depict the impulse response function with respect to a one standard deviation shock in the commodity consumer's wage endowment $z_{CC,t}$.

Figure 4: Impulse response functions: commodity trade



This figure depicts the impulse response functions in log deviations from the steady state in percentage points for the amount of commodity used in production $Q_{P,t}$, the value of commodities bought for production $S_t Q_{P,t}$ by the industrial producer, the value of commodities bought for consumption by the industrial producer $S_t Q_{IP,t}$, and the amount of commodity bought by the commodity consumer for consumption $Q_{CC,t}$ for our four model specifications. The graphs in Panel A depict the impulse response function with respect to a one standard deviation shock in the industrial producer's capital endowment k_t . The graphs in Panel B depict the impulse response function with respect to a one standard deviation shock in the agricultural producer's commodity endowment q_t . The graphs in Panel C depict the impulse response function with respect to a one standard deviation shock in the financial speculator's wage endowment $z_{FS,t}$. The graphs in Panel D depict the impulse response function with respect to a one standard deviation shock in the commodity consumer's wage endowment $z_{CC,t}$.

Figure 5: Impulse response functions: agents' consumption bundles



This figure depicts the impulse response functions in log deviations from the steady state in percentage points for the agents' consumption bundle levels $\tilde{C}_{i,t}$, where $i \in \{IP, AP, FS, CC\}$, for our four model specifications. The graphs in Panel A depict the impulse response function with respect to a one standard deviation shock in the industrial producer's capital endowment k_t . The graphs in Panel B depict the impulse response function with respect to a one standard deviation shock in the agricultural producer's commodity endowment q_t . The graphs in Panel C depict the impulse response function with respect to a one standard deviation shock in the financial speculator's wage endowment $z_{FS,t}$. The graphs in Panel D depict the impulse response function with respect to a one standard deviation shock in the commodity consumer's wage endowment $z_{CC,t}$.

A Technical Appendix

A.1 IP's Optimization Problem

The optimization problem of the industrial producer is to maximize her lifetime utility by choosing non-commodity good consumption, commodity allocation for production and consumption and the next period's bond holdings such that her budget constraint and the commodity resource constraint are satisfied. The objective function is

$$\max_{\{C_{IP,t}, Q_{P,t}, Q_{IP,t}, B_{IP,t+1}\}} \left\{ (1 - \beta_{IP}) \left[\left(\tilde{C}_{IP,t} \right)^{\frac{1-\gamma_{IP}}{\theta_{IP}}} \right] + \beta_{IP} \left(\mathbb{E}_t [(U_{IP,t+1})^{1-\gamma_{IP}}] \right)^{\frac{1}{\theta_{IP}}} \right\}^{\frac{\theta_{IP}}{1-\gamma_{IP}}},$$

subject to

$$\begin{aligned} \tilde{C}_{IP,t} &= (\phi_{IP} C_{IP,t}^{\rho_{IP}} + (1 - \phi_{IP}) Q_{IP,t}^{\rho_{IP}})^{\frac{1}{\rho_{IP}}}, \\ C_{IP,t} &= Y_t - S_t Q_{P,t} - S_t Q_{IP,t} + B_{IP,t} R_{f,t-1} - B_{IP,t+1} - \frac{\nu_1}{e^{\mu_q t}} B_{IP,t+1}^2. \end{aligned}$$

Thus, the Lagrangian with Lagrange multiplier $\lambda_{IP,t}$ is given by

$$\begin{aligned} \mathcal{L} &= \left\{ (1 - \beta_{IP}) \left[(\phi_{IP} C_{IP,t}^{\rho_{IP}} + (1 - \phi_{IP}) Q_{IP,t}^{\rho_{IP}})^{\frac{1-\gamma_{IP}}{\rho_{IP}\theta_{IP}}} \right] + \beta_{IP} \left(\mathbb{E}_t [(U_{IP,t+1})^{1-\gamma_{IP}}] \right)^{\frac{1}{\theta_{IP}}} \right\}^{\frac{\theta_{IP}}{1-\gamma_{IP}}} \\ &\quad + \lambda_{IP,t} \left(Y_t - S_t Q_{P,t} - S_t Q_{IP,t} + B_{IP,t} R_{f,t-1} - B_{IP,t+1} - \frac{\nu_1}{e^{\mu_q t}} B_{IP,t+1}^2 - C_{IP,t} \right). \end{aligned}$$

The first order condition with respect to consumption $C_{IP,t}$ implies

$$\frac{\partial U_{IP,t}}{\partial C_{IP,t}} = \lambda_{IP,t}.$$

Plugging in and rearranging terms gives

$$\begin{aligned} 0 &= (1 - \beta_{IP}) U_{IP,t}^{1-\frac{1-\gamma_{IP}}{\theta_{IP}}} \phi_{IP} C_{IP,t}^{\rho_{IP}-1} (\phi_{IP} C_{IP,t}^{\rho_{IP}} + (1 - \phi_{IP}) Q_{IP,t}^{\rho_{IP}})^{\frac{1-\gamma_{IP}}{\rho_{IP}\theta_{IP}}-1} - \lambda_{IP,t} \\ \lambda_{IP,t} &= (1 - \beta_{IP}) U_{IP,t}^{1-\frac{1-\gamma_{IP}}{\theta_{IP}}} \phi_{IP} C_{IP,t}^{\rho_{IP}-1} \left(\frac{1}{\phi_{IP} C_{IP,t}^{\rho_{IP}} + (1 - \phi_{IP}) Q_{IP,t}^{\rho_{IP}}} \right)^{\frac{\rho_{IP}\theta_{IP} + \gamma_{IP} - 1}{\rho_{IP}\theta_{IP}}} \\ \lambda_{IP,t} &= (1 - \beta_{IP}) \phi_{IP}^{\frac{1-\gamma_{IP}}{\rho_{IP}\theta_{IP}}} U_{IP,t}^{1-\frac{1-\gamma_{IP}}{\theta_{IP}}} C_{IP,t}^{\frac{1-\gamma_{IP}-\theta_{IP}}{\theta_{IP}}} x_{IP,t}^{\frac{\rho_{IP}\theta_{IP} + \gamma_{IP} - 1}{\rho_{IP}\theta_{IP}}} \\ \lambda_{IP,t} &= (1 - \beta_{IP}) \phi_{IP}^{\frac{1-\gamma_{IP}}{\rho_{IP}\theta_{IP}}} U_{IP,t}^{\frac{1}{\psi_{IP}}} C_{IP,t}^{-\frac{1}{\psi_{IP}}} x_{IP,t}^{-\xi_{IP}}, \end{aligned} \tag{A1}$$

where $\xi_{IP} = \frac{\psi_{IP}(1-\rho_{IP})-1}{\psi_{IP}\rho_{IP}}$ and $x_{IP,t} = \frac{\phi_{IP} C_{IP,t}^{\rho_{IP}}}{\phi_{IP} C_{IP,t}^{\rho_{IP}} + (1-\phi_{IP}) Q_{IP,t}^{\rho_{IP}}}$. The pricing kernel for the

industrial producer $M_{t,t+1}^{(IP)}$ is therefore given by

$$\begin{aligned}
M_{t,t+1}^{(IP)} &= \frac{\partial U_{IP,t}/\partial C_{IP,t+1}}{\partial U_{IP,t}/\partial C_{IP,t}} \\
&= \frac{\beta_{IP} U_{IP,t}^{1-\frac{1-\gamma_{IP}}{\theta_{IP}}} (\mathbb{E}_t[U_{IP,t+1}^{1-\gamma_{IP}}])^{\frac{1}{\theta_{IP}}-1} \mathbb{E}_t \left[U_{IP,t+1}^{-\gamma_{IP}} \frac{\partial U_{IP,t+1}}{\partial C_{IP,t+1}} \right]}{\lambda_{IP,t}} \\
&= \frac{\beta_{IP} U_{IP,t}^{1-\frac{1-\gamma_{IP}}{\theta_{IP}}} (\mathbb{E}_t[U_{IP,t+1}^{1-\gamma_{IP}}])^{\frac{1}{\theta_{IP}}-1} \mathbb{E}_t \left[U_{IP,t+1}^{-\gamma_{IP}} \lambda_{IP,t+1} \right]}{\lambda_{IP,t}} \\
&\stackrel{(A1)}{=} \beta_{IP} \left(\frac{C_{IP,t+1}}{C_{IP,t}} \right)^{-\frac{1}{\psi_{IP}}} \left(\frac{x_{IP,t+1}}{x_{IP,t}} \right)^{-\xi_{IP}} \left(\frac{U_{IP,t+1}^{1-\gamma_{IP}}}{\mathbb{E}_t[U_{IP,t+1}^{1-\gamma_{IP}}]} \right)^{1-\frac{1}{\theta_{IP}}}. \quad (A2)
\end{aligned}$$

Next, the first order condition with respect to the allocation of the commodity good towards production $Q_{P,t}$ is given by

$$\frac{\partial U_{IP,t}}{\partial Q_{IP,t}} = \lambda_{IP,t} \frac{\partial Y_t}{\partial Q_{P,t}}.$$

Plugging in and using the definition of production output (8) gives

$$(1 - \phi_{IP}) (Q_{IP,t})^{\rho_{IP}-1} \stackrel{(A1)}{=} (\eta K_t^\nu + (1 - \eta) Q_{P,t}^\nu)^{\frac{1}{\nu}-1} Q_{P,t}^{\nu-1} (1 - \eta) \phi_{IP} C_{IP,t}^{\rho_{IP}-1},$$

or stated differently

$$\frac{\partial U_{IP}}{\partial Q_{IP,t}} = \frac{\partial Y_t}{\partial Q_{P,t}} \frac{\partial U_{IP}}{\partial C_{IP,t}},$$

where U_{IP} denote the utility function of the industrial producer. The first order condition with respect to $Q_{IP,t}$ implies

$$\frac{\partial U_{IP,t}}{Q_{IP,t}} - \lambda_{IP,t} S_t = 0,$$

and thus the spot price solves

$$S_t = \frac{1 - \phi_{IP}}{\phi_{IP}} \left(\frac{Q_{IP,t}}{C_{IP,t}} \right)^{\rho_{IP}-1}.$$

We will use the following implication from the envelope theorem in the following

$$\frac{\partial U_{IP,t+1}^*}{\partial B_{IP,t+1}} = \left. \frac{\partial \mathcal{L}}{\partial B_{IP,t+1}} \right|_{B_{IP,t+1}=B_{IP,t+1}^*},$$

where B^* denotes optimal bond holdings and U^* the value function, i.e. the optimized

utility function. Finally, the first order condition with respect to $B_{IP,t+1}$ is

$$\begin{aligned}
0 &= \beta_{IP} U_{IP,t}^{1-\frac{1-\gamma_{IP}}{\theta_{IP}}} (\mathbb{E}_t[U_{IP,t+1}^{1-\gamma_{IP}}])^{\frac{1}{\theta_{IP}}-1} \mathbb{E}_t \left[U_{IP,t+1}^{-\gamma_{IP}} \frac{\partial U_{IP,t+1}}{\partial B_{IP,t+1}} \right] - \lambda_{IP,t} - \lambda_{IP,t} 2\nu_1 \hat{B}_{IP,t+1} \\
1 &= \mathbb{E}_t \left[\frac{\beta_{IP} U_{IP,t}^{\frac{1}{\psi_{IP}}} (\mathbb{E}_t[U_{IP,t+1}^{1-\gamma_{IP}}])^{\frac{1}{\theta_{IP}}-1} U_{IP,t+1}^{-\gamma_{IP}} \frac{\partial \mathcal{L}}{\partial B_{IP,t+1}}}{\lambda_{IP,t}} - 2\nu_1 \hat{B}_{IP,t+1} \right] \\
1 &= \mathbb{E}_t \left[\frac{\beta_{IP} U_{IP,t}^{\frac{1}{\psi_{IP}}} (\mathbb{E}_t[U_{IP,t+1}^{1-\gamma_{IP}}])^{\frac{1}{\theta_{IP}}-1} U_{IP,t+1}^{-\gamma_{IP}} \{\lambda_{IP,t+1} (R_{f,t})\}}{\lambda_{IP,t}} - 2\nu_1 \hat{B}_{IP,t+1} \right] \\
1 &\stackrel{(A1)}{=} \mathbb{E}_t \left[\beta_{IP} \left(\frac{C_{IP,t+1}}{C_{IP,t}} \right)^{-\frac{1}{\psi_{IP}}} \left(\frac{x_{IP,t+1}}{x_{IP,t}} \right)^{-\xi_{IP}} \left(\frac{U_{IP,t+1}^{1-\gamma_{IP}}}{\mathbb{E}_t[U_{IP,t+1}^{1-\gamma_{IP}}]} \right)^{1-\frac{1}{\theta_{IP}}} (R_{f,t}) - 2\nu_1 \hat{B}_{IP,t+1} \right] \\
1 &= \mathbb{E}_t \left[M_{t,t+1}^{(IP)} R_{f,t} - 2\nu_1 \hat{B}_{IP,t+1} \right],
\end{aligned}$$

where $\hat{B}_{IP,t+1} = \frac{B_{IP,t+1}}{e^{\mu_q t}}$ is normalized bond holdings of the industrial producer and $M_{t,t+1}^{(IP)}$ the pricing kernel of the industrial producer for payoffs expressed in units of the non-commodity consumption good (see equation (A2) or also (3)). The model is solved in these stationary normalized variables as our economy is growing.

A.2 AP's Optimization Problem

The optimization problem of the agricultural producer is to maximize her lifetime utility by choosing non-commodity good consumption, commodity consumption and the next period's futures holdings such that her budget constraint and the commodity resource constraint are satisfied. The objective function is

$$\max_{\{C_{AP,t}, Q_{AP,t}, n_{AP,t+1}\}} \left\{ (1 - \beta_{AP}) \left[\left(\tilde{C}_{AP,t} \right)^{\frac{1-\gamma_{AP}}{\theta_{AP}}} \right] + \beta_{AP} (\mathbb{E}_t [(U_{IP,t+1})^{1-\gamma_{AP}}])^{\frac{1}{\theta_{AP}}} \right\}^{\frac{\theta_{AP}}{1-\gamma_{AP}}},$$

subject to

$$\begin{aligned}
\tilde{C}_{AP,t} &= (\phi_{AP} C_{AP,t}^{\rho_{AP}} + (1 - \phi_{AP}) Q_{AP,t}^{\rho_{AP}})^{\frac{1}{\rho_{AP}}}, \\
C_{AP,t} &= (Q_t - Q_{AP,t}) S_t + n_{AP,t} S_t - n_{AP,t+1} F_t - \frac{\nu_2}{e^{\mu_q t}} n_{AP,t+1}^2.
\end{aligned}$$

Note that $Q_t = Q_{IP,t} + Q_{AP,t} + Q_{P,t} + Q_{CC,t}$ as given in Equation (21).

Thus, the Lagrangian with Lagrange multiplier $\lambda_{AP,t}$ is given by

$$\mathcal{L} = \left\{ (1 - \beta_{AP}) \left[(\phi_{AP} C_{AP,t}^{\rho_{AP}} + (1 - \phi_{AP}) Q_{AP,t}^{\rho_{AP}})^{\frac{1-\gamma_{AP}}{\rho_{AP}\theta_{AP}}} \right] + \beta_{AP} (\mathbb{E}_t [(U_{AP,t+1})^{1-\gamma_{AP}}])^{\frac{1}{\theta_{AP}}} \right\}^{\frac{\theta_{AP}}{1-\gamma_{AP}}} + \lambda_{AP,t} \left((Q_t - Q_{AP,t}) S_t + n_{AP,t} S_t - n_{AP,t+1} F_t - \frac{\nu_2 n_{AP,t+1}^2}{e^{\mu_q t}} - C_{AP,t} \right).$$

The first order condition with respect to consumption $C_{AP,t}$ implies

$$\frac{\partial U_{AP,t}}{\partial C_{AP,t}} = \lambda_{AP,t}.$$

Plugging in and rearranging terms gives (similarly as for the industrial producer in Section A.1)

$$\lambda_{AP,t} = (1 - \beta_{AP}) \phi_{AP}^{\frac{1-\gamma_{AP}}{\rho_{AP}\theta_{AP}}} U_{AP,t}^{\frac{1}{\psi_{AP}}} C_{AP,t}^{-\frac{1}{\psi_{AP}}} x_{AP,t}^{-\xi_{AP}}, \quad (\text{A3})$$

where $\xi_{AP} = \frac{\psi_{IP}(1-\rho_{IP})-1}{\psi_{IP}\rho_{IP}}$ and $x_{AP,t} = \frac{\phi_{AP} C_{AP,t}^{\rho_{AP}}}{\phi_{AP} C_{AP,t}^{\rho_{AP}} + (1-\phi_{AP}) Q_{AP,t}^{\rho_{AP}}}$. The pricing kernel for the agricultural producer $M_{t,t+1}^{(AP)}$ is therefore given by

$$\begin{aligned} M_{t,t+1}^{(AP)} &= \frac{\partial U_{AP,t}/\partial C_{AP,t+1}}{\partial U_{AP,t}/\partial C_{AP,t}} \\ &= \frac{\beta_{AP} U_{AP,t}^{1-\frac{1-\gamma_{AP}}{\theta_{AP}}} (\mathbb{E}_t [U_{AP,t+1}^{1-\gamma_{AP}}])^{\frac{1}{\theta_{AP}}-1} \mathbb{E}_t \left[U_{AP,t+1}^{-\gamma_{AP}} \frac{\partial U_{AP,t+1}}{\partial C_{AP,t+1}} \right]}{\lambda_{AP,t}} \\ &= \frac{\beta_{AP} U_{AP,t}^{1-\frac{1-\gamma_{AP}}{\theta_{AP}}} (\mathbb{E}_t [U_{AP,t+1}^{1-\gamma_{AP}}])^{\frac{1}{\theta_{AP}}-1} \mathbb{E}_t \left[U_{AP,t+1}^{-\gamma_{AP}} \lambda_{AP,t+1} \right]}{\lambda_{AP,t}} \\ &\stackrel{(\text{A3})}{=} \beta_{AP} \left(\frac{C_{AP,t+1}}{C_{AP,t}} \right)^{-\frac{1}{\psi_{AP}}} \left(\frac{x_{AP,t+1}}{x_{AP,t}} \right)^{-\xi_{AP}} \left(\frac{U_{AP,t+1}^{1-\gamma_{AP}}}{\mathbb{E}_t [U_{AP,t+1}^{1-\gamma_{AP}}]} \right)^{1-\frac{1}{\theta_{AP}}}. \end{aligned} \quad (\text{A4})$$

Next, the first order condition with respect to $Q_{AP,t}$ implies

$$\frac{\partial U_{AP,t}}{\partial Q_{AP,t}} - \lambda_{AP,t} S_t = 0,$$

and thus the spot price solves

$$S_t = \frac{1 - \phi_{AP}}{\phi_{AP}} \left(\frac{Q_{AP,t}}{C_{AP,t}} \right)^{\rho_{AP}-1}.$$

Coupled with the condition for the spot price of the previous section we obtain

$$\frac{1 - \phi_{IP}}{\phi_{IP}} \left(\frac{Q_{IP,t}}{C_{IP,t}} \right)^{\rho_{IP}-1} = \frac{1 - \phi_{AP}}{\phi_{AP}} \left(\frac{Q_{AP,t}}{C_{AP,t}} \right)^{\rho_{AP}-1}.$$

We will use the following implication from the envelope theorem in the following

$$\frac{\partial U_{AP,t+1}^*}{\partial n_{AP,t+1}} = \frac{\partial \mathcal{L}}{\partial n_{AP,t+1}} \Big|_{n_{AP,t+1} = \phi_{AP,t+1}^*},$$

where ϕ^* denotes optimal bond holdings and U^* the value function, i.e. the optimized utility function. Finally, the first order condition with respect to $n_{AP,t+1}$ is

$$\begin{aligned} F_t &= \beta_{AP} U_{AP,t}^{1 - \frac{1-\gamma_{AP}}{\theta_{AP}}} \left(\mathbb{E}_t[U_{AP,t+1}^{1-\gamma_{AP}}] \right)^{\frac{1}{\theta_{AP}}-1} \mathbb{E}_t \left[U_{AP,t+1}^{-\gamma_{AP}} \frac{\partial U_{AP,t+1}}{\partial n_{AP,t+1}} \right] - \lambda_{AP,t} 2\nu_2 \hat{n}_{AP,t+1} \\ F_t &= \mathbb{E}_t \left[\frac{\beta_{AP} U_{AP,t}^{\frac{1}{\psi_{AP}}} \left(\mathbb{E}_t[U_{AP,t+1}^{1-\gamma_{AP}}] \right)^{\frac{1}{\theta_{AP}}-1} U_{AP,t+1}^{-\gamma_{AP}} \{ \lambda_{AP,t+1}(S_{t+1}) \}}{\lambda_{AP,t}} - 2\nu_2 \hat{n}_{AP,t+1} \right] \\ F_t &\stackrel{(A3)}{=} \mathbb{E}_t \left[\beta_{AP} \left(\frac{C_{AP,t+1}}{C_{AP,t}} \right)^{-\frac{1}{\psi_{AP}}} \left(\frac{x_{AP,t+1}}{x_{AP,t}} \right)^{-\xi_{AP}} \frac{U_{AP,t+1}^{\frac{1}{\psi_{AP}} - \gamma_{AP}}}{\left(\mathbb{E}_t[U_{AP,t+1}^{1-\gamma_{AP}}] \right)^{1 - \frac{1}{\theta_{AP}}}} (S_{t+1}) - 2\nu_2 \hat{n}_{AP,t+1} \right] \\ F_t &= \mathbb{E}_t \left[\beta_{AP} \left(\frac{C_{AP,t+1}}{C_{AP,t}} \right)^{-\frac{1}{\psi_{AP}}} \left(\frac{x_{AP,t+1}}{x_{AP,t}} \right)^{-\xi_{AP}} \left(\frac{U_{AP,t+1}^{1-\gamma_{AP}}}{\mathbb{E}_t[U_{AP,t+1}^{1-\gamma_{AP}}]} \right)^{1 - \frac{1}{\theta_{AP}}} (S_{t+1}) - 2\nu_2 \hat{n}_{AP,t+1} \right] \\ F_t &= \mathbb{E}_t \left[M_{t,t+1}^{(AP)} S_{t+1} - 2\nu_2 \hat{n}_{AP,t+1} \right], \end{aligned}$$

where $\hat{n}_{AP,t+1} = \frac{n_{AP,t+1}}{e^{\mu q \cdot t}}$ is normalized futures holdings of the agricultural producer and $M_{t,t+1}^{(AP)}$ the pricing kernel of the agricultural producer for payoffs expressed in units of the non-commodity consumption good (see equation (A4) or also (3)).

A.3 FS's Optimization Problem

The optimization problem of the financial speculator is to maximize her lifetime utility by choosing (non-commodity good) consumption, the next period's bond holdings and the next period's futures holdings such that her budget constraint is satisfied. The objective function is

$$\max_{\{C_{FS,t}, B_{FS,t+1}, n_{FS,t+1}\}} \left\{ (1 - \beta_{FS}) \left[\tilde{C}_{FS,t}^{\frac{1-\gamma_{FS}}{\theta_{FS}}} \right] + \beta_{FS} \left(\mathbb{E}_t \left[(U_{FS,t+1})^{1-\gamma_{FS}} \right] \right)^{\frac{1}{\theta_{FS}}} \right\}^{\frac{\theta_{FS}}{1-\gamma_{FS}}},$$

subject to

$$\tilde{C}_{FS,t} = C_{FS,t}$$

$$C_{FS,t} = Z_{FS,t} + B_{FS,t}R_{f,t-1} + n_{FS,t}S_t - n_{FS,t+1}F_t - B_{FS,t+1} - \frac{\nu_1}{e^{\mu_q \cdot t}}B_{FS,t+1}^2 - \frac{\nu_2}{e^{\mu_q \cdot t}}n_{FS,t+1}^2.$$

Thus, the Lagrangian with Lagrange multiplier $\lambda_{FS,t}$ is given by

$$\begin{aligned} \mathcal{L} = & \left\{ (1 - \beta_{FS}) \left[\tilde{C}_{FS,t}^{\frac{1-\gamma_{FS}}{\theta_{FS}}} \right] + \beta_{FS} \left(\mathbb{E}_t \left[(U_{FS,t+1})^{1-\gamma_{FS}} \right] \right)^{\frac{1}{\theta_{FS}}} \right\}^{\frac{\theta_{FS}}{1-\gamma_{FS}}} \\ & + \lambda_{FS,t} \left(Z_{FS,t} + B_{FS,t}R_{f,t-1} + n_{FS,t}S_t - n_{FS,t+1}F_t - B_{FS,t+1} \right. \\ & \left. - \frac{\nu_1}{e^{\mu_q \cdot t}}B_{FS,t+1}^2 - \frac{\nu_2}{e^{\mu_q \cdot t}}n_{FS,t+1}^2 - C_{FS,t} \right). \end{aligned}$$

The first order condition with respect to consumption $C_{FS,t}$ implies

$$(1 - \beta_{FS})U_{FS,t}^{1-\frac{1-\gamma_{FS}}{\theta_{FS}}} C_{FS,t}^{-\frac{1}{\psi_{FS}}} - \lambda_{FS,t} = 0 \Rightarrow \lambda_{FS,t} = (1 - \beta_{FS})U_{FS,t}^{\frac{1}{\psi_{FS}}} C_{FS,t}^{-\frac{1}{\psi_{FS}}}. \quad (\text{A5})$$

The pricing kernel for the financial speculator $M_{t,t+1}^{(FS)}$ is therefore given by

$$\begin{aligned} M_{t,t+1}^{(FS)} &= \frac{\partial U_{FS,t} / \partial C_{FS,t+1}}{\partial U_{FS,t} / \partial C_{FS,t}} \\ &= \frac{\beta_{FS} U_{FS,t}^{1-\frac{1-\gamma_{FS}}{\theta_{FS}}} \left(\mathbb{E}_t [U_{FS,t+1}^{1-\gamma_{FS}}] \right)^{\frac{1}{\theta_{FS}}-1} \mathbb{E}_t \left[U_{FS,t+1}^{-\gamma_{FS}} \frac{\partial U_{FS,t+1}}{\partial C_{FS,t+1}} \right]}{\lambda_{FS,t}} \\ &= \frac{\beta_{FS} U_{FS,t}^{1-\frac{1-\gamma_{FS}}{\theta_{FS}}} \left(\mathbb{E}_t [U_{FS,t+1}^{1-\gamma_{FS}}] \right)^{\frac{1}{\theta_{FS}}-1} \mathbb{E}_t \left[U_{FS,t+1}^{-\gamma_{FS}} \lambda_{FS,t+1} \right]}{\lambda_{FS,t}} \\ &\stackrel{(\text{A5})}{=} \beta_{FS} \left(\frac{C_{FS,t+1}}{C_{FS,t}} \right)^{-\frac{1}{\psi_{FS}}} \left(\frac{U_{FS,t+1}^{1-\gamma_{FS}}}{\mathbb{E}_t [U_{FS,t+1}^{1-\gamma_{FS}}]} \right)^{1-\frac{1}{\theta_{FS}}}. \quad (\text{A6}) \end{aligned}$$

We will use the following implication from the envelope theorem in the following

$$\frac{\partial U_{FS,t+1}}{\partial B_{FS,t+1}} = \frac{\partial \mathcal{L}}{\partial B_{FS,t+1}}.$$

The equivalent condition holds for the derivative with respect to $n_{FS,t+1}$. Secondly, the first order condition with respect to $B_{FS,t+1}$ is

$$0 = \beta_{FS} U_{FS,t}^{1-\frac{1-\gamma_{FS}}{\theta_{FS}}} \left(\mathbb{E}_t [U_{FS,t+1}^{1-\gamma_{FS}}] \right)^{\frac{1}{\theta_{FS}}-1} \mathbb{E}_t \left[U_{FS,t+1}^{-\gamma_{FS}} \frac{\partial U_{FS,t+1}}{\partial B_{FS,t+1}} \right] - \lambda_{FS,t} - \lambda_{FS,t} 2\nu_1 \hat{B}_{FS,t+1}$$

$$\begin{aligned}
1 &= \mathbb{E}_t \left[\frac{\beta_{FS} U_{FS,t}^{\frac{1}{\psi_{FS}}} (\mathbb{E}_t[U_{FS,t+1}^{1-\gamma_{FS}}])^{\frac{1}{\theta_{FS}}-1} U_{FS,t+1}^{-\gamma_{FS}} \frac{\partial \mathcal{L}}{\partial B_{FS,t+1}}}{\lambda_{FS,t}} - 2\nu_1 \hat{B}_{FS,t+1} \right] \\
1 &\stackrel{(A5)}{=} \mathbb{E}_t \left[\frac{\beta_{FS} (\mathbb{E}_t[U_{FS,t+1}^{1-\gamma_{FS}}])^{\frac{1}{\theta_{FS}}-1} U_{FS,t+1}^{-\gamma_{FS}} \left\{ (1 - \beta_{FS}) U_{FS,t+1}^{\frac{1}{\psi_{FS}}} C_{FS,t+1}^{-\frac{1}{\psi_{FS}}} (R_{f,t}) \right\}}{(1 - \beta_{FS}) C_{FS,t}^{-\frac{1}{\psi_{FS}}}} - 2\nu_1 \hat{B}_{FS,t+1} \right] \\
1 &= \mathbb{E}_t \left[\beta_{FS} \left(\frac{C_{FS,t+1}}{C_{FS,t}} \right)^{-\frac{1}{\psi_{FS}}} \left(\frac{U_{FS,t+1}^{1-\gamma_{FS}}}{\mathbb{E}_t[U_{FS,t+1}^{1-\gamma_{FS}}]} \right)^{1-\frac{1}{\theta_{FS}}} (R_{f,t}) - 2\nu_1 \hat{B}_{FS,t+1} \right] \\
1 &= \mathbb{E}_t \left[M_{t,t+1}^{(FS)} R_{f,t} - 2\nu_1 \hat{B}_{FS,t+1} \right].
\end{aligned}$$

Finally, the first order condition with respect to $n_{FS,t+1}$ is

$$\begin{aligned}
F_t &= \mathbb{E}_t \left[\beta_{FS} U_{FS,t}^{1-\frac{1}{\psi_{FS}}} (\mathbb{E}_t[U_{FS,t+1}^{1-\gamma_{FS}}])^{\frac{1}{\theta_{FS}}-1} \mathbb{E}_t \left[U_{FS,t+1}^{-\gamma_{FS}} \frac{\partial U_{FS,t+1}}{\partial n_{FS,t+1}} \right] - 2\nu_2 \lambda_{FS,t} \hat{n}_{FS,t+1} \right] \\
F_t &= \mathbb{E}_t \left[\frac{\beta_{FS} U_{FS,t}^{\frac{1}{\psi_{FS}}} (\mathbb{E}_t[U_{FS,t+1}^{1-\gamma_{FS}}])^{\frac{1}{\theta_{FS}}-1} U_{FS,t+1}^{-\gamma_{FS}} \{ \lambda_{FS,t+1} (S_{t+1}) \}}{\lambda_{FS,t}} - 2\nu_2 \hat{n}_{FS,t+1} \right] \\
F_t &\stackrel{(A5)}{=} \mathbb{E}_t \left[\beta_{FS} \left(\frac{C_{FS,t+1}}{C_{FS,t}} \right)^{-\frac{1}{\psi_{FS}}} \left(\frac{U_{FS,t+1}^{1-\gamma_{FS}}}{\mathbb{E}_t[U_{FS,t+1}^{1-\gamma_{FS}}]} \right)^{1-\frac{1}{\theta_{FS}}} (S_{t+1}) - 2\nu_2 \hat{n}_{FS,t+1} \right], \\
F_t &= \mathbb{E}_t \left[M_{t,t+1}^{(FS)} S_{t+1} - 2\nu_2 \hat{n}_{FS,t+1} \right],
\end{aligned}$$

where $\hat{n}_{FS,t+1} = \frac{n_{FS,t+1}}{e^{\mu q \cdot t}}$ is normalized futures holdings of the financial speculator and $M_{t,t+1}^{(FS)}$ the pricing kernel of the financial speculator (see equation (A6) or also (3)).

A.4 CC's Optimization Problem

CC's optimization problem is to maximize her lifetime utility by choosing the amount of commodities she is buying from the producing agent (this amount is equal to her consumption as she only derives utility from commodity consumption):

$$\max_{\{\tilde{C}_{CC,t}\}} \left\{ (1 - \beta_{CC}) \left[\tilde{C}_{CC,t}^{\frac{1-\gamma_{CC}}{\theta_{CC}}} \right] + \beta_{CC} (\mathbb{E}_t [(U_{CC,t+1})^{1-\gamma_{CC}}])^{\frac{1}{\theta_{CC}}} \right\}^{\frac{\theta_{CC}}{1-\gamma_{CC}}},$$

subject to

$$\tilde{C}_{CC,t} = Q_{CC,t}.$$

The commodity consumer is endowed with $Z_{CC,t}$ units of the non-commodity good. Since she only derives utility from commodity consumption $Q_{CC,t}$ and has no option to use the non-commodity good elsewhere (there are no financial markets available for the commodity consumer), she will invest her whole endowment into commodities and thus it must hold in equilibrium

$$\tilde{C}_{CC,t} = Q_{CC,t} = \frac{Z_{CC,t}}{S_t},$$

since $1/S_t$ is the price of the non-commodity good in commodity units.

A.5 Pricing Kernel in Terms of the Return on Wealth

As in Cochrane (2006), pp. 91-92, we now rewrite IP's pricing kernel²⁵ here in terms of the return on wealth which is necessary to implement the pricing kernel in Dynare. The agricultural producer's pricing kernel is rewritten analogously. We define wealth as the claim to effective consumption

$$\tilde{C}_{IP,t} = (\phi_{IP} C_{IP,t}^{\rho_{IP}} + (1 - \phi_{IP}) Q_{IP,t}^{\rho_{IP}})^{\frac{1}{\rho_{IP}}},$$

and thus wealth is defined as

$$W_{IP,t} = \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j}^{(IP)} \tilde{C}_{IP,t+j} \right].$$

The utility function in terms of this effective consumption is defined as

$$U_{IP,t} = \left\{ (1 - \beta_{IP}) \tilde{C}_{IP,t}^{\frac{1-\gamma_{IP}}{\theta_{IP}}} + \beta_{IP} (\mathbb{E}_t[U_{IP,t+1}^{1-\gamma_{IP}}])^{\frac{1}{\theta_{IP}}} \right\}^{\frac{\theta_{IP}}{1-\gamma_{IP}}},$$

which is the standard EZ utility function when consumption is \tilde{C} . We can thus almost proceed one-to-one as in Cochrane (2006). This utility function is linearly homogeneous in \tilde{C} and thus

$$\begin{aligned} U_{IP,t} &= \sum_{j=0}^{\infty} \frac{\partial U_{IP,t}}{\partial \tilde{C}_{IP,t+j}} \tilde{C}_{IP,t+j} = \mathbb{E}_t \left[\sum_{j=0}^{\infty} \frac{\partial U_{IP,t}}{\partial \tilde{C}_{IP,t+j}} \tilde{C}_{IP,t+j} \right] \\ \frac{U_{IP,t}}{\partial U_{IP,t} / \partial \tilde{C}_{IP,t}} &= \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j}^{(IP)} \tilde{C}_{IP,t+j} \right] = W_{IP,t}. \end{aligned}$$

²⁵For the financial speculator the pricing kernel in terms of the return on wealth is directly given without alteration in Cochrane (2006), pp. 91-92.

From the definition of the utility function we obtain

$$\left(\mathbb{E}_t [U_{IP,t+1}^{1-\gamma_{IP}}]\right)^{\frac{1}{1-\gamma_{IP}}} = \left(\frac{1}{\beta_{IP}}\right)^{\frac{1}{1-\frac{1}{\psi_{IP}}}} \left[U_{IP,t}^{1-\frac{1}{\psi_{IP}}} - (1-\beta_{IP})\tilde{C}_{IP,t}^{1-\frac{1}{\psi_{IP}}} \right]^{\frac{1}{1-\frac{1}{\psi_{IP}}}}.$$

We obtain that we can rewrite wealth as

$$W_t = \frac{U_{IP,t}}{\partial U_{IP,t}/\partial \tilde{C}_{IP,t}} = \frac{U_{IP,t}}{U_{IP,t}^{\frac{1}{\psi_{IP}}} (1-\beta_{IP})\tilde{C}_{IP,t}^{-\frac{1}{\psi_{IP}}}} = \frac{1}{1-\beta_{IP}} U_{IP,t}^{1-\frac{1}{\psi_{IP}}} \tilde{C}_{IP,t}^{\frac{1}{\psi_{IP}}}.$$

By solving for U_t we obtain

$$U_{IP,t} = \left[W_{IP,t} (1-\beta_{IP}) \tilde{C}_{IP,t}^{-\frac{1}{\psi_{IP}}} \right]^{\frac{1}{1-\frac{1}{\psi_{IP}}}}.$$

Starting from the pricing kernel $M_{t,t+1}^{(IP)}$ given in equation (A2) or also (3) we obtain with the just derived relations

$$\begin{aligned} M_{t,t+1}^{(IP)} &= \beta_{IP} \left(\frac{U_{IP,t+1}}{\left(\mathbb{E}_t [U_{IP,t+1}^{1-\gamma_{IP}}]\right)^{\frac{1}{1-\gamma_{IP}}}} \right)^{\frac{1}{\psi_{IP}}-\gamma_{IP}} \left(\frac{C_{IP,t+1}}{C_{IP,t}} \right)^{-\frac{1}{\psi_{IP}}} \left(\frac{x_{IP,t+1}}{x_{IP,t}} \right)^{-\xi_{IP}} \\ &= \beta_{IP}^{\theta_{IP}} \left(\frac{W_{IP,t+1} \tilde{C}_{IP,t+1}^{-\frac{1}{\psi_{IP}}}}{W_{IP,t} \tilde{C}_{IP,t}^{-\frac{1}{\psi_{IP}}} - \tilde{C}_{IP,t}^{1-\frac{1}{\psi_{IP}}}} \right)^{\frac{1}{\psi_{IP}}-\gamma_{IP}} \left(\frac{C_{IP,t+1}}{C_{IP,t}} \right)^{-\frac{1}{\psi_{IP}}} \left(\frac{x_{IP,t+1}}{x_{IP,t}} \right)^{-\xi_{IP}} \\ &= \beta_{IP}^{\theta_{IP}} \left(\frac{W_{IP,t+1}}{W_{IP,t} - \tilde{C}_{IP,t}} \right)^{\theta_{IP}-1} \left(\frac{C_{IP,t+1}}{C_{IP,t}} \right)^{-\frac{1}{\psi_{IP}}} \left(\frac{\tilde{C}_{IP,t+1}}{\tilde{C}_{IP,t}} \right)^{-\frac{1}{\psi_{IP}}(\theta_{IP}-1)} \left(\frac{x_{IP,t+1}}{x_{IP,t}} \right)^{-\xi_{IP}} \\ &= \beta_{IP}^{\theta_{IP}} (R_{IP,t}^W)^{\theta_{IP}-1} \left(\frac{C_{IP,t+1}}{C_{IP,t}} \right)^{-\frac{1}{\psi_{IP}}} \left(\frac{\phi_{IP} C_{IP,t+1}^{\rho_{IP}} + (1-\phi_{IP}) Q_{t+1}^{\rho_{IP}}}{\phi_{IP} C_{IP,t}^{\rho_{IP}} + (1-\phi_{IP}) Q_{AP,t}^{\rho_{IP}}} \right)^{\frac{1-\theta_{IP}}{\rho_{IP}\psi_{IP}}} \left(\frac{x_{IP,t+1}}{x_{IP,t}} \right)^{-\xi_{IP}} \\ &= \beta_{IP}^{\theta_{IP}} (R_{IP,t}^W)^{\theta_{IP}-1} \left(\frac{C_{IP,t+1}}{C_{IP,t}} \right)^{-\frac{1}{\psi_{IP}}} \left(\frac{\phi_{IP} C_{IP,t+1}^{\rho_{IP}}/x_{IP,t+1}}{\phi_{IP} C_{IP,t}^{\rho_{IP}}/x_{IP,t}} \right)^{\frac{1-\theta_{IP}}{\rho_{IP}\psi_{IP}}} \left(\frac{x_{IP,t+1}}{x_{IP,t}} \right)^{-\xi_{IP}} \\ &= \beta_{IP}^{\theta_{IP}} (R_{IP,t}^W)^{\theta_{IP}-1} \left(\frac{C_{IP,t+1}}{C_{IP,t}} \right)^{-\frac{1}{\psi_{IP}} + \frac{1-\theta_{IP}}{\psi_{IP}}} \left(\frac{x_{IP,t+1}}{x_{IP,t}} \right)^{-\xi_{IP} - \frac{1-\theta_{IP}}{\psi_{IP}\rho_{IP}}} \\ &= \beta_{IP}^{\theta_{IP}} (R_{IP,t}^W)^{\theta_{IP}-1} \left(\frac{C_{IP,t+1}}{C_{IP,t}} \right)^{-\frac{\theta_{IP}}{\psi_{IP}}} \left(\frac{x_{IP,t+1}}{x_{IP,t}} \right)^{-\frac{\xi_{IP}\psi_{IP}\rho_{IP} + \theta_{IP}-1}{\psi_{IP}\rho_{IP}}}. \end{aligned}$$

A.6 Equilibrium Conditions

A.6.1 IP

The following collection of equations comprise the equilibrium for the industrial producer.

$$\begin{aligned}
U_{IP,t} &= \left\{ (1 - \beta_{IP}) \tilde{C}_{IP,t}^{\frac{1-\gamma_{IP}}{\theta_{IP}}} + \beta_{IP} (\mathbb{E}_t[U_{IP,t+1}^{1-\gamma_{IP}}])^{\frac{1}{\theta_{IP}}} \right\}^{\frac{\theta_{IP}}{1-\gamma_{IP}}} \\
(1 - \phi_{IP}) Q_{IP,t}^{\rho_{IP}-1} &= (\eta K_t^\nu + (1 - \eta) Q_{P,t}^\nu)^{\frac{1}{\nu}-1} Q_{P,t}^{\nu-1} (1 - \eta) \phi_{IP} C_{IP,t}^{\rho_{IP}-1} \\
1 &= \mathbb{E}_t \left[M_{t,t+1}^{(IP)} R_{f,t} - 2\nu_1 \frac{B_{IP,t+1}}{e^{\mu_q t}} \right] \\
S_t &= \frac{1 - \phi_{IP}}{\phi_{IP}} \left(\frac{Q_{IP,t}}{C_{IP,t}} \right)^{\rho_{IP}-1} \\
x_{IP,t} &= \frac{\phi_{IP} C_{IP,t}^{\rho_{IP}}}{\phi_{IP} C_{IP,t}^{\rho_{IP}} + (1 - \phi_{IP}) Q_t^{\rho_{IP}}} \\
k_t &= (1 - \varphi) \bar{k} + \varphi k_{t-1} + \varepsilon_{k,t} \\
K_t &= e^{\mu_k t + k_t} \\
C_{IP,t} &= Y_t - S_t Q_{P,t} - \bar{S}_t Q_{IP,t} + B_{IP,t} R_{f,t-1} - B_{IP,t+1} - \frac{\nu_1}{e^{\mu_q t}} B_{IP,t+1}^2 \\
Y_t &= Y(K_t, Q_{P,t}) = (\eta K_t^\nu + (1 - \eta) Q_{P,t}^\nu)^{\frac{1}{\nu}} \\
\tilde{C}_{IP,t} &= (\phi_{IP} C_{IP,t}^{\rho_{IP}} + (1 - \phi_{IP}) Q_{IP,t}^{\rho_{IP}})^{\frac{1}{\rho_{IP}}} \\
W_{IP,t} &= \tilde{C}_{IP,t} + \mathbb{E}_t[M_{t,t+1}^{(IP),aggr} W_{IP,t+1}] \\
R_{IP,t}^W &= \frac{W_{IP,t}}{W_{IP,t-1} - \tilde{C}_{IP,t-1}} \\
M_{t,t+1}^{(IP),aggr} &= \beta_{IP}^{\theta_{IP}} (R_{IP,t+1}^W)^{\theta_{IP}-1} \left(\frac{\tilde{C}_{IP,t+1}}{\tilde{C}_{IP,t}} \right)^{-\frac{\theta_{IP}}{\psi_{IP}}} \tag{A7} \\
M_{t,t+1}^{(IP)} &= \beta_{IP}^{\theta_{IP}} (R_{IP,t+1}^W)^{\theta_{IP}-1} \left(\frac{C_{IP,t+1}}{C_{IP,t}} \right)^{-\frac{\theta_{IP}}{\psi_{IP}}} \left(\frac{x_{IP,t+1}}{x_{IP,t}} \right)^{\frac{-\xi_{IP} \psi_{IP} \rho_{IP} + \theta_{IP} - 1}{\psi_{IP} \rho_{IP}}}.
\end{aligned}$$

A.6.2 AP

The following collection of equations comprises the equilibrium for the agricultural producer.

$$U_{AP,t} = \left\{ (1 - \beta_{AP}) \tilde{C}_{AP,t}^{\frac{1-\gamma_{AP}}{\theta_{AP}}} + \beta_{AP} (\mathbb{E}_t[U_{AP,t+1}^{1-\gamma_{AP}}])^{\frac{1}{\theta_{AP}}} \right\}^{\frac{\theta_{AP}}{1-\gamma_{AP}}}$$

$$\begin{aligned}
Q_t &= Q_{AP,t} + Q_{AP,t} + Q_{P,t} + Q_{CC,t} \\
F_t &= \mathbb{E}_t \left[M_{t,t+1}^{(AP)} S_{t+1} - 2\nu_2 \frac{n_{AP,t+1}}{e^{\mu_q \cdot t}} \right] \\
S_t &= \frac{1 - \phi_{AP}}{\phi_{AP}} \left(\frac{Q_{AP,t}}{C_{AP,t}} \right)^{\rho_{AP}-1} \\
x_{AP,t} &= \frac{\phi_{AP} C_{AP,t}^{\rho_{AP}}}{\phi_{AP} C_{AP,t}^{\rho_{AP}} + (1 - \phi_{AP}) Q_{AP,t}^{\rho_{AP}}} \\
q_t &= (1 - \varphi) \bar{q} + \varphi q_{t-1} + \varepsilon_{q,t} \\
Q_t &= e^{\mu_q \cdot t + q_t} \\
C_{AP,t} &= S_t (Q_t - Q_{AP,t}) + n_{AP,t} S_t - n_{AP,t+1} F_t - \frac{\nu_2}{e^{\mu_q \cdot t}} n_{AP,t+1}^2 \\
\tilde{C}_{AP,t} &= (\phi_{AP} C_{AP,t}^{\rho_{AP}} + (1 - \phi_{AP}) Q_{AP,t}^{\rho_{AP}})^{\frac{1}{\rho_{AP}}} \\
W_{AP,t} &= \tilde{C}_{AP,t} + \mathbb{E}_t [M_{t,t+1}^{(AP),aggr} W_{AP,t+1}] \\
R_{AP,t}^W &= \frac{W_{AP,t}}{W_{AP,t-1} - \tilde{C}_{AP,t-1}} \\
M_{t,t+1}^{(AP),aggr} &= \beta_{AP}^{\theta_{AP}} (R_{AP,t+1}^W)^{\theta_{AP}-1} \left(\frac{\tilde{C}_{AP,t+1}}{\tilde{C}_{AP,t}} \right)^{-\frac{\theta_{AP}}{\psi_{AP}}} \\
M_{t,t+1}^{(AP)} &= \beta_{AP}^{\theta_{AP}} (R_{AP,t+1}^W)^{\theta_{AP}-1} \left(\frac{C_{AP,t+1}}{C_{AP,t}} \right)^{-\frac{\theta_{AP}}{\psi_{AP}}} \left(\frac{x_{AP,t+1}}{x_{AP,t}} \right)^{\frac{-\xi_{AP} \psi_{AP} \rho_{AP} + \theta_{AP} - 1}{\psi_{AP} \rho_{AP}}} .
\end{aligned} \tag{A8}$$

A.6.3 FS

The following collection of equations comprises the equilibrium for the financial speculator.

$$\begin{aligned}
U_{FS,t} &= \left\{ (1 - \beta_{FS}) \tilde{C}_{FS,t}^{\frac{1-\gamma_{FS}}{\theta_{FS}}} + \beta_{FS} (\mathbb{E}_t [U_{FS,t+1}^{1-\gamma_{FS}}])^{\frac{1}{\theta_{FS}}} \right\}^{\frac{\theta_{FS}}{1-\gamma_{FS}}} \\
1 &= \mathbb{E}_t \left[M_{t,t+1}^{(FS)} R_{f,t} - 2\nu_1 \frac{B_{FS,t+1}}{e^{\mu_q \cdot t}} \right] \\
F_t &= \mathbb{E}_t \left[M_{t,t+1}^{(FS)} S_{t+1} - 2\nu_2 \frac{n_{FS,t+1}}{e^{\mu_q \cdot t}} \right] \\
z_{FS,t} &= (1 - \varphi) \bar{z}_{FS} + \varphi z_{FS,t-1} + \varepsilon_{FS,t} \\
Z_{FS,t} &= e^{\mu_{FS} \cdot t + z_{FS,t}} \\
\tilde{C}_{FS,t} &= C_{FS,t} \\
C_{FS,t} &= Z_t + B_{FS,t} R_{f,t-1} + n_{FS,t} S_t - n_{FS,t+1} F_t - B_{FS,t+1} - \frac{\nu_1}{e^{\mu_q \cdot t}} B_{FS,t+1}^2 - \frac{\nu_2}{e^{\mu_q \cdot t}} n_{FS,t+1}^2 \\
W_{FS,t} &= \tilde{C}_{FS,t} + \mathbb{E}_t [M_{t,t+1}^{(FS),aggr} W_{FS,t+1}]
\end{aligned}$$

$$\begin{aligned}
R_{FS,t}^W &= \frac{W_{FS,t}}{W_{FS,t-1} - \tilde{C}_{FS,t-1}} \\
M_{t,t+1}^{(FS),aggr} &= M_{t,t+1}^{(FS)} = \beta_{FS}^{\theta_{FS}} (R_{FS,t+1}^W)^{\theta_{FS}-1} \left(\frac{\tilde{C}_{FS,t+1}}{\tilde{C}_{FS,t}} \right)^{-\frac{\theta_{FS}}{\psi_{FS}}} .
\end{aligned} \tag{A9}$$

A.6.4 CC

The following collection of equations comprise the equilibrium for the consumer.

$$\begin{aligned}
U_{CC,t} &= \left\{ (1 - \beta_{CC}) \tilde{C}_{CC,t}^{\frac{1-\gamma_{CC}}{\theta_{CC}}} + \beta_{CC} (\mathbb{E}_t[U_{CC,t+1}^{1-\gamma_{CC}}])^{\frac{1}{\theta_{CC}}} \right\}^{\frac{\theta_{CC}}{1-\gamma_{CC}}} \\
z_{CC,t} &= (1 - \varphi) \bar{z}_{CC} + \varphi z_{CC,t-1} + \varepsilon_{CC,t} \\
Z_{CC,t} &= e^{\mu_{CC} \cdot t + z_{CC,t}} \\
\tilde{C}_{CC,t} &= Q_{CC,t} \\
Z_{CC,t} &= Q_{CC,t} S_t .
\end{aligned}$$

A.7 Normalized Equilibrium Conditions

Since the economy is growing we need to rescale it. We will choose $e^{\mu_q \cdot t}$ as the normalization variable for all (growing) variables and denote these variables by e.g. $\hat{C}_{AP,t} = \frac{C_{AP,t}}{e^{\mu_q \cdot t}}$, $\hat{\tilde{C}}_{AP,t} = \frac{\tilde{C}_{AP,t}}{e^{\mu_q \cdot t}}$ and so forth. The only exception is that we normalize the utility functions and wealth levels by its respective consumption, i.e. $\hat{U}_{i,t} = \frac{U_{i,t}}{C_{i,t}}$ and $\hat{W}_{i,t} = \frac{W_{i,t}}{C_{i,t}}$ for all $i \in \{IP, AP, FS, CC\}$. The bond and futures holdings are normalized by lagged $e^{\mu_q \cdot t}$, i.e. $\hat{B}_{i,t+1} = \frac{B_{i,t+1}}{e^{\mu_q \cdot t}}$ and $\hat{\phi}_{i,t+1} = \frac{\phi_{i,t+1}}{e^{\mu_q \cdot t}}$ for $i = IP, AP$ or FS . The full set of normalized equilibrium conditions is then given by

$$\begin{aligned}
\hat{U}_{IP,t} &= \left\{ (1 - \beta_{IP}) + \beta_{IP} \left(\mathbb{E}_t \left[\left(\hat{U}_{IP,t+1} \frac{\hat{\tilde{C}}_{IP,t+1}}{\hat{\tilde{C}}_{IP,t}} e^{\mu_q} \right)^{1-\gamma_{IP}} \right] \right)^{\frac{1}{\theta_{IP}}} \right\}^{\frac{\theta_{IP}}{1-\gamma_{IP}}} \\
(1 - \phi_{IP}) \hat{Q}_{IP,t}^{\rho-1} &= \left(\eta \hat{K}_t^\nu + (1 - \eta) \hat{Q}_{P,t}^\nu \right)^{\frac{1}{\nu}-1} \hat{Q}_{P,t}^{\nu-1} (1 - \eta) \phi_{IP} \hat{C}_{IP,t}^{\rho_{IP}-1} \\
\hat{Q}_t &= \hat{Q}_{IP,t} + \hat{Q}_{AP,t} + \hat{Q}_{P,t} + \hat{Q}_{CC,t} \\
1 &= \mathbb{E}_t \left[M_{t,t+1}^{(IP)} R_{f,t} - 2\nu_1 \hat{B}_{IP,t+1} \right] \\
S_t &= \frac{1 - \phi_{IP}}{\phi_{IP}} \left(\frac{\hat{Q}_{IP,t}}{\hat{\tilde{C}}_{IP,t}} \right)^{\rho_{IP}-1}
\end{aligned}$$

$$\begin{aligned}
x_{IP,t} &= \frac{\phi_{IP}\hat{C}_{IP,t}^{\rho_{IP}}}{\phi_{IP}\hat{C}_{IP,t}^{\rho_{IP}} + (1 - \phi_{IP})\hat{Q}_{IP,t}^{\rho_{IP}}} \\
\hat{C}_{IP,t} &= \left(\eta\hat{K}_t^\nu + (1 - \eta)\hat{Q}_{P,t}^\nu\right)^{\frac{1}{\nu}} - S_t\hat{Q}_{IP,t} - S_t\hat{Q}_{P,t} + \hat{B}_{IP,t}e^{-\mu_q}R_{f,t-1} - \hat{B}_{IP,t+1} - \nu_1\hat{B}_{IP,t+1}^2 \\
\hat{C}_{IP,t} &= \left(\phi_{IP}\hat{C}_{IP,t}^{\rho_{IP}} + (1 - \phi_{IP})\hat{Q}_{IP,t}^{\rho_{IP}}\right)^{\frac{1}{\rho_{IP}}} \\
\hat{W}_{IP,t} &= 1 + \mathbb{E}_t \left[M_{t,t+1}^{(IP),aggr} \hat{W}_{IP,t+1} \left(\frac{\hat{C}_{IP,t+1}}{\hat{C}_{IP,t}} e^{\mu_q} \right) \right] \\
R_{IP,t}^W &= \frac{\hat{W}_{IP,t} \frac{\hat{C}_{IP,t+1}}{\hat{C}_{IP,t}} e^{\mu_q}}{\hat{W}_{IP,t-1} - 1} \\
M_{t,t+1}^{(IP),aggr} &= \beta_{IP}^{\theta_{IP}} (R_{IP,t+1}^W)^{\theta_{IP}-1} \left(\frac{\hat{C}_{IP,t+1}}{\hat{C}_{IP,t}} e^{\mu_q} \right)^{-\frac{\theta_{IP}}{\psi_{IP}}} \\
M_{t,t+1}^{(IP)} &= \beta_{IP}^{\theta_{IP}} (R_{IP,t+1}^W)^{\theta_{IP}-1} \left(\frac{\hat{C}_{IP,t+1}}{\hat{C}_{IP,t}} e^{\mu_q} \right)^{-\frac{\theta_{IP}}{\psi_{IP}}} \left(\frac{x_{IP,t+1}}{x_{IP,t}} \right)^{\frac{\psi_{IP}(\rho_{IP}-1) + \theta_{IP}}{\psi_{IP}\rho_{IP}}} \\
k_t &= (1 - \varphi)\bar{k} + \varphi k_{t-1} + \varepsilon_{k,t} \\
\hat{K}_t &= e^{(\mu_k - \mu_q) \cdot t + k_t} \\
\hat{U}_{AP,t} &= \left\{ (1 - \beta_{AP}) + \beta_{AP} \left(\mathbb{E} \left[\left(\hat{U}_{AP,t+1} \frac{\hat{C}_{AP,t+1}}{\hat{C}_{AP,t}} e^{\mu_q} \right)^{1-\gamma_{AP}} \right] \right)^{\frac{1}{\theta_{AP}}} \right\}^{\frac{\theta_{AP}}{1-\gamma_{AP}}} \\
F_t &= \mathbb{E}_t \left[M_{t,t+1}^{(AP)} S_{t+1} - 2\nu_2 \hat{n}_{AP,t+1} \right] \\
S_t &= \frac{1 - \phi_{AP}}{\phi_{AP}} \left(\frac{\hat{Q}_{AP,t}}{\hat{C}_{AP,t}} \right)^{\rho_{AP}-1} \\
x_{AP,t} &= \frac{\phi_{AP}\hat{C}_{AP,t}^{\rho_{AP}}}{\phi_{AP}\hat{C}_{AP,t}^{\rho_{AP}} + (1 - \phi_{AP})\hat{Q}_{AP,t}^{\rho_{AP}}} \\
\hat{C}_{AP,t} &= S_t(\hat{Q}_t - \hat{Q}_{AP,t}) + \hat{n}_{AP,t}e^{-\mu_q}S_t - \hat{n}_{AP,t+1}F_t - \nu_2\hat{n}_{AP,t+1}^2 \\
\hat{C}_{AP,t} &= \left(\phi_{AP}\hat{C}_{AP,t}^{\rho_{AP}} + (1 - \phi_{AP})\hat{Q}_{AP,t}^{\rho_{AP}}\right)^{\frac{1}{\rho_{AP}}} \\
\hat{W}_{AP,t} &= 1 + \mathbb{E}_t \left[M_{t,t+1}^{(AP),aggr} \hat{W}_{AP,t+1} \left(\frac{\hat{C}_{AP,t+1}}{\hat{C}_{AP,t}} e^{\mu_q} \right) \right] \\
R_{AP,t}^W &= \frac{\hat{W}_{AP,t} \frac{\hat{C}_{AP,t+1}}{\hat{C}_{AP,t}} e^{\mu_q}}{\hat{W}_{AP,t-1} - 1}
\end{aligned}$$

$$\begin{aligned}
M_{t,t+1}^{(AP),aggr} &= \beta_{AP}^{\theta_{AP}} (R_{AP,t+1}^W)^{\theta_{AP}-1} \left(\frac{\hat{C}_{AP,t+1}}{\hat{C}_{AP,t}} e^{\mu_q} \right)^{-\frac{\theta_{AP}}{\psi_{AP}}} \\
M_{t,t+1}^{(AP)} &= \beta_{AP}^{\theta_{AP}} (R_{AP,t+1}^W)^{\theta_{AP}-1} \left(\frac{\hat{C}_{AP,t+1}}{\hat{C}_{AP,t}} e^{\mu_q} \right)^{-\frac{\theta_{AP}}{\psi_{AP}}} \left(\frac{x_{AP,t+1}}{x_{AP,t}} \right)^{\frac{\psi_{AP}(\rho_{AP}-1)+\theta_{AP}}{\psi_{AP}\rho_{AP}}} \\
q_t &= (1-\varphi)\bar{q} + \varphi q_{t-1} + \varepsilon_{q,t} \\
\hat{Q}_t &= e^{q_t} \\
1 &= \mathbb{E}_t \left[M_{t,t+1}^{(FS)} R_{f,t} - 2\nu_1 \hat{B}_{FS,t+1} \right] \\
F_t &= \mathbb{E}_t \left[M_{t,t+1}^{(FS)} S_{t+1} - 2\nu_2 \hat{n}_{FS,t+1} \right] \\
\hat{U}_{FS,t} &= \left\{ (1-\beta_{FS}) + \beta_{FS} \left(\mathbb{E}_t \left[\left(\hat{U}_{FS,t+1} \frac{\hat{C}_{FS,t+1}}{\hat{C}_{FS,t}} e^{\mu_q} \right)^{1-\gamma_{FS}} \right] \right)^{\frac{1}{\theta_{FS}}} \right\}^{\frac{\theta_{FS}}{1-\gamma_{FS}}} \\
z_{FS,t} &= (1-\varphi)\bar{z}_{FS} + \varphi z_{FS,t-1} + \varepsilon_{FS,t} \\
\hat{Z}_{FS,t} &= e^{(\mu_{FS}-\mu_q) \cdot t + z_{FS,t}} \\
\hat{\tilde{C}}_{FS,t} &= \hat{C}_{FS,t} \\
\hat{C}_{FS,t} &= \hat{Z}_t + \hat{B}_{FS,t} e^{-\mu_q} R_{f,t-1} + \hat{n}_{FS,t} e^{-\mu_q} S_t - \hat{n}_{FS,t+1} F_t \\
&\quad - \hat{B}_{FS,t+1} - \nu_1 \hat{B}_{FS,t+1}^2 - \nu_2 \hat{n}_{FS,t+1}^2 \\
\hat{W}_{FS,t} &= 1 + \mathbb{E}_t \left[M_{t,t+1}^{(FS),aggr} \hat{W}_{FS,t+1} \left(\frac{\hat{C}_{FS,t+1}}{\hat{C}_{FS,t}} e^{\mu_q} \right) \right] \\
R_{FS,t}^W &= \frac{\hat{W}_{FS,t} \frac{\hat{C}_{FS,t+1}}{\hat{C}_{FS,t}} e^{\mu_q}}{\hat{W}_{FS,t-1} - 1} \\
M_{t,t+1}^{(FS),aggr} &= M_{t,t+1}^{(FS)} = \beta_{FS}^{\theta_{FS}} (R_{FS,t+1}^W)^{\theta_{FS}-1} \left(\frac{\hat{C}_{FS,t+1}}{\hat{C}_{FS,t}} e^{\mu_q} \right)^{-\frac{\theta_{FS}}{\psi_{FS}}} \\
\hat{U}_{CC,t} &= \left\{ (1-\beta_{CC}) + \beta_{CC} \left(\mathbb{E}_t \left[\left(\hat{U}_{CC,t+1} \frac{\hat{C}_{CC,t+1}}{\hat{C}_{CC,t}} e^{\mu_q} \right)^{1-\gamma_{CC}} \right] \right)^{\frac{1}{\theta_{CC}}} \right\}^{\frac{\theta_{CC}}{1-\gamma_{CC}}} \\
z_{CC,t} &= (1-\varphi)\bar{z}_{CC} + \varphi z_{CC,t-1} + \varepsilon_{CC,t} \\
\hat{Z}_{CC,t} &= e^{(\mu_{CC}-\mu_q) \cdot t + z_{CC,t}} \\
\hat{\tilde{C}}_{CC,t} &= \hat{C}_{CC,t} \\
\hat{C}_{CC,t} &= \hat{Q}_{CC,t} \\
\hat{Z}_{CC,t} &= \hat{Q}_{CC,t} S_t.
\end{aligned}$$

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