

Briskorn, Dirk

Working Paper — Digitized Version

Feasibility of Home-Away-Pattern sets: A necessary condition

Manuskripte aus den Instituten für Betriebswirtschaftslehre der Universität Kiel, No. 616

Provided in Cooperation with:

Christian-Albrechts-University of Kiel, Institute of Business Administration

Suggested Citation: Briskorn, Dirk (2007) : Feasibility of Home-Away-Pattern sets: A necessary condition, Manuskripte aus den Instituten für Betriebswirtschaftslehre der Universität Kiel, No. 616, Universität Kiel, Institut für Betriebswirtschaftslehre, Kiel

This Version is available at:

<https://hdl.handle.net/10419/147669>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Manuskripte
aus den
Instituten für Betriebswirtschaftslehre
der Universität Kiel

No. 616

Feasibility of Home–Away–Pattern Sets: A Necessary Condition

Dirk Briskorn

January 2007

Dirk Briskorn
Christian-Albrechts-Universität zu Kiel,
Institut für Betriebswirtschaftslehre,
Olshausenstr. 40, 24098 Kiel, Germany,
<http://www.bwl.uni-kiel.de/bwlinstitute/Prod>
briskorn@bwl.uni-kiel.de

Abstract

When scheduling a round robin tournament (RRT) a Home-Away-Pattern (HAP) set defines for each team i and each period p whether i plays at home or away in p . So far the complexity of the decision problem whether a RRT can be arranged based on a given HAP set or not is open. We give a necessary condition which can be checked in polynomial time. We conjecture this condition to be sufficient and confirm this by a computational study.

Keywords: Round robin tournaments, Home-Away-Pattern set feasibility problem

1 Problem Definition

We consider sports leagues having a set T of an even number $n = |T|$ of teams. The single round robin tournament (RRT) structure is defined such that each pair of teams meets exactly once and such that each team plays exactly once per period $p \in P$, $|P| = n - 1$. Scheduling a RRT is a hard combinatorial problem as soon as additional constraints such as stadium availability are to be considered. There are several decomposition schemes based on the idea to separate the decision about the period when two teams compete and the decision about the venue where a match of two teams is carried out; see Easton et al. [1], Nemhauser and Trick [5], and Post and Woeginger [6], for example.

A Home-Away-Pattern (HAP) h_i for team i is a string of length $n - 1$ containing 0 in slot p if team i plays at home in period p (1 otherwise). The collection of HAPs of all teams is called HAP set. An example for a HAP set for $n = 6$ teams is provided in table 1.

| team | 1 | 2 | 3 | 4 | 5 |
|------|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 | 1 |
| 2 | 1 | 0 | 0 | 0 | 1 |
| 3 | 1 | 1 | 0 | 0 | 0 |
| 4 | 0 | 1 | 1 | 0 | 0 |
| 5 | 0 | 0 | 1 | 1 | 0 |
| 6 | 1 | 1 | 1 | 1 | 1 |

Table 1: HAP set for $n = 6$

A HAP set restricts the set of possible matches such that no two teams having the same entry in slot p can compete in period p . Obviously, there is no guarantee that a random HAP set $h \in \{0, 1\}^{n \times (n-1)}$ allows one single RRT to be arranged. In the following, a HAP set h is called feasible if and only if at least one RRT can be scheduled based on h .

We define the HAP set feasibility problem as follows:

Input: A HAP set $h \in \{0, 1\}^{n \times (n-1)}$.

Question: Is h feasible?

Miyashiro et al. [4] provide necessary condition (1) for HAP sets to be feasible. Let $c_0(T', p)$ and $c_1(T', p)$ be the number of zeros and ones, respectively, corresponding to a subset $T' \subseteq T$ of teams and column p .

$$\sum_{p \in P} \min(c_0(T', p), c_1(T', p)) - \binom{|T'|}{2} \geq 0 \quad \forall T' \subseteq T \quad (1)$$

A break for team i in period p occurs if i plays twice at home or twice away, respectively, in periods $p - 1$ and p . As shown in Miyashiro et al. [4] for HAP sets having the minimum number of breaks necessary condition (1) can be checked in polynomial time and, moreover, it is conjectured to be sufficient. However, sufficiency is not proven and, therefore, complexity of HAP set feasibility problem is open so far for arbitrary HAP sets as well as for HAP sets having the minimum number of breaks.

2 A New Necessary Condition

We formulate an IP model in order to check feasibility of a given HAP set h . Binary variable $x_{i,j,p}$, $i, j \in T$, $j < i$, $p \in P$, is equal to 1 if and only if teams i and j compete in period p

HAP-set-feasibility

$$\max z_h = \sum_{i \in T} \sum_{j \in T, j < i} \sum_{p \in P} x_{i,j,p} \quad (2)$$

$$\text{s.t.} \quad \sum_{p \in P} x_{i,j,p} \leq 1 \quad \forall i, j \in T, j < i \quad (3)$$

$$\sum_{j \in T, j < i} x_{i,j,p} + \sum_{j \in T, j > i} x_{j,i,p} \leq 1 \quad \forall i \in T, p \in P \quad (4)$$

$$x_{i,j,p} \leq |h_{i,p} - h_{j,p}| \quad \forall i, j \in T, j < i, p \in P \quad (5)$$

$$x_{i,j,p} \in \{0, 1\} \quad \forall i, j \in T, j < i, p \in P \quad (6)$$

Objective function (2) represents the goal to maximize the number of matches while constraints (3) and (4) assure a single RRT. More precisely, constraint (3) forces each pair of teams to meet at most once while constraint (4) restricts the number of matches per team and period to be less than or equal to one. The entry of HAP set h corresponding to team i and period p is denoted by $h_{i,p}$. Then, (5) takes care of the HAP set h such that no pair of teams can meet in a period where both of them play at home or away, respectively.

Clearly, h is feasible if and only if $z_h = \frac{n(n-1)}{2}$. Let \bar{z}_h be the maximum objective value to the LP-relaxation of HAP-set-feasibility. Then, $\bar{z}_h \geq z_h$ and, hence, $\bar{z}_h = \frac{n(n-1)}{2}$ is a necessary condition for h to be feasible. Furthermore, we can decide whether $\bar{z}_h = \frac{n(n-1)}{2}$ or not in polynomial time, see Garey and Johnson [2].

Theorem 1. Condition $\bar{z}_h = \frac{n(n-1)}{2}$ is strictly stronger than (1).

Proof. First, we show that each HAP set h being infeasible according to condition (1) is infeasible according to condition $\bar{z}_h = \frac{n(n-1)}{2}$, as well. Suppose $\bar{z}_h = \frac{n(n-1)}{2}$. We count the matches between teams in $T' \subseteq T$ in period p as $\bar{z}_{h,p}^{T'}$. Obviously, $\sum_{p \in P} \bar{z}_{h,p}^{T'} = \frac{|T'|(|T'|-1)}{2}$. Moreover, $\min(c_0(T', p), c_1(T', p)) \geq \bar{z}_{h,p}^{T'}$ and, thus, $\sum_{p \in P} \min(c_0(T', p), c_1(T', p)) \geq \binom{|T'|}{2}$ holds.

Second, we consider a HAP set h provided by Matsui and Kashiwabara [3] with $n = 14$ shown in table 2. HAP set h fulfills (1) but has $\bar{z}_h = 90 < 91 = \frac{n(n-1)}{2}$:

| team | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|------|---|---|---|---|---|---|---|---|---|----|----|----|----|
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| 2 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| 3 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 4 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 5 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 7 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 8 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 9 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 10 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 |
| 11 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 12 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 13 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 14 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 2: Infeasible HAP set for $n = 14$

Consequently, h is infeasible which is detected by $\bar{z}_h < \frac{n(n-1)}{2}$ but not by (1).

□

Moreover, we conjecture $\bar{z}_h = z_h$ for each $h \in \{0, 1\}^{n \times (n-1)}$. Unfortunately, we can not provide a formal proof. In order to support the conjecture we have carried out computational tests for two different classes of HAP sets solving HAP-set-feasibility and the corresponding LP-relaxation (using Cplex). Given n these classes of HAP sets are generated as follows:

- I Each entry is randomly chosen from $\{0, 1\}$.
- II For each slot p we randomly choose $\frac{n}{2}$ teams having 0 in p . The remaining teams have 1 in p .

Obviously, for HAP sets according to class II the probability to be feasible is much higher since identical amounts of zeros and ones in each slot are necessary for a HAP set to be feasible. We carried out test runs for 10.000 instances for each class and each $n \in \{6, \dots, 30\}$. Summarizing the results of the experiments our conjecture is true for each of the 260.000 considered instances.

If $\bar{z}_h = z_h$ holds for each $h \in \{0, 1\}^{n \times (n-1)}$ then $\bar{z}_h = \frac{n(n-1)}{2}$ is not only a necessary but also a sufficient condition for h to be feasible. Then, the HAP set feasibility problem is solvable in polynomial time via solving the LP-relaxation of HAP-set-feasibility to optimality.

3 Conclusion

We propose a new necessary condition for HAP sets to be feasible which can be checked in polynomial time. We prove this condition to be stronger than the well known necessary condition of Miyashiro et al. [4]. Moreover, we conjecture this condition to be sufficient and we support this claim by the results of a large-scale computational study.

In the future we aim at providing a formal proof for $\bar{z}_h = z_h$.

References

- [1] K. Easton, G. Nemhauser, and M. Trick. Sports Scheduling. In J. Leung, editor, *Handbook of Scheduling*, pages 52.1–52.19. CRC Press, 2004.
- [2] M. R. Garey and D. S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. Freeman, New York, 1979.
- [3] T. Matsui and K. Kashiwabara. Personal communication.
- [4] R. Miyashiro, H. Iwasaki, and T. Matsui. Characterizing Feasible Pattern Sets with a Minimum Number of Breaks. In E. Burke and P. de Causmaecker, editors, *Proceedings of the 4th international conference on the practice and theory of automated timetabling*, Lecture Notes in Computer Science 2740, pages 78–99. Springer, Berlin, Germany, 2003.
- [5] G. L. Nemhauser and M. A. Trick. Scheduling a Major College Basketball Conference. *Operations Research*, 46:1–8, 1998.
- [6] G. F. Post and G. J. Woeginger. Sports Tournaments, Home–Away–Assignments, and the Break Minimization Problem. *Discrete Optimization*, 3:165–173, 2006.