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Buffer stock savings in a New-Keynesian business cycle model

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FINANCIAL DISTORTIONS AND MACROECONOMIC PERFORMANCE: EXPECTATIONS, CONSTRAINTS AND INTERACTION OF AGENTS

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TITLE

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Buffer stock savings in a New-Keynesian business cycle model*

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June 22, 2016

Abstract

We introduce the tractable buffer stock savings setup of Carroll and Toche (2009 NBER Working Paper) into an otherwise conventional New-Keynesian dynamic stochastic general equilibrium model with financial frictions. The introduction of a precautionary saving motive arising from an uninsurable risk of permanent income loss, affects the model's properties in a number of interesting ways: it produces a more hump-shaped reaction of consumption in response to both supply (technology) and demand (monetary) shocks, and more pronounced reactions in response to demand shocks. Adoption of the buffer stock savings setup thus offers a more microfounded way, compared to, e.g., habit preferences in consumption, to introduce Keynesian features into the model, serving as a device to curbing excessive consumption smoothing, and to attributing a higher role to demand driven fluctuations. We also discuss steady state effects, determinacy properties as well as other practical issues.

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1 Introduction

We introduce Carroll and Toche's (2009) tractable buffer stock setup into an otherwise standard Dynamic Stochastic General Equilibrium (DSGE) model with monopolistic competition, nominal rigidities as well as equilibrium saving and borrowing along the lines of Kiyotaki and Moore (1997). In this setup, the household faces an uninsurable risk of permanent income loss, which gives rise to a precautionary savings motive. A large literature following the seminal contributions of, among others, Aiyagari (1994) as well as Krusell et al. (1998) has to heavily rely on computationally intensive numerical methods to study the effect of precautionary motives on macroeconomic variables. The tractability of Carroll and Toche's setup, however, allows to sustain the use of standard local approximation methods, as still predominantly used in the DSGE literature, especially in medium-scale, estimated, policy-oriented models. The aim of the present paper is to provide the reader with an understanding how the buffer stock precautionary motive alters and affects the characteristics of an otherwise conventional DSGE model, and to serve as a user guide for researchers interested in adopting the precautionary saving setup into general equilibrium models.[1] The model with Carroll and Toche's buffer stock mechanism (referred to as *precautionary model* hereafter) is nested and converges to a standard New-Keynesian model (referred to as the *conventional model*) in the limit as the risk of permanent income loss vanishes. Whenever the uninsurable risk of income loss is substantial, the precautionary model displays a consumption behavior, that deviates from the permanent income hypothesis: in response to supply (technology) shocks, the consumption path displays a more hump-shaped pattern, and the effect of demand (monetary) shocks is more pronounced compared to the conventional model. We also document that the determinacy-indeterminacy properties of the model (Taylor principle) are (mildly) affected by the presence of the precautionary motive.[2]

The standard certainty-equivalence life-cycle consumption theory as popularized by Modigliani and Brumbergh (1954) and Friedman (1957) predicts excessive consumption smoothing has long been criticized in the empirical literature (Flavin 1981). It has been challenged by its inability to sufficiently capture the co-movement of current income and consumption. On the one hand, there exists persuasive evidence suggesting that temporary income shocks considerably affect consumption (cf. Souleles 1999, Stephens 2003). One approach seeking to address this puzzle is to introduce, to the life-cycle consumption theory, various forms of uncertainty, for instance, regarding the length of life, income or required expenses such as medical services in the context of incomplete insurance markets (Hubbard et al. 1994, 1995). Due to an uninsurable risk households choose to accumulate precautionary savings. In a seminal line of research Carroll (1997), Carroll and Jeanne (2009), Carroll and Toche (2009) have shown that sufficiently impatient consumers facing an exogenous risk of permanent income loss attempt to accumulate a target wealth-income ratio in equilibrium in order to hedge against the case of damage. Then, the Euler equation of the household's intertemporal problem implies consumption to depend on current income. For a significant share of the population, precautionary saving motives seem to be a relevant factor for wealth accumulation (cf.

[1] Unlike in the original setup of Carroll and Toche (2009), the tractable buffer stock setup here is cast in general equilibrium, which implies that the interest rate is affected by the presence of the precautionary motive, and thus nonconstant.

[2] There exists a literature documenting that the Taylor principle may be affected when deviating from constant relative risk aversion preferences, e.g. in the presence of habit-preferences or with rule-of-thumb consumers, see, e.g. Bilbiie (2008), Gali et al. (2004a), Motta and Tirelli (2010).

Dardanoni 1991, Carroll and Samwick 1998, Kazarosian 1997, Zhou 2003, Mishra et al. 2012).[3] Despite the downsides of life-cycle consumption theory and the existence of alternative consumption theories, standard New-Keynesian business cycle models still largely rely on the former (cf. Smets and Wouters 2003, Woodford 2005).[4] Understating the role of current income for the determination of consumption, however, may lead to understate an important source of business fluctuations. The aim of the present paper is thus to fill this gap.

The present paper is related to Challe et al. (2015) who introduce a precautionary saving motive to an estimated medium-scale DSGE model with involuntary unemployment. Precautionary saving arises from incomplete insurance against labor income risk. Their model differs in two crucial aspects which come at the cost of tractability: First, the uninsurable risk inducing households to accumulate buffer savings is endogenous. Second, the model exhibits wealth heterogeneity across households. In contrast to that, the aim of the present paper is to introduce the Carroll and Toche (2009) setup to the DSGE literature.

The remainder of the paper is organized as follows: Section 2 presents the model, with the focus on the household sector. Section 3 discusses parameterization and solution method. Section 4 discusses results: section 4.1 provides insights into the workings of the buffer stock setup in a general equilibrium model by comparing and contrasting steady state results, section 4.2 turns to impulse responses of precautionary and conventional model. Section 4.3 studies how determinacy regions and the Taylor principle are affected by the presence of the precautionary motive. Section 4.4 performs extensive sensitivity analysis with respect to key parameters. Section 5 concludes.

2 The model

The model versions 'conventional' and 'precautionary' share a common model structure, in all aspects but the buffer stock savings setup. Both model economies comprise a household sector that consists of saving and borrowing households, a final good firm, intermediate good firms, capital producers, and the government (the monetary authority). Intermediate good firms are monopolistically competitive; they each produce a variety of similar goods, and set an optimal price facing nominal rigidities in the form of Rotemberg (1982) quadratic price adjustment costs. The final good firm is competitive, and combines varieties of the intermediate goods into a final

[3] Dardanoni (1991) study a cross section of British households and find that about 60% of savings are due to precautionary savings motives. Hubbard et al. (1994) find that introducing a precautionary saving motive contributes considerably in explaining important short-run time series properties of consumption and saving. Carroll and Samwick (1998) approximate income risk by the variance of observed income processes and argue that up to 50% of the wealth in their sample originates from income uncertainty differentials. Kazarosian (1997) finds that precautionary saving motives are more relevant for farm households than for households engaging in occupations with less income uncertainty. Lusardi (1998) finds for Italian households that the variance of income contributes to explaining wealth accumulation mainly for individuals close to retirement. Income variance cannot explain the saving of the very rich. Zhou (2003) attributes about 65% of the wealth accumulation of households in occupations with high income volatility to a precautionary saving motive. Mishra et al. (2012) find that precautionary savings account for over 50% of total wealth accumulation. Other studies attribute only low significance to precautionary saving in explaining wealth accumulation. See, for instance, Skinner (1988), Guiso et al. (1992), Lusardi (1998), Arrondel (2002), Jensen and Pope (2004), Kennickell and Lusardi (2004).

[4] Standard medium-scale DSGE models designed to match core moments of macroeconomic time series, however, introduce a considerable income sensitivity of consumption by assuming that a share of households is liquidity constrained (Gali et al. 2004b).

good, which is used for consumption and investment, the latter carried out by capital producers. The monetary authority follows a standard Taylor rule, reacting to contemporaneous inflation. The household sector in both economies, consists of patient households who own the intermediate goods firms and of impatient households who own the capital stock. In equilibrium, patient households end up being savers, while impatient households, who value consumption early on, become equilibrium borrowers, similar to Kiyotaki and Moore (1997). Impatient households are restricted in their ability to borrow by the expected next period's value of the collateral they pledge. Inclusion of such a financial friction allows us to pin down a well-defined level of savings and borrowings even in the conventional model, where a precautionary motive is absent.[5] Household borrowers are modeled identically in the precautionary and conventional version.

The key difference across the two model varieties lies in the modeling of the patient, saving households. While in the conventional model version patient households have standard constant relative risk aversion preferences and are employed over their entire (infinite) lifetime, in the precautionary model we adopt Carroll and Toche's (2009) tractable buffer stock saving setup. Patient households consist of so-called active households, that earn wage and profit income but each period face a constant probability U to drop out of the labor force permanently and lose both wage and profit income, and thus become inactive households. It should be emphasized that this risk of permanent income loss is assumed to be uninsurable. Inactive households consume out of their previously accumulated wealth until they die.

The economy exhibits constant exogenous growth over time, arising from labor augmented productivity increases. In particular, denote the period t level of labor productivity by Z_t, which grows at deterministic rate $\frac{Z_t}{Z_{t-1}} = \Gamma$.[6]

In the following, we discuss in detail the household sector and continue with a brief overview of firm behavior and policy which are standard features shared by both model versions.

2.1 Patient households

This section documents the modeling of patient households. Section 2.1.1 presents the setup for the conventional model, with the standard representative agent problem of an infinitely-lived agent that is always employed. Section 2.1.2 then turns to the precautionary model: in this case, members of the patient household are subject to Carroll and Toche's setup of tractable buffer stock savings, where agents face a risk of permanent income loss.

[5] Introducing a mechanism that induces households to save in equilibrium poses the question who acts as the provider of these assets. In the basic New Keynesian model, the Modigliani and Miller (1958)-theorem applies and the supply of assets is indeterminate. This is why the baseline model adapted here features patient as well as impatient households similar to Kiyotaki and Moore (1997). The financial frictions in this model variant imply a well-defined financial structure of the borrower, and thus has well-defined asset-holdings of the saver, both in the precautionary and in the conventional model.

[6] In order to solve a stationary version of the model, we will thus need to deflate all growing variables by the level of labor productivity. See appendix A for details.

2.1.1 Patient households in the conventional model

In case of the conventional model, there is a representative patient household with standard preferences over consumption, c_t^s, and labor, n_t. The patient household finances its consumption expenditure, and its next period asset holdings, $b_{t+1}^s \frac{\Pi_{t+1}}{R_t}$, from labor income, $W_t n_t$, past asset holdings, b_t^s, and dividends or profits, d_t, from the intermediate goods firms it owns. The problem reads

$$\max_{c_t^s, b_{t+1}^s, n_t} \mathrm{E}_0 \sum_{t=0}^{\infty} (\beta^s)^t \left(\frac{(c_t^s)^{1-\rho}}{1-\rho} - Z_t^{1-\rho} \psi \frac{n_t^{1+\eta}}{1+\eta} \right),$$
$$s.t. \quad b_{t+1}^s \frac{\Pi_{t+1}}{R_t} = b_t^s + W_t n_t + d_t - c_t^s$$

where β^s is the patient household's discount factor, ρ the coefficient of relative risk aversion, ψ is a scaling parameter, η the inverse of the Frisch elasticity, Π_t the gross inflation rate, and R_t the gross interest rate. The aggregated and normalized optimality conditions are

$$\Gamma \tilde{B}_{t+1}^s \frac{\Pi_{t+1}}{R_t} = \tilde{B}_t^s + \tilde{W}_t N_t + \tilde{D}_t - \tilde{C}_t^s, \tag{CV-01}$$

$$\left(\tilde{C}_t^s\right)^{-\rho} = \Gamma^{-\rho} \beta^s \mathrm{E}_t \left[\left(\tilde{C}_{t+1}^s\right)^{-\rho} \frac{R_t}{\Pi_{t+1}} \right], \tag{CV-02}$$

$$\psi N_t^{\eta} = \tilde{W}_t \left(\tilde{C}_t^s\right)^{-\rho}. \tag{CV-03}$$

Note that $\tilde{X}_t \equiv \frac{X_t}{\Gamma^t}$ for any aggregated variable X_t. Eqs. (CV-01) to (CV-04) are the aggregated budget constraint, the aggregated consumption Euler equation derived from combining the FOCs w.r.t. consumption and assets, the aggregated labor supply equation derived from combining the FOCs w.r.t. consumption and labor, and the stochastic discount factor of the patient households, respectively.

2.1.2 Patient households in the precautionary model

In the precautionary model, the patient household sector is more complex. The household sectors follows loosely Carroll (1997), Carroll and Jeanne (2009), Carroll and Toche (2009). A detailed derivation of an individual patient household's problem in the precautionary model, as well as aggregation, is discussed in appendix A. Let Θ_t^a and Θ_t^i denote the population sizes of active and inactive households, respectively. Active households are born into generations of size one and face a per-period risk U of becoming inactive. This poses an uninsurable risk. Once the household is inactive, i.e. has left the labor force, it cannot return. The law of motion of the active population size then is $\Theta_t^a - \Theta_{t-1}^a = 1 - U\Theta_{t-1}^a$. Inactive households face a per-period risk D of dying. Hence, the law of motion of the inactive population size is $\Theta_t^i - \Theta_{t-1}^i = U\Theta_{t-1}^a - D\Theta_{t-1}^i$. The steady-state size of each household type is then $\Theta^a = 1/U$ and $\Theta^i = 1/D$, respectively.

Inactive households Let us derive the first order conditions (FOCs) and budget constraint of an individual inactive household. Inactive households do not obtain labor or profit income, face a

per-period probability, D, of death and have access to a Blanchard (1985) insurance market. As shown in Appendix A, the problem of a single inactive household reads

$$\max_{c_t^i, b_{t+1}^i} \mathrm{E}_0 \sum_{t=0}^{\infty} \left(\beta^s (1-D)\right)^t \frac{(c_t^i)^{1-\rho}}{1-\rho},$$
$$s.t. \quad b_{t+1}^i = \frac{R_t}{\Pi_{t+1}} \frac{1}{1-D} \left(b_t^i - c_t^i\right).$$

Combining the FOCs w.r.t. consumption and wealth leads to

$$(c_t^i)^{-\rho} = \beta \mathrm{E}_t \frac{R_t}{\Pi_{t+1}} (c_{t+1}^i)^{-\rho}.$$

In contrast to the setting in Carroll (1997), Carroll and Jeanne (2009), Carroll and Toche (2009), the inactive household does not face a perfect-foresight problem with constant interest rate, but, in our setting, accumulates wealth that earns the general equilibrium real interest rate, which is generally time-varying. Thus, the inactive household equates the marginal utility of consumption today, with the *expected* marginal utility tomorrow times the real interest rate (discounted to today). Nevertheless, up to a first order approximation and under certainty equivalence, one can, similarly to the perfect-foresight case of Carroll, link the inactive household's consumption to its wealth level. As shown in the appendix, iterating forward the budget constraint, using the consumption Euler equation, aggregating over inactive households and normalizing by productivity yields[7]

$$\left(\tilde{C}_t^i\right)^{\rho} = \frac{1}{\check{\kappa}_t} \tilde{B}_t^i, \tag{PS-01}$$

where $\check{\kappa}_t \equiv \kappa_t \left(\frac{\Theta^i}{\Gamma^t}\right)^{1-\rho}$ can be recursively defined as

$$\check{\kappa}_t = (\tilde{C}_t^i)^{1-\rho} + (1-D)\beta \Gamma^{1-\rho} \mathrm{E}_t \check{\kappa}_{t+1}. \tag{PS-02}$$

The aggregation of the inactive households' budget constraint requires some caution. On the one hand, one needs to sum over all inactive households, so we define $B_t^i \equiv \Theta^i b_t^i$ and $C_t^i \equiv \Theta^i c_t^i$ for period t aggregate wealth and consumption. On the other hand, one needs to take into account that tomorrow's aggregate wealth of inactive households is diminished by a share D of inactive households that dies; and is fed by the aggregated wealth of households which were active at the beginning of t and inactive at the end of t, denoted B_{t+1}^{ai}. The aggregate budget constraint of inactive households then reads (after normalization by productivity)

$$\tilde{B}_{t+1}^i = \frac{R_t}{\Pi_{t+1}} \frac{1}{\Gamma} \left(\tilde{B}_t^i - \tilde{C}_t^i\right) + \tilde{B}_{t+1}^{ai}. \tag{PS-03}$$

Active households Each member of the active household maximizes its lifetime expected discounted stream of utilities. Period utility is given by

$$U(c_t^a, n_t) = \frac{(c_t^a)^{1-\rho}}{1-\rho} - Z_t^{1-\rho} \psi \frac{n_t^{1+\eta}}{1+\eta}$$

[7] In the case of perfect-foresight, the relation between current consumption and wealth is given by $\tilde{C}_t^i = \frac{1}{\kappa_t} \tilde{B}_t^i$, and the propensity to consume out of wealth, $\frac{1}{\kappa_t}$, is given by the constant $\frac{1}{\kappa_t} = \frac{1}{\kappa} = 1 - \frac{(\beta R)^{1/\rho}(1-D)}{R}$ (cf. Carroll and Toche 2009).

and the budget constraint is

$$b^a_{t+1} = \frac{R_t}{\Pi_{t+1}} \left(W_t n_t + d_t + b^a_t - c^a_t - \tau^a_t \right)$$

where τ^a_t represents a transfer from non-newborn to newborn (which is born without any wealth) active households, such that both have an identical level of wealth (cf. Carroll and Jeanne 2009). The presence of this transfer simplifies aggregation of active households (see the appendix). The household faces a risk U of permanent income loss. We assume that the active household cannot loose income, i.e. become inactive, and die in the same period. It is convenient to set up the household's problem as a dynamic program:

$$v^a_t(b^a_t) = \max_{c^a_t, b^a_{t+1}, n_t} \begin{bmatrix} \frac{(c^a_t)^{1-\rho}}{1-\rho} - Z_t^{1-\rho} \psi \frac{n_t^{1+\eta}}{1+\eta} + \\ +\beta(1-U) E_t v^a_{t+1}(b^a_{t+1}) + \beta U E_t v^i_{t+1}(b^a_{t+1}) \end{bmatrix}$$

where $v^a_t(b^a_t)$ is the value function in t. Note that $v^i_{t+1}(b^a_{t+1})$ is the value function of a household in $t+1$ that has become inactive between t and $t+1$. Substituting out c^a_t using the budget constraint and taking the FOC w.r.t. wealth, b_{t+1}, implies,

$$v^{a\prime}_t = \beta \mathrm{E}_t \left[\frac{R_t}{\Pi_{t+1}} \left((1-U) v^{a\prime}_{t+1} + U v^{i\prime}_{t+1} \right) \right],$$

where from the envelope condition

$$v^{a\prime}_t = (c^a_t)^{-\rho}$$
$$v^{i\prime}_t = (c^i_t)^{-\rho}.$$

Because $v^{i\prime}_t = (c^i_t)^{-\rho}$ denotes the marginal utility of a person that was active in period t but has become inactive in period $t+1$and because for the newly inactive household $(c^i_t)^{\rho} = \frac{1}{\kappa_t} b^a_t$, we can write the active household's Euler equation (after aggregation) as:

$$(\tilde{C}^a_t)^{-\rho} = \beta \mathrm{E}_t \left[\frac{R_t}{\Pi_{t+1}} \left((1-U)(C^a_{t+1})^{-\rho} + U \frac{\check{\kappa}_{t+1}}{\tilde{B}^a_{t+1}} \left(\frac{D}{U} \right)^{-\rho} \right) \right]. \tag{PS-04}$$

The aggregated FOC w.r.t. n_t reads

$$\psi U^{\rho+\eta} N_t^{\eta} = \tilde{W}_t \left(\tilde{C}^a_t \right)^{-\rho}. \tag{PS-05}$$

Let us turn to deriving the active households' aggregate budget constraint: we can derive aggregate wealth of active households at period t, of both newborn and non-newborn members as $B^a_t \equiv \Theta^a(b^a_t - \tau^a b^a_t)$, where the transfer is assumed to be proportional to wealth $\tau^a_t = \tau^a b^a_t$, and which ensures that every active household has the same stock wealth. Further define aggregate profits $D_t \equiv \Theta^a d_t$. Period $t+1$ aggregate wealth of active households' requires accounting for a share U of active households that become inactive, diminishing aggregate wealth of active households, and carrying over wealth B^{ai}_{t+1} to inactive households in the aggregate. Hence, $B^a_{t+1} = \Theta^a b^a_{t+1} - B^{ai}_{t+1}$. Solving this equation for $\Theta^a b^a_{t+1}$ and substituting it into the left hand side of the aggregated budget constraint above, yields (after normalizing by productivity)

$$\tilde{B}^a_{t+1} = \frac{R_t}{\Pi_{t+1}} \frac{1}{\Gamma} \left(\tilde{W}_t N_t + \tilde{D}_t + \tilde{B}^a_t - \tilde{C}^a_t \right) - \tilde{B}^{ai}_{t+1}. \tag{PS-06}$$

B^{ai}_{t+1}, the aggregate wealth of those active households which become inactive at the beginning of $t+1$, is

$$\tilde{B}^{ai}_{t+1} = U\frac{R_t}{\Pi_{t+1}}\frac{1}{\Gamma}\left(\tilde{W}_t N_t + \tilde{D}_t + \tilde{B}^a_t - \tilde{C}^a_t\right). \tag{PS-07}$$

Having in hand expressions for aggregate active and inactive households' consumption and wealth positions, let us define the total aggregate consumption and wealth of patient households in the precautionary model as

$$\tilde{C}^s_t \equiv \tilde{C}^a_t + \tilde{C}^i_t, \tag{PS-08}$$

$$\tilde{B}^s_t \equiv \tilde{B}^a_t + \tilde{B}^i_t. \tag{PS-09}$$

2.2 Impatient households

Impatient households are assumed to be the owners of the economy's capital stock and, for simplicity, do not earn labor income. At the end of each period, they sell the old non-depreciated capital to capital producers, and buy the repaired and newly installed capital at the beginning of the period. To finance the acquisition of the capital stock they can, in addition to using their own resources, borrow from patient households. Impatient households face a collateral constraint that limits the amount of loans they can take out by the value of their pledgable collateral.

Facing budget and borrowing constraints, the impatient household chooses inter-temporal paths for consumption, c^b_t, loans, b^b_{t+1}, and capital, k_{t+1}, to maximize expected discounted future utility. The problem reads

$$\max_{c^b_t, b^b_{t+1}, k_{t+1}} \mathrm{E}_0 \sum_{t=0}^{\infty} \left(\beta^b\right)^t \left(\frac{\left(c^b_t\right)^{1-\rho}}{1-\rho}\right),$$

$$s.t. \quad b^b_t - b^b_{t+1}\frac{\Pi_{t+1}}{R_t} = R_{Kt}k_t - Q_{Kt}\left(k_{t+1} - (1-\delta)\,k_t\right) - c^b_t,$$

$$b^b_t \leq m\mathrm{E}_t\left(Q_{Kt+1}k_{t+1}\frac{\Pi_{t+1}}{R_t}\right)$$

where R_{Kt} is the gross rental rate of capital, Q_{Kt} is the shadow price of capital, δ the rate of capital depreciation, and m the collateralizable fraction of capital. We will assume that impatient households discount future utility more heavily than patient households, so that $\beta^b < \beta^s$. Solving this optimization problem, aggregating and normalizing by labor productivity, the optimality conditions of the impatient households can be obtained as

$$\tilde{B}^b_t - \Gamma\tilde{B}^b_{t+1}\frac{\Pi_{t+1}}{R_t} = R^k_t\tilde{K}_t - Q_{Kt}\left(\Gamma\tilde{K}_{t+1} - (1-\delta)\,\tilde{K}_t\right) - \tilde{C}^b_t, \tag{1}$$

$$\tilde{B}^b_{t+1} = m\mathrm{E}_t\left[Q_{Kt+1}\tilde{K}_{t+1}\frac{\Pi_{t+1}}{R_t}\right], \tag{2}$$

$$Q_{Kt}\left(\tilde{C}^b_t\right)^{-\rho} = \Gamma^{-\rho}\beta^b\mathrm{E}_t\left[\left(\tilde{C}^b_{t+1}\right)^{-\rho}\left(R_{Kt+1} + Q_{Kt+1}\left(1-\delta\right)\right)\right] + \mu_t m\mathrm{E}_t Q_{Kt+1}, \tag{3}$$

$$\left(\tilde{C}^b_t\right)^{-\rho} = \Gamma^{-\rho}\beta^b\mathrm{E}_t\left[\left(\tilde{C}^b_{t+1}\right)^{-\rho}\frac{R_t}{\Pi_{t+1}}\right] + \mu_t\frac{R_t}{\Pi_{t+1}}, \tag{4}$$

where μ_t is the Lagrangian multipliers of the borrowing constraint. Eqs. (1) and (2) are the aggregated budget constraint and borrowing constraints. Eq. (3) results from the FOC w.r.t. consumption and capital, and (4) from the FOC w.r.t. consumption and loans.

2.3 Final good firm

A perfectly competitive firm produces final good, Y_t, used for consumption and investment. It does so by aggregating differentiated intermediate goods, $Y_t(j)$, into a final good according to a constant elasticity of substitution (CES) production function.

$$Y_t = \int_0^1 \left[Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Taking the price, $P_t(j)$, as given, profit maximization implies for the demand for variety j,

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} Y_t.$$

2.4 Intermediate good firms

There is a continuum of intermediate good firms which we assume to be owned by the patient households. Each firm produces a quantity, $Y_t(j)$, of a differentiated good j using capital and labor as inputs, and selling it on a monopolistically competitive market. In setting its optimal price, firm j faces quadratic costs of price adjustment a la Rotemberg (1982). Production follows a Cobb-Douglas production function which, after aggregation and normalization, reads

$$\tilde{Y}_t = A_t \tilde{K}_t^{\alpha} N_t^{1-\alpha}, \tag{5}$$

where A_t is total factor productivity following a first-order autoregressive process in logs, i.e.

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_{A,t}, \tag{6}$$

where ρ_A captures the persistence of the technology process and $\varepsilon_{A,t}$ its exogenous innovations which are distributed as $\mathrm{N}(0, \sigma_A^2)$.

We can decompose the intermediate good firm's optimization problem into a static cost minimization and an intertemporal profit maximization part. Cost minimization, choosing optimally the labor and capital demands, $N_t(j)$ and $K_t(j)$, results, after aggregation, in the following expressions for the real wage rate, W_t, and the real rate of return on capital, R_{Kt}:

$$\tilde{W}_t = MC_t (1-\alpha) \frac{\tilde{Y}_t}{N_t}, \tag{7}$$

$$R_{Kt} = MC_t \alpha \frac{\tilde{Y}_t}{\tilde{K}_t}, \tag{8}$$

where MC_t are real marginal costs. With the cost-minimizing choice of inputs, firm j maximizes expected discounted intertemporal profits by setting an optimal path of future prices, $\{P_t(j)\}$, taking as given the demand for its product and facing quadratic Rotemberg (1982) price adjustment

costs, $Z_t P_t \frac{\phi_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)}-1\right)^2$. The FOC with respect to the price decision, after imposing that all firms are identical and after aggregation and normalization, can be written as

$$\tilde{Y}_t\left((\varepsilon-1)-\varepsilon MC_t\right)+\phi_p\left(\Pi_t-1\right)\Pi_t=\mathrm{E}_t\Lambda^s_{t,t+1}\phi_p\left(\Pi_{t+1}-1\right)\Pi_{t+1} \tag{9}$$

where

$$\Lambda^s_{t,t+1}=\Gamma^{-\rho}\beta^s\mathrm{E}_t\left(\frac{\tilde{C}^s_{t+1}}{\tilde{C}^s_t}\right)^{-\rho} \tag{CV-04}$$

is the stochastic discount factor in the conventional model and

$$\Lambda^s_{t,t+1}=\Gamma^{-\rho}\beta^s\mathrm{E}_t\frac{(1-U)(C^a_{t+1})^{-\rho}+U\frac{\check{\kappa}_{t+1}}{\check{B}^a_{t+1}}\left(\frac{D}{U}\right)^{-\rho}}{\left(\tilde{C}^a_t\right)^{-\rho}} \tag{PS-10}$$

in the precautionary model. We define real profits as

$$\tilde{D}_t=\tilde{Y}_t-R_{Kt}\tilde{K}_t-\tilde{W}_tN_t-\frac{\phi_p}{2}\left(\Pi_t-1\right)^2. \tag{10}$$

2.5 Capital producers

Capital producers add final goods to the existing capital stock in order to produce new capital goods, I_t. Capital production is subject to quadratic adjustment costs. Capital producers choose the level of I_t that maximizes their profits, $Q_{Kt}I_t-\left(I_t+\frac{\phi_k}{2}\left(\frac{I_t}{K_t}-(\Gamma-(1-\delta))\right)^2K_t\right)$, where ϕ_k governs the slope of the capital producers adjustment cost function. From profit maximization, the relative price of capital, Q_{Kt}, is

$$Q_{Kt}=1+\phi_k\left(\frac{\tilde{I}_t}{\tilde{K}_t}-(\Gamma-(1-\delta))\right). \tag{11}$$

In the absence of investment adjustment costs, Q_{Kt} is constant and equal to one. The usual capital accumulation equation holds, i.e.

$$\Gamma K_{t+1}=(1-\delta)K_t+I_t. \tag{12}$$

2.6 Monetary authority

Monetary policy is assumed to follow a Taylor rule, by which the nominal interest rate, R_t, responds to deviations of inflation from its target value. M_t is an unexpected disturbance to the monetary rule, following an exogenous first-order autroregressive process:

$$\frac{R_t}{R}=\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi}M_t, \tag{13}$$

$$\log M_t=\rho_M\log M_{t-1}+\varepsilon_{M,t}. \tag{14}$$

2.7 Market clearing and equilibrium

In equilibrium, labor, capital, asset and goods market clear. Asset market clearing is given by $\tilde{B}_t^s = \tilde{B}_t^b$ which then implies the goods market to clear as well. Adding over the budget constraints in our model yields

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t \tag{15}$$

where

$$\tilde{C}_t = \tilde{C}^s + \tilde{C}^b. \tag{16}$$

Overall, the conventional model comprises 20 equations ((1)-(16) and (CV-01)-(CV-04)) in 20 variables ($\tilde{Y}$, $\tilde{C}$, $\tilde{I}$, $\tilde{C}^s$, $\tilde{C}^b$, N, $\tilde{K}$, $\tilde{B}$, $\tilde{B}^s$, $\tilde{B}^b$, $\tilde{D}$, $\tilde{W}$, R, Π, Q_K, μ, Λ^s, MC, A, M). The precautionary model comprises 26 equations ((1)-(16) and (PS-01)-(PS-10)) in 26 variables ($\tilde{Y}$, $\tilde{C}$, $\tilde{I}$, $\tilde{C}^s$, $\tilde{C}^b$, N, $\tilde{K}$, $\tilde{B}$, $\tilde{B}^s$, $\tilde{B}^b$, $\tilde{D}$, $\tilde{W}$, R, Π, Q_K, μ, Λ^s, MC, A, M, $\tilde{C}^a$, $\tilde{C}^i$, $\tilde{B}^a$, $\tilde{B}^i$, $\tilde{B}^{ai}$, $\check{\kappa}$).

3 Parameterization and solution method

Table 1 summarizes the choice of baseline parameters. We first focus on the production side. Parameter $\alpha = 0.33$ implies a share of capital in output equal to one third, the quarterly depreciation rate of capital, δ, is set to 0.025. The elasticity of substitution between varieties of intermediate goods, ε, of 6, implies a (net) markup of prices over costs, $\frac{\varepsilon}{\varepsilon-1} - 1$, of 20%. The gross rate of exogenous, labor-augmenting growth, Γ^{cv}, is set to 1.004 per quarter. Parameter ϕ_k in the quadratic capital adjustment cost function is set to 0 in the baseline. Finally, the parameter of the quadratic Rotemberg price adjustment costs, ϕ_p, is taken to be 50. This choice roughly translates into a Calvo parameter of 0.75 in the setup of price stickiness a la Calvo (1983), to which the Rotemberg price adjustment cost setup is equivalent up to a first order approximation.[8] A Calvo parameter of about 0.75 in turn implies that prices remain constant for an average of $\frac{1}{1-0.75} = 4$ quarters, a standard value in the literature.

We now turn to the preference side. The coefficient of relative risk aversion, ρ, is set to 2, which implies that the elasticity of intertemporal substitution, the inverse, is equal to 1/2. This lies well in the range typically used in the macroeconomic literature, where it takes on values between 1 and 1/5. The coefficient on labor in the utility function, equal to the inverse of the Frisch elasticity of labor supply, is set to 1. Parameter ψ pins down the steady state labor supply, and is set such that workers use one third of their time endowment for supplying their labor to the labor market. The discount factors of patient and impatient households are set to 0.99 and 0.975, respectively. Note that the requirement $\beta^b < \beta^s$ is fulfilled, so that impatient households discount future consumption more heavily, thus have a preference for consumption early on and for becoming equilibrium borrowers, with binding borrowing constraint.[9] In a conventional DSGE

[8] In particular, the precise mapping between Calvo parameter, ξ, and ϕ_P is given by $\phi_P = \frac{\xi(\varepsilon-1)\overline{Y}}{(1-\xi)(1-\xi\beta^s)}$.

[9] Strictly speaking, $\beta^b < \beta^s$, only guarantees that the borrowing constraint binds at the *non-stochastic* steady state. We follow a large literature in assuming that the stochastic shocks hitting the economy are small enough, such that the borrowing constraint also continues to bind in the stochastic setting, so that standard first-order perturbation methods can be applied to solve the model.

Table 1: Parameterization

Parameters common to conventional and precautionary model		
Cobb-Douglas coefficient on capital	α	0.33
Depreciation rate of capital stock	δ	0.025
Elasticity of subst., intermed. goods	ϵ	6
Capital adj. cost parameter	ϕ_k	0
Rotemberg price adj. cost parameter	ϕ_p	50
Coeff. of relative risk aversion	ρ	2
Coeff. on labor in utility function	η	1
Steady state share of time in labor mkt.	$\bar{N}$	0.33
Discount factor, patient households	β^s	0.99
Discount factor, impatient households	β^b	0.975
Borrowing constraint parameter	m	0.50
Inflation coeff., Taylor rule	ϕ_π	1.50
Autocorrelation, prod.	ρ_A	0.979
Standard Deviation, prod.	σ_A	0.007
Autocorrelation, mon.	ρ_M	0.500
Standard Deviation, mon.	σ_M	0.005
Parameters specific to conventional model		
Growth rate in conventional model	Γ^{cv}	1.004
Parameters specific to precautionary model		
Probability of permanent income loss	U	0.015
Probability of death	$D = U/0.2$	0.075
Growth rate of expected permanent income	$\Gamma^{ps} = \Gamma^{cv}/(1-U)$	1.019

model without precautionary motive, the choice of the saver's discount factor, together with the economy's growth rate, directly pin down the steady state interest rate at the non-stochastic steady state. In particular, from Euler equation (CV-02) it follows that the steady state gross quarterly real interest rate in the conventional model is given by $\frac{\Gamma^{\rho}}{\beta^{s}}$, which is equal to 1.018 under the parameterization reported here. Note that this is slightly higher than the annual interest rate of 4% often targeted in the macroeconomics literature. However, we rather target the interest rate in the precautionary model – because of the presence of a precautionary savings motive, the interest rate in the precautionary model is lower and close to a rate of 4% annually. We discuss steady state results in detail in the next section. The borrowing constraint parameter, m, is set to 0.5, which implies that impatient agents can take out loans up to 50% of the value of the collateral they pledge.

Monetary policy is described by a Taylor rule, targeting inflation with coefficient $\phi_{\Pi} = 1.5$, a standard value. Autocorrelations of the productivity and monetary shock processes are 0.979 and 0.5, the standard deviation of their innovations equal 0.007 and 0.005 respectively.

Finally, we discuss the parameters that are specific to the buffer stock savings mechanism in the precautionary model. Parameter U, the parameter that governs the risk of permanent income loss, is set to 0.015 per quarter. This parameter choice is somewhat higher than the calibration in Carroll and Jeanne (2009), who set $U = 0.025$ in an annual calibration. Taking U strictly as the risk of permanent income loss, this value would imply an average duration of active households remaining active of $1/0.015$ quarters, which implies an average duration in the workforce of 16.7 years before becoming inactive. We choose to, instead of sticking strictly to this interpretation, see U mostly as a device to introduce more realistic consumption patterns into DSGE models, that are less subject to excessive consumption smoothing as conventional agents with CRRA preferences that are infinitely-lived and infinitely-working.[10] In addition, section 4.1 performs sensitivity analysis with respect to this crucial parameter. Parameter D, the per quarter probability of an inactive household dying, is set in proportion to the parameter choice of U. In particular, D is set equal to U times the inverse of the old-age dependency ratio which is about 0.2. Our parameter choice for U then implies an average length of 13.3 quarters of being inactive before death. Finally, the relevant growth rate of the economy Γ^{ps} in the precautionary model is equal to the growth rate Γ^{cv} in the conventional model, elevated by factor $1/(1-U)$, so that while labor income will grow by factor $\Gamma^{ps} = \Gamma^{cv}/(1-U)$, the *expected* labor income growth factor for employed consumers is equal to Γ^{cv}, as would be the case in the no-risk perfect foresight case (cf. Carroll and Jeanne 2009).[11]

The model is solved with a standard first order perturbation method (log-linearization). The big advantage of Carroll's tractable buffer stock saving approach is that one can separate the risk of permanent income loss, that gives rise to precautionary motives from other, aggregate macro, shocks. This is in contrast to the large existing literature that heavily relies on numerical methods to understand the role of precautionary saving motives for macroeconomics (cf. Aiyagari 1994 and Krusell et al. 1998), as precautionary motives are typically captured in higher order moments, for which one needs accurate (global or higher order) methods. Adoption of Carroll's tractable buffer stock setup allows integrating a role for precautionary motives in macroeconomic dynamics,

[10] An example of a more ad hoc device is, e.g., the assumption of habit preferences, widely used in macroeconomics.

[11] This means that an increase in U is a pure increase in risk with no effect on the present discounted value of expected labor income; thus, any change in behavior that results from a change in U can be interpreted solely as reflecting an effect on uncertainty, and not as an effect of a change in human wealth (cf. Carroll and Jeanne 2009).

where standard methods, such as (log-)linearization, still widely used in macroeconomics, especially medium-scale (estimated) policy models that, because of a large number of state variables are still hard or infeasible to solve with global methods, suffice.

4 Results

4.1 Implications of the precautionary saving mechanism and steady state results

Table 2 summarizes results for the non-stochastic steady state, under the baseline parameterization discussed in section 3, for both precautionary and conventional model. The word 'non-stochastic' for the precautionary model, may need additional clarification: in particular, 'non-stochastic' refers to the fact that the economy's uncertainty coming from the aggregate shock processes is shut down, i.e. that the level of total factor productivity, A_t, and the shift term in the Taylor rule, M_t, are constant and take on their mean values. It does, however, not imply that the risk of permanent income loss is shut down; since the latter is independent of the stochastics of aggregate shocks of the DSGE model, we, however, continue to refer to it as the 'non-stochastic' steady state.

Several findings on the steady state results are noteworthy. One, as we let U become increasingly smaller, the steady state of the precautionary model converges to the steady state of the conventional model. That is, the conventional model is indeed a special case of the precautionary model, or, the latter is nested in the former. Two, we want to provide intuition for what explains the differences in steady state values across models, with a focus on the steady state (real) interest rate and consumption variables. To do so, we summarize the main mechanism and intuition of Carroll's buffer stock mechanism. For this reason, it is illustrative to consider the growth-deflated consumption Euler equations of the patient (in the precautionary model, patient active) households. They are, respectively, for conventional and precautionary model:

$$\begin{aligned}
\left(\tilde{C}_t^s\right)^{-\rho} &= \Gamma^{-\rho}\beta^s \mathrm{E}_t\left[\frac{R_t}{\Pi_{t+1}}\left(\tilde{C}_{t+1}^s\right)^{-\rho}\right], \\
(\tilde{C}_t^a)^{-\rho} &= \Gamma^{-\rho}\beta^s \mathrm{E}_t\left[\frac{R_t}{\Pi_{t+1}}\left((1-U)\left(\tilde{C}_{t+1}^a\right)^{-\rho} + U\left(\tilde{C}_{t+1}^i\right)^{-\rho}\left(\frac{\Theta^a}{\Theta^i}\right)^{-\rho}\right)\right].
\end{aligned}$$

At the non-stochastic steady state, $\frac{R_t}{\Pi_{t+1}}$ is constant, and, since we consider a model without steady state inflation (with $\Pi = 1$), equal to R. We can then follow Carroll in defining the growth patience factor, $\Þ_\Gamma$, as , $\Þ_\Gamma \equiv (R\beta^s)^{\frac{1}{\rho}}/\Gamma$. This is the factor by which growth-deflated consumption would grow in absence of labor income risk. In particular, in the conventional model, a steady state in which the gross (net) consumption growth rate equals one (zero), $\tilde{C}_{t+1}^s/\tilde{C}_t^s = \tilde{C}^s/\tilde{C}^s = 1$, implies a growth patience factor equal to one ($\Þ_\Gamma = 1$) and a steady state interest rate that is equal to Γ^ρ/β^s, which under the baseline parameterization is equal to 1.018, as shown in the right column of Table 2. Instead, a steady state consumption of (active) households in the precautionary model is reached as a result of two offsetting forces; on the one hand, the (active) consumer needs to be sufficiently impatient, and her growth impatience factor will thus be smaller than 1 (or, more precisely, smaller than $(1-U)^{-\frac{1}{\rho}}$, (cf. Carroll and Jeanne 2009)). This condition implies that a consumer would be sufficiently impatient to want to consume early on and to run down her growth-deflated wealth over

time, implying a consumption growth rate that is falling over time. Table 2 shows that, indeed, this is the case, and the growth patience factor in the precautionary model lies, with $\Þ_\Gamma = 0.981$, below one. On the other hand, the balancing force to growth impatience is the consumer's desire to accumulate wealth for precautionary reasons, which is reflected in the second term on the right hand side of the (rewritten) steady state consumption Euler equation of the active household below having to be larger than one at steady state:

$$\frac{\tilde{C}^a_{t+1}}{\tilde{C}^a_t} = \Þ_\Gamma \left((1-U) + U \left(\frac{\tilde{C}^a_{t+1}}{\tilde{C}^i_{t+1}} \right)^\rho \left(\frac{\Theta^a}{\Theta^i} \right)^{-\rho} \right)^{\frac{1}{\rho}}.$$

The term in parenthesis shows that the consumption growth rate depends on the employment outcome: since consumption next period if the household remains active, $\tilde{C}^a_{t+1}$, is greater than consumption if the consumer becomes unemployed the next period, $\tilde{C}^i_{t+1}$, the term in parenthesis is larger than one.[12] This means that the presence of unemployment risk boosts consumption growth, and tends to lead to a consumption growth rate that increases over time. The two offsetting forces, growth impatience which tends to lead to consumption growth rates of the active households that are falling over time, and the precautionary motive coming from uninsurable risk of income loss, which tends to lead to consumption growth rates that are increasing over time, exactly balance at the steady state, such that consumption growth is constant and $\tilde{C}^a_{t+1} = \tilde{C}^a_t = \tilde{C}^a$. As shown in Table 2, the tendency of a higher consumption growth rate coming from the precautionary motive, goes in hand with a lower consumption level, so that $\tilde{C}^s$ is found to be lower in the precautionary compared to the conventional model. Also, in line with the broader precautionary savings literature (when there are precautionary motives other than arising from risk of permanent income loss such as in Carroll's tractable buffer stock savings setup), the steady state interest rate is below the rate it would take on in the model version as $U \longrightarrow 0$. This finding was first observed by Aiyagari (1994), who argued that, as a result of the motives to hold precautionary buffer asset, when the gross interest rate would be at its certainty equivalent level, Γ^ρ/β^s, there would be an excess demand for savings. Under uncertainty (or, respectively, in Carroll's setup, under positive risk of permanent income loss), therefore, the asset price needs to be higher relative to its certainty-equivalent level to clear the asset market, or, equivalently, the real interest rate needs to be lower than in a non-stochastic world.

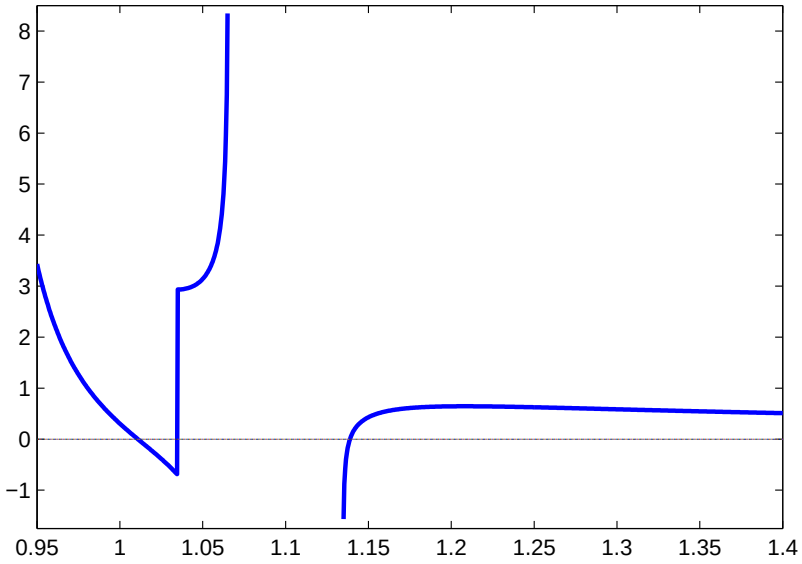
Finally, we want to comment on practical issues when taking the Carroll tractable buffer stock savings setup into a dynamic stochastic general equilibrium model. In particular, we find that, numerically, multiplicity of steady states exists. This issue can best be seen, when reducing the steady state system of the precautionary model (evaluated at the non-stochastic steady state) to a single equation in the steady state interest rate, R. This single equation is a polynomial equation, whose zeros can be solved for numerically. Figure 1 shows the corresponding plot for our baseline parameterization. The figure shows that, in practice, when the interest rate is not constant and taken as given, as in the partial equilibrium setup of Carroll and Jeanne (2009), but has to be solved for in a general equilibrium setting, one has to be careful in selecting the 'economically meaningful' steady state. The steady state reported in Table 2 is the, we claim, economically meaningful one: it, in particular, is the one consistent with the above described properties of a precautionary motive

[12] Note that this line of argumentation holds despite the term $\left(\frac{\Theta^a}{\Theta^i}\right)^{-\rho}$ being < 1. In particular, it holds as long as for an *individual* household consumption while being active, c^a_t is higher that her consumption when inactive, c^i_t, which is always the case in Carroll's buffer stock savings setup.

Table 2: Steady state results

Variable	precautionary model	conventional model
$\tilde{Y}$	0.689	0.768
$\tilde{I}$	0.133	0.121
$\tilde{C}$	0.556	0.647
$\tilde{C}^b$	0.070	0.064
$\tilde{C}^s$	0.485	0.583
$\tilde{C}^a$	0.467	
$\tilde{C}^i$	0.019	
$\tilde{K}$	3.010	4.162
$\tilde{B}^b$	1.489	2.044
$\tilde{B}^s$	1.489	2.044
$\tilde{B}^a$	1.306	
$\tilde{B}^i$	0.182	
MC	0.833	0.833
$\tilde{W}$	1.160	1.293
R_K	0.064	0.051
Π	0.115	0.128
R	1.011	1.018
Absolute Patience Factor, Þ	1.000	1.004
Return Patience Factor, Þ_R	0.990	0.986
Growth Patience Factor, Þ_Γ	0.981	1.000

Figure 1: Multiplicity of steady state interest rate



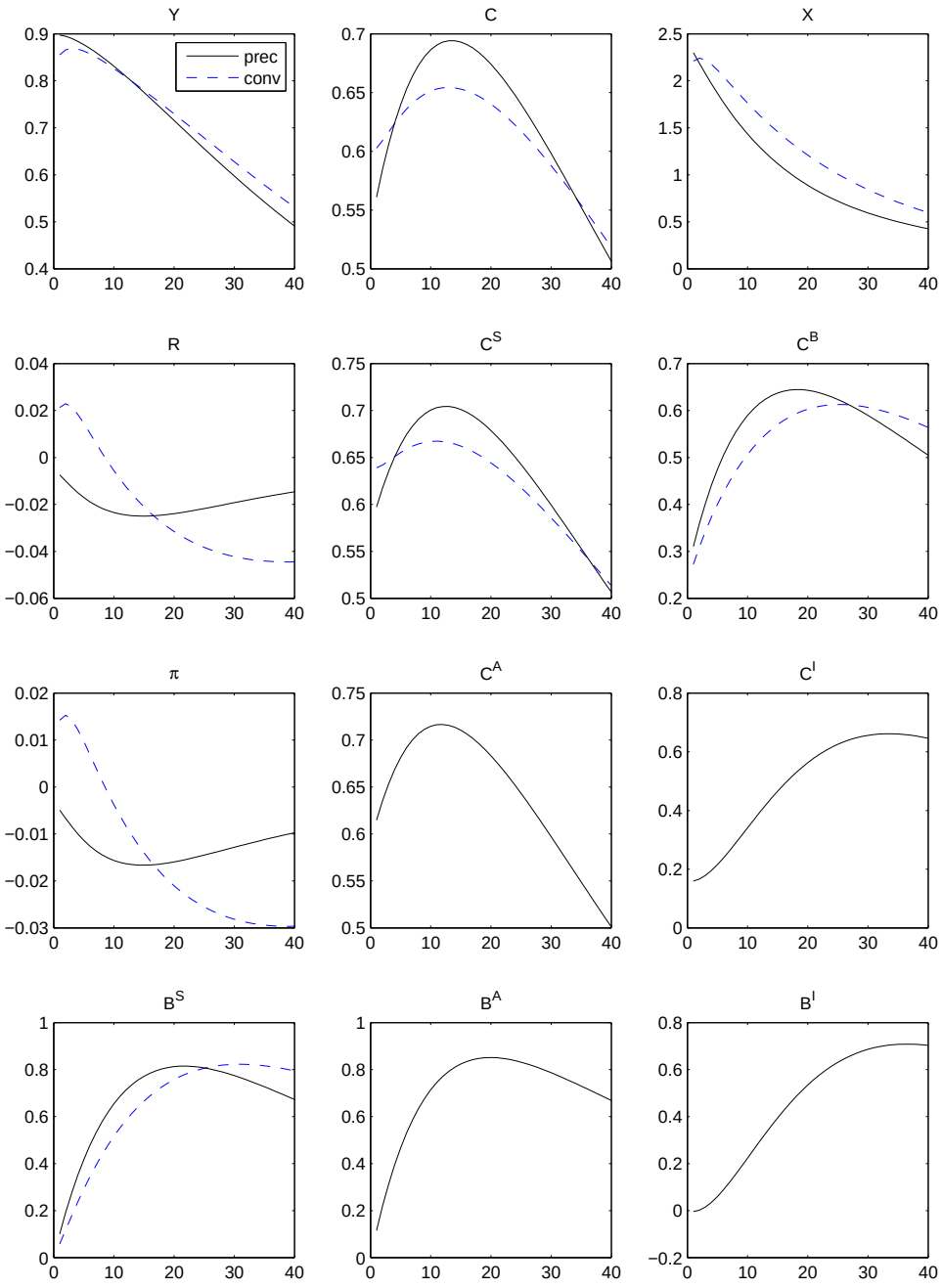
present, and is the one that one obtains when starting at the steady state of the conventional model (when $U = 0$) and increasing U in small steps.[13]

4.2 Impulse response analysis

This section progresses to illustrate the effects of the tractable buffer stock savings setup on the dynamics of our macroeconomic model. To this reason, figure 2 plots responses of the model economies to a one percent positive productivity shock. All variables are expressed in terms of percentage deviations from steady state. Responses for both model economies are shown for output, $\tilde{Y}_t$, aggregate consumption, $\tilde{C}_t$, investment, $\tilde{I}_t$, the nominal interest rate, R_t, consumptions of patient and impatient agent, $\tilde{C}^s_t$ and $\tilde{C}^b_t$, inflation, Π_t, and asset holdings, $\tilde{B}^s_t$. In addition, for the precautionary model, the subcomponents of aggregate consumption of savers, consumption of active households, $\tilde{C}^a_t$, and consumption of inactive households, $\tilde{C}^i_t$, are reported, and similarly for asset holdings, $\tilde{B}^a_t$ and $\tilde{B}^i_t$. As figure 2 shows, macroeconomic dynamics are broadly similar across the two model versions. Output increases, as a direct result of the higher productivity, and, because the technology improvement is persistent, as a result of the additional capital stock that is built up. Part of the output increase is consumed, but since agents care about a smooth consumption path, a part of the output increase also leads to increased savings, both in the form of investments in building up the capital stock, or savings in issuing loans to borrowing households. Patient households' asset holdings increase both because patient households want to accumulate

[13] In particular, other steady states that a numerical solver may obtain with more arbitrarily chosen initial values, may imply cases with growth patience factors larger than one, and counterfactually high interest rates.

Figure 2: Impulse responses to a one percent productivity increase



savings as a device to transfer consumption to later periods, but also because impatient households see the value of their collateral, the capital stock, increase, which allows them to increase their loans from patient households. The broadly similar dynamics across the two model versions are hardly surprising, given that the model versions are identically specified and parameterized in all aspects but for the buffer stock savings part. However, as can be expected, some noticeable differences in the consumption dynamics arise. In response to the positive productivity shock, we observe a more hump-shaped consumption pattern in the precautionary model compared to the conventional model without buffer stock savings. This is because the positive technology shock has increased the active households' income relative to their wealth position (which, as a state variable, is predetermined at the impact of the shock). Thus its wealth-to-income ratio target is, now, temporarily too low. To realign the ratio with its target, the active household thus has to temporarily save more, to increase its wealth; to achieve this, it is allowed only a smaller consumption increase relative to the model without a precautionary motive present. That means that the consumption response in the precautionary model is, on impact, less pronounced compared to a conventional model. After a few periods, additional wealth has been accumulated (and in addition, the temporarily higher income is converging back to normal as the technology shock declines out), so the wealth-to-income target is re-reached. The higher level of accumulated wealth now enables the household to consume more compared to the conventional case. Overall, the presence of the precautionary motive works to increase the humpshapedness of the consumption response. Since the active household's consumption is the predominant part of aggregate consumption, the pattern of the active households' consumption dynamics translates also into a more hump-shaped response of aggregate consumption. The presence of the tractable buffer stock savings mechanism thus gives rise to similar effects on the consumption impulse response to technology shock as, e.g., the assumption of habit-preferences in consumption. The tractable buffer stock setup may thus be seen as a, compared to the ad-hoc assumption of habit preferences, more microfounded mechanism to generate more realistic consumption patterns which are less susceptible to excessive consumption smoothing.

Figure 3 presents impulse response for the same set of variables in response to an expansionary monetary shock that implies a decrease in the monetary shock variable in the Taylor rule, M_t, by one standard deviation. Again, broadly speaking, the macroeconomic dynamics triggered by the shock are similar across the two model versions. Because of frictions in price adjustment, prices are lower and increase only gradually compared to a flexible price world, and the monetary expansion works to increase output and its components, consumption and investment. Because of the strong positive inflation response, the nominal interest rate increases despite the decrease in M_t, however keeps its expansionary effect.[14] However, we again observe that the consumption response in the precautionary model is different: we find the consumption response to be both more pronounced and, as with the productivity shock, more hump-shaped than in the conventional model. As before, we interpret the increased humpshapedness as a successful device to curb the excessive consumption smoothing behavior implied by infinitely-lived and infinitely working agents in more standard settings. In addition, the more pronounced consumption response can be interpreted as the model becoming, for a same given level of price rigidity, more demand-determined, i.e. more 'Keynesian'. The next section documents that this also has implications for the Taylor principle and determinacy issues, particularly when nominal rigidities are large.

[14]Note that this is a result of the assumption of the impatient household owning all of the economy's capital stock and no capital adjustment costs.

Figure 3: Impulse responses to an expansionary monetary shock

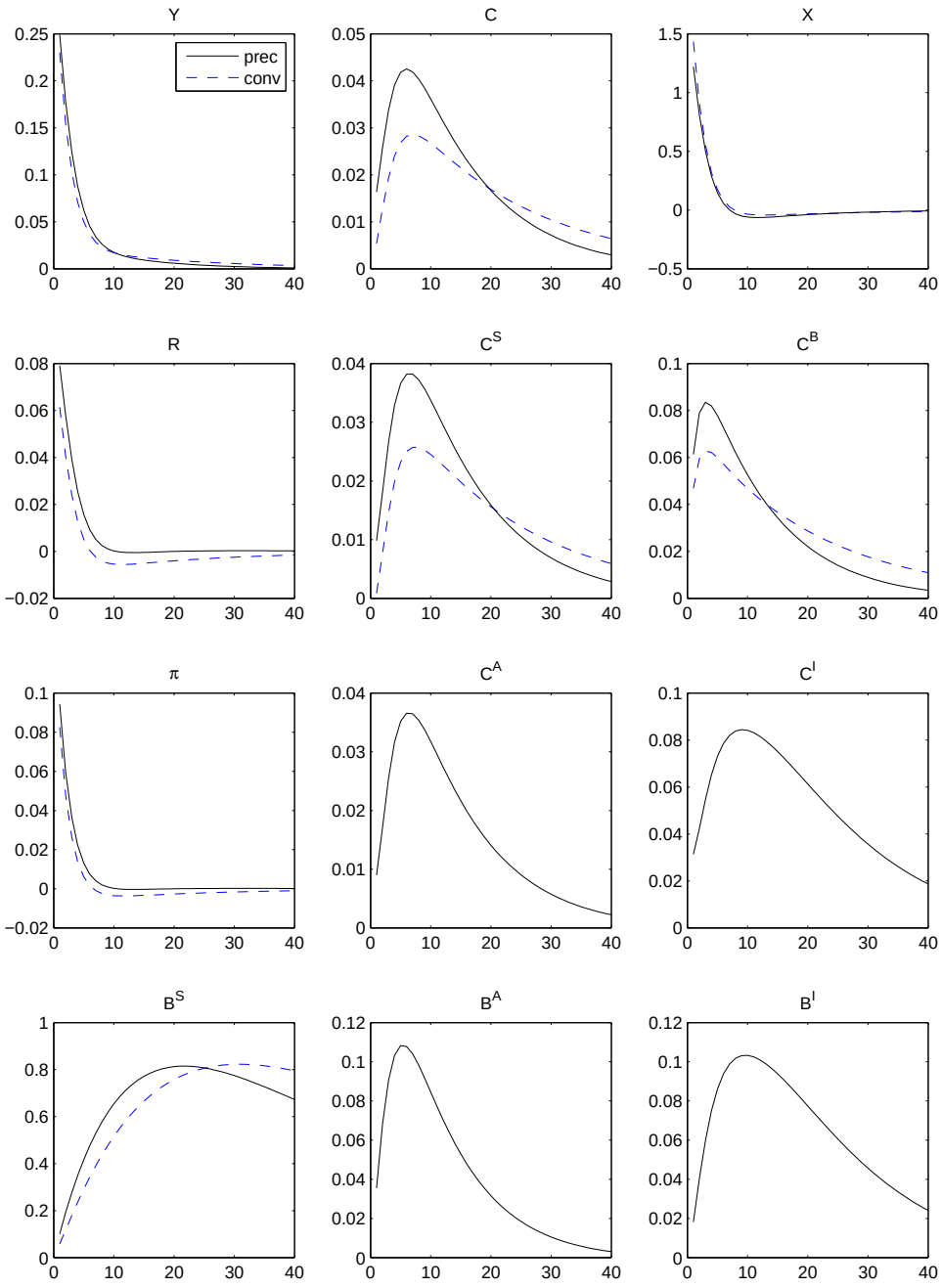
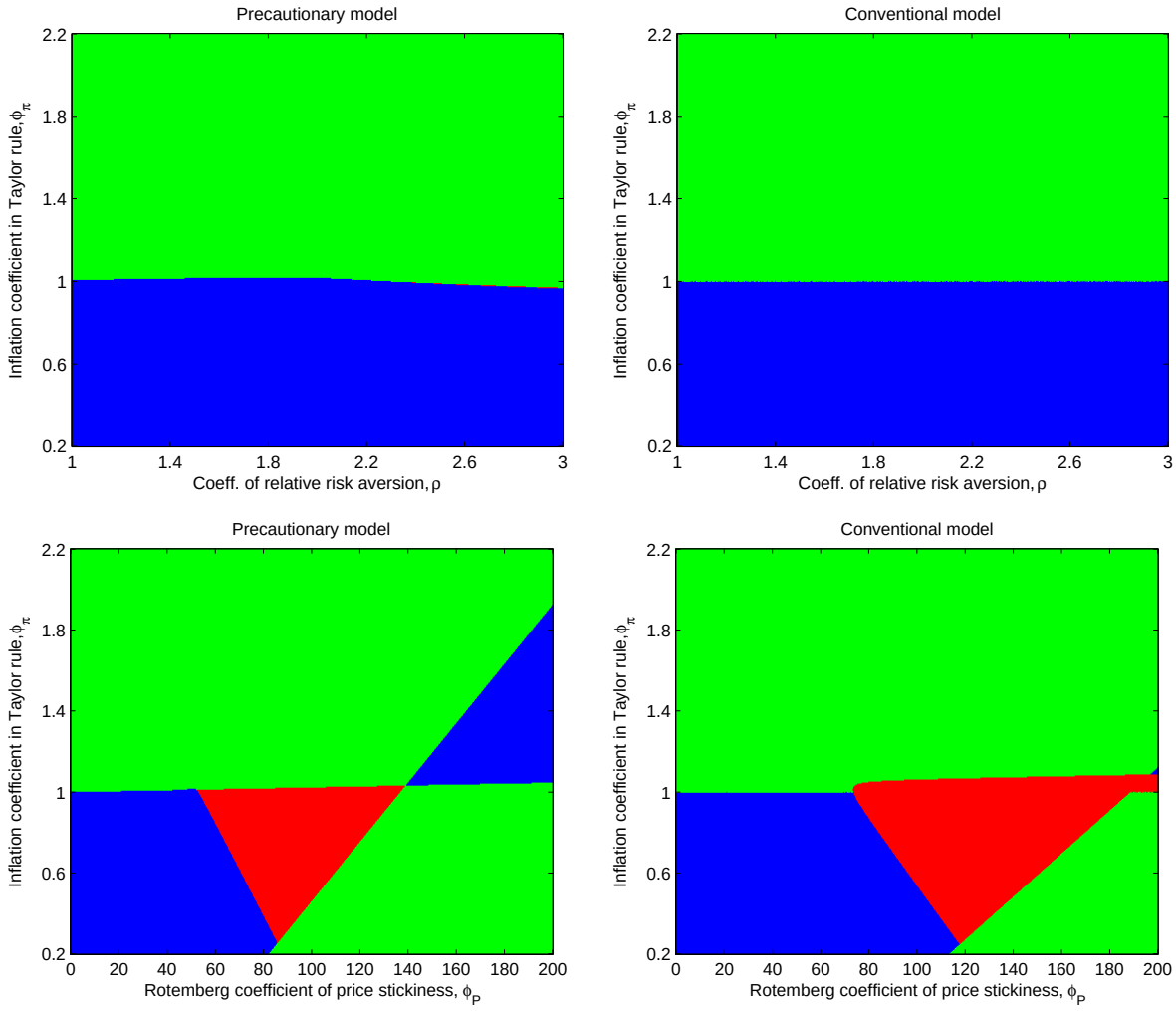


Figure 4: Determinacy regions, precautionary and conventional model

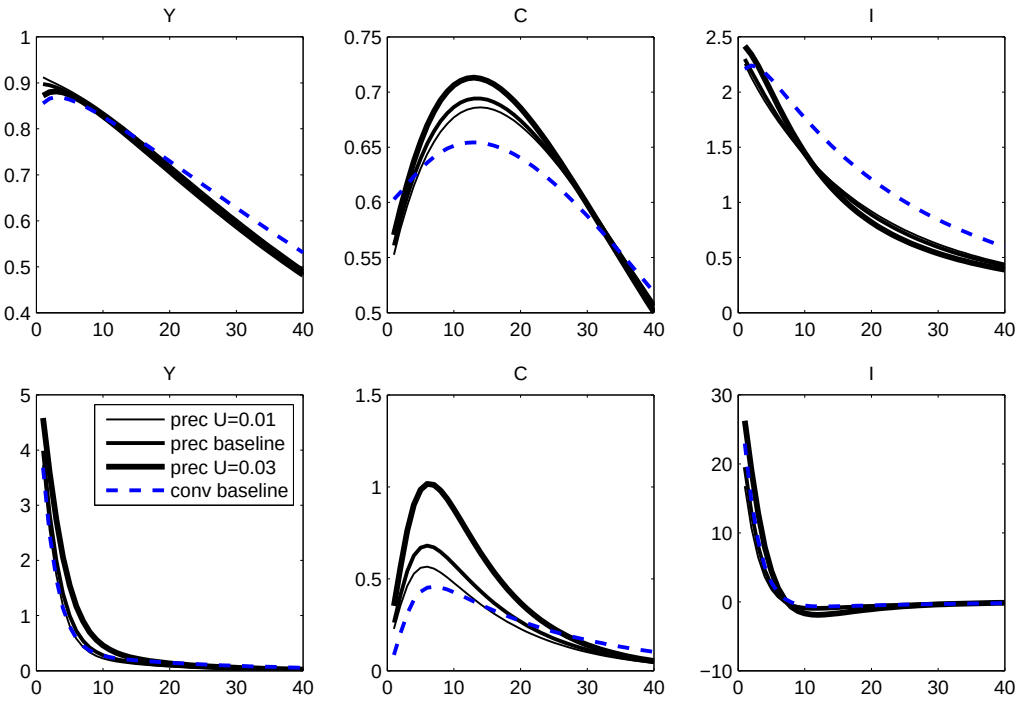


4.3 Determinacy regions

For the precautionary and conventional models, the plots in Figure 4 show the determinacy regions for the inflation coefficient in the Taylor rule ϕ_π against the coefficient of relative risk aversion ρ (upper panels) and against the Rotemberg coefficient of price stickiness ϕ_p (lower panels). The red, green and blue areas indicate parameter combinations for which no unique equilibrium exists, one unique equilibrium exists and infinitely many equilibria exist, respectively. For the baseline calibration, the standard Taylor principle holds. In the precautionary set up the the monetary policy response has to be slightly more pronounced than in the conventional model for low ρ's. For high levels of price rigidity, the inflation coefficient in the monetary policy rule has to increase considerably for a unique solution to exist.

Note that the green region with a low inflation coefficient and high price rigidity present in both the precautionary and conventional setup involves a complex eigenvalue causing the impulse

Figure 5: Sensitivity analysis with respect to variations in the risk of permanent income loss, U. Row 1: technology shock, row 2: monetary shock



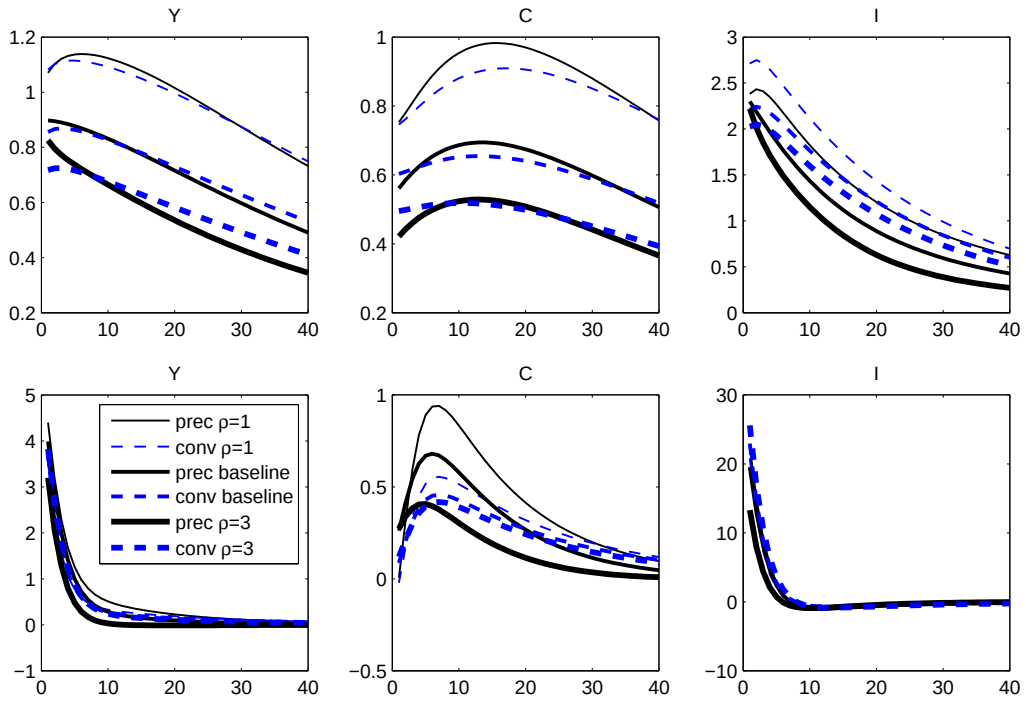
responses to oscillate.

4.4 Sensitivity analysis

Figure 5 presents sensitivity with respect to variations in parameter U, the risk of permanent income loss. The size of U thus directly governs the strength of the precautionary motive and is the key parameter behind the buffer stock savings motive. The figure reports the output, (aggregate) consumption, and investment response to both a technology shock (row 1) and a monetary shock (row 2), of the same size as reported in section 4.2. Sensitivity is shown with respect to three different values of the risk of permanent income loss: $U = 0.01$, $U = 0.015$ (baseline), and $U = 0.03$. Unsurprisingly, the humpshapedness of the consumption response becomes more pronounced, the higher parameter U, as the mechanism discussed in section 4.2 increases in importance.

Figure 6 presents sensitivity with respect to another key parameter, varying the coefficient of relative risk aversion, ρ. Unlike in the previous figure, variations in ρ affect responses in both the precautionary and conventional model. The differences across the two model versions increase for higher ρ, as the role of the precautionary motive is intensified. This can be seen also from the consumption Euler equation of the precautionary model, equation (PS-04), where the term $\left(\frac{\tilde{C}^a_{t+1}}{\tilde{C}^i_{t+1}}\right)^{\rho}$ is raised by a higher power for increasing ρ. This directly implies that the force giving rise to consumption growth rates that are increasing over time becomes more important.

Figure 6: Sensitivity analysis with respect to variations in coefficient of relative risk aversion, ρ. Row 1: technology shock, row 2: monetary shock



5 Conclusion

The paper documents and analyzes the effects of adopting Carroll's and Toche's (2009) setup of tractable buffer stock savings into a standard New-Keynesian dynamic stochastic general equilibrium model with financial frictions. For that reason, we compare and contrast two model versions of an otherwise identical model: one with buffer stock savings setup ('precautionary model'), one without ('conventional model'). We find that the presence of a precautionary motive, that stems from an uninsurable risk of permanent income loss, has both interesting steady state and dynamic effects on the models' properties. As for the steady state effects, consistent with Carroll's findings, the precautionary motive, which implies that consumption growth rates are higher over time compared to a conventional model without such motive, leads to a lower steady state consumption level. Also, because of the general equilibrium nature of the model, the precautionary motive is also reflected in a higher price for savings (asset holdings), or a lower equilibrium interest rate, consistent with the broader precautionary savings literature. Macroeconomic dynamics are also affected in nontrivial ways: Carroll's buffer stock saving setup translates into a more hump-shaped reaction of consumption in response to both supply (technology) and demand (monetary) shocks, and produces more pronounced reactions in response to demand shocks. Adoption of the buffer stock savings setup thus offers a more microfounded way, compared to, e.g., habit preferences in consumption, to introduce Keynesian features into the model, serving as a device to curbing excessive consumption smoothing, and to attributing a higher role to demand driven fluctuations. With the aim of serving as a user guide to researchers interested in adopting this setup, we also discuss

determinacy properties as well as other practical issues.

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A Appendix A: The patient household problem in the precautionary model

A.1 Inactive households

The problem of the inactive household is

$$\max_{c_t^i, b_{t+1}^i} \mathrm{E}_0 \sum_{t=0}^{\infty} (\beta(1-D))^t \frac{(c_t^i)^{1-\rho}}{1-\rho}$$
$$s.t. \quad b_{t+1}^i = \frac{R_t}{\Pi_{t+1}} \left(\tau_t^i + b_t^i - c_t^i\right)$$

where β, ρ, c_t^i, b_t^i, R_t and Π_t are the discount rate, the inverse of the elasticity of intertemporal substitution of consumption, real consumption of the inactive household, real wealth of the inactive household, the gross nominal interest rate and the gross inflation rate, respectively. The zero-profit condition of a Blanchard (1985) insurance company implies

$$0 = D \left(\frac{R_t}{\Pi_{t+1}} (\tau_t^i + b_t^i - c_t^i) \right) - \frac{R_t}{\Pi_{t+1}} \tau_t^i$$

Solving for τ_t^i and substituting into the inactive household's budget constraint yields

$$b_{t+1}^i = \frac{R_t}{\Pi_{t+1}} \frac{1}{1-D} \left(b_t^i - c_t^i\right).$$

The Lagrangian characterizing this problem is

$$L = \mathrm{E}_0 \sum_{t=0}^{\infty} (\beta(1-D))^t \left(\frac{(c_t^i)^{1-\rho}}{1-\rho} + \lambda_t^i \left(\frac{R_t}{\Pi_{t+1}} \frac{1}{1-D} \left(b_t^i - c_t^i\right) - b_{t+1}^i \right) \right).$$

The FOCs w.r.t. consumption and wealth are

$$\lambda_t^i \mathrm{E}_t \frac{R_t}{\Pi_{t+1}} \frac{1}{1-D} = (c_t^i)^{-\rho}$$

and

$$\lambda_t^i = \beta(1-D) \mathrm{E}_t \frac{R_{t+1}}{\Pi_{t+2}} \frac{1}{1-D} \lambda_{t+1}^i,$$

respectively. Combining the two FOCs leads to

$$(c_t^i)^{-\rho} = \beta \mathrm{E}_t \frac{R_t}{\Pi_{t+1}} (c_{t+1}^i)^{-\rho}$$
$$(c_t^i)^{-\rho} = \beta^n \mathrm{E}_t \prod_{k=0}^{n-1} \frac{R_{t+k}}{\Pi_{t+k+1}} (c_{t+n}^i)^{-\rho}$$

The budget constraint can be rearranged and iterated forward and, then, the previous result can be used to get

$$\begin{aligned}
b_t^i &= (1-D)\left(\frac{R_t}{\Pi_{t+1}}\right)^{-1} b_{t+1}^i + c_t^i \\
&= \sum_{n=0}^{\infty}(1-D)^n \prod_{k=0}^{n-1}\left(\frac{R_{t+k}}{\Pi_{t+k+1}}\right)^{-1} c_{t+n}^i \\
&= \sum_{n=0}^{\infty}(1-D)^n \prod_{k=0}^{n-1}\left(\frac{R_{t+k}}{\Pi_{t+k+1}}\right)^{-1} c_{t+n}^i \beta^n \mathrm{E}_t \prod_{k=0}^{n-1}\frac{R_{t+k}}{\Pi_{t+k+1}}(c_{t+n}^i)^{-\rho}(c_t^i)^{\rho} \\
&= \sum_{n=0}^{\infty}(1-D)^n \beta^n \mathrm{E}_t (c_{t+n}^i)^{1-\rho}(c_t^i)^{\rho} \\
b_t^i &= \kappa_t (c_t^i)^{\rho} \\
b_t^i \frac{\Theta^i}{\Gamma^t} &= \kappa_t \left(\frac{\Theta^i}{\Gamma^t}\right)^{1-\rho}\left(c_t^i \frac{\Theta^i}{\Gamma^t}\right)^{\rho} \\
\tilde{B}_t^i &= \check{\kappa}_t (\tilde{C}_t^i)^{\rho}
\end{aligned}$$

where

$$\begin{aligned}
\kappa_t &= (c_t^i)^{1-\rho} + (1-D)\beta \mathrm{E}_t \kappa_{t+1} \\
\kappa_t \left(\frac{\Theta^i}{\Gamma^t}\right)^{1-\rho} &= \left(c_t^i \frac{\Theta^i}{\Gamma^t}\right)^{1-\rho} + (1-D)\beta\Gamma^{1-\rho}\mathrm{E}_t \kappa_{t+1}\left(\frac{\Theta^i}{\Gamma^{t+1}}\right)^{1-\rho} \\
\check{\kappa}_t &= (\tilde{C}_t^i)^{1-\rho} + (1-D)\beta\Gamma^{1-\rho}\mathrm{E}_t \check{\kappa}_{t+1}
\end{aligned}$$

Note that we have assumed $\prod_{k=0}^{n-1}\left(\frac{R_{t+k}}{\Pi_{t+k+1}}\right)^{-1} c_{t+n}^i \mathrm{E}_t \prod_{k=0}^{n-1}\frac{R_{t+k}}{\Pi_{t+k+1}}(c_{t+n}^i)^{-\rho} = \mathrm{E}_t (c_{t+n}^i)^{1-\rho}$ which holds only up to a first-order approximation $\forall k = 0, \ldots, n$ and $\forall n$ because of certainty equivalence. The aggregation of the inactive household's budget constraint requires some caution. Summing over all inactive households, the budget constraint reads

$$\Theta^i b_{t+1}^i = \frac{R_t}{\Pi_{t+1}}\left(\Theta^i \tau_t^i + B_t^i - C_t^i\right)$$

where we define $B_t^i \equiv \Theta^i b_t^i$ and $C_t^i \equiv \Theta^i c_t^i$. Note, however, that, in general, $B_{t+1}^i \neq \Theta^i b_{t+1}^i$. This is because the evolution of B_t^i needs to take into account that a share U of active households becomes inactive and a share D of inactive households dies, whereas the individual evolution of b_t^i is conditional on staying alive. Hence, $\Theta^i b_{t+1}^i$ is the aggregated wealth of inactive households tomorrow *before* a share D of them dies. When they die, $D\Theta^i b_{t+1}^i = DR_t/\Pi_{t+1}(\Theta^i \tau_t^i + B_t^i - C_t^i)$ of wealth goes to the insurance company as accidental bequests which, due to the zero profit condition, are fully transferred to the remaining inactive households. Hence, $R_t/\Pi_{t+1}\Theta^i \tau_t^i = DR_t/\Pi_{t+1}(\Theta^i \tau_t^i + B_t^i - C_t^i)$. Using this equation to substitute out $\Theta^i \tau_t^i$ from the aggregated budget constraint above, we get, similar to the individual budget constraint,

$$\Theta^i b_{t+1}^i = \frac{R_t}{\Pi_{t+1}}\frac{1}{1-D}\left(B_t^i - C_t^i\right)$$

The aggregated wealth in $t+1$ of the households which have been inactive in t and are still inactive in $t+1$, i.e. have not died, is $(1-D)\Theta^i b^i_{t+1}$. Yet, note that $B^i_{t+1} \neq (1-D)\Theta^i b^i_{t+1}$ either, as tomorrow's inactive wealth needs to include the wealth of those households which were active in t and become inactive in $t+1$, as well. Let B^{ai}_t denote the aggregated wealth of households which were active at the beginning of t and inactive at the end of t. Then, $B^i_{t+1} = (1-D)\Theta^i b^i_{t+1} + B^{ai}_{t+1}$. Solving this equation for $\Theta^i b^i_{t+1}$ and substituting it into the left hand side of the aggregated budget constraint above, yields

$$\begin{aligned}
\frac{1}{1-D}(B^i_{t+1} - B^{ai}_{t+1}) &= \frac{R_t}{\Pi_{t+1}} \frac{1}{1-D}\left(B^i_t - C^i_t\right) \\
B^i_{t+1} &= \frac{R_t}{\Pi_{t+1}}\left(B^i_t - C^i_t\right) + B^{ai}_{t+1} \\
\tilde{B}^i_{t+1} &= \frac{R_t}{\Pi_{t+1}} \frac{1}{\Gamma}(\tilde{B}^i_t - \tilde{C}^i_t) + \tilde{B}^{ai}_{t+1}.
\end{aligned}$$

A.2 Active households

The active household's problem is

$$\begin{aligned}
&\max_{c^a_t, b^a_{t+1}, n_t} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(c^a_t)^{1-\rho}}{1-\rho} - Z_t^{1-\rho} \psi \frac{n_t^{1+\eta}}{1+\eta} \right) \\
&\quad s.t. \quad b^a_{t+1} = \frac{R_t}{\Pi_{t+1}} \left(W_t n_t + d_t + b^a_t - c^a_t - \tau^a_t \right)
\end{aligned}$$

where n_t, W_t, u_t, d_t, τ^a_t, ψ, and η denote labor supply, the real wage, the unemployment rate, distributed profits, a non-distortionary transfer, a scaling parameter and the inverse of the Frisch elasticity, respectively. The variable τ^a_t needs some clarification (cf. Carroll and Jeanne 2009): Note that without any transfer from non-newborn to newborn active households, the latter will not have any wealth. The older households are, the more wealth they will have accumulated. This makes aggregation impossible. To simplify, we assume τ^a_t to be such that wealth is distributed equally across active households at any point in time. Let us interpret the budget constraint as seen from a newborn household: It receives a transfer of $b^a_t - \tau^a_t$. Now, as seen from a non-newborn household, the budget constraint states that it must give away τ^a_t of its wealth. The crucial implication is that both newborn and non-newborn households face the same budget constraint after the transfer. Assuming that the transfer is financed by a tax on wealth, i.e. $\tau^a_t = \tau^a b^a_t$, the required tax rate can be shown to be $\tau^a = U$. This follows from the fact that the payments aggregated over all non-newborn active households with mass $1/U - 1$ must equal the receipts aggregated over all newborn active households with mass one, i.e. $(1/U-1)\tau^a b^a_t = b^a_t - \tau^a b^a_t$.

The household faces a risk U of permanent income loss. We assume that the active household cannot loose income and die in the same period. It is convenient to set up the household's problem as a dynamic program:

$$\begin{aligned}
v^a_t(b^a_t) = \max_{c^a_t, b^a_{t+1}, n_t} &\left[\begin{array}{c} \frac{(c^a_t)^{1-\rho}}{1-\rho} - Z_t^{1-\rho}\psi \frac{n_t^{1+\eta}}{1+\eta} + \\ +\beta(1-U)\mathrm{E}_t v^a_{t+1}(b^a_{t+1}) + \beta U \mathrm{E}_t v^i_{t+1}(b^a_{t+1}) \end{array} \right] \\
&s.t. \quad b^a_{t+1} = \frac{R_t}{\Pi_{t+1}} \left(W_t n_t + d_t + b^a_t - c^a_t - \tau^a_t \right)
\end{aligned}$$

where $v_t^a(b_t^a)$ is the value function in t. $v_{t+1}^i(b_{t+1}^a)$ is the value function of a newly inactive household. Substituting out c_t^a using the budget constraint and, the FOC w.r.t. to wealth, b_{t+1}, implies after applying the envelop condition,

$$
\begin{aligned}
v_t^{a\prime} &= \beta \mathrm{E}_t \frac{R_t}{\Pi_{t+1}} \left((1-U) v_{t+1}^{a\prime} + U v_{t+1}^{i\prime} \right) \\
v_t^{a\prime} (\Theta^a \Theta^i)^{-\rho} &= \beta \mathrm{E}_t \frac{R_t}{\Pi_{t+1}} \left((1-U) v_{t+1}^{a\prime} (\Theta^a \Theta^i)^{-\rho} + U v_t^{i\prime} (\Theta^a \Theta^i)^{-\rho} \right) \\
v_t^{a\prime} \left(\frac{\Theta^a \Theta^i}{\Gamma^t} \right)^{-\rho} &= \beta \Gamma^{-\rho} \mathrm{E}_t \frac{R_t}{\Pi_{t+1}} \left((1-U) v_{t+1}^{a\prime} \left(\frac{\Theta^a \Theta^i}{\Gamma^{t+1}} \right)^{-\rho} + U v_t^{i\prime} \left(\frac{\Theta^a \Theta^i}{\Gamma^{t+1}} \right)^{-\rho} \right) \\
\tilde{v}_t^{a\prime} (\Theta^i)^{-\rho} &= \beta \Gamma^{-\rho} \mathrm{E}_t \frac{R_t}{\Pi_{t+1}} \left((1-U) \tilde{v}_{t+1}^{a\prime} (\Theta^i)^{-\rho} + U \tilde{v}_{t+1}^{i\prime} (\Theta^a)^{-\rho} \right) \\
\tilde{v}_t^{a\prime} &= \beta \Gamma^{-\rho} \mathrm{E}_t \frac{R_t}{\Pi_{t+1}} \left((1-U) \tilde{v}_{t+1}^{a\prime} + U \left(\frac{\Theta^a}{\Theta^i} \right)^{-\rho} \tilde{v}_{t+1}^{i\prime} \right) \\
\tilde{v}_t^{a\prime} &= \beta \Gamma^{-\rho} \mathrm{E}_t \frac{R_t}{\Pi_{t+1}} \left((1-U) \tilde{v}_{t+1}^{a\prime} + U \left(\frac{D}{U} \right)^{-\rho} \tilde{v}_{t+1}^{i\prime} \right)
\end{aligned}
$$

where

$$
\begin{aligned}
v_t^{a\prime} &= (c_t^a)^{-\rho} \\
v_t^{a\prime} &= \left(\frac{C_t^a}{\Theta^a} \right)^{-\rho} \\
v_t^{a\prime} (\Theta^a)^{-\rho} &= (C_t^a)^{-\rho} \\
v_t^{a\prime} \left(\frac{\Theta^a}{\Gamma^t} \right)^{-\rho} &= \left(\frac{C_t^a}{\Gamma^t} \right)^{-\rho} \\
\tilde{v}_t^{a\prime} &= (\tilde{C}_t^a)^{-\rho}
\end{aligned}
$$

with $\tilde{v}_t^{a\prime} \equiv v_t^{a\prime} \left(\frac{\Theta^a}{\Gamma^t} \right)^{-\rho}$, and, using the FOC of the inactive household as well as recalling that v_t^i is the value function of the newly inactive household,

$$
\begin{aligned}
v_t^{i\prime} &= (c_t^i)^{-\rho} \\
v_t^{i\prime} &= \left(\frac{C_t^i}{\Theta^i} \right)^{-\rho} \\
v_t^{i\prime} (\Theta^i)^{-\rho} &= \left(C_t^i \right)^{-\rho} \\
v_t^{i\prime} \left(\frac{\Theta^i}{\Gamma^t} \right)^{-\rho} &= \left(\frac{C_t^i}{\Gamma^t} \right)^{-\rho} \\
\tilde{v}_t^{i\prime} &= (\tilde{C}_t^a)^{-1} \\
\tilde{v}_t^{i\prime} &= (\kappa \tilde{B}_t^a)^{-1}
\end{aligned}
$$

with $\tilde{v}_t^{i\prime} \equiv v_t^{i\prime} \left(\frac{\Theta^i}{\Gamma^t} \right)^{-\rho}$. To derive the active household's aggregate budget constraint note that we have assumed a transfer which ensures that every active household has the same stock wealth.

Aggregation of wealth over newborn and non-newborn households allows us to define $B_t^a \equiv b_t^a - \tau^a b_t^a + (1/U - 1)(b_t^a - \tau^a b_t^a) = (1/U)(b_t^a - \tau^a b_t^a) = \Theta^a(b_t^a - \tau^a b_t^a)$. Summing over all inactive households, the budget constraint reads

$$\Theta^a b_{t+1}^a = \frac{R_t}{\Pi_{t+1}} \left(W_t N_t + D_t + B_t^a - C_t^a\right)$$

where we define $N_t \equiv \Theta^a n_t$, $D_t \equiv \Theta^a d_t$ and $C_t^a \equiv \Theta^a c_t^a$. Again, $B_{t+1}^a \neq \Theta^a b_{t+1}^a$ as a share U of the active household will become inactive carrying over wealth B_{t+1}^{ai} in the aggregate. Hence, $B_{t+1}^a = \Theta^a b_{t+1}^a - B_{t+1}^{ai}$. Solving this equation for $\Theta^a b_{t+1}^a$ and substituting it into the left hand side of the aggregated budget constraint above, yields

$$\begin{aligned}
B_{t+1}^a &= \frac{R_t}{\Pi_{t+1}} \left(W_t N_t + D_t + B_t^a - C_t^a\right) - B_{t+1}^{ai} \\
\tilde{B}_{t+1}^a &= \frac{R_t}{\Pi_{t+1}} \frac{1}{\Gamma} (\tilde{W}_t N_t + \tilde{D}_t + \tilde{B}_t^a - \tilde{C}_t^a) - \tilde{B}_{t+1}^{ai}.
\end{aligned}$$

What is $\tilde{B}_{t+1}^{ai}$? It is the aggregate wealth of those active households which become inactive at the beginning of $t+1$. Hence,

$$\tilde{B}_{t+1}^{ai} = U \frac{R_t}{\Pi_{t+1}} \left(\tilde{W}_t N_t + \tilde{D}_t + \tilde{B}_t^a - \tilde{C}_t^a\right)$$

The FOC of the active household w.r.t. labor supply implies

$$\begin{aligned}
Z_t^{1-\rho} \psi n_t^\eta &= \lambda_t \mathrm{E}_t \frac{R_t}{\Pi_{t+1}} W_t \\
\Gamma^{t(1-\rho)} \psi n_t^\eta &= (c_t^a)^{-\rho} W_t \\
\Gamma^{t(1-\rho)} \psi \left(\frac{N_t}{\Theta^a}\right)^\eta &= \left(\frac{C_t^a}{\Theta^a}\right)^{-\rho} W_t \\
\psi (\Theta^a)^{-(\rho+\eta)} N_t^\eta &= (\tilde{C}_t^a)^{-\rho} \tilde{W}_t \\
\psi U^{\rho+\eta} N_t^\eta &= (\tilde{C}_t^a)^{-\rho} \tilde{W}_t.
\end{aligned}$$