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**Working Paper — Digitized Version**

## Shortcomings of MRP II and a new planning meta-method

Manuskripte aus den Instituten für Betriebswirtschaftslehre der Universität Kiel, No. 404

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*Suggested Citation:* Kimms, Alf; Drexl, Andreas (1996) : Shortcomings of MRP II and a new planning meta-method, Manuskripte aus den Instituten für Betriebswirtschaftslehre der Universität Kiel, No. 404, Universität Kiel, Institut für Betriebswirtschaftslehre, Kiel

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## Shortcomings of MRP II and a New Planning Meta-Method

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September 1996

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## Abstract

Lot sizing when done for the short-term heavily interacts with the sequencing decisions for the operations to be performed. Especially for real-world situations where capacities are scarce, demand is dynamic, and precedence relations among the operations have to be taken into account the MRP II logic which is implemented in most production planning systems does not satisfy. In this paper, we will reveal the shortcomings of MRP II by means of an example. A mixed-integer programming model is then defined to specify the problem of capacitated, dynamic, multi-level lot sizing and scheduling. Also, we present a generic solution method (a so-called meta-method) which may be used as a basis of more advanced implementations that may replace the traditional MRP II systems.

## 1 Problem Context

Consider the organization of an in-house production system. Typically, the architecture of such a system is build up from several production cells, so-called segments, which may be implemented in different fashions (flow lines or work centers for instance). This macro-structure further refines into a micro-structure as each segment provides the capability to perform a bunch of operations.

Raw materials and component parts are floating concurrently through this complex system in order to be processed and assembled until a final product comes out being ready for deliverance.

Production planning is one of the most challenging subjects for the management there. It appears to be a hierarchical process ranging from long- to medium- to short-term decisions (see for instance [6, 10, 12, 32, 33, 34, 36]). Our focus will be the short-term scope which links to medium-term decisions via the master production schedule (MPS). The MPS defines the external (or independent) demand, i.e. due dates and order sizes for final products. The goal now is to find a feasible production plan which meets the requests and provides release dates and amounts for all products including component parts. For economical reasons, finding a feasible plan is not sufficient. In the usual case, production plans can be evaluated by means of an objective function (e.g. a function which measures the setup and the holding costs). Then, the aim is to find a feasible production plan with optimum (or close to optimum) objective function value.

## 2 Problem Outline

Let the manufacturing process be triggered by orders which originate from customers or from other facilities. Suppose now, that the output of the *make-to-order* system under concern is or at least includes a set of *non-customized*

*products*. Certainly, this is a valid assumption for many firms no matter what industry they belong to and no matter of what size they are.

To motivate a planning activity, we first need to identify a subject of concern that is worth (in terms of economical rationale) to be considered. A first clue are large inventories. Due to the opportunity costs of capital and the direct costs of storing goods, holding items in inventory and thus causing holding costs should be avoided. On the other hand, if different parts are making use of common resources, say machines, and a setup action must take place to prepare proper operation, then opportunity costs (i.e. setup costs) are incurred since production is delayed. Another aspect of sharing resources is that the production of such parts cannot coincide if different setup states are required. Hence, orders must be sequenced. If production planning is about the timing of production and not about what to produce (e.g. make-or-buy decisions), then production costs need not to be considered as long as they are time invariant. In summary, we have a trade-off between low setup costs (favoring large production lots) and low holding costs (favoring a lot-for-lot-like production where sequence decisions have to be made due to sharing common resources). Essentially, the problem of short-term production planning turns out to be a *lot sizing and scheduling problem* then.

If we ask about how to solve this production planning problem, we first need a deeper understanding of its basic attributes. The first key element we have to remember is the stream of component parts floating through a complex production system. Operations may be executed only if parts being subject of these particular operations are indeed available. In other words, a production plan must respect the precedence relations of operations. Hence, *multi-level* structures must be taken into account. For the sake of convenience, we do not further distinguish between operations and items (also called products or parts). Each operation produces an item, and each item is the output of an operation. Both terms are used as synonyms. Apparently, we face a *multi-item* problem here.

The second key element of our problem is the presence of scarce capacity. As usual in in-house production systems, producing an item requires a certain amount of one or more resources (e.g. manpower, machine time, energy, ...) with limited capacity per time unit. Thus, production planning must take *scarce capacity* into account. Following the terminology in the literature, we also say machine instead of resource.

Furthermore, we have the following situation: The time interval which is the focus of the planning process is *finite* and subdivided into several *discrete time periods*. To refer to these periods we number them consecutively beginning with period 1. The length of the overall time interval is called the planning horizon, or horizon for short, and is counted in number of time periods. This properly reflects the real-world situation where we face a planning horizon of say four weeks (or 20 days or 40 shifts) and discrete time periods are naturally given. The (known or estimated) external demand (given by the MPS) is given in units per

item per period. It is to be met promptly at the end of each period. Backlogging and shortages are not allowed here which enforces a high service level. The demand may vary from period to period. This is called *dynamic* demand. All relevant data for the planning process is assumed to be *deterministic* which is justified by having a short-term planning problem on hand.

### 3 Current Practice

The basic working principle of today’s decision support systems for Manufacturing Resource Planning (MRP II) is more or less the same in all current implementations. It should be reviewed here by means of a small example (see also [8]). For an overview of more than a hundred modern MRP II software packages we refer to [1, 2] where a detailed list of features is enclosed.

Assume the following data: Three items  $j = 1, \dots, 3$  are to be produced sharing a single machine. The gozinto-structure<sup>1</sup> of these items is given in Figure 1. The planning horizon is 10 periods ( $t = 1, \dots, 10$ ) long and inventory is empty. Table 1<sup>2</sup> provides the MPS (with external demands  $d_{jt}$ ), the capacity limit per period  $C_t$ , the need of capacity per item  $p_j$ , and the item-specific setup costs  $s_j$  and holding costs per period  $h_j$ . We assume that the minimum lead time is zero.

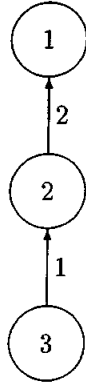
| $d_{jt}$ | $t = 1$ | ... | 6   | 7   | 8   | 9   | 10  | $h_j$ | $s_j$ | $p_j$ |
|----------|---------|-----|-----|-----|-----|-----|-----|-------|-------|-------|
| $j = 1$  |         |     | 20  |     | 20  |     | 20  | 25    | 900   | 1     |
| $j = 2$  |         |     |     |     |     |     |     | 10    | 850   | 1     |
| $j = 3$  |         |     |     |     |     |     |     | 10    | 800   | 1     |
| $C_t$    | 100     | ... | 100 | 100 | 100 | 100 | 100 |       |       |       |

Table 1: Data of the Example

Starting with the MPS, lot sizes for all items are determined by a level-by-level approach disregarding capacity constraints. In our example, we start to compute lot sizes for item 1 using some lot sizing rule. Let us say, we decide to use a lot-for-lot policy. Then, we derive internal demands and due dates for the next level which is item 2. As a result we have an internal demand for 40 units of item 2 in periods 6, 8 and 10. Again, we employ some lot sizing rule, but, this time considering item 2. Suppose, we decide to produce the demands

<sup>1</sup>The term gozinto-structure was coined by Vazsonyi [35] who gave a reference to an italian mathematician named Zepartzat Gozinto. This person is pure fiction. But, if you read his name in English pronunciation, it turns out what is meant: The product-structure. Nodes represent items, and arcs depict precedence relations between the items. Arc weights are production coefficients.

<sup>2</sup>Missing entries in tables are assumed to be zero throughout the text.




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Figure 1: A Gozinto-Structure with Three Items

of periods 8 and 10 in one lot in period 8. Following the same lines, we then compute internal demands for item 3 and again employ some lot sizing rule. As a result we might get what is shown in Table 2 where  $q_{jt}$  denotes the production quantity of item  $j$  in period  $t$ . Note, this is not a valid production plan since capacity restrictions are violated in period 8.

| $q_{jt}$ | $t = 1$ | 2 | 3 | 4 | 5 | 6  | 7 | 8  | 9 | 10 |
|----------|---------|---|---|---|---|----|---|----|---|----|
| $j = 1$  |         |   |   |   |   | 20 |   | 20 |   | 20 |
| $j = 2$  |         |   |   |   |   | 40 |   | 80 |   |    |
| $j = 3$  |         |   |   |   |   | 40 |   | 80 |   |    |

Table 2: A Production Plan with Capacity Constraint Violations

In a next step, the intermediate result is modified to find a plan without requiring an excess of capacity. This is done by shifting lots to the left or to the right until the capacity profile is met. In our example for instance, this can be achieved by shifting the lot for item 2 in period 8 to the left (see Table 3).

But, because precedence constraints are not taken into account when lots are shifted in order to find a plan that does not violate the capacity restrictions, the conventional approach obviously fails. Looking at the example reveals, it would never be possible to produce item 2 in period 7 since we lack a sufficient amount of item 3 there.

The traditional way to overcome this is to introduce lead times. To make

| $q_{jt}$ | $t = 1$ | 2 | 3 | 4 | 5 | 6  | 7  | 8  | 9 | 10 |
|----------|---------|---|---|---|---|----|----|----|---|----|
| $j = 1$  |         |   |   |   |   | 20 |    | 20 |   | 20 |
| $j = 2$  |         |   |   |   |   | 40 | 80 |    |   |    |
| $j = 3$  |         |   |   |   |   | 40 |    | 80 |   |    |

Table 3: A Production Plan with Precedence Relation Violations

this idea clear, let us start with the MPS again (see Table 1). Going through the level-by-level approach as described above, we now use an offset of, say, two periods — the so-called lead time — when we compute the due dates of the internal demand. The outcome of the procedure is given in Table 4.

| $q_{jt}$ | $t = 1$ | 2  | 3 | 4  | 5 | 6  | 7 | 8  | 9 | 10 |
|----------|---------|----|---|----|---|----|---|----|---|----|
| $j = 1$  |         |    |   |    |   | 20 |   | 20 |   | 20 |
| $j = 2$  |         |    |   | 40 |   | 80 |   |    |   |    |
| $j = 3$  |         | 40 |   | 80 |   |    |   |    |   |    |

Table 4: A Production Plan with Positive Lead Times

Note, this plan is infeasible again. Once more, we are off capacity limits. Shifting the lot of item 2 in period 4 to the left, leads us to the solution in Table 5.

| $q_{jt}$ | $t = 1$ | 2  | 3  | 4  | 5 | 6  | 7 | 8  | 9 | 10 |
|----------|---------|----|----|----|---|----|---|----|---|----|
| $j = 1$  |         |    |    |    |   | 20 |   | 20 |   | 20 |
| $j = 2$  |         |    | 40 |    |   | 80 |   |    |   |    |
| $j = 3$  |         | 40 |    | 80 |   |    |   |    |   |    |

Table 5: A Feasible Production Plan

Now we are done. The result represents a feasible production plan. The sum of setup and holding costs is 9,800. Due to the introduction of positive lead times we had more flexibility when shifting lots which helped. And, indeed this is what can be observed in practice. Known as the lead-time-syndrome [37], planners tend to increase lead times arbitrarily whenever they detect backorders or high work-in-process inventories. But, as a result most firms suffer from long makespans and large total holding costs. To be convinced, compare the feasible plan in Table 5 with the optimum plan in Table 6. Figure 2 shows the schedule of the optimum solution by means of a Gantt-chart. The sum of setup and holding costs is 6,700.



| $q_{jt}$ | $t = 1$ | 2 | 3 | 4 | 5 | 6  | 7  | 8  | 9 | 10 |
|----------|---------|---|---|---|---|----|----|----|---|----|
| $j = 1$  |         |   |   |   |   | 20 |    | 20 |   | 20 |
| $j = 2$  |         |   |   |   |   | 40 |    | 80 |   |    |
| $j = 3$  |         |   |   |   |   | 40 | 80 |    |   |    |

Table 6: An Optimum Production Plan

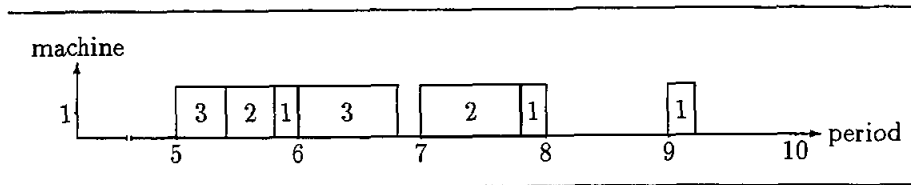


Figure 2: An Optimum Schedule

The reason for this dilemma in production planning apparently is the traditional level-by-level approach. This widely used method does not take capacity constraints and precedence relations simultaneously into account. Unfortunately, good alternatives are not available yet. This gives the motivation to develop such.

## 4 Basic Assumptions

Several items are to be produced in order to meet some known (or estimated) dynamic demand without backlogs and stockouts. Precedence relations among these items define an acyclic gozinto-structure of the general type. In contrast to many authors who allow demand for end items only, now, demand may occur for all items including component parts. The finite planning horizon is subdivided into a number of discrete time periods. Positive lead times are given due to technological restrictions such as cooling or transportation for instance. Furthermore, items share common resources. Some (maybe all) of them are scarce. The capacities may vary over time. Producing one item requires an item-specific amount of the available capacity. All data are assumed to be deterministic.

Items which are produced in a period to meet some future demand must be stored in inventory and thus cause item-specific holding costs. Most authors assume that the holding costs for an item must be greater than or equal to the sum of the holding costs for all immediate predecessors. They argue that holding costs are mainly opportunity costs for capital which occurs no matter a component part is assembled or not. Two reasons persuade us to make no particular assumptions for holding costs. First, as it is usual in the chemical industry for

instance, keeping some component parts in storage may require ongoing additional effort such as cooling, heating, or shaking. While these parts need no special treatment when processed, storing component parts might be more expensive than storing assembled items. Second, operations such as cutting tin mats for instance make parts smaller and often easier to handle. The remaining “waste” can often be sold as raw material for other manufacturing processes. Hence, opportunity costs may decrease when component parts are assembled. However, it should be made clear that the assumption of general holding costs is the most unrestrictive one. All models and methods developed under this assumption work for more restrictive cases as well.

Each item requires at least one resource for which a setup state has to be taken into account. Production can only take place if a proper state is set up. Setting a resource up for producing a particular item incurs item-specific setup costs which are assumed to be sequence independent. Setup times are not considered. Once a certain setup action is performed, the setup state is kept up until another setup changes the current state. Hence, same items which are produced having some idle time in-between do not enforce more than one setup action. To get things straight, note that some authors use the word *changeover* instead of *setup* in this context.

The most fundamental assumption here is that for each resource at most one setup may occur within one period. Hence, at most two items sharing a common resource for which a setup state exists may be produced per period. Due to this assumption, the problem is known as the *proportional lot sizing and scheduling problem (PLSP)* [7, 15, 26]. By choosing the length of each time period appropriately small, the PLSP is a good approximation to a continuous time axis. It refines the well-known discrete lot sizing and scheduling problem (*DLSP*) [5, 11, 18, 28, 31] as well as the continuous setup lot sizing problem (*CSLP*) [3, 20, 19]. Both assume that at most one item may be produced per period. All three models could be classified as *small bucket models* since only a few (one or two) items are produced per period. In contrast to this, the well-known *capacitated lot sizing problem (CLSP)* [4, 9, 13, 17, 27, 29, 30] represents a *large bucket model* since many items can be produced per period. Remember, the CLSP does not include sequence decisions and is thus a much “easier” problem. An extension of the single-level CLSP with partial sequence decisions can be found in [14]. In [16] a large bucket single-level lot sizing and scheduling model is discussed.

## 5 A Mixed-Integer Programming Model

An important variant of the PLSP is the one with multiple machines (*PLSP-MM*). Several resources (machines) are available and each item is produced on an item-specific machine. This is to say that there is an unambiguous mapping from items to machines. Of course, some items may share a common machine.

Special cases are the single-machine problem for which models and methods are given in [24, 21, 25, 23], and the problem with dedicated machines where items do not share a common machine. For the latter optimal solutions can be easily computed with a lot-for-lot policy [22].

Let us first introduce some notation. In Table 7 the decision variables are defined. Likewise, the parameters are explained in Table 8. Note, from these data, we can easily derive the net requirement  $nr_j$  for an item  $j$  and the set  $\mathcal{P}_j$  of all immediate predecessors of an item  $j$ . Both will later on be needed for describing a solution meta-method. Using this notation, we are now able to present a MIP-model formulation.

| Symbol   | Definition  |
|----------|---|
| $I_{jt}$ | Inventory for item $j$ at the end of period $t$ .   |
| $q_{jt}$ | Production quantity for item $j$ in period $t$ .  |
| $x_{jt}$ | Binary variable which indicates whether a setup for item $j$ occurs in period $t$ ( $x_{jt} = 1$ ) or not ( $x_{jt} = 0$ ).                     |
| $y_{jt}$ | Binary variable which indicates whether machine $m_j$ is set up for item $j$ at the end of period $t$ ( $y_{jt} = 1$ ) or not ( $y_{jt} = 0$ ). |

Table 7: Decision Variables for the PLSP-MM

$$\min \sum_{j=1}^J \sum_{t=1}^T (s_j x_{jt} + h_j I_{jt}) \quad (1)$$

subject to

$$I_{jt} = I_{j(t-1)} + q_{jt} - d_{jt} - \sum_{i \in \mathcal{S}_j} a_{ji} q_{it} \quad \begin{array}{l} j = 1, \dots, J \\ t = 1, \dots, T \end{array} \quad (2)$$

$$I_{jt} \geq \sum_{i \in \mathcal{S}_j} \sum_{\tau=t+1}^{\min\{t+v_j, T\}} a_{ji} q_{i\tau} \quad \begin{array}{l} j = 1, \dots, J \\ t = 0, \dots, T-1 \end{array} \quad (3)$$

$$\sum_{j \in \mathcal{J}_m} y_{jt} \leq 1 \quad \begin{array}{l} m = 1, \dots, M \\ t = 1, \dots, T \end{array} \quad (4)$$

$$x_{jt} \geq y_{jt} - y_{j(t-1)} \quad \begin{array}{l} j = 1, \dots, J \\ t = 1, \dots, T \end{array} \quad (5)$$

$$p_j q_{jt} \leq C_{m_j t} (y_{j(t-1)} + y_{jt}) \quad \begin{array}{l} j = 1, \dots, J \\ t = 1, \dots, T \end{array} \quad (6)$$

| Symbol          | Definition  |
|-----------------|---|
| $a_{ji}$        | “Gozinto”-factor. Its value is zero if item $i$ is not an immediate successor of item $j$ . Otherwise, it is the quantity of item $j$ that is directly needed to produce one item $i$ . |
| $C_{mt}$        | Available capacity of machine $m$ in period $t$ .   |
| $d_{jt}$        | External demand for item $j$ in period $t$ .  |
| $h_j$           | Non-negative holding cost for having one unit of item $j$ one period in inventory.  |
| $I_{j0}$        | Initial inventory for item $j$ .  |
| $\mathcal{J}_m$ | Set of all items that share the machine $m$ ,<br>i.e. $\mathcal{J}_m \stackrel{def}{=} \{j \in \{1, \dots, J\} \mid m_j = m\}$ .  |
| $J$             | Number of items.  |
| $M$             | Number of machines.   |
| $m_j$           | Machine on which item $j$ is produced.  |
| $p_j$           | Capacity needs for producing one unit of item $j$ .   |
| $s_j$           | Non-negative setup cost for item $j$ .  |
| $\mathcal{S}_j$ | Set of immediate successors of item $j$ ,<br>i.e. $\mathcal{S}_j \stackrel{def}{=} \{i \in \{1, \dots, J\} \mid a_{ji} > 0\}$ .   |
| $T$             | Number of periods.  |
| $v_j$           | Positive and integral lead time of item $j$ .   |
| $y_{j0}$        | Unique initial setup state.   |

Table 8: Parameters for the PLSP-MM

$$\sum_{j \in \mathcal{J}_m} p_j q_{jt} \leq C_{mt} \quad \begin{array}{l} m = 1, \dots, M \\ t = 1, \dots, T \end{array} \quad (7)$$

$$y_{jt} \in \{0, 1\} \quad \begin{array}{l} j = 1, \dots, J \\ t = 1, \dots, T \end{array} \quad (8)$$

$$I_{jt}, q_{jt}, x_{jt} \geq 0 \quad \begin{array}{l} j = 1, \dots, J \\ t = 1, \dots, T \end{array} \quad (9)$$

The objective (1) is to minimize the sum of setup and holding costs. Equations (2) are the inventory balances. At the end of a period  $t$  we have in inventory what was in there at the end of period  $t - 1$  plus what is produced minus external and internal demand. To fulfill internal demand we must respect positive lead times. Restrictions (3) guarantee so. Constraints (4) make sure that the setup state of each machine is uniquely defined at the end of each period. Those periods in which a setup happens are spotted by (5). Note that idle periods may occur in order to save setup costs. Due to (6) production can only take place if there is a proper setup state either at the beginning or at the end of

a particular period. Hence, at most two items can be manufactured on each machine per period. Capacity constraints are formulated in (7). Since the right hand side is a constant, overtime is not available. (8) define the binary-valued setup state variables, while (9) are simple non-negativity conditions. The reader may convince himself that due to (5) in combination with (1) setup variables  $x_{jt}$  are indeed zero-one valued. Hence, non-negativity conditions are sufficient for these. For letting inventory variables  $I_{jt}$  be non-negative backlogging cannot occur.

## 6 A Meta-Method

There is a generic construction scheme that forms the basis of potential methods. It is a backward oriented procedure which schedules items period by period starting with period  $T$  and ending with period one. We choose here a recurrent representation which enables us to develop the underlying ideas in a stepwise fashion. Now, let us assume that  $construct(t, \Delta t, m)$  is the procedure to be defined. Keep in mind that  $t + \Delta t$  is the period and  $m$  is the machine under concern. We use  $\Delta t \in \{0, 1\}$  where  $\Delta t = 1$  indicates that the setup state for machine  $m$  at the beginning of period  $t + 1$  is to be fixed next and  $\Delta t = 0$  indicates that we already have chosen a setup state at the end of period  $t$ . In the specification below,  $j_{mt}$  will denote the setup state for machine  $m$  at the end of period  $t$ . Assume  $j_{mt} = 0$  for  $m = 1, \dots, M$  and  $t = 1, \dots, T$  initially.

Before the construction mechanism starts, the decision variables  $y_{jt}$  and  $q_{jt}$  are assigned zero for  $j = 1, \dots, J$ ,  $m = 1, \dots, M$ , and  $t = 1, \dots, T$ . Remember, given the values for  $y_{jt}$  and  $q_{jt}$  the values for  $x_{jt}$  and  $I_{jt}$  are implicitly defined. Furthermore, assume auxiliary variables  $\bar{d}_{jt}$  and  $CD_{jt}$  for  $j = 1, \dots, J$  and  $t = 1, \dots, T$ . The former ones represent the entries in the demand matrix and thus are initialized with  $\bar{d}_{jt} = d_{jt}$ . The latter ones stand for the cumulative future demand for item  $j$  which is not been met yet. As we will see, the cumulative demand can be efficiently computed while moving on from period to period. For the sake of convenience we introduce  $CD_{j(T+1)} = 0$  for  $j = 1, \dots, J$ . The remaining capacity of machine  $m$  in period  $t$  is denoted as  $RC_{mt}$ . Initially,  $RC_{mt} = C_{mt}$  for  $m = 1, \dots, M$  and  $t = 1, \dots, T$ .

The initial call is  $construct(T, 1, 1)$  and initiates the fixing of setup states at the end of period  $T$ . Table 9 gives all the details.

The choice of  $j_{mT}$  needs to be refined, but at this point we do not need any further insight and suppose that the selection is done somehow. All we need to know is that  $\mathcal{I}_{mt} \subseteq \mathcal{J}_m \cup \{0\}$  for  $m = 1, \dots, M$  and  $t = 1, \dots, T$  is the set of items among which items are chosen. Item 0 is a dummy item. As one can see, once a setup state is chosen for all machines at the end of period  $T$ , a call of  $construct(T, 0, 1)$  is made. Table 10 provides a recipe of how to evaluate such calls.

The situation when calling  $construct(t, 0, m)$  is that the setup state  $j_{mt}$  has

---

choose  $j_{mT} \in \mathcal{I}_{mT}$ .  
 if ( $j_{mT} \neq 0$ )  
      $y_{j_{mT}T} := 1$ .  
 if ( $m = M$ )  
      $construct(T, 0, 1)$ .  
 else  
      $construct(T, 1, m + 1)$ .

---

Table 9: Evaluating  $construct(T, 1, \cdot)$

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for  $j \in \mathcal{J}_m$   
      $CD_{jt} := \min \left\{ CD_{j(t+1)} + \tilde{d}_{jt}, \max\{0, nr_j - \sum_{\tau=t+1}^T q_{j\tau}\} \right\}$ .  
 if ( $j_{mt} \neq 0$ )  
      $q_{j_{mt}t} := \min \left\{ CD_{j_{mt}t}, \frac{RC_{mt}}{p_{j_{mt}}} \right\}$ .  
      $CD_{j_{mt}t} := CD_{j_{mt}t} - q_{j_{mt}t}$ .  
      $RC_{mt} := RC_{mt} - p_{j_{mt}} q_{j_{mt}t}$ .  
     for  $i \in \mathcal{P}_{j_{mt}}$   
         if ( $t - v_i > 0$  and  $q_{j_{mt}t} > 0$ )  
              $\tilde{d}_{i(t-v_i)} := \tilde{d}_{i(t-v_i)} + a_{ij_{mt}} q_{j_{mt}t}$ .  
 if ( $m = M$ )  
      $construct(t - 1, 1, 1)$ .  
 else  
      $construct(t, 0, m + 1)$ .

---

Table 10: Evaluating  $construct(t, 0, \cdot)$  where  $1 \leq t \leq T$

already been chosen. Remarkable to note, how easy it is to take initial inventory into account. This is due to the backward oriented scheme. Evaluating

$$\min \left\{ CD_{j(t+1)} + \tilde{d}_{jt}, \max\{0, nr_j - \sum_{\tau=t+1}^T q_{j\tau}\} \right\} \quad (10)$$

makes sure that for an item  $j$  no more than the net requirement  $nr_j$  is produced. Note, cumulating the production quantities is an easy task which can be done very efficiently. Given the cumulative demand  $CD_{j_{mt}t}$ , production quantities  $q_{j_{mt}t}$  can be determined with respect to capacity constraints. Afterwards, we simply update the  $\tilde{d}_{jt}$ -matrix to take internal demand into account and proceed. Table 11 describes how to evaluate  $construct(t, 1, \cdot)$ -calls.

These lines closely relate to what is defined in Table 10. Differences lie in the

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choose  $j_{mt} \in \mathcal{I}_{mt}$ .  
 if ( $j_{mt} \neq 0$ )  
      $y_{j_{mt}t} := 1$ .  
     if ( $j_{mt} \neq j_{m(t+1)}$ )  
          $q_{j_{mt}(t+1)} := \min \left\{ CD_{j_{mt}(t+1)}, \frac{RC_{m(t+1)}}{p_{j_{mt}}} \right\}$ .  
          $CD_{j_{mt}(t+1)} := CD_{j_{mt}(t+1)} - q_{j_{mt}(t+1)}$ .  
          $RC_{m(t+1)} := RC_{m(t+1)} - p_{j_{mt}} q_{j_{mt}(t+1)}$ .  
         for  $i \in \mathcal{P}_{j_{mt}}$   
             if ( $t+1 - v_i > 0$  and  $q_{j_{mt}(t+1)} > 0$ )  
                  $\bar{d}_{i(t+1-v_i)} := \bar{d}_{i(t+1-v_i)} + a_{ij_{mt}} q_{j_{mt}(t+1)}$ .  
 if ( $m = M$ )  
      $construct(t, 0, 1)$ .  
 else  
      $construct(t, 1, m+1)$ .

---

Table 11: Evaluating  $construct(t, 1, \cdot)$  where  $1 \leq t < T$

fact that a setup state is chosen for the end of period  $t$  but items are scheduled in period  $t+1$ . For computing production quantities we must therefore take into account that item  $j_{m(t+1)}$  may already be scheduled in period  $t+1$ .

Note, the combination of what is given in Tables 10 and 11 enforces that every item  $j_{mt}$  that is produced at the beginning of a period  $t+1$  is also produced at the end of period  $t$  if there is any positive cumulative demand left.

Turning back to the specification of the *construct*-procedure, it remains to explain what shall happen when the first period is reached. Table 12 describes how to schedule those items in period 1 for which the machines are initially set up for. In contrast to what is given in Table 11 the initial setup state is known and thus needs not to be chosen.

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if ( $j_{m0} \neq j_{m1}$ )  
      $q_{j_{m0}1} := \min \left\{ CD_{j_{m0}1}, \frac{RC_{m1}}{p_{j_{m0}}} \right\}$ .  
      $CD_{j_{m0}1} := CD_{j_{m0}1} - q_{j_{m0}1}$ .  
 if ( $m = M$ )  
      $construct(0, 0, 1)$ .  
 else  
      $construct(0, 1, m+1)$ .

---

Table 12: Evaluating  $construct(0, 1, \cdot)$

A call to  $construct(0, 0, \cdot)$  terminates the construction phase. What is left is a final feasibility test where

$$nr_j = \sum_{t=1}^T q_{jt} \quad (11)$$

must hold for  $j = 1, \dots, J$  for being a feasible solution. Eventually, the objective function value of a feasible solution can be determined.

If we have no initial inventory, we can also perform a capacity check testing

$$\sum_{j \in \mathcal{J}_m} \sum_{i \in \{\bar{\mathcal{P}}_j \cup \{j\}\} \cap \mathcal{J}_m} p_i d_{ji} C D_{j(t+\Delta t)} > \sum_{\tau=1}^{t+\Delta t} C_{m\tau} \quad (12)$$

which must be false for  $m = 1, \dots, M$  if period  $t + \Delta t$  is under concern and thus, when true, indicates an infeasible solution.

The procedure is a meta-method only, since some details are left unspecified. The rule to select an item is, for instance, not discussed any further. For concrete implementations which are based on this meta-method, these details need to be defined. In [26] we find some meta-method instances which work well. All of them are sampling methods which run the construction phase several times and come up with the best solution that was found. Good suboptimal results, say within a 10% range from the optimum objective function value, are reported to be available after a few seconds when running on a modern personal computer.

## 7 Conclusion

We have pointed out that the underlying concept of modern MRP II systems is insufficient. Many things, such as long lead times and high work-in-process, practitioners complain about are not as fate would have it, but are inherent in modern production planning systems. A mathematical programming model is defined to specify the multi-level lot sizing and scheduling problem which is to be solved in order to avoid these shortcomings. Moreover, we have given a generic method that outlines efficient solution procedures to attack the planning problem. Future work should integrate concrete implementations of the meta-method into computer based planning tools and compare the results with those of existing MRP II systems.

## Acknowledgement

This work was done with partial support from the DFG-project Dr 170/4-1.



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