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## Perturbed best response dynamics in a hawk-dove game

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# DISCUSSION PAPER

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# Perturbed Best Response Dynamics in a Hawk–Dove Game

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## Abstract

We examine the impact of behavioral noise on equilibrium selection in a hawk-dove game with a model that linearly interpolates between the one- and two-population structures in an evolutionary context. Perturbed best response dynamics generates two hypotheses in addition to the bifurcation predicted by standard replicator dynamics. First, when replicator dynamics suggests mixing behavior (close to the one-population model), there will be a bias against hawkish play. Second, polarizing behavior as predicted by replicator dynamics in the vicinity of the two-population model will be less extreme in the presence of behavioral noise. We find both effects in our data set.

Keywords: evolutionary game theory, perturbed best response dynamics, experiment in continuous time, hawk-dove game.

JEL Classification: C62, C73, C91, C92.

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## 1. Introduction

Evolutionary models provide key insights for the understanding of central aspects of strategic interactions. For instance, the *mass-action* interpretation of mixed Nash equilibria emerges naturally in the evolutionary context (Björnerstedt and Weibull, 1996; Young, 2011). The aggregate strategy of a population can be interpreted as a mixed strategy even though each individual agent chooses a pure strategy. Equilibrium selection is another cornerstone of these models. It is possible to discern whether an equilibrium is more or less likely to be selected depending on the structure of the population (Friedman, 1991; Weibull, 1995).

The family of *perturbed best response dynamics* (PBR) can take into account departures from the best-response paradigm, which is not possible with the standard model of replicator dynamics. The PBR models introduce a random component in the definition of the best-response correspondences. Players are assumed to behave as myopic best responders—just as in replicator dynamics—but with the additional feature that

they may tremble in their decisions (Blume, 1993). In the limit where the impact of the error term approaches zero, the PBR predictions converge toward those of replicator dynamics. However, for moderate levels of noise, PBR models can account for some behavioral patterns that deviate from the standard approach (see Hofbauer and Hopkins, 2005; Hofbauer and Sandholm, 2002; Hopkins, 2002, for technical details).

In this note, we apply a PBR model to a hawk-dove game. The hawk-dove game is a symmetric two-strategy game with three Nash equilibria: a symmetric one in mixed strategies and two asymmetric ones in pure strategies.

There are two common (and simplified) ways to implement this interaction as a population game. In the *one-population* case, the interaction only takes place between agents *within* the groups, while in the *two-population* case, the interaction occurs exclusively *between* the groups. Basic intuition in population games argues that mixing behavior emerges when the game is played within the population (one-population matching) because only symmetric equilibria can survive. The polarized case is more likely to be observed in the two-population matching (Oprea et al., 2011).

A recent experiment by Benndorf et al. (2016)

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relaxes the assumptions for the matching discussed above by introducing a *coupled* model. This allows to linearly interpolate between both extreme structures (one- and two-population models) with a coupling parameter  $\kappa \in [0, 1]$ . This parameter is a measure for the relative importance of the interaction between the populations. In a discrete analogy, it can also be interpreted as the probability that an agent is matched with an agent from the other group. Note that  $\kappa = 0$  and  $\kappa = 1$  correspond to the one- and two-population cases, respectively. The coupled model unveils the transition regime from symmetric mixing to polarized behavior in pure strategies. Given the payoff parameters in the experiment, replicator dynamics predicts symmetric mixed play with  $\frac{2}{3}$  of hawk for  $\kappa < \frac{1}{2}$ . A sudden bifurcation occurs at  $\kappa = \frac{1}{2}$  such that one population plays pure hawk and the other plays a mixed strategy with  $\frac{1}{3}$  of hawk. Separation (difference in the share of hawk play) increases monotonically with  $\kappa$  in the interval  $\kappa \in [\frac{1}{2}, \frac{2}{3}]$ . Finally, the system is fully polarized for  $\kappa > \frac{2}{3}$ . The experiment by Benndorf et al. (2016) largely confirms these predictions, but the authors also report some subtle discrepancies between the data and the replicator model.

In the present paper, we complement their analysis with the study of logit response dynamics as a natural extension of the standard replicator model.<sup>3</sup> This is a common implementation of a PBR model and assumes that the random component follows the logistic distribution. The PBR model applied to our experimental setting makes two predictions that go beyond the scope of the best-response paradigm of replicator dynamics. First, the share of hawk choices in the symmetric mixed equilibrium will be lower than  $\frac{2}{3}$ . Second, PBR implies that the impact of the polarizing forces on the behavior of the system will be weaker than suggested by replicator dynamics. This effect has two interpretations (see further explanation of the model below). The separation between the two populations will be lower than predicted by replicator dynamics. An alternative perspective is that the value of the coupling parameter  $\kappa$  for which the system transits from the mixed regime to the asymmetric configuration will be higher than  $\frac{1}{2}$ . This noisy decision rule accounts for the discrepancies between the standard replicator predictions and the behavioral patterns observed in the experiment.

<sup>3</sup>Traulsen et al. (2010) provide evidence supporting this method of strategy updating in human behavior. Alós-Ferrer and Netzer (2010) and Zhuang et al. (2014) characterize some theoretical long run properties of the model.

## 2. PBR model

We consider two populations of players ( $X$  and  $Y$ ) in a two-strategy environment. Let  $S^X = \{(s_1, s_2) : s_1^X + s_2^X = 1\}$  such that any point in it represents the share of each strategy among population  $X$  (equivalent definition for population  $Y$ ). The pair  $(x, y)$  gives the state of the system with  $x = s_1^X$  and  $y = s_1^Y$ . Then,  $s_2^X$  and  $s_2^Y$  are given by  $1 - x$  and  $1 - y$ , respectively.

We interpolate the play of the game between the one- and the two-population models with a coupling parameter  $\kappa \in [0, 1]$ . Recall that when  $\kappa = 0$ , a player only participates in interactions within her own population. If  $\kappa = 1$ , the player interacts only with the agents of the other population. Intermediate values of  $\kappa$  correspond to simultaneous interactions at the intra- and intergroup level (Benndorf et al., 2016, Section 3). The instantaneous payoff earned by a player in population  $X$  choosing strategy  $s_i$  for a given state of the system  $(x, y)$  is  $\pi_X(s_i; x, y) = (1 - \kappa)[\pi_{i1}x + \pi_{i2}(1 - x)] + \kappa[\pi_{i1}y + \pi_{i2}(1 - y)]$  where  $\pi_{ij}$  are the elements of a  $2 \times 2$  payoff matrix.

According to the logit response function, a player in population  $X$  who observes a choice profile in the populations  $(x, y)$ , and given the chance to revise the play, chooses action  $s_1$  with probability

$$p_X(s_1; x, y) = \frac{1}{1 + e^{-\lambda \Delta \pi_X(x, y)}}. \quad (1)$$

$\Delta \pi_X(x, y) = \pi_X(s_1; x, y) - \pi_X(s_2; x, y)$  is the payoff advantage (in population  $X$ ) of strategy  $s_1$  over strategy  $s_2$ . Analogous for  $Y$ . The comparison of profits influences the dynamics of the system weighted by  $\lambda \in [0, \infty)$ . This parameter captures deviations from the best response function. If  $\lambda = 0$ , the revision mechanism is independent from the payoff structure of the game and the system evolves toward an equal share of strategies in the populations. When  $\lambda \rightarrow \infty$ , PBR approaches replicator dynamics.

We define the action set  $S = \{s_1, s_2\}$  such that  $s_1$  corresponds to strategy hawk, and  $s_2$  to dove. Then, the hawk-dove game in matrix notation is

$$\Pi = \begin{pmatrix} a + \frac{1}{2}(v - c) & a + v \\ a & a + \frac{1}{2}v \end{pmatrix}. \quad (2)$$

This game represents a conflict of cost  $c$  over a scarce resource of value  $0 < v < c$ , and  $a > 0$  is an endowment of the players. With these parameters (and the payoff function above) we obtain the fitness function  $\Delta \pi_X(x, y) = \frac{1}{2}[v - c(x + \kappa(y - x))]$ .  $\Delta \pi_Y$  is defined analogously.

The logit response dynamics is given by the following system of coupled differential equations:

$$\begin{cases} \dot{x} = p_X(s_1; x, y) - x \\ \dot{y} = p_Y(s_1; x, y) - y, \end{cases} \quad (3)$$

with  $p_X$  and  $p_Y$  defined in (1). A rest point of (3) corresponds to the logit quantal response equilibrium (McKelvey and Palfrey, 1995) for the given value of the parameter  $\lambda$ .

We illustrate the predictions of the PBR model in Figure 1. Panel (a) contains several cuts of the stable manifold of (3) for different values of the rationality parameter  $\lambda$  that show the shape of the bifurcation as a function of  $\kappa$ . The prediction for  $\lambda = 0$  is independent of the coupling condition and corresponds to uniform randomization. When  $\lambda$  increases, the bifurcation diagram of the system becomes closer to the prediction with replicator dynamics the higher the value of  $\lambda$ .

For every sufficiently high value of the exponent  $\lambda$  (representing low levels of noise in the best response correspondences of the players), there exists a critical value  $\kappa_{\text{crit}}$  such that the equilibrium stability shifts from the mixed configuration toward a polarized one. We compute  $\kappa_{\text{crit}}$  as a function of  $\lambda$  in panel (b). This value converges monotonically toward  $\frac{1}{2}$  when the PBR model degenerates in the replicator dynamics ( $\lambda \rightarrow \infty$ ). The same logic applies to the share of the hawk choices in the populations for the regime with low coupling ( $\kappa < \kappa_{\text{crit}}$ ). We illustrate in panel (c) how the level of hawk play monotonically increases with  $\lambda$  and converges to the mixed NE,  $v/c = \frac{2}{3}$ , when  $\lambda \rightarrow \infty$ .

From this discussion, we see that the PBR model generates two testable hypotheses about human behavior in the experiment:

*H1.* The share of hawk choices in the populations  $X$  and  $Y$  for treatments with  $\kappa < \frac{1}{2}$  will be lower than  $v/c = \frac{2}{3}$  and higher than  $\frac{1}{2}$ .

*H2.* The observed separation between populations (difference between hawk play in groups  $X$  and  $Y$ ) for the treatment with  $\kappa = 0.6$  will be lower than  $\frac{5}{6}$ .

The first hypothesis mirrors regular findings regarding mixing behavior in the quantal response literature (Goeree et al., 2016), but the second formulation deserves some explanation. Replicator dynamics makes a sharp prediction concerning the location of the splitting point ( $\kappa_{\text{crit}} = \frac{1}{2}$ ). By contrast, the presence of noise in the best response function shifts the location of the critical level of coupling  $\kappa_{\text{crit}}$  for which polarization begins. In the noisy model, this point is generally higher than  $\frac{1}{2}$  for low levels of  $\lambda$  (more noise) and

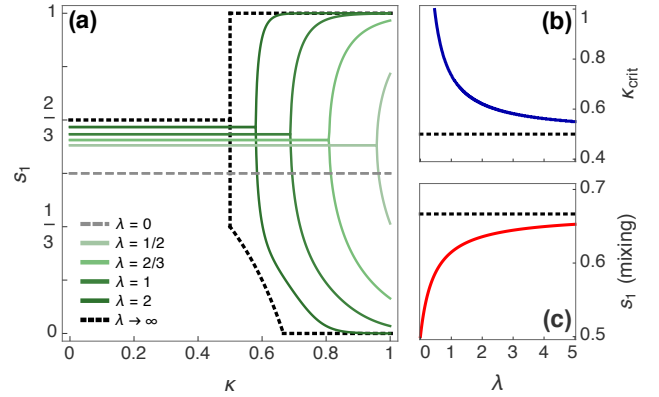


Figure 1: PBR predictions (parameters  $a = 3$ ,  $v = 12$  and  $c = 18$ ). (a) Bifurcation diagrams for different levels of  $\lambda$ . (b) Location of  $\kappa_{\text{crit}}(\lambda)$  in the PBR model (blue) vs. replicator limit (dotted black). (c) Share of  $s_1(\lambda)$  in mixed equilibrium in the PBR model (red) vs. replicator limit (dotted black).

decreases monotonically toward the replicator prediction as  $\lambda \rightarrow \infty$ . It is not possible to make an ex-ante point prediction for  $\lambda$  and one cannot cover all possible values of  $\kappa$  as a treatment variable. Therefore, the exact point  $\kappa_{\text{crit}}$  cannot be directly observed in an experiment; however, we can still identify the effect of the possible upward shift of such a splitting point. For this, we measure the separation between groups for  $\kappa = 0.6$  (the first one that we observed above  $\frac{1}{2}$ ) and compare it to the prediction with replicator dynamics. According to the PBR model, the higher the  $\kappa_{\text{crit}}$ , the smaller the separation reached by the populations for a fixed level of coupling. See Figures 1 and 2-(b).

### 3. Experiment

In this note, we utilize the dataset generated in an experiment by Benndorf et al. (2016). Subjects played the hawk-dove game with parameters  $a = 3$ ,  $v = 12$ , and  $c = 18$ . The payoff entries are  $\pi_{11} = 0$ ,  $\pi_{12} = 15$ ,  $\pi_{21} = 3$ , and  $\pi_{22} = 9$ .

The treatment variable was the coupling parameter  $\kappa$  and took the six values from 0 to 1 with step  $\Delta\kappa = 0.2$ . The experiment varied the treatments within subjects. All participants played all six treatments consecutively and the order of these treatments was randomized at the session level. A total of 24 players participated in each session, always divided into two groups of equal size. The composition of the groups was randomized at the treatment level and players were independently and randomly assigned their initial actions in each treatment. The instantaneous payoff flow earned by each subject was determined by confronting the subject's current strategy with the current aggregate (average) strategy of their own group and the other group,

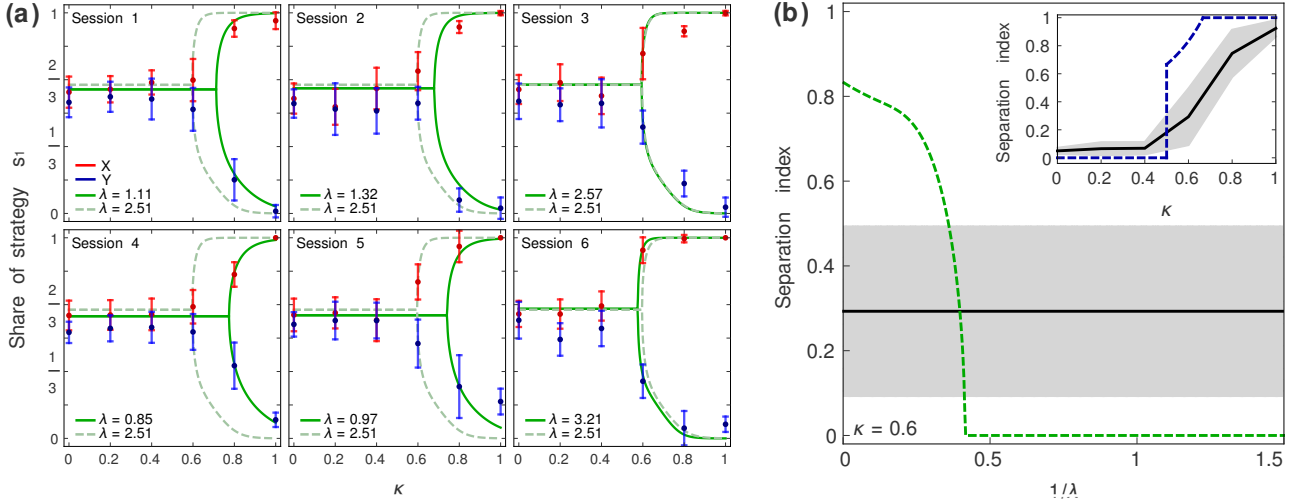


Figure 2: Experimental results. (a) Steady states for each  $\kappa$  and fit of PBR dynamics, by session. (b) Separation index for  $\kappa = 0.6$ . PBR prediction as a function of  $\lambda$  (dashed green) and experimental observation (solid black). Inset shows the observed separation for all treatments (solid black) compared to the replicator prediction (dashed blue). Gray bands indicate standard deviations.

weighted by  $(1 - \kappa)$  and  $\kappa$ , respectively. Each treatment lasted 210 seconds of play and treatment payoffs were the cumulative (integral) of the payoff flow over the period length. Only one treatment was paid, selected with a random draw at the end of the session. There were six experimental sessions and we employed a total of 144 participants. All sessions took place at the DICELab for experimental economics in Düsseldorf, in Spring 2015.

The experiment was conducted in (virtually) continuous time with ConG (Pettit et al., 2014). This environment is relevant to experiments in evolutionary dynamics because it allows for asynchronous choice making by the players and implements real time updating of the information set displayed to the agents.<sup>4</sup>

#### 4. Results

Panel (a) in Figure 2 reports the experimental results at the session level. We considered the last 60 seconds of play for the computation of the steady state of the system in each treatment (total length is 210 seconds). The two scatter plots (red and blue) show the share of strategy hawk in the two populations together with the error bar. Label  $X$  is arbitrarily assigned to the more ‘hawkish’ population in the steady state. As anecdotal information, we show a fit<sup>5</sup> of the PBR model at the

session level (solid green), with the estimated values of  $\lambda$  as inset. For the sake of completeness, the best fit when considering a unique value of  $\lambda$  for the whole data set is for 2.51 (dashed grayish). We observe a certain degree of heterogeneity across the six experimental sessions. The variation of the fitted values of  $\lambda$  (at the session level) captures such differences in the level of noise between sessions.

The two hypotheses stated by our PBR model can be tested directly from the experimental data set and are independent of any consideration about the fit of the parameter  $\lambda$ . In order to test the first hypothesis, we take the steady states reached during the three treatments with  $\kappa \in \{0, 0.2, 0.4\}$  in the six sessions. In total, we have 36 measurements of the share of hawk play in the range  $[0.493, 0.660]$ . However, our experimental design generates only one independent observation per session. Each session gives six data points: two populations,  $X$  and  $Y$ , times three treatments  $\kappa \in \{0, 0.2, 0.4\}$ . Thus, in order to perform quantitative tests, we average the steady state of the two populations  $X$  and  $Y$  across the three mentioned treatments to get one observation per session. These six data points lie in the range  $[0.551, 0.597]$ . One-sided sign tests confirm the hypothesis  $H1$  of the PBR model in that the play of hawk strategy in the mixed regime belongs to the interval  $(1/2, 2/3)$ . For our sample, average play of hawk is 0.583 with standard deviation of 0.017.

<sup>4</sup>Further details about other procedures can be found in Bendorf et al. (2016, Section 4).

<sup>5</sup>Grid search on the values of  $\lambda$  with resolution  $\Delta\lambda = 10^{-2}$  considering the distance  $\sum_{\kappa} [x^*(\kappa) - \bar{s}_1^X(\kappa)]^2 + [y^*(\kappa) - \bar{s}_1^Y(\kappa)]^2$  as the objective function to minimize, where  $(x^*, y^*)$  is the PBR predic-

tion (as a function of  $\lambda$ ) and  $\bar{s}_1$  is the observed share of hawk play in the steady state for the corresponding population and treatment.

Regarding the second hypothesis, we have six measures of the separation between populations for the treatment with  $\kappa = 0.6$ . We define the separation index  $\Delta s(\kappa) \in [0, 1]$  as  $\bar{s}_1(\kappa, X) - \bar{s}_2(\kappa, Y)$ . The separation index is the share of strategy hawk in the more hawkish population ( $X$ ) minus the share of strategy hawk in the more dovish group ( $Y$ ), for a given treatment  $\kappa$ . The observations lie in the range  $[0.126, 0.653]$ , with average of 0.293 and standard deviation of 0.201. A one-sided sign test rejects the null hypothesis that the median of the data points is greater than or equal to  $\frac{5}{6}$  ( $\approx 0.833$ ) with a p-value of 0.0156 and cannot reject the null hypothesis that the median of the data points is lower than or equal to  $\frac{5}{6}$  with  $p > 0.999$ . The data set confirms  $H2$  in that the observed separation between the populations for  $\kappa = 0.6$  is below the prediction under perfect rationality. Figure 2-(b) compares the separation observed in the treatment with  $\kappa = 0.6$  to the corresponding model predictions as a function of the level of noise (inverse of  $\lambda$ ). The inset also shows the separation observed throughout all the treatments in comparison with the sharp step function predicted by replicator dynamics. The departure from the best-response paradigm in the way subjects played the games is clear, given our two experimental results being highly significant.

Our second hypothesis illustrates that polarizing behavior is not as extreme as predicted by replicator dynamics. The treatments with  $\kappa \in \{0.8, 1\}$  provide additional evidence for this effect. In these cases, replicator dynamics suggests that group behavior should be fully polarized:  $\Delta s(0.8) = \Delta s(1) = 1$ . See inset of Figure 2-(b). By contrast, PBR dynamics predicts that separation for moderate levels of noise should be higher the higher the value of the coupling parameter:  $\Delta s(0.8) < \Delta s(1)$ . See Figure 1-(a). Our experimental data favors the latter prediction. The separation index for  $\kappa = 0.8$  is significantly smaller than the one for  $\kappa = 1$  (one-sided Wilcoxon signed-rank test,  $p = 0.031$ ).

## 5. Discussion

In this note, we tested two deviations from replicator dynamics of qualitative nature. We characterized two traits of human behavior in a dynamic environment—a bias against hawk strategy in the symmetric mixed configuration, and less polarization in the asymmetric outcomes—that are consistent with PBR dynamics and that represent a systematic departure from the best response assumption that underlies the definition of

replicator dynamics. Our results relate to other applications of PBR models, for example, to the experimental study of limit cycles in rock-paper-scissors games by Cason et al. (2014); and the recent characterizations of noise in behavioral dynamics in experimental games by Mäs and Nax (2016) and Lim and Neary (2016).

To conclude, we acknowledge the ongoing debate concerning the informative value of estimating the parameter  $\lambda$  to compare point predictions between quantal response equilibrium and Nash equilibrium. See, for example, Goeree et al. (2005), Haile et al. (2008), and the series of papers by Selten and Chmura (2008), Brunner et al. (2011), and Selten et al. (2011) on this issue. We would like to emphasize that the two experimental results presented in this note are independent of any numerical fit of the parameter  $\lambda$ . The tests that we performed rely purely on the experimental observations; we estimated  $\lambda$  in Figure 2 (a) only for illustrative purposes.

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