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## Efficient pricing of large value interbank payment systems

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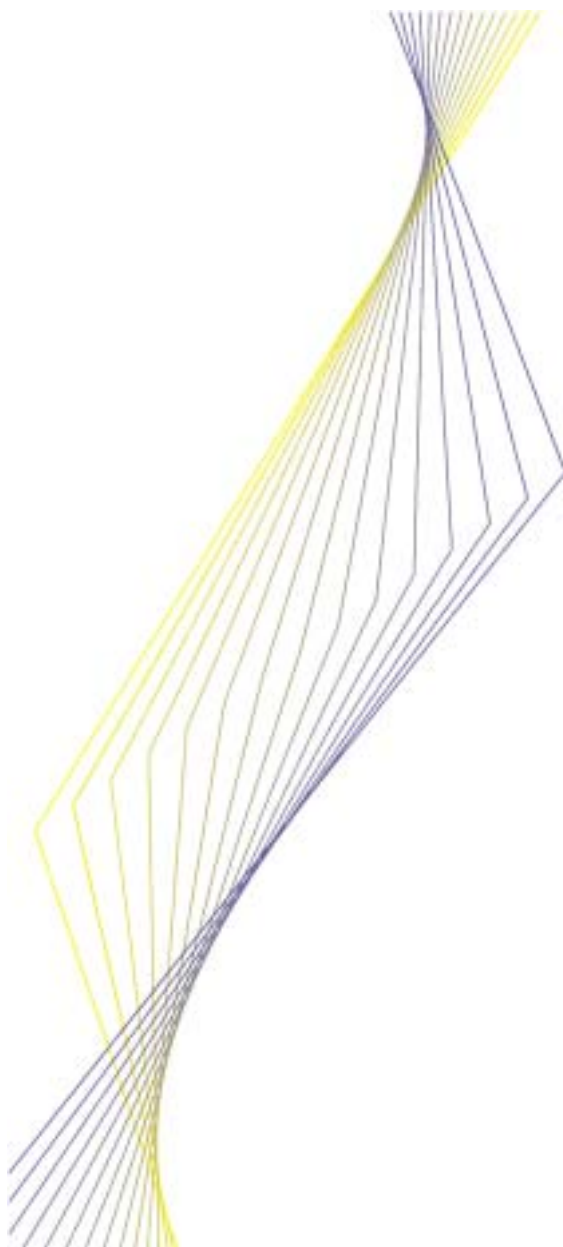
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**BY CORNELIA HOLTHAUSEN  
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**October 2002**

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**BY CORNELIA HOLTHAUSEN<sup>2</sup>  
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**October 2002**

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# Contents

Abstract	4
Non-technical summary	5
1 Introduction	7
2 The model	9
3 Optimal pricing for a monopoly LVPS	11
3.1 Discrete distribution of types	11
3.2 Continuous distribution of types	13
4 Optimal pricing in a mixed duopoly	15
5 Competition between national RTGSs	19
5.1 Efficient pricing with by-passing	20
5.2 By-passing is deterred	21
6 Concluding remarks	23
Appendix: proofs	24
References	29
European Central Bank working paper series	31

## **Abstract**

This paper studies the efficient pricing of large-value payment systems in the presence of unobservable heterogeneity about banks' future payment volumes. It is shown that the optimal pricing scheme for a public monopoly system involves quantity discounts in the form of a decreasing marginal fee. This is also true when the public system competes with a private system characterized by a lower marginal cost. However in this case, optimal marginal fees in the public system are lower than its marginal cost, and fixed fees have to be levied. We also study the case of competition between several public systems. The structure of the optimal tariff depends on the willingness of Central Banks to allow by-pass.

**JEL Codes:** G28, E58, D42, D43

**Keywords:** payment systems, non-linear pricing, public monopoly, mixed duopoly

## Non-Technical Summary

Large-value payment systems are characterized by large economies of scale: in order to provide payment services, large fixed costs arise related to the setting-up and maintenance of the system. Contrarily, each individual payment generates very small costs. Such scale economies present a problem for efficient pricing. Standard economic theory requires that the price of each unit purchased should equal its marginal cost (marginal cost pricing). In the presence of fixed costs, however, marginal cost pricing would create a deficit because the fixed cost could not be recovered. Therefore, if cost recovery is a requirement, a different pricing rule needs to be found. The goal of this paper is to find the optimal, welfare-maximizing pricing rule for a large-value payment system.

There are several possibilities for pricing schemes that achieve cost recovery. The first would be a simple linear tariff, in which each payment is priced at the average cost. That is, the fixed cost would be divided and split among the total number of payments made. The drawback of such a scheme is that for high fixed costs, the marginal fee would need to be substantially above the marginal cost. This would discourage banks to undertake a large number of payments. But with a low payment volume, it will be even more difficult to recover costs. A second possibility is to apply a fixed fee only, and no charge on each individual payment. This tariff has the advantage of being very simple, and of yielding a high payment volume for banks who participate in the system. However, it may be politically undesirable, and it will discourage banks with a low expected volume of payments to participate in the system.

A better solution might be to design a pricing scheme that combines the advantages of these alternatives. This can be achieved by having a pricing scheme in which marginal fees vary with the payment volume of each individual bank. Such a "non-linear pricing" scheme could, in its simplest form, take the form of a two-part tariff, i.e. a combination of a fixed fee and a constant marginal fee. A more advanced scheme could consist of a block tariff, or of a menu of two-part tariffs. Indeed, this paper shows that efficiency generally requires that marginal fees decrease with the payment volume of each system participant. In this way, a high fixed fee is avoided, and at the same time, banks with a large payment demand will not be discouraged from routing these payments through the system.

The paper starts from a situation in which there is only one public large value payment system. The fee structure is chosen with the aim to maximize the aggregate welfare of the system participants, under the constraint that the payment system breaks even. It is shown that the optimal tariff implies that low payment volumes are priced above marginal cost, the higher volumes are priced at marginal cost, and that there is no fixed fee. The optimal "distortion" of lower payment volumes, i.e. its divergence from marginal cost pricing, results from the following trade-off: on the one hand, a low distortion is more efficient and encourages a high payment volume. On the other hand, a high distortion, i.e. a high marginal price, makes cost recovery easier.

In the remaining part of the paper, a duopoly situation is studied in which a public payment system coexists with another system. First, we assume that the public system competes with a private non-profit payment system. Here, it is assumed that the private system has lower marginal costs and is therefore able to attract those banks with the highest payment volume. Then, we take the case of two national payment systems competing against each other by allowing banks to by-pass their national system and use the foreign system instead. It is shown that the pricing scheme remains degressive. However, competition for banks with a high payment volume may require that high volumes are subsidized at the margin, in order to ensure that these banks remain in the national public system.



# 1 Introduction

This paper analyzes the efficient pricing of large-value payment systems. These systems, which are used by banks and large financial service providers to channel payments to each other, have changed substantially over the last decade. In particular, the volumes transferred in these systems have increased dramatically, mainly because of growing financial integration, new financial products and technological developments. This trend raised concerns among regulators because of its potential implications for systemic stability, and led to a number of regulatory initiatives with the aim to reduce risk-exposures between participants (see Bank for International Settlements 1990). These included the promotion of real-time gross settlement (RTGS) systems, which replaced netting systems in many countries.

In Europe, the advent of Economic and Monetary Union played an equally important role for the development of large value payment systems. For a long time, one or several payment systems existed in each country, all organized differently. Most cross-border transactions were done via correspondent banks. In order to facilitate the implementation of the single monetary policy in Europe, it was decided in 1995 to build a European wide payment system, TARGET<sup>4</sup>, connecting national RTGS systems of all EU member states.<sup>5</sup> These national components of TARGET were harmonized only to some degree, e.g. in terms of operating hours. Since the use of TARGET is compulsory for settlement of ECB monetary policy operations, all banks wishing to obtain liquidity from the central bank are required to participate directly or indirectly in TARGET.

Because monetary policy transactions of the European Central Bank are conducted using TARGET, all banks wishing to obtain liquidity from the Central Bank are required to have an account in TARGET.

Naturally, TARGET does not provide the only way to make large value interbank payments in Europe. Bilateral agreements between banks and correspondent banking arrangements continue to exist. Furthermore, a few, privately run, netting and hybrid systems are in place, mostly operating at domestic levels. The biggest of these systems is EURO1, a net system that offers mainly cross-border payment services. EURO1 is run by the Euro Banking Association (EBA). In order to use EURO1, a bank must be member of the EBA and fulfil certain requirements such as good credit rating and a minimum

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<sup>4</sup>This is an acronym for Trans-European Automated Real-time Gross settlement Express Transfer.

<sup>5</sup>see ECB (2001).

balance sheet size.

The aim of this paper is to characterize the efficient pricing structure for a public payment system, depending on the type of competition it faces. This question is relevant not only for the European case discussed above, but equally well for the US, where a public RTGS system, Fedwire, exists side by side with CHIPS, a private system<sup>6</sup>. So far, the literature on payment systems was mostly concerned with issues of systemic risk. For instance, Freixas and Parigi (1996) and also Kahn and Roberds (1998) analyze the trade-off between a more efficient liquidity management in RTGS systems versus higher systemic risk in netting systems. Few papers have addressed the co-existence of private and public systems. Exceptions are Rochet and Tirole (1996), who argue in favor of a close co-operation in the controlling of risk-exposures in both systems, and Holthausen and Rønde (2001) who analyze access regulation to netting systems.

To our knowledge, there exists no theoretical study so far on the pricing of large value payment services. There are, however, a few empirical studies on the pricing of Fedwire (the US equivalent to TARGET). Hancock et al (1999) analyze the pricing on interbank transactions by testing for scale economies in payments processing. McAndrews (1998) estimates demand elasticities for Fedwire payment services. The modelling approach taken in this paper is closely connected to the literature on natural monopolies and Ramsey pricing (see e.g. Bös 1994). It is also related to issues of competition in telecommunications and other network industries (see e.g. Laffont and Tirole 2000).

In Section 2 of this paper, we develop a simple model of a banking industry where banks make payments through a large value payment system. Section 3 analyzes the benchmark case of a public monopoly payment system. The optimal pricing scheme is the one that maximizes aggregate surplus of the banking industry under the constraint that the public payment system breaks even. In section 4, the more interesting case of a mixed duopoly is studied, in which banks are free to choose whether to route the payments through a public or a private payment system.

As noted above, national RTGS systems in the EU are not fully harmonized. In particular, the fee structure for payments within a country differs across systems. This leads to a situation where these systems compete for payments, because larger banks that are incorporated in several countries may be able to choose which national system to use

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<sup>6</sup>CHIPS for many years operated as a net settlement system. Since 2001, it has revised its operational framework and is now better described as a hybrid system.

to route payments.<sup>7</sup> To capture such an environment, in Section 5 of this paper we study the optimal pricing when two public systems compete for customers.

## 2 The Model

We model a perfectly competitive banking industry over a fixed period of time (say a month or a quarter) used as the invoice period by the large value interbank payment system (in short, LVPS).

Each bank has a demand for making payments over the LVPS, denoted  $q$ . Payments are of the same size. The payment demand depends on the tariff  $T(q)$  that is charged by the LVPS. In particular, we assume that each payment generates some random monetary value  $\tilde{m}$  to the bank. This value is interpreted as the monetary surplus that is generated by routing the payment through the LVPS instead of delaying it and bundling it with other payments, or instead of finding alternative ways to send payments, for instance by using a correspondent bank. Notice that the realization of  $\tilde{m}$ ,  $m$ , will differ for each payment and each bank, but the distribution is the same.

Each payment will be undertaken if and only if its monetary value is at least as high as the marginal fee charged by the LVPS,  $T'(q)$ , that is if  $\tilde{m} \geq T'(q)$ . Assuming that the time period is large enough so that the law of large numbers can be applied, the payment demand  $q$  is then the proportion of potential payments that will be routed on the LVPS by a given bank,  $q = \text{Proba} [\tilde{m} \geq T'(q)]$ . This “demand function” of the bank will be denoted  $D(p)$ , and its inverse  $P(q)$ .

The “utility” function of the bank is then by definition

$$v(q) = \int_0^q P(s) ds$$

and individual surplus for a bank (and its customers) is then equal to  $v(q) - T(q)$ .<sup>8</sup>

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<sup>7</sup>In fact, banks are also able to use other RTGS systems connected to TARGET by *remote access*. However, this possibility is not widely used, and it is disregarded in the modelling.

<sup>8</sup>In the sequel, we will not analyze the repartition of the surplus between the bank and its customers. The only assumption that we need is that  $q$  is chosen so as to maximize this surplus, which is the case if banks are perfectly competitive (this is our assumption) but also if banks are mutuals who act in the best interest of their customers or at the other extreme if banks are monopolistic and extract all the consumers’ surplus. The case where the banks and the consumers share this surplus in fixed proportions is also compatible with our analysis.

Banks will typically differ in their payment demands. To model this, we assume that the distributions of monetary values across banks are identical up to a translation

$$\tilde{m} = \theta + \tilde{\varepsilon},$$

where  $\theta$  is an unobservable heterogeneity parameter that characterizes the bank (the “type” of the bank) and the  $\tilde{\varepsilon}$ s are independently and identically distributed across banks. For instance,  $\theta$  could be related to the bank’s size, its efficiency, or to its engagement in a certain type of financial activity.

This linear structure implies a simple parametrization of banks’ demand and utility functions:

$$D(\theta, T') = \text{Proba} [\theta + \tilde{\varepsilon} \geq T'] = D(0, T' - \theta),$$

and

$$v(\theta, q) = \int_0^q \{\theta + D^{-1}(0, s)\} ds = \theta q + v(0, q).$$

That is, a bank’s utility depends linearly on its type. The advantage of this parametrization is that it satisfies the familiar “single crossing property” since

$$\frac{\partial^2 v}{\partial \theta \partial q} \equiv 1 > 0. \tag{1}$$

This implies that, independently of the pricing structure, the volume of transactions  $q(\theta)$  chosen by a bank of type  $\theta$  will be increasing with  $\theta$ . The additive parametrization of heterogeneity is only made for convenience: all our results go through under the more general single crossing property.

As is usual in the nonlinear pricing literature, we assume that the value of  $\theta$  is privately observed by each bank, and that the Central Bank only knows the statistical distribution of  $\theta$  among banks. The problem of determining the efficient pricing schedule would be trivial if the individual banks’ payment demand were perfectly observable. In that case, the best tariff would be first best optimal and would consist of a marginal fee equal to the marginal cost and a “personalized” fixed fee that would depend on the surplus obtained by each bank<sup>9</sup>. In the more realistic case where tariffs cannot be conditioned on  $\theta$ , the second best schedule involves volume based pricing, as we now analyze in detail.

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<sup>9</sup>In fact there would be an infinity of Pareto optimal tariffs, each associated to a different sharing of fixed costs among banks. Not only such a “discriminatory” tariff would be politically difficult to implement, but it would require that the Central Bank possesses very precise information on characteristics that individual banks can easily alter.

### 3 Optimal Pricing for a Monopoly LVPS

In this section, we derive the (second best) optimal pricing rule for a public LVPS when it does not face any competition, either in the same country by a privately run LVPS (this is studied in Section 4) or in a neighboring country by a publicly run LVPS (this is studied in Section 5). The pricing rule developed here is a particular case of Ramsey pricing for a public monopoly subject to a budget constraint (see Bös 1994).

#### 3.1 Discrete distribution of types

To begin with, we adopt a simple discrete distribution of banks' types. We assume that there are only two possible types,  $\theta_L$  and  $\theta_H$ , where  $\theta_L < \theta_H$ . A fraction  $f_k$  of banks is of type  $\theta_k$ ,  $k \in \{L, H\}$ , and  $f_L + f_H = 1$ .

Denote  $T(q)$  the tariff charged by the Central Bank for a volume  $q$  of transactions. Without loss of generality, the tariff  $T(q)$  is supposed to be piecewise differentiable, and whenever it exists, its derivative is denoted by  $T'(q)$ . Since banks are not obliged to participate, we require  $T(0) = 0$  (individual rationality condition). Notice that this is not incompatible with a fixed fee  $\bar{T}$  which is only incurred for a strictly positive transaction value (see also footnote 11).

The indirect "utility" function of a bank of type  $k$  is defined as the maximum surplus  $u_k$  obtained by a bank of type  $\theta_k$  (or its customers) when the tariff charged by the Central Bank is  $T$ . By definition:

$$u_k = \max_{q_k \geq 0} \{v(\theta_k, q_k) - T(q_k)\}, \quad (2)$$

where the maximum for a type  $k$  bank is obtained for a volume  $q_k = q(\theta_k)$  and a payment  $T_k = T(q_k)$ . Whenever  $T$  is differentiable at  $q(\theta)$ , the first order condition implies:

$$v_q(\theta, q(\theta)) = T'(q(\theta)), \quad (3)$$

where  $v_q$  denotes the partial derivative of  $v$  with respect to  $q$ . Given that there are only two types of banks, the Central Bank problem can be reduced to finding two pairs  $(q_k, T_k)$  that satisfy the agents' incentive compatibility constraints:

$$(IC_H) : \quad u_H = v(\theta_H, q_H) - T_H \geq v(\theta_H, q_L) - T_L \quad (4)$$

$$(IC_L) : \quad u_L = v(\theta_L, q_L) - T_L \geq v(\theta_L, q_H) - T_H \quad (5)$$

and individual rationality constraints, which guarantee that banks obtain their reservation utility (which we normalized to 0):

$$(IR_H) : \quad u_H \geq 0 \quad (6)$$

$$(IR_L) : \quad u_L \geq 0 \quad (7)$$

The total volume of transactions in the LVPS is  $Q = f_L q_L + f_H q_H$ . The cost function of the LVPS is denoted  $C(Q)$ . We will adopt the following simple specification:

$$C(Q) = a + cQ, \quad (8)$$

where  $a$  denotes the fixed cost, and the marginal cost  $c$  is taken to be constant. We assume that the Central Bank is constrained to choosing a tariff structure that satisfies budget balance<sup>10</sup>:

$$\sum_{k=L,H} f_k T(q_k) = C(Q). \quad (9)$$

The central bank's problem consists of finding the optimal quantities for both types of banks  $q_L$  and  $q_H$ , together with tariffs  $T_L = T(q_L)$  and  $T_H = T(q_H)$  so that aggregate surplus  $S$  is maximized.  $S$  is given by

$$S = \sum_{k=L,H} f_k v(\theta_k, q_k) - C(Q) = \sum_{k=L,H} f_k s(\theta_k, q_k) - a.$$

where we have defined  $s(\theta_k, q_k) \equiv v(\theta_k, q_k) - cq_k$ .

The Central Bank therefore needs to maximize  $\sum_k f_k \{v(\theta_k, q_k) - cq_k\}$  under the constraints (3) - (7) and (9).

The solution of this problem is given by the following proposition.

**Proposition 1** *For a monopoly LVPS, the second best optimal tariff satisfies*

$$\begin{aligned} T'(q_H) &= v_q(\theta_H, q_H) = c \\ T'(q_L) &= v_q(\theta_L, q_L) = c + \frac{\lambda}{1 + \lambda} \frac{f_H}{f_L} (\theta_H - \theta_L) \end{aligned}$$

where  $\lambda$  is the Lagrange multiplier associated with the break-even constraint (9).

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<sup>10</sup>Cost recovery is a statutory requirement for many payment systems, for instance TARGET or Fed-wire.

From Proposition 1, we see that, as soon as the break-even constraint is binding ( $\lambda > 0$ ), the optimal tariff is volume-based: the marginal price  $T'$  charged to bank depends on the total number of payments sent by the bank. In particular, banks with higher volumes face a lower marginal price, i.e. the scheme is degressive. Notice that high type banks  $\theta_H$ , who choose quantity  $q_H$ , obtain marginal cost pricing, which implies that their transaction volume is not distorted. However, low type banks  $\theta_L$  pay marginal fees above marginal cost, which implies that they send less payments than what would be efficient (downward distortion).

When the fixed cost of the system is small, the break-even constraint is not binding ( $\lambda = 0$ ) and the optimal tariff is first-best efficient: marginal fees equal marginal cost for all types. However, in the more realistic case where the fixed cost is high, the need to achieve budget balance introduces price distortions: marginal fees for low volumes are set above marginal cost, at the minimum level that deters high type banks to reduce their transaction volume. Notice that the property that high transaction volumes are not distorted (“no distortion at the top”), is a standard result in the adverse-selection literature.

This simple two-type case illustrates the problem of finding the optimal pricing scheme and thus allows us to find some essential characteristics of the optimal tariff. Still, it is not sufficient to obtain a complete characterization of the tariff structure, since there is an infinity of functions  $T(\cdot)$  satisfying the conditions of Proposition 1. In particular, we are not able to determine uniquely the optimal value of the fixed fee. Moreover it is important to check the robustness of the features of the solution to more general distributions of unobservable heterogeneity. This is why we now turn to the case of a continuous distribution of types.

## 3.2 Continuous distribution of types

We now take the distribution of types to be continuous with density  $f$  and c.d.f.  $F$  on a bounded support  $[\underline{\theta}, \bar{\theta}]$ . The density  $f$  is supposed to be log concave. Using this distribution, all aggregate variables need to be redefined. The total volume of transactions over all banks is now

$$Q = \int_{\theta_0}^{\bar{\theta}} q(\theta) dF(\theta), \quad (10)$$

where  $\theta_0$  is the participation threshold, defined implicitly by  $u(\theta_0) = 0$ . Budget balance of the LVPS is given by

$$\int_{\theta_0}^{\bar{\theta}} T(q(\theta))dF(\theta) = C(Q), \quad (11)$$

where  $Q$  is given by (10) and  $q(\cdot)$  satisfies (3).

Analogously to the two-type case, the optimal pricing for a monopoly LVPS is obtained by choosing the tariff  $T(\cdot)$  and the banks' reaction function  $q(\cdot)$  that maximize aggregate surplus:

$$S = \int_{\theta_0}^{\bar{\theta}} v(\theta, q(\theta))dF(\theta) - C(Q), \quad (12)$$

under constraints (2) to (8), (10) and (11), plus the condition that  $q(\cdot)$  is non-decreasing, which we will only check ex-post. Since this problem is standard, we immediately state the form of the solution (all proofs are in the appendix).

**Proposition 2** *When the distribution of types is continuous, the optimal tariff for a monopoly LVPS is characterized by the absence of a fixed fee<sup>11</sup> ( $\bar{T} = 0$ ) and a decreasing marginal fee  $T'(q)$ . This marginal fee and the transaction volume  $q(\theta)$  are jointly determined by two equations:*

$$T'(q(\theta)) = c + \frac{\lambda}{1 + \lambda} \frac{1 - F}{f}(\theta) = v_q(\theta, q(\theta)), \quad (13)$$

where  $\lambda$  is the Lagrange multiplier associated to the break-even constraint (11).

As in the two-type case, we obtain a non-linear, degressive pricing scheme. The marginal fee is above the marginal cost, except at the top of the distribution of banks ( $\theta = \bar{\theta}$ ) for which  $T'(q(\bar{\theta})) = c$  (no distortion at the top). The increment is proportional to the inverse hazard rate  $\frac{1-F}{f}$  which is decreasing in  $\theta$  since  $f$  is log concave (for a proof see Bagnoli and Bergstrom 1989). Since  $q(\cdot)$  is an increasing function of  $\theta$ ,  $T'(q)$

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<sup>11</sup>The fixed fee (which is here optimally set to zero) is not to be confused with the connection fee paid by the bank depending on the type of connection it requires (see Hancock et al. (1999) footnote 7). Notice that  $T(0) = 0$  by construction (individual rationality condition) but that  $T$  can a priori be discontinuous at 0 if there is a fixed fee, which we characterize by

$$\bar{T} \equiv T(0^+) = \lim_{\substack{q \rightarrow 0 \\ q > 0}} T(q).$$



is decreasing with respect to  $q$ . This is illustrated in figure 1.  $T'$  is also increasing with respect to  $\lambda$ , the Lagrange multiplier associated to the break-even constraint. Extreme values of  $\lambda$  correspond to marginal cost pricing (for  $\lambda = 0$ ) and private monopoly pricing (for  $\lambda = +\infty$ ). In practice, the optimal  $T$  can be approximated by a menu of two part tariffs.

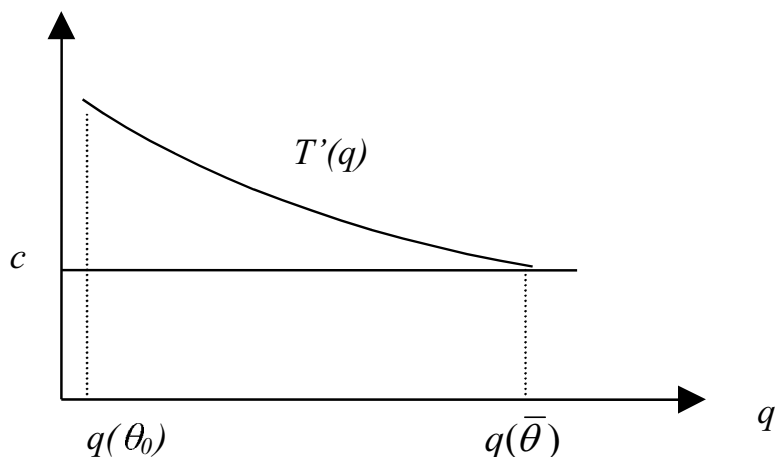


Figure 1: Optimal Pricing in a Public Monopoly

Let us now move to the more original problem of optimally pricing a public LVPS that has a private competitor.

## 4 Optimal pricing in a mixed duopoly

After the monopoly benchmark solution characterized in Section 3, we consider now the mixed duopoly problem posed by cross border transactions within the EU, which can be routed alternatively on TARGET or its private competitor EURO1. Such a mixed duopoly situation is also present in the US, where the Federal Reserve runs the gross system FEDWIRE, while large commercial banks run the hybrid system CHIPS. In fact, this situation exists also, for national transactions, in all the countries (including EU countries) who have implemented a national RTGS, while allowing commercial banks to organize simultaneously a competing payment system.<sup>12</sup> For the moment we focus on the mixed duopoly situation, which we illustrate by cross border transactions within the EU.

<sup>12</sup>However, an additional complexity appears in EMU countries, given that the banks that are incorporated in several countries can also route their transactions on the national RTGSs of these different countries: this is analyzed in Section 5.

Notice that a unique payment system would probably be more cost-effective (at least in the short run) due to the property of increasing returns to scale of the payment technology (see Hancock et al., 1999, for an empirical test of this property). It is also clear that the coexistence of a private payment system, often organized on a net basis, increases the potential for systemic risk (see for instance Rochet and Tirole, 1996, and the references therein for discussions of systemic risk issues in payment systems).<sup>13</sup> However, it would be extremely difficult for any Central Bank to oblige commercial banks to route all their transactions on a unique system. Moreover, from the point of view of long term efficiency the co-existence of two systems is probably a good way to provide incentives for cost minimization and promote innovation (yardstick competition).

We model the private system by considering that the “biggest” or “most efficient” banks move all their transactions to a different, private LVPS. We believe this assumption to be consistent with the actual situation, since members of the private systems tend to represent the largest banks of their respective countries<sup>14</sup>.

The private LVPS is assumed to be characterized by a cost function

$$C_1(Q_1) = a_1 + c_1 Q_1$$

where  $c_1 < c$ . This assumption reflects evidence from EU countries suggesting that the marginal cost for cross-border payment processing is lower in EURO1 than in the public RTGS systems.

For technical reasons we will restrict ourselves to the case of discrete distributions, and assume that there are three types of banks,  $\theta_L < \theta_M < \theta_H$ , which occur with frequency  $f_k$ ,  $k = L, M, H$ , with  $f_L + f_M + f_H = 1$ . Several configurations might occur, whereby the three types of banks use one system or the other. For the sake of conciseness, we focus on the most interesting regime where the private system is under the control of the high type banks only, and the binding participation constraint for the Central Bank is to prevent medium type banks from also moving their transactions to the private system.<sup>15</sup>

We assume that the private system is run like a mutual representing the collective

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<sup>13</sup>Some private systems have recently adopted a hybrid structure (such as CHIPS) and thereby reduced systemic risk.

<sup>14</sup>Still, in practice even the largest banks use the publicly run RTGS for routing some of their transactions, related for example to money market and foreign exchange operations. It may be the case that for such transactions, which are often of very large value, immediacy and risk considerations may dominate pricing issues. These issues are left outside the scope of the present paper.

<sup>15</sup>When this constraint is not binding, the solution is the same as in Section 3: the presence of a private competitor does not affect the pricing policy of the Central Bank. There is also a more complex case

interests of the member banks and their customers, i.e., that they choose a tariff  $T_1$  that maximizes the aggregate surplus generated by the member banks under the break even constraint

$$f_H T_H \geq C_1(Q_1).$$

Since there is only one type using the private system, the tariff structure is simple: the system needs to recover costs with a payment volume of  $Q_1 = f_H q_H$ . This is achieved with average cost pricing: the fixed fee is shared among participants proportionally, and marginal fees are equal to the marginal cost. The tariff charged by the private system is therefore

$$T_1(q) = \frac{a_1}{f_H} + c_1 q. \quad (14)$$

Notice that with this pricing structure, banks choose the transaction volume  $q$  that maximizes surplus

$$s_H^1 \equiv \max_q [v(\theta_H, q) - c_1 q]. \quad (15)$$

We model the public system as a Stackelberg leader: we assume that it has the power to commit to a tariff  $T(\cdot)$  and is able to anticipate the reaction of its private competitor. The objective of the public system is thus to maximize aggregate surplus

$$\max \sum_{k=L,M,H} f_k s(\theta_k, q_k(\theta_k)),$$

under a condition of balanced budget, which requires

$$\sum_{k=L,M} f_k T_k \geq a + c \sum_{k=L,M} f_k q_k.$$

As before, the Central Bank can design combinations  $(q_k, T_k)$  for each type of bank using its system. The incentive constraints of the banks participating in the public system are, similar to section 3,

$$(\theta_M - \theta_L) q_L \leq u_M - u_L \leq (\theta_M - \theta_L) q_M,$$

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where the Central Bank faces two binding participation constraints: that of low types, and the constraint that medium types are not attracted by the private system.

and the participation constraints  $u_L \geq 0$  and  $u_M \geq 0$ . Furthermore, one needs to ensure that banks of type  $\theta_M$  have no interest in switching to the private system. This requires  $u_M \geq u_M^1$  where

$$u_M^1 \equiv \max_q [v(\theta_M, q) - T(q)] = s_M^1 - \frac{a_1}{f_H} \quad (16)$$

where  $s_M^1$ , which is defined analogously to (15), denotes the welfare the medium type would have obtained in the private system. This illustrates the difference to the above monopoly situation analyzed in section 3: now, there is competition "from above" which requires the introduction of the additional constraint,  $u_M \geq u_M^1$ . As already discussed, we focus on the most interesting case in which this is the binding participation constraint. This is the case for

$$s_M^1 - \frac{a_1}{f_H} > (\theta_M - \theta_L) q_M. \quad (17)$$

The optimal pricing is characterized by the following proposition.

**Proposition 3** *Suppose that the highest types  $\theta_H$  participate in a private system, the types  $\theta_M$  and  $\theta_L$  in the public system, and (17) holds. The optimal marginal tariff in the public LVPS is then given by*

$$\begin{aligned} T'(q_M) &= v_q(\theta_M, q_M) = c - \frac{\lambda}{1 + \lambda} \frac{f_L}{f_M} (\theta_M - \theta_L) \\ T'(q_L) &= v_q(\theta_L, q_L) = c. \end{aligned}$$

Therefore, in the case of a mixed duopoly, we again obtain a degressive pricing scheme. Contrary to the monopoly case however, it is now the lower type who obtains marginal cost pricing, while the higher type faces a distortion. Moreover, the medium bank types ( $\theta_M$ ) face prices that are *below* marginal cost. This pricing scheme results from competition from the private system: the private system has marginal fees  $c_1$  that are below marginal cost of the public system,  $c$ . In order to give incentives for banks of type  $\theta_M$  to use the public system, there must be a subsidy at the margin. For the low types, marginal cost pricing is now attainable. This is because of condition (17), which implies that the medium types do not have any incentives to choose the payment volume of the lower types, but rather move to the private system. Therefore, there is no need to distort the lower type's marginal fee upwards.

The pricing scheme of Proposition 3 necessarily involves a positive fixed fee. Indeed, since marginal costs are below or equal to the marginal cost  $c$ , budget balance can only be

achieved with a fixed fee, high enough to recover not only the fixed cost but also the subsidies to medium sized banks. This pricing scheme is only viable if  $v(\theta_L, q_L) - T(q_L) \geq 0$  is still satisfied, i.e. if the lower type's participation constraint is not violated. In particular, there is an upper limit to the maximal fixed cost  $a$  that can be sustained with the tariff schedule developed here. For very high  $a$ , (or for high  $\lambda$ ), the Central Bank might have to depart from the cost recovery principle or else relax the incentive constraints by allowing the bank types  $\theta_M$  to use the private system. In the more general case of a continuous distribution of types, the pattern of the optimal tariff is more complex, involving marginal prices above marginal cost for low volumes, and below for large volumes, combined with a positive fixed fee.

## 5 Competition Between National RTGSs

We now consider a different type of competition between LVPSs, namely between the public systems of two different countries (denoted by the index  $i = 1, 2$ ). We model the banking sectors and the LVPSs of the two countries exactly as before: in both countries, there is a proportion  $f_k^i$ ,  $i = 1, 2$ , of types  $k = \{L, H\}$ . The cost functions of the LVPS in country  $i$  are given as

$$C_i = a_i + c_i Q_i.$$

where w.l.o.g., we assume that  $c_1 < c_2$  and  $a_1 > a_2$ . If all banks were using their own national LVPS, each country could choose a locally optimal fee structure, as given in Proposition 1:

$$\begin{aligned} T'_i(q_H^i) &= v_q(\theta_H, q_H^i) = c_i \\ T'_i(q_L^i) &= v_q(\theta_L, q_L^i) = c_i + \frac{\lambda_i}{1 + \lambda_i} \frac{f_H^i}{f_L^i} (\theta_H - \theta_L). \end{aligned}$$

Different characteristics of the banking sectors (e.g. a different distribution of bank types), and differences in the cost structures of the LVPS would lead to different fee structures across countries. Such an environment may create incentives for banks to bypass their own system and route their payments through the system of a neighboring country instead. In the above example, for instance, for the higher types  $\theta_H$  the marginal fee is lower in country 1 than in country 2. Banks may be in a position to choose which of the national payment systems to use either because they have subsidiaries in several

countries and therefore access to several national LVPS, or because remote access<sup>16</sup> is possible.

The possibility of by-passing raises many questions for the design of an efficient pricing scheme, such as whether the different systems should co-ordinate their fee structures in order to either deter or to encourage by-passing, and whether a uniform fee structure throughout the area could be optimal. This section tries to address some of these issues. We will analyse both cases, first one where by-passing is allowed, and a second case where the LVPSs adjust their pricing in order to make by-passing unattractive to banks. A welfare comparison of the two outcomes would determine which is the more desirable way to go.

We will focus on the case that remote access is not possible. Instead, we assume that only some banks can by-pass because they have branches in the other country, those with the higher demand for payments. It is assumed that there is a cost  $\gamma$  that needs to be borne by the banks who by-pass. Furthermore, we assume for simplicity that a bank needs to route all its payments through one single system, i.e. it cannot split payments so they are routed through two systems at the same time. Finally, we require that the cost-recovery constraint has to be satisfied for each individual LVPS.

## 5.1 Efficient pricing with by-passing

Suppose that the cost of bypass is sufficiently low so it is worthwhile for banks of type  $\theta_H$  in country 2 to use the LVPS in country 1. The problem of finding the efficient pricing structure for that LVPS is derived analogously to the previous sections, the only difference being that the demand for a high payment volume in country 1 has now increased. The optimal marginal tariff in country 1 is given in the following proposition:

**Proposition 4** *Suppose that  $\theta_H$ -banks from country 2 by-pass. The optimal tariff in country 1 has to satisfy*

$$\begin{aligned} T'_1(q_H^1) &= T'_1(q_H^2) = v_q(\theta_H, q_H^1) = c_1 \\ T'_1(q_L^1) &= v_q(\theta_L, q_L^1) = c_1 + \frac{\lambda}{1 + \lambda} \frac{f_H^1 + f_H^2}{f_L^1} (\theta_H - \theta_L). \end{aligned}$$

The efficient pricing in country 1 is therefore very similar to the monopoly case. In country 2, on the other hand, there is now only one bank type with a positive payment

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<sup>16</sup>Remote access is a possibility in TARGET, however, at present it is not widely used.

demand. Therefore, the LVPS can simply apply average cost pricing and charge a tariff

$$T_2(q) = \frac{a_1}{f_L} + c_2q.$$

Allowing by-pass has one attractive feature: more banks use the system where the marginal cost of sending payments is lowest (country 1). However, as usual we need to check whether the participation constraint of the  $\theta_L$ -types is satisfied. This might pose a problem here: because the number of participants in the LVPS of country 2 has now diminished, the banks with a low payment demand have to bear the entire fixed cost of running the system,  $a_2$ , among themselves. Their participation constraint might therefore be violated, and the system would break down. Thus, there are cases where it can be justified to deter by-pass. This is what we study now.

## 5.2 By-passing is deterred

The alternative to allowing by-pass to happen is to change the pricing structure of the national LVPS in order to make by-pass non-profitable for banks. One possibility here is to require a full harmonization of tariffs. In that case, a positive cost of bypass  $\gamma$  would make by-pass unattractive. However, we have argued above that this solution cannot be efficient: prices should vary across countries and reflect differences in the structure of banking sectors and possibly the underlying cost parameters of each LVPS.

A less extreme situation is one in which there are price differences across countries but they are limited in order to deter by-pass. Since it is the high type banks that are more likely to by-pass, we just have to introduce the additional constraint that banks of type  $\theta_H$  in country 2 prefer to use the home country LVPS:

$$u_H^2 \geq u_H^1 - \gamma.$$

We focus on the case where this constraint is binding. The analysis also depends crucially on the level of  $\gamma$ , the cost of bypass. We focus on the case where  $\gamma$  is relatively low (for details, see the proof in the appendix). Denote by  $\lambda_1$  and  $\lambda_2$  Lagrange multipliers of the break even constraints of the two LVPS. For this case, we obtain

**Proposition 5** *For a low cost of bypass  $\gamma$ , the optimal pricing scheme that deters by-pass*

in country 1 needs to satisfy:

$$\begin{aligned} T_1'(q_H^1) &= v_q(\theta_H, q_H^1) = c_1 \\ T_1'(q_L^1) &= v_q(\theta_L, q_L^1) = c_1 + \frac{\lambda_1 f_H^1 + \lambda_2 (f_L^2 + f_H^2)}{(1 + \lambda_1) f_L^1} (\theta_H - \theta_L), \end{aligned}$$

and in country 2:

$$\begin{aligned} T_2'(q_H^2) &= v_q(\theta_H, q_H^2) = c_2 - \frac{\lambda_2}{1 + \lambda_2} \frac{f_L^2}{f_H^2} (\theta_H - \theta_L) \\ T_2'(q_L^2) &= v_q(\theta_L, q_L^2) = c_2. \end{aligned}$$

If by-pass is to be deterred, the pricing schemes in the two countries therefore have to have different features. In the country with the lower marginal cost (i.e. country 1), the optimal tariff is similar to the one of the monopoly case: it is degressive, and only banks with a high payment volume pay a marginal fee equal to marginal cost. In the country facing the risk of by-pass (i.e. country 2), on the other hand, the marginal fee for the high payment volume needs to be lowered in order to make by-pass unattractive.

Notice, however, that also here we need to verify whether the lower type's participation constraints are satisfied: because marginal fees are below marginal cost for some types, a fixed fee is needed in order to achieve budget balance. If the fixed fee is too high, or if the marginal subsidy for the high types is too large, then the system is not viable. Still, because the high types participate in the system, budget balance is easier to satisfy than in the case where by-pass is allowed.

A judgement whether by-pass should be allowed or not is not possible in general terms. This will depend on the specific parameters of the model. Propositions 4 and 5 imply that in both cases marginal prices need to be distorted away from marginal costs. The resulting inefficient levels of payment demand  $q_k$  need to be taken into account in the welfare analysis. Still, by-passing has the advantage that more banks use the payment technology with the lower marginal cost. Bypassing in itself might however introduce social welfare cost which are not possible to capture in a model like ours. One such cost could be related to the possible breakdown of one of the systems in case of by-pass. Therefore, the optimal tariff will depend on the willingness of central banks to allow this to happen, or to possibly subsidize one of the systems.



## 6 Concluding Remarks

We have analyzed the optimal pricing in a Large Value Payment System when full cost recovery is required. We have shown that if the payment system operates as a monopoly, then the optimal tariff involves quantity discounts, but no fixed fee. This particular form of volume-based pricing is the optimal way to achieve recovery of fixed costs, given the impossibility to implement personalized tariffs. Quantity discounts are important to ensure that banks with a potentially high payment volume don't decide to lower the number of payments sent through the payment system. Moreover, the absence of a fixed fee ensures participation of low type banks.

The situation changes when the public system faces competition from a private system which, because of lower marginal costs, is able to attract the banks with the highest payment demand. The optimal pricing scheme of the public system then needs to be adjusted. This paper shows that the public system needs to decrease its marginal fees below marginal costs in order to keep a sufficient volume of transactions; as a result, fixed fees have to be introduced.

We also analyze the situation where two national public payment systems compete for the domestic transactions of large banks who have branches in both countries and are able to route their payments through either system. We characterize the optimal tariffs and show that they depend on whether such a by-pass is deterred or not. When it is not, the optimal tariffs exhibit the same qualitative properties as in the monopoly case: marginal fees are above marginal costs. If by-pass is to be deterred on the contrary, the features of the optimal tariffs are similar to the case of competition by a private system: the country that is subject to the risk of by-pass has to decrease its marginal fees below marginal costs, while the opposite is true for the other country.

One important limitation of our analysis is that we have considered homogenous payments, so that volume is the variable that can be used to differentiate prices. It would be important to introduce heterogeneous payments, so as to take into account other characteristics such as size. This would fit more accurately the present situation where large commercial banks typically use the private system for small to medium size transactions and the public system for large transactions for which immediate finality is crucial. It is planned to address this extension in a separate paper.

## APPENDIX: PROOFS

**Proof of Proposition 1:** We decompose the Central Bank's problem into two parts: first, for fixed quantities, it chooses the optimal  $u_k$ , and in a second step, it chooses the quantities  $q_k$ .

It is easy to see that  $(IR_H)$  is not binding, since  $(IC_H)$  and  $(IR_L)$  imply  $u_H \geq u_L$ . Furthermore, the incentive-compatibility constraints can be simplified to

$$(\theta_H - \theta_L)q_L \leq u_H - u_L \leq (\theta_H - \theta_L)q_H$$

The break-even constraint can be re-stated  $\sum_k f_k (s_k - u_k) \geq a$ . From here, it is immediate that maximizing surplus is equivalent to minimizing the sum of individual utilities

$$\min \sum_{k=L,H} f_k u_k$$

such that

$$\begin{aligned} (\theta_H - \theta_L)q_L &\leq u_H - u_L \leq (\theta_H - \theta_L)q_H \\ u_L &\geq 0. \end{aligned}$$

It is easy to see that, as usual, the lower type's participation constraint needs to be binding,  $u_L = 0$ , as well as the higher type's incentive compatibility constraint,  $u_H = (\theta_H - \theta_L)q_L$ . Using these values of  $u_k$ , we can in the second step re-state the problem of the central bank of finding the optimal quantities  $q_L$  and  $q_H$  that maximize surplus under the break-even constraint, using the Lagrangian

$$\begin{aligned} L &= \sum_{k=L,H} f_k s_k + \lambda \left\{ \sum_{k=L,H} f_k s_k - a - \sum_{k=L,H} f_k u_k \right\} \\ &= (1 + \lambda) \sum_{k=L,H} f_k \{v(\theta_k, q_k) - cq_k\} - \lambda \{a + f_H(\theta_H - \theta_L)q_L\} \end{aligned}$$

Maximizing w.r.t.  $q_H$  and  $q_L$ , we find

$$v_q(\theta_H, q_H) = c$$

and  $(1 + \lambda)f_L v_q(\theta_L, q_L) = c + \lambda(\theta_H - \theta_L)$ , or

$$v_q(\theta_L, q_L) = c + \frac{\lambda}{1 + \lambda} \frac{f_H}{f_L} (\theta_H - \theta_L).$$

Together with equation (3), this establishes Proposition 1.

**Proof of Proposition 2:** The indirect “utility” function of the banks is defined in (2), and (3) holds. A simple revealed preference argument, together with the single crossing property that  $v_{\theta q} > 0$ , implies that  $q(\cdot)$  must be everywhere non-decreasing. A classical result (for a proof, see for instance Fudenberg and Tirole) shows that conversely any non-decreasing function  $q(\cdot)$  can be implemented by a tariff satisfying (3). The envelope principle applied to condition (2) then implies that for almost every  $\theta$ :

$$u'(\theta) = v_{\theta}(\theta, q(\theta)). \quad (\text{A.1})$$

Adopting the “dual” approach developed by Mirrlees, we use  $u(\cdot)$  and  $q(\cdot)$  as policy variables (instead of  $T$ ).  $T$  will later be derived implicitly by the formula:

$$T[q(\theta)] = v(\theta, q(\theta)) - u(\theta).$$

We therefore have to find  $q(\cdot)$  and  $u(\cdot)$  that solve

$$\begin{aligned} \max \int_{\theta_0}^{\bar{\theta}} \{v(\theta, q(\theta)) - cq(\theta)\} dF(\theta) \\ u'(\theta) = v_{\theta}(\theta, q(\theta)) \quad \theta \in ]\underline{\theta}, \bar{\theta}[ \end{aligned} \quad (\text{A.2})$$

$$\int_{\underline{\theta}}^{\bar{\theta}} \{v(\theta, q(\theta)) - cq(\theta) - u(\theta)\} dF(\theta) \geq a \quad (\text{A.3})$$

$$u(\theta_0) = 0. \quad (\text{A.4})$$

The solution to the above problem can be obtained by maximizing its Lagrangian:

$$L = \int_{\theta_0}^{\bar{\theta}} [(1 + \lambda)\{v(\theta, q(\theta)) - cq(\theta)\} - \lambda u(\theta)] f(\theta) d\theta,$$

under (A.1) and (A.3). The term in  $u$  can be eliminated by an integration by parts:

$$\int_{\theta_0}^{\bar{\theta}} u(\theta) f(\theta) d\theta = \int_{\theta_0}^{\bar{\theta}} q(\theta) (1 - F(\theta)) d\theta$$

which gives

$$L = \int_{\theta_0}^{\bar{\theta}} [(1 + \lambda)\{v(\theta, q(\theta)) - cq(\theta)\} f(\theta) - \lambda q(\theta) (1 - F(\theta))] d\theta,$$

which is maximized when

$$v_q(\theta, q(\theta)) = c + \frac{\lambda}{1 + \lambda} \frac{1 - F}{f}(\theta). \quad (\text{A.5})$$

The optimal marginal fee  $T'(q)$  is thus determined jointly with the optimal volume  $q(\theta)$  by (A4) and the condition

$$T'(q(\theta)) = v_q(\theta, q(\theta)).$$

Since  $\frac{1-F}{f}$  decreases in  $\theta$  (since  $f$  is log-concave), total differentiation of the first equation gives

$$v_{\theta q} + v_{qq}q'(\theta) < 0,$$

which implies  $q'(\theta) > 0$  (as was to be checked) since  $v_{\theta q} > 0$  and  $v_{qq} < 0$ . Similarly, by differentiating the second equation we see that  $T'$  is decreasing.

The participation threshold is determined by maximizing  $L$  with respect to  $\theta_0$ . We obtain:

$$(1 + \lambda)\{v(\theta_0, q(\theta_0)) - cq(\theta_0)\}f(\theta_0) = \lambda q(\theta_0)(1 - F(\theta_0)).$$

Thus

$$\frac{v(\theta_0, q(\theta_0))}{q(\theta_0)} = c + \frac{\lambda}{1 + \lambda} \frac{1 - F(\theta_0)}{f(\theta_0)} = v_q(\theta_0, q(\theta_0)).$$

Given the strict concavity of  $v$  in  $q$  (and the fact that  $v(\theta_0, 0) = 0$ ) this is only possible when  $q(\theta_0) = 0$ . This implies in turn that the optimal fixed fee is zero, since it is defined as

$$\bar{T} = \lim_{q \rightarrow 0^+} T(q) = \lim_{\theta \rightarrow \theta_0} T(q(\theta)).$$

But  $T(q(\theta)) = v(\theta, q(\theta)) - u(\theta)$  and both terms tend to zero when  $\theta \rightarrow \theta_0$ . ■

**Proof of Proposition 3:** As before, we divide the problem into two steps, and as a first step, find the optimal  $u_k$  for given quantities  $q_L$  and  $q_M$ . Again, maximization of surplus requires minimization of  $u_k$  under the constraints

$$\begin{aligned} u_L &\geq 0 \\ u_M &\geq s_M^1 - \frac{a_1}{f_H} \\ (\theta_M - \theta_L)q_L &\leq u_M - u_L \leq (\theta_M - \theta_L)q_M. \end{aligned}$$

where type  $\theta$ 's lower participation constraint ( $u_M \geq 0$ ) is left out as it is not binding.

Under assumption (17), it is easy to see that the minimal  $u_k$  satisfy:

$$\begin{aligned} u_M &= s_M^1 - \frac{a_1}{f_M} \\ u_L &= u_M - (\theta_M - \theta_L) q_M. \end{aligned}$$

Assumption (17) implies that in order to induce the middle types not to change to the private system,  $u_M$  needs to be so high that  $u_L = 0$  is no longer attainable because this would violate the lower types incentive constraint not to pass off as a middle type.

Given the optimal values of  $u_k$ , the second step of the maximization problem is to find the optimal quantities  $q_L$  and  $q_M$ . The budget balance restriction can be rewritten as:

$$f_L s_L + f_M s_M \geq a + f_L u_L + f_M u_M.$$

Denoting by  $\lambda$  the multiplier associated with the budget constraint of the public system, the Lagrangian is

$$\begin{aligned} L &= \sum_{k=L,M} f_k s_k + \lambda \left\{ \sum_{k=L,M} f_k s_k - a - f_L u_L - f_H u_H \right\} \\ &= (1 + \lambda) \sum f_k s_k - \lambda \left[ a + (f_L + f_H) \left( s_M^1 - \frac{a_1}{f_H} \right) - c q_M \right] \end{aligned}$$

Here, we have left out the surplus of the  $\theta_H$ -types, as it is constant from point of view of the public system. Maximization with respect to  $q_L$  and  $q_M$  yields

$$\begin{aligned} v_q(\theta_L, q_L) &= c \\ v_q(\theta_M, q_M) &= c - \frac{\lambda}{1 + \lambda} \frac{f_L}{f_H} (\theta_M - \theta_L). \end{aligned}$$

Together with the bank's optimality condition  $T'(q_k) = v_q(\theta_k, q_k)$ , we can establish Proposition 3.<sup>17</sup>

**Proof of Proposition 5:** As before, the first step of the maximization problem consists in minimizing expected rents subject to the incentive compatibility- and individual rationality constraints, i.e.

$$\max - \sum_{i,k} f_k^i u_k^i$$

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<sup>17</sup>The Lagrange multiplier  $\lambda$  can be determined together with  $q_M$  by using the second FOC and the budget constraint. Here, it is useful to work with the "virtual type"  $\theta_M \equiv \theta_M + \frac{\lambda}{1+\lambda} \frac{f_L}{f_M} (\theta_M - \theta_L)$ .

such that

$$\begin{aligned}
u_L^i &\geq 0 && (\alpha_1^i) \\
u_H^i - u_L^i &\leq (\theta_H - \theta_L)q_H^i && (\alpha_2^i) \\
(\theta_H - \theta_L)q_L^i &\leq u_H^i - u_L^i && (\alpha_3^i) \\
u_H^1 - \gamma &\leq u_H^2 && (\alpha_4)
\end{aligned}$$

are satisfied for  $i = 1, 2$ , and  $\alpha_j^i$  denote the Lagrange multipliers associated with the constraints. We focus on the case where the last constraint is binding, so that  $u_H^2 = u_H^1 - \gamma$ . Furthermore, there are two possibilities, depending on whether

$$u_H^1 - \gamma \geq (\theta_H - \theta_L)q_H^2 \quad (\text{A.6})$$

is satisfied. Suppose that (A.6) holds. The minimal values for  $u_k^i$ ,  $i = 1, 2$ ,  $k = L, H$ , that satisfy all seven constraints are then

$$\begin{aligned}
u_L^1 &= 0 & u_L^2 &= (\theta_H - \theta_L)(q_L^1 - q_H^2) - \gamma \\
u_H^1 &= (\theta_H - \theta_L)q_L^1 & u_H^2 &= (\theta_H - \theta_L)q_L^1 - \gamma.
\end{aligned}$$

As before, these values are used in the second step of finding the optimal quantities  $q_k^i$ . The Lagrangian of the problem is

$$\begin{aligned}
\mathcal{L} &= \sum_{i=1,2} \sum_{k=L,H} f_k^i s_k^i + \sum_{i=1,2} \lambda_i \left\{ \sum_{k=L,H} f_k^i s_k^i - \sum_{k=L,H} f_k^i u_k^i \right\} \\
&= \sum_{i=1,2} (1 + \lambda_i) \sum_{k=L,H} f_k^i s_k^i - \lambda_1 f_H^1 (\theta_H - \theta_L) q_L^1 - \lambda_2 (f_L^2 + f_H^2) [(\theta_H - \theta_L)q_L^1 - \gamma] + \lambda_2 q_H^2.
\end{aligned}$$

The first order conditions yield

$$\begin{aligned}
v_q(\theta_L, q_L^1) &= c_1 + \frac{\lambda_1 f_H^1 + \lambda_2 (f_L^2 + f_H^2)}{(1 + \lambda_1) f_L^1} (\theta_H - \theta_L) \\
v_q(\theta_H, q_H^1) &= c_1 \\
v_q(\theta_L, q_L^2) &= c_2 \\
v_q(\theta_H, q_H^2) &= c_2 - \frac{\lambda_2}{1 + \lambda_2} \frac{f_L^2}{f_H^2} (\theta_H - \theta_L).
\end{aligned}$$

■

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