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## Consumption, habit persistence, imperfect information and the lifetime budget constraint

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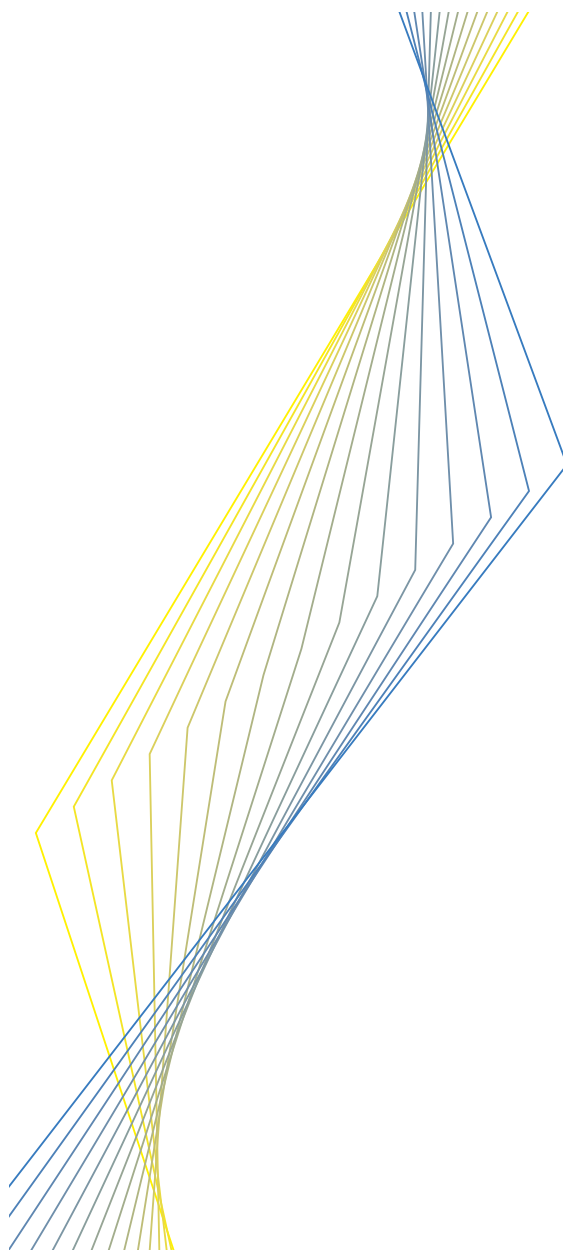
**WORKING PAPER NO. 251**

**CONSUMPTION, HABIT  
PERSISTENCE, IMPERFECT  
INFORMATION AND THE  
LIFETIME BUDGET CONSTRAINT**

**BY ALPO WILLMAN**

**AUGUST 2003**

**EUROPEAN CENTRAL BANK**  
**WORKING PAPER SERIES**



**WORKING PAPER NO. 25 I**

**CONSUMPTION, HABIT  
PERSISTENCE, IMPERFECT  
INFORMATION AND THE  
LIFETIME BUDGET CONSTRAINT<sup>1</sup>**

**BY ALPO WILLMAN<sup>2</sup>**

**AUGUST 2003**

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**Abstract:**

Based on the households' utility maximisation, a closed form approximation of the consumption function is derived and the deep parameters of the consumption function are estimated using aggregate euro area data. The novel element in our approach is the parameterisation of the information content regarding future income changes. In addition to the information regarding time series properties of the historical development of labour income, consumers have also period-specific information on future income realisations. Estimation results support the hypothesis that, although front-loaded, consumers have a lot of information on future income changes, but that also lagged consumption, through habit formation, plays an important role.

*Keywords:* consumption, wealth, life-cycle hypothesis

*JEL classification:* D12, E21

## **Non-technical summary**

In this paper we specify a closed form approximation of the consumption function derived from the households' utility maximisation problem and, thereafter, estimate the consumption function using aggregate euro area data. The novel element in our approach is the parameterisation of the information content regarding future income changes. We assume that, in addition to the information regarding time series properties of the historical development of labour income, consumers have also period-specific information on future income realisations. We show that, if the amount of period-specific information is front-loaded and it follows a geometrically declining pattern, the information effect can be captured by one parameter and human wealth can be expressed in terms of current income and ex post observable future income changes. Furthermore, approximating the stochastic variation of the future income stream by a risk premium, we can solve an intrinsically stochastic optimisation problem of the household by standard techniques of deterministic optimisation. Accounting for "habit formation" in the utility function and allowing for alternatively infinite or finite horizons, we end up with the consumption function, which allows quite general dynamics for consumption and the identification of the underlying deep parameters of the consumption function. According to the derived consumption function, current-period consumption depends on one period lead and lag of consumption as well as on current-period income, real financial wealth and the real interest rate as fundamental variables. This specification captures the stylised features of the data, i.e. strong dependency of consumption on current income and delayed effects from shocks to consumption.

We apply our specifications to the euro area data and estimate the deep parameters of the model based on the assumption that the rate of subjective time preference equals the long-run equilibrium real interest rate. Estimation results based on both infinite and finite horizon specifications are reasonable and broadly in line with our priors, although the finite horizon specification, encompassing the infinite horizon specification, is favoured by the data. The results also indicate that, although deviating significantly from perfect foresight, consumers have quite a lot of information on future income changes, and this has a significant impact on current-period consumption. However, the amount of forward information is front-loaded. Our estimation results support the view that, through habit formation, lagged consumption is almost as important a determinant of current consumption as the lead of consumption.

## 1. Introduction

It is well known that, owing to non-linearities, a closed-form solution to consumption cannot be derived from the optimisation problem of the representative household with stochastic labour income and constant relative risk-averse utility. This is one reason why, after the seminal paper by Hall (1978), the major part of empirical research on consumption has been concentrated on estimating the Euler equation, which rather than being a consumption function, is an equilibrium condition relating consumption at two different points in time, i.e. the level of consumption today relative to the level of consumption tomorrow.<sup>3</sup> Consequently this approach does not explicitly solve the optimisation problem of the household and focuses instead on a specific first order condition implied by that problem. The Euler approach nonetheless allows one to estimate structural parameters of the life cycle/ permanent income model and to test some of the implications of the model, e.g. whether consumption is a “martingale process” as Hall (1978) argued. On the other hand, the drawback of the Euler approach is that it is unable to say anything about how consumption reacts to unexpected changes e.g. in income, taxes and interest rates, although these are precisely the issues which policy makers are most interested in. Therefore, it is no wonder that empirical research using the "closed-form" life-cycle/permanent income consumption function originated by Modigliani and Brumberg (1954 and 1979) and Friedman (1953) has remained a topic of sustained interest.

In deriving a closed-form consumption function the problems associated with income uncertainty have to be dealt with somehow. The simplest ways are to assume that the utility function is quadratic, implying certainty equivalence as in Gali (1990), or, simply, to neglect uncertainty. Another and, as argued by, for example, Nagatani (1972), Hayashi (1982) and Zeldes (1989), also realistic way to account for income uncertainty is to add a risk premium term in discounting the future income stream. In addition, to get the closed form solution it is necessary to define the stochastic nature of the marginal process of income determination. Only after this, can the resolved consumption function be derived.

The economic rationale of the error correction approach to modelling consumption, as proposed by Davidson et al (1978), can be based on this line of argument. The merits of the error correction approach, after being augmented by adjustment costs or, alternatively, by habit formation behaviour, lie in allowing very rich dynamics and in clarifying statistical problems associated with working with non-stationary time series data, such as consumption, income and wealth. However, as marginal processes are defined as autoregressive and moving average processes, the costs of this approach are that the (possible) forward looking nature of consumption is lost and estimated parameters in the consumption function are reduced form parameters. This implies that, instead of being stable, the reduced form parameters of the consumption function may shift in responses to changes in policy regimes. Hence, the estimated consumption function of this type is open to the "Lucas critique".

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<sup>3</sup> A paper on estimating Euler equations, see e.g. Attanasio and Low (2000).



More recently Sefton and in't Veld (1999) and Fuhrer (1998, 2000) have estimated consumption functions derived explicitly from optimisation. The merit of both of these works lies in the fact that the forward-looking nature of consumption is contained in estimated specifications, and that, in addition, Fuhrer (1998, 2000) introduces habit formation explicitly into the optimisation framework.

Sefton and in't Veld (1999) apply the Blanchard (1985) overlapping generation model with positive probability of death. This assumption implies that in defining human wealth the future income stream is discounted at a rate exceeding the market rate of interest by a premium equalling the probability of being alive in the following period. This results in the aggregated consumption function, where current consumption is a function of future consumption, with a coefficient of less than one, and also of financial wealth, including current-period income. Outside the optimisation framework, Sefton and in't Veld (1999) add current income to the estimated relation by assuming that a percentage of consumers are liquidity-constrained, i.e. rule-of-thumb consumers, the modification proposed first by Flavin (1981) and Hayashi (1982). However, although it includes death probability, an unrealistic aspect in the framework of Sefton and in't Veld (1999) is that the existence of income uncertainty is neglected. Neither includes habit formation in their model, implying that the non liquidity constrained part of consumption acts like a "jump variable". Accordingly, consumption jumps immediately in response to current news about life-time resources.

Fuhrer (1998, 2000), in the framework of the representative consumer, accounts for habit formation by assuming that a consumers' current utility is determined by current consumption relative to a reference level of consumption. As in Campbell and Mankiw (1989), the approximate consumption function is obtained by solving households' optimisation problem under the log-linearised intertemporal budget constraint, where the income term captures only capital income. These simplifications allow one to reduce income uncertainty to the random rate of return on capital. Moreover, outside the optimising framework, a rule-of-thumb behaviour reflecting liquidity constraints, is added to the estimated equation, or, rather, to the system of equations, to capture the empirical fact that the predictable component of current income is correlated with current consumption.

In this paper, we present an alternative to rule-of-thumb behaviour for explaining the observed close correlation between current income and consumption by focusing on the information content regarding future income changes. The advantage of this approach is that it can be incorporated into the optimisation framework.

We assume that, in addition to the information on time series properties of past income realisations, consumers may also have period-specific information on future realisations. Furthermore, we assume that the amount of period-specific information may be front-loaded so that, with the lengthening of the projection horizon, expected income changes converge to those implied by random walk.<sup>4</sup> Subject to these assumptions, we show that the joint probability distribution associated with future realisations is not

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<sup>4</sup> This is a simplification the first order autoregressive process in first differences that has become a popular representation for income generation process in the macroliterature, see e.g. discussion by Deaton (1985).

symmetric. It may be bimodal, having, possibly, modes both at the point implied by the random walk model for the expected income change and in the ex- post actual realisation. Hence, the mathematical expectation of the joint distribution coincides with neither mode, except in the limiting cases of perfect foresight and random walk. Instead, we show that expected income changes can be expressed as weighted averages of ex post realisations and expected values implied by the random walk model for income changes. Finally, if the amount of front-loaded, period-specific information follows a geometrically declining pattern, the information effect can be captured by one parameter and human wealth can be expressed in terms of current income and ex post observable income changes with geometrically declining weights. Accounting for stochastic uncertainty associated with the time series representation of the income generating process by risk premium, and allowing finite as well as infinite horizons, a closed form approximation for the consumption function can be derived by the technique of deterministic optimisation. Moreover, if we include habit formation, we end up with a consumption function where current consumption depends on one period lead and lag of consumption as well as on fundamental variables, i.e. on real financial wealth, current-period real labour income and the real interest rate. As the deep parameters of the derived equations are identified, the empirical relevance of the underlying hypothesis can be tested and the consumption function containing quite general dynamics can be estimated.

We apply our specifications to the euro area data and estimate the deep parameters of the model based on the assumption that the rate of subjective time preference is assumed to equal the long-run equilibrium real interest rate. Estimation results based on both infinite and finite horizon specifications are reasonable and broadly in line with our priors, although the finite horizon specification, which encompasses the infinite horizon specification as a special case, is favoured by the data. Furthermore, the results indicate that, although deviating significantly from perfect foresight, consumers have quite a lot of information regarding their future income changes, although this information is front-loaded. However, our estimation results support the view that, through habit formation, lagged consumption is almost as important a determinant of current consumption as the lead of consumption.

The structure of this paper is as follows. Section 2 examines the lifetime budget or resource constraint and argues for the information parameter to be introduced into the resource constraint. In section 3, consumption functions in the infinite and finite horizon frameworks, allowing for habit persistence, are derived. Our empirical results are presented in Section 4 and our conclusions are given in Section 5.

## 2. The life-time budget constraint

Assume that the representative household faces the following period to period budget constraint:

$$(1) \quad V_t = (1 + r_t)[V_{t-1} + y_t - c_t] ,$$

where  $V_t$  is the financial wealth in the end of period  $t$ ,  $y_t$  is labour income (net of taxes minus transfers),  $c_t$  is consumption and  $r_t$  is the real interest rate in period  $t$ .

Under perfect foresight and an infinite horizon the period to period budget constraint (1) implies the following life-time resource constraint in period  $t$ :

$$(2) \quad \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) c_{t+i} = V_{t-1} + \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) y_{t+i} = W_t$$

where  $R_t = 1/(1+r_t)$  and  $W_t$  is total wealth available in period  $t$ .<sup>5</sup>

Hayashi (1982) argued that it is not realistic to solve the household's optimal consumption path without taking into account uncertainty associated with labour income. However, if income uncertainty is explicitly introduced into the utility maximisation framework of consumers, a closed form expression for optimal consumption is possible to derive only in two special cases. The utility function has to be either quadratic implying certainty equivalence (see e.g. Zeldes (1989) and Gali (1990)) or based on constant absolute risk aversion, as shown by Merton (1971). Another alternative is to follow the recommendation of e.g. Nagatani (1972), Hayashi (1982) and Zeldes (1989), who have argued that, without hampering the derivation of a closed-form expression for optimal consumption, income uncertainty can be reasonably approximated by adding a risk premium in discounting expected future incomes.<sup>6</sup> In this case the flow budget constraint relevant for the expected utility maximisation can be written as,

$$(3) \quad V_{t+i} = R_{t+i}^{-1} [V_{t+i-1} + z_{t+i} - c_{t+i}]$$

Where  $z_{t+i}$  denotes the risk adjusted expected income,

$$(4) \quad z_{t+i} = \theta^i E_t y_{t+i} = y_t + \theta^i E_t \sum_{j=1}^i \Delta y_{t+j} \quad ; \quad 0 \leq \theta \leq 1$$

where  $1-\theta$  is the risk premium,  $\Delta$  is the difference operator and  $E_t$  is the expectation operator conditional on information available at period  $t$ . Now the infinite horizon budget constraint can be written as follows:<sup>7</sup>

$$(5) \quad \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) c_{t+i} = V_{t-1} + \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) \theta^i E_t y_{t+i} \quad ; \quad 1 \geq \theta > 0$$

It is worth noting that the resource constraint as in (5) leaves open the question of how much the future realisations of labour income changes affect the consumption path planned at period  $t$ . That depends completely on the information content possessed by households concerning future labour income, as shown by the following identity:

$$(6) \quad y_{t+i} = y_t + \sum_{j=1}^i E_t \Delta y_{t+j} + \sum_{j=1}^i \varepsilon_{t+j}$$

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<sup>5</sup> Note that  $\prod_{j=0}^i R_{t-1}^{-1} R_{t+i-1} = \begin{cases} 1 & \text{when } i=1 \\ R_t \cdot R_{t+1} \cdot \dots \cdot R_{t+i-2} \cdot R_{t+i-1} & \text{when } i \geq 1 \end{cases}$

<sup>6</sup> Nagatani (1972) showed that this is a reasonable approximation for all utility functions with a positive third derivative. All popular utility functions satisfy this assumption with the obvious exception of the quadratic function.

<sup>7</sup> Also the left hand variable  $c_{t+i}$  describes the expected or planned consumption path. To keep notation simple expectation operator is omitted.

where  $\varepsilon_{t+j}$  is stochastic error term. Under perfect foresight  $\varepsilon_{t+j} = 0$  and  $E_t \Delta y_{t+j} = \Delta y_{t+j}$  and the life-time budget constraint (5) reduces to (2). If, in turn,  $E_t \Delta y_{t+j} = 0$  for all  $j$  and  $\varepsilon_{t+j}$  is  $N(0, \sigma^2)$  distributed, then (6) reduces to a random walk process. In this case equation (5) reduces to,

$$(5a) \quad \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) c_{t+i} = V_{t-1} + y_t \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) \rho^i$$

Hence, (5a) implies that the planned consumption path is independent from future realisations of labour income. That results from the fact that households are assumed to possess no information on their expected future income changes except the time series properties of the past income realisations. Although popular in empirical applications, we think that this assumption is equally unrealistic as the opposite polar case assumption of perfect foresight.

We think that it is reasonable to assume that, in addition to current income and the stochastic properties of the income generating process, consumers have also some period-specific information concerning future income changes. Albeit imperfect, households may have quite a lot of information on their future professional careers, family sizes, wage increases etc. factors affecting the future realisations of (per capita) households' labour income. In the context of wage increases, they typically are well aware of, which part of wage increase is permanent (annual wage and salary increases) and which is transitory (bonuses etc.). On the other hand, it is quite likely that the amount of actual information is front-loaded, i.e. the information content is much wider concerning income changes in the near future than regarding longer planning horizons.

In the following we formalise the idea described above. We start by presenting a simplified example, where income is a discrete random variable. Thereafter we generalise the example by treating income as a continuous variable and accounting for the aggregation across population. We show that the future expected income changes can be expressed as weighted averages of the future realisations, which ex post are available for an econometrician, and the drift term implied by the random walk process representing the time series properties of past income realisations.

In our example we denote the expected income change in the absence of period-specific information as  $E(\Delta y_{t+i} | {}_t I_{t+i} = 0)$  and the expected income change conditional on all available information as  $E(\Delta y_{t+i} | {}_t I_{t+i} \neq 0)$ , where  ${}_t I_{t+i}$  refers to the amount of period-specific information available at time  $t$  concerning the income change at period  $t+i$ , (with the integer  $i \geq 1$ ). Assume that the labour income based on historical data can be modelled as a random walk and that any future realisation of  $\Delta y_{t+i}$  (for all  $t+i$ ) will be the outcome from the set  $\{x_1, x_2, \dots, x_n\}$  that is distributed symmetrically around the drift term of the random walk process. Probabilities  $p_j$  ( $j=1, \dots, n$ ) are associated with each alternative realisations. For notational simplicity we assume that the drift parameter equals zero.<sup>8</sup> Hence, we can write  $E(\Delta y_{t+i} | {}_t I_{t+i} = 0) = \sum_{j=1}^n p_j x_j = 0$ . In the next step, assume that the period-specific information

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<sup>8</sup> A drift term would be easy to incorporate into our framework, as is shown in Appendix 1. However, in order to not unnecessarily complicate our notation and because in our empirical experiments the inclusion of the drift term had hardly any effect on estimation results, we have abstracted the drift term from the analysis.

increases the probability weight of actual realisation (e.g.  $\Delta y_{t+i} = x_k$ ) and, correspondingly, decreases the weights associated with other possible realisations. In that case the à priori probability weight associated with the future actual realisation can be denoted as  $\gamma_{t+i} + (1 - \gamma_{t+i})p_k$  with  $0 \leq \gamma_{t+i} \leq 1$ , where the size of parameter  $\gamma_{t+i}$  is positively related to the amount of the period-specific information  ${}_t I_{t+i}$  available at period  $t$ . It is easy to see that  $p_k \leq \gamma_{t+i} + (1 - \gamma_{t+i})p_k \leq 1$ .

As  $\gamma_{t+i}$  is the measure of the available period specific information, it is a predetermined parameter for the consumer. If no period-specific information is available, i.e. the set  ${}_t I_{t+i}$  is empty, then  $\gamma_{t+i} = 0$ . Perfect foresight is another polar case with  $\gamma_{t+i} = 1$  and all the probability mass is concentrated on  $x_k = \Delta y_{t+i}$ . In general, however,  $\gamma_{t+i}$  deviates from unity and zero, which implies that the probability distribution, accounting for all available information, is asymmetric with the mathematical expectation deviating from  $x_k = \Delta y_{t+i}$  and zero. Instead, the mathematical expectation, implied by the probability distribution accounting for all available information, is as follows:

$$\begin{aligned}
 (7) \quad E_t(\Delta y_{t+i} \mid {}_t I_{t+i} \neq \emptyset) &= [\gamma_{t+i} + (1 - \gamma_{t+i})p_k]x_k + [1 - \gamma_{t+i} - (1 - \gamma_{t+i})p_k] \sum_{\substack{j=1 \\ j \neq k}}^n \frac{p_j x_j}{1 - p_k} \\
 &= \gamma_{t+i}x_k + (1 - \gamma_{t+i})p_k x_k + (1 - \gamma_{t+i})(1 - p_k) \sum_{\substack{j=1 \\ j \neq k}}^n \frac{p_j x_j}{1 - p_k} \\
 &= \gamma_{t+i}x_k + (1 - \gamma_{t+i}) \underbrace{\sum_{j=1}^n p_j x_j}_{=0} = \gamma_{t+i} \Delta y_{t+i}
 \end{aligned}$$

Hence, the expected income change including all available information can be expressed as a weighted average of the future, ex post observable, realisation and the random walk expectation (equalling zero). The increase of period-specific information, i.e. the rise in parameter  $\gamma_{t+i}$ , shifts the probability mass of the probability distribution and, accordingly, the mathematical expectation towards the future realisation. We also see that, if the realisation  $\Delta y_{t+i} = 0$ , equalling the random walk forecast, then the increase in period-specific information does not change the expected value of income change. Anyway, even in this case the increase of the period-specific information decreases the variance around the expected value, i.e. the probability mass is more concentrated on the expected value.

Although the above simple example illustrates the basic idea, it can be criticised for containing unrealistically restrictive assumptions. For instance, it would be more realistic to assume that the à priori available period-specific information increases the probabilities of outcomes within a range in the neighbourhoods of ex-post realisations instead of increasing the outcome probability of a single point, coinciding with the future realisation. In the following generalised presentation, we, however, show that at the aggregate level relation (7) holds, although the expected values of the probability distributions implied by the à priori available period-specific information would deviate from ex-post realisations at the individual level. The necessary condition is that across the population the expectation errors of individual consumers cancel each other, i.e. on the average the period-specific information is concentrated around the correct values.

Denote by superscript  $j$  individuals ( $j = 1, 2, \dots, N$ ), by  $\Delta y_{t+i}^j$  the actual realisations of income changes and by  $x_{t+i}^j$  the set of the ex-ante possible outcomes of future income changes. The probability density function of income changes based on all available information is  $f(x_{t+i}^j | \gamma_{t+i}^j)$ . If no period-specific information is available, i.e. the information parameter  $\gamma_{t+i}^j = 0$ , then  $f(x_{t+i}^j | \gamma_{t+i}^j = 0)$  coincides with the probability density function  $h(x_{t+i}^j)$  implied by the historical random walk process. With the value of information parameter  $\gamma_{t+i}^j = 1$ , the probability distribution  $f(x_{t+i}^j | \gamma_{t+i}^j = 1)$  coincides with the probability density function  $g(x_{t+i}^j)$  implied by the à priori available period-specific information. Unlike in our simple example above, the period specific information need not be concentrated on a single point but can be distributed over any interval in the range of feasible outcomes. Moreover, we do not assume that the expected values of  $x_{t+i}^j$  for all  $t+i$ , implied by the probability distribution function  $g(x_{t+i}^j)$ , are equal to future realisation  $\Delta y_{t+i}^j$ . Generally, with any value of  $0 \leq \gamma_{t+i}^j \leq 1$ , the probability density function  $f(x_{t+i}^j | \gamma_{t+i}^j)$  can be expressed as a weighted average of probability density functions  $g(x_{t+i}^j)$  and  $h(x_{t+i}^j)$ ,

$$(8) \quad f(x_{t+i}^j | \gamma_{t+i}^j) = \gamma_{t+i}^j g(x_{t+i}^j) + (1 - \gamma_{t+i}^j) h(x_{t+i}^j)$$

where the weights  $\gamma_{t+i}^j$  and  $1 - \gamma_{t+i}^j$  show the shares of probability mass related to period specific information and to the random-walk process, respectively.

We see that, if the mean of the density function  $g(x_{t+i}^j)$ , based on à priori available period-specific information, is  $\bar{x}_g^j \neq 0$ , and the mean of the density function  $h(x_{t+i}^j)$ , based on historical time series properties, is  $\bar{x}_h^j = 0$ , then, with values of the information parameter  $\gamma_{t+i}^j$  different from zero and unity, the density function  $f(x_{t+i}^j | \gamma_{t+i}^j)$  is asymmetric and possibly bimodal. Equation (8) implies that at period  $t$  expected income change for period  $t+i$  is:

$$(9) \quad E_t(\Delta y_{t+i}^j) = \int_{-\infty}^{\infty} x_{t+i}^j f(x_{t+i}^j | \gamma_{t+i}^j) dx_{t+i}^j \\ = \gamma_{t+i}^j \int_{-\infty}^{\infty} x_{t+i}^j g(x_{t+i}^j) dx_{t+i}^j + (1 - \gamma_{t+i}^j) \int_{-\infty}^{\infty} x_{t+i}^j h(x_{t+i}^j) dx_{t+i}^j \\ = \gamma_{t+i}^j \bar{x}_{g,t+i}^j + (1 - \gamma_{t+i}^j) \bar{x}_{h,t+i}^j \\ = \gamma_{t+i}^j \bar{x}_{g,t+i}^j$$

Equation (9) corresponds with (7) with the exception that  $\bar{x}_{g,t+i}^j$  needs not equal to  $\Delta y_{t+i}^j$ . The next step is to aggregate (9) across the population. Denote the population averages  $\bar{\gamma}_{t+i} = \frac{1}{N} \sum_{j=1}^N \gamma_{t+i}^j$  and  $\Delta \bar{y}_{t+i} = \sum_{j=1}^N \Delta y_{t+i}^j / N = \Delta y_{t+i} / N$ , and denote the expectation error based on the probability density function  $g(x_{t+i}^j)$  by  $v_{t+i}^j$ , i.e.  $\Delta y_{t+i}^j = \bar{x}_{g,t+i}^j + v_{t+i}^j$ . In aggregating across population, we assume that  $\bar{v}_{t+i} = \frac{1}{N} \sum_{j=1}^N v_{t+i}^j \approx 0$ . Aggregating (9) across the population we obtain,

$$(10) \quad E_t(\Delta y_{t+i}) = \sum_{j=1}^N \gamma_{t+i}^j \Delta y_{t+i}^j - \sum_{j=1}^N \gamma_{t+i}^j v_{t+i}^j$$

Linearising the right-hand product terms of (10) around the population averages  $\bar{\gamma}_{t+i}$ ,  $\Delta \bar{y}_{t+i}$  and  $\bar{v}_{t+i} = 0$ , we end up with:

$$(11) \quad E_t(\Delta y_{t+i}) \approx N \bar{\gamma}_{t+i} \Delta \bar{y}_{t+i} + \bar{\gamma}_{t+i} \underbrace{\sum_{j=1}^N (\Delta y_{t+i}^j - \Delta \bar{y}_{t+i})}_{=0} + \Delta \bar{y}_{t+i} \underbrace{\sum_{j=1}^N (\gamma_{t+i}^j - \bar{\gamma}_{t+i})}_{=0}$$

$$= \bar{\gamma}_{t+i} \Delta y_{t+i}$$

Relation (11) states that with rather general assumptions on the distribution of the probability density function implied by the period-specific information available at period  $t$  expected aggregate income changes can be expressed in terms of ex-post realisations. However, to be empirically applicable an additional assumption concerning the evolving of the period specific information over the planning horizon is needed. For that purpose we utilise the assumption that the information content is much wider concerning income changes in the near future than regarding longer planning horizons. More specifically we assume that the information parameter  $\gamma_{t+i}$  (for simplicity denoted without the bar) is determined by the following simple process  $\gamma_{t+i} = \gamma^i$ , where superscript  $i$  refers now to the power function.<sup>9</sup> Although overly simplistic at the individual level, we believe that this deterministic process captures reasonably well the accumulation of the period-specific information, when averaged over the population, which our representative agent approach aims to mimic. Now the assumption  $\gamma_{t+i} = \gamma^i$  and (7) or (11) imply that equation (4), defining the risk adjusted expected income  $z_{t+i}$ , can be written as,<sup>10</sup>

$$(12) \quad z_{t+i} = \theta^i \left( y_t + \sum_{j=1}^i \gamma^j \Delta y_{t+j} \right)$$

By treating, for presentational purposes, the real interest rate provisionally as a constant, the flow budget constraint (3) implies the following lifetime budget constraint:

$$(13) \quad \sum_{i=0}^{\infty} R^i c_{t+i} = V_{t-1} + \sum_{i=0}^{\infty} R^i z_{t+i}$$

$$= V_{t-1} + \sum_{i=0}^{\infty} (R\theta)^i \left( y_t + \sum_{j=1}^i \gamma^j \Delta y_{t+j} \right)$$

$$= V_{t-1} + \frac{1}{1-R\theta} \left( y_t + \sum_{i=1}^{\infty} (R\theta)^i \Delta y_{t+i} \right)$$

<sup>9</sup> This simplifying assumption is analogous to the fixed depreciation rate of the capital stock or to the fixed probability to reset prices in the Calvo sticky-price model, see Calvo (1983).

<sup>10</sup> If the time series representation of income process were a random walk with a drift ( $\mu$ ), then relation (12) would be

$$z_{t+i} = \theta^i \left( y_t + \sum_{j=1}^i \gamma^j \Delta y_{t+j} + \mu \sum_{j=1}^i (1-\gamma^j) \right) = \theta^i \left( y_t + \sum_{j=1}^i \gamma^j (\Delta y_{t+j} - \mu) + i \cdot \mu \right)$$

$$= V_{t-1} + \underbrace{\frac{1-R\gamma\theta}{1-R\theta} \sum_{i=0}^{\infty} (R\gamma\theta)^i y_{t+i}}_{=H_t}$$

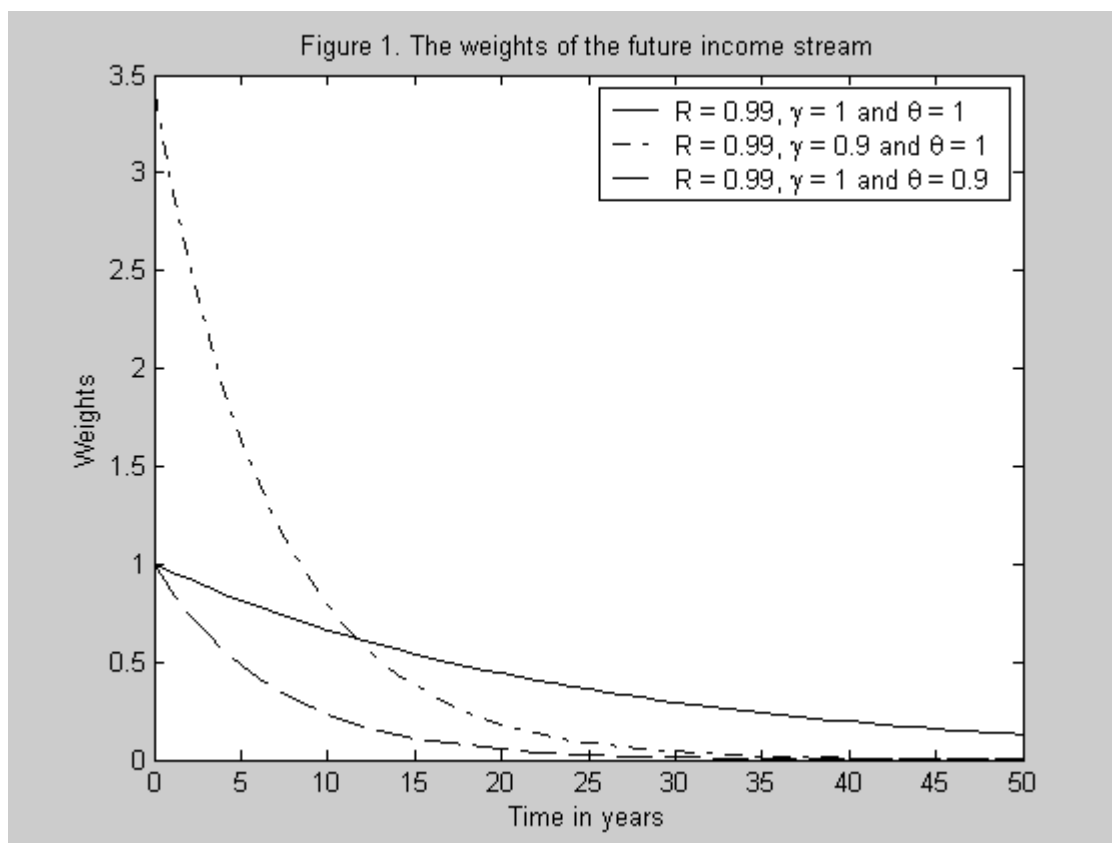
Although technically deterministic, the lifetime budget constraint (13) is, through parameters  $\gamma$  and  $\theta$ , closely related to the uncertainty associated with the future income streams of households. Parameter  $\gamma$  is associated with the amount of period-specific information concerning future realisations, i.e. the closer to unity  $\gamma^i$ , the more symmetrically the joint probability distribution, including also available period-specific information, is distributed around the future realisation. Another limiting case is obtained with  $\gamma$  approaching zero, when no period-specific information is available, and expected income changes equal the random walk expectation and, hence, human wealth  $H_t$  reduces to  $y_t/(1-R\theta)$  compatible with (5a). Parameter  $\theta$ , in turn, can be envisaged to be related inversely to the variance of the probability distribution conditional on all available information and to the rate of risk aversion.

The scaling factor  $\frac{1-R\gamma\theta}{1-R\theta}$  has an important role in differentiating the effects of risk premium and information content parameter on human wealth. A decrease in  $\theta$ , i.e. the increase of the random component of the future changes of labour income, decreases human wealth but the size of  $\gamma$ , in turn, affects human wealth only to the extent that it reflects non-constancy of the anticipated income profile. This results from the fact that, while a rise in  $\gamma$  decreases the sum of the discounted income stream, it also raises the scaling factor, which neutralises the effect of the deviation of  $\gamma$  from unity on human wealth. To give an example assume for simplicity  $y_{t+i} = \bar{y}$  for all  $i$ . Now human wealth is  $H_t = \frac{1-R\gamma\theta}{1-R\theta} \sum_i (R\gamma\theta)^i y_{t+i} = \frac{\bar{y}}{1-R\theta}$  and it is independent of information parameter  $\gamma$ ; but an increase in risk premium (i.e. a decrease in parameter  $\theta$ ), just like an increase in the discounting interest rate, decreases human wealth.

This is illustrated in Figure 1, where the weights of future income realisations are presented with two alternative values of  $\gamma$  (1 and 0.9) and  $\theta$  (1 and 0.9). The discount factor  $R$  is assumed in all alternatives to equal 0.96, corresponding to the annual discount rate of 4 per cent. In calculating the benchmark weights (the solid line) only  $R$  deviates from unity. In calculating two alternative weighting schemes, parameters  $\gamma$  and  $\theta$  get in turn the value of 0.9. In both cases the weighting schemes turn to be more front-loaded than in the benchmark case. However, in the case where the information parameter  $\gamma=0.9$  the sum of weights (the area between the curve and the time axes) remains the same as in the benchmark, while in the case, where the risk premium parameter  $\theta=0.9$ , also the sum of weights (the area below the respective curve) is markedly smaller than in the benchmark case.

For the purposes of next section we rewrite the lifetime budget constraint (13) allowing the real interest rate to change. However, because the interest rate in the scaling factor is related to the weighted average of the real interest rate over the life-cycle we associate it with the long-term equilibrium real interest rate  $\bar{r}$  and, hence,  $\bar{R} = 1/(1+\bar{r})$ .





$$(14) \quad \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) c_{t+i} = V_{t-1} + \underbrace{\frac{1 - \bar{R}\gamma\theta}{1 - \bar{R}\theta} \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right)}_{H_t} (\theta\gamma)^i y_{t+i} = W_t$$

where  $H_t$  denotes human wealth conditional on information available at time  $t$ . When solving the intertemporal utility maximisation problem of the representative household subject to the flow budget constraint (3) and the resource constraint (14), a closed-form solution for the optimal consumption path is obtained. In next section we derive that relation first in the infinite horizon life cycle-permanent income framework and thereafter in the Blanchard (1985) overlapping generation framework. Both models are extended to include habit persistence.

### 3. The consumption function with habit persistence

In this section we derive forward looking consumption functions in infinite and finite horizon frameworks accounting for habit persistence. The habit formation implies non-separability in utility over time. In internal-habit models, habit depends on a household's own past consumption and the household takes

account of this when choosing how much to consume as e.g. in Muellbauer (1988), Muellbauer and Lattimore (1995), Sundaresan (1989) and Constantinides (1990).<sup>11</sup> The simplest treatment of habits is to replace the  $c_{t+i}$  argument in the utility function by  $c_{t+i}^* = c_{t+i} - ac_{t+i-1}$ , where  $c_{t+i}$  is consumption in period  $t+i$ , parameter  $a$  measures habit persistence with  $a > 0$  and the term  $ac_{t+i-1}$  is the time-varying habit level of consumption.

### 3.1. Aggregate consumption with Infinitely living households

Households are assumed to maximise the discounted utility of current and future consumption. With constant relative risk aversion the maximised function is:

$$(15) \quad \max U_t = \sum_{i=0}^{\infty} \left( \frac{1}{1+\rho} \right)^i \frac{\sigma}{\sigma-1} (c_{t+i}^*)^{\frac{\sigma-1}{\sigma}}$$

where  $\rho$  is the rate of subjective time preference and  $\sigma$  is the elasticity of intertemporal substitution. Following Muellbauer (1988) (see also Appendix 1), after substituting  $c_{t+i}^* + ac_{t+i-1}$  for  $c_{t+i}$ , the life-time resource constraint (14) can be written in the form:<sup>12</sup>

$$(16) \quad \sum_{i=0}^{\infty} \left( \prod_0^i R_{t-1}^{-1} R_{t+j-1} \right) c_{t+i}^* = (1 - a\bar{R})W_t - a \cdot c_{t-1}$$

As a first-order condition of the above maximisation problem defined by (15) and (16) we get:

$$(17) \quad \left( \frac{c_{t+i+1}^*}{c_{t+i}^*} \right)^{\frac{1}{\sigma}} = \frac{1}{(1+\rho)R_{t+i}}$$

After substituting (21) into (16) and solving for  $c_t$  we obtain:

$$(18) \quad c_t = \kappa(1 - a\bar{R})W_t + a(1 - \kappa)c_{t-1} \\ = \kappa[1 - \bar{R}a] \left[ V_{t-1} + \frac{1 - \bar{R}\theta\gamma}{1 - \bar{R}\theta} \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) (\theta\gamma)^i y_{t+i} \right] + a(1 - \kappa)c_{t-1}$$

<sup>11</sup> In external-habit models such as those in Abel (1990, 1996) and Campbell and Cochrane (1995), habit depends on aggregate consumption which is unaffected by any one agent's decisions.

<sup>12</sup> In deriving (16) the approximation  $\sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t+k-1}^{-1} R_{t+k+j-1} \right) a^i \approx \sum_0^{\infty} (a\bar{R})^i = \frac{1}{1 - a\bar{R}}$  for all integer  $k \geq 0$

where  $H_t$  is human wealth and parameter  $\kappa$  is defined as

$$(19) \quad \kappa = \frac{1}{\sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t+j}^{1-\sigma} \right) (1+\rho)^{-i\sigma}} \approx 1 - \bar{R}^{1-\sigma} (1+\rho)^{-\sigma} = 1 - \frac{(1+\bar{r})^{\sigma-1}}{(1+\rho)^{\sigma}}.$$

It is worth noting that, if the long-run average of the real interest rate is constant, then also parameter  $\kappa$  is not affected by the short run variations of the real interest rate. Further, under the alternative assumptions that the intertemporal rate of substitution  $\sigma=1$  or that, independently from the size of  $\sigma$ , the rate of subjective time preference equals to the equilibrium real interest rate, i.e.  $\rho = \bar{r}$ , equation (19) reduces to,

$$(19a) \quad \kappa = \rho / (1 + \rho).$$

It is worth noting that, except the problem associated with the fact that human wealth requires observations up to infinity, equation (18) is defined in terms of ex post observable variables. To eliminate the infinite sum we utilise the fact that our formulation for human wealth allows us to express current period human wealth in terms of current income and next period human wealth as follows,

$$(20) \quad H_t = \frac{1 - \bar{R}\theta\gamma}{1 - \bar{R}\theta} y_t + R_t \gamma \theta \frac{1 - \bar{R}\theta\gamma}{1 - \bar{R}\theta} \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t+j}^{-1} R_{t+j} \right) (\theta\gamma)^i y_{t+1+i} = \frac{1 - \bar{R}\theta\gamma}{1 - \bar{R}\theta} y_t + R_t \theta \gamma H_{t+1}$$

Now by adding also error terms capturing possible measurement errors and the fact that for period  $t$  planned and materialised consumption are not necessarily equal, we can write equations for current and next period consumption as follows,

$$(21a) \quad c_t = \kappa (1 - a\bar{R}) \left[ V_{t-1} + \frac{1 - \bar{R}\theta\gamma}{1 - \bar{R}\theta} y_t + R_t \theta \gamma H_{t+1} \right] + a(1 - \kappa)c_{t-1} + \varepsilon_t$$

$$(21b) \quad c_{t+1} = \kappa (1 - a\bar{R}) [V_t + H_{t+1}] + a(1 - \kappa)c_t + \varepsilon_{t+1}$$

After multiplying (21b) by the term  $R_t \theta \gamma$  and subtracting (21b) from (21a) and, thereafter, utilising (3) to eliminate current period financial wealth  $V_t$ , we end up with:

$$(22) \quad \left[ 1 + \gamma \theta (R_t a (1 - \kappa) - \kappa (1 - a\bar{R})) \right] c_t = R_t \gamma \theta c_{t+1} + a(1 - \kappa)c_{t-1} \\ + (1 - \bar{R}a)\kappa \left\{ (1 - \gamma \theta)(V_{t-1} + y_t) + \left( \frac{1 - \bar{R}\theta\gamma}{1 - \bar{R}\theta} - 1 \right) y_t \right\} + \varepsilon_t - R_t \gamma \theta \varepsilon_{t+1}$$

Equation (22) covers a wide range of alternative cases. In the case of no habit persistence and perfect information, i.e.  $a = 0$  and  $\theta = \gamma = 1$ , equation (22) reduces to the Euler condition, where current period

consumption is completely determined by next period consumption.<sup>13</sup> The habit persistence introduces lagged consumption into the relation. Further, if the anticipated stream of labour income is associated with risk, then through  $\theta < 1$ , financial wealth is introduced into (22). The introduction of wealth effect reflects the fact that the expected future income is discounted by higher than the market rate of interest and accounts for feed-back effects from precautionary saving. Finally, current income is also introduced into the relation, if the anticipated future income stream deviates from that of perfect foresight. Then also  $\gamma < 1$  reflecting e.g. imperfect information concerning future personal working career and family size and other factors affecting the future income profile. Simultaneously, along with the decrease in parameters  $\theta$  and  $\gamma$ , the weight of next period consumption is decreased in (22) reflecting imperfect information. In the limiting case where  $\gamma = 0$ , the lead of consumption disappears, reflecting the fact that no period-specific information on future incomes is available.

### 3.2. Aggregate consumption with finitely lived overlapping generations

Following Blanchard (1985), instead of being infinitely lived, agents face each period a constant probability of death  $\pi$ . Because of uncertain lifetime resulting from constant probability of death  $\pi$  there exist life insurance companies. Agents may contract to receive a payment  $(\pi/(1-\pi))V_{k,t-1}$  if they do not die, and pay  $V_{k,t-1}$  if they do die in the beginning of period  $t$ . Thus, preserving the same notation as above, except subscript  $k$  referring individuals born in period  $k$ , the dynamic budget constraint of an individual in the  $k^{\text{th}}$  cohort is,

$$(23) \quad V_{k,t} = R_t^{-1} \left( \frac{V_{k,t-1}}{1-\pi} + y_{k,t} - c_{k,t} \right)$$

As shown in Appendix 2 the lifetime resource constraint implied by (23) can be written as,

$$(24) \quad \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) (1-\pi)^i c_{k,t+i} = \\ \left( 1 - (1-\pi)a\bar{R} \right) \cdot \left[ \frac{V_{k,t-1}}{1-\pi} + \frac{1-(1-\pi)\bar{R}\gamma\theta}{1-(1-\pi)\bar{R}\theta} \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) (\gamma\theta)^i y_{k,t+i} \right] - ac_{k,t-1}$$

The certainty equivalent of the expected utility can be written as,

$$(25) \quad E_t[u_{k,t}] = E_t \left( \sum_{i=0}^{\infty} \left( \frac{1}{1+\rho} \right)^i \frac{\sigma}{\sigma-1} (c_{k,t+i}^*)^{\left( \frac{\sigma-1}{\sigma} \right)} \right)$$

---

<sup>13</sup> It is worth noting that in the Euler equation (17) future period consumption is defined in terms of information available at period  $t$ , whilst in equation (22) the lead of consumption refers to the actual realisation as implied by (21b), which is based on information available at period  $t+1$ . Hence, differences between the Euler equation (17) and the equation (22) reflect information differences.

$$= \sum_{i=0}^{\infty} \left( \frac{1-\pi}{1+\rho} \right)^i \frac{\sigma}{\sigma-1} (c_{k,t+i}^*)^{\frac{\sigma-1}{\sigma}}$$

Maximising (25) subject to the lifetime resource constraint (24) results in,

$$(26) \quad c_{k,t} = \kappa \left[ 1 - (1-\pi)\bar{R}a \right] \left[ \frac{V_{k,t-1}}{1-\pi} + \frac{1-(1-\pi)\bar{R}\theta\gamma}{1-(1-\pi)\bar{R}\theta} \sum_{i=-\infty}^t \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) \left( (1-\pi)\theta\gamma \right)^i y_{k,t+i} \right] + a(1-\kappa)c_{k,t-1}$$

where

$$(27) \quad \kappa = \frac{1}{\sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t+j}^{1-\sigma} \right) \left[ (1-\pi)(1+\rho)^{-\sigma} \right]^i}$$

$$\approx 1 - (1-\pi)\bar{R}^{1-\sigma} (1+\rho)^{-\sigma} = 1 - \frac{(1-\pi)(1+\bar{r})^{\sigma-1}}{(1+\rho)^{\sigma}}$$

Under the alternative assumptions that the rate of subjective time preference  $\rho$  equals to the equilibrium real interest rate  $\bar{r}$  or that the intertemporal rate of substitution  $\sigma = 1$ , relation (27) determining  $\kappa$  reduces to the form,

$$(27a) \quad \kappa = (\pi + \rho)/(1 + \rho).$$

Assume that the size of each cohort when born is  $\pi$ . Accordingly in period  $t$  the size of cohort born in period  $k$  is  $\pi(1-\pi)^{t-k}$  and the size of population is  $\sum_{k=-\infty}^t \pi(1-\pi)^{t-k} = 1$ . This results in the following aggregation rule:  $x_t = \sum_{k=-\infty}^t \pi(1-\pi)^{t-k} x_{k,t}$  and  $x_{t-1} = \sum_{k=-\infty}^{t-1} \pi(1-\pi)^{t-1-k} x_{k,t-1}$  with  $x = c, y$  and  $V$ . If we further assume that labour income is equally distributed across agents, i.e.  $y_{k,t+i} = y_{t+i}$ , after aggregation equation (26) (and adding the measurement error  $\varepsilon_t$ ) results in,

$$(28) \quad c_t = \kappa \left[ 1 - (1-\pi)\bar{R}a \right] \left[ \frac{V_{t-1}}{1-\pi} + \frac{1-(1-\pi)\bar{R}\theta\gamma}{1-(1-\pi)\bar{R}\theta} \sum_{i=-\infty}^t \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) \left( (1-\pi)\theta\gamma \right)^i y_{t+i} \right] + a(1-\kappa)c_{t-1} + \varepsilon_t$$

We follow the same steps as in previous section. We subtract from (28) its one period lead multiplied by the term  $R_t(1-\pi)\theta\gamma$  and utilise the aggregate level dynamic budget constraint  $V_t = (1+r_t)[V_{t-1} + y_t - c_t]$  implied by (23), as shown in Appendix 3. We end up with,

$$(29) \quad \left\{ 1 + \gamma\theta \left[ (1-\pi)(1-\kappa)aR_t - \kappa \left( 1 - (1-\pi)a\bar{R} \right) \right] \right\} c_t = (1-\pi)\gamma\theta R_t c_{t+1} + a(1-\kappa)c_{t-1}$$

$$+ (1-(1-\pi)\bar{R}a)\kappa \left\{ \left( \frac{1}{1-\pi} - \gamma\theta \right) (V_{t-1} + y_t) + \left( \frac{1-(1-\pi)\bar{R}\theta\gamma}{1-(1-\pi)\bar{R}\theta} - \frac{1}{1-\pi} \right) y_t \right\} + \varepsilon_t - R_t(1-\pi)\gamma\theta\varepsilon_{t+1}$$

It is straightforward to see that, in the case  $\pi = 0$ , equation (29) reduces to the infinite horizon equation (22).

## 4. Empirical results

In this section we first describe briefly the euro area data that we use in estimation and, thereafter, we present our estimation results.

### 4.1. Data

In our estimation we use the aggregated euro area quarterly data, i.e. the updated Fagan, Henry and Mestre (2001) data. Our sample covers the period 1970:1–2000:4. Consumption is aggregated private consumption and labour income is proxied by the sum of compensation to employees and transfers to households net of direct taxes. The real interest rate is measured alternatively in terms of the short-term interest rate and the long-term government bond rate. From theoretic point of view the use of short term interest rate is more preferable but as in empirical applications the long term interest rate it is widely used we also apply it as an alternative to the short term interest rate. In calculating real interest rates inflation is expressed in terms of the annualised one-quarter ahead consumption deflator inflation.

In defining the financial wealth, we assume that the private sector wealth is owned by households and, therefore, the private sector wealth is also the household sector wealth. The private sector wealth in nominal terms (FWN) can be calculated as a sum of the private sector nominal capital stock, government net debt and private the stock of net foreign assets corresponding the accumulated private sector savings. Nominal private capital stock is evaluated at repurchasing value and, hence, the deflator of private fixed investment is relevant price also for the capital stock.

### 4.2. Estimation results

We first estimate infinite horizon specification and then finite horizon overlapping generation model. Both cases are estimated without and with the assumption of habit formation. We also assume that subjective time preference and the long run real interest rate equal to 0.01 (corresponding 4% annual rate), which is quite conventional a priori assumption in empirical applications. Hence, as was shown in Sections 3.1 and 3.2, intertemporal substitution parameter disappears from estimated equation, which alleviates the estimation of other structural parameters of the model. Now (19a) implies that in the infinite horizon specification (22) the marginal propensity to consume out of wealth is  $\kappa = 0.01/1.01$ , i.e. it is predetermined. When annualised it correspond around 0.04, which is the range of typical

estimates obtained for the marginal propensity to consume out of wealth.<sup>14</sup> Equation (27a), in turn, implies that in the finite horizon specification (29) the marginal propensity parameter  $\kappa = (0.01 + \pi)/1.01$ , i.e. the shorter the life horizon the higher  $\kappa$ .

Expressing explicitly, estimated equations are as follows,

A) *Infinite horizon without habit formation*

$$\left(1 + \gamma\theta \frac{0.01}{1.01}\right) \frac{c_t}{y_t} c_t - \gamma\theta \frac{R_t c_{t+1}}{y_t} - \frac{0.01}{1.01} \left[ (1 - \gamma\theta) \left( \frac{V_{t-1}}{y_t} + 1 \right) + \left( \frac{1 - 0.99 \cdot \theta\gamma}{1 - 0.99 \cdot \theta} - 1 \right) \right] = u_t$$

where  $u_t = \varepsilon_t - R_t \gamma\theta \varepsilon_{t+1}$

B) *Infinite horizon with habit formation*

$$\left\{ 1 + \gamma\theta \left[ \left( 1 - \frac{0.01}{1.01} \right) a R_t - \frac{0.01}{1.01} (1 - 0.99 \cdot a) \right] \right\} \frac{c_t}{y_t} - \gamma\theta \frac{R_t c_{t+1}}{y_t} - a \left( 1 - \frac{0.01}{1.01} \right) \frac{c_{t-1}}{y_t} - (1 - 0.99a) \frac{0.01}{1.01} \left\{ (1 - \gamma\theta) \left( \frac{V_{t-1}}{y_t} + 1 \right) + \left( \frac{1 - 0.99 \cdot \theta\gamma}{1 - 0.99 \cdot \theta} - 1 \right) \right\} = u_t$$

where  $u_t = \varepsilon_t - R_t \gamma\theta \varepsilon_{t+1}$

C) *Finite horizon without habit formation*

$$\left[ 1 + \gamma\theta \left( \frac{0.01 + \pi}{1.01} \right) \right] \frac{c_t}{y_t} c_t - \gamma\theta (1 - \pi) \frac{R_t c_{t+1}}{y_t} - \left( \frac{0.01 + \pi}{1.01} \right) \left[ \left( \frac{1}{1 - \pi} - \gamma\theta \right) \left( \frac{V_{t-1}}{y_t} + 1 \right) + \left( \frac{1 - 0.99 \cdot \theta\gamma}{1 - 0.99 \cdot \theta} - \frac{1}{1 - \pi} \right) \right] = u_t$$

where  $u_t = \varepsilon_t - R_t (1 - \pi) \gamma\theta \varepsilon_{t+1}$

D) *Finite horizon with habit formation*

$$\left\{ 1 + \gamma\theta \left[ (1 - \pi) \left( 1 - \frac{0.01 + \pi}{1.01} \right) a R_t - \frac{0.01 + \pi}{1.01} (1 - 0.99 \cdot a (1 - \pi)) \right] \right\} \frac{c_t}{y_t} - (1 - \pi) \gamma\theta \frac{R_t c_{t+1}}{y_t} - a \left( 1 - \frac{0.01 + \pi}{1.01} \right) \frac{c_{t-1}}{y_t}$$

<sup>14</sup> For instance, recent estimations of the long-run relations between consumption, income and wealth with the US aggregate data indicate that the marginal propensity to consume of of wealth is in the range 0.03-0.08. More closely see Mehra (2001), Davis Palumbo (2001) and Palumbo, Rudd and Whelan (2002)

$$-[1 - 0.99(1 - \pi)a] \left( \frac{0.01 + \pi}{1.01} \right) \left\{ \left( \frac{1}{1 - \pi} - \gamma\theta \right) \left( \frac{V_{t-1}}{y_t} + 1 \right) + \left( \frac{1 - 0.99 \cdot (1 - \pi)\theta\gamma}{1 - 0.99 \cdot (1 - \pi)\theta} - \frac{1}{1 - \pi} \right) \right\} = u_t$$

where  $u_t = \varepsilon_t - R_t(1 - \pi)\gamma\theta\varepsilon_{t+1}$

In estimating we use the generalised method of moments (GMM), as described in Hansen (1982) and Hansen and Singleton (1982). Following the general practice in the GMM estimations, Hansen's J statistic of over-identifying restrictions together with associated p-values is used as main statistical criterion in evaluating how well the estimated model fits the observed data. To take into account serial correlation of residuals, the modified Bartlett weights proposed by Newey and West (1987) were used in calculating the weighting matrix of the minimised objective function.<sup>15</sup> Also standard errors, reported in brackets, are Newey-West corrected. As instruments, we used, in addition to constant, from two to four period lags of the dependent variable and the real interest rate as well as from one to four period lags of the wealth to income ratio.

Our estimation period covers the interval 1971:1 - 2000:3. The estimation results of infinite horizon specifications (equations A and B) are presented in Table 1 and the results of finite horizon specifications results (equations C and D) in Table 2. In both tables rows (1)-(2) present results, when the real interest rate is measured in terms of the short-term interest rate, and rows (3) - (4) present results, when the real interest rate is measured in terms of the long-term interest rate. We see that our estimation results are quite insensitive with respect to these alternative real interest rate measures.

As can be seen from Table 1, estimation results based on infinite horizon specifications quite uniformly imply that households possess a lot of information on future income changes. However, point estimates of information parameter  $\gamma$  are well below unity (around 0.8 without habit formation and around 0.85 with habit formation) deviating significantly from both zero and unity. These point estimates imply that period-specific information is front-loaded with the major part of information concentrating on the nearest 2 years. On the other hand, also the very marked deviation of  $\gamma$  from zero solves the puzzle of 'excess smoothness' of consumption (Deaton, 1987). That is because consumers have essentially more information on future income variation than what expectations based only on unit-root process for income would imply. Therefore, human wealth and, accordingly, consumption may vary less than labour income. Regarding the point estimates of risk premium parameter  $\theta$ , they are slightly below unity although their deviations from unity are statistically significant.

These point estimates imply that annualised risk premium, which is used in discounting expected future labour income, would be around 1-5%. These estimates are quite moderate and markedly below the estimate of risk premium used e.g. in the FRB/US model, where households are estimated to discount

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<sup>15</sup> To account for the possibility that also the measurement error  $\varepsilon_t$  is serially correlated, the lag length was allowed to be determined by data and the lag length of 5 quarters were ended up.



their expected future income at the rate of 25 per year (Brayton et. al, 1997).<sup>16</sup> The estimates are not sensitive with respect to habit formation hypothesis. However, habit formation hypothesis is strongly supported by the data. Estimates for the habit persistence parameter  $a$  are high, i.e. in the range of slightly below and slightly above 0.9 being well in line with estimates presented e.g. by Fuhrer (2000) for the US economy (0.8-0.9).

Table 2 present estimation results based on the finite horizon specifications. Parameter estimates of the probability of death are highly significant, supporting the finite horizon specification. The implied estimates for the expected remaining life-time are in the range 48-63 quarters (12-16 years) and, jointly with the equilibrium real interest rate, the estimates for the (annualised) marginal propensity to consume out of wealth are around 10-12%. Point estimates of information parameter are marginally higher, i.e. in the range of 0.84-0.88, than in the case of infinite horizon specification shown in Table 1. Point estimates of risk parameter  $\theta$  deviate now more from unity than in the case of infinite horizon, although the deviation from unity is not in every case statistically significant. Implied annualised risk premium estimates are now in the range of 9-18%, i.e. they are much closer to the estimate used in the FRB/US model. Estimates for the habit persistence parameter  $a$  are high (0.6-0.7) although smaller than in the case in the infinite horizon.

At least broadly taken these results can be thought reasonable although the probability of death and perhaps also risk premium estimates, although broadly in line with the FRB/US model, are somewhat on the high side of what one might expect on a priori bases. We can see that the estimates of the risk parameter  $\theta$ , and especially the standard errors, are affected most by the introduction of the parameter for the probability of death into estimated specifications compared to the infinite horizon results presented in Table 1. When in context of infinite horizon estimations the standard error of  $\theta$  was in the very narrow range of (0.00005-0.0007), in the context of finite horizon estimations the standard error rose to the range of (0.018-0.037). Therefore, as an experiment of sensitivity analysis, we present in Table 2 also estimation results (rows 1a, 2a, 3a and 4a), where the size of the risk parameter  $\theta$  is constrained by one standard error above corresponding freely estimated values. These constraints imply annualised risk premiums of 6.4-8 % in discounting the expected income stream, when the real interest rate is expressed in terms of the short-term interest rate and of around 12 %, when the real interest rate is expressed in terms of the long-term interest rate.

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<sup>16</sup> When coupled with the 4 per cent annual real interest this implies around 21 per cent risk premium in discounting the expected future income stream.

Table 1. Infinite horizon

	Parameters			Test <sup>17</sup>
	$\gamma$	$\theta$	$a$	J
$r_t$ = Short-term real interest rate				
(1)	0.814 (0.016)	0.987 (0.0002)		9.99 [0.351]
(2)	0.850 (0.046)	0.996 (0.0004)	0.868 (0.070)	11.12 [0.268]
$r_t$ = Long-term real interest rate				
(3)	0.787 (0.021)	0.996 (0.00005)		11.64 [0.235]
(4)	0.868 (0.06)	0.997 (0.0007)	0.924 (0.057)	11.71 [0.230]

<sup>17</sup> Estimation period is 1971:1-2000:3. Figures in square brackets are p-values.

Table 2. Finite horizon

	Parameters				Expected remaining life-time	Test <sup>18</sup> J
	$\gamma$	$\theta$	$\pi$	$a$		
$r_t =$ Short-term real interest rate						
(1)	0.845 (0.027)	0.966 (0.018)	0.016 (0.005)		62.5 (18.7)	11.6 [0.171]
(1a)	0.818 (0.016)	0.984 n.a.	0.009 (0.0009)		105.5 (1.0)	10.2 [0.331]
(2)	0.876 (0.056)	0.943 (0.037)	0.021 (0.006)	0.710 (0.086)	48.3 (13.7)	8.09 [0.424]
(2a)	0.835 (0.030)	0.980 n.a.	0.008 (0.0002)	0.0.759 (0.006)	129.7 (0.006)	0.021 [0.292]
$r_t =$ Long-term real interest rate						
(3)	0.835 (0.037)	0.947 (0.023)	0.020 (0.004)		50.3 (10.0)	9.64 [0.291]
(3a)	0.801 (0.017)	0.970 n.a.	0.015 (0.00007)		68.1 (0.3)	8.55 (0.479)
(4)	0.851 (0.045)	0.941 (0.027)	0.021 (0.004)	0.565 (0.115)	47.5 (9.3)	8.75 [0.364]
(4a)	0.821 (0.027)	0.968 n.a.	0.012 (0.0002)	0.703 (0.111)	84.6 (1.2)	11.5 [0.243]

<sup>18</sup> Estimation period is 1971:1-2000:3. Figures in square brackets are p-values.

Estimates of the information parameter  $\gamma$  are not strongly affected by the imposed constraint. They are in the range of 0.8 - 0.835, whilst the estimates for the probability of death are roughly halved. Accordingly the implied estimates for the expected remaining lifetime are almost doubled compared to the corresponding unconstrained estimates. They are in the ranges of 26-32 years and 17-22 years, respectively, when the real interest rate is expressed alternatively in terms of the short-term or the long-term interest rate. Especially the former estimates can be thought very reasonable. Correspondingly the implied (annualised) marginal propensities to consume out of wealth are also, especially in the former case, in the very reasonable range of 7.2-7.8 %. Again the estimates of the habit persistence parameter  $a$  are quite high (0.7-0.8) and highly significant.

## 5. Conclusions

In this paper we have derived and estimated a closed form approximation of the consumption function derived from the households' utility maximisation problem. The novel element in our approach is the parameterisation of the information content regarding future income changes. Supplemented with a risk premium parameter associated with stochastic variation of the future income stream, we were able to present an approximation of the life-time budget constraint, where uncertainty and imperfect information regarding future income changes are reduced to two parameters. This allowed us to solve an intrinsically stochastic optimisation problem of the household using standard techniques of deterministic optimisation. Accounting for "habit formation" in the utility function we ended up with the consumption function, which allows quite general dynamics for consumption. According to the derived consumption function, current consumption depends on one period lead and lag of consumption as well as on fundamentals, i.e. on real financial wealth, current period real labour income and the real interest rate. We estimated the deep parameters of the model based on the assumption that the rate of subjective time preference is assumed to equal the long-run equilibrium real interest rate. In the infinite horizon specification this assumption fixed the marginal propensity to consume out of wealth to the assumed equilibrium real interest rate, but in the finite horizon specification the marginal propensity also depended on the estimated parameter for the probability of death.

All estimation alternatives fitted the euro area data well and gave reasonable estimates for deep parameters. Estimation results based on both infinite and finite horizon specifications are reasonable, although the finite horizon specification, encompassing the infinite horizon specification, is favoured by the data. Our estimation results also support strongly the view that consumers have much more information regarding future labour income than would be implied by a simple random-walk model. Accordingly, consumption behaviour is forward-looking, although the deviation from the perfect foresight solution is marked. Habit formation also plays an important role, implying that consumers have a motive to smooth changes in consumption as well as the level of consumption.

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## APPENDIX 1. Lifetime resource constraint under habit persistence

Define period  $t$  consumption as

$$(A.1.1) \quad c_t = c_t^* + ac_{t-1}$$

and lifetime budget constraint

$$(A.1.2) \quad \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) c_{t+i} = V_{t-1} + \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) y_{t+i}$$

(A.1.1) implies that

$$\begin{aligned} (A.1.3) \quad \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) c_{t+i} &= c_t^* + ac_{t-1} \\ &+ (c_{t+1}^* + ac_t^* + a^2 c_{t-1}^*) R_t \\ &+ (c_{t+2}^* + ac_{t+1}^* + a^2 c_t^* + a^3 c_{t-1}^*) R_t R_{t+1} \\ &+ \dots + (c_{t+k}^* + ac_{t+k-1}^* + \dots + a^k c_t^* + a^{k+1} c_{t-1}^*) R_t R_{t+1} \dots R_{t+k} + \dots \\ &= \left( ac_{t-1} + \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) c_{t+i}^* \right) \sum_{i=0}^{\infty} a^i \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \\ &\approx \left( ac_{t-1} + \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) c_{t+i}^* \right) \frac{1}{1-a\bar{R}} \end{aligned}$$

Hence (A.1.2) can be written in the form,

$$(A.1.4) \quad \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) c_{t+i}^* = (1-a\bar{R}) \left[ V_{t-1} + \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) y_{t+i} \right] - ac_{t-1}$$



**APPENDIX 2. The derivation of the lifetime resource constraint in the overlapping generation framework**

Let  $c_{k,t}$  denote the level of consumption,  $y_{k,t}$  the level of income and  $V_{k,t}$  the level of financial wealth in period  $t$  of an individual born in period  $k$ . Because of uncertain lifetime resulting from constant probability of death  $\pi$  there exist life insurance companies. Agents may contract to receive a payment  $(\pi/(1-\pi))V_{k,t-1}$  if they do not die and pay  $V_{k,t-1}$  if they do die in the beginning of period  $t$ . As in the main text let  $z_{k,t+i}$  denote risk adjusted expected income, (5) implies

$$(A.2.1) \quad z_{k,t+i} = \theta^i \left( y_{k,t} + \sum_{j=1}^i \gamma^j \Delta y_{k,t+j} \right)$$

and the flow budget constraint of the optimising consumer is

$$(A.2.2) \quad V_{k,t+i} = R_{t+i}^{-1} \left[ \frac{V_{k,t+i-1}}{1-\pi} + z_{k,t+i} - c_{k,t+i} \right]$$

Equation (A.2.2) implies the following lifetime budget constraint,

$$(A.2.3) \quad \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) (1-\pi)^i c_{k,t+i} = \frac{V_{k,t-1}}{1-\pi} + \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) (1-\pi)^i z_{k,t+i}$$

$$= \frac{V_{k,t-1}}{1-\pi} + \frac{1-(1-\pi)\bar{R}\gamma\theta}{1-(1-\pi)\bar{R}\theta} \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) (\gamma\theta)^i y_{k,t+i}$$

On the bases of Appendix 1, accounting for habit formation  $c_{k,t} = c_{k,t}^* + ac_{k,t-1}$ , resource constraint

(A.2.3) can be written in the form,

$$(A.2.4) \quad \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) (1-\pi)^i c_{k,t+i}^* =$$

$$(1-(1-\pi)a\bar{R}) \cdot \left[ \frac{V_{k,t-1}}{1-\pi} + \frac{1-(1-\pi)\bar{R}\gamma\theta}{1-(1-\pi)\bar{R}\theta} \sum_{i=0}^{\infty} \left( \prod_{j=0}^i R_{t-1}^{-1} R_{t+j-1} \right) (\gamma\theta)^i y_{k,t+i} \right] - ac_{k,t-1}$$

### APPENDIX 3. Aggregation of the dynamic budget constraint

Assume that the size of each cohort when born is  $\pi$ . Accordingly in period  $t$  the size of cohort born in period  $k$  is  $\pi(1-\pi)^{t-k}$  and the size of population is  $\sum_{k=-\infty}^t \pi(1-\pi)^{t-k} = 1$ . This results in the following aggregation rules with  $x=(c, y, V)$ :

$$(A.3.1) \quad x_t = \sum_{k=-\infty}^t \pi(1-\pi)^{t-k} x_{k,t}$$

$$(A.3.2) \quad x_{t-1} = \sum_{k=-\infty}^{t-1} \pi(1-\pi)^{t-1-k} x_{k,t-1}.$$

Hence,

$$\begin{aligned} (A.3.3) \quad V_t - V_{t-1} &= \sum_{k=-\infty}^t \pi(1-\pi)^{t-k} V_{k,t} - \sum_{k=-\infty}^{t-1} \pi(1-\pi)^{t-1-k} V_{k,t-1} \\ &= \sum_{k=-\infty}^t \pi(1-\pi)^{t-k} (1+r_t) \left[ \frac{V_{k,t-1}}{1-\pi} + y_{k,t} - c_{k,t} \right] - \sum_{k=-\infty}^{t-1} \pi(1-\pi)^{t-1-k} V_{k,t-1} \\ &= (1+r_t) \sum_{k=-\infty}^t \pi(1-\pi)^{t-k} [y_{k,t} - c_{k,t}] + \frac{\pi}{1-\pi} \underbrace{V_{t,t-1}}_{=0} + r_t \sum_{k=-\infty}^{t-1} \pi(1-\pi)^{t-1-k} V_{k,t-1} \\ &= r_t V_{t-1} + (1+r_t)(y_t - c_t) \end{aligned}$$

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