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## Can adjustment costs explain the variability and counter-cyclical of the labour share at the firm and aggregate level?

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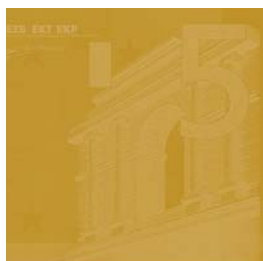
**CAN ADJUSTMENT  
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OF THE LABOUR SHARE  
AT THE FIRM AND  
AGGREGATE LEVEL?**

by Philip Vermeulen



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## WORKING PAPER SERIES

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# CAN ADJUSTMENT COSTS EXPLAIN THE VARIABILITY AND COUNTER-CYCLICALITY OF THE LABOUR SHARE AT THE FIRM AND AGGREGATE LEVEL? <sup>1</sup>

by Philip Vermeulen <sup>2</sup>

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<sup>1</sup> The opinions reflected in the paper are entirely my own, they do not necessarily reflect those of the ECB. I thank Klaus Adam, Gabriel Fagan, Giovanni Lombardo and Frank Smets for useful comments. Any remaining errors are my own.

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## Abstract

This paper shows that adjustment costs modelled as firing costs of moderate size go a long way in explaining the variability and counter-cyclicality of the labour share at the firm and aggregate level. Firing costs cause firms to fire less in recessions and hire less in booms causing wage costs to fluctuate less cyclically than output, thus inducing variability and countercyclicality in the labour share. The paper develops a dynamic labour demand model with firing costs. The model is then calibrated using moments derived from 1634 French manufacturing firms and aggregate French manufacturing data. The calibrated model is able to closely match the variability and counter-cyclicality of the labour share at the firm level while it also generates a countercyclical aggregate labour share with a variability 60 % of that in French aggregate manufacturing.

*Keywords:* labour share, labor adjustment costs, firing costs, real business cycles

*JEL classification:* D21, E25.

## Non Technical Summary

This paper studies the role of adjustment costs as a mechanism that causes variability and fluctuations of the labour share (i.e. the share of labour costs in output). The labour costs (or labour income) as a share of GDP is countercyclical in many countries (i.e. labour costs as a share of GDP go down in booms and up in recessions). Adjustment costs, in this paper modelled as firing costs form potentially an explanation. Firing costs cause firms to fire less in recessions and hire less in booms causing wage costs to fluctuate less cyclically than output, thus inducing variability and countercyclicity in the labour share. Standard economic models (the so called real business cycle models) however do not mimic that fact. They rather stress the long-run stylized fact of growth that the labour share is constant in the long-run. A few authors have augmented real business cycle models with mechanisms to create the countercyclical movements in the labour share that are observed in the data. This paper uses firing costs to create realistic counter-cyclical fluctuations both at the firm and aggregate level. The emphasis of this paper is on the simultaneous explanation of labour share movements at the firm and aggregate level and testing it with real data. I first model the dynamic labour demand of an individual monopolistic competitive firm with linear firing costs. The aggregate labour share is obtained by simply aggregating the behaviour of many firms. The parameters of the model are then calibrated so that the behavior of the model matches the average behavior of 1634 French manufacturing firms and aggregate French manufacturing data. Firing costs of around 3 months of an average yearly wage go a long way in explaining the variability and counter-cyclicity of the labour share at the firm and aggregate level. The calibrated model is able to closely match the variability and counter-cyclicity of the labour share at the firm level while it also generates a countercyclical aggregate labour share with a variability 60 % of that in French aggregate manufacturing.

# 1 Introduction

This paper studies the role adjustment costs as a mechanism that causes variability and counter-cyclical fluctuations of the labour share (i.e. the share of labour costs in output) simultaneously at the firm and aggregate level. Adjustment costs are a standard modelling device used simultaneously in the micro-econometrics literature and the macro literature to generate real world like fluctuations. Usually though, the macro literature does not check formally if the macro fluctuations are generated by plausible fluctuations at the micro level and vice versa the micro literature seldomly checks whether micro fluctuations are relevant for aggregate ones. I model adjustment costs as firing costs and show that firing costs at the firm level of a moderate amount (around 3 months of pay) are sufficient to explain the variability and countercyclicality of the labour share of the average firm whereas simultaneously the counter-cyclical movements of the labour share at the aggregate level is explained. Firing costs however only explain 60% of the variability of the aggregate labour share. The paper demonstrates the above using a large panel of 1634 French manufacturing firms jointly with aggregate manufacturing data. A dynamic labour demand model with firing costs is calibrated so that its moments match both firm level moments calculated from the panel and moments calculated from aggregate French manufacturing.

The motivation of this paper extends beyond the French manufacturing sector. The labour income as a share of GDP is countercyclical in many countries. Kydland and Prescott (1990) report a variability of 0.47 (% standard deviation) and a negative correlation of the labour share with real GNP for the U.S. of -0.39 over the period 1954-1989. Boldrin and Horvath (1995) report a correlation of -0.32 for the longer period 1947-1990. The countercyclicality extends to the OECD countries and is even more pronounced in most countries compared to the U.S. Gomme and Greenwood (1995) report a correlation of -0.85 for Japan and -0.72 for France for the period 1971-1989. Standard real business cycle models however do not mimic that fact. They rather stress the so called Kaldor long-run stylized fact of growth that the labour share is constant in the long-run. Under Cobb-Douglas assumptions, absence of adjustment costs, and competitive markets, the labour share is indeed constant both in long and short run.

A few authors have augmented real business cycle models with mechanisms to create the countercyclical movements in the labour share that are



observed in the data. Boldrin and Horvath (1995) and Gomme and Greenwood (1995) include employment contracting so that the equilibrium wage is not any longer set as in the competitive case. Wages in productive states are below the marginal productivity of labour whereas they are above the marginal productivity in low productivity states. This induces a negative correlation between the labour costs and output and hence a countercyclical labour share. Hansen and Prescott (2005) develop an RBC model with capital that is occasionally binding. The occasional binding causes it to earn rents in boom periods. Therefore in their model the labour share also fluctuates countercyclically as rents crowd out labour earnings.

This paper uses firing costs to create realistic counter-cyclical fluctuations both at the firm and aggregate level. The firing costs are meant to summarise all the adjustment costs, both of a monetary and non-monetary nature, i.e. not just legal severance payments. The emphasis of this paper is on the simultaneous explanation of labour share movements at the firm and aggregate level and testing it with real data. The key research question is really how far simple firing cost can go in explaining the variability and counter-cyclicity of the labour share at both aggregate and micro level. It turns out, pretty far. Labour demand is modelled in a partial equilibrium framework. I first model the dynamic labour demand of an individual monopolistic competitive firm with linear firing costs. The model is a modified version of the discrete time dynamic labour demand model of Bertola (1990). The modelling device of firing costs is used as a proxy for total adjustment costs. The production of the firm is affected both by an aggregate and an idiosyncratic productivity shock. The firing costs affects the firms labour demand in reaction to the productivity shocks. An increase in firing costs, reduces the employment reaction to productivity shocks. Thus, because of reduced hiring and firing, firing costs cause wage costs to fluctuate less than output, thus inducing countercyclicity in the labour share of the firm. I then model aggregate manufacturing labour demand as the simple aggregation of many individual firms labour demand, each firm facing the same aggregate productivity shock but uncorrelated (across firms) idiosyncratic productivity shocks. Finally, the parameters of the model of the individual firm are calibrated so that the average behaviour of simulated individual firms and their aggregation matches respectively the observed average firm level as well as the observed aggregate manufacturing level dynamics of employment, output and labour share. More precisely, the calibration uses on the one hand the average firm level of the standard deviation of employment growth, the

standard deviation of output growth, the standard deviation of labour share changes and the correlation of output with the labour share calculated from a panel of 1634 French manufacturing firms and on the other hand it uses the same moments calculated from aggregate French manufacturing data. The data moments are matched with the moments derived from simulation using a minimum distance criterion.

A calibration of the firm level adjustment cost of around 3 months of pay results in a close matching of both individual firm moments and aggregate moments. The counter-cyclical of the labour share both at the firm and aggregate level can plausibly be caused by the presence of moderate adjustment costs. In addition, the variability of the labour share at the firm level in the model matches closely with that in the data. However, firing costs generate a variability of the aggregate labour share that is only 60% of that in the data. Therefore one should conclude that other mechanisms add to the variability of the aggregate labour share. Although labour demand is modeled here in a partial equilibrium framework, the results suggest that adding firing costs in a general equilibrium framework might prove useful in matching the counter-cyclical labour share.

The paper is also related to the work that explains European labour dynamics with labour adjustment costs. Labour adjustment cost have been found to play a role in European employment dynamics. For instance Bentalila and Bertola (1990) show that high firing costs can rationalize the dynamic behaviour of European employment in the seventies and eighties. The presence of labour adjustment costs has implications for business cycle models based on New Keynesian inflation theories as well. If adjustment costs are present, the representation of the so-called New Keynesian Phillips curve (NKPC) is affected. Sbordone (2005) has argued that a full understanding of the Phillips curve can only be reached through an understanding of the dynamics of labor costs, and how these relate to output dynamics. This paper should help therefore in shedding light onto this relationship. For instance, Batini et al. (2005) assume quadratic labour adjustment costs and show that the New Keynesian Phillips curve is augmented with employment terms. They find that for the UK adjustment costs are important in estimating the Phillips curve and hence in the determination of inflation.

The rest of the paper is structured as follows. In section two, a brief overview of the literature that concerns itself with the dynamics of the labour share is given, in section three the data is described, in section four a theoretical model of dynamic labour demand is developed. Sections five and six provide the simulation and calibration results. Section seven concludes.

## 2 Labour share dynamics: the literature

There is sufficient available empirical evidence to conclude that the labour share, in OECD countries, is countercyclical.<sup>1</sup> Kydland and Prescott (1990) report a negative correlation of the labour share with real GNP for the U.S. of -0.39 for the period 1954-1989. Gomme and Greenwood (1995) report a correlation of -0.37 for the same period. Boldrin and Horvath (1995) report a correlation of -0.32 for the period 1947-1990. For the U.S. further evidence of the countercyclicality can be found in Rotemberg and Woodford (1999). Gomme and Greenwood (1995) and Giammarioli et al. (2002) also extend the evidence to OECD countries.

Traditionally the labour share does not play an important role in the real business cycle literature. The counter-cyclicality is usually not one of the features RBC models are trying to match. The standard real business cycle model has a constant labour share. This stems from Cobb-Douglas technology, competitive markets and the absence of adjustment costs. It is therefore not surprising that most of the real business cycle literature does not even mention the labour share movements as an important feature of the business cycle to be matched. For instance the canonical labour market behaviour RBC model of Hansen (1985) does not consider labour share movements.

Only a few authors have augmented the real business cycle model with mechanisms to create movements in the labour share that are observed in the data. Boldrin and Horvath (1995) and Gomme and Greenwood (1995) include employment contracting so that the equilibrium wage is not any longer set as in the competitive case.

In Boldrin and Horvath (1995) and Gomme and Greenwood (1995) wage contracts have an insurance component so that wages in productive states are below the marginal productivity of labour whereas they are above the marginal productivity in low productivity states. This induces a negative

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<sup>1</sup>The emphasis of this section is on cyclical movement of the labour share. Macroeconomists have also investigated long and medium run movements in the labour share in conjunction with explaining long and medium trends in unemployment and the capital-output ratio: e.g. Caballerro and Hamour (1998) and Blanchard (1997).

correlation between the labour costs and output. Where both papers do a good job in matching the countercyclicality, both papers have however difficulty in matching the variability of the labour share when calibrating it on quarterly U.S. data. In the case of Boldrin and Horvath the calibrated model shows too low a standard deviation of the labour share (a standard deviation of 0.50 versus 1.08 in the data) whereas in Gomme and Greenwood, the calibrated model shows too high a variance of the labour share (a standard deviation between 1.31 and 1.63 versus 0.80 in the data). This seems to imply that additional mechanisms are needed to match the variability of the labour share.

Hansen and Prescott (2005) develop a RBC model with two types of capital. One type of capital is "capacity" that occasionally binds. The occasional binding causes it to earn rents in boom periods. Therefore in their model the labour share also fluctuates countercyclically as rents crowd out labour earnings. They conduct three experiments, where in all of them also the model shows too low a variability of the labour share movements with respect to the data.

The labour share also plays an important role in the new-Keynesian literature. The canonical New Keynesian Phillips curve derived in this literature posits a relationship between inflation, expected inflation and real marginal costs. When the economy operates under a Cobb-Douglas production function, real marginal costs are identical (up to a multiplicative factor) to the labour share (see e.g. Gali and Gertler, 1999). A number of authors have used the labour share in estimating a New Keynesian Phillips curve (see e.g. Gali and Gertler (1999), Batini et al. (2000,2005), Woodford (2001), Gali et al. (2001), Sbordone (2002,2005), Mcadam and Willman (2004)). Rotemberg and Woodford (1999) argue that adjustment costs of labour (and other frictions) should be taken into account. Rudd and Whelan (2005) have argued that the counter-cyclicality of the labour share poses serious problems for policy makers using New Keynesian Phillips curves. They state: "*Because the labor share has spiked upward in every modern U.S. recession, a policy rule based on this variable would have called for higher interest rates during each of these episodes*". Batini et al (2000) assume quadratic labour adjustment costs and variable markups so that the New Keynesian Phillips curve contains additional terms in the elasticity of demand and the adjustment cost parameter. Important is that the presence of adjustment costs alters the theoretical Phillips curve relationship.

However, clearly additional research is needed to understand the frictions that make the labour share move the way it does. These frictions have to be found at the micro level and simultaneously not vanish upon aggregation. This paper attempts to provide insight in labour adjustment costs at the firm as a potential plausible cause of labour share movements at the aggregate level. However, very few papers have investigated this so far. Giannarioli et al. (2002) document the counter-cyclical of the labour share for Europe and the US. They link this to institutional factors such as firing costs but do not go further than showing that some aggregate measures of labour market rigidity are correlated with the counter-cyclical of the labour share. Some others have focused not on aggregate but industry level labour share. Bentolila and Saint-Paul (2003) dissect the movement in the labour share at the industry level for 12 countries stemming from movements in the capital-labour ratio (which itself moves due to changes in relative wage-rental prices), and factors such as non-labour embodied technical change, price of imports, changes in markups, labour adjustment costs and changes in bargaining power. They however don't measure labour adjustment costs directly but proxy them by the growth rate of employment (low growth rates proxy high adjustment costs).

On a theoretical level, Kessing (2003) derives two interesting neutrality results. First, in a stochastic labour demand model with linear adjustment costs and Cobb-Douglas technology, he shows that labour share movements depend only on the size of the adjustment costs, but are independent on the size of demand or wage shocks. He further shows that the labour share at the firm level is unambiguously counter-cyclical in the presence of linear adjustment costs.

### 3 Data

When investigating the labour share at the sector or firm level it is more natural to focus on the labour share as a fraction of total output rather than value added. Total output at the firm level can be proxied by sales, whereas at the aggregate level output data is readily available. Figure 1 depicts the labour share (as a fraction of output) for the manufacturing sector (compensation of employees divided by output) of France over the period 1978-2004 (using yearly data).<sup>2</sup> The labour share fluctuates around

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<sup>2</sup>The labour share is constructed using data from the French national statistical office (INSEE) for the manufacturing sector (exclusive of energy). The total nominal wage bill



0.205 in the eighties and early nineties, then drops in the late nineties to start fluctuating around a lower level around 0.18. Of more interest than the level are the movements of the labour share over the business cycle. The movements of the labour share are depicted in Figure 2 together with output growth in the manufacturing sector (both calculated as log differences). Over this period the correlation of output growth and labour share movements is -0.82. This correlation is in line with the aggregate number of -0.72 for France reported by Gomme and Greenwood (1995) for the period 1971-1989 based on quarterly data. Figure 2 clearly demonstrates that when output growth is strong, the labour share falls and vice versa .

The paper exploits yearly data on the labour share, employment and sales growth of 1634 French manufacturing firms. The firms are observed over the period 1995-2003, i.e. 9 years. The employment, wage bill and sales data is obtained from annual accounts data (AMADEUS database). The data are trimmed to remove outliers (see the appendix for the details about trimming.) Also over this smaller period of nine years of data the labour share was counter-cyclical in the 1634 firms. Figure 3 depicts the labour share changes and sales growth for all 1634 firms individually. Again the counter-cyclicality is quite clear. The correlation (of the 1634 firms) of labour share changes with sales growth is -0.65. There is a clear negative relationship indicating counter-cyclical behaviour of the labour share at both the aggregate and at the firm level.

## 4 A model of firing costs and the labour share

### 4.1 The model

A discrete time stochastic labour demand model with linear firing costs is developed in which employment adjusts in reaction to productivity shocks. It is a modified version of the model by Bertola (1990).<sup>3</sup> Consider a repre-

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is divided by nominal output. The nominal wage bill is constructed from INSEE Table 2.212: Rémunération des salariés par branche (DB industrie minus EG Energy). Total nominal output is given in INSEE Table 2.101 Production par branche à prix courants (DB industrie minus EG Energy)

<sup>3</sup>In Bertola (1990), the revenue function is shocked by a two state Markov proces, here in this paper, production is shocked by two continuous shocks that form an AR(1) process.

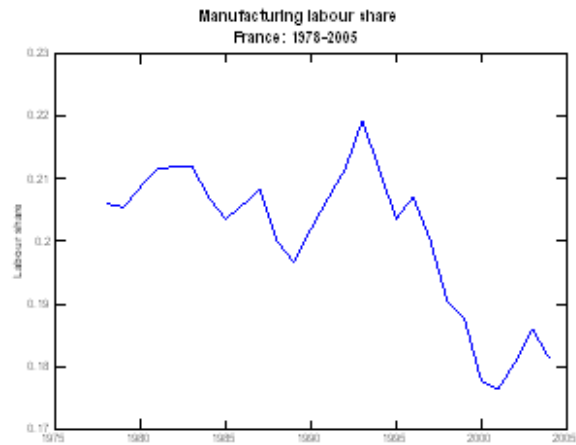


Figure 1: Labour share French Manufacturing 1978-2005



Figure 2: Labour share movements (dotted) and output growth (solid)

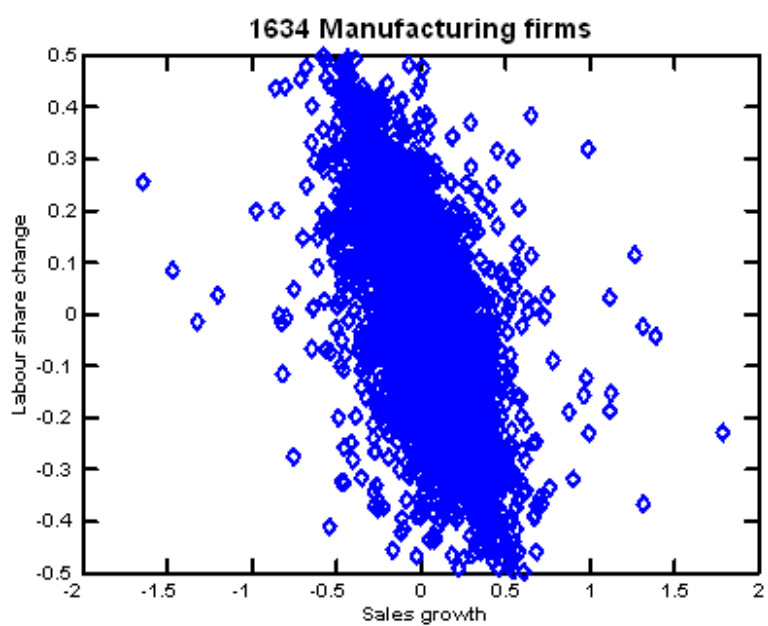


Figure 3: Labour share change and sales growth: France 1995-2003, 1634 Manufacturing firms



sentative imperfectly competitive firm with costly employment adjustment. The firm's real output is given by the following production function:

$$Y_{it} \equiv Y(a_{it}, L_{it}) = Ae^{a_{it}} K^{1-\alpha} L_{it}^{\alpha} \quad (1)$$

where  $Y_{it}$  is real output,  $A$  is the productivity level of the firm and  $a_{it}$  a productivity shock and  $K$  is a fixed level of all other inputs (capital and intermediate inputs). The productivity shock is exogenous and follows an AR(1) process<sup>4</sup>

$$a_{it} = \rho_a a_{it-1} + \eta_t + \epsilon_{it}, \quad (2)$$

Productivity is both affected by aggregate shock  $\eta_t$  (i.e. the shock is identical across firms and uncorrelated across time) and idiosyncratic shock  $\epsilon_{it}$  (i.e. the shock is uncorrelated with all other firms and uncorrelated across time). Both  $\epsilon_{it}$  and  $\eta_t$  are distributed normal with mean zero and variance  $\sigma_{\epsilon}^2$  and  $\sigma_{\eta}^2$  respectively.

The firm can adjust its number of employees  $L_{it}$  with a linear cost of firing  $Fp_{ot} > 0$ :

$$C(L_{it-1}, L_{it}) = I(L_{it} - L_{it-1})Fp_{ot}(L_{it-1} - L_{it}) \quad (3)$$

with the indicator function  $I(L_{it} - L_{it-1}) = 1$  in case of firing, i.e. if  $L_{it} < L_{it-1}$  and  $I(L_{it} - L_{it-1}) = 0$  otherwise and  $p_{ot}$  is the aggregate price level. In the model here the firing cost will be assumed to belong to the wage costs and therefore directly influence the labour share.

Demand for the firms output is given by the isoelastic demand function.

$$p_{it} = Y_{it}^{\xi-1} C p_{ot} \quad (4)$$

with constant  $C$ ,  $0 < \xi < 1$ . The price elasticity of demand is equal to  $\frac{1}{\xi-1}$ .

So output times price is given by  $p_{it} Y_{it} = Y_{it}^{\xi-1} C p_{ot} Y_{it} = C p_{ot} (Y_{it})^{\xi}$ . Nom-

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<sup>4</sup>Linear labour adjustment cost models are not uncommon in the labour literature to explain labour adjustment at the firm level. For instance, Bertola (1990) analyzes a Markov shock that can take two values, Bentolila and Saint Paul (1994) analyze a related labour demand model with a quadratic revenue function and serially uncorrelated shocks.

inal Profits<sup>5</sup> at time  $t$  are given by

$$\Pi_{it} = Cp_{ot}(Ae^{a_{it}}K^{1-\alpha}L_{it}^\alpha)^\xi - Wp_{ot}L_{it} - mp_{ot}K - C(L_{it-1}, L_{it}) \quad (5)$$

where it is assumed that wages and intermediate input prices all move with the aggregate price level. This last assumption implies constant real wages. This is a simplifying assumption that has three benefits. First, the purpose of the model is to focus on adjustment costs as driver of the dynamics of employment implying a countercyclical labour share, not on employment movements due to wage/ rental prices. By keeping real wages fixed, employment dynamics are solely caused by the interaction of productivity shocks and adjustment costs. Second, the assumption is not that far from the truth as generally, one of the stylized facts of real business cycle theory is that real wages are much less volatile than output and have only a small correlation with output. Third the state variables of the model will not include the wage which makes solving and simulating the model much less involved.

The firm maximizes the present value of the future flow of real profits  $\Pi_{it}/p_{0t}$  discounted with real discount factor  $\beta$ . The value function of the firm can be written as a function of employment and the productivity shock only.

$$V(a_{it}, L_{it-1}) = \max_{\{L_{it}\}} C(AK^{1-\alpha})^\xi (e^{a_{it}})^\xi L_{it}^{\alpha\xi} - WL_{it} - mK \quad (6)$$

$$-I(L_{it} - L_{it-1})F(L_{it-1} - L_{it}) + \beta E_{a_{it+1}|a_{it}} V(a_{it+1}, L_{it}) \quad (7)$$

Note that the value function is a function of the productivity shock  $a_{it}$  and not of the firm idiosyncratic  $\epsilon_{it}$  and aggregate shock  $\eta_t$  separately. The solution to the above value function  $V(a_{it}, L_{it-1})$  is identical to the solution of the value function  $V^*(a_{it}^*, L_{it-1})$  below where profits are normalized by the real wage  $W$  and the constant  $mK$  is removed from the problem (since  $mK$  is constant it doesn't influence the optimal solution and can be dropped):

$$V^*(a_{it}^*, L_{it-1}) = \max_{\{L_{it}\}} A^*(e^{a_{it}^*})L_{it}^{\alpha^*} - L_{it} \quad (8)$$

$$-I(L_{it} - L_{it-1})F^*(L_{it-1} - L_{it}) + \beta E_{a_{it+1}|a_{it}} V^*(a_{it+1}^*, L_{it}) \quad (9)$$

<sup>5</sup>Instead of productivity shocks one could alternatively model demand shocks leading to an identical profit function.

$$a_{it}^* = \rho_a^* a_{it-1}^* + \epsilon_{it}^* + \eta_t^*, \quad (10)$$

with  $A^* = C(AK^{1-\alpha})^\xi / W$ ,  $\alpha^* = \alpha\xi$ ,  $F^* = F/W$  and  $(e^{a_{it}})^\xi = e^{a_{it}^*}$ , so that the transformed productivity shock  $a_{it}^*$  ( $= a_{it}\xi$ ) has parameters  $\rho_a^* = \xi\rho_a$  and  $\sigma_\epsilon^* = \xi\sigma_\epsilon$  and  $\sigma_\eta^* = \xi\sigma_\eta$ ,

Adjustment costs  $F^*$  are measured in terms of yearly real wages.

The value function  $V^*(a_{it}^*, L_{it-1})$  has two state variables, current period productivity shock  $a_{it}^*$  and last period employment  $L_{it-1}$ . Employment this period  $L_{it}$  is the only control variable. The value function depends on seven parameters:  $\Theta = \{\alpha^*, \beta, A^*, \rho_a^*, \sigma_\epsilon^*, \sigma_\eta^*, F^*\}$ .

## 4.2 First order conditions in the absence and presence of linear firing costs

Without firing costs (i.e.  $F^* = 0$ ), the firm adjusts its employment period by period to equalize the value of marginal product with marginal cost. This implies that the labour share will be constant. Consider the first order condition of the firm's problem in the absence of firing costs

$$\alpha^* A^* (e^{a_{it}^*}) L_{it}^{\alpha^*-1} - 1 = 0 \quad (11)$$

which implies

$$L_{it} = [\alpha^* A_i^* (e^{a_{it}^*})]^{1-\alpha^*} \quad (12)$$

The labour share (as a fraction of output) is then:  $\alpha^*$ . See the Appendix for derivation.

In the presence of firing costs, optimal employment is not any longer a one period problem and there exist no longer an analytical solution for employment  $L_{it}$ . Bertola (1990) describes the solution for a model with linear hiring and firing costs.<sup>6</sup> The firms entire marginal revenue generated by the marginal employment decision will match the marginal hiring (in this case zero) or firing cost ( $F$ ). The expected marginal revenue stream is equal to:

<sup>6</sup>Bentolila and Bertola (1990) describe the solution for a continuous time version of the model.

$$MR = E_t \left\{ \sum_{j=0}^{\infty} \beta^j (\alpha^* A_i^* (e^{\alpha_{it+j}^*}) L_{it+j}^{\alpha^*-1} - 1) \right\} \quad (13)$$

The first order conditions are:

$$-F^* \preceq MR \preceq 0 \quad (14)$$

$$MR = 0 \text{ if } L_{it} - L_{it-1} > 0 \quad (15)$$

$$MR = -F \text{ if } L_{it} - L_{it-1} < 0 \quad (16)$$

In the appendix a derivation of these conditions can be found. The firm has three possible regimes: hiring, inaction or firing. When the firm hires, it sets the expected discounted future stream of marginal revenues (marginal product minus wage) equal to zero. When the firm fires, the marginal worker still has an expected discounted future stream of marginal revenues that is negative and equal to minus the firing cost. The firm is then indifferent between keeping the marginal worker or firing him. When the firm does not hire nor fire, the expected marginal revenue stream has to be between  $-F$  and zero. These conditions imply that large positive shocks will make the firm hire and large negative shocks will make the firm fire and small shocks will make the firm to take no action. The firing costs reduces the reaction of employment to the productivity shock.

Kessing (2003) solves analytically the simple case in which the productivity shock can take only two values and shows that the labour share becomes counter-cyclical. It is easy to provide intuition on why linear firing costs causes a countercyclical labour share. Consider inaction periods caused by those firing costs. The labour share is  $\frac{W p_{ot} L_{it}}{p_{ot} C(A e^{\alpha_{it}} K^{1-\alpha} L_{it}^{\alpha})^{\xi}}$  or simply  $\frac{W L_{it}}{C(A e^{\alpha_{it}} K^{1-\alpha} L_{it}^{\alpha})^{\xi}}$ . In periods of inaction, the numerator ( $W L_{it}$ ) remains constant and hence the labour share is inversely related to the shocks  $\alpha_{it}^*$  in the denominator. Small positive shocks lead to a drop in the labour share, and vice versa for small negative shocks.

### 4.3 Finding the policy function

The first order conditions do not give an analytical expression of the solution of the model. The solution of the model is given by the policy

function which describes the current level of employment given last periods level of employment and this periods productivity shock, i.e. the function  $L_{it} = f(L_{it-1}, a_{it}^*, \Theta)$ . For a given numerical value of the set of parameter values  $\Theta = \{\alpha^*, \beta, A^*, \rho_a^*, \sigma_\epsilon^*, \sigma_\eta^*, F^*\}$  the policy function is found numerically using value function iteration (see e.g. Judd, 1998, page 412). The value function iteration is performed as following. First, the state space  $\{L_{it-1}, a_{it}^*\}$  is discretized. Employment  $\{L_{it-1}\}$  is discretized as the subset of the natural numbers from  $\{1, \dots, 105\}$ <sup>7</sup> The AR(1) productivity shock process  $\{a_{it}^*\}$  is defined as  $a_{it}^* = \rho_a^* a_{it-1}^* + \mu_{it}^*$  with  $\mu_{it}^*$  normal distributed with mean zero and standard deviation  $\sigma_\epsilon^* + \sigma_\eta^*$ . The shock process  $\{a_{it}^*\}$  is transformed into a discrete Markov process on a very fine grid of 51 points using Tauchen (1986). (Note that the grid is a function of the numerical values of  $\rho_a^*, \sigma_\epsilon^*, \sigma_\eta^*$ ). So the discretized state space contains 5355 (105 times 51) elements.

## 5 Calibration and simulation

Table 1 gives moments of the micro firm and aggregate manufacturing data. The variability of employment, output and labour share are measured by the standard deviation of employment growth  $\sigma_l$ , the standard deviation of output growth  $\sigma_y$  and the standard deviation of the labour share growth  $\sigma_s$ .<sup>8</sup> All three moments are measured for the sample of 1634 manufacturing firms and aggregate manufacturing. The dynamics of the labour share are captured by the correlation of the labour share growth with output growth  $corr(y, s)$  both at the firm and aggregate level.

<sup>7</sup>The upper bound on the employment state space here taken to be 105 is large enough so that the solution under reasonable parameter values is not influenced by them. Note that average employment in the dataset is 23 employees.

<sup>8</sup>The moments are standard deviations of growth rates for the following reason. In standard RBC modelling, variables are usually detrended using some form of filtering (generally by way of a Hodrick-Prescott filter). As the firm level data is only 9 year long and annual we use a simple filter, the log-first difference. We use the same filter for the annual manufacturing data.

Table 1: Data moments

	Firm level	Aggregate
$\bar{L}$	3.14	
$\bar{S}$	0.31	
$\sigma_l$	0.11	0.016
$\sigma_y$	0.15	0.022
$\sigma_s$	0.12	0.027
$corr(y, s)$	-0.65	-0.82

note: Firm level moments based on 9 years for 1634 firms  
 Aggregate moments based aggregate French manufacturing  
 on period 1978-2004 (27 years)  
 $\bar{L}$  is average employment in logs,  $\bar{S}$  is average labour share,  
 $\sigma_l, \sigma_y, \sigma_s$  are standard deviation of employment growth,  
 output growth and labour share growth respectively

The model parameters  $\Theta$  are calibrated such that the model dynamics match as closely as possible the 10 moments of Table 1. The sum of squared percentage deviations of data moments  $\{M_{jN}\}_{j=1}^{10}$  from simulated moments  $\{\widehat{M}_{jN}\}_{j=1}^{10}$  of the model is minimized. I.e. the following quadratic loss function is used:

$$\min_{\Theta} [M_{1N} - \widehat{M}_{1N}(\Theta) \quad M_{2N} - \widehat{M}_{2N}(\Theta) \dots M_{10N} - \widehat{M}_{10N}(\Theta)] \Omega \quad (17)$$

$$[M_{1N} - \widehat{M}_{1N}(\Theta) \quad M_{2N} - \widehat{M}_{2N}(\Theta) \dots M_{10N} - \widehat{M}_{10N}(\Theta)]' \quad (18)$$

with  $\Omega$  having the inverse of the squared datamoments on the diagonal, i.e.  $(1/M_{jN})^2$ .<sup>9</sup>

The model simulation is performed using the policy function. Once the policy function is found numerically, an individual firm can be simulated by drawing two samples of i.i.d. standard normal random variables. Multiplying

<sup>9</sup>This way of minimizing a criterion function is identical as using the indirect inference method as explained in Gourieroux, et al. (1993). However since both aggregate and firm level moments are used (which in a statistical sense have different asymptotic behaviour), this can not be strictly called the indirect inference method. Also, the across firm observations are correlated due to the aggregate shock so the usual asymptotic theory of indirect inference does not apply.

the first sample by  $\sigma_\epsilon^*$  one obtains a simulation of the idiosyncratic shocks  $\{\widehat{\epsilon}_{i1}, \dots, \widehat{\epsilon}_{iT}\}$  and multiplying the second one by  $\sigma_\eta^*$  one obtains a simulation of the aggregate shocks  $\{\widehat{\eta}_1^*, \dots, \widehat{\eta}_T^*\}$ . Employing  $\widehat{a}_{it}^* = \rho_a^* \widehat{a}_{it-1}^* + \widehat{\epsilon}_{it}^* + \widehat{\eta}_t^*$  and setting initial  $\widehat{a}_{i0}^*$  equal to zero one obtains a simulated series  $\{\widehat{a}_1^*, \dots, \widehat{a}_T^*\}$ . Combining the simulated series  $\{\widehat{a}_1^*, \dots, \widehat{a}_T^*\}$  with an initial value for  $L_0$  one obtains a simulated series of employment  $\{\widehat{L}_1^*, \dots, \widehat{L}_T^*\}$  through the policy function. The corresponding simulated series of (normalized) nominal output and labour share can be readily obtained. Normalized (and deflated by aggregate price level) output  $p_{it}Y_{it}/p_{ot}W$  is equal to  $A^*(e^{a_{it}^*})L_{it}^{\alpha^*}$ . The costs of firing are treated as belonging into the total wage costs. In case of firing, the labour share is equal to  $(Wp_{ot}L_{it} + Fp_{ot}(L_{it-1} - L_{it})) / (p_{it}Y_{it})$  which is equal to employment plus the cost of firing divided by normalized output  $(L_{it} + F^*(L_{it-1} - L_{it})) / (p_{it}Y_{it}/Wp_{ot}) = (L_{it} + F^*(L_{it-1} - L_{it})) / [A^*(e^{a_{it}^*})L_{it}^{\alpha^*}]$ . In the case of hiring the labour share is equal to  $(Wp_{ot}L_{it}) / (p_{it}Y_{it})$  or  $L_{it} / [A^*(e^{a_{it}^*})L_{it}^{\alpha^*}]$ .

Similarly, a whole sample of  $N$  firms can be simulated by drawing  $N$  series of i.i.d. standard normal random samples and constructing  $\{\widehat{\epsilon}_{i1}, \dots, \widehat{\epsilon}_{iT}\}_{i=1}^N$  while keeping the aggregate shocks  $\{\widehat{\eta}_1^*, \dots, \widehat{\eta}_T^*\}$  the same for all firms. When searching over the space  $\Theta$ , the idiosyncratic shocks  $\{\widehat{\epsilon}_{i1}, \dots, \widehat{\epsilon}_{iT}\}_{i=1}^N$  and aggregate shocks  $\{\widehat{\eta}_1^*, \dots, \widehat{\eta}_T^*\}$  are kept constant.

In practice, to construct the simulated moments, for each  $\Theta$ , 244 year of 1634 firms are simulated. Only the last 162 years are taken to calculate firm level moments and aggregate moments. That is, for each  $\Theta$  the firm level moments are calculated 18 times and then averaged, i.e. 18 periods of 9 year. Aggregate simulated data is obtained by aggregating the 1634 firms individual labour, output, etc. to obtain aggregate employment, output, etc. For each  $\Theta$  the aggregate moments are calculated 6 times and then averaged, i.e. 6 periods of 27 year)<sup>10</sup>. The discount factor  $\beta$  is set equal to 0.93 throughout the whole exercise.

## 6 Calibration results

The calibrated parameters obtained by minimizing the criterion are presented in Table 2. The parameter  $\alpha^*$  which determines the labour share in the

<sup>10</sup>The number of periods over which moments were averaged might seem low. However, the simulated moments differ very little from period to period so that averaging over more periods brings very little except an increase in computing time

absence of firing costs is 0.27. The productivity shocks are strongly autocorrelated with  $\rho^*$  equal to 0.94. The standard deviation of the idiosyncratic shocks  $\sigma_\epsilon^*$  is equal to 0.14, the standard deviation of the aggregate shocks  $\sigma_\eta^*$  is equal to 0.02. If the aggregate shocks are interpreted as TFP shocks, their magnitude seems reasonable. The linear firing cost is equal to 0.26. This represents 26% of a yearly wage, which could be interpreted as a total adjustment cost slightly over 3 months of pay. It would be tempting to interpret those firing costs as true severance payments to the workers. However, such a strict interpretation would almost certainly be wrong. Whereas it is certainly the case that actual severance payments are large in France, i.e according to the World Bank severance payments in France are on average 32 weeks (i.e around 7 months of pay)<sup>11</sup>, the firing costs here are a modelling device capturing both monetary and non-monetary costs of adjustment. The firing costs capture the average cost of reducing employment at the firm level.<sup>12</sup>

Table 2 Calibrated Parameters

$A^*$	$\alpha^*$	$\rho^*$	$\sigma_\epsilon^*$	$\sigma_\eta^*$	$F$
28.46	0.27	0.94	0.14	0.20	0.26

Table 3 compares the data moments with the moments of the calibrated model. Overall, the calibrated model creates moments that are close to the data moments. The variability of employment, output and labour share at the firm level are all closely matched. The negative correlation of the labour share with output at the firm level is somewhat stronger than in the data (-0.78 versus -0.65). The aggregate moments are not all matched equally well. The best matched is the aggregate variability of employment. The variability of aggregate output in the model is somewhat larger than in the data (0.027 versus 0.022), whereas the variability of the labour share in the model is only 63% of the variability in the data (0.017 versus 0.027).

Overall, however, the model can explain a counter-cyclical labour share but with a variability that is too small. Interestingly, other mechanisms that have been suggested in the literature to match the countercyclicality generally also either generate a too low or too high variability of the labour

<sup>11</sup>The weeks of severance pay can be found on <http://www.doingbusiness.org>.

<sup>12</sup>In the sense that adjustment of employment might be relatively costless in the cases of retirement, voluntary leaves, etc.. it is reassuring that the calibrated firing costs of 3 months as an average cost is below the World Bank measure of an average of 7 months of severance payments for France.



share. The RBC model of Boldrin and Horvath (1995) includes a risk sharing component to wages that also induces counter-cyclicalities but also with an aggregate variability of the labour share that is only 46% of that in the data ( They calculate a standard deviation of the labour share of 1.08 (see their Table 1 page 974) on US Hodrick Prescott filtered data, whereas their model simulation imply a standard deviation of 0.50) . Similarly, Hansen and Prescott (2005) can explain countercyclicalities with capacity constraints but also with a variability of the labour share in the simulated model that is 70% of that in the data (See their table 3 where the simulated labour share has a standard deviation of 0.32 versus 0.46 in the data). The insurance mechanism in the RBC model of Gomme and Greenwood generally generates a too high variability of the labour share (164% of the variability in the data) (See their benchmark model standard deviation of 1.31 versus 0.80 in the data). Possibly a combination of a number of mechanisms can generate the aggregate variability of the labour share in RBC models that matches the actual one in the data.

Table 3: Data moments versus moments of the calibrated model

	Firm level		Aggregate level	
	Data	Calibrated Model	Data	Calibrate Model
$\bar{L}$	3.14	3.00		
$\bar{S}$	0.31	0.32		
$\sigma_l$	0.11	0.10	0.016	0.015
$\sigma_y$	0.15	0.17	0.022	0.027
$\sigma_s$	0.12	0.12	0.027	0.017
$corr(y, s)$	-0.65	-0.78	-0.82	-0.87

The calibrated model shows that a small amount of firing costs induce a large degree of countercyclicalities. This begs the question how the degree of firing cost and the degree of countercyclicalities are related. Figure 4 depicts what happens to the correlation between output and the labour share as a function of firing costs. For this experiment, the model is simulated and all parameters beside the firing cost are set to their calibrated value found in table 2. The firing costs are varied from 0 to 0.4. The correlation very rapidly becomes highly negative both at the firm and the aggregate level. It drops faster at the aggregate level. A feature of aggregation is that the aggregate is not simply mimicking the average firm but rather the firms with the most

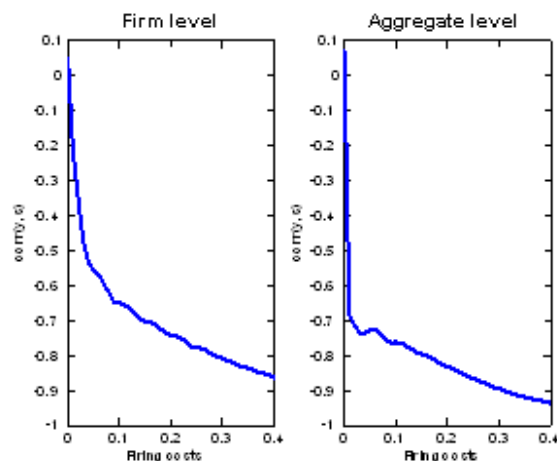


Figure 4: Correlation of output growth and labour share growth as function of firing costs

negative correlation of output with labour share. The intuition is simple, if two firms are added, say one with a constant labour share (imagine the firm has received zero productivity shocks) and one with a negatively correlated labour share, the aggregate will be negatively correlated.

One further can ask what happens to the volatility of employment growth, output growth and the labour share as a function of firing costs. Higher firing costs also reduce the volatility of employment growth and output growth, both at the aggregate and firm level whereas firing cost have the opposite effect on the volatility of the labour share. Figures 5 and 6 depict the standard deviation of employment growth and of output growth respectively as a function of firing costs. Figure 7 depicts the standard deviation of labour share growth as a function of firing cost. Figure 8 shows 6 examples of simulations of the aggregate labour share over a period of 27 year.

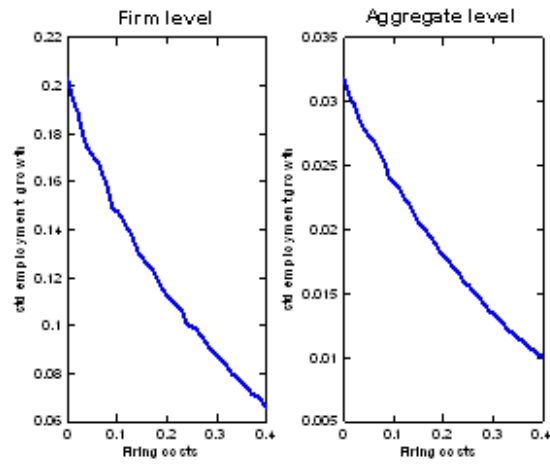


Figure 5: Standard deviation of employment growth as a function of firing costs

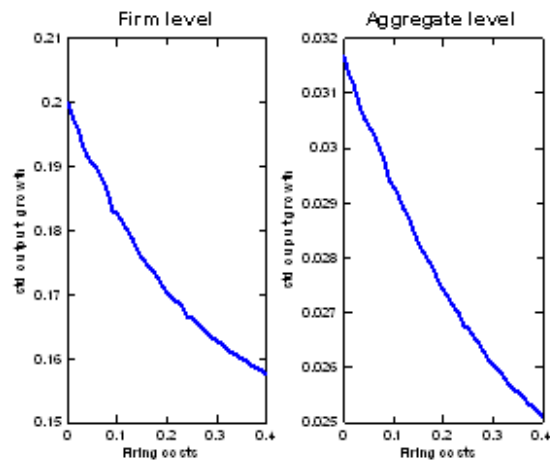


Figure 6: Standard deviation of output growth as a function of firing costs

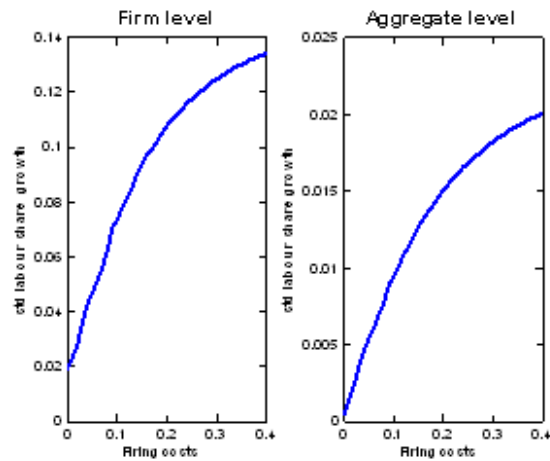


Figure 7: Standard deviation of labour share growth as a function of firing costs

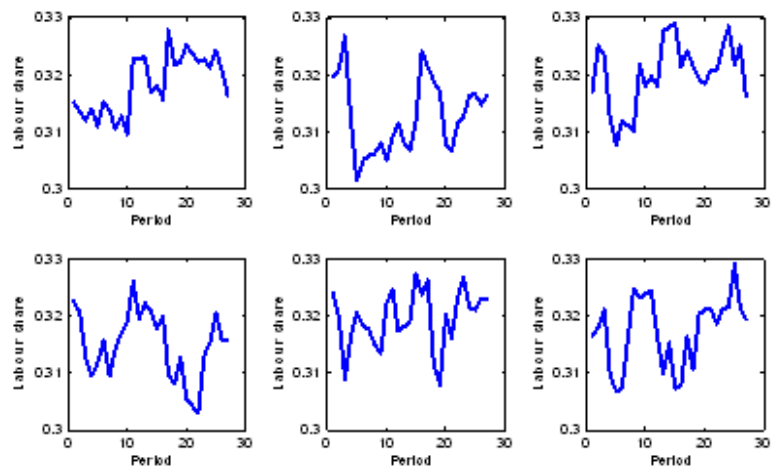


Figure 8: The aggregate labour share: 6 simulation examples of 27 periods

## 7 Conclusion

A simple stochastic dynamic model of labour demand with firing costs of moderate magnitude (around 3 months of pay) can mimic quite closely the variability of employment and output both at the aggregate and firm level while simultaneously matching the counter-cyclical behaviour of the labour share. However, the model generates a too low variability of the labour share at the aggregate level. Adjustment costs alone can not make the labour share volatile enough. This is also true for other mechanisms that lead to counter-cyclical labour share, such as employment insurance in Boldrin and Horvath (1995) and capacity constraints as in Hansen and Prescott (2005). The findings in this paper are relevant for both real and monetary business cycle modelling. First, real business cycle models could potentially match the counter-cyclical behaviour of the labour share when introducing labour adjustment costs rather than occasionally binding capital or insurance mechanisms. Firing costs are shown to also match the behaviour at the firm level, so they are truly micro-founded. Second, New Keynesian business cycle models explanations of inflation often imply that the labour share is a good measure of marginal cost. To gauge the extent to which movements in the labour share are caused by adjustment costs rather than true marginal cost changes is therefore important in estimating New Keynesian Philips curves relationships. The results in this paper show that a substantial amount of the variability of the labour share can be explained by firing costs alone and not by movements in real marginal costs. Future research is needed to assess if a mix of different mechanisms can explain the variability of the labour share simultaneously with the counter-cyclical behaviour.

## References

- [1] Batini N., Jackson B. and S. Nickell, 2000, Inflation dynamics and the labour share in the UK, External MPC Unit Discussion Paper No 2.
- [2] Batini N., Jackson B. and S. Nickell, 2005, An open-economy new Keynesian Phillips curve for the U.K., *Journal of Monetary Economics*, 1061-1071
- [3] Bentolila S. and G. Bertola, 1990, Firing costs and Labour demand. How bad is Eurosclerosis, *Review of Economic Studies*, 57, 381-402.
- [4] Bentolila S. and G. Saint-Paul, 1994, A model of labor demand with linear adjustment costs, *Labour Economics*, 1, 303-326
- [5] Bentolila S. and G. Saint-Paul, 2003, "Explaining movements in the labor share", *Contribution to Macroeconomics* ", Vol 3, Issue 1, Article 9.
- [6] Bertola G., 1990, "Job security, employment and wages ", *European Economic Review*, 34, 851-886.
- [7] Blanchard O., 1997, "The medium run", *Brookings papers on economic activity*, Vol No2, 89-158.
- [8] Boldrin M, and M. Horvath, 1995, Labor contracts and business cycles, *Journal of Political Economy*, Vol 103, N05, 972-1004
- [9] Caballero R. and M. Hammour, 1998, "Jobless growth: appropriability, factor substitution, and unemployment", *Carnegie-Rochester Conference Series on Public policy* 48, 51-94.
- [10] Gagnon E. and H. Kahn, 2005, "New Phillips curve under alternative production technologies for Canada, the United States, and the Euro area", *European Economic Review*, 49, 1571-1602.
- [11] Gali, J., Gertler, M. and D. Lopez-Salido, 2001, "European Inflation Dynamics", *European Economic Review*, 45, 1237-1270.
- [12] Gali, J. and M. Gertler, 1999, "Inflation dynamics: A structural Econometric Analysis", *Journal of Monetary Economics*, 44, pp 195-222.

- [13] Giammarioli, N., Messina J., Steinberger T. and C. Strozzi, 2002, European labor Share dynamics: an institutional perspective, *EUI Working Paper*, No. 2002/13.
- [14] Gomme, P. and J. Greenwood, 1995, On the cyclical allocation of risk, *Journal of Economic dynamics and control*, 19, 91-124.
- [15] Gourieroux C., Monfort A. and E. Renault, 1993, Indirect Inference, *Journal of applied econometrics* 8, 85-118.
- [16] Kessing S., 2003, "A note on the determinants of labour share movements", *Economics Letters* 81, pp 9-12.
- [17] Kydland, F.E. and E.C. Prescott, 1990, "Business-Cycles: Real facts and a monetary Myth", *Federal Reserve Bank of Minneapolis Quarterly Review*, 154(2), 3-18.
- [18] Judd K. J., 1998, *Numerical Methods in Economics*, MIT Press.
- [19] Mcadam P. and A. Willman, 2004, New Keynesian Phillips Curves: a reassessment using euro-area data, *Oxford Bulletin of Economics and Statistics*, 2004, 66, 637-670.
- [20] Rotemberg, J. and M. Woodford, 1999. The Cyclical Behavior of Prices and Costs. In *Handbook of Macroeconomics*, Vol. 1B, edited by John Taylor, and Michael Woodford, pp. 1051–1135. Amsterdam: North-Holland.
- [21] Rudd J. and K. Whelan, 2005, Does Labor's share drive inflation?, *Journal of Money credit and banking*, Vol 37, no 2.
- [22] Sbordone, A. 2002. Prices and Unit Labor Costs: A New Test of Price Stickiness. *Journal of Monetary Economics*, Vol 49, 265–292.
- [23] Sbordone, A., 2005. "Do expected future marginal costs drive inflation dynamics?", *Journal of Monetary Economics*, vol.52 (6), 1183-1197;
- [24] Tauchen, G., 1986, 'Finite State Markov chain Approximations to univariate and vector autoregressions', *Economics Letters*, Vol 20 , 177-181.
- [25] Woodford, M., 2001, The Taylor Rule and Optimal Monetary Policy. *American Economic Review* 91 232–237.

## 8 Appendix A.

The dataset was constructed using the AMADEUS database. This database contains balance sheet statement and profit and loss accounts of European firms. First all French manufacturing firms were selected. From this selection, firms with complete data on employment, profit, sales and capital stock for 9 years were kept. To make sure that the selected firms were truly plants and not headquarters without a physical plant, firms that had more than 50% of fixed assets under a different form as tangible assets (i.e. financial fixed assets, which are equity in other firms) were dropped. Firms were also dropped if they had a large outlier observation in the 9 year period. The outliers were defined as a growth rate of more than 200% in employment in a given year, a growth rate of sales more than 200% in a given year. Further, firms which employment grew more than 60% over the whole period were removed. This last criterion is done as the model in the paper has a long run stationary employment level of the firm as its solution.

## 9 Appendix B.

(1) *Derivation of the labour share (in term of output) in the absence of firing costs:*

The first order condition of the firm's problem in the absence of firing costs

$$\alpha^* A_i^* (e^{a_{it}^*}) L_{it}^{\alpha^* - 1} - 1 = 0$$

or  $L_{it} = [\alpha^* A_i^* (e^{a_{it}^*})]^{\frac{1}{1-\alpha^*}}$ . The labour share is then:  $\frac{W_{pot} L_{it}}{C_{pot} (A e^{a_{it}} K^{1-\alpha} L_{it}^\alpha)^\xi} =$   
 $\frac{L_{it}}{C (A e^{a_{it}} K^{1-\alpha} L_{it}^\alpha)^\xi / W} = \frac{L_{it}}{A^* (e^{a_{it}^*}) L_{it}^{\alpha^*}} = \frac{L_{it}^{1-\alpha^*}}{A_i^* (e^{a_{it}^*})} = \alpha^*$



## 10 Appendix C.

### *Characterizing the solution with linear firing costs*

The indicator function  $I(L_{it}-L_{it-1})$  is not differentiable at  $L_{it}-L_{it-1} = 0$ . To characterize the solution it helps to rewrite the value function of the original problem into a problem of hiring and firing.

Define  $p$  to be the probability of hiring,  $H$  the number of people hired, and  $F$  ( $F^*$  is still the cost of firing) the number of people fired. So depending on hiring or firing we have  $L_{it} = L_{it-1} + H$  or  $L_{it} = L_{it-1} - F$ . (When hiring is zero just set  $H=0$ ).

To simplify notation, remove time subscripts and stars (except for  $F^*$ ) and using a prime  $'$  to signify next period (or first derivative depending on context). Consider  $L$  to be the initial level of employment, we can rewrite the original value function as.

$$V(a, L) = \max_{\{p, H, F\}} p[A(e^a)(L + H)^\alpha - (L + H)] \quad (19)$$

$$+ (1 - p)[A(e^a)(L - F)^\alpha - (L - F) - F^*F] \quad (20)$$

$$+ p\beta EV(a', L + H) + (1 - p)\beta EV(a', L - F) \quad (21)$$

So the problem is rewritten as a choice in the probability of hiring and firing  $p$ , together with the number of hiring  $H$  and firing  $F$ .

The first order conditions w.r.t.  $H$  and  $F$  are

$$p[\alpha A(e^a)(L + H)^{\alpha-1} - 1] + p\beta EV'(a', L + H) = \quad (22)$$

$$(1 - p)[- \alpha A^*(e^a)(L - F)^{\alpha-1} + 1 - F] - (1 - p)\beta EV'(a', L - F) = \quad (23)$$

Differentiating the value function

$$V'(a, L) = p[\alpha A(e^a)(L + H)^{\alpha-1} - 1] \quad (24)$$

$$+ (1 - p)[\alpha A(e^a)(L - F)^{\alpha-1} - 1] \quad (25)$$

$$+ p\beta EV'(a', L + H) + (1 - p)\beta EV'(a', L - F) \quad (26)$$

Rewriting the first order conditions w.r.t. H and F:

$$-p[\alpha A(e^a)(L + H)^{\alpha-1} - 1] = p\beta EV'(a', L + H) \quad (27)$$

$$(1 - p)[- \alpha A^*(e^a)(L - F)^{\alpha-1} + 1 - F] = (1 - p)\beta EV'(a', L - F) \quad (28)$$

Substituting the first order conditions w.r.t. H and F into the above we get  $V'(a, L) = (1 - p)(-F)$  i.e.  $V'(a, L) = -F$  when  $p = 0$  i.e.  $F > 0$  (firing) and  $V'(a, L) = 0$  when  $p=1$  i.e.  $H > 0$  (hiring)

And

$$V'(a, L) = E[\alpha A(e^a)(L')^{\alpha-1} - 1] \quad (29)$$

$$+ \beta EV'(a', L') \quad (30)$$

By repeated substitution:  $V'(a, L) = E \sum_{j=0}^{\infty} \beta^j (\alpha A(e^a)(L_j)^{\alpha-1} - 1)$

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