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PRODUCT INNOVATION
UNDER VERTICAL DIFFERENTIATION
AND THE PERSISTENCE OF MONOPOLY

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Abstract

The incentives to innovate for the incumbent and the entrant in a vertically differentiated market are analised, in the absence of uncertainty. It turns out that if consumers' marginal willingness to pay for quality is sufficiently low, the efficiency effect observationally works so as to favour innovation by the entrant, i.e., competition. Otherwise, it operates to the advantage of the incumbent who acquire the right to innovate, preempting thus the rival.

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1. Introduction

We avail of a relatively large literature on the optimal choice of product quality by oligopolists under either Bertrand or Cournot competition and simultaneous play (Shaked and Sutton, 1982, 1983; Bonanno, 1986, *inter alia*). The sequential introduction of new products has been analysed by Donnenfeld and Weber (1992, 1995) without explicitly taking into account R&D efforts, which are instead the focus of the contributions due to Beath *et al*. (1987), Motta (1992) and Rosenkranz (1995a,b). Beath *et al*. (1987) extend the analysis of Vickers (1986) to the case of product innovation under vertical differentiation when there is a repeated patent auction for innovations. Leaving the issue of the strategic timing of innovation outside the main analysis, they find how the conditions for the persistence of quality leadership look like. Along related lines, Rosenkranz (1995b) shows that the low-quality firm has a higher incentive to innovate if the adjustment cost is sufficiently high. She also shows that a change of leadership can be observed whenever adjustment cost is high or the degree of differentiation at the outset is high enough. Motta (1992) investigate the impact of cooperation in the R&D stage on the overall level of research effort and market structure. In his model, R&D is simply a tool for improving product quality provided firms have already entered the market. Firms enter costlessly and simultaneously, so that the timing of innovation is exogenous. In Rosenkranz (1995a) both the quality and the timing of innovations are endogenous, and the author investigates the conditions under which firms are incentivated to form a research joint venture. In the context of horizontal differentiation, R&D competition between two firms both racing for the optimal monopolistic location, i.e., the midpoint of the linear city, has been analysed by Harter (1993).

Product proliferation as a preemptive strategy has been investigated in countless contribution, dealing mainnly with spatial differentiation (Hay, 1976; Eaton and Lipsey, 1979; Bonanno, 1987, *inter alia*).[1] To the best of my knowledge, the issue of entry deterrence through

1. See chapter 8 in Tirole (1988), and Gilbert (1989) for exhaustive accounts of this literature.

the strategic choice of product quality has been tackled only by Donnenfeld and Weber (1995). Though, they do not specifically deal with the related problem of the persistence of monopoly which has been largely debated in the R&D literature (Gilbert and Newbery, 1982, 1984; Reinganum, 1983, to mention only the seminal contributions). Provided that a monopolist can at least replicate the oligopolists' performance, these contributions generally lead to the conclusion that monopoly persists since the incumbent has a greater incentive to innovate or preempt than the rival has to enter.

I will focus on the issue of the persistence of monopoly by analysing an adapted version of the so called *efficiency effect* (Fudenberg and Tirole, 1986; Tirole, 1988) in a vertically differentiated market where it is assumed that all consumers buy one unit of the differentiated good independently of the number of firms or goods. I shall assume that at a certain date an incumbent, which is already supplying a vertically differentiated product whose quality cannot be changed, and a potential entrant participate to an auction for a new product characterized by a different quality. The firm who can bid more for the innovation acquires the right to produce a good whose quality is then optimally defined according to the existing one, in order to maximize either monopolistic profit, if the monopolist is the winner, or the entrant's profit in the opposite case. Then, prices are set according to the market structure previously emerged.

I will show that, due to the assumption of full market coverage, which may appear plausible for instance when public utilities are considered, the efficiency effect operates so as to favour the persistence of monopoly only if consumers are sufficiently rich. In such a case, the incumbent is indifferent between introducing the new quality from below or above the existing one. Otherwise, the incumbent offering two varieties can at most mimic the duopolists' performance, so that the outcomes associated with innovation by either firm are observationally equivalent, in that both lead to a noncooperative duopoly where the innovation is characterized by a higher quality as compared to the already existing variety.

The remainder of the paper is organized as follows. Section 2 is devoted to the description

of the model as well as the optimal price and quality choices by a single product monopolist. In Section 3 I describe the choice of a second quality by the incumbent. The entrant's behaviour is investigated in Section 4. The issue of the persistence of monopoly is then addressed in Section 5. Finally, Section 6 provides concluding comments.

2. Single product monopolist

Consider a single product monopolist operating in a market for a vertically differentiated good where consumers are characterized by a marginal willingness to pay for quality $\theta \in [\underline{\theta}, \overline{\theta}]$, $\underline{\theta} > 0$, $\overline{\theta} = \underline{\theta} + 1$. The population of consumer is uniformly distributed over the interval, with density 1, so that the total number of consumers in this market is also 1. Individuals derive a net surplus from consumption defined as follows:

$$U = \theta q - p \geq 0, \qquad (1$$

where q is the quality level and p is the price at which such a quality is being supplied. It is assumed that all of them buy one unit of the differentiated product, i.e., that (1) is satisfied at least as an equality for all consumers.

Technology is described by a cost function which is convex in quality and linear in quantity, while the introduction of a product involves a sunk cost, or equivalently a fixed entry feefor each variety, given by k and independent of the quality level, so that total costs for each product are:

$$C = tq^2 x + k, \quad k < \frac{1}{27t}, \qquad (2$$

where x is the output level and t is a positive parameter. The condition imposed on k is such that

the incumbent finds it advantageous to innovate independently of any entry threat, and she does not gain any profit from keeping the patent asleep. The presence of a sunk cost also implies that quality cannot be changed as a reaction to the introduction of a new variety, either by the same firm or by a rival. Provided the market is completely covered, the monopolist's profit function is the following:

$$\pi_M = p_M - t q_M^2 - k. \tag{3}$$

It can be immediately verified that

$$\frac{\partial \pi_M}{\partial p_M} > 0, \quad \frac{\partial \pi_M}{\partial q_M} < 0; \tag{4}$$

this can be given the following intuitive explanation: since by assumption all consumers buy a unit of the good, the monopolist sets the maximum price consistent with this assumption, and since costs are convex in quality, supplies the minimum quality in the range of consumers' preferred qualities, $[\underline{\theta}/2t, \overline{\theta}/2t]$ which can be defined considering the quality levels required by the maximization of consumer surplus (1) when each variety is sold at marginal cost (see Cremer and Thisse, 1994).

Notice that the maximum price is given by

$$p_M^{\max} = \underline{\theta} q_M. \tag{5}$$

Substituting (5) into (3), the monopolist's profit function is defined in terms of q_M only:

$$\pi_M = (\underline{\theta} - tq_M)q_M - k; \qquad (6$$

function (6) is concave and single peaked, with the maximum $\pi_M^* = (\overline{\theta} - 1)^2/(4t) - k$ at $q_M^* = \underline{\theta}/(2t)$, which denotes the quality preferred by the consumer characterized by the lowest marginal willingness to pay for quality, $\underline{\theta}$. Furthermore, notice that a social planner aiming at the maximization of social welfare would supply the quality preferred by the average consumer characterized by $\hat{\theta} = (\overline{\theta} + \underline{\theta})/2$, i.e., $q_{SP}^* = (2\overline{\theta} - 1)/(4t)$.[2]

3. Innovation by the incumbent

I shall focus here on the introduction of a new variety by the monopolist, given that the quality already being supplied is fixed at $\underline{\theta}/(2t)$. Generally speaking, the monopolist will offer a high and a low quality product, segmenting the market in such a way that the demand for the the two products are:

$$x_H = \overline{\theta} - \frac{(p_H - p_L)}{q_H - q_L}, \quad x_L = \frac{(p_H - p_L)}{q_H - q_L} - (\overline{\theta} - 1), \qquad (7$$

and the profit function is

$$\pi_M = (p_H - tq_H^2)x_H + (p_L - tq_L^2)x_L - 2k. \qquad (8$$

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2. Here drastically emerges once more the well known result according to which the private monopolist's behaviour takes into account the marginal consumer, while the social planner's one takes into account the average consumer (see Spence, 1975, and Tirole, 1988, ch. 2). It can be shown that when the number of varieties being supplied tends to infinity (or quality becomes a continuous variable) the monopolist ends up offering the socially optimal top quality, while she keeps undersupplying all the other qualities. Indeed, the distance between the monopolist's lowest quality and the social planner's one is maximized when the number of varieties tends to infinity (see Mussa and Rosen, 1978; Itoh, 1983; and Besanko *et al*., 1987).

The price of the low quality good is $p_L = \underline{\theta} q_L = (\bar{\theta} - 1) q_L$, while that of the high quality good is obtained from the first order condition for profit maximization:

$$\frac{\partial \pi_M}{\partial p_H} = 0 \quad \Rightarrow \quad p_H^* = \frac{\bar{\theta} q_H + t q_H^2 - 2 q_L + \bar{\theta} q_L - t q_L^2}{2}. \tag{9}$$

Substituting prices into (8), the monopolist's profit can be written as a function of qualities only:

$$\pi_M = \frac{\bar{\theta}^2 q_H - 2\bar{\theta} t q_H^2 + t^2 q_H^3 - 4 q_L + 4\bar{\theta} q_L - \bar{\theta}^2 q_L + t^2 q_H^2 q_L - 4 t q_L^2 + 2\bar{\theta} t q_L^2 - t^2 q_H q_L^2 - t^2 q_L^3}{4} \tag{10}$$

I am now in a position to investigate which kind of quality the monopolist would assign to a new product. Assume first that the monopolist introduces a new variety *from above*, i.e., $q_L = (\bar{\theta} - 1)/(2t)$. The candidate new variety has to be selected between the critical points of the first order condition:

$$\frac{\partial \pi_M}{\partial q_H} \Big|_{q_L = \frac{\bar{\theta} - 1}{2t}} = \frac{(3\bar{\theta} - 1 - 6 t q_H)(\bar{\theta} + 1 - 2 t q_H)}{16} = 0, \tag{11}$$

which are $q_{H1} = (3\bar{\theta} - 1)/(6t)$ and $q_{H2} = (\bar{\theta} + 1)/(2t)$. Since only q_{H1} satisfies the second order condition for a maximum, $q_H = (3\bar{\theta} - 1)/(6t)$ is the candidate high quality, yielding a profit gross of fixed costs equal to $\pi_M = (27\bar{\theta}^2 - 54\bar{\theta} + 31)/(108t)$. Prices are $p_H = (9\bar{\theta}^2 - 12\bar{\theta} + 7)/(18t)$ and $p_L = (\bar{\theta} - 1)^2/(2t)$, which are both positive given the viability condition $\bar{\theta} > 1$. Demands are x_H=1/3 and x_L=2/3. Finally, the degree of differentiation, measured by the distance between the two goods along the quality spectrum, is q_H-q_L=1/(3t).

Consider now what happens if the monopolist starts producing a good whose quality is *lower* than that already available, i.e., assume $q_H = (\bar{\theta} - 1)/(2t)$. Proceeding as above, from the first order condition

$$\frac{\partial \pi_M}{\partial q_L} \Big|_{q_H = \frac{\bar{\theta}-1}{2t}} = \frac{(3\bar{\theta} - 5 - 6tq_L)(\bar{\theta} - 3 - 2tq_L)}{16} = 0, \qquad (12$$

I obtain $q_{L1} = (\bar{\theta} - 3)/(2t)$ and $q_{L2} = (3\bar{\theta} - 5)/(6t)$ as the two critical points. Since only q_{L2} satisfies the second order condition, it represents the candidate innovation from below, yielding $\pi_M = (27\bar{\theta}^2 - 54\bar{\theta} + 31)/(108t)$ as the profits gross of fixed costs accruing to the two-product monopolist. It appears immediately that they coincide with those obtained in the previous case, so that the choice between introducing the new variety from above or from below depends upon the conditions on $\bar{\theta}$ ensuring the positivity of prices. In this setting, prices are $p_H = (9\bar{\theta}^2 - 18\bar{\theta} + 11)/(18t)$, which is always positive given that the above viability condition must be met, and $p_L = (3\bar{\theta} - 5)(\bar{\theta} - 1)/(6t)$, which is positive if $\bar{\theta} > 5/3$. The same condition must be satisfied in order for q_{L2} to be positive. Demands are x_H=2/3 and x_L=1/3 and the degree of differentiation is the same as above. Post-innovation momopoly profits are strictly greater than pre-innovation gross profits for all admissible values of $\bar{\theta}$. The same holds for net profits if k<1/27t. Hence, if $\bar{\theta} \geq 5/3$ the monopolist is indifferent between the two available alternatives, while if $\bar{\theta} \in]1, 5/3[$ she innovates *from above*, obtaining the same profit in either case.

4. Innovation by the entrant

I focus now on the setting where the entrant introduces a new variety and then plays simultaneously in prices against the incumbent. The profits, gross of fixed costs, associated with the two products are

$$\pi_H = (p_H - tq_H^2)x_H, \quad \pi_L = (p_L - tq_L^2)x_L, \tag{13}$$

where demands x_H and x_L are defined as in (7). The problem can be solved as usual by backward induction. From the first order conditions

$$\frac{\partial \pi_H}{\partial p_H} = \bar{\theta} - \frac{(p_H - p_L)}{q_H - q_L} - \frac{(p_H - tq_H^2)}{q_H - q_L} = 0, \tag{14}$$

$$\frac{\partial \pi_L}{\partial p_L} = \frac{(p_H - p_L)}{q_H - q_L} - \frac{(p_L - tq_L^2)}{q_H - q_L} - (\bar{\theta} - 1) = 0. \tag{15}$$

the following noncooperative equilibrium prices can be obtained:

$$p_H = \frac{q_H + \bar{\theta}q_H + 2tq_H^2 - q_L - \bar{\theta}q_L + tq_L^2}{3}; \quad p_L = \frac{2q_H - \bar{\theta}q_H + tq_H^2 - 2q_L + \bar{\theta}q_L + 2tq_L^2}{3}. \tag{16}$$

Hence, the profit functions (13) at the quality stage look as follows:

$$\pi_H = \frac{(q_H - q_L)(tq_H - tq_L - \bar{\theta} - 1)^2}{9}, \quad \pi_L = \frac{(q_H - q_L)(2 - \bar{\theta} + tq_H + tq_L)^2}{9}. \tag{17}$$

Provided the incumbent produces a good of quality $(\bar{\theta} - 1)/(2t)$, the entrant has to choose between competing from above or from below. Assume she decides to enter with a higher quality product. In such a case, adding indexes *i* and *e* to identify the incumbent and the entrant, respectively, the incumbent supplies $q_{Li} = (\bar{\theta} - 1)/(2t)$. Substituting the latter in (17) and differentiating π_H w.r.t. q_{He}, the first order condition for the entrant obtains:

$$\frac{\partial \pi_{He}}{\partial q_{He}}\Big|_{q_{Li}=\frac{(\bar{\theta}-1)}{2t}} = \frac{(3\bar{\theta}+1-6tq_{He})(\bar{\theta}+3-2tq_{He})}{36} = 0, \qquad (18$$

which yields the critical points $q_{He1} = (3\bar{\theta}+1)/(6t)$ and $q_{He2} = (\bar{\theta}+3)/(2t)$. The second order condition is satisfied only by q_{Le1}, yielding gross profits $\pi_{He} = 32/(243t)$ and $\pi_{Li} = 50/(243t)$. Demands are x_{He}=4/9 and x_{Li}=5/9, while finally prices are $p_{He} = (27\bar{\theta}^2 + 18\bar{\theta} + 35)/(108t)$ and $p_{Li} = (27\bar{\theta}^2 - 54\bar{\theta} + 67)/(108t)$, with $p_{He}>p_{Li}>0$ by virtue of the overall viability condition according to which $\bar{\theta} > 1$. Notice that, as it could be expected from the outset, innovation by the entrant implies an increase in the degree of differentiation as compared to what obtains if the innovation is introduced by the incumbent.

The opposite perspective, where the entrants innovates from below, can be quickly dealt with. Qualities are q_{Hi} and q_{Le}, respectively. The unique maximum of π_{Le} is at $q_{Le} = (3\bar{\theta}-5)/(6t)$, yielding $\pi_{Hi} = 49/(243t)$ and $\pi_{Le} = 4/(243t)$ as the duopolistic profits. Since the latter are both lower than those obtained in the previous case, it appears that not only the entrant would choose to compete from above,[3] but both firms would strictly agree on that.

5. The persistence of monopoly

I am now able to assess the relative size of the incentives to innovate for the incumbent and the entrant, in order to understand whether in a vertically differentiated market like the one described in the previous sections a monopoly regime can be expected to persist under sequential product innovation. The outcome of the auction for the product innovation is summarized by the following

3. This result sharply contrasts with a large part of the existing literature, e.g., Shaked and Sutton (1982, 1983), where firms fill the product range starting from the top quality. The different behaviour of the two models is due to the fact that here production costs are convex in quality.

PROPOSITION. If the marginal willingness to pay for quality is sufficiently high and the incumbent maximizes the joint profits associated with the two varieties, she is able to bid more for the innovation and thus remains a monopolist. Otherwise, irrespectively of the innovator's identity, innovation by either firm yields a noncooperative duopoly outcome, where observationally the entrant bids more for the innovation and enters the market with a variety of higher quality than the incumbent's.

PROOF. Consider first the relative performances in terms of profits of a monopolist offering two products versus two oligopolists offering a single variety each. The incumbent will bid more for the innovation, and thus remain a monopolist, if the following inequality is met:

$$\pi_M(q_H, q_L) \geq \pi_i(q_{Li}) + \pi_e(q_{He}), \tag{19}$$

that is

$$\frac{27\bar{\theta}^2 - 54\bar{\theta} + 31}{108t} \geq \frac{50}{243t} + \frac{32}{243t}, \tag{20}$$

which, after simple manipulations, yields

$$\frac{243\bar{\theta}^2 - 486\bar{\theta} - 49}{972t} \geq 0. \tag{21}$$

Condition (21) is satisfied for $\bar{\theta} \geq 1 + 2\sqrt{219}/27$. This implies that the incumbent bids more than the entrant and remains thus a monopolist when consumers are sufficiently rich. In such a case,

the incumbent is also indifferent between introducing the new variety from above or from below. Otherwise, if $\bar{\theta} \in]1, 1+2\sqrt{219}/27[$, the incumbent is at best able to replicate the profit performance of non colluding duopolists,[4] by resorting to the noncooperative maximization of the profits separately accruing to the two varieties, and virtually giving up her identity as a monopolist her price-and-quantity setting behaviour is concerned. Hence, either of the two firms may innovate, yielding a duopolistic market where the quality of the new product is higher than the quality of the one already available, and variety innovation leads to an outcome which is observationally equivalent in either case.[5] *Q.E.D.*

A few comments are now in order. First, the effect I have taken into account, although adapted to a context of endogenous differentiation, closely reflects the efficiency effect, as defined by Fudenberg and Tirole (1986) and Tirole (1988) following Arrow (1962), analogous to the incentive to pre-empt of Katz and Shapiro (1987) and the competitive threat effect of Beath *et al.* (1989). Contrarily to the results emerging from these contributions, where under competition with homogeneous products the efficiency effect univocally favours the persistence of monopoly, here the assumption of full market coverage makes it possible for the efficiency effect to work in the opposite direction, since the monopolist would prefer not to serve the entire market, provided that revenues increase linearly in quality while costs increase with the square of quality (see Mussa and Rosen, 1978; Besanko *et al.*, 1987; and the Appendix), while duopolists may prefer either market regime, depending on the level of the marginal willingness to pay (see the Appendix). Consequently, there exist an interval for $\bar{\theta}$ as defined above, where the efficiency

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4. Notice that even in such an extreme situation the incumbent still have an incentive to innovate, not only as a preemptive strategy but also because if $\bar{\theta} \in]1, 1+2\sqrt{219}/27]$ the overall profits accruing to non colluding duopolists are strictly higher than the monopolistic profit when a single variety is supplied, provided $k<1/27t$.

5. It is worth stressing that this mirrors what happens with homogeneous goods when firms bids for a *drastic* process innovation. In such a case, though, monopoly persists independently of the identity of the innovator.

effect is such to operate so as to bring about a change in market structure from the previous monopoly into a differentiated duopoly where the high quality good is supplied by the firm who either has entered the market later or has acquired the right to innovate by winning the auction. Incidentally, notice that this argument also implies that that when consumers are sufficiently poor firms cannot gain from collusion, as there exists an interval for the marginal willingness to pay where cartel profits are strictly lower than the sum of the noncooperative ones.

Second, it might be easily checked that in any other setting the efficiency effect would work univocally to the advantage of the incumbent, leading to the persistence of monopoly for all parameters values. This would be particularly evident if firms were able to adjust both qualities at the time of adoption of the innovation, i.e., in the absence of sunk costs. Furthermore, it would intuitively apply if firms were allowed to choose whether to serve the entire market or restrict output according to the relative profit performance attainable in the two cases. The persistence of monopoly would obtain as well if the incumbent firm could anticipate the auction for the innovation at some time in the future and adjusted the quality of the existing product accordingly in advance, thus being able to bid more than the entrant.

Finally, in the light of the above analysis, the motivation at the basis of the full market coverage assumption is twofold. On the one hand, a regulator may oblige the monopolist to serve all consumers so as to increase social welfare. On the other, the regulator may introduce such a rule anticipating that this may favour innovation so as to *create* market competition irrespectively of the innovator's identity.

6. Conclusions

In a model of vertical product differentiation with convex variable production costs of quality, I have addressed the question whether in a market for endogenously differentiated goods the incumbent has a higher incentive to innovate as compared to the potential entrant, under full market coverage. I have also assumed that quality is chosen once and for all, i.e., it represents

a long run commitment, so that the quality level of the existing variety cannot be adjusted as a reaction to the introduction of an innovation.

In order to answer the above question, an adapted version of the well known efficiency effect has been evaluated. Due to the fact that the market is completely covered, the direction of the efficiency effect depends on the distribution of income, in that it leads to the conclusion that the incumbent is more incentivated to innovate than the entrant only if the marginal willingness to pay characterizing consumers is sufficiently high to justify the introduction of a new variety. Otherwise, it works in such a way that market competition is observed once the innovation has been introduced, independently of which of the two firm has won the auction.

According to the above results, it is possible to conclude that if the marginal willingness to pay for quality is sufficiently low, the "observational" outcome is that the entrant has a higher incentive than the incumbent and bids more for the innovation, introducing a variety of higher quality than the one already being supplied by the incumbent. Thus, full market coverage appears as an indirect tool that a regulator may adopt so as to positively affect competition and social welfare, provided that the incumbent firm has tied her own hands by producing a certain quality without anticipating the auction for an innovation.

Finally, it appears desirable for future research to investigate the alternative settings where (i) technology is characterized by a different degree of convexity; and/or (ii) the distribution of consumers is non-uniform, e.g., the number of consumers characterized by a low marginal willingness to pay for quality is larger than the number of consumers whose marginal willingness to pay is high; and/or (iii) the incumbent anticipates the possibility of an innovation, but there is uncertainty about the auction date.

Appendix

First, I am going to show that a single-product monopolist weakly prefers partial to full market coverage. This result extends intuitively to the case of a multiproduct monopolist. I start by considering a monopolist who is only partially serving the market, selling a single variety. The demand for her product is:

$$x = \overline{\theta} - \frac{p}{q}, \qquad (a1$$

so that the monopolist's profit function is:

$$\pi^m = (p - tq^2)x. \qquad (a2$$

Observe that, potentially, the monopolist could choose not to exclude any individual from consumption by setting the price-quality ratio below $\underline{\theta}$, so as to serve the entire market. If this does not obtains at equilibrium, it implicitly means that the monopolist prefers quantity restriction to quality distortion. Optimal quality and price can be obtained by solving the first order conditions (it can be easily shown that second order conditions are also satisfied):

$$\frac{\partial \pi^m}{\partial q} = tp + \frac{p^2}{q^2} - 2\overline{\theta}qt = 0; \qquad (a3$$

$$\frac{\partial \pi^m}{\partial p} = \overline{\theta} - \frac{2p}{q} + qt = 0; \qquad (a4$$

yielding $p^m = 2\overline{\theta}^2/(9t)$ and $q^m = \overline{\theta}/(3t)$. The equilibrium quantity is $x^m = \overline{\theta}/(3t)$ and profit amounts to $\pi^m = \overline{\theta}^3/(27t)$.

The behaviour of the single-product monopolist under full market coverage is described in Section 2. I am now in a position to compare the performance of the monopolist in the two alternative settings. The following obtains:

$$\frac{\bar{\theta}^3}{27t} \geq \frac{(\bar{\theta}-1)^2}{4t} \qquad (a5$$

for all $\bar{\theta} \geq 1$. In particular, condition (a5) holds as a strict inequality for all values of $\bar{\theta}$ except $\bar{\theta} = 3$, for which it holds as an equality. Analogous calculations are needed to obtain the same results when more than one product is supplied.

Consider now the oligopoly problem. Here I shall confine myself to a duopoly, showing that if both prices and qualities are optimally set by duopolists, there exists an interval for the relevant parameter where firms strictly prefer to serve all consumers, while the opposite holds outside such interval. Consider a duopoly made up by single product firms. Under the full market coverage assumption, provided $\bar{\theta} > 5/4$, it can be shown that duopolists symmetrically locate their respective varieties outside the range defined by consumers' preferred qualities and obtain $\pi_H = \pi_L = 3/(16t)$ (see Cremer and Thisse, 1994). Under the partial market coverage assumption, the poorest consumers are not being served, so that demands are defined as follows:

$$x_H = \bar{\theta} - \frac{(p_H - p_L)}{q_H - q_L}; \quad x_L = \frac{(p_H - p_L)}{q_H - q_L} - \frac{p_L}{q_L}. \qquad (a6$$

Profits are defined as in (13). The standard solution concept is the subgame perfect equilibrium in two stages. Proceeding as usual by backward induction, one can solve for the simultaneous equilibrium in prices and then in qualities, whose generic expression is

$$q_i(\bar{\theta}) = \bar{\theta} q_i(\bar{\theta} = 1), \quad i = H, L. \qquad (a7$$

Finally, equilibrium profits are $\pi_H = 0.0164 \frac{\bar{\theta}^3}{t}$ and $\pi_L = 0.0122 \frac{\bar{\theta}^3}{t}$, or, in general, $\pi_i(\bar{\theta}) = \bar{\theta}^3 \pi_i(\bar{\theta} = 1)$. It is now possible to verify that the high (low) quality firm prefers partial to full market coverage if $\bar{\theta} > 2.2525$ (2.4897). Analogous conclusions can be reached when more than two varieties are produced.

References

Arrow, K. (1962), Economic Welfare and the Allocation of Resources for Invention, in R. Nelson (ed.), *The Rate and Direction of Inventive Activity*, Princeton, NJ, Princeton University Press.

Beath, J., Y. Katsoulacos and D. Ulph (1987), Sequential Product Innovation and Industry Evolution, *Economic Journal*, (Conference Papers), 97, pp.32-43.

Beath, J., Y. Katsoulacos and D. Ulph (1989), Strategic R&D Policy, *Economic Journal*, (Conference Papers), 99, pp.74-83.

Besanko, D., S. Donnenfeld and L. White (1987), Monopoly and Quality Distortion: Effects and Remedies, *Quarterly Journal of Economics*, 102, pp.743-68.

Bonanno, G. (1986), Vertical Differentiation with Cournot Competition, *Economic Notes*, 15, pp.68-91.

Bonanno, G. (1987), Location Choice, Product Proliferation and Entry Deterrence, *Review of Economic Studies*, 54, pp.37-46.

Cremer, H. and J.-F. Thisse (1994), Commodity Taxation in a Differentiated Oligopoly, *International Economic Review*, 35, pp.613-33.

Donnenfeld, S. and S. Weber (1992), Vertical Product Differentiation with Entry, *International Journal of Industrial Organization*, 10, pp.449-72.

Donnenfeld, S. and S. Weber (1995), Limit Qualities and Entry Deterrence, *Rand Journal of Economics*, 26, pp.113-30.

Eaton, B.C., and R. Lipsey (1979), The Theory of Market Preemptition: The Persistence of Excess Capacity and Monopoly in Growing Spatial Models, *Economica*, 46, pp.149-58.

Fudenberg, D. and J. Tirole (1986), *Dynamic Models of Oligopoly*, London, Harwood.

Gilbert, R. (1989), Mobility Barriers and the Value of Incumbency, in R. Schmalensee and R. Willig (eds.), *Handbook of Industrial Organization*, Amsterdam, North-Holland.

Gilbert, R. and D. Newbery (1982), Preemptive Patenting and the Persistence of Monopoly, *American Economic Review*, 72, pp.514-26.

Gilbert, R. and D. Newbery (1984), Preemptive Patenting and the Persistence of Monopoly: Comment, *American Economic Review*, 74, pp.238-42.

Harter, J. (1993), Differentiated Products with R&D, *Journal of Industrial Economics*, 41, pp.19-28.

Hay, D. (1976), Sequential Entry and Entry-Deterring Strategies in Spatial Competition, *Oxford Economic Papers*, 28, pp.240-57.

Itoh, M. (1983), Monopoly, Product Differentiation and Economic Welfare, *Journal of Economic Theory*, 31, pp.88-104.

Katz, M. and C. Shapiro (1987), R&D Rivalry with Licensing or Imitation, *American Economic Review*, 77, pp.402-20.

Motta, M. (1992), Cooperative R&D and Vertical Product Differentiation, *International Journal of Industrial Organization*, 10, pp.643-61.

Mussa, M. and S. Rosen (1978), Monopoly and Product Quality, *Journal of Economic Theory*, 18, pp.301-17.

Reinganum, J. (1983), Uncertain Innovation and the Persistence of Monopoly, *American Economic Review*, 73, pp.741-8.

Rosenkranz, S. (1995a), Innovation and Cooperation under Vertical Product Differentiation, *International Journal of Industrial Organization*, 13, pp.1-22.

Rosenkranz, S. (1995b), Quality Improvement and Cooperation under Vertical Product Differentiation, *International Journal of Industrial Organization*, forthcoming.

Shaked, A. and J. Sutton (1982), Relaxing Price Competition through Product Differentiation, *Review of Economic Studies*, 49, pp.3-13.

Shaked, A. and J. Sutton (1983), Natural Oligopolies, *Econometrica*, 51, pp.1469-83.

Spence, M. (1975), Monopoly, Quality and Regulation, *Bell Journal of Economics*, 6, pp.417-29.

Tirole, J. (1988) *The Theory of Industrial Organization*, Cambridge, MA, MIT Press.

Vickers, J. (1986), The Evolution of Market Structure when there is a Sequence of Innovations, *Journal of Industrial Economics*, 35, pp.1-12.