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**Working Paper**

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Center Discussion Paper, No. 357

**Provided in Cooperation with:**
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*Suggested Citation:* Binswanger, Hans P.; Quizon, Jaime B. (1980) : Factor and Output Market Effects of Technical Change and Public Investment Policies in Agriculture, Center Discussion Paper, No. 357, Yale University, Economic Growth Center, New Haven, CT

This Version is available at:
https://hdl.handle.net/10419/160282

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New Haven, Connecticut

CENTER DISCUSSION PAPER NO. 357

# FACTOR AND OUTPUT MARKET EFFECTS OF TECHNICAL CHANGE AND PUBLIC INVESTMENT POLICIES IN AGRICULTURE

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August 1980

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## Introduction

This paper presents a class of partial equlibrium models which investigate the effects of technical change and shifts in factor supplies and output demand on the equilibrium prices and quantities of output and factors of production in a particular sector of the economy. While the models can be applied to any sector they have been built with agriculture in mind and we will use this sector to talk about the models.

The models are extension of earlier work by Evenson and Welch (1974) and Evenson (1978) who treated a case of a sector with one producing region and two factors of production. The key idea of that model was to trace simultaneously the effects of technical change, factor supply shifts and output demand shifts on equilibrium prices and quantities in the land, labor and output markets. In each of these markets both demand and supply are assumed to be price responsive.

This early effort provided many insights. However since it used a production function framework it was difficult to extend to more than two factors of production, more than one region or more than one sector. Binswanger (1978) reformulated the model in terms of cost functions which made it amenable to such extensions and also put it into a framework in which was easier to estimate the required

parameters for empirical implementation.

The paper here is part of a larger project to formulate and estimate a model of factor and output price determination for the rural sector of India with several regions and a substantial number of factors of production, using the basic approach of the Evenson-Welch model. It is clear that for a many factor-many region model the number of analytically derivable solutions will not be very large. In this paper the basic equation system for this class of models is developed, starting with systems of factor demand and output supply equations which can be estimated empirically. This will be done in section 1. (In later papers we intend to apply this model to India and the Philippines to evaluate the income distribution impact of policies aimed at changing rates and biases of technical change and influencing factor supplies and output demand in different regions.)

Here we want to use the same systems of equations to derive analytical results analogous to the Evenson-Welch model at the largest possible level of generality, by increasing the number of factors of production and the number of regions. The general system of equation derived first forms a class of models of which we will consider several submodels. The submodels are distinguished by (1) the number of factors in price responsive supply, (2) the number of factors in exogenous (fixed) supply, (3) the number of factors in infinitely elastic supply (or factors with fixed prices), and (4) the number of regions. The models are described in the table below

with the sections in which they are discussed.

| Section No. | Model Class | No.of Price Responsive Factors | No.of Factors With Fixed Supply | No.of Factors With Fixed Price | Regions |
|---|---|---|---|---|---|
| 2 | Evenson/Welch | 2 | 0 | 0 | 1 |
| | 2/0/m | 2 | 0 | m | 1 |
| 3 | 2/1/0 | 2 | 1 | 0 | 1 |
| 4 | 1/1/0/ML | 1 | 1 | 0 | 2, with mobile labor |
| 5 | 1/1/0/IL | 1 | 1 | 0 | 2, with immobile labor |

In agriculture it is often useful to consider the supply of land as fixed and that is the rationale for the development of those models above where land is fixed. Furthermore, intermediate inputs are often supplied to the agricultural sector from other sectors that are able to expand production easily and whose prices can therefore be considered to be technologically determined or fixed exogenously. One of the submodels, therefore, treats the case wherein such factors are in infinitely elastic supply.

Sections 1 and 2 discuss the approach and develop the first model with a verbal discussion of the results. Sections 3, 4 and 5 are mainly proofs, repeating the pattern of sections 1 and 2 for different models. Sections 6 and 7 return more to economic implications.

Note that this paper is an inventory of distributional effects for a large class of cases. Its use would primarily be for

readers interested in particular special cases for which they can make use of specific equations. Furthermore, the approach is far from exhausted. For example, we do not specifically develop the equations for the quantity effects. However these effects can be solved for by combining the factor supply equations of the models with the factor price solutions, and the output demand equations with the output price solutions in a straightforward manner.

Sections 1 and 2 and Appendix A follow closely a set of unpublished notes by Binswanger (1979) while Section 3 and Appendix B are largely drawn from chapters 2 and 4 of Quizon (1980).

Finally note that a complete list of symbols is given in table 4 at the end of this paper. Furthermore many of the proofs involve a set of relationships which has been summarized in Table 1 for easy reference.

TABLE 1

RELATIONSHIPS BETWEEN DIFFERENT CONCEPTS OF ELASTICITIES OF FACTOR DEMAND AND OUTPUT SUPPLY

| A. Profit Maximization With One Factor in Fixed Supply (1) | (2) | (3) | A-1 Two Factor Specialization (4) | B. Profit Maximization With Elastic Land Supply (5) | (6) |
|---|---|---|---|---|---|
| Elasticities | | Assumptions | | | |
| $\beta_{ij} = \eta_{ij} - \eta_{Zj} + \frac{s_j}{s_Z}(\eta_{ZZ} - \eta_{iZ})$ | $\lesseqgtr$ | 0 | | $\gamma_{ij} = \beta_{ij} - \frac{s_j}{s_Z}\epsilon_Z$ | |
| $\beta_{ii} = \eta_{ii} - \eta_{Zi} + \frac{s_i}{s_Z}(\eta_{ZZ} - \eta_{iZ})$ | $\leq$ | 0 | convexity | $\beta_{LL} = -\frac{1}{s_Z}\sigma = -\beta_{LY}$ | $\gamma_{ii} = \beta_{ii} - \frac{s_i}{s_Z}\epsilon_Z \leq 0$ |
| $\beta_{iY} = \frac{1}{s_Z}(\eta_{iZ} - \eta_{ZZ})$ | $\geq$ | 0 | noninferiority | $\beta_{LY} = \frac{1}{s_Z}\sigma = -\beta_{LL}$ | $\gamma_{iY} = \beta_{iY} + \frac{1}{s_Z}\epsilon_Z \geq 0$ |
| $\beta_{Yj} = \frac{s_j}{s_Z}(\eta_{ZZ} - \eta_{jZ})$ | $\leq$ | 0 | noninferiority | $\beta_{YL} = \frac{s_L}{s_Z}\sigma = -\beta_{YY}$ | $\gamma_{Yj} = \beta_{Yj} - \frac{s_j}{s_Z}\epsilon_Z \leq 0$ |
| $\beta_{YY} = -\frac{1}{s_Z}\eta_{ZZ}$ | $\geq$ | 0 | convexity | $\beta_{YY} = \frac{s_L}{s_Z}\sigma = -\beta_{YL}$ | $\gamma_{YY} = \beta_{YY} + \frac{1}{s_Z}\epsilon_Z \geq 0$ |
| Technology Shifts | | | | | Technology Shifts |
| $E_i' = \beta_{iY}T' + A_Z' - A_i'$ | | | | | $E_i' = \gamma_{iY}T' + A_Z' - A_i'$ |
| $E_Y' = \beta_{YY}T' + A_Z'$ | | | | | $E_Y' = \gamma_{YY}T' + A_Z'$ |

Notes:

a) Further relations used extensively in proofs

1) $\beta_{iY} \geq 0$, i.e., $\eta_{ii} - \eta_{ji} \leq 0$

2) $\eta_{ii} - \eta_{ji} = -\sigma$ is the two factor case

3) $\eta_{ji} = (s_i/s_j)\eta_{ij}$

4) $\sum_j \eta_{ij} = 0$

5) $\beta_{ij} = \frac{s_j}{s_i}\beta_{ij}$

6) $\beta_{yj} = -s_j\beta_{jy}$

7) $\sum_j \beta_{ij} + \beta_{iy} = 0$; $\sum_j \beta_{yj} + \beta_{yy} = 0$

b) Definitions

1) $\eta_{ij}$ are cost function factor demand elasticities

2) $T'$ is the rate of technical change

3) $A_i'$ are the factoral rates of technical change of the cost function derived factor demand curves

4) $s_i$ are factor shares in total product

5) $\beta_{ij}$ are profit function factor demand or output supply elasticities when one factor is in fixed supply

6) $\gamma_{ij}$ are factor demand and output supply elasticities when no factor is in fixed supply

7) $\sigma$ is the elasticity of substitution in the two factor case

8) Z is the fixed factor land

1) The basic model and its relation to cost function and technical change parameters.

The models of this paper are based on profit functions which correspond uniquely to production functions. However, for the interpretation of technical change parameters we also need the link between the profit function and cost functions and we breifly digress on this issue.

The correspondence between cost functions and variable profit functions.

We start from a production function $Y = F(V,t)$ where Y is output, V is a vector of factors of production and t is a technology shifter. The following conditions are imposed on the production function: (a) It is twice differentiable in V and (b) homogeneous of degree 1 in V; (c) strictly increasing in V; (d) strictly concave in V over its effective domain; (e) and Y is finite for all finite V and unbounded as V approaches infinity. For a discussion of these conditions see Jorgenson and Lau (1974). Profit functions also exist under weaker conditions, but the conditions above are necessary for the correspondences to hold on which this paper is based.

Let U be the prices of factors of production which are exogenous to the firm and let producers minimize cost of production $C^* = V' U$. Then a unique cost function exists $C = C^*(Y, U, t)$ obeying certain regularity conditions. A unique set of factor demand curves and monotonicity conditions is given by Shephards lemma stating that

$$(1.1)\quad \frac{\partial C}{\partial U_i} = V_i = V_i(Y, U, t) > 0$$

The cost function and the factor demand curves defined by the production function and the cost minimizing problem correspond to each other and to the production function in a one to one fashion. (For a

full discussion of these one to one relationships see Jorgenson and Lau, 1974). This means that information about the characteristics of the production process can be recovered from either the factor demand curves or the cost function.

The vector of inputs can be partitioned into variable and fixed inputs $V = (X, Z)$ where X are the variable inputs and Z the fixed inputs. The corresponding vector of factor prices might be rewritten as $U = (W,S)$ where W and S are the prices of variable and fixed factors respectively. Enterpreneurs, instead of minimizing costs of all factors with a fixed output, may maximize variable profits $\Pi^* = PY - X'W$ subject to $F(X, Z, t)$ (where F is the same function as the one used for the cost minimization problem). A unique variable profit function $\Pi^* (P, W, Z, t)$ corresponds to this problem with Shephard's lemma providing the following unique factor demand and output supply equations and monotonicity conditions.

$$(1.2) \quad -X_i = \Pi_i (W,P,Z,t) \leq 0$$

$$Y = \Pi_Y (W,P,Z,t) \geq 0$$

where $\Pi_Y$ is the derivative with respect to the output price P and $\Pi_i$ is the derivative with respect to input price $W_i$.

Production function variable profit function and factor demand curves correspond to each other in a unique one-to-one fashion, i.e. information about the technology can be recovered from the variable profit function or the output supply and factor demand curves and vice versa. The uniqueness of the duality relation between the production function and the set of cost minimizing factor demand curves (1.1) and the uniqueness of the set of the duality relations between the production function and the profit

maximizing output supply and factor demand curves in (1.2) implies further that (1.1) and (1.2) are related to each other in a one-to-one fashion, i.e. that we can recover all information in (1.1) from (1.2) and vice versa. We now return to stating the model in its profit function version.

The Profit Function Formulation

These factor demand and output supply equations in (1.1) have the following slopes and symmetry conditions, in addition to the monotonicity conditions discussed before.

$$(1.3) \qquad \frac{\partial X_i}{\partial W_j} = \frac{\partial X_j}{\partial W_i} = -\Pi_{ij} = -\Pi_{ji}$$

$$(1.4) \qquad \frac{\partial Y}{\partial W_i} = -\frac{\partial X_i}{\partial P} = \Pi_{Yi} = \Pi_{iY}$$

where the subscripts denote derivatives of the profit function with respect to the prices of the variables indicated in the subscript.

Differentiating the equations in (1.2) totally and changing signs

$$(1.5) \qquad dX_i = -\sum_{j=1}^{n-1} \Pi_{ij}\, dW_j - \Pi_{iY}\, dP + \frac{\partial X_i}{\partial Z}\, dZ + \frac{\partial X_i}{\partial t}\, dt$$

$$dY = \sum_{j=1}^{n-1} \Pi_{Yj}\, dW_j + \Pi_{YY}\, dP + \frac{\partial Y}{\partial Z}\, dZ + \frac{\partial Y_i}{\partial t}\, dt$$

Now for any variable Q let its rate of change over time be denoted as $Q' = \frac{\partial Q}{\partial t}\frac{1}{Q}$

Then the above equations can be transformed into rates of changes.

$$(1.6) \qquad X_i' = \sum_{j=1}^{n-1} \beta_{ij} W_j' + \beta_{iY} P' + \beta_{iZ} Z^* + E_i'$$

$$Y' = \sum_{j=1}^{n-1} \beta_{Yj} W_j' + \beta_{YY} P' + \beta_{YZ} Z^* + E_Y'$$

The notation is explained in (1.7) where the right hand side also provides relations among elasticities which follow directly from the symmetry constraints (1.3).

(1.7)

| | |
|---|---|
| Elasticities of factor demand with respect to factor prices | $\beta_{ij} = -\Pi_{ij} W_j/X_i = \frac{s_j}{s_i} \beta_{ji}$ |
| Elasticities of factor demand with respect to output prices | $\beta_{iY} = -\Pi_{iY} P/X_i = -\frac{1}{s_i} \beta_{Yi}$ |
| Output supply elasticities with respect to input prices | $\beta_{Yj} = \Pi_{Yj} W_j/Y = -s_j \beta_{jY}$ |
| Output supply elasticities with respect to output prices | $\beta_{YY} = \Pi_{YY} P/Y$ |
| Share of factor i in total output | $s_i = \frac{W_i X_i}{PY}$ |
| Factor demand and output supply shifts due to technical change given output and factor prices | $\begin{cases} E_i' = \frac{\partial X_i}{\partial t} \frac{1}{X_i} \\ E_Y' = \frac{\partial Y}{\partial t} \frac{1}{Y} \end{cases}$ |
| Factor demand and output supply shifts due to a shift in supply of the fixed factor. | $\beta_{iZ}$ and $\beta_{YZ}$ |

Note further that profit functions are homogeneous of degree one in input and output prices. Therefore the factor demand and output supply equations (1.2) are homogeneous of degree zero in input and output prices, which implies that

(1.8) $$\sum_j \beta_{ij} + \beta_{iY} = 0$$

$$\sum_j \beta_{Yj} + \beta_{YY} = 0$$

For the theoretical discussion we admit only one fixed factor Z, which we will call land. There may of course be more than one fixed factor in any empirical application and then this can be handled straightforwardly. For theory purposes we can also regard Z as an appropriate index of all fixed factors with the fixed factors separable from the variable factors.

If the production process is homogeneous of degree one in all factors of production (variable and fixed), then the profit function is homogeneous of degree 1 in the fixed factor (Diewert 1978). This implies that

$$(1.9) \quad \beta_{iZ} = \beta_{YZ} = 1$$

and this will be assumed throughout.

System (1.6) can be closed by first adding n factor supply equations in rates of changes

$$(1.10) \quad X_i' = \varepsilon_i W_i' + X_i^*$$

where $\varepsilon_i$ is a factor supply elasticity and $X_i^*$ are exogenously given rates of increase in factor supply. Second we add an output demand curve

$$(1.11) \quad Y' = \alpha P' + D^*$$

where $\alpha$ is the output demand elasticity and $D^*$ is an exogenous demand shifter. Now consider the three factor case with one output where

L = labor, K = capital, W = wage rate and R = capital rental rate, and Z is the third factor land. Cases with n factors are obvious generalizations. Combining (1.6) with (1.10) and (1.11) leads to the following matrix formulation

(1.12)
$$\begin{bmatrix} \beta_{LL} - \varepsilon_L & \beta_{LK} & \beta_{LY} \\ \beta_{KL} & \beta_{KK} - \varepsilon_K & \beta_{KY} \\ \beta_{YL} & \beta_{YK} & \beta_{YY} - \alpha \end{bmatrix} \begin{bmatrix} W' \\ R' \\ P' \end{bmatrix} = \begin{bmatrix} L^* - Z^* - E_L' \\ K^* - Z^* - E_K' \\ D^* - Z^* - E_Y' \end{bmatrix}$$

The extension of (1.12) to the case of many factors is obvious and can be written in more compact notation as

(1.13) $GW' = K^*$

where G is called the <u>excess elasticity matrix</u> since it has excess elasticities on the diagonal. Note that

(1.14) $G = [\beta - \varepsilon]$

where $\beta = [\beta_{ij}]$ is the elasticity matrix and $\varepsilon = \text{diag}\,[\varepsilon_K, \varepsilon_L, \alpha]$. $W'$ is the vector of endogenous rates of price changes and $K^*$ is the vector of exogenous rates of changes in factor supplies, output demands and technology shifters.

The solution to the system obviously is

(1.15) $W' = G^{-1} K^*$

which exists G is nonsingular.[1]

With numerical estimates of the relevant parameters one can therefore always solve for the implied changes in factor prices and output prices of any known combination of changes in factor supply, output demand and technology shifters. Given the price changes one can find the changes in input levels and output levels via equations (1.10) and (1.11.) Using equation (1.23) given below one can also solve for the implied change in the land rent, S, i.e. for s'.

Before proceeeding we briefly point out the link of the analysis of the quantity and price effects with the analysis of producer incomes and their income distribution. It is possible to measure the nominal income impact of the change in factor prices on a specific producer. Let $\delta_{ik}$ be the share of income of producer k arising out of factor i, and let $M_k = L_k W + K_k R + Z_k S$ be income where $L_k$, $K_k$ and $Z_k$ are the fixed quantities of factors owned by the producer and used in the sector considered his income change then is[2]

(1.16) $M'_k = \delta_{Lk} W' + \delta_{Kk} R' + \delta_{Zk} S'$

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[1] G will usually be nonsingluar if at least one of the elements of $(\varepsilon_i, \alpha)$ is nonzero. The $\beta$ matrix is derived from the Hessian matrix of the profit function in the appendix. If there are n inputs and one output the Hessian matrix of the profit function usuallly is singular of rank n. It is of course possible to impose separatability constraints on the production process which will reduce the rank of the Hessian matrix below n, in which case more than one of the elements of $\varepsilon_i$, $\alpha$) must be non-zero to ensure a solution to 1.15. In empirical application such separatability restrictions are unlikely to hold.

[2] If the proper experiences changes in factor endowments, equation (1.16) will also include terms in $L'_k K'_k$ and $Z'_k$ which must be derived from group specific factor supply equations.

where S' is derived from W', R' and P' via equation (1.23). This income is counted in units of the numeraire which in this case is nonagricultural commodities. Suppose that, for income group k, we know the shares of expenditures $\mu_{hk}$ on the different commodities, where h is a commodity index (h = 1 for agriculture and h = 2 for nonagriculture). Then we can compute an income group specific price index

$$(1.17) \qquad \bar{P}_k' = \mu_{1k} P_1' + \mu_{2k} P_2' = \mu_{1k} P_1'$$

because $P_2' = 0$.

The change in real income of producer group k then becomes

$$(1.18) \qquad \bar{M}_k' = M_k' - \bar{P}_k'$$

Note that this approach is an extension of the way in which Hayami and Herdt (1977) considered income effects of the Green Revolution on small and large farmers. The same price index can also be used to deflate individual factor prices, for example the real wage rate change for producer k is $\bar{W}_k' = W' - \bar{P}_k'$. Below we will give analytical solutions for one special case.

In the following section we will push the model for analytical solutions of as much generality as we can. Obviously, as the number of factors or regions gets larger, the number of general analytical results gets smaller. Before we can turn to that note, however, that

the technology shifters $E_i'$ and $E_Y'$ in equations (1.6) or (1.17) are not directly interpretable in terms of traditional technical change concepts such as rates and biases of technical change. We therefore need to relate the equations in (1.6) to the cost function formulation.

The Cost Function Formulation

We rewrite the cost function with Z and S still denoting land and its price but land assumed variable whereas output Y is assumed fixed, i.e. $C = C(Y,W,S,t)$. Hence the factor demand equations (1.1) now read $X_i = X_i(Y,W,S,t)$ and

$$Z = Z(Y,W,S,t)$$

Differentiating these equations totally and converting them to rates of changes as in the case of the profit function leads to the following factor demand equations:

$$(1.19) \quad X_i' = Y' + \sum_{j=1}^{n-1} \eta_{ij} W_j' + \eta_{iZ} S' - A_i' \qquad \forall\ i < n$$

$$(1.20) \quad Z' = Y' + \sum_{j=1}^{n-1} \eta_{Zj} W_j + \eta_{ZZ} S' - A_Z'$$

$\eta_{ij}$ are factor demand elasticities with fixed outputs and

$A_i'$ are factoral rates of technical change, i.e.

$A_i' = -(\partial X_i/\partial t)(1/X_i)$, given factor prices and output levels.[1]

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[1] For a discussion of factoral rates of technical change see Binswanger and Ruttan (1978, Chapters 4 and 5).

Under competition the following relation will hold for the output price:

$$(1.21) \quad P' = \sum_{j=1}^{n-1} s_j W_j' + s_Z S' - T'$$

where $s_i$ are factor shares in value of output. Recall from the beginning of this section that 1.1 and 1.2 correspond to each other uniquely. Therefore the system 1.19 and 1.20 corresponds uniquely to the system 1.6 and it is possible to express 1.6 in terms of the parameters and variables of equations 1.19 and 1.20. To do this we hold $Z'$ fixed in equation 1.20 i.e. we replace it by Z*. We can then solve it for Y' as follows:

$$(1.22) \quad Y' = -\sum_{j=1}^{n-1} \eta_{Zj} W_j' - \eta_{ZZ} S' + A_Z' + Z^*$$

Furthermore we can solve equation (1.21) for S', the rate of change in the land rent

$$(1.23) \quad S' = \frac{1}{s_Z} (T' + P' - \sum_{j=1}^{n-1} s_j W_j')$$

(In all further models equation (1.23), or variations thereof, will be used to determine the land rent residually, once the models have been solved for the output price changes and the input price changes. Note that the rate of change in land rents is equal to the rate of technical change plus the rate of increase in output prices minus the share weighted sum of increases in all other factor prices.)

Now replace (1.23) into (1.22) and find the following supply equation[1]:

$$(1.24) \qquad Y' = \sum_{j=1}^{n-1} \frac{s_j}{s_Z} (\eta_{ZZ} - \eta_{jZ}) W_j' - \frac{\eta_{ZZ}}{s_Z} P' - \frac{\eta_{ZZ}}{s_Z} T' + A_Z' + Z^*$$

Note that if technical change is Hicks neutral with respect to all factors $A_Z' = T'$ and the expression for technical change simplifies somewhat.

To find the factor demand equations, set equations (1.23) and (1.24) into equations (1.19)

$$(1.25) \qquad X_i' = \sum_{j=1}^{n-1} \left[ \eta_{ij} + \eta_{Zj} + \frac{s_j}{s_Z} (\eta_{ZZ} - \eta_{iZ}) \right] W_j' + \frac{1}{s_Z} (\eta_{iZ} - \eta_{ZZ}) P'$$

$$+ \frac{1}{s_Z} (\eta_{iZ} - \eta_{ZZ}) T' + A_Z' - A_i' + Z^*$$

We thus have transformed to a system of one supply equation and n-1 factor demand equations. These equations are uniquely related to those derived from the profit function which corresponds to the profit maximizing problem with n-1 variable factors, one variable output and one fixed factor of production Z, and the underlying linear homogenous production function. Therefore the coefficients of $W'$ in equations (1.24) and (1.25) are equal to $\beta_{ij}$ and $\beta_{Yj}$ of equations (1.6). Similarly the coefficients of $P'$, (and by extension those of $T'$) are equal to

---

[1]Note that in the proof you use the relation $\frac{s_j}{s_Z} \eta_{jZ} = \eta_{Zj}$

$\beta_{ij}$ and $\beta_{Yj}$ respectively. Therefore the $E_i'$ variables are now interpreted as follows:

$$(1.26) \quad E_i' = \beta_{iY} T' + A_Z' - A_i'$$

$$E_Y' = \beta_{YY} T' + A_Z'$$

All these relations between elasticities and technology shifters are summarized in panel A of Table 1. Note that the technology shifter in each factor demand curve of the profit function is the rate of technical change weighted by the input demand elasticity of an output price change, minus the bias of the factor i relative to land, i.e. $B_{iZ} = A_i - A_Z$.

Noninferiority

In all models discussed below, not many conclusions can be reached whenever the number of factors of production exceeds two, unless we impose additional constraints on the profit function. The basic reason for sign indeterminacy is that factors of production can be complements. We know from empirical studies that complementarity relations are not infrequent in production processes (Binswanger 1973),
However, especially in agriculture, it is unlikely that inferior factors of production exist whose input is reduced when the scale of output is increased, unless an extraordinary level of disaggregation of factors of production is used. We therefore make the following noninferiority assumption

$$(1.26) \quad \beta_{iY} = \frac{1}{s_Z} (\eta_{iZ} - \eta_{ZZ}) \geq 0$$

More generally, treating all factors of production as potential fixed factors we assume

$$(1.27) \quad \eta_{ii} - \eta_{ji} \leq 0$$

Complementarity of factors of production would mean that all $\eta_{ji} \geq 0$. However, here we admit complementarity but restrict the size of $\eta_{ji}$ (when it is negative) to be of smaller absolute size than $\eta_{ii}$. For a five factor model of U.S. agriculture Binswanger (1973) has found this constraint to hold for all pairs of factors of production which he considered.

Land in elastic supply

The Evenson-Welch model is one with two factors of production in elastic supply and no other factors. An important case below will be an extension of that model. We can derive the case where land is also in elastic supply from the cost function formulation as well. When land is not supplied exogenously equations (1.19) and (1.21) are unaffected but equation (1.20) has to incorporate the land supply relationship $Z' = \varepsilon_Z S' + Z^*$, and becomes

$$(1.28) \quad Z^* = Y' + \sum_{j=1}^{n-1} \eta_{Zj} W_j + (\eta_{ZZ} - \varepsilon_Z) S' - A_Z'$$

where $\eta_{ZZ} - \epsilon_Z$ is the excess demand elasticity of land when output is fixed.

Going through the same substitutions then before, with equation (1.28) replacing equation (1.20) in the system (1.19), (1.20), (1.21), leads to the expressions for factor demand and output supply elasticities and technology shifters given in Panel B of Table 1. Note that we simply replace all $\eta_{ZZ}$ values by $(\eta_{ZZ} - \epsilon_Z)$ and all steps are the same. We then get a system of input demands and output supply equations as follows

$$(1.29) \qquad X_i' = \sum_{j=1}^{n-1} \gamma_{ij} W_j' + \gamma_{iY} P' + Z^* + \tilde{E}_i'$$

$$Y_i' = \sum_{j=1}^{n-1} \gamma_{Yj} W_j' + \gamma_{YY} P' + Z^* + \tilde{E}_Y'$$

This is the most general form of output supply and factor demand functions of which equations (1.6) of the profit function with exogenously fixed land are a special case.

## 2) An Extension of The Evenson-Welch Two-Factor Model.

The Evenson-Welch model deals with two primary factors in price responsive supply. In this section we add to the two primary factors m factors which are in perfectly elastic supply, i.e. whose prices are fixed.

Consider the three factor case of equations 1.12 and assume that land is also in price responsive supply, i.e. that all the $\beta$ in these equations are replaced by $\gamma$ and the $E'$ terms by $\hat{E}'$. Suppose that capital were in infinitely elastic supply. This additional constraint provides the solution for $R'$, i.e. $R' = 0$. Therefore the system of equation reduces to 2 equations which would be solved for $W'$ and $P'$. The excess elasticity matrix reduces to

$$G = \begin{bmatrix} \gamma_{LL} - \varepsilon_L & \gamma_{LY} \\ \gamma_{YL} & \gamma_{YY} - \alpha \end{bmatrix}$$

Note that by similar reasoning the two equation system so found can be considered as corresponding to a system in which there are two primary factors in price responsive supply and m factors in infinitely elastic supply, i.e. for which we already know that the corresponding price changes are zero. Note further that the earlier equation system 1.12 can also be considered as a genuine 2 factor case or a case of two primary factors and m intermediate factors in infinitely elastic supply.

The solution to the Evenson-Welch model with m factors of production with fixed prices is given in equation 2.1

$$(2.1) \qquad \begin{bmatrix} W' \\ P' \end{bmatrix} = \frac{1}{|G|} \begin{bmatrix} \gamma_{YY} - \alpha & -\gamma_{LY} \\ -\gamma_{YL} & \gamma_{LL} - \varepsilon_L \end{bmatrix} \begin{bmatrix} L^* - Z^* - \gamma_{LY} T' - A_Z' + A_L' \\ D^* - Z^* - \gamma_{YY} T' - A_Z' \end{bmatrix}$$

Expanding $|G|$ according to the formulas on Table 1 we find that

$$(2.2) \qquad |G| = \varepsilon_L \alpha - \alpha\gamma_{LL} - \varepsilon_L \gamma_{YY}$$

$$+ \beta_{LL} \beta_{YY} - \beta_{YL} \beta_{LY}$$

$$+ \frac{\varepsilon_Z}{s_Z} (s_L \beta_{LY} - s_L \beta_{YY} + \beta_{LL} - \beta_{YL}) < 0$$

The first line of (2.2) is negative because all terms are negative. The second line is the determinant of a principle minor the $\beta$ matrix which is negative according to rules 1 and 3A of Appendix A which discusses the signs of all other determinants. The third line reduces to $\eta_{LL} \leq 0$ when expanded using the formulas of Panel A of Table 1.

Shifts in Factor Supply: From (2.1) and (2.2) it follows immediately that

$$(2.3) \qquad \frac{\partial W'}{\partial L^*} = |G|^{-1} (\gamma_{YY} - \alpha) = |G|^{-1} (\beta_{YY} + \frac{1}{s_Z} \varepsilon_Z - \alpha) \leq 0$$

$$(2.4) \qquad \frac{\partial P'}{\partial L^*} = -|G|^{-1} \gamma_{YL} = - |G|^{-1} (\beta_{YL} - \frac{s_L}{s_Z} \varepsilon_Z) \leq 0$$

A positive shift in labor supply will reduce the wage rate measured in units of the nonagricultural good.[1] When noninferiority holds, it will also reduce the output price of agricultural output.

In the developed country context most agricultural laborers will consume a commodity bundle consisting primarily of nonagricultural goods and the wage rate measured in those goods is a good welfare indicator. In developing countries, however, agricultural laborer's expenditures consist primarily of agricultural commodities. Therefore it might be more appropriate to consider the change in wages measured in agricultural commodities, i.e. deflated by the agricultural price. This we can do as follows

$$(2.5)\quad \frac{\partial (W/P)'}{\partial L^*} = \frac{\partial W'}{\partial L^*} - \frac{\partial P'}{\partial L^*} = |G|^{-1} (\gamma_{YY} + \gamma_{YL} - \alpha)$$

$$= |G|^{-1} (\beta_{YY} + \beta_{YL} + \frac{1-s_L}{s_Z} \varepsilon_Z - \alpha)$$

$$= |G|^{-1} (-\sum_{K=1}^{m} \beta_{YK} + \frac{1-s_L}{s_Z} \varepsilon_Z - \alpha) \leq 0$$

The expression on the third line (which follows from (1.8) is negative because noninferiority implies that all $\beta_{YK}$, the supply elasticities with respect to factors with fixed prices, are less than zero. In the Evenson-Welch model there are no such factors and all $\beta_{YK}$ are zero. Thus the expression reduces to $\frac{\partial (W/P)'}{\partial L^*} = |G|^{-1} [\varepsilon_Z - \alpha]$.

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[1] In our verbal discussions we will often not mention the fact that we are talking about effects of a change in the rate of a variable (say the labor supply $L^*$) on the risk of growth of another variable (say the wage rate $W^*$). This would unduly complicate the language without adding any substantive element.

Equations (2.3) and (2.5) thus imply that positive shifts in labor supply hurt rural labor regardless of the numeraire used. However, the loss in terms of nonagricultural goods is larger than the loss in terms of agricultural goods. Note that these results carry through all models of the later sections. Since this particular model treats land and labor symmetrically we also know that shifts in land supply will reduce land rents, however they may be measured, i.e.

$$\partial S'/\partial Z^* \leq 0, \qquad \frac{\partial (S/P)'}{\partial Z^*} \leq 0.$$

Positive shifts in land supply can affect the wage rate either way, i.e. the cross-supply effects are indeterminate.

$$(2.6) \qquad \frac{\partial W'}{\partial Z^*} = -|G|^{-1} (\gamma_{YY} - \gamma_{LY} - \alpha) = -|G|^{-1}(\beta_{YY} - \beta_{LY} - \alpha) \qquad 2/0/m$$

$$= |G|^{-1} [\sigma_{LZ} + \alpha] \gtrless 0 \qquad 2/0/0$$

An increase in land supply will lead to an increase in the nominal wage if the elasticity of final demand exceeds the elasticity of substitution between land and labor in absolute value.

The output price effect of the land supply increase is negative because of noninferiority.

$$(2.7)\qquad \frac{\partial P'}{\partial Z^*} = |G|^{-1}\,[\gamma_{YL} - \gamma_{LL} + \varepsilon_L] = |G|^{-1}(\beta_{YL} - \beta_{LL} + \varepsilon_L)$$

$$= |G|^{-1}\,[\eta_{ZL} - \eta_{LL} + \varepsilon_L] \leq 0 \qquad 2/0/m$$

$$= |G|^{-1}\,[\sigma + \varepsilon_L] \leq 0 \qquad 2/0/0$$

The second line is the 2/0/m case with additional fixed-price factors, while the third line is the 2/0/0 Evenson-Welch case. In both cases the output price effect is negative.

Combining equations (2.6) and (2.7) as before leads again to the wage effect in terms of agricultural goods.

$$(2.8)\qquad \frac{\partial (W/P)'}{\partial Z^*} = |G|^{-1}\,[\beta_{LY} - \beta_{YY} + \beta_{LL} - \beta_{YL} + \alpha - \varepsilon_L]$$

$$= |G|^{-1}\,[\sum_{K=1}^{m} \beta_{YK} - \sum_{K=1}^{m} \beta_{LK} + \alpha - \varepsilon_L] \gtrless 0 \qquad 2/0/m$$

$$= |G|^{-1}\,[\alpha - \varepsilon_L] \gtrless 0 \qquad 2/0/0$$

While the Evenson-Welch model (third-line) predicts that wages in terms of agricultural goods will rise as land supply increases, this cannot be shown as soon as more factors of production are included, in which case the labor demand elasticities and output supply elasticities with respect to "left out" factors become important as well, and only knowledge of these elasticities will allow signing of equation (2.8).

Output Demand Effects

The effects on wages and prices of a shift in final demand are both positive as can be read off (2.1) directly

$$(2.9) \qquad \frac{\partial W'}{\partial D^*} = -|G|^{-1} \gamma_{LY} = -|G|^{-1} (\beta_{LY} + \frac{1}{s_Z} \varepsilon_Z) \geq 0$$

$$(2.10) \qquad \frac{\partial P'}{\partial D^*} = |G|^{-1} (\gamma_{LL} - \varepsilon_L) = |G|^{-1} (\beta_{LL} - \frac{s_L}{s_Z} \varepsilon_Z - \varepsilon_L) \geq 0$$

The sign of (2.9) depends on the noninferiority assumption.

The wage effect in terms of agricultural prices is given by

$$(2.11) \qquad \frac{\partial (W/P)'}{\partial D^*} = -|G|^{-1} (\gamma_{LL} + \gamma_{LY} - \varepsilon_L)$$

$$= -|G|^{-1} (\beta_{LL} + \beta_{LY} + \frac{1-s_L}{s_Z} \varepsilon_Z - \varepsilon_L)$$

$$= -|G|^{-1} (-\sum_{K=1}^{m} \beta_{LK} + \frac{1-s_L}{s_Z} \varepsilon_Z - \varepsilon_L) \gtrless 0 \qquad 2/0/m$$

$$= |G|^{-1} (\varepsilon_L - \varepsilon_Z) \gtrless 0 \qquad 2/0/0$$

The sign depends on the difference of the supply elasticities of land and labor and, when "left out" factors exist, on the labor demand elasticities with respect to these factors as well. A special case which will be important later is the case in which $\varepsilon_Z$ is zero and in which the third line of (2.11) is negative. The wage rate rise is not as large as the output price rise and labor loses if agricultural commodities are the numeraire. The greatest gains go to the factor in inelastic supply.

Neutral Technical Change

In the neutral technical change case $A_Z' = A_L' = T'$. Consider first the price reduction associated with technical change, and expand the determinant

$$(2.12) \quad \frac{\partial P'}{\partial T'} = -\frac{(\gamma_{YY}+1)(\gamma_{LL}-\varepsilon_L) - \gamma_{LY}\gamma_{YL}}{(\gamma_{YY}-\alpha)(\gamma_{LL}-\varepsilon_L) - \gamma_{LY}\gamma_{YL}} \leq 0$$

$$= -\frac{(1+\alpha)(\gamma_{LL}-\varepsilon_L) + (\gamma_{YY}-\alpha)(\gamma_{LL}-\varepsilon_L) - \gamma_{LY}\gamma_{YL}}{(\gamma_{YY}-\alpha)(\gamma_{LL}-\varepsilon_L) - \gamma_{LY}\gamma_{YL}}$$

$$= -|G|^{-1}(1+\alpha)(\gamma_{LL}-\varepsilon_L) - 1$$

$$= -(1+\alpha)\frac{\partial P}{\partial D^*} - 1$$

The first line of (2.12) establishes the sign, because the numerator and denominator are of opposite sign but their size is the same except that the $-\alpha$ of the denominator becomes $+1$ in the numerator. Therefore it follows that

$$(2.13) \quad \frac{\partial P'}{\partial T'} \gtreqless -1 \Rightarrow |\alpha| \gtreqless 1$$

i.e. the price falls by less than the rate of technical change if the elasticity of final demand exceeds 1 and by more than the rate of technical change if final demand is inelastic. This condition is independent of any factor supply elasticities and the number of

factors in infinitely elastic supply. It will also carry through to the genuine n-factor case discussed in Appendix B. The other lines of (2.12) are a further decomposition used below.

Since the rate of technical change is equal to the rate of unit cost reduction, factors of production engaged in agriculture must lose in terms of prices of nonagricultural commodities if final demand is inelastic since--at constant factor prices--the cost reduction will be smaller than the output price reduction and some of the price responsive factors must experience a decline.

This is borne out by the wage rate equation:

$$(2.14) \qquad \frac{\partial W'}{\partial T'} = |G|^{-1} \left[ - (\gamma_{YY} - \alpha) \gamma_{LY} + (\gamma_{YY} + 1) \gamma_{LY} \right]$$

$$= |G|^{-1} \gamma_{LY} (\alpha + 1) \gtrless 0$$

$$= -(\alpha + 1) \frac{\partial W'}{\partial D^*} \gtrless 0$$

The second line of this expression shows--as expected--that the wage effect of technical change is positive if the final demand is elastic ($|\alpha| > 1$) and negative if it is inelastic. Furthermore we can express the technical change effect as a constant multiple (of opposite sign) of the final demand effect with the constant depending only on the final demand elasticity.

Combining (2.12) and (2.14) leads to the real wage effect

$$(2.15)\quad \frac{\partial(W/P)'}{\partial T'} = 1 - (\alpha+1)\left[\frac{\partial W'}{\partial D^*} - \frac{\partial P}{\partial D^*}\right] = 1 - (\alpha+1)\frac{\partial(W/P)'}{\partial D^*}$$

$$= 1 - |G|^{-1}(\alpha+1)\left(-\sum_{K=1}^{m}\beta_{LK} + \frac{1-s_L}{s_Z}\varepsilon_Z - \varepsilon_L\right) \gtrless 0$$

This condition is not signable. However, it is more likely to be positive because there is an important positive element coming from the +1. The sign of the effect depends on the relative supply elasticities of land and labor. The one special case for which a sign of (2.15) can be established is the case when there is no fixed price factor and when $\varepsilon_Z = 0$, and all $\gamma$ reduce to $\beta$ terms.

Then

$$(2.15a)\quad \frac{\partial(W/P)'}{\partial T'} = |G|^{-1}(\beta_{YY}+1)(\beta_{LL}-\varepsilon_L) - \beta_{LY}\beta_{YL} + \beta_{LY}(\alpha+1)$$

$$= |G|^{-1}[\beta_{LY}\alpha - (\beta_{YY}+1)\varepsilon_L] \gtrless 0 \qquad 1/1/0$$

We will encounter this case again in the regional models.

Labor Saving Technical Change

In considering biases we will look at cases in which the rate of technical change stays constant, i.e. in which

$$(2.16)\quad dT' = s_L dA_L' + s_Z dA_Z' + s_K dA_K' = 0$$

where K stands for all factors in infinietly elastic supply. While

increasing $A_L'$ we must therefore reduce either $A_K'$ or $A_Z'$. Let us call the case when technical change saves labor at the expense of capital the LK bias and when it is at the expense of land the LZ bias. The LK bias is treated first. From 2.16 we than have

$$(2.17)\quad dA_K' = -\frac{s_L}{s_K}\, dA_L' \quad \text{and} \quad dA_Z' = 0$$

From equations (2.1) it follows immediately that

$$(2.18)\quad \left.\frac{\partial W'}{\partial A_L'}\right|_{\text{LK bias}} = \frac{\partial W'}{\partial L^*} = |G|^{-1}\,[\gamma_{YY} - \alpha] \leq 0$$

$$(2.19)\quad \left.\frac{\partial P'}{\partial A_L'}\right|_{\text{LK bias}} = \frac{\partial P'}{\partial L^*} = -|G|^{-1}\,\gamma_{YL} \leq 0$$

And therefore

$$(2.20)\quad \left.\frac{\partial (W/P)'}{\partial A_L'}\right|_{\text{LK bias}} = \frac{\partial (W/P)'}{\partial L^*} \leq 0$$

LK-type biases act exactly in the same <u>manner</u> and <u>magnitude</u> than increases in labor supply. That a bias which saves factors of production which are price responsive rather than in infinitely elastic supply should depress output price according to equation (2.19) makes sense: The technical change saves the factors which

are in relatively inelastic supply and thus allow a reduction in output price in addition to the reduction which would occur if technical change were neutral and not directed specifically at saving the factors which are not in perfectly elastic supply. This idea is the key feature of the Induced Innovation Hypothesis.

LZ-type biases save labor relative to land, while leaving the capital-rate of technical change unaffected, i.e.

$$(2.21) \quad dA_K' = 0', \quad dT' = 0 \quad \text{and} \quad A_Z' = \frac{s_L}{s_Z} A_L'$$

Now replace $A_Z'$ in equations (2.1) by $-\frac{s_L}{s_Z} A_L'$ according to (2.21) and take the derivatives with respect to $A_L'$

$$(2.22) \quad \left.\frac{\partial W'}{\partial A_L'}\right|_{\text{LZ-bias}} = |G|^{-1} \left[ \left( \frac{s_L}{s_Z} + 1 \right) (\gamma_{YY} - \alpha) - \frac{s_L}{s_Z} \gamma_{LY} \right]$$

$$= |G|^{-1} \frac{1}{s_Z} \left[ \varepsilon_Z - (s_L + s_Z)\alpha - \eta_{ZZ} - \eta_{ZL} \right] \gtrless 0 \qquad 2/0/m$$

$$= |G|^{-1} \frac{1}{s_Z} [\varepsilon_Z - \alpha] \leq 0 \qquad 2/0/0$$

The second line is the solution for the 2/0/m case and is not signable because the "left out factors" could be complements. However, when there are no such factors, a Z-type bias will result in a reduction of wage rate in terms of nonagriculatural goods. The output price effects of an LZ bias is not signable and depends on the factor supply elasticities

$$(2.23)\quad \left.\frac{\partial P'}{\partial A_L'}\right|_{LZ\text{-bias}} = |G|^{-1}\left[\frac{s_L}{s_Z}(\gamma_{LL}-\varepsilon_L) - \gamma_{YL}\left(1+\frac{s_L}{s_Z}\right)\right] \gtrless 0$$

$$= |G|^{-1}\ \frac{s_L}{s_Z}(\eta_{LL}+\eta_{LZ}-\eta_{ZZ}-\eta_{ZL}+\varepsilon_Z-\varepsilon_L) \qquad 2/0/m$$

$$= |G|^{-1}\ \frac{s_L}{s_Z}(\varepsilon_Z-\varepsilon_L) \gtrless 0 \qquad 2/0/0$$

The second line is the 2/0/m case and the third line is the specialization to the 2/0/0 case. In the last case, if labor supply is inelastic relative to land supply $(\varepsilon_Z - \varepsilon_L) > 0$ then the output price will drop with a shift from neutral to labor-saving technical change, since the technical change saves especially the factor in more inelastic supply.

The price effect in terms of agricultural commodities is

$$(2.24)\quad \left.\frac{\partial (W/P)'}{\partial A_L'}\right|_{LZ\text{-bias}} = |G|^{-1}\ \frac{1}{s_Z}\Big[-(1-s_L)(\varepsilon_Z+\sum_K \eta_{ZK}+s_L(\varepsilon_L+\sum_K \eta_{LK})$$

$$-(s_L+s_Z)\ \alpha\Big] \gtrless 0 \qquad 2/0/m$$

$$= |G|^{-1}\ \frac{1}{s_Z}\ (s_Z\varepsilon_Z + s_L\varepsilon_L - \alpha) \lesseqgtr 0 \qquad 2/0/0$$

In the 2/0/m case the sign will be negative, unless the left out factor(s) K are complements with Z and L (i.e. $\eta_{ZK} \lesseqgtr 0$, $\eta_{LK} \lesseqgtr 0$). $\varepsilon_Z + \eta_{ZK}$ is the excess supply elasticity of land and similarly for labor. For the sign of (2.24) to be positive when left out factors are complements, these excess supply elasticities must be negative, i.e. the absolute

values of the cost function demand elasticities for land and labor with respect to capital must exceed the supply elasticities of these factors. We can summarize this section by stating that for both types of biases labor will usually lose, no matter how we measure the wage rate. An exception is possible for LZ-type biases if there are additional factors (capital) with fixed prices which are very good complements to the factors which have price responsive supply (land and labor).

Relative Wages

Wages relative to the land price are another distributional measure which is often considered. To find them we have first to determine the land rent residually from equation (1.23), which, because of the price fixity of "left out" factors, reduces to $S' = \frac{1}{s_Z}(P' + T' - s_L W')$.

Thus

$$(2.25)\quad W' - S' = W' - \frac{1}{s_Z}(P' + T' - s_L W')$$

$$= \frac{1}{s_Z}\left[(1-\Sigma_K s_K)\, W' - P' - T'\right] \qquad 2/0/m$$

$$= \frac{1}{s_Z}\left[(W/P)' - T'\right] \qquad 2/0/0$$

In the 2/0/m case no signs can be proved because of possible complementarity of the left out factors. Therefore we confine ourselves here to the Evenson-Welch 2/0/0 case.

The factor supply effects are

$$(2.26)\quad \frac{\partial(W/S)'}{\partial Z^*} = \frac{1}{s_Z}\frac{\partial(W/P)'}{\partial Z^*} = |G|^{-1} s_Z^{-1}[\alpha-\varepsilon_L] \gtrless 0$$

and by symmetry of the problem it follows that $\frac{\partial(W/S)'}{\partial L^*} \lesseqgtr 0$. Increases in a factor of production hurt its relative wage and vice versa. The only other effect which is signable is the labor-saving bias at the expense of land (Z-type) where we have

$$(2.27)\quad \left.\frac{\partial(W/S)'}{\partial A_L'}\right|_{\text{LZ bias}} = \frac{1}{s_Z}\frac{\partial(W/P)'}{\partial A_L'} \lesseqgtr 0$$

Again a labor-saving LZ bias hurts labor by this measure of welfare as well. No other effects have determinate signs.

3) The 2/1/0 Model

Equation (1.12) describes the case where there are two factors L and K, whose supplies are price responsive and one factor Z in fixed supply. Note the departure of this case from the previous case where Z was treated as a price responsive factor as well. The solution to this new case reads

$$(3.1)\qquad \begin{bmatrix} W' \\ R' \\ P' \end{bmatrix} = |G|^{-1} \begin{bmatrix} G_{LL} & G_{LK} & G_{LY} \\ G_{KL} & G_{KK} & G_{KY} \\ G_{YL} & G_{YK} & G_{YY} \end{bmatrix} \begin{bmatrix} L^* - Z^* - \beta_{LY}T' - A_Z' + A_L' \\ K^* - Z^* - \beta_{KY}T' - A_Z' + A_K' \\ D^* - Z^* - \beta_{YY}T' - A_Z' \end{bmatrix}$$

where the $G_{ij}$ are the signed co-factors of the excess elasticity matrix G. From rules established in Appendix A, we note that

$$(3.2)\quad |G| > 0,\ G_{LL}, G_{KK} \leq 0 \text{ and } G_{YY} \geq 0$$

Moreover,

$$(3.3)\quad G_{LY} = \beta_{LK}\beta_{KY} - \beta_{LY}\,(\beta_{KK} - \varepsilon_K)$$

or by homogeneity (equation 1.8)

$$\begin{aligned} G_{LY} &= (-\beta_{LY} - \beta_{LL})(\beta_{KK} - \beta_{KL}) - \beta_{LY}(\beta_{KK} - \varepsilon_K) \\ &= \beta_{KL}(\beta_{LY} + \beta_{LL}) + \beta_{LL}\beta_{KK} + \beta_{LY}\varepsilon_K \\ &= -\beta_{KL}\beta_{LK} + \beta_{LL}\beta_{KK} + \beta_{LY}\varepsilon_K \geq 0 \end{aligned}$$

This last line of (3.3) is nonnegative since the first two terms are equal to $G_{YY} \geq 0$ with $\varepsilon_L$ and $\varepsilon_K = 0$ and since the last term, $\beta_{LY}\varepsilon_K$, is also nonnegative. By similar proofs, it can likewise be shown that

(3.4)

$$G_{KY} = \beta_{LY}\beta_{KL} - \beta_{KY}(\beta_{LL} - \varepsilon_L) \gtreqless 0$$
$$G_{YL} = \beta_{KL}\beta_{YK} - \beta_{YL}(\beta_{KK} - \varepsilon_K) \lesseqgtr 0$$
$$G_{YK} = \beta_{LK}\beta_{YL} - \beta_{YK}(\beta_{LL} - \varepsilon_L) \lesseqgtr 0$$

However, no signs can be established for the cofactors $G_{LK}$ and $G_{KL}$, i.e.,

(3.5)

$$G_{KL} = \beta_{KY}\beta_{YL} - \beta_{KL}(\beta_{YY} - \alpha) \gtreqless 0$$
$$G_{LK} = \beta_{LY}\beta_{YK} - \beta_{LK}(\beta_{YY} - \alpha) \gtreqless 0$$

From equations (3.2) to (3.5), we can immediately establish the following effects of increases in factor supplies,

$$(3.6)\quad \frac{\partial W'}{\partial L^*} = |G|^{-1} G_{LL} \lesseqgtr 0$$

$$(3.7)\quad \frac{\partial P'}{\partial L^*} = |G|^{-1} G_{YL} \lesseqgtr 0$$

$$(3.8)\quad \frac{\partial \left(\frac{W}{P}\right)'}{\partial L^*} = |G|^{-1}(G_{LL} - G_{YL})$$
$$= |G|^{-1}[(\beta_{KK} - \varepsilon_K)(\beta_{YY} - \alpha) - \beta_{YK}\beta_{KY} - \beta_{KL}\beta_{YK} + \beta_{YL}(\beta_{KK} - \varepsilon_K)]$$
$$= |G|^{-1}[(\beta_{KK} - \varepsilon_K)(\beta_{YY} + \beta_{YL} - \alpha) - \beta_{YK}(\beta_{KY} + \beta_{KL})]$$

or by homogeneity (equation (1.8)),

$$= |G|^{-1}[(\beta_{KK}-\epsilon_K)(-\beta_{YK}-\alpha) + \beta_{YK}\beta_{KK}]$$

$$= |G|^{-1}[\epsilon_K\beta_{YK}-\alpha(\beta_{KK}-\epsilon_K)] \leq 0$$

Also, by again using the condition of homogeneity, we can establish that

$$(3.9)\quad \frac{\partial\left(\frac{W}{R}\right)'}{\partial L^*} = |G|^{-1}\,(G_{LL}-G_{KL})$$

$$= |G|^{-1}[\alpha\beta_{KY}-\epsilon_K(\beta_{YY}-\alpha)] \leq 0$$

Finally, we have

$$(3.10)\quad \frac{\partial W'}{\partial Z^*} = |G|^{-1}[-G_{LL}-G_{LK}-G_{LY}] \gtrless 0$$

$$(3.11)\quad \frac{\partial\left(\frac{W}{P}\right)'}{\partial Z^*} = |G|^{-1}[-G_{LL}-G_{LK}-G_{LY}+G_{YL}+G_{YK}+G_{YY}]$$

$$(3.12)\quad \frac{\partial W'}{\partial K^*} = |G|^{-1}G_{LK} \gtrless 0$$

$$(3.13)\quad \frac{\partial\left(\frac{W}{P}\right)'}{\partial K^*} = |G|^{-1}[G_{LK}-G_{YK}] \gtrless 0$$

Equations (3.6) and (3.7) tell us that an expansion in the supply of labor reduces labor wages and also the output price. Equations (3.8) and (3.9) show that real wages in terms of the agricultural output prices and labor wages relative to capital rents will likewise decline with increases in the supply of labor. Indeed, by the symmetry of the problem, it is also true that

$$\frac{\partial R'}{\partial K^*},\ \frac{\partial P'}{\partial K^*},\ \frac{\partial\left(\frac{R}{P}\right)'}{\partial L^*},\ \text{and}\ \frac{\partial\left(\frac{R}{W}\right)'}{\partial K^*} \leq 0.$$

Finally equations (3.10) to (3.13) show that the effects of an expansion of land and of capital on nominal wages and real wages (in terms of the agricultural output price) remain ambiguous. Again, by the symmetry of the problem, the signs of the cross effects $\frac{\partial R'}{\partial Z^*}$, $\frac{\partial \left(\frac{R}{P}\right)'}{\partial Z^*}$, $\frac{\partial R'}{\partial L^*}$ and $\frac{\partial \left(\frac{R}{P}\right)'}{\partial L^*}$ remain indeterminate.

Wages will increase with increase in output demand and so will input prices since

$$(3.14)\quad \frac{\partial W'}{\partial D^*} = |G|^{-1} G_{LY} \geq 0$$

and

$$(3.15)\quad \frac{\partial P'}{\partial D^*} = |G|^{-1} G_{YY} \geq 0$$

However, the real wage effect of increases in final output demand remains ambiguous, i.e.

$$(3.16)\quad \frac{\partial \left(\frac{W}{P}\right)'}{\partial D^*} = |G|^{-1}(G_{LY} - G_{YY})$$

$$= |G|^{-1}[\varepsilon_L(\beta_{KK} - \varepsilon_K) - \varepsilon_K \beta_{LK}] \gtrless 0$$

This effect will clearly be negative if $\beta_{LK}$, the elasticity of labor demand with repsect to the price of capital services, is nonzero. Again, by the symmetry of the problem, it is also true that $\frac{\partial R'}{\partial D^*} \geq 0$ and $\frac{\partial \left(\frac{R}{P}\right)'}{\partial D^*} \gtrless 0$.

The Neutral Technical Change Case

In the 2/1/0 case the effects of neutral technical change can be obtained in a similar manner to the 2/0/m case. The derivations of these conditions are rather straight forward. We can establish that

$$(3.17)\quad \frac{\partial W}{\partial T}' = -(\alpha+1)\frac{\partial W}{\partial D^*}'$$

$$(3.18)\quad \frac{\partial P}{\partial T}' = \frac{\beta_{LY}G_{YL} + \beta_{KY}G_{YK} - (\beta_{YY}+1)G_{YY}}{\beta_{LY}G_{YL} + \beta_{KY}G_{YK} - (\beta_{YY}-\alpha)G_{YY}} \leq 0$$

$$(3.19)\quad \frac{\partial\left(\frac{W}{P}\right)}{\partial T}' = 1 - (\alpha+1)\frac{\partial(W/P)}{\partial T}' \gtrless 0$$

Equation (3.17) is the same as equation (2.14) for the 2/0/m case. Also, equations (3.18) and (3.19) bear close resemblance to equations (2.12) and (2.15) respectively. While (3.18) is signable, (3.19) is not, though we can presume that the positive 1 in this last equation will tend to make $\frac{\partial\left(\frac{W}{P}\right)}{\partial T}'$ nonnegative.

Labor Saving Technical Change

As in the 2/0/m case explored earlier, in this section we consider the output price and the nominal and real factor price effects of an LK-type biased technical change. We continue to assume equations (2.16) and (2.17) for the LK-type bias and work with the 2/1/0 case summarized by (3,1).

For the LK-type bias, it follows directly from (2.17) and (3.1) that

$$(3.20)\quad \frac{\partial w'}{\partial A_L'}\bigg|_{\text{LK bias}} = |G|^{-1}\left(G_{LL} - \frac{s_L}{s_K} G_{LK}\right)$$

$$= |G|^{-1}(\alpha\beta_{KY} - \varepsilon_K\beta_{YY} + \alpha\varepsilon_K) \lesseqgtr 0$$

$$(3.21)\quad \frac{\partial P'}{\partial A_L'}\bigg|_{\text{LK bias}} = |G|^{-1}\left(G_{YL} - \frac{s_L}{s_K} G_{YK}\right)$$

$$= |G|^{-1} s_L(\varepsilon_L\beta_{KY} - \varepsilon_K\beta_{LY}) \gtreqless 0$$

$$(3.22)\quad \frac{\partial\left(\frac{W}{P}\right)'}{\partial A_L'}\bigg|_{\text{LK bias}} = |G|^{-1}\left[G_{LL} - \frac{s_L}{s_K} G_{LK} - G_{YL} + \frac{s_L}{s_K} G_{YK}\right]$$

$$= |G|^{-1}[\beta_{KY}(\alpha - s_L\varepsilon_L) + \varepsilon_K\beta_{YK} + \alpha\varepsilon_K] \lesseqgtr 0$$

Equation (3.20) shows that an LK bias will decrease labor wages. That an LK bias will increase interest payments to capital can also be inferred from (3.20), since this problem is parallel to the question of establishing the sign of $\frac{\partial w'}{\partial A_K'}\Big|_{\text{KL bias}} = |G|^{-1}\frac{s_K}{s_L}\left(\frac{s_L}{s_K} G_{LK} - G_{LL}\right)$, which from (3.20) is $\gtrless 0$. The effect of an LK bias on output prices are uncertain as equation (3.21) suggests, though this effect will be clearly negative (positive) if the elasticity of labor (capital) supply is sufficiently small, or if the elasticity of demand for capital (labor) with respect to output price is close to zero. All these effects are not unusual and are in fact features of the induced innovation hypothesis. Technical change that saves a particular factor relative to another factor will tend to decrease this particular factor's price

and increase that of the other factor. Further, the output price effects of biased technical change would depend on factor demand and supply elasticities as equation (3.21) suggests.

Equation (3.22) shows that real wages in terms of the price of agricultural output will decline with an LK bias. Indeed under LK bias conditions, laborers should be unwilling to accept any labor saving technical change.

As in the 2/0/m case we are unable to attach signs to the effects of LZ-type biased technical change on output price and on real and nominal factor prices. However, in the 2/1/m case where labor saving technical change occurs at the expense of one or more factors with fixed prices, it follows straightforwardly from (3.1) that

$$(3.23)\quad \frac{\partial W_1'}{\partial A_L}\bigg|_{Lm} = |G|^{-1} G_{LL} \leq 0$$

$$(3.24)\quad \frac{\partial P'}{\partial A_L}\bigg|_{Lm} = |G|^{-1} G_{YL} \leq 0$$

4) Two regions with mobile labor

The regional cases considered in this and the following section are not closed general equilibrium regional trade models. We are considering the agricultural sector only of an economy. Production takes place in several regions because land is an essential factor of production and cannot be moved from region to region. The regions supply one single national market. The regional dimension of this market is suppressed since much of the demand for the agricultural commodities comes from urban sectors and the rural nonfarm sector. We have sketched elsewhere the extensions of the approach of this paper to a genuine one region - two sector trade model and noted the difficulties of empirically implementing it (Binswanger 1978). Generalizing that model to several regions is possible but would lead to few insights in the absence of parameter estimates for all sectors and regions considered and for intersectoral and inter-regional factor mobility conditions.

In extending the partial equilibrium model to more than one region we first consider the case where output is traded and labor is perfectly mobile between regions. Therefore there will be only one wage rate and one output price to consider. However, land rents in the two regions will differ. A discussion of the land rent impact will, however, be deferred to section 6. One further simplification is the assumption that land supply in each region is fixed. We therefore deal with a 1/1/m specialization of the 2/0/m model of section 2.

Each region therefore has a labor demand function and an output supply function as in (1.6) where r = 1, 2 is the region index

$$L_r' = \beta_{LLr} W_r' + \beta_{LYr} (P' + T_r') + A_{Zr}' - A_{Lr}' + Z_r^*$$

$$Y_r' = \beta_{LYr} W_r' + \beta_{YYr} (P' + T_r') + A_{Zr}' + Z_r^*$$

In this particular case $W_1' = W_2' = W'$.[1/] These equations hold also when there are m factors which are mobile between the regions and are in infinitely elastic supply. Therefore the model below covers the 1/1/m case as well.

Since total labor is $L = L_1 + L_2$ and total output is $Y = Y_1 + Y_2$, their total rates of changes are share-weighted sums of the rates of changes of the individual regions. These rates correspond to the supply and output demand functions (1.10) and (1.11) and are written as

---

1/ Rates of changes of these wages must be the same, but not the wages which can differ by a constant multiple $W_1 = k W_2$

$$L' = \epsilon_L W + L^* = \lambda_1 L_1' + \lambda_2 L_2'$$

$$Y' = \alpha P + D^* = \nu_1 Y_1' + \nu_2 Y_2' \tag{4.2}$$

where the shares in total labor and output are

$$\lambda_r = L_r/L \text{ and } \nu_r = Y_r/Y \tag{4.3}$$

One therefore can add up the factor demand and output supply equations by weighting them by these respective shares and setting them equal to the total changes.

$$\begin{aligned} \epsilon_L W + L^* = {} & (\lambda_1 \beta_{LL1} + \lambda_2 \beta_{LL2})\, W' + (\lambda_1 \beta_{LY1} + \lambda_2 \beta_{LY2})\, P' \\ & + \lambda_1 (\beta_{LY1} T_1' + A_{Z1}' - A_{L1}') + \lambda_2 (\beta_{LY2} T_2' + A_{Z2}' - A_{L2}') \\ & + \lambda_1 Z_1' + \lambda_2 Z_2' \\ = {} & \bar{\beta}_{LL} W' + \bar{\beta}_{LY} P' + \lambda_1 E_{L1}' + \lambda_2 E_{L2}' + \lambda_1 Z_1^* + \lambda_2 Z_2^* \end{aligned} \tag{4.4}$$

$$\begin{aligned} \alpha P + D^* = {} & (\nu_1 \beta_{YL1} + \nu_2 \beta_{YL2})\, W' + (\nu_1 \beta_{YY1} + \nu_2 \beta_{YY2})\, P' \\ & + \nu_1 (\beta_{YY1} T' + A_{Z1}') + \nu_2 (\beta_{YY2} T_2' + A_{Z2}') \\ & + \nu_1 Z_1^* + \nu_2 Z_2^* \\ = {} & \bar{\beta}_{YL} W' + \bar{\beta}_{YY} P' + \nu_1 E_{Y1}' + \nu_2 E_{Y2}' + \nu_1 Z_1^* + \nu_2 Z_2^* \end{aligned} \tag{4.5}$$

In these equations $\bar{\beta}$ coefficients are the overall factor demand and

output supply elasticities as defined in these equations. Since they are sums, they have the same signs as the individual region's elasticities, i.e.

$$(4.6)\quad \bar{\beta}_{LL} \leq 0,\ \bar{\beta}_{YL} \leq 0,\ \bar{\beta}_{LY} \geq 0,\ \bar{\beta}_{YY} \geq 0$$

Rearranging terms we get an equation system analoguous to (1.12), which has the following solution.

$$(4.7)\quad \begin{bmatrix} W' \\ P' \end{bmatrix} = \begin{bmatrix} \bar{\beta}_{YY}-\alpha & -\bar{\beta}_{LY} \\ -\bar{\beta}_{YL} & \bar{\beta}_{LL}-\varepsilon_L \end{bmatrix} \begin{bmatrix} L^*-\lambda_1 Z_1^*-\lambda_2 Z_2^*-\lambda_1 E_{L1}'-\lambda_2 E_{L2}' \\ D^*-\nu_1 Z_1^*-\nu_2 Z_2^*-\nu_1 E_{Y1}'-\nu_2 E_{Y2}' \end{bmatrix}$$

From Appendix A we know that the determinant $|G|$ has a negative sign. Therefore, and by (4.6) we find that the <u>own factor supply effects</u> are as before

$$(4.8)\quad \frac{\partial W'}{\partial L^*} = |G|^{-1}(\bar{\beta}_{YY}-\alpha) \leq 0$$

$$(4.9)\quad \frac{\partial P'}{\partial L^*} = |G|^{-1}(\bar{\beta}_{YL}) \leq 0$$

$$(4.10)\quad \frac{\partial (W/P)'}{\partial L^*} = |G|^{-1}(\bar{\beta}_{YY}+\bar{\beta}_{YL}-\alpha) = -|G|^{-1}(\Sigma_K \bar{\beta}_{YK}+\alpha) \leq 0$$

where $\bar{\beta}_{YK}$ are the share weighted factor demand elasticities with respect to the left out factors and where in (4.10) we use the familiar adding up constraint (1.8) and the fact that all $\beta_{YKr} \leq 0$. Thus the own factor supply effects are as in the one region case.

The cross supply effects are

$$(4.11)\quad \frac{\partial W'}{\partial Z_r^*} = -|G|^{-1}[\lambda_r(\bar{\beta}_{YY}-\alpha) - \nu_r(\bar{\beta}_{LY})] \gtrless 0$$

This equation looks like (2.6) which is not signable in the one region case and much less so in the two region case. However the price effect is signable

$$(4.12)\quad \frac{\partial P'}{\partial Z_r^*} = |G|^{-1}[\lambda_r\bar{\beta}_{YL} - \nu_r(\bar{\beta}_{LL}-\varepsilon_L)] \lesseqgtr 0$$

Proof: Note first the following relationship, where we use the symmetry relations of equations (1.7.)

$$\bar{\beta}_{YL} = \frac{Y_1}{Y}\beta_{YL1} + \frac{Y_2}{Y}\beta_{YL2} = -\frac{Y_1}{Y}s_{L1}\beta_{YL1} - \frac{Y_2}{Y}s_{L2}\beta_{YL2}$$

$$(4.13) = \left[-\frac{Y_1}{Y}\frac{WL_1}{PY_1}\beta_{LY1} - \frac{Y_2}{Y}\frac{WL_2}{PY_2}\beta_{LY2}\right]\frac{L}{L} = -\frac{WL}{PY}(\lambda_1\beta_{LY1} + \lambda_2\beta_{LY2})$$

$$= -\bar{s}_L\bar{\beta}_{LY}$$

where $\bar{s}_L = \frac{WL}{PY}$ is the share of total labor (both regions) in total output and $s_{Lr}$ are the regional labor shares. Further note that in the genuine 2 factor case $\bar{\beta}_{LL} = -\bar{\beta}_{LY}$. Setting this into (4.12) we have

$$(4.14)\quad \frac{\partial P'}{\partial Z_r^*} = |G|^{-1}\left[-\frac{L_r}{L}\frac{LW}{PY}\bar{\beta}_{LY} + \frac{Y_r}{Y}\bar{\beta}_{LY} + \nu_r\varepsilon_L\right] \qquad 1/1/0$$

$$= |G|^{-1}\left[\frac{1}{PY}\bar{\beta}_{LY}(-L_rW + Y_rP) + \nu_r\varepsilon_L\right] \lesseqgtr 0$$

The sign follows because the value of output is greater than labor cost i.e. $Y_r P > L_r W$, and therefore all terms in the parathesis are positive. However, this case is restricted to the 1/1/0 case. A similar proof with "left out" factors can only be shown for the case when those factors are substitutes with labor. For the same 1/1/0 case we can also show that

$$(4.15)\quad \frac{\partial (W/P)'}{\partial Z_r}^{*} = |G|^{-1}[\lambda_r(\alpha - \bar{\beta}_{YY} - \bar{\beta}_{YL}) + \nu_r(\bar{\beta}_{LL} + \bar{\beta}_{LY} - \varepsilon_L)]$$

$$= |G|^{-1}[\lambda_r \alpha - \nu_r \varepsilon_L] \gtrless 0 \qquad 1/1/0$$

This holds only when there are no left out factors because then all $\beta$ terms cancel due to homogeneity in (1.8).

The output demand effects are straightforward

$$(4.16)\quad \frac{\partial W'}{\partial D^{*}} = - |G|^{-1}\bar{\beta}_{LY} \gtrless 0$$

$$(4.17)\quad \frac{\partial P'}{\partial D^{*}} = |G|^{-1}(\bar{\beta}_{LL} - \varepsilon_L) \gtrless 0$$

$$(4.18)\quad \frac{\partial (W/P)'}{\partial D^{*}} = |G|^{-1}(\sum_K \bar{\beta}_{LK} + \varepsilon_L)$$

which is negative only if left out factors are substitutes on balance or there are no such factors and the $\bar{\beta}_{LK}$ terms vanish. In comparing (4.18) with (2.11), note that in that case a sign could not be established because of the presence of $\varepsilon_Z$ which here is assumed zero. Therefore note that the fixity of land supply is a crucial assumption to establish the sign of (4.18).

Finally, the technical change effects are as follows:

$$(4.19)\quad \frac{\partial W'}{\partial T'_1} = |G|^{-1}\left[(\bar{\beta}_{YY} - \alpha)(-\lambda_1\beta_{LY1}) - \bar{\beta}_{LY}(-\nu_1\beta_{YY1} - \nu_1)\right] \gtrless 0$$

$$(4.20)\quad \frac{\partial P'}{\partial T'_1} = |G|^{-1}\left[(-\bar{\beta}_{YL})(-\lambda\,\beta_{LY1}) + (\bar{\beta}_{LL} - \varepsilon_L)\,(-\nu_1\beta_{YY1} - \nu_1)\right]$$

$$= |G|^{-1}\left[\nu_1(\bar{\beta}_{LY}\beta_{YL1} - \bar{\beta}_{LL}\beta_{YY1}) + \varepsilon_L(\nu_1\beta_{YY1} + \nu_1)\right.$$

$$\left. -\nu_1\bar{\beta}_{LL}\right] \leq 0$$

The first term inside the bracket sign in (4.20) is nonnegative because this is a negatively signed principal minor of the excess elasticities matrix of (4.7) with $\alpha$, $\varepsilon_L$ and $\nu_2$ equal to zero. The remaining two terms inside the bracket sign are likewise nonnegative, making the overall effect of a neutral technical change in one region on output prices nonpositive. The same, however, cannot be said with regard to the effect of neutral technical change in one region on wages. In this instance, the effect remains ambiguous (equation 4.19).

In the genuine 1|1|0 case, we can also show that neutral technical change in any one region is likely to improve real wages since

$$(4.21)\quad \frac{\partial\left(\frac{W}{P}\right)'}{\partial T'} = |G|^{-1}\left[\lambda_1\alpha\beta_{LY1} + \varepsilon_L\,(-\nu_1\beta_{YY1} - \nu_1)\right] > 0$$

For the 1|1|m case, similar signs can be established when the "left out" factors are on balance, substitutes with labor.

5) Two Regions With Immobile Labor

When labor is immobile between regions, each region has its own labor demand equation in (4.1) equated to its own labor supply function $L_r' = \epsilon_{Lr} W_r' + L_r^*$, and these equations are not added up. Only the output supply equations are added up as before. The resulting equations again admit "left out" factors K with fixed prices and have the form

$$(5.1)\quad \begin{bmatrix} W_1' \\ W_2' \\ P' \end{bmatrix} = \begin{bmatrix} \beta_{LL1}-\epsilon_{L1} & 0 & \beta_{LY2} \\ 0 & \beta_{LL2}-\epsilon_{L2} & \beta_{LY2} \\ \nu_1\beta_{YL1} & \nu_2\beta_{YL2} & \bar{\beta}_{YY}-\alpha \end{bmatrix}^{-1} \begin{bmatrix} L_1^*-Z_1^*-E_{L1}' \\ L_2^*-Z_2^*-E_{L2}' \\ D^*-\nu_1 Z_1^*-\nu_2 Z_2^*-\nu_1 E_{Y1}'-\nu_2 E_{Y2}' \end{bmatrix}$$

where the notation is the same as before. In Appendix A it is shown that the inverse $|G|^{-1} > 0$ since $|G|$ can be generated from a sum of nonnegative definite matrices. Since G has some zero elements we can write out its inverse directly

$$(5.2)\quad G^{-1} = |G|^{-1} \begin{bmatrix} G_{11} & G_{12} & G_{1Y} \\ G_{21} & G_{22} & G_{2Y} \\ G_{Y1} & G_{Y2} & G_{YY} \end{bmatrix} =$$

$$= |G|^{-1} \begin{bmatrix} \overset{-}{(\beta_{LL2}-\epsilon_{L2})(\bar{\beta}_{YY}-\alpha)} - \nu_2\beta_{YL2}\beta_{LY1} & \overset{-}{\nu_2\beta_{YL2}\beta_{LY1}} & \overset{+}{-\beta_{LY1}(\beta_{LL2}-\epsilon_{L2})} \\ \overset{-}{\nu_1\beta_{YL1}\beta_{LY2}} & \overset{-}{(\beta_{LL1}-\epsilon_{L1})(\bar{\beta}_{YY}-\alpha)-\nu_1\beta_{YL1}\beta_{LY1}} & \overset{+}{-\beta_{LY2}(\beta_{LL1}-\epsilon_{L1})} \\ \overset{-}{-\nu_1\beta_{YL1}(\beta_{LL2}-\epsilon_{L2})} & \overset{-}{-\nu_2\beta_{YL2}(\beta_{LL1}-\epsilon_{L1})} & \overset{+}{(\beta_{LL1}-\epsilon_{L1})(\beta_{LL2}-\epsilon_{L2})} \end{bmatrix}$$

From Appendix A we can determine that

$$(5.3) \quad G_{11} \leq 0, \quad G_{22} \leq 0, \quad G_{YY} \geq 0$$

which is a consequence of convexity of the profit function. Using the signs in table 1, i.e. the noninferiority assumption, we can also establish the signs of all other cofactors,

$$(5.4) \quad G_{12} \leq 0, \; G_{21} \leq 0, \; G_{1Y} \geq 0, \; G_{2Y} \geq 0$$

$$G_{Y1} \leq 0, \; G_{Y2} \leq 0$$

In what follows we will consider only effects of changes on the wage in region one. Since regions are treated symmetrically the results for region two can be found by interchanging indices.

The <u>own factor supply effects</u> are as follows.

$$(5.5) \quad \frac{\partial W_1'}{\partial L_1^*} = |G|^{-1} G_{11} \leq 0$$

$$(5.6) \quad \frac{\partial P'}{\partial L_1^*} = |G|^{-1} G_{Y1} \leq 0$$

$$(5.7) \quad \frac{\partial (W_1/P)'}{\partial L_1^*} = |G|^{-1}(G_{11}-G_{Y1})$$

$$= |G|^{-1}[(\beta_{LL2}-\varepsilon_{L2})(\bar{\beta}_{YY}-\alpha + \nu_1\beta_{YL1}) - \nu_2\beta_{YL2}\beta_{LY2}]$$

$$= |G|^{-1}[(\beta_{LL2}-\varepsilon_{L2})(\nu_1\beta_{YY1}+\nu_1\beta_{YL1}+\nu_2\beta_{YY2}-\alpha)-....]$$

$$= |G|^{-1}[\nu_2(\beta_{LL2}\beta_{YY2}-\beta_{YL2}\beta_{LY2}) - \nu_2\varepsilon_{L2}\beta_{YY2}$$

$$-(\beta_{LL2}-\varepsilon_{L2})(\nu_1\textstyle\sum_K\beta_{YK1}+\alpha)] \leq 0$$

This expression is negative because the first term in the parenthesis is a principal minor of a $\beta$ matrix which is negative by Appendix A, while the second and third terms are negative by noninferiority.

Now consider the effect on wage rates of increases in labor supply in the other region.

$$(5.8)\quad \frac{\partial W_1'}{\partial L_2^*} = |G|^{-1} G_{12} \lesseqgtr 0$$

$$(5.9)\quad \frac{\partial (W_1/P)'}{\partial L_2^*} = |G|^{-1}(G_{12}-G_{Y2})$$

$$= |G|^{-1} \nu_2 \beta_{YL2}[\beta_{LY1} + \beta_{LL1} - \varepsilon_{L1}]$$

$$= |G|^{-1} \nu_2 \beta_{YL2} [-\Sigma_K \beta_{LK} - \varepsilon_{L1}] \gtreqless 0$$

This condition will be <u>positive</u> if their are no "left out" factors or those factors are on balance substitutes. Therefore its sign is determined only for the genuine 1/1/0 case. It is important to note here that an increase in labor supply in the second region clearly <u>depresses</u> the wage rate in the first region if it is measured in <u>nonagricultural goods</u> but is more likely to <u>increase</u> the wage rate as measured in terms of <u>agricultural commodities,</u> because its output price effect is likely to dominate the input demand effect in the first region.

Now consider the <u>cross supply effects</u>. As in all previous models $\partial W_1'/\partial Z_1^*$ cannot be signed

$$(5.10)\quad \frac{\partial W_1'}{\partial Z_1^*} = -|G|^{-1}(G_{11}+\nu_1 G_{1Y}) \gtrless 0$$

$$(5.11)\quad \frac{\partial P'}{\partial Z_1^*} = -|G|^{-1}(G_{Y1}+\nu_1 G_{YY}) = |G|^{-1}[\nu_1(\beta_{LL2}-\varepsilon_{L2})(\beta_{YL1}-\beta_{LL1}+\varepsilon_{L1})] \lesseqgtr 0$$

The sign of this expression follows from equation (2.7) and the fact that the $|G|^{-1}$ has a negative sign in (2.7). It differs from equation (2.7) for the one region case in that the price effect is first weighted by the share of region one in total output and by the excess elasticity of labor demand in the second region. The more price responsive the labor market in the second region, the higher the price effect of an increase in land in the first region.

$$(5.12)\quad \frac{\partial(W_1/P)'}{\partial Z_1^*} = |G|^{-1}(G_{Y1}+\nu_1 G_{YY}-G_{11}-\nu_1 G_{1Y})$$

$$= |G|^{-1}\{\nu_2\beta_{YL2}\beta_{LY2}+(\beta_{LL2}-\varepsilon_{L2})[\alpha-\nu_2\beta_{YY2}$$

$$-\nu_1(\beta_{YY1}+\beta_{YL1}-\beta_{LY1}-\beta_{LL1}+\varepsilon_{L1})]$$

$$= |G|^{-1}\{\nu_2(\beta_{YL2}\beta_{LY2}-\beta_{LL2}\beta_{YY2}+\varepsilon_{L2}\beta_{YY2})$$

$$+(\beta_{LL2}-\varepsilon_{L2})[\alpha+\nu_1(\textstyle\sum_K \beta_{YK}-\sum_K \beta_{LK}-\varepsilon_{L1})]\}$$

$$\text{usually} \quad \gtrless 0$$

The first term multiplied by $\nu_2$ is clearly positive since it is the negative of a principle minor which, according to Appendix A, is negative. The term on the second line is also positive unless the term $\sum_K \beta_{LK}$ is negative. Therefore the <u>wage effect</u> in terms of agricultural

commodities of an increase in land supply in the own region is positive for the 1/1/0 model, or if on balance the "left out" factors are substitutes. This result is entirely analoguous to the one region case.

Now consider the cross supply effects when land increases in the other region

$$(5.13)\quad \frac{\partial W_1'}{\partial Z_2^*} = -|G|^{-1}(G_{12}+\nu_2 G_{1Y}) = |G|^{-1}\nu_2\beta_{LY1}(\beta_{LL2}-\beta_{YL2}-\epsilon_{L2}) \lesseqgtr 0$$

This expression has a negative sign because the term in brackets is analoguous to equation (2.7), where, however $|G|^{-1}$ has a negative sign instead of a positive one. We note by reference to equation (5.8) that any increase in factor supply in the second region, whether in fixed or in price responsive supply, reduces the wage rate in the first region.

$$(5.14)\quad \frac{\partial (W_1/P)'}{\partial Z_2^*} = -|G|^{-1}(G_{12}+\nu_2 G_{1Y}-G_{Y2}-\nu_2 G_{YY})$$

$$= |G|^{-1}\nu_2[\beta_{LY1}(\beta_{LL2}-\beta_{YL2}-\epsilon_{L2})(\beta_{LL1}-\epsilon_{L1})(\beta_{YL2}-\beta_{LL2}+\epsilon_{L2})]$$

$$= |G|^{-1}\nu_2[(-\Sigma_K \beta_{LK1}-\epsilon_{L1})(\beta_{LL2}-\beta_{YL2}-\epsilon_{L2})] \text{ usually } \gtrsim 0$$

The first term in brackets is negative unless the left out factors, on balance, are complements. The second term is negative by (5.13) so that the whole expression is positive, unless strong complementarities exist. Considering both (5.14) and (5.9) note that increases in factor supply (whether fixed factors or price responsive factors) in the other region usually lead to an increase in the wage measured in agricultural

commodities of the laborers in the first region.

The final demand effects are straightforward

$$(5.15)\quad \partial W_1'/\partial D^* \gtrless 0,\ \partial P'/\partial D^* \gtrless 0$$

$$\frac{\partial (W_1/P)'}{\partial D^*} = |G|^{-1}(G_{1Y}-G_{YY})$$

$$= -|G|^{-1}(\beta_{LL2}-\varepsilon_{L2})(\beta_{LL1}+\beta_{LY1}-\varepsilon_{L1})$$

$$= -|G|^{-1}(\beta_{LL2}-\varepsilon_{L2})(\sum_K \beta_{LK1}-\varepsilon_{L1}) \quad \text{usually} \lesseqgtr 0$$

Again this is less than zero, except if the left out factors are complements, on balance. These results are totally in line with the one region case.

Now consider the relative position of workers in the two regions when labor supply increases

$$(5.16)\quad \frac{\partial (W_1/W_2)'}{\partial L_2^*} = |G|^{-1}(G_{12}-G_{22})$$

$$= |G|^{-1}[\nu_2\beta_{YL2}\beta_{LY1}-(\beta_{LL1}-\varepsilon_{L1})(\bar{\beta}_{YY}-\alpha) + \nu_1\beta_{YL1}\beta_{LY1}]$$

$$= |G|^{-1}[\nu_2(\beta_{YL2}\beta_{LY1}-\beta_{LL1}\beta_{YY2}) - \nu_1(\beta_{LL1}\beta_{YY1}-\beta_{YL1}\beta_{LY1}) + \beta_{LL1}\alpha + \bar{\beta}_{YY}\varepsilon_{L1}-\varepsilon_{L1}\alpha]$$

$$\gtrless 0 \text{ for } 1/1/0$$

The second term in brackets derives from a principal minor and is negative so that it contributes to a positive sign. The terms on the

last line are all positive as well. But the term in the first brackets weighted by $\nu_2$ cannot be signed unless there are no "left out" factors. In that genuine 1/1/0 case the term vanishes and the overall expression is positive, which intuitively makes sense. Labor in the region experiencing the supply increase suffers more than labor in the first region where supply does not increase.

The technical change effects are as follows:

$$(5.17)\quad \partial W_1'/\partial T_1' = |G|^{-1}[-\beta_{LY1}G_{11}\bar{+}\nu_1(\beta_{YY_1}+1)G_{1Y}]$$

$$= |G|^{-1}\beta_{LY1}[\nu_2(\beta_{YL2}\beta_{LY2}-\beta_{YY2}\beta_{LL2}+\beta_{YY2}\varepsilon_{L2}) +$$

$$(\alpha+\nu_1)(\beta_{LL2}-\varepsilon_{L2})] \gtrless 0$$

This expression is derived by full expansion of terms of the first line and by cancelling and collecting all the terms together again. The second line term is positive since the first two terms in brackets are the negative of a negative principal minor of the $\beta$ matrix. The third line term is negative only if $|\alpha| \leq \nu_1$ i.e. final demand is inelastic. Therefore, the wage effect is much more likely to be positive than in the one region case.

$$(5.18)\quad \frac{\partial P'}{\partial T_1'} = |G|^{-1}[-\beta_{LY1}G_{Y1}-\nu_1(\beta_{YY1}+1)\,G_{YY}]$$

$$= |G|^{-1}\nu_1[\beta_{LL2}-\varepsilon_{L2}][\beta_{LY1}\beta_{YL1}-\beta_{YY1}\beta_{LL1}+\beta_{YY1}\varepsilon_{L1}-\beta_{LL1}+\varepsilon_{L1}] \leq 0$$

which is less than zero by a reasoning analoguous to the one for the

the second line of (5.17).

For the genuine 1/1/0 case the principal minors in expressions (5.17) and (5.18) are zero and by leaving them out to form the next expression we have

$$(5.19)\quad \frac{\partial(W_1/P)'}{\partial T_1'} = \frac{\partial W_1'}{\partial T_2'} - \frac{\partial P'}{\partial T_1'} =$$

$$= |G|^{-1}\{\nu_2\beta_{LY1}\beta_{YY2}\varepsilon_{L2} + \alpha\,\beta_{LY1}(\beta_{LL2}-\varepsilon_{L2}) + \nu_1\varepsilon_{L1}[(\varepsilon_{L2}-\beta_{LL2})(\beta_{YY1}+1)]\}$$

$$= |G|^{-1}\{\nu_2\beta_{LY1}\beta_{YY2}\varepsilon_{L2} + [(\beta_{LL2}-\varepsilon_{L2})(\alpha\beta_{LY1}-(\beta_{YY1}+1)\nu_1\varepsilon_{L1})]\} \geq 0$$

The expression is derived by multiplying out completely, cancelling terms and noting that in the 1/1/0 case $\beta_{LY1} = -\beta_{LL1}$. Note further, that if we included the principal minors for the 1/1/m case in forming (5.19) they would both contribute further to the positive sign. Labor benefits from a technical change in region one if measured in agricultural goods just as in the one region case when the supply elasticity of land is zero.

Technical change in the second region reduces wages in terms of agricultural goods in the first region:

$$(5.20)\quad \partial W_1'/\partial T_2' = |G|^{-1}[-\beta_{LY2}G_{12} - \nu_2(\beta_{YY2}+1)G_{1Y}]$$

$$= |G|^{-1}\nu_2\beta_{LY1}[\beta_{YY2}\beta_{LL2} - \beta_{LY2}\beta_{YL2} + \beta_{LL2} - \beta_{YY2}\varepsilon_{L2} - \varepsilon_{L2}] \leq 0$$

This term is negative because the first two terms come from a negative cofactor and all other terms are negative. For the genuine 1/1/0 case

we also have:

$$(5.21)\quad \frac{\partial (W_1/P)'}{\partial T_2'} = \frac{\partial W_1'}{\partial T_2'} - \frac{\partial P}{\partial T_2'} = |G|^{-1} \nu_1 \varepsilon_{L1} (\beta_{YY2}\varepsilon_{L2} - \beta_{LL2} + \varepsilon_{L2}) \gtrless 0$$

Note that these signs cannot be established for the 1/1/m case. But for the 1/1/0 case technical change in region two raises the wage rate in region one measured in terms of agricultural commodities. Finally note that the sign of the technical change effect on the wage in region one versus that of region two cannot be signed i.e.

$$(5.22)\quad \frac{\partial (W_1/W_2)'}{\partial T_1'} \gtrless 0$$

6) The Regional Land Rent Effects

The land rent effects are among the most interesting in the two region case and are found in a residual fashion as in equation (1.23) For each region we know that $P' = s_{Lr} W_r' + s_{Zr} S_r' - T_r'$ even if these are in factors in infinitely elastic supply because $s_{Zr} R' = 0$.

In the mobile labor case there is only one wage rate change and the solution for region r is

$$(6.1) \quad S_r' = \frac{1}{s_{Zr}} \quad (P' - s_{Lr} W' + T_r')$$

Into this equation one can set the partial effects of any exogenous change in the output price and the wage rate determined in section 4. In principle we could use (6.1) to solve analytically for the partial effects of exogenous changes on the land price as is done previously for the wage rate in terms of agricultural goods. However, that exercise is unlikely to lead to major insights.

Instead we focus on the land rent changes in region one relative to those in region two to understand why regions push technical changes, even though it may result into losses for land rents if all the regions pursue technical change simultaneously. The equation for the relative regional land rents reads

$$(6.2)\quad (S_1/S_2)' = S_1' - S_2' =$$

$$= \frac{s_{Z2} - s_{Z1}}{s_{Z1}s_{Z2}} P' + \left( \frac{s_{L2}}{s_{Z2}} - \frac{s_{L1}}{s_{Z1}} \right) W'$$

$$+ \frac{T_1'}{s_{Z1}'} - \frac{T_2'}{s_{Z2}'}$$

This equation contains two endogenous changes, $P'$ and $W'$, and the exogenous changes $T_r'$. If we want to consider the overall effect of technical change we should also consider the partials $\partial P'/\partial T_r'$ and $\partial W'/\partial T_r'$. But note that, if the regional labor and land shares are roughly equal, the terms in $P'$ and $W'$ are close to zero, and what is left is the terms in $T_r'$ and they will dominate the equations. We can therefore state that the change in relative land rents is roughly proportional to the difference in rates of technical change, whatever the price and wage rate effects of those technical changes are. Therefore, whether land owners ultimately gain or loose from any configuration of regional technical changes, their position relative to other regions is the better, the faster their own rate of technical change relative to that of other regions. As long as they cannot stop the investment for technical change in other regions, they must attempt to maximize their own rate of technical changes to maximize their gains or minimize their losses.

When labor is immobile the situation is somewhat more complicated for the land owners. The analoguous of equation (6.2) then reads

$$(6.3)\quad (S_1/S_2)' = \frac{s_{Z2}-s_{Z1}}{s_{Z1}s_{Z2}} P' + \frac{s_{L2}}{s_{Z2}} W_2' - \frac{s_{L1}}{s_{Z1}} W_1' + \frac{T_1'}{s_{Z1}} - \frac{T_2'}{s_{Z2}}$$

Since technical change in region r can affect $W_1$ and $W_2$ in the opposite direction and in different magnitudes, we cannot expect the $W_r'$ terms to come close to cancelling each other. Nevertheless, it remains true that under a broad range of conditions it pays landowners to maximize their own rate of technical change if they are concerned with their position relative to landowners in other regions. However, from the aggregate regional income point of view, which a regional government would espouse, it makes more sense to look at the rate of change of factor rewards $F_r' = s_{Lr}W_r' + s_{Zr}S_r' = P' - T_r'$. Forming the ratio of these factor rewards we find that in all cases,

$$(6.4)\quad (F_1/F_2)' = F_1' - F_2' = T_1' - T_2'$$

which holds for both the mobile and immobile labor case. Thus a regions overall agricultural income position relative to other regions is directly proportional to the difference in its rate of technical change relative to all other regions. Even though in the end technical change in all regions may lead to losses for all agricultural producers, each region must try to maximize its rate of technical change. This is also true if there are m

factors of production is infinitely elastic supply because $\sum_K s_{Kr} R'_{Kr} = 0$ in that case and equation (6.1) to (6.4) still hold.

## 7) An overview

Tables two and three give a summary of the signs of the effects which we have derived so far. For the Evenson-Welch model they also give (the negative of) the numerator of the sign condition as an indication of which parameters determine the sign. It will be most convenient to discuss these tables in terms of the effects on particular groups.

High income workers have a consumption pattern which gives a low weight to agricultural commodities in their income group specific price index. We can, therefore, discuss the approximate impact on their incomes by looking at the $dW'$, the change in the wage rate in terms of non-agricultural goods.

Throughout all the models high income workers would be hurt by any expansion in labor supply. Even if labor is immobile across regions and labor increases in other regions than their own, this would be the case since $\partial W/\partial L_j \leq 0$ in column 6. Increases in other factors of production, however, can either hurt these workers or benefit them. The condition for $\partial W'/\partial Z^*$ in column 1 suggests that the lower the substituability of labor for other factors, and the higher the final demand elasticity, the more likely will high income labor gain from an increase in the supply of other factors of production.

An exception to this is the case where labor is immobile across regions (column (6) and (7)) and where an increase in land in region 2

Table 2: Signs for the Input Supply and the Output Demand Effects

| | One Region Cases | | | Two Region Cases | | | |
|---|---|---|---|---|---|---|---|
| | | | | Mobile Labor | | Immobile Labor | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Supply responsive factors | 2 | 2 | 2 | 1 | 1 | 1 | 1 |
| Factors with fixed quantities | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| Factors with fixed prices | 0 | m | 0 | 0 | m | 0 | m |
| **Own Factor Supply Effects** | | | | | | | |
| $\frac{\partial W'}{\partial L^*}$ | $\alpha - \beta_{YY} - \epsilon_Z/s_Z \leq 0$ | − | − (convex) | − | − | $\frac{\partial W_i'}{\partial L_i^*}$ | − | − |
| $\frac{\partial P'}{\partial L^*}$ | $\beta_{YL} - \epsilon_Z(s_L/s_Z) \leq 0$ | − | − (noninf) | − | − | $\frac{\partial P'}{\partial L_i^*}$ | − | − |
| $\frac{\partial (W/P)'}{\partial L^*}$ | $\alpha - \epsilon_Z \leq 0$ | − | − (noninf) | − | − | $\frac{\partial (W_i/P)'}{\partial L_i^*}$ | − | − |
| | | | | | | $\frac{\partial W_i'}{\partial L_j^*}$ | − | − |
| | | | | | | $\frac{\partial (W_i/P)'}{\partial L_j^*}$ | +[a] | ±[b] |
| $\frac{\partial (W/S)'}{\partial L^*}$ | $\alpha - \epsilon_L \leq 0$ | ±[b] | $\frac{\partial (W/R)'}{\partial L^*}$ −2/1/0(noninf) | | | $\frac{\partial (W_i/W_i)'}{\partial L_i^*}$ | − | ±[b] |
| **Cross Factor Supply Effects** | | | | | | | |
| $\frac{\partial W'}{\partial Z^*}$ | $-\gamma - \alpha \geq 0$ | ±[b] | ± | $\frac{\partial W_i'}{\partial Z_i^*}$ | ± | ± | $\frac{\partial W_i'}{\partial Z_i^*}$ | ± | ± |
| $\frac{\partial (W/P)'}{\partial Z^*}$ | $\epsilon_L - \alpha \geq 0$ | ±[b] | ± | $\frac{\partial (W_i/P)'}{\partial Z_i^*}$ | + | ±[b] | $\frac{\partial (W_i/P)'}{\partial Z_i^*}$ | + | ±[b] |
| | | | $\frac{\partial W'}{\partial K^*}$ ± | | | | $\frac{\partial W_i'}{\partial Z_j^*}$ | − | − (noninf) |
| | | | $\frac{\partial (W/P)'}{\partial K^*}$ ± | | | | $\frac{\partial (W_i/P)'}{\partial Z_j^*}$ | + | ±[b] |
| **Final Demand Effects** | | | | | | | |
| $\frac{\partial W'}{\partial D^*}$ | $\beta_{LY} + \epsilon_Z/s_Z \geq 0$ | + | + (noninf) | + | + | | + | + |
| $\frac{\partial P'}{\partial D^*}$ | $\epsilon_L + \epsilon_Z s_L/s_Z - \beta_{LL} \geq 0$ | + | + (convex) | + | + | | + | + |
| $\frac{\partial (W/P)'}{\partial D^*}$ | $(\epsilon_L - \epsilon_Z) \gtrless 0$ | ± | ± | −[a] | ±[b] | | −[a] | ±[b] |

Footnotes:

a) This sign is determined only because land is in fixed supply and $\epsilon_{Zi} = 0$. Otherwise it would not be signable.

b) If the left out factors are substitutes on balance, the sign is the same as in the previous column.

convex: Convexity is the minimum assumption to establish sign.

noninf: Noninferiority is the minimum assumption to establish sign.

− sign is nonpositive
+ sign is nonnegative
± sign is ambiguous

Table 3: Signs of the Technical Change Effects

| | One Region Cases | | | Two Region Cases | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Mobile Labor | | Immobile Labor | | | |
| | (1) | (2) | (3) | (4) | (5) | (6) | | (7) | |
| Supply responsive factors | 2 | 2 | $\geq$ | 1 | 1 | 1 | | 1 | |
| Factors with fixed quantities | 0 | 0 | 1 | 1 | 1 | 1 | | 1 | |
| Factors with fixed prices | 0 | $n$ | 0 | 0 | $n$ | 0 | | $n$ | |
| **Neutral technical change** | | | | | | | | | |
| $\frac{\partial P'}{\partial T'}$ | $\leq 0; (\frac{\geq}{<} -1 \longleftrightarrow \lvert a \rvert \frac{\geq}{<} 1)$ | $-$ | $-$ | $\frac{\partial P'}{\partial T_i'}$ | | $\frac{\partial P'}{\partial T_i'}$ | $-$ | $-$ | |
| $\frac{\partial W'}{\partial T'}$ | $-(a+1)\frac{\partial W}{\partial D^*} \gtrless 0$ | $\pm$ | $\pm$ | $\frac{\partial W'}{\partial T_i'}$ | $\pm$ $\pm$ | $\frac{\partial W_i'}{\partial T_i'}$ | $\pm$ | (involves $a+v_1$ and $\epsilon_{Lj}$) | $\pm$ |
| $\frac{\partial (W/P)'}{\partial T'}$ | $\gtrless 0$ (positive if $\epsilon_Z = 0$) | $\pm$ | $\pm$ | $\frac{\partial (W/P)'}{\partial T_i'}$ | $+^{a)}$ $\pm^{b)}$ | $\frac{\partial (W_i/P)'}{\partial T_i'}$ | $+^{a)}$ | | $\pm^{b)}$ |
| | | | | | | $\frac{\partial W_i'}{\partial T_j'}$ | $-$ | | $-$ |
| | | | | | | $\frac{\partial (W_i/P)'}{\partial T_j'}$ | $-$ | | $-$ |
| | | | | | | $\frac{\partial (W_i/W_j)'}{\partial T_j'}$ | $\pm$ | | $\pm$ |

**Biases**

| | at expense of land | at expense of land | 2/1/0 case at expense of capital |
|---|---|---|---|
| $\frac{\partial W'}{\partial A_L'}$ | $= \frac{1}{s_Z}\frac{\partial (W/P)}{\partial L^*} = \frac{1}{s_Z}(a - \epsilon_Z) \lesseqgtr 0$ | $\pm^{b)}$ | $-$ (noninf) |
| $\frac{\partial P'}{\partial A_L'}$ | $= \frac{s_L}{s_Z}(\epsilon_L - \epsilon_Z) \gtrless 0$ | $\pm$ | $\pm$ |
| $\frac{\partial (W/P)'}{\partial A_L'}$ | $= \frac{1}{s_Z}\left[a - s_L\epsilon_L - s_Z\epsilon_Z\right] \lesseqgtr 0$ | $\pm^{b)}$ | $-$ (noninf) |

| | | at expense of factors with fixed prices | $n$/1/$n$ case at expense of factors with fixed prices |
|---|---|---|---|
| $\frac{\partial W'}{\partial A_L'}$ | n.a. | $= \frac{\partial W'}{\partial L^*} \leq 0$ | $\leq 0$ (convex) |
| $\frac{\partial P'}{\partial A_L'}$ | n.a. | $= \frac{\partial P'}{\partial L^*} \leq 0$ | $\leq 0$ (noninf) |
| $\frac{\partial (W/P)'}{\partial A_L'}$ | n.a. | $= \frac{\partial (W/P)'}{\partial L^*} \leq 0$ | $\leq 0$ (noninf) |

**Footnotes:**

a) This sign is determined only because land is in fixed supply and $\epsilon_{Zi} = 0$. Otherwise it would not be signable.

b) The sign is determined if the factors with fixed prices are substitutes of the other factors.

convex: Convexity is the minimum assumption to establish sign.

noninf: Noninferiority is the minimum assumption to establish sign.

$-$ sign is nonpositive
$+$ sign is nonnegative
$\pm$ sign is ambiguous

will clearly hurt labor in region 1.

Now consider low income laborers who spend most of their income on food, i.e., look at the effects on $(W/P)'$. Again these workers will lose from an increase in labor supply as long as it occurs in their own region. However, since the labor supply increase also reduces the agricultural price, their losses would usually be proportionately less than those of high income workers. The proportionate loss of high income workers exceeds that of low income workers by $\frac{\partial P'}{\partial L^*} \leq 0$.

Note here, we discuss the case where high income laborers spend all their income on non-foods and low income laborers all their income on food. If, however, high income workers spend a share $\mu_H$ of their income on food while low income workers spend $\mu_L$ on food, with $\mu_H < \mu_L$ we can form rates of changes of price indices $(\bar{P})$ such that

$$\bar{P}_H' = \mu_H P' \text{ and } \bar{P}_L' = \mu_L P'$$

Therefore their wage effects will always differ by $\left(\mu_H - \mu_L\right)P' \leq 0$ instead of by $P'$.

Returning to the extreme case, when labor is immobile and land is in fixedsupply (column 6) poor laborers in region 1 will gain from an increase in labor in another region, which is in contrast to the high income workers. When more factors are added (column 7), they may either gain or lose, but we can presume that in many cases they will continue to gain while high income laborers can never gain. The beneficial effects of the price

drop often will outweigh the detrimental effect on the wage rate.

A similar situation arises when other factors than labor increase. In column (1) we see that if land supply increases the poor laborers will gain while the high income laborers lose. This is also the case in both regional cases when there are only two factors of production. Only if more factors of production are added (columns (2), (5) and (7)) can we no longer be sure that the poor laborers will gain because complementarities of additional factors may work against them.

Expansion in final demand always benefits high income laborers, but it will definitely lead to losses for low income laborers when there are only two factors of production and the land supply is exogenous (column 1 with $\epsilon_z = 0$ and column (4) and (6) ). Only when complementary factors of production are added or land is in elastic supply may the poor workers not lose from an expansion in final demand.

Neutral technical change may benefit or hurt high income labor. In all cases highly elastic final demand will lead to gains for labor because the saving of labor made possible by the technical change is offset by the more than proportional output expansion. In the regional case with immobile labor the expressions in the text suggest that the smaller the region experiencing the technical change, the more likely its labor is to gain.

Output prices almost always fall when technical change occurs. In the one region case the price drop is proportional to the rate of technical change when the final demand elasticity is equal to minus one and exceeds it if final demand is more elastic than that (column (1), but also for 2 and 3). The fact that prices are reduced makes gains for low income workers more likely than for high income workers, especially if land has zero supply

elasticity and there are no additional complementary factors of production. In that case the poor workers tend to gain regardless of the final demand elasticity while the high income workers may still lose.

Wherever we can establish a sign for the biases, high income and low income workers both lose from labor-saving technical change. Note, however, that in a model with many factors of production we cannot establish signs for an increase in the labor-saving bias if it goes at the expense of land. Also recall that for the genuine n/1/0 case we cannot establish the effects of any biases.

Large owners and capital owners. We will only discuss the income effects in terms of nonagricultural goods by assuming that the land and capital owners are usually among the wealthier groups consuming mostly nonagricultural goods.

In the 2/0/0 and 2/0/m models land is treated symmetrically to labor and the effects on the land owners can be found by inverting the role of land and labor in the equations corresponding to these models. Thus land owners lose when land is expanded, they may gain or lose when labor is expanded and they gain if final demand is expanded. By looking at the conditions for technical change it is clear that landowners and laborers both lose or gain together when technical change occurs, a fact which was noted already in Binswanger (1978)[1].

For the 2/1/0 model capital and labor are treated symmetrically and the conditions for the capital owners can be derived from the equations in the text by interchanging the role of capital and labor. But for this model and the regional models, land is fixed in supply and the land rent effects have to be determined residually. It is, however, clear that land

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[1] Note that this is not the case when we consider general equilibrium models with more than one sector. See Binswanger (1978).

owners will lose if land expands in their own region, that they may gain or lose if other factors of production expand in their own region, that they may gain or lose if other factors of production expand and that they will gain with expansion in final demand. Furthermore, technical change will lead to losses or gains according to the final demand elasticities.

Consumers in other sectors will gain from any changes which reduces agricultural prices. Expansion in the supply of any factor of production anywhere will do that. Similarly, any increase in technical change will do it as well, unless final demand is infinitely elastic (possible exception). Even without changes in the rate of technical change, a shift of the bias of technical change in favor of saving factors in relatively elastic supply will often tend to reduce the output price and benefit nonagricultural consumers. Consider $\frac{\partial P'}{\partial A_L}$ in the different models. In the 2/0/0 case (column 1), this will be positive if land is in elastic supply relative to labor and the bias shifts from saving land more to saving labor. When complementary factors are possible (2/0/n and 2/1/0 cases) however, we can no longer be sure of this sign. However, if there are factors in infinitely elastic supply or with fixed prices (2/0/m and 2/1/n cases) a shift of the bias from saving those factors towards saving labor will reduce the output price since labor is then the less elastic factor. That shifts of biases towards factors in relatively inelastic supply will tend to reduce output price is another demonstration of the induced innovation hypothesis which states that society gains from directing technical change towards factors in inelastic supply.

One observation which may be appropriate to conclude this paper is that evaluating distributional impacts of policies which affect factor supplies,

output demand and technical change in agriculture is not straight forward at all. With the exception of the own factor demand effects, the impacts depend on the conditions in the factor and output markets and on the position of a region where the changes occur relative to other regions. The observation that a particular policy or technical change has had a favorable distributional effect when occurring in one environment just is not sufficient to advocate the technical change in another environment on distributional gounds. We hope, however, that the results of this paper may be helpful in understanding under what kinds of conditions similar results can be expected and when not, and to assist in evaluating likely distributional effects in an ex ante framework.

## APPENDIX

### Sign Proofs for Determinants

To prove signs of inverse elements of the Excess Elasticities Matrices one has to trace those matrices back to the matrix of second order derivatives of the profit function and to minors thereof. Consider the (singular) Elasticities Matrix $\beta$ for the two factor-one output case

(A-1)

$$\beta = \begin{bmatrix} \beta_{LL} & \beta_{LK} & \beta_{LY} \\ \beta_{KL} & \beta_{KK} & \beta_{KY} \\ \beta_{YL} & \beta_{YK} & \beta_{YY} \end{bmatrix} = \begin{bmatrix} \frac{\partial L}{\partial W}\frac{W}{L} & \frac{\partial L}{\partial R}\frac{R}{L} & \frac{\partial L}{\partial P}\frac{P}{L} \\ \frac{\partial K}{\partial W}\frac{W}{K} & \frac{\partial K}{\partial R}\frac{R}{K} & \frac{\partial K}{\partial P}\frac{P}{K} \\ \frac{\partial Y}{\partial W}\frac{W}{Y} & \frac{\partial Y}{\partial R}\frac{R}{Y} & \frac{\partial Y}{\partial P}\frac{P}{Y} \end{bmatrix}$$

The first two rows are factor demand rows and the last row is the output supply row. Now compare it to the matrix $\Pi$ of second order derivatives of the cost function with respect to the prices of the factors of production in the subsrcipts.

(A-2)

$$\Pi = \begin{bmatrix} \Pi_{LL} & \Pi_{LK} & \Pi_{LY} \\ \Pi_{KL} & \Pi_{KK} & \Pi_{KY} \\ \Pi_{YL} & \Pi_{YK} & \Pi_{YY} \end{bmatrix} = \begin{bmatrix} -\frac{\partial L}{\partial W} & -\frac{\partial L}{\partial R} & -\frac{\partial L}{\partial P} \\ -\frac{\partial K}{\partial W} & -\frac{\partial K}{\partial R} & -\frac{\partial K}{\partial P} \\ \frac{\partial Y}{\partial W} & \frac{\partial Y}{\partial R} & \frac{\partial Y}{\partial P} \end{bmatrix}$$

It is clear that $\beta$ can be derived from $\pi$ by dividing the labor demand row by $(-L)$, the capital demand row by $(-K)$, the output supply row by Y and multiplying each of the columns by W, R and P respectively.

If we multiply a row or a column of a matrix A by a constant k, its determinant $|A|$ becomes $k|A|$. Therefore

$$(A\text{-}3) \qquad |\beta| = \frac{W\,R\,P}{(-K)(-L)Y}\,|\Pi|$$

and $|\beta|$ would have the same sign than $\Pi$.[1] Similarly if we knew the sign of the determinant of any minor of $\Pi$, say $|\Pi|_{LL} = \Pi_{KK}\,\Pi_{YY} - \Pi_{KY}\,\Pi_{YK}$ It would follow that

$$(A\text{-}4) \qquad |\beta|_{LL} = \frac{WP}{(-L)Y}\,|\Pi|_{LL}$$

i.e. $|\beta|_{LL}$ would be of the opposite sign than $|\Pi|_{LL}$.

Generally to the n-factor and m-goods case, we can have the following rule:

Rule 1: The determinant of an elasticity matrix (or of any minor thereof) has the same sign than the determinant of the corresponding matrix (or minor) of second order derivatives of the profit function if the number of factor demand rows involved in the matrix (or minor) is even. It has the opposite sign if the number of factor demand rows involved is odd. (The number of

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[1] In this case $|\Pi| = 0$, therefore $|\beta| = 0$.

output supply rows involved is immaterial.)

But we are interested in signing the determinant and minors of the excess elasticity matrix $[\beta + \varepsilon]$. In the two factor-one good case discussed above let H be the matrix of factor supply and output demand slopes.

$$(A.5) \qquad H = \begin{bmatrix} h_L & 0 & \\ & h_k & \\ 0 & & -h_Y \end{bmatrix} = \begin{bmatrix} \frac{\partial L}{\partial W} & & 0 \\ & \frac{\partial K}{\partial R} & \\ 0 & & -\frac{\partial Y}{\partial P} \end{bmatrix}$$

which is non-negative definite. Note that $[\beta + \varepsilon]$ is constructed from $(\Pi + H)$ in exactly the same way that $\beta$ is constructed from $\Pi$. Therefore Rule 2 holds:

Rule 2: The determinant of an excess elasticity matrix (or of any of its minors) has the same sign than the determinant of the corresponding matrix (or minor) of the $[\Pi + H]$ matrix if the number of factor demand rows involved in the matrix (or minor) is even. It has the opposite sign if the number of factor demand rows involved is odd.

We therefore confine our attention to the signs of the determinants and minors of the $[\Pi + H]$ matrix and note the following well known facts:

Rule 3A: The matrix $\Pi$ is non-negative definite and all its principal minors have non-negative determinants.

Rule 3B: The matrix H is nonnegative definite, since all its diagonal elements are positive or zero in the one good case. (In the m-goods case the output demand submatrix would also be non-negative definite and the matrix H remains non-negative definite.)

Rule 3C: The sum of two non-negative definite matrices is non-negative definite and therefore $[\Pi + H]$ is non-negative definite. All its principal minors have non-negative determinants.

Now consider the regional cases with mobile labor where the national elasticity matrix has the form

$$(A\text{-}6) \qquad \bar{\beta} = \begin{bmatrix} \bar{\beta}_{LL} & \bar{\beta}_{LY} \\ \bar{\beta}_{YL} & \bar{\beta}_{YY} \end{bmatrix} = \begin{bmatrix} \lambda_1\beta_{LL1} + \lambda_2\,\beta_{LL2}; & \lambda_1\,\beta_{LY1} + \lambda_2\,\beta_{LY2} \\ \nu_1\beta_{YL1} + \nu_2\,\beta_{YL2}; & \nu_1\,\beta_{YY1} + \nu_2\,\beta_{YY2} \end{bmatrix}$$

with $\lambda_i = \frac{L_i}{L}$ and $\nu_i = \frac{Y_i}{Y}$. Compare it with the sum of the second order derivatives of the profit function in each region, which is non-negative definite because each of the regional matrices is non-negative definite.

$$(A\text{-}7) \qquad \bar{\Pi} = [\Pi_1 + \Pi_2] = \begin{bmatrix} \Pi_{LL1} + \Pi_{LL2} \; ; & \Pi_{LY1} + \Pi_{LY2} \\ \Pi_{YL1} + \Pi_{YL2} \; ; & \Pi_{YY1} + \Pi_{YY2} \end{bmatrix}$$

It is clear that (A-6) can be derived from (A-7) by dividing the first row by (-L), the second row by Y, and by multiplying the first column by W and the second column by P. Therefore $|\bar{\beta}| = -(WP/LY)\ |\bar{\Pi}| \leq 0$. Rule 1 continues to apply in the case of regional models with an arbitrary number of mobile factors of production.

Now consider the case of immobile labor where the excess elasticity matrix has the form:

$$(A\text{-}8) \qquad [\beta + \varepsilon] = \begin{bmatrix} \beta_{LL1} - \varepsilon_{L1} & 0 & \beta_{LY1} \\ 0 & \beta_{LL2} - \varepsilon_{L2} & \beta_{LY2} \\ \nu_1 \beta_{YL1} & \nu_2 \beta_{YL2} & \bar{\beta}_{YY} - \alpha \end{bmatrix}$$

Compare it with the following matrices

$$(A\text{-}9) \quad [\Pi_1^* + \Pi_2^* + h^*] = \begin{bmatrix} \Pi_{LL1} & 0 & \Pi_{LY1} \\ 0 & 0 & 0 \\ \Pi_{YL1} & 0 & \Pi_{YY1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Pi_{LL2} & \Pi_{LY2} \\ 0 & \Pi_{YL2} & \Pi_{YY2} \end{bmatrix}$$

$$+ \begin{bmatrix} h_{L1} & & \\ & h_{L2} & \\ & & -h_Y \end{bmatrix}$$

$\Pi_1^*$, $\Pi_2^*$ and h* are all non-negative definite therefore their sum is non-negative definite. The $[\beta + \epsilon]$ matrix is derived from $[\Pi_1^* + \Pi_2^* + h^*]$ by multiplying the first column by $W_1$, the second by $W_2$ and the third by P and by dividing the first row by $(-L_1)$, the second by $(-L_2)$ and the third by Y. Therefore

$$|\beta + \epsilon| = \frac{W_1 W_2 P}{(-L_1)(-L_2)Y} \left| \Pi_1^* + \Pi_2^* + h^* \right| \geq 0$$

Rule 2 again carries through to the regional case with immobile labor where labor in region 1 gets counted as a separate factor of production from labor in region 2 for the purposes of establishing signs. Note that for determining signs of determinants we could apply the regional cases to as many mobile and immobile factors of production and goods as we want, i.e. develop mixed cases.

Table 4: LIST OF SYMBOLS

$A_K'$, $A_L'$, $A_Z'$ : Factoral rates of technical change under the cost function

$C$ : Cost of production

$D^*$ : Final demand shifters

$B_K'$, $B_L'$, $B_Y'$ : Shifts in factor demands and output supplied with fixed land profit function

$K$, $K^*$ : Capital, capital supply shifter

$L$, $L^*$ : Labor, labor supply shifter

$M$ : Per capita product or income

$M_L$, $M_K$, $M_Z$ : Labor, capital, land income

$N$ : Population

$P$ : Output prices, $\bar{P}$ are price indexes

$Q$ : Biases

$R$ : Capital rental rate

$S$ : Land rent

$s_i$ : Share of factor i in value of output

$T'$ : Rates of technical change.

$W$ : Wage rates (or factor prices in general many factor case).

$Y$ : Outputs

$Z$ : Quantity of land

$\alpha$ : Commodity demand elasticities

$\sigma$ : Elasticity of substitution

$\epsilon$ : Factor supply elasticities

$\eta$ : Factor demand elasticities

$\lambda_\ell$ : Share of labor force in sector $\ell$

$\mu_\ell$ : Share of sector $\ell$ in national product

$\xi_\ell$ : Share of capital in sector $\ell$

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$\lambda_r$ : Share of labor force in region r

$\nu_r$ : Share of region r in output

$\xi_r$ : Share of capital in region r

i = 1.....n factors or goods

j = 1.....n factors or goods

$\ell$ = 1,2 Sectors

r = 1,2 Regions

k = 1.....K income groups

$\mu_h^k$ = expenditure share of income groups k on good h

$\beta$ = factor demand and output supply elasticities for profit functions

$\delta_{ih}$ = Share of income arising out of factor i for income group k