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## The Economic Role of Commodity Storage

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THE ECONOMIC ROLE OF COMMODITY STORAGE

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THE ECONOMIC ROLE OF COMMODITY STORAGE

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ABSTRACT

This paper presents a model of competitive, profit-maximizing storage of a commodity with economically responsive, although stochastic, supply. By comparing the distributions of market variables with and without storage, we show that several intuitive notions about the role of storage are misleading. Rather than stabilizing production, storage actually accentuates its variability. Rather than being most effective at eliminating short-falls in consumption, storage is more effective at reducing the incidence of exceedingly high consumption. Even so, a welfare analysis shows storage is favorable to consumers over a wide range of demand specifications and supply elasticities.

# THE ECONOMIC ROLE OF COMMODITY STORAGE

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One of the earliest, and most successful, examples of economic policy is the oft-quoted Biblical account of Joseph's interpretation of the Pharaoh's dream as implying that seven years of abundant harvests would precede seven years of drought, and Joseph's recommendation that the Pharaoh should accumulate grain during the good years. Since that time the central role of storage in stabilizing the economy in the face of exogenous disturbances has been obvious, but our understanding of the nature of that role has not greatly advanced.

Without divine assistance in forecasting stochastic production, the storage decision is considerably more complex than the one Joseph faced, and the role of storage quite different. In fact, several commonly held impressions about the role of storage of commodities such as grains are incorrect. Rather than stabilizing production, storage actually accentuates its variability. Rather than causing a mean-price-preserving decrease or a mean-output-preserving decrease in the dispersion of price, storage generally causes a more complex modification of the distribution of price. Rather than being most effective at eliminating short-falls in consumption, storage actually is more effective at eliminating the incidence of exceedingly high consumption.

In this paper we explore the role of storage in a model where production is stochastic and both production and storage are performed by competitive profit-maximizers who form rational expectations about the returns to their

activities. We derive the subtle but very important interactions among production, price expectations, and storage, which simpler models cannot capture. Finally, we make a comparative statics assessment of the distributional implications of storage. These results, while confirming the importance of the specification of the demand function and the supply elasticity identified in recent analytical studies (e.g. Wright (1979) and Newbery and Stiglitz (1979)), are surprisingly favorable to consumers, considering the asymmetric nature of the effects of storage on consumption and price.

We start with a closed competitive economy, in which all consumers are assumed to be identical. The inverse consumption demand for the single commodity in question is

$$(1) \quad P_t = P(q_t)$$

where  $P_t$  is the price at time  $t$  and  $q_t$  is the quantity consumed.

$$(2) \quad q_t = I_t - S_t$$

where  $S_t$  is the amount stored to period  $t + 1$ , and  $I_t$  is the amount on hand,

$$(3) \quad I_t = x_t + aS_{t-1}$$

where  $x_t$  is production in period  $t$  and  $a$  is the proportion of  $S_{t-1}$  available at time  $t$ , after "shrinkage" or wastage of  $(1 - a)S_{t-1}$ .

Production in each period is subject to a random disturbance. Common sources of production instability are likely to have multiplicative effects on output, rather than the additive effects assumed in much of the literature on storage and market stabilization. In grain production for example, because weather determines the yield of a particular acre, the more acres planted the

greater will be the variation in total output. Accordingly, the supply function is

$$(4) \quad x_t = \hat{x}(P_t^r) [1 + v_t]$$

where  $v_t$  is the random production disturbance with a probability distribution  $f(v)$  of finite variance. The disturbance is assumed to be serially uncorrelated and is the same for each producer.<sup>1</sup>  $P_t^r$  is the producer incentive at time  $t - 1$ , when planned production,  $\hat{x}(P_t^r)$ , must be selected for time  $t$ . Under this specification short-run (same period) production is perfectly inelastic.

We assume that producers and all others in the model maximize profits and have rational expectations in the Muthian sense. Both the structure of the model and the distribution of  $v_t$  are in the common information set  $\Omega_{t-1}$  in period  $t-1$ . Revenue of producer  $i$  when his realized production is  $x_{it}$  is

$$(5) \quad r_{it} = x_{it} P(q_t)$$

The producer maximizes expected profits

$$(6) \quad E[\Pi_{it}] = E[r_{it}] - H(\hat{x}_{it})$$

where  $H$  is total cost and  $E$  denotes the conditional expectation given  $\Omega_{t-1}$ . Under atomistic competition, each producer is a price-taker, but he recognizes the perfect correlation between the disturbance in his own production and the disturbance in aggregate production. Hence the first order condition for competitive profit maximization is

$$(7) \quad \frac{\partial E[\Pi_{it}]}{\partial \hat{x}_{it}} = \frac{\partial E[r_{it}]}{\partial \hat{x}_{it}} - H'(\hat{x}_{it})$$

Thus a producer's incentive is the marginal return per unit of planned production,  $P_t^r$ , where (remembering that he is a price-taker),

$$(8) \quad P_t^r = \frac{\partial E[r_{it}]}{\partial \hat{x}_{it}} = \frac{\partial}{\partial \hat{x}_{it}} E[ [1 + v_t] x_{it} P([1 + v_t] \hat{x}(P_t^r) + S_{t-1} - S_t) ]$$

$$= \frac{E[P_t x_{it}]}{\hat{x}_{it}}$$

### 1. The Competitive Profit-Maximizing Storage Rule

To complete the market model, we must consider the cost of storage, which can be viewed as a productive activity transferring units of the commodity available in period  $t$  to units available in period  $t + 1$ . The cost of storing  $S_t$  units in period  $t$  is

$$(9) \quad K(S_t) = \xi(S_t) + (1 - a)P_t S_t + rP_t S_t$$

where  $\xi(S_t)$  is the net cost of physical storage services,  $(1 - a)$  is the shrinkage factor, that is, the physical depreciation of the stored commodity, and  $r$  is the interest rate. Both  $a$  and  $r$  are, for simplicity, assumed constant over time, with  $r > 0$  and  $0 < a \leq 1$ . All prices and costs are expressed in real terms. For commodities such as grains, empirical evidence indicates that marginal physical storage costs are fairly constant over the relevant range (Paul 1970). Accordingly, we specify physical storage costs in (9) as

$$(10) \quad \xi(S_t) = kS_t, \quad k \geq 0$$

Empirical research on the "price of storage" relating grain stocks at the end of the crop year to the difference between the nearest futures price and the spot price shows that the net cost of storage includes an offsetting accessibility value or "convenience yield" to users which makes the net marginal cost of storage negative at low levels of  $S$ . (See Working (1949), Brennan (1958), or Telser (1958).) This accessibility value, which is related to stochastic elements in distribution and demand, is discussed elsewhere at length by one of the authors (Williams 1980). Here we assume away any accessibility value of stocks, and focus on the role of storage in mitigating the effects of aggregate production disturbances.<sup>2</sup>

Private storage, like production, is assumed to be a competitive profit-maximizing activity. Given current inventory  $I_0$ , and conditional on storage of  $S_T$  in some future year  $T$ , the optimal storage in the current period is the solution to a stochastic dynamic programming problem, in the tradition of Gustafson (1958a and b). (See also Johnson and Summer (1976), Newbery and Stiglitz (forthcoming).) As Samuelson (1971) shows, given an individualistic social welfare function and appropriate regularity and transversality conditions,<sup>3</sup> welfare-maximizing storage,  $S_t^*$ , in an undistorted economy with infinite horizon is a function of the amount available,

$$(11) \quad S_t^* = f(I_t) \quad , \quad 0 \leq f' \leq 1$$

and the Kuhn-Tucker necessary conditions are

$$(12) \quad 0 \leq (1+r)^{-1} a EP_{t+1} - (P_t + k)$$

$$0 = S_t [(1+r)^{-1} a EP_{t+1} - (P_t + k)]$$



These conditions can be reinterpreted as the following competitive profit-maximizing arbitrage condition: Profit-maximizing competitive storage, if positive, will equate current period price with expected price in the next period, less the marginal cost of storage services, shrinkage, and interest on capital invested.

The non-negativity of storage means that there is a fundamental discontinuity in the storage rule. Although it is possible to store for the future, it is physically impossible to borrow from the future. This asymmetry has crucial implications for the effect of storage on consumption and price.<sup>4</sup>

## 2. The Effects of Storage on Market Demand

The rule for profit-maximizing storage depends upon the particular specifications of supply and consumption demand, as well as on the degree of shrinkage, the cost of storage services, and the interest rate. To allow for examination of a wide range of specifications of the consumption demand function, the following general form is used:

$$(13) \quad P = \alpha + \beta q^{1-C}, \quad \alpha \geq 0$$

This form includes the linear ( $\alpha > 0, C = 0, \beta \leq 0$ ) and constant elasticity ( $\alpha = 0, \beta > 0, C > 1$ ) as special cases. In what follows, the relative curvature of a given demand curve is measured by  $C$ , which is the Pratt-Arrow measure of curvature used to assess relative risk aversion:

$$(14) \quad C \equiv - \frac{qP''(q)}{P'(q)}$$

If  $C$  is greater than 2.0, the demand curve displays "commodity risk aversion", because consumers would pay for a mean-preserving decrease of the dispersion in price. If  $C$  is less than 2.0, the demand curve has commodity risk preference.

Derivation of the optimal storage rule in this model is analytically intractable. Fortunately profit-maximizing storage rules can be derived numerically using a process of successive approximation described in the Appendix. The storage function illustrated in Figure 1 was derived using this numerical method. It represents the case where the elasticity of demand  $\eta^D$  is -0.2, the elasticity of supply  $\eta^S$  is 0, the interest rate is 0.05, and supply in the absence of the stochastic disturbance (i.e. at  $v_t = 0$ ) is 100. The distribution of the multiplicative disturbance is a normal density function with a mean of zero and a standard deviation of 0.05. Physical storage costs are set at zero, and shrinkage is assumed to be zero. Notice that when a quantity less than an amount  $\bar{I}$  (equal to 99.09 in this example) is on hand, from current output and previous storage, all of the available commodity is consumed. Any excess above  $\bar{I}$  is divided between current consumption and storage. The marginal propensity to store changes only slowly over a large range of  $I$ , for  $I > \bar{I}$ . This is characteristic of the storage functions derived for a wide range of sets of parameters.<sup>5</sup> In the example behind the storage function illustrated, because mean production and consumption are 100, and mean storage is 3.4, mean availability is close to 103.4.

Under profit-maximizing storage, current price can be expressed as function of the amount in store. Using equation (1), the inverse consumption demand function, and equations (2) and (11),

$$(15) \quad P_t = P(f^{-1}(S_t^*) - S_t^*) = \phi(S_t^*)$$

This expression is the inverse demand function for storage. More precisely, it is the inverse derived demand for the input of the commodity into the storage process; accordingly, it is a function of the costs of the other inputs into that process, including the costs of shrinkage, storage services, and capital. The derived demand for storage corresponding to the storage function in Figure 1 meets the price axis at  $\bar{P} = 104.7$ . When current price exceeds  $\bar{P}$ , expected future price net of all storage costs is less than current price, so that there is no profit in even the first unit of storage.

Horizontal addition of the storage demand function to the consumption demand function yields the market demand function shown in Figure 2. At price  $\bar{P}$  the elasticity of market demand changes from 0.20 to 0.48. This augmentation of consumption demand below  $\bar{P}$  by the storage demand function may explain the (admittedly tentative) conclusion of Hillman, Johnson, and Gray (1975) that the demand curve for corn is highly nonlinear, being much less elastic at high prices than at lower prices. Their measurements, relating price changes to changes in availability rather than in consumption, may reflect the demand for storage, rather than any nonlinearity in the underlying consumption demand curve. This distinction is important, since, as we show below, the welfare effects of stabilization are crucially dependent on the curvature of the consumption demand curve, not of the market demand curve. The failure of Hillman et al. to draw this distinction is shared by several studies of price stabilization that quote their conclusion, including Reutlinger (1976) and Just et al. (1978).

### 3. The Effects of Storage with Zero Supply Elasticity.

As observed above, a rule for optimal storage has the property that below some level of availability  $\bar{I}$ , no storage is carried over from one production period to the next. Above  $\bar{I}$ , consumption and storage generally both increase as  $\bar{I}$  increases, and the marginal propensity to store also rises.

These simple qualities of the storage rule actually have strong implications for the effects of storage. To show this, we used the example behind the storage rule illustrated above, with constant elasticity of demand  $\eta^D = -0.2$  ( $C = 6$ ), and elasticity of supply  $\eta^S = 0.0$ . Starting with nothing in store, we applied the storage rule in a simulation of 10,004 periods, drawing from the random distribution of the production disturbance, and saved all market data beyond the fourth period. (For a sample of this size, the distributions of the variables of interest should closely follow the population distributions.)

The distribution of storage is shown in Figure 3. It is clearly bi-modal and highly skewed. No storage occurs 24.4% of the time. Mean storage is 3.4 and the standard deviation is 3.5. (Sample means and standard deviations for the distributions discussed in this and the following sections are displayed in Table 1.) We also simulated the same number of periods with the identical string of random numbers holding storage at zero. A comparison of these two simulations provides an instructive illustration of the effects of storage on market variables.

### 3.1 Effects of Storage on Price

Storage causes a large, asymmetric and possibly counter-intuitive change in the distribution of price. The distribution of price in the absence of storage is shown in Figure 4. Although the production disturbance is symmetric, the distribution of price is not, because of the nonlinearity in the constant elasticity demand curve. Figure 5 shows the distribution of price for the same production sequence when storage is possible. A comparison of the distributions in Figure 4 and Figure 5 indicates that first of all, storage lowers the mean price, in this example by 2.4%. Because the mean of the distribution changes, the total effect of storage is not a mean-price-preserving

decrease in the dispersion of price, in the terminology of Rothschild and Stiglitz (1970 and 1971).

To isolate the changes in dispersion from this change in the mean, we shifted the distribution in Figure 5 by the difference in the mean price and subtracted the densities in Figure 4 from those in Figure 5 to obtain Figure 6. With the help of Figure 6, we can see that storage affects price dispersion mainly by shifting probability mass from the lower tail towards the mean. Thus the effects of storage on the price distribution are asymmetric in a fashion that contradicts popular notions about storage. We tend to think of storage primarily as protection of consumers against commodity shortages and high prices. But the type of storage considered here is much more dependable in precluding commodity gluts and low prices. The greater the inconvenience to consumers of a shortage (reflected in the demand curve) the higher will be the expected price and the larger the incentive to store. Even so, optimal storage will not be large enough to ensure that there will never be a shortage. Indeed as Townsend (1977) has shown, any finite store will be emptied with probability one in finite time.

### 3.2 Effects of Storage on Consumption

Because the elasticity of supply is zero in this example, storage does not affect average consumption. Without storage, consumption has the same distribution as production. Under perfectly inelastic supply, storage causes a large mean-preserving decrease in the dispersion of consumption. In the sample of 10,000 periods, storage reduces the standard deviation of consumption from 5.0 to 3.0; by that measure it goes forty percent of the distance to complete stabilization. But this decrease in dispersion is

clearly asymmetric, as the resulting distribution is significantly skewed to the left. Figure 7 shows the difference between the frequency of consumption with and without storage.

#### 4. The Implications of Responsive Supply

##### 4.1 Effects of Storage on Planned Production

Once storage is introduced in the model, the assumption of perfectly inelastic supply becomes a very important restriction. In the absence of storage, the elasticity of supply is in fact irrelevant. Because there is no serial correlation in the production disturbances, a shortage or glut in one season has no effect on price in the next. Hence  $P_t^F$ , the economic incentive for production in year  $t$  as of year  $t - 1$ , when production must be planned, is constant from year to year.

Storage effects the production incentive,  $P_t^F$ , in a given period in two ways. First, for a given current output of the commodity, the demand for current storage increases price by augmenting the consumer demand curve as shown previously in Figure 2. Second, for a given output, any carryover from the previous year depresses the realized price. The relative strength of these two effects on the incentive to produce varies from period to period, so that  $P_t^F$  is sometimes higher, sometimes lower than it would be without storage. This interaction of storage and production is quite important to the effects of storage, a fact missed by other models in the tradition of Gustafson (1958a and b) in which elasticity of supply is fixed at zero.<sup>6</sup> We illustrate the net result of storage on  $P_t^F$  using the previous example modified so that supply elasticity  $\eta^S$  is constant at 1.0 and supply is linear within the observed range of planned production.

Under responsive supply, the marginal propensity to store is greater than in the storage rule illustrated in Figure 1 for  $\eta^S = 0.0$ , and the level of availability at which storage begins,  $\bar{I}$ , is larger. Although storage is generally thought of as a market-stabilizing mechanism, it clearly destabilizes planned production, as can be seen in the distribution of planned production under storage as shown in Figure 8. In fact, the coefficient of variation of planned production rises from 0% under completely inelastic supply to 41% of that of realized production, which is in turn 10% higher than the coefficient of variation of production without storage. It is obvious that the derived demands for production inputs (which are not explicitly considered here) are also destabilized by storage. Rather than being regarded as a means of stabilizing production, competitive storage should be thought of as a way of efficiently dispersing the effects of a disturbance throughout an (undistorted) economy.

In effect, storage acts as a substitute for production. When current supplies are abundant and the price of the commodity put into storage is low, it is more economical to deliver supplies next period through storage rather than through production. On the other hand, if current supplies are expensive, production is relatively more attractive. When production is more responsive, these two substitutes will each display greater variation, but their combined action results in more stable consumption.

Besides increasing the dispersion of production, storage also changes its mean, in this case by  $-0.4\%$ . The direction of this change is related to the curvature of the demand curve, measured by the relative commodity risk aversion parameter  $C$ . As Table 1 shows, if the example is changed so that  $C$  equals  $0.0$ , its value when demand is linear, while the demand elasticity (at the equilibrium consumption in the nonstochastic case)

remains  $-0.2$ , storage increases mean planned production when  $\eta^S = 1$ . For demand curves with intermediate values of  $C$ , but the same elasticity, the direction of change of mean planned production when  $\eta^S = 1$  depends on the degree of stabilization of consumption effected by storage, which is itself a function of the cost of storage.<sup>7</sup>

The contribution of responsive production to this process, however, is asymmetric. Maximum planned production, at 102.3, is the level of planned production whenever storage is zero, which occurs in 27.9% of all years. Therefore responsive production is poor insurance against a run of particularly bad harvests, since it provides a maximum offsetting increase in expected availability of only 2.3%. Production response is much more flexible in compensating for abnormally good years; minimum planned production in the sample is 9.2% below the mean. This may explain why Gustafson (1958b) indicates observed yields per acre of field crops are significantly skewed to the left, while Day (1965) concludes yields in controlled experiments are skewed to the right if at all. Through its effects on economic incentives, storage may alter realized production asymmetrically not only through acres planted but through yields.

#### 4.2 Effects of Responsive Supply on Storage, Price, and Consumption

Responsive supply greatly accentuates the effects of storage on price and consumption, though it scarcely changes the first two moments of the storage distribution. In the standard example, ( $\eta^D = -0.2$ ,  $C = 6$ ) mean storage is higher by only 1.3% for  $\eta^S = 1$  relative to  $\eta^S = 0$ , while the standard deviation is virtually unaltered. But under the more responsive supply the distribution is much less skewed, and the maximum amount in store in the sample is reduced from 24.8 to 20.3. Reductions in planned production moderate the build-up of storage in a string of good years.



The existence of responsive supply greatly enhances the decrease in the price dispersion caused by storage. The coefficient of variation is lower by 22.4% compared to the case illustrated in Figure 5. The distribution is much more highly skewed and minimum price is more than doubled in this sample.

The dramatic effect of responsive supply on the dispersion of consumption under storage is shown in Figure 9. The clustering effected by storage is greatly accentuated by a transfer of probability mass from areas both above and below the mean, reducing the coefficient of variation by 24%. The difference in mean consumption is negligible, but the new distribution is much more skewed and has much higher kurtosis. The most striking effect is that maximum consumption in the sample is reduced by 12.5%, even though maximum production is actually increased. **Maximum consumption is in fact only a miniscule 0.35% higher than maximum planned production, which occurs whenever storage is zero.**

The effect of responsive supply on maximum consumption can be explained as follows. When supply is perfectly inelastic, very high consumption levels occur after consecutive years of very high production. When supply is elastic, planned production is reduced after a good year, and profit-maximizing storage is increased; the net effect is a lower level of consumption in the current year and the next, relative to the situation with fixed long-run supply. The same kind of compensation does not occur in a string of very bad years, however, because below the level of availability at which storage is zero, further marginal shortfalls do not increase the price incentive  $P^r$ . This explains why minimum consumption in the sample is higher by only 0.7% under elastic supply, even though maximum consumption is so drastically reduced.

## 6. The Relevance of Demand Specification

Both the slope and curvature of the demand curve affect storage behavior. The less steep is the demand curve, the lower is average storage, and the less frequent is the occurrence of storage. For example, line 5 of Table 1 shows that at  $\eta^D = -0.5$ ,  $C = 6$ , and  $\eta^S = 0$ , mean storage is 1.12 (compared to 3.4 for  $\eta^D = -0.2$ ,  $C = 6$ ,  $\eta^S = 0$ ), storage occurs 49.0% of the time (compared to 75.6%) and the standard deviation of consumption is reduced by only 21% (compared to 40%). Indeed, further examples would show that for demand elasticities above unity the effects of storage are negligible.

The effects of demand curvature, measured by  $C$ , the degree of "relative commodity risk aversion," can be inferred from the cases summarized in Table 1. The higher risk aversion at  $C = 6$  is reflected in somewhat higher storage and lower variance of consumption. But the dispersion of prices is greater for  $C = 6$ , whether or not storage is possible. Although the magnitudes of these effects of demand curvature are not very great, the distributional implications are very important, as we shall now show.

## 7. The Distributional Effects of Storage

So far we have considered the effects of storage on prices and quantities. Many studies of storage consider nothing else. But the ultimate interest of the results depicted in the figures and in Table 1 lies in their implications for human welfare. There is a large analytical literature in the tradition of Waugh (1944), Oi (1964), and Massell (1969) which attempts to model the welfare effects of storage as a symmetric reduction in the dispersion of the production disturbance, implicitly or explicitly assuming away the non-negativity constraint on storage, which, as noted above, makes the problem analytically intractable. (See Turnovsky (1978) for a survey

of this literature on stabilization. More recent work includes Newbery and Stiglitz (1979) and Wright (1979).<sup>8</sup>) Because storage is much more reliable at eliminating gluts than in alleviating shortages, it might seem likely that the share of the allocative benefits accruing to consumers might be significantly lower than under the symmetric reduction in the dispersion of consumption effected by ideal stabilization. Further, from Table 1 one might guess that storage favors consumers most when it lowers consumption variance the most (line 4), or when it lowers price variance the most (line 2). In fact none of these deductions from the information presented thus far is correct.

To assess the comparative statics distributional implications of storage, we measured the mean changes in the present value of producer rents at the time of harvest (denoted by the shorthand term "land value") and the mean changes in present value of consumer surplus.<sup>9</sup> To make these measures meaningful, we expressed them as percentages of a common base, the expected annual value of production in a market without storage. This base was preferred to land value without storage, because land value is dependent on the specification of the entire supply function from zero to maximum production, that is, well beyond the relevant range here.<sup>10</sup>

The results for eight cases are presented in Table 2. It is immediately clear that the distributional effects are heavily dependent on the three parameters  $C$ ,  $\eta^D$ , and  $\eta^S$ . The direction of the effects depend largely on  $C$ . Consider first the cases where  $\eta^S = 0$ . Under linear demand ( $C = 0$ ) in which consumers are commodity risk-preferring, storage favors land holders at the expense of consumers, but under constant elasticity of demand ( $C = 6$ ), in which consumers are commodity risk averse, the reverse is true. In the intermediate case ( $C = 1.95$ , which approximates the hyperbolic demand specification ( $P = a + b q^{-1}$ )) in which consumers are commodity risk-neutral, storage has only a minor distributional impact.

If  $\eta^S = 1$ , storage always increases the expected welfare of consumers in Table 2, even if they are commodity-risk-preferring ( $C = 0$ ). It is also evident that responsive production greatly moderates the distributional impact of storage. Therefore, the assumption in most previous studies of either  $\eta^S = 0$  or an "irrational" response (e.g., adaptive expectations) in supply may result in misleading distributional inferences. Note also that responsive supply increases the sum of the changes in the expected present value of producer and consumer surplus so that, as the adverse distributional effects decline, the increase in net welfare is greater. The case in line 2 of Table 2 in which the reduction in the standard deviation of price is greatest (see Table 1) actually has the greatest net increase in welfare (in the comparative statics sense), but certainly does not confer the greatest benefit on consumers, as intuition might suggest. Two other perhaps counter-intuitive results are that the net gains are largest in the case when consumers have commodity risk preference, and that the reduction in the variance of consumption is not greatest when the net gains are largest.

Lines 7 and 8 in Table 2 show that storage has much less significance to welfare at higher elasticities of demand, in line with the less pronounced effects on price and consumption shown for  $\eta^D = -0.5$  in Table 1. At the higher elasticity of demand, consumers can more easily substitute other goods for the commodity in question during a shortage, so storage is of less importance.<sup>11</sup>

The second and fourth columns of Table 2 display the differential effects of ideal stabilization, that is, the complete absence of the production disturbance itself. Even though the storage modeled here has a very low cost (an interest rate of 5 percent being the only carrying charge) ideal stabilization has much greater distributional effects and net benefits. Furthermore,

lines 3 and 4 indicate that the sign of the effect on land value reverses at a higher value of  $C$  under ideal stabilization than under storage, so that over a certain range, ideal stabilization has an effect opposite to that of storage. In both storage and ideal stabilization, the distributive effects are almost linearly related to  $C$ .

The most noteworthy lesson to be drawn from Table 2 is that the asymmetric effects of storage, emphasized in previous sections, do not result in a greater share of the allocative benefits accruing to producers. Relative to the net gain, the differential gain to consumers is even greater under storage than under ideal stabilization, except in line 1. The explanation lies in the incompleteness of the stabilization effected by storage. Small symmetric reductions in variance always favor consumers for  $C > 1$ , even though larger reductions may favor producers.<sup>12</sup> Given  $C$ , the total distributive outcome depends on the extent of storage, which is a function of the cost of storage, the consumer demand elasticity, and the supply elasticity.

### Conclusion

Competitive storage of commodities that are subject to stochastic production disturbances is much more effective in eliminating excessive levels of consumption and low prices than in preventing low levels of consumption and high prices. This asymmetry stems from the constraint that storage must not be negative, and is greatly accentuated when producers, as well as storers, respond to incentives with rational expectations. When this is the case, the interaction between storage and responsive production is subtle and complex. Responsive production generally has relatively little effect on mean storage, and vice versa, so in this sense it is not clear whether the two activities

are substitutes or complements. Yet when combined, they stabilize consumption and market price in a highly complementary way, even though storage destabilizes planned and realized production.

The implications of storage for producers and consumers cannot be directly inferred from its effects on the distribution of consumption or price. A numerical welfare analysis shows that when demand is relatively inelastic the storage activity may have substantial effects on the expected present value of consumer surplus and of producer surplus, effects that are either positive or negative depending on the curvature of the demand curve and the supply elasticity. Given the asymmetric effects of storage on consumption and price which would seem to favor producers, it is noteworthy that the differential gains to consumers who are commodity risk averse are more favorable, relative to the net social gain, than they are under the symmetric elimination of the disturbance defined as ideal stabilization.

## APPENDIX

Solving for the Derived Demand for Storage

If there is an infinite horizon, the derived demand for storage is the same in all periods. Therefore, if a storage rule,  $S_t = f(I_t)$ , when applied, reproduces itself, the derived demand curve has been deduced. In the computer program, the storage rule is found by using the relation between  $EP_{t+1}$  and  $S_t$  implicit in the necessary conditions for profit-maximizing storage.

First a guess is made for a polynomial in  $S_t$  that approximates  $E[P_{t+1}(S_t)]$  for  $S_t \geq 0$ . Using the storage rule implied by this function and the competitive arbitrage conditions, expected price is calculated for a range of integer values of  $S$ . This calculation requires a determination of the particular planned production consistent with that  $S$  because the amount of production influences expected price. This is accomplished by guessing a planned production  $\hat{x}$  and calculating the various prices that occur with particular outcomes of the random probability distribution around that planned production. (A discrete approximation to the normal density function is used, with 80 possible values spanning four standard deviations each side of the mean.) The integer storage under consideration plus these random outcomes of production provide a distribution of amounts available. For each of these in turn, the current storage rule is used to compute storage and consumption. Expected price is calculated from the distribution of consumption using the inverse consumption demand function. The producer incentive price,  $P_t^R$ , is calculated along with expected price by weighting the price at particular outcomes by the ratio of realized production to planned production. If this  $P_t^R$ , when applied in the supply function, would yield the guess for planned

production,  $\hat{x}$ , a consistent set of  $S$ ,  $E[P_{t+1}]$ , and  $\hat{x}$  has been found. If not, another guess for  $\hat{x}$  is made.

Once the calculation of  $E[P_{t+1}]$  has been made for each of the integer values of  $S_t$ , expected price can be fitted to a polynomial in storage by the means of a least-squares regression (A fourth-order polynomial is used). If that polynomial has not changed significantly (as defined by the convergence criterion) from the one used to generate the values of expected price at various levels of storage, a stable storage demand curve has been found. If not, the most recently fitted polynomial replaces the previous guess, and the process is repeated. In effect, this procedure continues until the incorrect initial guess is no longer of any significance.



TABLE 1

## EXAMPLES OF THE MARKET EFFECTS OF STORAGE

Sample Means<sup>a</sup>

Case (C, $\eta^D$ , $\eta^S$ ) <sup>b</sup>	Production and Consumption Without Storage	Production and Consumption With Storage	Storage	Price	Price Without Storage
1. (0.0, -0.2, 0.0) <sup>c</sup>	100.0	100.0	3.2	100.1	100.1
2. (0.0, -0.2, 1.0) <sup>c</sup>	99.9	99.9	3.2	100.7	101.0
3. (6.0, -0.2, 0.0) <sup>d</sup>	100.0	100.0	3.4	101.6	104.0
4. (6.0, -0.2, 1.0) <sup>d</sup>	100.4	100.0	3.4	100.9	102.0
5. (6.0, -0.5, 0.0)	100.0	100.0	1.1	101.1	101.6
6. (6.0, -0.5, 1.0)	100.3	100.1	1.4	100.7	100.9

Sample Standard Deviations<sup>a</sup>

Case (C, $\eta^D$ , $\eta^S$ ) <sup>b</sup>	Production and Consumption Without Storage	Planned Production	Consumption	Storage	Price	Price Without Storage	
1. (0.0, -0.2, 0.0) <sup>c</sup>	5.0	0.0	5.0	3.1	3.5	15.3	25.1
2. (0.0, -0.2, 1.0) <sup>c</sup>	5.0	2.1	5.5	2.4	3.4	11.9	25.1
3. (6.0, -0.2, 0.0) <sup>d</sup>	5.0	0.0	5.0	3.0	3.5	17.9	27.3
4. (6.0, -0.2, 1.0) <sup>d</sup>	5.0	2.2	5.5	2.3	3.5	13.8	26.7
5. (6.0, -0.5, 0.0)	5.0	0.0	5.0	3.9	1.7	9.2	10.9
6. (6.0, -0.5, 1.0)	5.0	1.2	5.2	3.5	2.1	8.3	10.7

- Footnotes:
- The sample consists of a string of 10,004 periods, with the first four discarded. The same sequence of random disturbances was used for each case.
  - The symbols  $(C, \eta^D, \eta^S)$  denote the measure of demand curvature ( $\equiv -q^{P''}(q)/P'(q)$ ), the elasticity of consumption demand, and the (one period lagged) elasticity of supply, respectively. Both elasticities are measured at the point (100, 100).
  - Linear demand curve.
  - Constant elasticity demand curve.

TABLE 2

## DISTRIBUTIONAL IMPLICATIONS OF STORAGE AND IDEAL STABILIZATION

## DIFFERENCE FROM SITUATION WITHOUT STORAGE

(Percent of expected annual revenue without storage)<sup>a</sup>

Case (C, $\eta^D$ , $\eta^S$ ) <sup>b</sup>	Land Values:		Present Value of Consumer Surplus:	
	<u>Storage</u> Mean	<u>Ideal Stabilization</u>	<u>Storage</u> Mean	<u>Ideal Stabilization</u>
1.(0.0, -0.2, 0.0) <sup>c</sup>	12.2	24.7	-7.1	-12.4
2.(0.0, -0.2, 1.0) <sup>c</sup>	2.2	4.1	4.2	8.6
3.(1.95, -0.2, 0.0)	-3.1	0.6	7.9	11.7
4.(1.95, -0.2, 1.0)	-.6	0.1	6.6	12.2
5.(6.0, -0.2, 0.0) <sup>d</sup>	-33.6	-49.1	38.0	61.4
6.(6.0, -0.2, 1.0) <sup>d</sup>	-5.8	-8.3	11.6	20.6
7.(6.0, -0.5, 0.0)	-8.0	-19.9	8.7	24.9
8.(6.0, -0.5, 1.0)	-3.4	-6.6	4.3	11.6

- Footnotes: a. The sample consists of a string of 10,004 periods, with the first four discarded. The same sequence of random disturbances was used for each case.
- b. The symbols (C,  $\eta^D$ ,  $\eta^S$ ) denote the measure of demand curvature ( $\equiv -q^{P''}(q)/P'(q)$ ), the elasticity of consumption demand, and the (one period lagged) elasticity of supply, respectively. Both elasticities are measured at the point (100,100).
- c. Linear demand curve.
- d. Constant elasticity demand curve.

## FOOTNOTES

1. The presence of additional individual disturbances uncorrelated with aggregate production would not alter the results of this paper.

2. If we had included the accessibility value as a decreasing function of  $S$ , the storage rules derived below would have been bent upwards at the left, to indicate higher levels of storage at low levels of availability.

3. The following conditions (Samuelson 1971),

$$\lim_{T \rightarrow \infty} (1+r)^{-T} S_T = \lim_{T \rightarrow \infty} (1+r)^{-T} EP_t = 0,$$

rule out long-run speculative explosion of storage and expected price.

4. If the net cost of storage included a sufficiently high premium for accessibility at low levels of  $S$ , then some storage would always occur. However, results obtained under such a specification confirm that such essential "working stocks", being relatively small and unresponsive to economic incentives, do not greatly alter the general inferences discussed below because they play only a minor role in smoothing production.

5. Qualitatively similar storage functions were derived in the pioneering study of Gustafson (1958a). A linear storage rule is derived analytically in a starkly simplified model in Aiyagari, Eckstein, and Eichenbaum (1980).

6. An exception is Gardner (1979) who allows for responsive supply in a model with integer storage and additive disturbances in production.

7. For example, in the case where  $C = 1.95$ ,  $\eta^D = -0.2$ ,  $\eta^S = 1$ , (not reported in Table 1) mean planned production is less under storage. However, if the market is completely stabilized (i.e., if  $v$  is fixed at zero), planned production is slightly higher than in a stochastic market

without storage.

8. These studies, like this paper, take a comparative statics approach. They do not consider the welfare implications of the initial buildup of stocks upon the introduction of storage. Such dynamic effects are considered in Wright and Williams (1981).

9. Assuming an individualistic social welfare function, the change in the area under the uncompensated consumption demand curve is an exact measure of the change in welfare, only if the marginal utility of income is constant over the relevant range of price. This is true if  $R = \eta^Y$  over the range of prices considered where  $R$  is the coefficient of relative risk aversion with respect to income, and  $\eta^Y$  is the income elasticity of demand. This condition is fulfilled if, for example,  $R$  is constant and the indirect utility function has the additively separable form in each time period:

$$V = A(P) + F(Y)$$

where  $F(Y)$  is linear in  $\ln(Y)$  (in which case  $R = \eta^Y = 1$ ) or in  $Y$  (risk neutral,  $\eta^Y = 0$ ). More generally, the error involved in using the Marshallian demand curve is small if the commodity in question has a low share of the consumer budget or a low income elasticity of demand (Willig (1976)). Under these conditions, the measure of relative commodity risk aversion,  $C$ , is at least approximately independent of  $R$ .

For producers and storers, we have assumed either that  $R = 0$  or that they behave in a risk-neutral fashion because they have access to a competitive capital market, and because the coefficient of variation of the land price is

very small. (The coefficient of variation of the land prices in the examples in Table 2 is always below .03, assuming the land share is at least 0.3). The relaxation of the assumption of risk neutrality with respect to income is an obvious topic for further research. The implications of risk averse behavior on the part of producers is investigated in an analytical model of price stabilization by Newbery and Stiglitz (1979).

10. In the 10,000 sample observations for the set of cases considered, planned production ranged from 2% above to about 8% below the equilibrium output under ideal stabilization. Since nonlinearities in supply outside this range would have virtually no effect on the derivation of the storage rule or on the calculation of changes in land value, we have chosen a yardstick that does not impose an unnecessary restriction on the supply function outside the relevant range.

11. Further simulations (not reported here) show that the effect of storage on land value has a nonlinear relation to  $\eta^D$  and  $\eta^S$ . This can be shown analytically for ideal stabilization. From Wright (1979, p. 1025, equation 36) the annual expected gain in producer surplus relative to  $\bar{P} \bar{q}$  is approximated by

$$G_R = \frac{\sigma^2(1 - \frac{C}{2})}{|\eta^D| + \eta^S}$$

where  $|\eta^D|$  is the absolute value of  $\eta^D$ .

Thus

$$\frac{\partial G_R}{\partial |\eta^D|} = \frac{\partial G_R}{\partial \eta^S} = - \frac{\sigma^2(1 - \frac{C}{2})}{(|\eta^D| + \eta^S)^2}$$

and

$$\frac{\partial^2 G_R}{\partial (|\eta^D|)^2} = \frac{\partial^2 G_R}{\partial (\eta^S)^2} = \frac{2\sigma^2(1 - \frac{C}{2})}{(|\eta^D| + \eta^S)^2}$$

The numerical results for storage are qualitatively similar. They show that marginal increases in  $|\eta^D|$  or  $\eta^S$  moderate the positive or negative effects of storage on producer surplus, the effect decreasing as the absolute value of the elasticity in question increases.

12. This can be shown using a simplified analytical model of storage. The easiest cases to consider are those for which  $\eta^S = 0$ . Ignoring the non-negativity constraint on S, and assuming a constant marginal propensity to store s,  $0 \leq s \leq 1$ , the rational producer incentive  $P^r$  becomes, using a second order approximation to the inverse demand function evaluated at mean consumption  $\bar{q}$ ,

$$\begin{aligned} P^r(s) &= E \left\{ (1 + v) [P(\bar{q}) + v\bar{q}(1 - s)P'(\bar{q}) + (\frac{1}{2}) v^2 \bar{q}^2 (1 - s)^2 P''(\bar{q})] \right\} \\ &= P(\bar{q}) + \sigma_v^2 \bar{q}(1 - s)P' + (\frac{1}{2}) \sigma_v^2 \bar{q}^2 (1 - s)^2 P''(\bar{q}) \end{aligned}$$

For any particular marginal propensity to store  $\bar{s}$  the difference in mean producer surplus due to storage relative to expected revenue is  $G(s)$  where

$$G(\bar{s}) = \frac{\sigma_v^2}{\eta} \left[ C \left(\frac{\bar{s}}{2}\right)^2 - \bar{s} + \bar{s} \right]$$

The effect of a marginal increase in storage is given by

$$\frac{\partial G(\tilde{s})}{\partial \tilde{s}} = \frac{-\sigma^2 v}{D} [C(\tilde{s} - 1) + 1]$$

Thus when there is no storage ( $\tilde{s} = 0$ ), the introduction of a small amount of storage reduces producer surplus (and, since the net benefits are positive, must favor consumers) if  $C > 1$ . Producers always gain from marginal storage in the neighborhood of  $\tilde{s} = 1$ .

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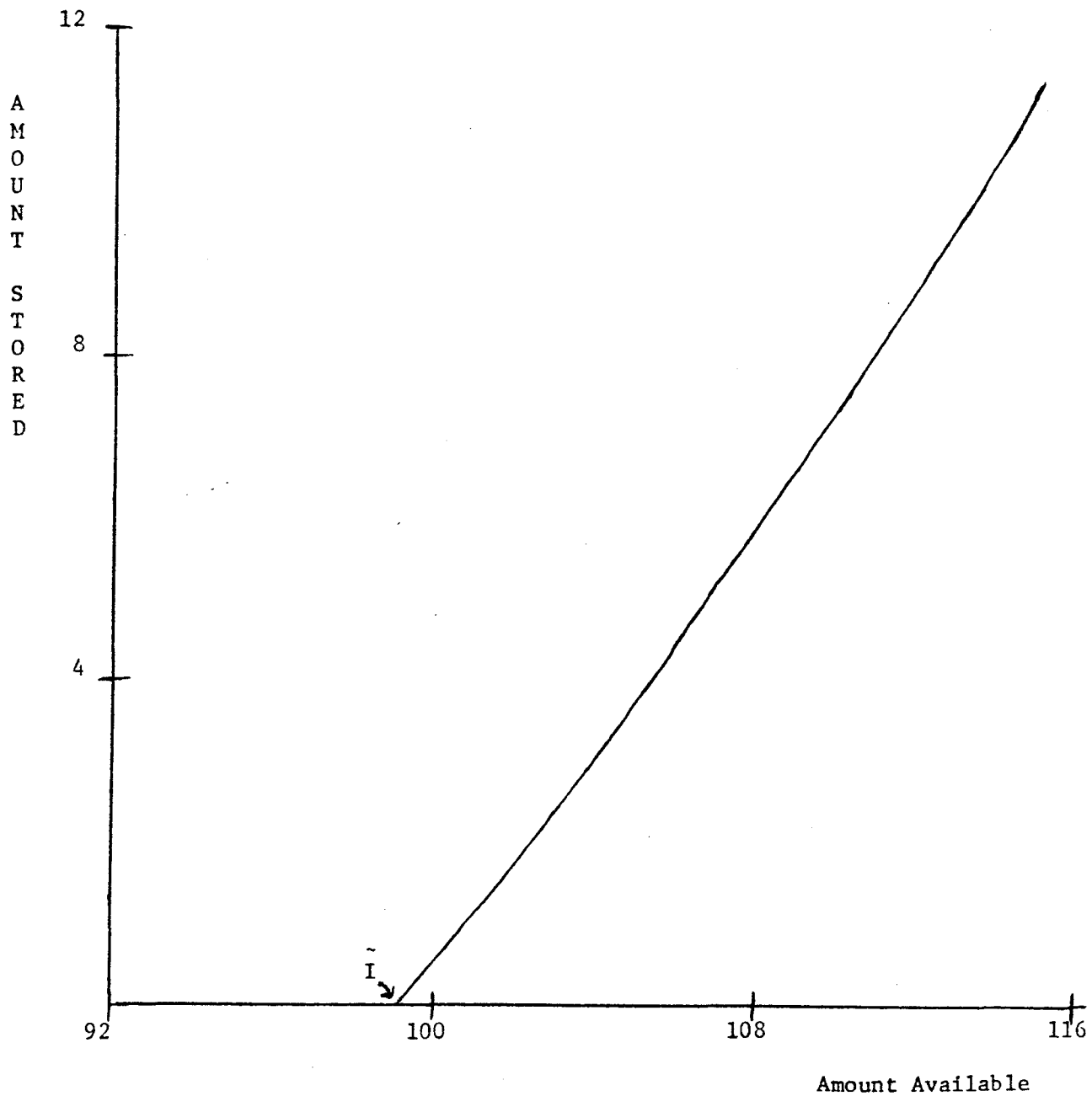


FIGURE 1  
STORAGE RULE

F2

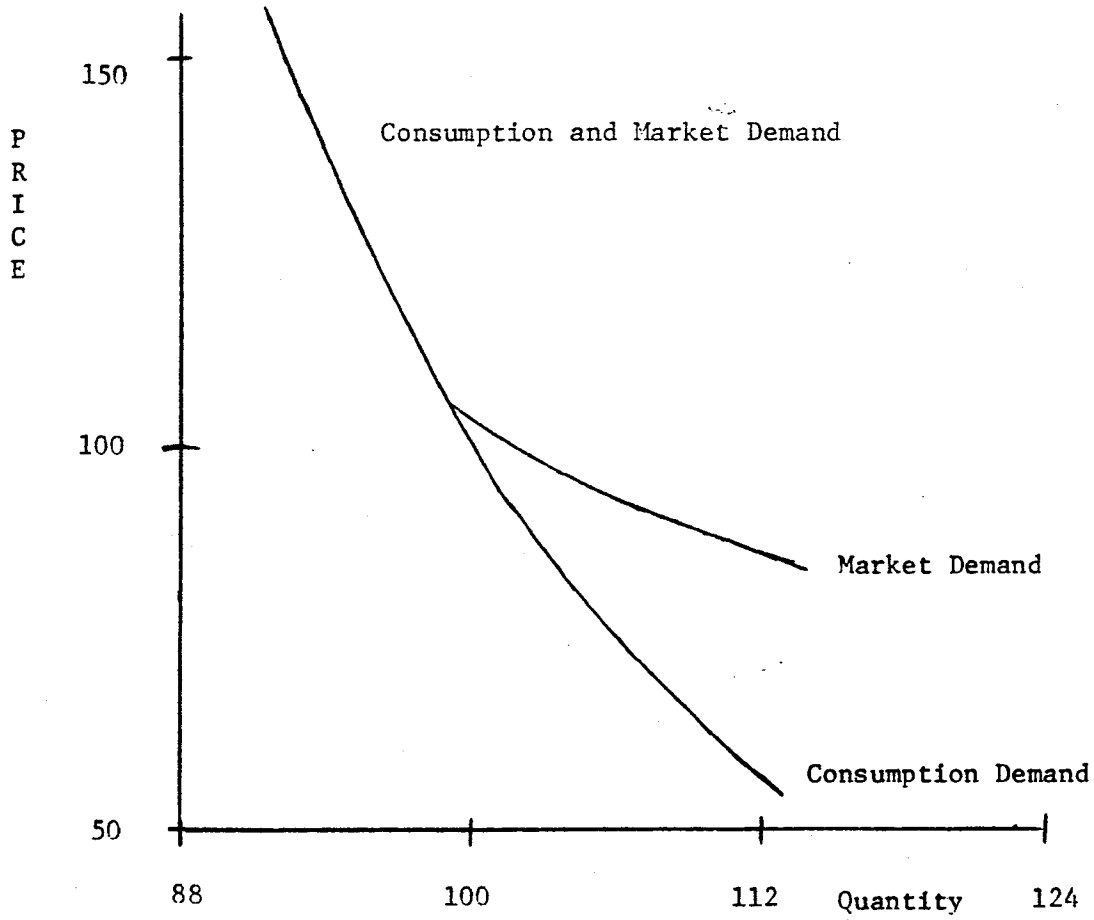


FIGURE 2

DEMAND CURVES

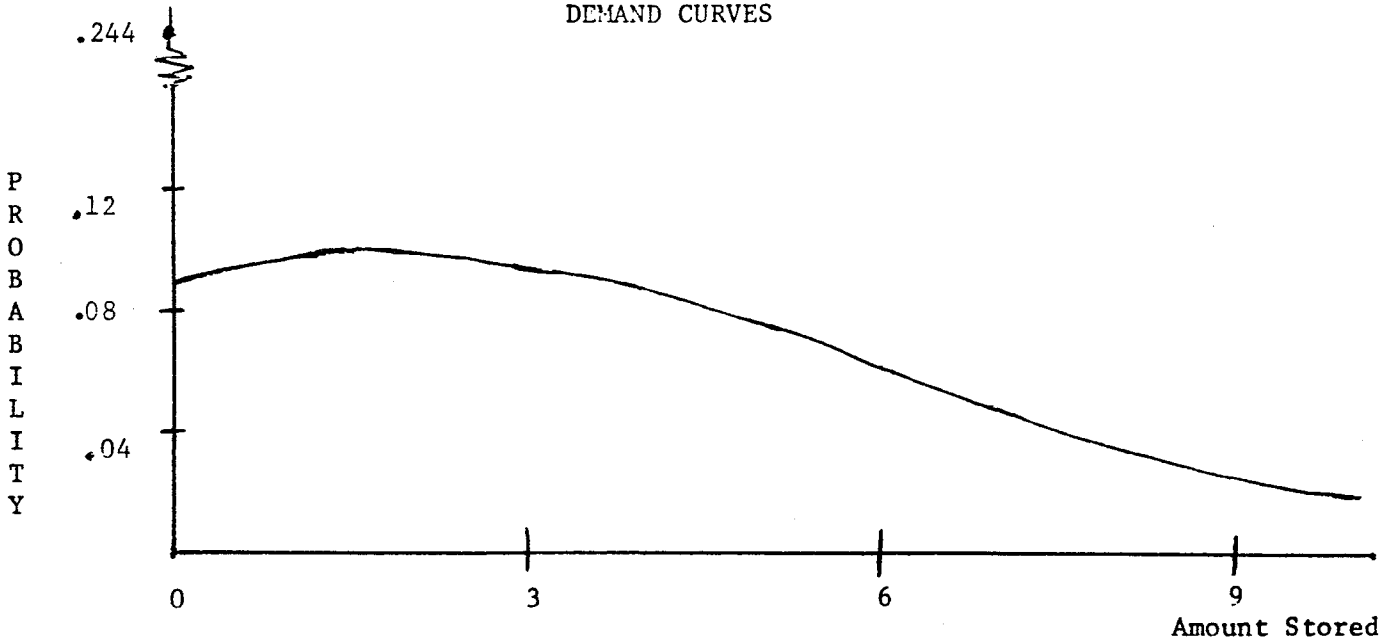


FIGURE 3

DISTRIBUTION OF STORAGE

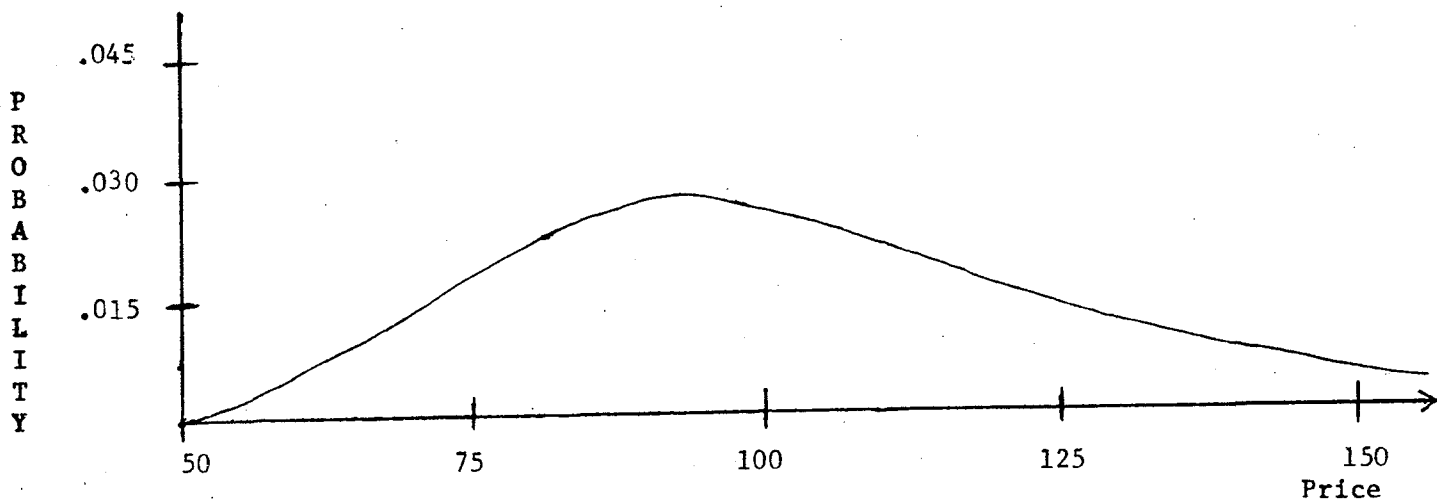


FIGURE 4  
PRICE WITHOUT STORAGE

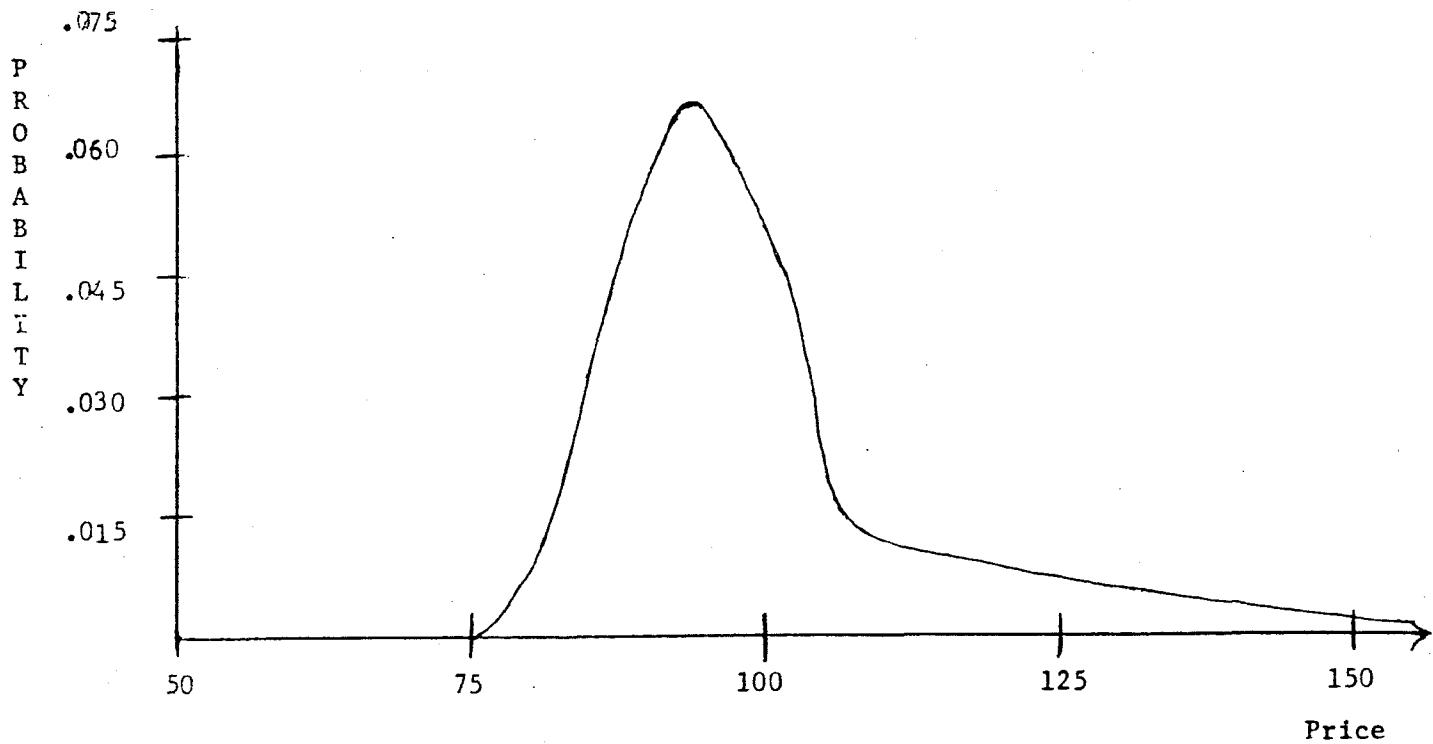


FIGURE 5  
PRICE WITH STORAGE

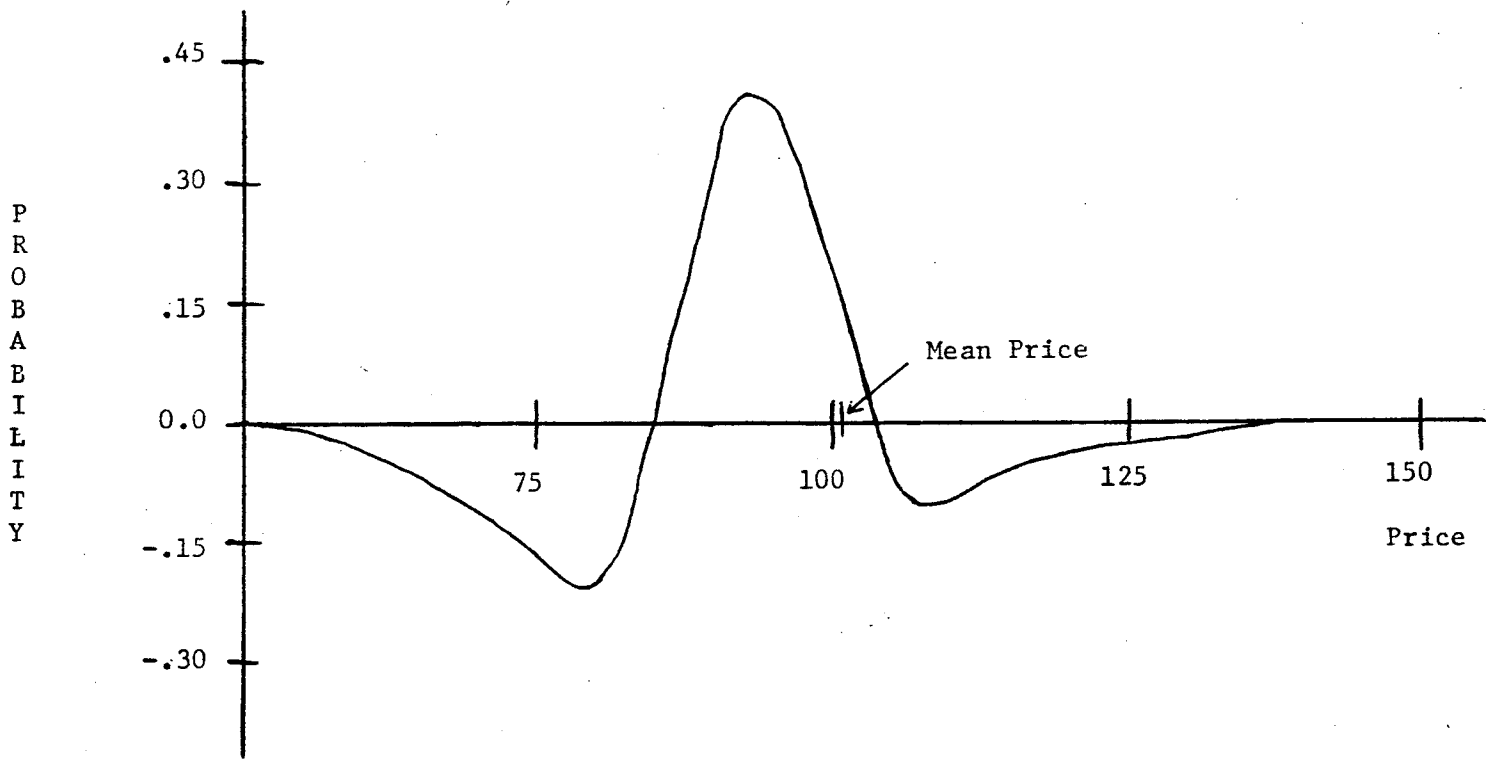


FIGURE 6  
EFFECT OF STORAGE ON PRICE  
(Mean Adjusted)

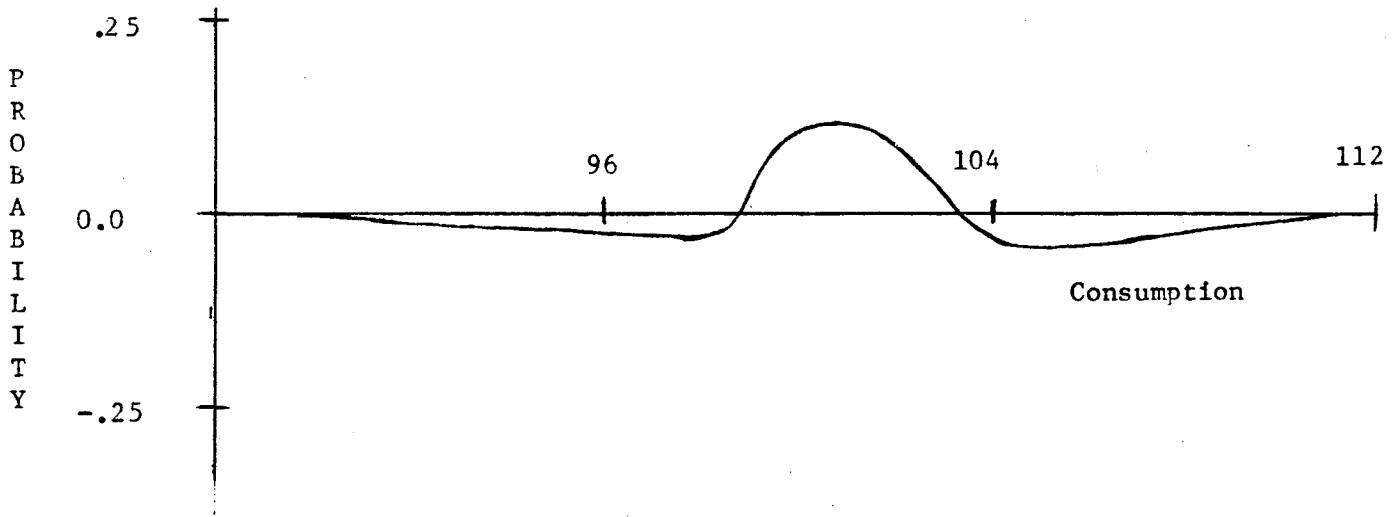


FIGURE 7  
EFFECT OF STORAGE ON CONSUMPTION

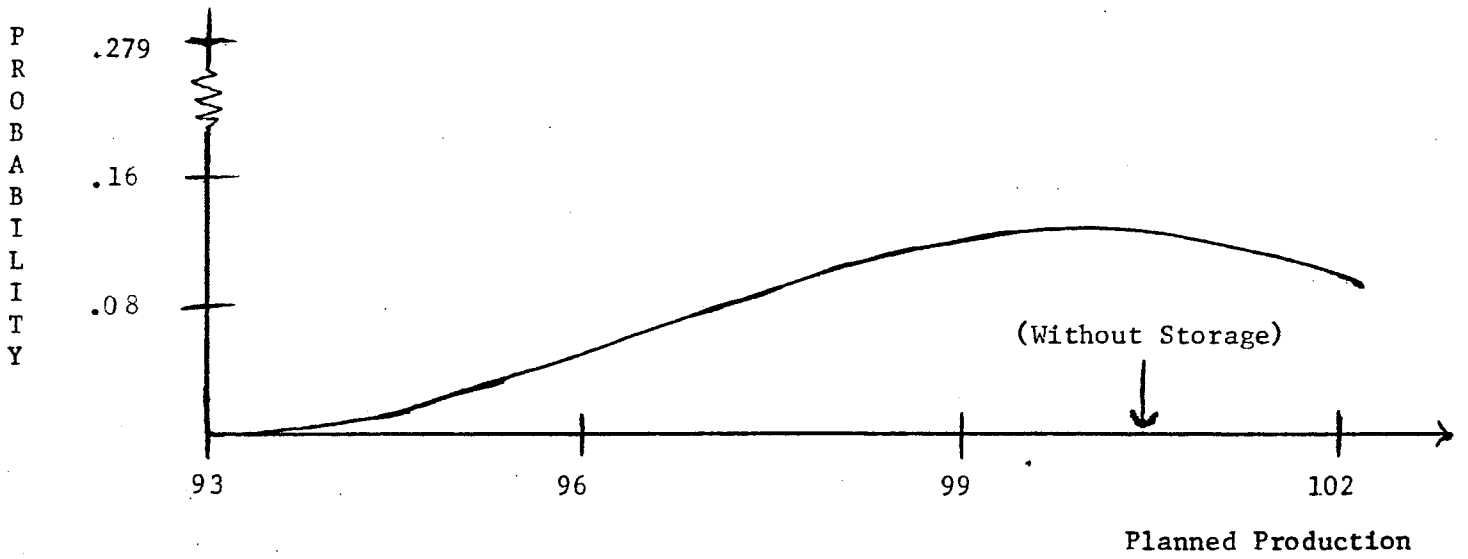


FIGURE 8  
DISTRIBUTION OF PLANNED PRODUCTION

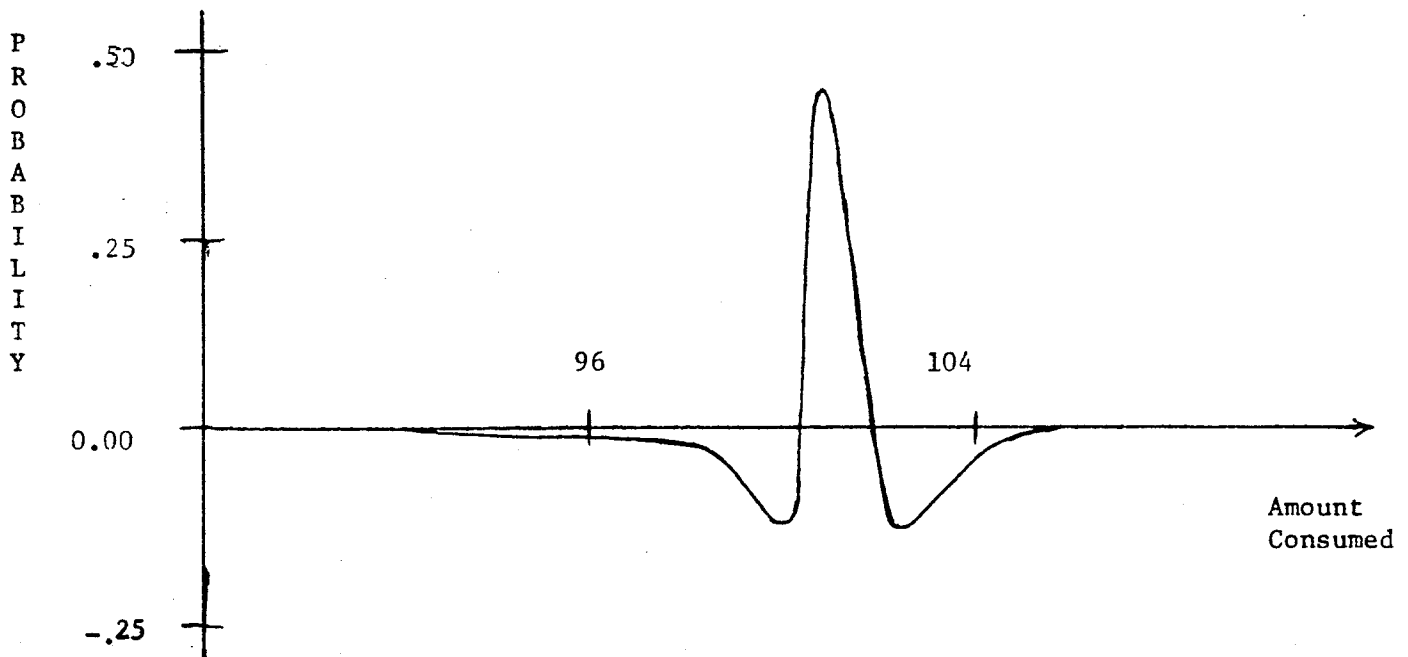


FIGURE 9  
EFFECT OF RESPONSIVE  
SUPPLY ON CONSUMPTION