# A Comparison of Parametric Models of Income Distribution

# Across Countries and Over Time

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## Abstract

The five-parameter generalized beta distribution and ten of its special cases are considered as models fore the size distribution of income. The models are fit to income data for 23 countries and various years—a total of 82 data sets. Of the models considered, the Weibull, Dagum and generalized beta of the second kind are best fitting of the models with two, three and four parameters for 62 percent, 84 percent, and 96 percent of the data sets, respectively. Increasing inequality with respect to pre-tax income is observed in most of the countries considered.

## I. Introduction

Vilfredo Pareto first proposed a model of income distribution in the form of a probability density function (pdf) in 1895. Pareto's analysis of income inequality, based on his model and economic data he gathered, stirred up the debate on the effect of economic growth on income inequality. In today's terms, are the rich getting richer and the poor getting poorer, or is the rising economic tide lifting all the boats? Corrado Gini disagreed with Pareto's opinion that economic growth leads to less inequality. To better study the phenomena, he proposed a unit-free measure of income inequality known as the Gini coefficient that is still commonly used today (Gini, 1912).

Later empirical studies showed that Pareto's distribution accurately modeled high levels of income, but did a poor job describing the low end of the distribution. As research continued, new distributions were proposed that better fit the observed data. Gibrat (1931) wrote a famous paper in which he suggested the two-parameter lognormal distribution, further examined by Aitchinson and Brown (1969). Ammon (1895) proposed the gamma distribution, which was more recently reintroduced and fit to US income data by Salem and Mount (1974). Bartels and van Metelel (1975) suggested another two-parameter pdf, the Weibull.

Even better fits could be obtained using three-parameter distributions. These include the generalized gamma (Amoroso, 1924-25 and Taille, 1981) and beta (Thurow, 1970) as well as two closely related models which are members of the Burr family of distributions: the Singh-Maddala (1976), known in statistics literature as the Burr 12, and the Dagum (1977), known as the Burr 3. These distributions allowed for intersecting Lorenz curves, a phenomenon observed in the data that could not be modeled by any of the two-parameter distributions considered.

McDonald (1984) introduced the generalized beta of the first and second kinds (GB1 and GB2), two four-parameter distributions that were not only very successful in fitting the data, but also included all of the previously mentioned distributions as special or limiting cases. McDonald and Mantrala (1996) found that the GB2 distribution provided a significantly better fit than its nested distributions when fit to income data from the United States. The empirical success of the GB2 was complemented by Parker's (1999) theoretical model of income generation, showing earnings to follow a GB2 distribution.

This paper compares many of the probability density functions that have been considered successful in describing the size distribution of income. While others have shown the goodness-of-fit of their distributional models using US income data, one may imagine that income in other countries is distributed very differently, particularly in countries formerly Communist or in earlier stages of economic development. By comparing a diverse set of countries, it is easier to draw conclusions about the quality of these models in describing the distribution of income generally and not just the characteristics observed in the United States. Section II of this paper discusses the methodology and functional forms used in this study, the data sets being considered are described in section III, the results are presented in section IV, and section V contains the authors' conclusions.

## II. Methodology

### A. The Generalized Beta Distribution Family

McDonald and Xu (1995) showed that all of the distributions previously mentioned could be represented as special or limiting cases of a very flexible, five-parameter distribution they termed the generalized beta (GB), whose probability distribution function can be written:

$$\mathrm{GB}(y;a,b,c,p,q) = \frac{|a| y^{ap-1}(1-(1-c)(y/b)^a)^{q-1}}{b^{ap}\mathrm{B}(p,q)(1+c(y/b)^a)^{p+q}} \quad \text{for } 0 < y^a < b^a$$

and zero otherwise, where $0 \le c \le 1,$ and $b, p, q > 0.$ Using this parameterization, it is easy to see the relationships within the generalized beta family of distributions. The GB2 can be derived by setting the c parameter equal to one.

$$\mathrm{GB2}(y;\, a,\, b,\, p,\, q) = \frac{|a|\, y^{ap-1}\left(1-(y/b)^a\right)^{q-1}}{b^{ap}\mathrm{B}\left(p,q\right)} = \mathrm{GB}(y;\, a,\, b,\, c=1,\, p,\, q)$$

The three-parameter Dagum and Singh-Maddala distributions correspond to the case

$$\mathrm{DAGUM}\ (\mathrm{y};\, a,\, b,\, p) = \mathrm{GB2}\ (y;\, a,\, b,\, p,\, q=1),$$

$$\mathrm{SM}\ (\mathrm{y};\, a,\, b,\, q) = \mathrm{GB2}\ (y;\, a,\, b,\, p=1,\, q).$$

The generalized gamma (GG) distribution is a limiting case defined as

$$\mathrm{GG}\ (y;\, a,\, \beta,\, p) = \lim_{q\to\infty} \mathrm{GB}\ (y;\, a,\, b = q^{1/a}\beta,\, c = 1,\, p,\, q).$$

A convenient way to visualize these relationships is the distribution tree in figure 1. These relationships can be seen in greater detail in McDonald and Xu (1995).

**Models for the Distribution of Income**

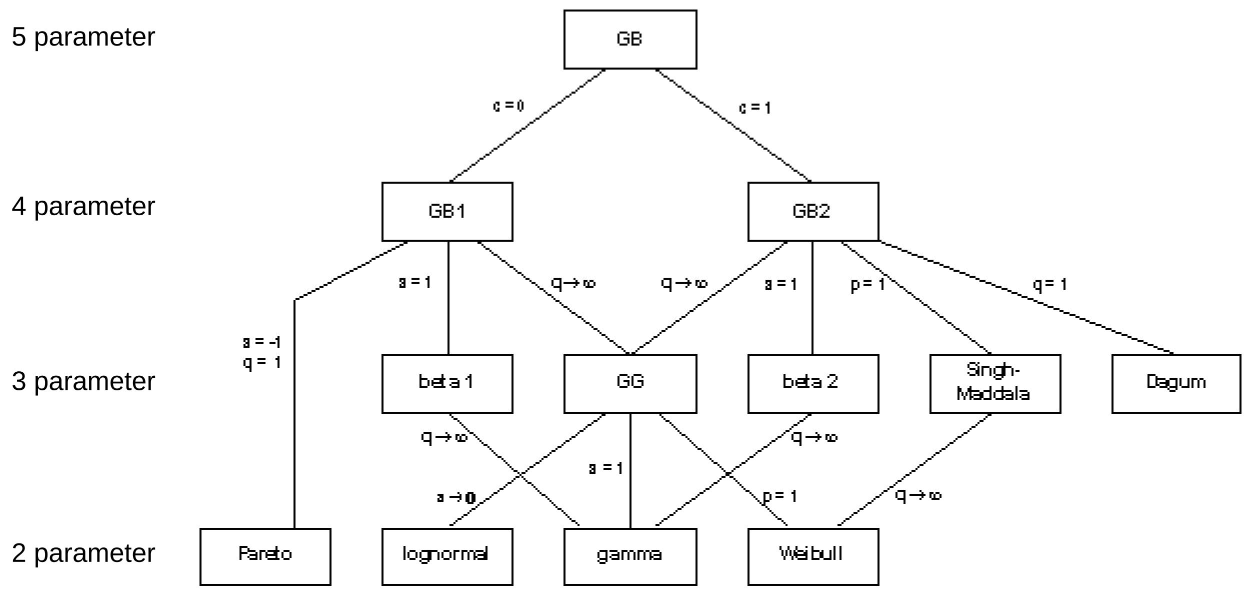


Figure 1

**B. Distributional Estimation**

Since the income data utilized in this study are grouped into twenty equal probability intervals, the distributional parameters are estimated using maximum likelihood techniques by maximizing the multinomial likelihood function

$$L(\theta) = N! \prod_{i=1}^{20} \frac{(p_i(\theta))^{n_i}}{n_i!}$$

where $p_i(\theta) = F(Y_i;\theta) - F(Y_{i-1};\theta)$, F( ) denotes the cumulative distribution function, θ is a vector containing the distributional parameters, $Y_i$ and $Y_{i-1}$ are the upper and lower bounds of the $i^{th}$ of twenty data groups, $n_i$ is the number of observations in the $i^{th}$ group, and N is the total number of observations.

There are no closed form solutions for the value of θ that maximizes the likelihood function so iterative techniques must be used. In this paper, the likelihood function is maximized using a simplex method from the Matlab optimization toolkit, finding the minimum to the negative log likelihood function with a convergence criterion of $10^{-8}$.

### C. Goodness-of-Fit

In order to evaluate relative performance of the eleven distributions, we evaluate the sum of squared errors (SSE), sum of absolute errors (SAE), and chi-square($\chi^2$) goodness-of-fit measures in addition to the log-likelihood value for the estimated model and data set. The SSE, SAE and $\chi^2$ are defined by

$$\mathrm{SSE} = \sum_{i=1}^{20} \left( \frac{n_i}{N} - p_i\left(\hat{\theta}\right) \right)^2,$$

$$\mathrm{SAE} = \sum_{i=1}^{20} \left| \frac{n_i}{N} - p_i\left(\hat{\theta}\right) \right|, \text{ and}$$

$$\chi^2 = N \sum_{i=1}^{20} \left[ \left( \frac{n_i}{N} - p_i\left(\hat{\theta}\right) \right)^2 \Big/ p_i\left(\hat{\theta}\right) \right],$$

where $\hat{\theta}$ denotes the estimated parameter vector. The $\chi^2$ is asymptotically distributed as a chi-square with degrees of freedom equal to the one less than the difference between the number of groups and the number of estimated parameters.

### D. The Gini Coefficient: a Measure of Inequality

The Gini coefficient is widely used as a measure of income inequality. It has many desirable properties that make it convenient to compare inequality across countries and time. For example, the Gini coefficient is not affected by changes in the unit of measurement (currency), allowing researchers to avoid issues of inflation and purchasing power.

Conveniently, the Gini coefficient can be expressed mathematically in terms of the distributional parameters. These expressions can be found in McDonald (1984) for all the distributions considered except the generalized beta and the Dagum. Dagum (1977) demonstrated that the Gini coefficient for the Dagum distribution could be represented as

$$G_{DAGUM} = \frac{\Gamma(p)\Gamma(2p+1/a)}{\Gamma(p+1/a)\Gamma(2p)} - 1,$$

where $\Gamma(.)$ denotes the gamma function. This result also follows from the relationship between the Dagum, Singh-Maddala and GB2 distributions.

Using these expressions, Gini coefficients were calculated for each distribution in every data set considered, showing the effects of assuming different distributional models.

## III. Description of the Income Data

Household income data was obtained from the Luxembourg Income Study (LIS) database for 23 countries, including both developed and developing economies. European countries are perhaps over-represented within the LIS database because of its quality and availability. LIS data is grouped into 5-year waves, which, for a few countries, went back as far as the 1970's. In total, there were 82 data sets. Table 1 summarizes the countries and years that were examined.

| Income Data Obtained From the Luxembourg Income Study | | | | | |
|---|---|---|---|---|---|
| | historical | wave I | wave II | wave III | wave IV |
| Australia | | 1981 | 1985 | 1989 | 1994 |
| Belgium | | | 1985 | 1988 | 1992,1997 |
| Canada | 1971,1975 | 1981 | 1987 | 1991 | 1994,1997 |
| Czech Rep. | | | | 1992 | 1996 |
| Denmark | | | 1987 | 1992 | 1995,1997 |
| Finland | | | 1987 | 1991 | 1995 |
| France | | 1979 | 1984 | 1989 | 1994 |
| Germany | 1973,1978 | 1981 | 1984 | 1989 | 1994 |
| Hungary | | | | 1991 | |
| Ireland | | | | 1987 | |
| Israel | | 1979 | 1986 | 1992 | 1997 |
| Italy | | | 1986 | | 1995 |
| Mexico | | | 1984 | 1989,1992 | 1994,1996 |
| Netherlands | | | 1983,1987 | 1991 | 1994 |
| Norway | | 1979 | 1986 | 1991 | 1995 |
| Poland | | | 1986 | 1992 | 1995 |
| Taiwan | | 1981 | 1986 | 1991 | 1995 |
| Russia | | | | 1992 | 1995 |
| Slovakia | | | | 1992 | |
| Spain | | 1980 | | 1990 | |
| Sweden | 1967,1975 | 1981 | 1987 | 1992 | 1995 |
| Switzerland | | 1982 | | 1992 | |
| United States | 1969,1974 | 1979 | 1986 | 1991 | 1994 |

Table 1

An advantage of using the LIS is that the data from each country is formatted as uniformly as possible, particularly concerning the definition of income, so that comparisons across countries and time are more plausible. For the purposes of this paper, we examine household data, defining income as gross wages and salaries, farm income, and any self-employment income. This definition of income seems appropriate since many of the distributions are based on models of pre-tax income generation. It is important to note that this is pre-tax household income, not including government transfer payments. Using different definitions of income can cause dramatic changes in the observed distribution of income.

Particularly, accounting for government redistribution will significantly alter the distribution of income in many of the countries considered.

In all cases income was measured in nominal local currency units. Because of government regulations and privacy laws, income data with individual observations are usually not available. In this analysis, the data was obtained in a grouped format with twenty equal probability intervals, corresponding to 5$^{th}$ through 95$^{th}$ percentiles.

## IV. Results

Each distribution was fit to the 82 data sets, and goodness-of-fit criteria were calculated for each distribution, including the log-likelihood (Log-L) value as well as the SSE, SAE, and $\chi^2$ value. The results for the United States 1997 are reported in table 2 and can be seen visually in figure 2.

The Weibull and Dagum are clearly the best fitting two and three parameter models, respectively. This holds true for either a SAE, SSE, $\chi^2$, or log-likelihood criteria. This also demonstrates the advantages of the three-parameter Dagum distribution over the two-parameter lognormal, gamma, and Weibull distributions. In this particular case, the four-parameter GB2 provides very marginal improvement relative to the Dagum. A comparison of the SSE, SAE, and $\chi^2$ values show they are very similar for the Dagum and GB2 distributions. Examining their plots in figure 2 confirms they are nearly observationally equivalent. The same could said of the GB relative to the GB2.

## United States 1997

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Gini |
|---|---|---|---|---|---|---|
| gamma | -508.82 | 46,792 | 0.0010 | 0.1153 | 868 | 0.424 |
| lognormal | -2844.48 | 52,892 | 0.0065 | 0.2917 | 6509 | 0.503 |
| Weibull | -380.13 | 46,445 | 0.0007 | 0.1031 | 586 | 0.413 |
| Dagum | -192.79 | 47,719 | 0.0003 | 0.0552 | 208 | 0.427 |
| Beta 1 | -440.62 | 46,372 | 0.0009 | 0.1133 | 706 | 0.414 |
| Beta 2 | -509.15 | 46,705 | 0.0010 | 0.1143 | 871 | 0.423 |
| GG | -372.77 | 46,384 | 0.0007 | 0.1032 | 566 | 0.412 |
| Singh-Maddala | -376.84 | 46,530 | 0.0007 | 0.1002 | 584 | 0.415 |
| GB1 | -372.77 | 46,384 | 0.0007 | 0.1032 | 566 | 0.412 |
| GB2 | -190.26 | 48,035 | 0.0003 | 0.0550 | 203 | 0.430 |
| GB | -187.77 | 46,359 | 0.0003 | 0.0538 | 198 | - |

Table 2

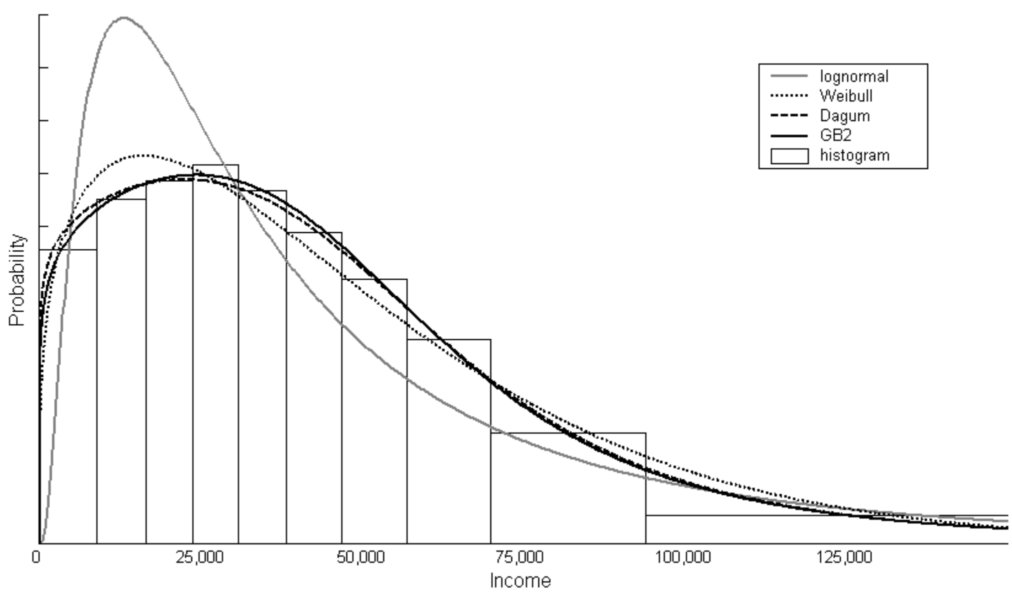


Figure 2

The nested relationship of the distributions guarantees the generalized distributions will fit the data at least as well as their special cases. However, this does not suggest their superiority as a descriptive model will be statistically significant. Nested models can be compared using the likelihood ratio (LR) test:

$$\mathrm{LR} = 2\left[\ln L(\theta_{\mathrm{ML}}) - \ln L(\theta_{\mathrm{R}})\right],$$

where $\theta_{ML}$ and $\theta_{R}$ represent parameter estimates of the general and the restricted models, respectively. In cases in which the parameters are not on the boundary of the parameter spaces, the LR is asymptotically distributed as a $\chi^2$ with degrees of freedom equal to the number of parameter restrictions imposed. This test cannot be used to compare non-nested models. Using this test at a one percent significance level to compare the fits obtained for the 1997 US data, the differences between the GB and GB2 and between the GB2 and Dagum are not significant. In this case, it is unclear if the improvements gained in adding parameters are of practical significance.

Detailed results for the other countries and time periods are found in the appendix. Table 3 summarizes these results by reporting the best fitting two-parameter, three-parameter, and four-parameter models for each of the 82 cases along with the corresponding log-likelihood values. Asterisks are placed next to log-likelihood values of the GB1,GB2 and GB distributions if adding the additional fourth or fifth parameter provided a statistically significant improvement, using the likelihood ratio test at the one percent level as outlined above.

Among the two-parameter models, the Weibull provides the best fit in sixty two percent of the cases. As in the case of the United States, introducing a third parameter provides a significantly improved fit. Among the three-parameter distributions, the Dagum is clearly the favorite. However, even though the Dagum distribution provides the best fit in 84 percent of the cases, it is not definitively the best of the three-parameter estimators in all cases. In fact, of the

## Best-Fitting Models

| country | year | two-parameter | | three-parameter | | four-parameter | | five-parameter | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|
| | | model | Log-L | model | Log-L | model | Log-L | model | Log-L | |
| ***Australia*** | 1981 | Weibull | -368.35 | Dagum | -252.61 | GB2 | -247.5* | GB | -242.38 | 0.333 |
| | 1985 | Weibull | -220.68 | Dagum | -155.47 | GB2 | -150.78* | GB | -149.14 | 0.337 |
| | 1989 | Weibull | -296.22 | Dagum | -205.63 | GB2 | -202.41 | GB | -202.41 | 0.349 |
| | 1994 | Weibull | -204.52 | Dagum | -153.58 | GB2 | -152.61 | GB | -152.17 | 0.361 |
| | | | | | 0 | | | | | |
| ***Belgium*** | 1985 | gamma | -235.41 | GG | -227.34 | GB1 | -227.33 | GB | -227.33 | 0.255 |
| | 1988 | gamma | -202.09 | GG | -200.79 | GB1 | -200.78 | GB | -200.78 | 0.254 |
| | 1992 | gamma | -83.68 | GG | -82.59 | GB2 | -82.59 | GB | -82.59 | 0.292 |
| | 1997 | gamma | -103.85 | Dagum | -93.69 | GB2 | -92.47 | GB | -91.69 | 0.327 |
| ***Canada*** | 1971 | Weibull | -611.61 | Dagum | -192.48 | GB2 | -166.75* | GB | -166.14 | 0.379 |
| | 1975 | Weibull | -481.71 | Dagum | -171.46 | GB2 | -159.42* | GB | -156.27 | 0.372 |
| | 1981 | Weibull | -273 | Dagum | -117.24 | GB2 | -116.5 | GB | -111.62* | 0.372 |
| | 1987 | Weibull | -251.37 | Dagum | -104.27 | GB2 | -99.55* | GB | -97.3 | 0.386 |
| | 1991 | Weibull | -286.14 | Dagum | -105.53 | GB2 | -105.26 | GB | -103.7 | 0.387 |
| | 1994 | Weibull | -479.16 | Dagum | -135.12 | GB2 | -132 | GB | -127.17* | 0.392 |
| | 1997 | Weibull | -364.79 | Dagum | -118.53 | GB2 | -117.9 | GB | -116.52 | 0.392 |
| ***Czech Rep.*** | 1992 | Weibull | -160.71 | Dagum | -127.72 | GB2 | -121.39* | GB | -121.39 | 0.303 |
| | 1996 | gamma | -125.71 | Singh-Maddala | -101.44 | GB2 | -100.17 | GB | -100.17 | 0.332 |
| ***Denmark*** | 1987 | Weibull | -584.64 | Dagum | -236.12 | GB2 | -233.65 | GB | -233.65 | 0.386 |
| | 1992 | Weibull | -710.57 | Dagum | -277.62 | GB2 | -276.69 | GB | -276.69 | 0.404 |
| | 1995 | Weibull | -754.11 | Dagum | -265.43 | GB2 | -257.38* | GB | -254.8 | 0.427 |
| | 1997 | gamma | -1669.06 | Dagum | -676.05 | GB2 | -664.5* | GB | -661.54 | 0.478 |
| ***Finland*** | 1987 | Weibull | -555.6 | Dagum | -289.9 | GB2 | -270.53* | GB | -258.62* | 0.401 |
| | 1991 | Weibull | -517.82 | Dagum | -267.57 | GB2 | -244.33* | GB | -238.45* | 0.417 |
| | 1995 | Weibull | -772.69 | Dagum | -341.4 | GB2 | -325.44* | GB | -322.37 | 0.433 |
| ***France*** | 1979 | lognormal | -414.26 | Dagum | -185.54 | GB2 | -133.5 | GB | -133.5 | 0.504 |
| | 1984 | Weibull | -193.93 | Singh-Maddala | -169.2 | GB2 | -166.8 | GB | -165.27 | 0.322 |
| | 1989 | gamma | -141.78 | Singh-Maddala | -117.92 | GB2 | -117.05 | GB | -117.05 | 0.334 |
| | 1994 | Weibull | -192.58 | Dagum | -142.42 | GB2 | -141.8 | GB | -141.79 | 0.387 |

Table 3

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ***Germany*** | 1973 | Weibull | -4951.58 | Dagum | -3502.02 | GB2 | -3303.37* | GB | -3291.03* | 0.311 |
| | 1978 | Weibull | -4555.74 | Dagum | -3213.12 | GB2 | -3030.9* | GB | -3022.61* | 0.319 |
| | 1981 | gamma | -105.82 | Dagum | -93.07 | GB2 | -93 | GB | -92.4 | 0.274 |
| | 1984 | gamma | -170.13 | Dagum | -125.63 | GB2 | -125.34 | GB | -124.78 | 0.299 |
| | 1989 | Weibull | -125.56 | Dagum | -105.87 | GB2 | -105.86 | GB | -104.78 | 0.295 |
| | 1994 | Weibull | -146.76 | Dagum | -104.35 | GB2 | -99.84* | GB | -99.4 | 0.343 |
| ***Hungary*** | 1991 | Weibull | -89.16 | Dagum | -72.32 | GB2 | -70.13 | GB | -69.86 | 0.428 |
| ***Ireland*** | 1987 | Weibull | -96.5 | Dagum | -84.06 | GB2 | -83.34 | GB | -80.57 | 0.408 |
| ***Israel*** | 1979 | Weibull | -69.97 | Singh-Maddala | -69.26 | GB2 | -68.22 | GB | -68.06 | 0.350 |
| | 1986 | gamma | -76.01 | Beta 2 | -73.24 | GB2 | -72.86 | GB | -72.83 | 0.388 |
| | 1992 | gamma | -82.34 | Singh-Maddala | -73.06 | GB2 | -73.02 | GB | -72.55 | 0.405 |
| | 1997 | gamma | -82.4 | Dagum | -75.98 | GB2 | -74.37 | GB | -73.97 | 0.430 |
| ***Italy*** | 1986 | gamma | -244.65 | Singh-Maddala | -233.17 | GB2 | -232.91 | GB | -227.51* | 0.306 |
| | 1995 | gamma | -239.59 | Dagum | -215.65 | GB2 | -214.55 | GB | -209.14* | 0.338 |
| ***Mexico*** | 1984 | gamma | -122.13 | Dagum | -90.73 | GB2 | -90.55 | GB | -88.57 | 0.466 |
| | 1989 | lognormal | -189.15 | Singh-Maddala | -104.6 | GB2 | -105.59 | GB | -104.59 | 0.494 |
| | 1992 | lognormal | -245.56 | Dagum | -107.74 | GB2 | -105.26 | GB | -103.2 | 0.546 |
| | 1994 | gamma | -357.1 | Dagum | -115.67 | GB2 | -108.3* | GB | -104.67* | 0.577 |
| | 1996 | lognormal | -287.09 | Dagum | -137.16 | GB2 | -133.91 | GB | -128.21* | 0.548 |
| ***Netherlands*** | 1983 | gamma | -155.7 | Dagum | -129.22 | GB2 | -128.59 | GB | -123.89* | 0.277 |
| | 1987 | gamma | -134.19 | Dagum | -113.59 | GB2 | -113.43 | GB | -109.2* | 0.288 |
| | 1991 | Weibull | -356.09 | Dagum | -248.86 | GB2 | -245.09* | GB | -243.69 | 0.326 |
| | 1994 | Weibull | -369.98 | Dagum | -230.55 | GB2 | -227.29 | GB | -223.88* | 0.340 |
| ***Norway*** | 1979 | gamma | -944.48 | Dagum | -625.37 | GB2 | -385.43* | GB | -385.43 | 0.528 |
| | 1986 | Weibull | -372.86 | Dagum | -219.58 | GB2 | -213.57* | GB | -210.84 | 0.343 |
| | 1991 | Weibull | -409.31 | Dagum | -231.58 | GB2 | -226.62* | GB | -225.21 | 0.362 |
| | 1995 | Weibull | -554.06 | Dagum | -258.06 | GB2 | -237.94* | GB | -231.54* | 0.373 |

Table 3 cont.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ***Poland*** | 1986 | Weibull | -875.59 | Dagum | -436.41 | GB2 | -421.29* | GB | -419.19 | 0.384 |
| | 1992 | Weibull | -236.14 | Dagum | -206.72 | GB2 | -206.71 | GB | -204.95 | 0.358 |
| | 1995 | gamma | -254.44 | Beta 2 | -148.55 | GB2 | -148.54 | GB | -148.54 | 0.362 |
| ***Taiwan*** | 1981 | lognormal | -227.66 | Dagum | -164.88 | GB2 | -164.86 | GB | -152.39* | 0.293 |
| | 1986 | gamma | -343.15 | Dagum | -174.1 | GB2 | -164.45* | GB | -149.09* | 0.314 |
| | 1991 | gamma | -231.49 | Dagum | -140.57 | GB2 | -139.96 | GB | -114.96* | 0.317 |
| | 1995 | Weibull | -338.26 | Dagum | -216.03 | GB2 | -211.46* | GB | -183.05* | 0.341 |
| ***Russia*** | 1992 | Weibull | -127.35 | Dagum | -104.93 | GB2 | -104.93 | GB | -104.01 | 0.466 |
| | 1995 | gamma | -91.02 | Dagum | -79.52 | GB2 | -79.05 | GB | -78.93 | 0.537 |
| ***Slovakia*** | 1992 | gamma | -140.79 | GG | -137.54 | GB1 | -134.44 | GB | -134.44 | 0.295 |
| ***Spain*** | 1980 | gamma | -516.38 | Dagum | -300.68 | GB2 | -272.21* | GB | -254.51* | 0.356 |
| | 1990 | gamma | -389.47 | Dagum | -261.5 | GB2 | -258.35 | GB | -242.13* | 0.348 |
| ***Sweden*** | 1967 | Weibull | -554.48 | Dagum | -328.11 | GB2 | -293.38* | GB | -293.43 | 0.399 |
| | 1975 | Weibull | -218.12 | Dagum | -169.93 | GB2 | -169.98 | GB | -144.6* | 0.385 |
| | 1981 | Weibull | -263.15 | Dagum | -142.91 | GB2 | -142.67 | GB | -138.33* | 0.395 |
| | 1987 | Weibull | -482.61 | Dagum | -173.54 | GB2 | -166.15* | GB | -165.32 | 0.384 |
| | 1992 | Weibull | -666.41 | Dagum | -256.1 | GB2 | -245.12* | GB | -245.04 | 0.405 |
| | 1995 | Weibull | -1179.37 | Dagum | -424.18 | GB2 | -399.51* | GB | -398.88 | 0.446 |
| ***Switzerland*** | 1982 | Weibull | -416.35 | Dagum | -270.07 | GB2 | -204.79* | GB | -204.79 | 0.360 |
| | 1992 | Weibull | -273.01 | Dagum | -181.71 | GB2 | -156.25* | GB | -151.24* | 0.373 |
| ***United States*** | 1969 | Weibull | -321.88 | Dagum | -170.59 | GB2 | -155.37* | GB | -154.32 | 0.368 |
| | 1974 | Weibull | -264.27 | Dagum | -144.99 | GB2 | -135.84* | GB | -134.17 | 0.379 |
| | 1979 | Weibull | -251.07 | Dagum | -154.41 | GB2 | -148.44* | GB | -146.57 | 0.392 |
| | 1986 | Weibull | -186.99 | Dagum | -109.58 | GB2 | -103.19* | GB | -101.34 | 0.408 |
| | 1991 | Weibull | -206.76 | Dagum | -137.27 | GB2 | -136.66 | GB | -133.86 | 0.417 |
| | 1994 | Weibull | -443.19 | Dagum | -229.76 | GB2 | -228.3 | GB | -226.1 | 0.425 |
| | 1997 | Weibull | -380.13 | Dagum | -192.79 | GB2 | -190.26 | GB | -187.77 | 0.430 |

Table 3 cont.

three-parameter distributions, all but the beta 1 outperform the others in certain cases. Within a given country, the relative ordering of the models in terms of their goodness-of-fit generally holds constant over time, but this is not always the case. For example, the distribution of income in Mexico (see table 3) is best modeled by the Dagum in 1984, 1992, 1994, and 1996, but in 1989, the Singh-Maddala distribution provides a better fit as measured by log-likelihood, SAE, or $\chi^2$ value. The GB2 is clearly the best four-parameter distribution. It's additional parameter makes it significantly better than the best three-parameter distribution in 44% of the data sets considered.

In almost every case, the four measures of goodness-of-fit (log-likelihood, SSE, SAE, and $\chi^2$) agree in the relative ranking of the models considered. In the few cases where they do not agree, the difference is negligible. In Australia 1989, for example, the GB has a log-likelihood value just barely better than that of the GB2 (by a difference of 0.15) but has a SAE value slightly worse than that of the GB2 (by a difference of 0.007).

In nearly all of the cases considered, the generalized beta family of distributions has the flexibility to provide a good fit to the data. This can be seen by a visual inspection of the histogram overlaid with the fitted pdf, an inspection of the SSE, SAE, or $\chi^2$ values, or a comparison of the sample mean with the expected value of the fitted pdf. In a few cases, no model provided a good fit. This is most obvious in the Scandinavian countries of Finland, Norway, and Sweden where the data shows a large clump of people earning very low incomes, giving the observed distribution a spike at the lowest income levels, which causes the sample mean to be significantly lower than the fitted (unimodal) models predict.

The choice of functional model strongly affects the predicted level of inequality as measured by the Gini coefficient. Generally, the Gini coefficients estimated by the lognormal are the highest, the Weibull and generalized gamma are the lowest, and the Dagum and GB2 find themselves somewhere in between. The disagreement results from inferior fits, particularly in the tails. It is also interesting that in almost every country, the Gini coefficients increase monotonically over time. It is important to note that these show inequality in income earned before transfer payments. Countries with progressive tax structures and large social programs and transfer payments may not observe such inequality after income redistribution—but looking at pre-tax earnings there is a general tendency towards greater inequality.

## V. Conclusions

This paper compares the ability of eleven probability distribution functions to fit income data for twenty three countries over time; in total, eighty two data sets were analyzed. Given the estimated distributional parameters, Gini coefficients were calculated for each country. Examining these Gini coefficients over multiple time periods reveals a general trend of increasing inequality for almost all the countries considered. Concerning functional form, the Weibull distribution is the best fitting two-parameter distribution in 62 percent of the data sets, the Dagum is the best fitting three-parameter distribution in 84 percent of the time, and the GB2 provides a statistically significant (at the one percent level) improvement over all the three-parameter distributions in 44 percent of the cases considered.

*Appendix*

## Australia 1981

| sample mean | sample obs |
|---|---|
| 21,338 | 10568 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -600.31 | 21,160 | 0.0052 | 0.2287 | 1164 | 8740 | 2.421 | - | - | - | 0.344 |
| lognormal | -1440.09 | 22,757 | 0.0134 | 0.3817 | 3718 | 9.758 | 0.742 | - | - | - | 0.400 |
| Weibull | -368.35 | 20,889 | 0.0029 | 0.1631 | 617 | 1.777 | 23475 | - | - | - | 0.323 |
| Dagum | -252.61 | 21,018 | 0.0018 | 0.1297 | 350 | 4.873 | 30808 | 0.296 | - | - | 0.328 |
| Beta 1 | -406.18 | 20,792 | 0.0035 | 0.2020 | 657 | 67716 | 1.798 | 4.058 | - | - | 0.324 |
| Beta 2 | -600.42 | 21,188 | 0.0052 | 0.2308 | 1156 | 2E+11 | 2.407 | 2E+07 | - | - | 0.345 |
| GG | -326.68 | 20,806 | 0.0026 | 0.1772 | 493 | 2.383 | 30284 | 0.635 | - | - | 0.322 |
| Singh-Maddala | -368.35 | 20,889 | 0.0029 | 0.1631 | 617 | 1.777 | 3E+07 | 3E+05 | - | - | 0.323 |
| GB1 | -326.68 | 20,806 | 0.0026 | 0.1771 | 493 | 2.383 | 4E+06 | 0.635 | 84327 | - | 0.322 |
| GB2 | -247.50 | 21,183 | 0.0018 | 0.1300 | 340 | 6.223 | 28340 | 0.228 | 0.6506 | - | 0.333 |
| GB | -242.38 | 20,630 | 0.0017 | 0.1211 | 330 | 7.71 | 26097 | 0.992 | 0.1838 | 0.422 | - |

## Australia 1985

| sample mean | sample obs |
|---|---|
| 29,439 | 5492 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -339.70 | 29,145 | 0.0050 | 0.2266 | 613 | 12260 | 2.377 | - | - | - | 0.347 |
| lognormal | -784.97 | 31,432 | 0.0130 | 0.3766 | 2020 | 10.07 | 0.751 | - | - | - | 0.404 |
| Weibull | -220.68 | 28,768 | 0.0028 | 0.1648 | 324 | 1.758 | 32309 | - | - | - | 0.326 |
| Dagum | -155.47 | 28,929 | 0.0017 | 0.1236 | 170 | 4.83 | 42482 | 0.296 | - | - | 0.331 |
| Beta 1 | -242.51 | 28,637 | 0.0035 | 0.2049 | 346 | 95573 | 1.783 | 4.169 | - | - | 0.327 |
| Beta 2 | -339.72 | 29,166 | 0.0050 | 0.2278 | 611 | 2E+11 | 2.37 | 2E+07 | - | - | 0.348 |
| GG | -199.16 | 28,650 | 0.0026 | 0.1781 | 255 | 2.349 | 41698 | 0.638 | - | - | 0.325 |
| Singh-Maddala | -220.68 | 28,768 | 0.0028 | 0.1648 | 324 | 1.758 | 4E+07 | 3E+05 | - | - | 0.326 |
| GB1 | -199.16 | 28,650 | 0.0026 | 0.1781 | 255 | 2.349 | 6E+06 | 0.639 | 105117 | - | 0.325 |
| GB2 | -150.78 | 29,236 | 0.0016 | 0.1190 | 160 | 6.74 | 38276 | 0.208 | 0.5687 | - | 0.337 |
| GB | -149.14 | 28,438 | 0.0016 | 0.1148 | 157 | 7.991 | 36104 | 0.995 | 0.1749 | 0.41 | - |

## Australia 1989

| sample mean | sample obs |
|---|---|
| 40,755 | 10629 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -454.81 | 39,755 | 0.0035 | 0.2002 | 835 | 17718 | 2.244 | - | - | - | 0.356 |
| lognormal | -1209.18 | 42,868 | 0.0101 | 0.3479 | 3044 | 10.37 | 0.768 | - | - | - | 0.413 |
| Weibull | -296.22 | 39,302 | 0.0021 | 0.1437 | 457 | 1.677 | 44006 | - | - | - | 0.339 |
| Dagum | -205.63 | 39,726 | 0.0013 | 0.1099 | 262 | 4.4 | 57344 | 0.318 | - | - | 0.345 |
| Beta 1 | -339.14 | 39,175 | 0.0026 | 0.1709 | 529 | 2E+05 | 1.776 | 5.266 | - | - | 0.339 |
| Beta 2 | -456.43 | 39,543 | 0.0034 | 0.1948 | 864 | 3E+11 | 2.296 | 2E+07 | - | - | 0.353 |
| GG | -279.08 | 39,199 | 0.0020 | 0.1528 | 405 | 2.035 | 53445 | 0.738 | - | - | 0.337 |
| Singh-Maddala | -296.22 | 39,302 | 0.0021 | 0.1437 | 457 | 1.677 | 4E+07 | 1E+05 | - | - | 0.339 |
| GB1 | -279.08 | 39,200 | 0.0020 | 0.1528 | 405 | 2.035 | 2E+07 | 0.738 | 154710 | - | 0.337 |
| GB2 | -202.41 | 40,012 | 0.0012 | 0.1083 | 256 | 5.317 | 53464 | 0.259 | 0.7172 | - | 0.349 |
| GB | -202.26 | 39,382 | 0.0012 | 0.1090 | 256 | 5.52 | 52166 | 0.994 | 0.2495 | 0.649 | - |

## Australia 1994

| sample mean | sample obs |
|---|---|
| 45,825 | 4619 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -289.04 | 44,760 | 0.0051 | 0.2216 | 487 | 22379 | 2 | - | - | - | 0.375 |
| lognormal | -693.82 | 49,471 | 0.0141 | 0.3783 | 1748 | 10.46 | 0.834 | - | - | - | 0.445 |
| Weibull | -204.52 | 44,175 | 0.0031 | 0.1560 | 292 | 1.592 | 49247 | - | - | - | 0.353 |
| Dagum | -153.58 | 44,513 | 0.0021 | 0.1399 | 167 | 4.575 | 68511 | 0.275 | - | - | 0.358 |
| Beta 1 | -210.53 | 43,884 | 0.0036 | 0.1909 | 280 | 2E+05 | 1.536 | 3.856 | - | - | 0.352 |
| Beta 2 | -289.05 | 44,793 | 0.0051 | 0.2223 | 485 | 4E+11 | 1.995 | 2E+07 | - | - | 0.375 |
| GG | -178.95 | 43,939 | 0.0028 | 0.1747 | 216 | 2.267 | 68116 | 0.583 | - | - | 0.350 |
| Singh-Maddala | -204.52 | 44,176 | 0.0031 | 0.1561 | 292 | 1.592 | 2E+08 | 4E+05 | - | - | 0.353 |
| GB1 | -178.95 | 43,939 | 0.0028 | 0.1748 | 216 | 2.267 | 1E+07 | 0.583 | 97488 | - | 0.350 |
| GB2 | -152.61 | 44,793 | 0.0021 | 0.1393 | 165 | 5.436 | 64046 | 0.229 | 0.7336 | - | 0.361 |
| GB | -152.17 | 43,815 | 0.0021 | 0.1379 | 165 | 6.094 | 60014 | 0.988 | 0.2039 | 0.557 | - |

## Belgium 1985

| sample mean | sample obs |
|---|---|
| 6,494 | 4357 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -235.41 | 6,484 | 0.0037 | 0.2229 | 366 | 1357 | 4.778 | - | - | - | 0.251 |
| lognormal | -236.42 | 6,556 | 0.0037 | 0.2228 | 362 | 8.676 | 0.473 | - | - | - | 0.262 |
| Weibull | -322.43 | 6,452 | 0.0057 | 0.2809 | 563 | 2.348 | 7280 | - | - | - | 0.256 |
| Dagum | -271.29 | 6,671 | 0.0047 | 0.2516 | 434 | 3.823 | 6374 | 0.83 | - | - | 0.275 |
| Beta 1 | -235.41 | 6,484 | 0.0037 | 0.2229 | 366 | 3E+09 | 4.777 | 3E+06 | - | - | 0.251 |
| Beta 2 | -229.42 | 6,511 | 0.0036 | 0.2145 | 349 | 21680 | 6.061 | 21.18 | - | - | 0.256 |
| GG | -227.34 | 6,510 | 0.0035 | 0.2123 | 345 | 0.507 | 21.05 | 17.79 | - | - | 0.255 |
| Singh-Maddala | -255.78 | 6,544 | 0.0042 | 0.2369 | 406 | 3.078 | 7581 | 1.745 | - | - | 0.262 |
| GB1 | -227.33 | 6,510 | 0.0035 | 0.2123 | 345 | 0.503 | 6E+08 | 17.99 | 5749.5 | - | 0.255 |
| GB2 | -227.39 | 6,510 | 0.0035 | 0.2123 | 345 | 0.523 | 1E+07 | 16.96 | 980.12 | - | 0.255 |
| GB | -227.33 | 6,510 | 0.0035 | 0.2123 | 345 | 0.503 | 6E+08 | 0 | 17.986 | 5755 | - |

## Belgium 1988

| sample mean | sample obs |
|---|---|
| 6,942 | 2518 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -202.09 | 7,000 | 0.0049 | 0.2590 | 324 | 1466 | 4.774 | - | - | - | 0.252 |
| lognormal | -211.24 | 7,083 | 0.0054 | 0.2621 | 338 | 8.753 | 0.475 | - | - | - | 0.263 |
| Weibull | -240.92 | 6,964 | 0.0064 | 0.3056 | 415 | 2.37 | 7858 | - | - | - | 0.254 |
| Dagum | -230.17 | 7,166 | 0.0059 | 0.2831 | 391 | 4.159 | 7491 | 0.675 | - | - | 0.270 |
| Beta 1 | -202.09 | 7,001 | 0.0049 | 0.2590 | 324 | 1E+10 | 4.771 | 8E+06 | - | - | 0.252 |
| Beta 2 | -201.61 | 7,011 | 0.0049 | 0.2584 | 322 | 74124 | 5.181 | 55.77 | - | - | 0.253 |
| GG | -200.79 | 7,015 | 0.0049 | 0.2569 | 320 | 0.732 | 357.1 | 8.657 | - | - | 0.254 |
| Singh-Maddala | -216.19 | 7,034 | 0.0054 | 0.2745 | 359 | 2.916 | 9366 | 2.301 | - | - | 0.258 |
| GB1 | -200.49 | 7,008 | 0.0048 | 0.2571 | 320 | 0.678 | 2E+05 | 9.144 | 89.501 | - | 0.253 |
| GB2 | -200.79 | 7,014 | 0.0049 | 0.2569 | 320 | 0.736 | 2E+08 | 8.56 | 15394 | - | 0.254 |
| GB | -200.78 | 7,014 | 0.0049 | 0.2569 | 320 | 0.731 | 5E+07 | 0 | 8.6649 | 5657 | - |

## Belgium 1992

| sample mean | sample obs |
|---|---|
| 12,231 | 2581 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -83.68 | 12,761 | 0.0008 | 0.1075 | 44 | 3601 | 3.544 | - | - | - | 0.289 |
| lognormal | -103.02 | 13,012 | 0.0016 | 0.1365 | 84 | 9.317 | 0.56 | - | - | - | 0.308 |
| Weibull | -118.30 | 12,696 | 0.0022 | 0.1663 | 114 | 2.021 | 14328 | - | - | - | 0.290 |
| Dagum | -93.05 | 13,081 | 0.0012 | 0.1296 | 63 | 3.755 | 14112 | 0.611 | - | - | 0.307 |
| Beta 1 | -83.68 | 12,761 | 0.0008 | 0.1076 | 44 | 4E+10 | 3.543 | 1E+07 | - | - | 0.289 |
| Beta 2 | -82.61 | 12,793 | 0.0008 | 0.0974 | 42 | 1E+05 | 3.883 | 36.23 | - | - | 0.292 |
| GG | -82.59 | 12,787 | 0.0008 | 0.0972 | 42 | 0.805 | 1560 | 5.315 | - | - | 0.292 |
| Singh-Maddala | -85.95 | 12,876 | 0.0009 | 0.1066 | 49 | 2.532 | 17010 | 2.171 | - | - | 0.297 |
| GB1 | -82.59 | 12,787 | 0.0008 | 0.0972 | 42 | 0.803 | 7E+07 | 5.329 | 5558.4 | - | 0.292 |
| GB2 | -82.59 | 12,788 | 0.0008 | 0.0972 | 42 | 0.843 | 1E+06 | 4.97 | 209.85 | - | 0.292 |
| GB | -82.59 | 12,788 | 0.0008 | 0.0972 | 42 | 0.844 | 1E+06 | 1 | 4.9604 | 203.9 | - |

## Belgium 1997

| sample mean | sample obs |
|---|---|
| 1,580,581 | 2906 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -103.85 | 1,652,354 | 0.0015 | 0.1224 | 81 | 6E+05 | 2.789 | - | - | - | 0.323 |
| lognormal | -185.57 | 1,713,748 | 0.0040 | 0.2065 | 276 | 14.14 | 0.651 | - | - | - | 0.355 |
| Weibull | -110.23 | 1,641,387 | 0.0016 | 0.1464 | 95 | 1.813 | 2E+06 | - | - | - | 0.318 |
| Dagum | -93.69 | 1,682,970 | 0.0010 | 0.1149 | 62 | 3.776 | 2E+06 | 0.479 | - | - | 0.333 |
| Beta 1 | -102.29 | 1,647,344 | 0.0014 | 0.1277 | 77 | 2E+07 | 2.587 | 27.99 | - | - | 0.320 |
| Beta 2 | -103.89 | 1,650,318 | 0.0015 | 0.1211 | 81 | 1E+13 | 2.807 | 2E+07 | - | - | 0.322 |
| GG | -100.38 | 1,646,775 | 0.0013 | 0.1277 | 74 | 1.285 | 1E+06 | 1.801 | - | - | 0.319 |
| Singh-Maddala | -94.61 | 1,657,103 | 0.0011 | 0.1077 | 63 | 2.089 | 3E+06 | 3.534 | - | - | 0.323 |
| GB1 | -100.38 | 1,646,787 | 0.0013 | 0.1277 | 74 | 1.285 | 2E+10 | 1.801 | 224989 | - | 0.319 |
| GB2 | -92.47 | 1,667,672 | 0.0010 | 0.1077 | 59 | 2.999 | 2E+06 | 0.629 | 1.5555 | - | 0.327 |
| GB | -91.69 | 1,638,410 | 0.0010 | 0.1044 | 58 | 3.661 | 2E+06 | 0.967 | 0.5075 | 0.842 | - |

## Canada 1971

| sample mean | sample obs |
|---|---|
| 8,633 | 20386 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -1004.16 | 8,224 | 0.0044 | 0.2480 | 2002 | 4805 | 1.711 | - | - | - | 0.401 |
| lognormal | -3039.14 | 9,420 | 0.0148 | 0.4304 | 7442 | 8.721 | 0.927 | - | - | - | 0.488 |
| Weibull | -611.61 | 8,104 | 0.0023 | 0.1746 | 1193 | 1.474 | 8957 | - | - | - | 0.375 |
| Dagum | -192.48 | 8,098 | 0.0005 | 0.0818 | 223 | 5.099 | 13799 | 0.214 | - | - | 0.371 |
| Beta 1 | -466.35 | 8,002 | 0.0020 | 0.1636 | 769 | 25410 | 1.285 | 2.796 | - | - | 0.369 |
| Beta 2 | -1004.16 | 8,223 | 0.0044 | 0.2479 | 2003 | 4E+10 | 1.712 | 8E+06 | - | - | 0.401 |
| GG | -306.15 | 8,021 | 0.0011 | 0.1266 | 446 | 2.694 | 14478 | 0.417 | - | - | 0.367 |
| Singh-Maddala | -611.62 | 8,104 | 0.0023 | 0.1746 | 1193 | 1.474 | 7E+07 | 6E+05 | - | - | 0.375 |
| GB1 | -306.15 | 8,021 | 0.0011 | 0.1266 | 446 | 2.694 | 1E+06 | 0.417 | 106495 | - | 0.366 |
| GB2 | -166.75 | 8,222 | 0.0004 | 0.0669 | 171 | 8.654 | 11927 | 0.123 | 0.4281 | - | 0.379 |
| GB | -166.14 | 8,010 | 0.0004 | 0.0656 | 169 | 9.409 | 11664 | 0.998 | 0.1134 | 0.37 | - |

## Canada 1975

| sample mean | sample obs |
|---|---|
| 14,052 | 21750 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -866.89 | 13,370 | 0.0035 | 0.2193 | 1698 | 7310 | 1.829 | - | - | - | 0.390 |
| lognormal | -2795.66 | 14,993 | 0.0126 | 0.4030 | 6774 | 9.227 | 0.881 | - | - | - | 0.467 |
| Weibull | -481.71 | 13,192 | 0.0016 | 0.1413 | 884 | 1.519 | 14634 | - | - | - | 0.366 |
| Dagum | -171.46 | 13,252 | 0.0004 | 0.0676 | 178 | 4.8 | 21578 | 0.241 | - | - | 0.367 |
| Beta 1 | -422.94 | 13,066 | 0.0016 | 0.1488 | 682 | 43902 | 1.392 | 3.286 | - | - | 0.363 |
| Beta 2 | -867.03 | 13,388 | 0.0035 | 0.2202 | 1690 | 6E+10 | 1.821 | 9E+06 | - | - | 0.391 |
| GG | -281.75 | 13,097 | 0.0009 | 0.1147 | 396 | 2.439 | 22031 | 0.495 | - | - | 0.361 |
| Singh-Maddala | -481.71 | 13,192 | 0.0016 | 0.1413 | 884 | 1.519 | 8E+07 | 5E+05 | - | - | 0.366 |
| GB1 | -281.76 | 13,097 | 0.0009 | 0.1147 | 396 | 2.44 | 2E+06 | 0.494 | 91916 | - | 0.361 |
| GB2 | -159.42 | 13,393 | 0.0004 | 0.0621 | 153 | 6.615 | 19359 | 0.172 | 0.5782 | - | 0.372 |
| GB | -156.27 | 13,027 | 0.0003 | 0.0589 | 147 | 7.839 | 18215 | 0.994 | 0.1448 | 0.418 | - |

## Canada 1981

| sample mean | sample obs |
|---|---|
| 24,969 | 12331 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -477.91 | 23,898 | 0.0032 | 0.2049 | 853 | 13331 | 1.793 | - | - | - | 0.393 |
| lognormal | -1536.26 | 26,818 | 0.0122 | 0.3948 | 3520 | 9.802 | 0.889 | - | - | - | 0.470 |
| Weibull | -273.00 | 23,589 | 0.0014 | 0.1302 | 428 | 1.498 | 26127 | - | - | - | 0.370 |
| Dagum | -117.24 | 23,740 | 0.0003 | 0.0661 | 81 | 4.828 | 39195 | 0.234 | - | - | 0.370 |
| Beta 1 | -218.98 | 23,349 | 0.0012 | 0.1285 | 287 | 76836 | 1.353 | 3.1 | - | - | 0.365 |
| Beta 2 | -477.92 | 23,888 | 0.0032 | 0.2048 | 854 | 2E+11 | 1.795 | 2E+07 | - | - | 0.393 |
| GG | -155.43 | 23,422 | 0.0006 | 0.0976 | 158 | 2.471 | 40078 | 0.478 | - | - | 0.364 |
| Singh-Maddala | -273.01 | 23,589 | 0.0014 | 0.1302 | 428 | 1.498 | 2E+08 | 5E+05 | - | - | 0.370 |
| GB1 | -155.43 | 23,423 | 0.0006 | 0.0976 | 158 | 2.471 | 5E+06 | 0.478 | 122069 | - | 0.364 |
| GB2 | -116.50 | 23,826 | 0.0003 | 0.0667 | 79 | 5.373 | 37435 | 0.209 | 0.8175 | - | 0.372 |
| GB | -111.62 | 23,232 | 0.0003 | 0.0610 | 69 | 7.315 | 32558 | 0.987 | 0.1527 | 0.427 | - |

## Canada 1987

| sample mean | sample obs |
|---|---|
| 38,275 | 8667 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -390.95 | 33,944 | 0.0035 | 0.2187 | 683 | 20641 | 1.644 | - | - | - | 0.408 |
| lognormal | -1211.14 | 38,991 | 0.0133 | 0.4094 | 2811 | 10.13 | 0.944 | - | - | - | 0.495 |
| Weibull | -251.37 | 33,485 | 0.0018 | 0.1504 | 396 | 1.43 | 36856 | - | - | - | 0.384 |
| Dagum | -104.27 | 33,577 | 0.0004 | 0.0714 | 61 | 4.866 | 57426 | 0.218 | - | - | 0.381 |
| Beta 1 | -193.71 | 33,068 | 0.0014 | 0.1371 | 242 | 1E+05 | 1.253 | 2.909 | - | - | 0.377 |
| Beta 2 | -390.97 | 33,916 | 0.0035 | 0.2184 | 685 | 1E+11 | 1.649 | 7E+06 | - | - | 0.408 |
| GG | -140.44 | 33,172 | 0.0008 | 0.1083 | 134 | 2.537 | 59747 | 0.434 | - | - | 0.375 |
| Singh-Maddala | -251.37 | 33,485 | 0.0018 | 0.1504 | 396 | 1.43 | 4E+08 | 7E+05 | - | - | 0.384 |
| GB1 | -140.44 | 33,173 | 0.0008 | 0.1083 | 134 | 2.537 | 6E+06 | 0.434 | 98095 | - | 0.375 |
| GB2 | -99.55 | 33,968 | 0.0003 | 0.0636 | 52 | 6.946 | 50974 | 0.15 | 0.5467 | - | 0.386 |
| GB | -97.30 | 32,903 | 0.0003 | 0.0589 | 47 | 9.189 | 47134 | 0.995 | 0.1131 | 0.339 | - |

## Canada 1991

| sample | |
|---|---|
| ***mean*** | ***obs*** |
| 45,589 | 15257 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -489.38 | 41,563 | 0.0027 | 0.1887 | 852 | 25145 | 1.653 | - | - | - | 0.407 |
| lognormal | -1758.46 | 47,208 | 0.0114 | 0.3818 | 3891 | 10.33 | 0.931 | - | - | - | 0.489 |
| Weibull | -286.14 | 41,060 | 0.0013 | 0.1281 | 438 | 1.421 | 45153 | - | - | - | 0.386 |
| Dagum | -105.53 | 41,377 | 0.0002 | 0.0459 | 53 | 4.63 | 69595 | 0.231 | - | - | 0.385 |
| Beta 1 | -218.66 | 40,610 | 0.0010 | 0.1157 | 276 | 1E+05 | 1.284 | 3.269 | - | - | 0.380 |
| Beta 2 | -489.39 | 41,563 | 0.0027 | 0.1887 | 852 | 6E+09 | 1.653 | 2E+05 | - | - | 0.407 |
| GG | -151.52 | 40,758 | 0.0005 | 0.0826 | 143 | 2.325 | 70577 | 0.484 | - | - | 0.378 |
| Singh-Maddala | -286.14 | 41,060 | 0.0013 | 0.1281 | 438 | 1.421 | 5E+08 | 6E+05 | - | - | 0.386 |
| GB1 | -151.52 | 40,759 | 0.0005 | 0.0826 | 143 | 2.325 | 1E+07 | 0.484 | 89571 | - | 0.378 |
| GB2 | -104.26 | 41,576 | 0.0002 | 0.0453 | 50 | 5.294 | 65805 | 0.2 | 0.7831 | - | 0.387 |
| GB | -103.70 | 40,779 | 0.0002 | 0.0430 | 49 | 5.918 | 61515 | 0.985 | 0.1787 | 0.594 | - |

## Canada 1994

| sample | |
|---|---|
| ***mean*** | ***obs*** |
| 47,549 | 28155 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -839.48 | 45,218 | 0.0026 | 0.1903 | 1578 | 28132 | 1.607 | - | - | - | 0.412 |
| lognormal | -3240.70 | 51,732 | 0.0116 | 0.3927 | 7391 | 10.4 | 0.948 | - | - | - | 0.497 |
| Weibull | -479.16 | 44,671 | 0.0013 | 0.1267 | 844 | 1.397 | 48998 | - | - | - | 0.391 |
| Dagum | -135.12 | 45,010 | 0.0002 | 0.0472 | 101 | 4.535 | 75904 | 0.232 | - | - | 0.390 |
| Beta 1 | -352.06 | 44,166 | 0.0010 | 0.1224 | 535 | 2E+05 | 1.257 | 3.285 | - | - | 0.384 |
| Beta 2 | -840.17 | 45,356 | 0.0027 | 0.1924 | 1569 | 4E+11 | 1.594 | 1E+07 | - | - | 0.414 |
| GG | -224.78 | 44,324 | 0.0005 | 0.0883 | 280 | 2.292 | 77266 | 0.482 | - | - | 0.383 |
| Singh-Maddala | -479.16 | 44,672 | 0.0013 | 0.1267 | 844 | 1.397 | 7E+08 | 6E+05 | - | - | 0.391 |
| GB1 | -224.79 | 44,324 | 0.0005 | 0.0884 | 280 | 2.291 | 1E+07 | 0.482 | 97755 | - | 0.383 |
| GB2 | -132.00 | 45,270 | 0.0002 | 0.0448 | 95 | 5.281 | 71142 | 0.198 | 0.7575 | - | 0.392 |
| GB | -127.17 | 44,069 | 0.0002 | 0.0391 | 85 | 6.588 | 63832 | 0.987 | 0.1577 | 0.471 | - |

## Canada 1997

| sample | |
|---|---|
| ***mean*** | ***obs*** |
| 50,742 | 25018 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -659.59 | 47,738 | 0.0022 | 0.1775 | 1225 | 29413 | 1.623 | - | - | - | 0.410 |
| lognormal | -2700.64 | 54,314 | 0.0104 | 0.3737 | 6281 | 10.46 | 0.939 | - | - | - | 0.493 |
| Weibull | -364.79 | 47,187 | 0.0010 | 0.1151 | 612 | 1.399 | 51769 | - | - | - | 0.391 |
| Dagum | -118.53 | 47,665 | 0.0001 | 0.0376 | 69 | 4.395 | 79112 | 0.243 | - | - | 0.391 |
| Beta 1 | -289.08 | 46,706 | 0.0008 | 0.1060 | 412 | 2E+05 | 1.285 | 3.578 | - | - | 0.385 |
| Beta 2 | -659.71 | 47,802 | 0.0022 | 0.1782 | 1221 | 7E+11 | 1.617 | 2E+07 | - | - | 0.411 |
| GG | -188.87 | 46,867 | 0.0004 | 0.0768 | 210 | 2.17 | 79031 | 0.52 | - | - | 0.383 |
| Singh-Maddala | -364.79 | 47,187 | 0.0010 | 0.1151 | 612 | 1.399 | 7E+08 | 6E+05 | - | - | 0.391 |
| GB1 | -188.87 | 46,867 | 0.0004 | 0.0768 | 210 | 2.17 | 2E+07 | 0.52 | 90672 | - | 0.383 |
| GB2 | -117.90 | 47,800 | 0.0001 | 0.0376 | 68 | 4.724 | 76517 | 0.225 | 0.8738 | - | 0.392 |
| GB | -116.52 | 46,824 | 0.0001 | 0.0371 | 65 | 5.402 | 69783 | 0.981 | 0.1958 | 0.618 | - |

## Czech Rep. 1992

| sample mean | sample obs |
|---|---|
| 964 | 11512 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -163.79 | 944 | 0.0008 | 0.1051 | 176 | 298.2 | 3.167 | - | - | - | 0.305 |
| lognormal | -470.84 | 973 | 0.0033 | 0.2220 | 847 | 6.696 | 0.607 | - | - | - | 0.332 |
| Weibull | -160.71 | 938 | 0.0007 | 0.1100 | 170 | 1.958 | 1058 | - | - | - | 0.298 |
| Dagum | -127.72 | 955 | 0.0004 | 0.0702 | 104 | 4.373 | 1214 | 0.419 | - | - | 0.308 |
| Beta 1 | -142.34 | 940 | 0.0006 | 0.0957 | 131 | 6041 | 2.738 | 14.87 | - | - | 0.299 |
| Beta 2 | -163.79 | 944 | 0.0008 | 0.1050 | 176 | 3E+09 | 3.171 | 9E+06 | - | - | 0.305 |
| GG | -135.13 | 940 | 0.0005 | 0.0909 | 117 | 1.468 | 719.4 | 1.626 | - | - | 0.299 |
| Singh-Maddala | -125.42 | 943 | 0.0004 | 0.0803 | 98 | 2.17 | 2052 | 4.872 | - | - | 0.301 |
| GB1 | -135.14 | 940 | 0.0005 | 0.0909 | 117 | 1.468 | 1E+06 | 1.626 | 49023 | - | 0.299 |
| GB2 | -121.39 | 947 | 0.0004 | 0.0698 | 90 | 3.046 | 1440 | 0.647 | 2.0118 | - | 0.303 |
| GB | -121.39 | 947 | 0.0004 | 0.0696 | 90 | 3.068 | 1432 | 1 | 0.6417 | 1.979 | - |

## Czech Rep. 1996

| sample mean | sample obs |
|---|---|
| 2,168 | 19442 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -125.71 | 2,042 | 0.0002 | 0.0551 | 88 | 752.3 | 2.714 | - | - | - | 0.327 |
| lognormal | -497.53 | 2,113 | 0.0020 | 0.1677 | 888 | 7.441 | 0.656 | - | - | - | 0.357 |
| Weibull | -248.77 | 2,031 | 0.0009 | 0.1065 | 337 | 1.771 | 2282 | - | - | - | 0.324 |
| Dagum | -117.56 | 2,091 | 0.0002 | 0.0496 | 74 | 3.596 | 2449 | 0.506 | - | - | 0.341 |
| Beta 1 | -125.71 | 2,042 | 0.0002 | 0.0551 | 88 | 3E+06 | 2.713 | 3469 | - | - | 0.327 |
| Beta 2 | -125.71 | 2,042 | 0.0002 | 0.0551 | 88 | 6E+08 | 2.715 | 8E+05 | - | - | 0.327 |
| GG | -124.88 | 2,041 | 0.0002 | 0.0545 | 86 | 1.054 | 873.2 | 2.471 | - | - | 0.326 |
| Singh-Maddala | -101.44 | 2,056 | 0.0001 | 0.0385 | 40 | 2.096 | 3407 | 2.959 | - | - | 0.331 |
| GB1 | -124.88 | 2,041 | 0.0002 | 0.0545 | 86 | 1.054 | 3E+07 | 2.471 | 63631 | - | 0.326 |
| GB2 | -100.17 | 2,060 | 0.0001 | 0.0360 | 38 | 2.391 | 3042 | 0.839 | 2.2023 | - | 0.332 |
| GB | -100.17 | 2,061 | 0.0001 | 0.0359 | 38 | 2.394 | 3040 | 1 | 0.8377 | 2.196 | - |

## Denmark 1987

| sample mean | sample obs |
|---|---|
| 222,676 | 9111 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -778.20 | 224,395 | 0.0080 | 0.3228 | 1549 | 2E+05 | 1.467 | - | - | - | 0.428 |
| lognormal | -2004.38 | 276,078 | 0.0235 | 0.5464 | 5192 | 11.98 | 1.051 | - | - | - | 0.543 |
| Weibull | -584.64 | 220,447 | 0.0050 | 0.2482 | 1232 | 1.374 | 2E+05 | - | - | - | 0.396 |
| Dagum | -236.12 | 218,040 | 0.0017 | 0.1464 | 335 | 6.256 | 4E+05 | 0.151 | - | - | 0.383 |
| Beta 1 | -332.18 | 215,681 | 0.0029 | 0.1938 | 524 | 6E+05 | 1.05 | 1.821 | - | - | 0.381 |
| Beta 2 | -778.21 | 224,547 | 0.0080 | 0.3232 | 1546 | 1E+12 | 1.465 | 9E+06 | - | - | 0.429 |
| GG | -256.58 | 216,146 | 0.0020 | 0.1599 | 374 | 3.641 | 4E+05 | 0.264 | - | - | 0.379 |
| Singh-Maddala | -584.64 | 220,450 | 0.0050 | 0.2481 | 1232 | 1.374 | 5E+09 | 8E+05 | - | - | 0.396 |
| GB1 | -256.58 | 216,149 | 0.0020 | 0.1599 | 374 | 3.641 | 9E+06 | 0.264 | 50415 | - | 0.379 |
| GB2 | -233.65 | 219,808 | 0.0017 | 0.1411 | 332 | 8.873 | 4E+05 | 0.105 | 0.5472 | - | 0.386 |
| GB | -233.65 | 219,779 | 0.0017 | 0.1411 | 332 | 8.842 | 4E+05 | 1 | 0.1058 | 0.55 | - |

## Denmark 1992

| sample mean | obs |
|---|---|
| 258,142 | 9104 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -854.99 | 261,240 | 0.0090 | 0.3454 | 1677 | 2E+05 | 1.277 | - | - | - | 0.454 |
| lognormal | -2190.64 | 342,297 | 0.0263 | 0.5792 | 5485 | 12.07 | 1.16 | - | - | - | 0.588 |
| Weibull | -710.57 | 256,863 | 0.0065 | 0.2970 | 1496 | 1.263 | 3E+05 | - | - | - | 0.422 |
| Dagum | -277.62 | 252,287 | 0.0021 | 0.1662 | 433 | 6.62 | 5E+05 | 0.129 | - | - | 0.402 |
| Beta 1 | -354.88 | 249,591 | 0.0030 | 0.2031 | 587 | 7E+05 | 0.92 | 1.597 | - | - | 0.400 |
| Beta 2 | -855.21 | 261,233 | 0.0090 | 0.3455 | 1678 | 1E+09 | 1.277 | 7084 | - | - | 0.454 |
| GG | -289.38 | 250,075 | 0.0022 | 0.1789 | 455 | 3.995 | 6E+05 | 0.216 | - | - | 0.398 |
| Singh-Maddala | -710.57 | 256,865 | 0.0065 | 0.2970 | 1496 | 1.263 | 1E+10 | 9E+05 | - | - | 0.422 |
| GB1 | -289.38 | 250,075 | 0.0022 | 0.1789 | 455 | 3.994 | 9E+06 | 0.216 | 79005 | - | 0.398 |
| GB2 | -276.69 | 253,780 | 0.0021 | 0.1606 | 434 | 8.537 | 5E+05 | 0.099 | 0.625 | - | 0.404 |
| GB | -276.69 | 253,775 | 0.0021 | 0.1606 | 434 | 8.537 | 5E+05 | 1 | 0.0991 | 0.625 | - |

## Denmark 1995

| sample mean | obs |
|---|---|
| 279,396 | 9289 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -850.05 | 281,377 | 0.0090 | 0.3572 | 1623 | 2E+05 | 1.144 | - | - | - | 0.474 |
| lognormal | -2270.97 | 391,087 | 0.0272 | 0.5915 | 5445 | 12.09 | 1.252 | - | - | - | 0.624 |
| Weibull | -754.11 | 277,268 | 0.0072 | 0.3103 | 1548 | 1.175 | 3E+05 | - | - | - | 0.446 |
| Dagum | -265.43 | 269,886 | 0.0021 | 0.1632 | 391 | 6.073 | 6E+05 | 0.132 | - | - | 0.420 |
| Beta 1 | -371.18 | 267,629 | 0.0033 | 0.2128 | 601 | 8E+05 | 0.854 | 1.621 | - | - | 0.419 |
| Beta 2 | -850.66 | 283,252 | 0.0091 | 0.3584 | 1611 | 3E+12 | 1.129 | 1E+07 | - | - | 0.477 |
| GG | -289.48 | 267,853 | 0.0023 | 0.1794 | 439 | 3.76 | 6E+05 | 0.214 | - | - | 0.416 |
| Singh-Maddala | -754.11 | 277,272 | 0.0072 | 0.3103 | 1548 | 1.175 | 5E+10 | 1E+06 | - | - | 0.446 |
| GB1 | -289.48 | 267,854 | 0.0023 | 0.1794 | 439 | 3.76 | 1E+07 | 0.214 | 102065 | - | 0.416 |
| GB2 | -257.38 | 274,268 | 0.0019 | 0.1536 | 378 | 12.12 | 5E+05 | 0.065 | 0.3316 | - | 0.427 |
| GB | -254.80 | 264,523 | 0.0019 | 0.1461 | 374 | 23.99 | 4E+05 | 1 | 0.0328 | 0.139 | - |

## Denmark 1997

| sample mean | obs |
|---|---|
| 286,881 | 9941 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -1669.06 | 292,836 | 0.0172 | 0.4953 | 3693 | 4E+05 | 0.771 | - | - | - | 0.551 |
| lognormal | -4280.06 | 754,740 | 0.0496 | 0.7872 | 13436 | 11.89 | 1.815 | - | - | - | 0.801 |
| Weibull | -1838.07 | 294,664 | 0.0166 | 0.4877 | 5444 | 0.921 | 3E+05 | - | - | - | 0.529 |
| Dagum | -676.05 | 262,846 | 0.0060 | 0.2716 | 1316 | 7.809 | 7E+05 | 0.078 | - | - | 0.470 |
| Beta 1 | -763.71 | 266,913 | 0.0070 | 0.3144 | 1474 | 8E+05 | 0.59 | 1.07 | - | - | 0.466 |
| Beta 2 | -1669.06 | 292,803 | 0.0172 | 0.4953 | 3693 | 4E+12 | 0.771 | 1E+07 | - | - | 0.551 |
| GG | -689.90 | 261,964 | 0.0061 | 0.2822 | 1341 | 5.644 | 7E+05 | 0.107 | - | - | 0.467 |
| Singh-Maddala | -1838.07 | 294,656 | 0.0166 | 0.4877 | 5443 | 0.921 | 9E+11 | 1E+06 | - | - | 0.529 |
| GB1 | -689.90 | 261,965 | 0.0061 | 0.2822 | 1341 | 5.644 | 5E+06 | 0.107 | 77555 | - | 0.467 |
| GB2 | -664.50 | 267,151 | 0.0059 | 0.2620 | 1292 | 19.69 | 5E+05 | 0.031 | 0.217 | - | 0.478 |
| GB | -661.54 | 252,964 | 0.0058 | 0.2542 | 1287 | 87.45 | 5E+05 | 1 | 0.007 | 0.039 | - |

## Finland 1987

| sample mean | sample obs |
|---|---|
| 116,212 | 10423 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -755.54 | 145,143 | 0.0063 | 0.2593 | 1608 | 94706 | 1.533 | - | - | - | 0.421 |
| lognormal | -2088.12 | 175,162 | 0.0198 | 0.4722 | 6259 | 11.56 | 1.016 | - | - | - | 0.528 |
| Weibull | -555.60 | 142,789 | 0.0039 | 0.1931 | 1226 | 1.391 | 2E+05 | - | - | - | 0.392 |
| Dagum | -289.90 | 142,065 | 0.0022 | 0.1480 | 436 | 4.703 | 2E+05 | 0.22 | - | - | 0.389 |
| Beta 1 | -468.97 | 140,695 | 0.0040 | 0.2223 | 811 | 5E+05 | 1.177 | 2.717 | - | - | 0.386 |
| Beta 2 | -755.81 | 145,608 | 0.0064 | 0.2616 | 1588 | 3E+12 | 1.519 | 3E+07 | - | - | 0.422 |
| GG | -361.07 | 140,765 | 0.0029 | 0.1920 | 577 | 2.585 | 3E+05 | 0.406 | - | - | 0.383 |
| Singh-Maddala | -555.60 | 142,791 | 0.0039 | 0.1930 | 1226 | 1.391 | 2E+09 | 6E+05 | - | - | 0.392 |
| GB1 | -361.07 | 140,766 | 0.0029 | 0.1920 | 577 | 2.585 | 2E+07 | 0.406 | 65960 | - | 0.383 |
| GB2 | -270.53 | 145,194 | 0.0020 | 0.1385 | 394 | 8.863 | 2E+05 | 0.115 | 0.364 | - | 0.401 |
| GB | -258.62 | 138,195 | 0.0019 | 0.1231 | 370 | 14.05 | 2E+05 | 1 | 0.0721 | 0.192 | - |

## Finland 1991

| sample mean | sample obs |
|---|---|
| 149,614 | 10160 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -672.53 | 180,148 | 0.0055 | 0.2552 | 1391 | 1E+05 | 1.423 | - | - | - | 0.434 |
| lognormal | -1963.75 | 221,851 | 0.0186 | 0.4679 | 5740 | 11.75 | 1.063 | - | - | - | 0.548 |
| Weibull | -517.82 | 177,451 | 0.0037 | 0.1989 | 1108 | 1.322 | 2E+05 | - | - | - | 0.408 |
| Dagum | -267.57 | 176,644 | 0.0019 | 0.1431 | 394 | 4.452 | 3E+05 | 0.221 | - | - | 0.404 |
| Beta 1 | -440.93 | 174,718 | 0.0037 | 0.2226 | 754 | 6E+05 | 1.125 | 2.924 | - | - | 0.401 |
| Beta 2 | -671.40 | 178,791 | 0.0054 | 0.2473 | 1429 | 2E+13 | 1.451 | 1E+08 | - | - | 0.430 |
| GG | -341.76 | 174,832 | 0.0027 | 0.1900 | 540 | 2.408 | 3E+05 | 0.417 | - | - | 0.397 |
| Singh-Maddala | -517.82 | 177,454 | 0.0037 | 0.1989 | 1108 | 1.322 | 4E+09 | 5E+05 | - | - | 0.408 |
| GB1 | -341.76 | 174,832 | 0.0027 | 0.1900 | 540 | 2.408 | 5E+07 | 0.417 | 148398 | - | 0.397 |
| GB2 | -244.33 | 181,483 | 0.0017 | 0.1326 | 345 | 9.59 | 3E+05 | 0.1 | 0.3136 | - | 0.417 |
| GB | -238.45 | 171,736 | 0.0017 | 0.1235 | 332 | 13.97 | 2E+05 | 1 | 0.0686 | 0.19 | - |

## Finland 1995

| sample mean | sample obs |
|---|---|
| 143,892 | 7913 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -867.08 | 177,156 | 0.0105 | 0.3625 | 1844 | 2E+05 | 1.142 | - | - | - | 0.475 |
| lognormal | -2298.47 | 258,059 | 0.0311 | 0.6241 | 7050 | 11.63 | 1.287 | - | - | - | 0.637 |
| Weibull | -772.69 | 174,225 | 0.0079 | 0.2914 | 1957 | 1.185 | 2E+05 | - | - | - | 0.443 |
| Dagum | -341.40 | 168,830 | 0.0035 | 0.2064 | 558 | 5.473 | 3E+05 | 0.149 | - | - | 0.422 |
| Beta 1 | -475.14 | 168,144 | 0.0054 | 0.2802 | 828 | 5E+05 | 0.869 | 1.745 | - | - | 0.421 |
| Beta 2 | -867.09 | 177,346 | 0.0105 | 0.3631 | 1839 | 2E+12 | 1.14 | 1E+07 | - | - | 0.475 |
| GG | -377.53 | 167,760 | 0.0040 | 0.2372 | 626 | 3.373 | 4E+05 | 0.242 | - | - | 0.418 |
| Singh-Maddala | -772.69 | 174,232 | 0.0079 | 0.2914 | 1957 | 1.185 | 2E+10 | 8E+05 | - | - | 0.443 |
| GB1 | -377.53 | 167,760 | 0.0040 | 0.2372 | 626 | 3.373 | 1E+07 | 0.242 | 96676 | - | 0.418 |
| GB2 | -325.44 | 172,857 | 0.0033 | 0.1864 | 525 | 14.62 | 3E+05 | 0.055 | 0.2343 | - | 0.433 |
| GB | -322.37 | 164,938 | 0.0032 | 0.1804 | 519 | 28.54 | 3E+05 | 1 | 0.0281 | 0.107 | - |

## France 1979

| sample mean | sample obs |
|---|---|
| 68,143 | 9162 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -466.71 | 84,486 | 0.0044 | 0.2422 | 826 | 52247 | 1.617 | - | - | - | 0.411 |
| lognormal | -414.26 | 89,542 | 0.0039 | 0.2106 | 714 | 11.03 | 0.863 | - | - | - | 0.458 |
| Weibull | -590.89 | 84,546 | 0.0058 | 0.2945 | 1082 | 1.271 | 91110 | - | - | - | 0.420 |
| Dagum | -185.54 | 89,230 | 0.0013 | 0.1212 | 219 | 2.376 | 79915 | 0.698 | - | - | 0.454 |
| Beta 1 | -466.75 | 84,489 | 0.0044 | 0.2422 | 826 | 2E+09 | 1.617 | 34740 | - | - | 0.411 |
| Beta 2 | -297.01 | 87,224 | 0.0027 | 0.1855 | 436 | 1E+05 | 2.378 | 4.658 | - | - | 0.440 |
| GG | -350.71 | 85,979 | 0.0034 | 0.2110 | 540 | 0.405 | 350.5 | 8.596 | - | - | 0.431 |
| Singh-Maddala | -215.94 | 90,260 | 0.0016 | 0.1337 | 285 | 1.903 | 76216 | 1.296 | - | - | 0.457 |
| GB1 | -350.78 | 85,956 | 0.0034 | 0.2111 | 540 | 0.407 | 7E+11 | 8.544 | 6004.7 | - | 0.431 |
| GB2 | -133.50 | 98,557 | 0.0007 | 0.0906 | 117 | 5.238 | 69820 | 0.278 | 0.3362 | - | 0.504 |
| GB | -133.50 | 98,557 | 0.0007 | 0.0906 | 117 | 5.238 | 69820 | 1 | 0.2779 | 0.336 | - |

## France 1984

| sample mean | sample obs |
|---|---|
| 113,568 | 7875 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -193.93 | 111,602 | 0.0017 | 0.1302 | 240 | 36459 | 3.061 | - | - | - | 0.310 |
| lognormal | -220.37 | 114,141 | 0.0022 | 0.1378 | 286 | 11.46 | 0.604 | - | - | - | 0.331 |
| Weibull | -328.44 | 111,146 | 0.0032 | 0.2076 | 532 | 1.846 | 1E+05 | - | - | - | 0.313 |
| Dagum | -179.07 | 115,599 | 0.0014 | 0.1344 | 211 | 3.223 | 1E+05 | 0.728 | - | - | 0.336 |
| Beta 1 | -193.97 | 111,687 | 0.0017 | 0.1301 | 239 | 1E+12 | 3.048 | 4E+07 | - | - | 0.310 |
| Beta 2 | -172.11 | 112,447 | 0.0015 | 0.1148 | 188 | 4E+05 | 3.851 | 13.61 | - | - | 0.317 |
| GG | -175.77 | 112,189 | 0.0016 | 0.1173 | 194 | 0.597 | 3178 | 8.077 | - | - | 0.315 |
| Singh-Maddala | -169.20 | 113,823 | 0.0013 | 0.1182 | 189 | 2.494 | 1E+05 | 1.619 | - | - | 0.326 |
| GB1 | -175.77 | 112,189 | 0.0016 | 0.1173 | 194 | 0.597 | 1E+13 | 8.077 | 471592 | - | 0.315 |
| GB2 | -166.80 | 113,241 | 0.0013 | 0.1112 | 181 | 1.937 | 1E+05 | 1.444 | 2.5972 | - | 0.322 |
| GB | -165.27 | 111,643 | 0.0013 | 0.1142 | 179 | 2.485 | 1E+05 | 0.961 | 1.0688 | 1.202 | - |

## France 1989

| sample mean | sample obs |
|---|---|
| 137,949 | 5942 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -141.78 | 136,015 | 0.0012 | 0.1203 | 144 | 47585 | 2.858 | - | - | - | 0.319 |
| lognormal | -159.06 | 139,450 | 0.0016 | 0.1225 | 177 | 11.65 | 0.627 | - | - | - | 0.343 |
| Weibull | -241.64 | 135,493 | 0.0029 | 0.1941 | 355 | 1.777 | 2E+05 | - | - | - | 0.323 |
| Dagum | -124.17 | 141,087 | 0.0009 | 0.1095 | 109 | 3.149 | 1E+05 | 0.706 | - | - | 0.347 |
| Beta 1 | -141.82 | 136,135 | 0.0012 | 0.1204 | 144 | 1E+12 | 2.845 | 2E+07 | - | - | 0.320 |
| Beta 2 | -122.03 | 137,212 | 0.0010 | 0.0985 | 100 | 4E+05 | 3.635 | 12.14 | - | - | 0.328 |
| GG | -125.36 | 136,845 | 0.0011 | 0.1008 | 106 | 0.575 | 3360 | 8.076 | - | - | 0.326 |
| Singh-Maddala | -117.92 | 138,940 | 0.0008 | 0.0974 | 95 | 2.397 | 2E+05 | 1.641 | - | - | 0.337 |
| GB1 | -125.36 | 136,840 | 0.0011 | 0.1008 | 106 | 0.575 | 3E+10 | 8.084 | 9972.8 | - | 0.326 |
| GB2 | -117.05 | 138,402 | 0.0008 | 0.0956 | 92 | 2.013 | 2E+05 | 1.287 | 2.2561 | - | 0.334 |
| GB | -117.05 | 138,402 | 0.0008 | 0.0957 | 92 | 2.013 | 2E+05 | 1 | 1.2873 | 2.256 | - |

## France 1994

| sample mean | sample obs |
|---|---|
| 159,343 | 7880 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -225.73 | 156,665 | 0.0022 | 0.1424 | 303 | 81474 | 1.923 | - | - | - | 0.382 |
| lognormal | -645.17 | 169,882 | 0.0074 | 0.2723 | 1364 | 11.7 | 0.825 | - | - | - | 0.441 |
| Weibull | -192.58 | 155,361 | 0.0017 | 0.1341 | 231 | 1.498 | 2E+05 | - | - | - | 0.370 |
| Dagum | -142.42 | 158,960 | 0.0010 | 0.0961 | 136 | 3.554 | 2E+05 | 0.377 | - | - | 0.384 |
| Beta 1 | -210.22 | 155,477 | 0.0020 | 0.1432 | 265 | 1E+06 | 1.733 | 12.26 | - | - | 0.373 |
| Beta 2 | -225.73 | 156,664 | 0.0022 | 0.1424 | 303 | 1E+11 | 1.923 | 1E+06 | - | - | 0.382 |
| GG | -192.40 | 155,416 | 0.0017 | 0.1329 | 230 | 1.461 | 2E+05 | 1.042 | - | - | 0.371 |
| Singh-Maddala | -183.64 | 156,183 | 0.0016 | 0.1205 | 217 | 1.592 | 6E+05 | 8.446 | - | - | 0.374 |
| GB1 | -192.40 | 155,416 | 0.0017 | 0.1329 | 230 | 1.461 | 6E+08 | 1.041 | 163562 | - | 0.371 |
| GB2 | -141.80 | 159,799 | 0.0009 | 0.0963 | 135 | 3.905 | 2E+05 | 0.34 | 0.847 | - | 0.387 |
| GB | -141.79 | 157,903 | 0.0009 | 0.0965 | 135 | 3.954 | 2E+05 | 0.997 | 0.3354 | 0.816 | - |

## Germany 1973

| sample mean | sample obs |
|---|---|
| 28,312 | 37614 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -7208.28 | 29,189 | 0.0201 | 0.4622 | 24536 | 11717 | 2.491 | - | - | - | 0.340 |
| lognormal | -13126.27 | 32,843 | 0.0385 | 0.6442 | 74310 | 10.09 | 0.789 | - | - | - | 0.423 |
| Weibull | -4951.58 | 28,525 | 0.0119 | 0.3320 | 17243 | 1.96 | 32173 | - | - | - | 0.298 |
| Dagum | -3502.02 | 28,100 | 0.0101 | 0.3025 | 8082 | 6.192 | 42782 | 0.241 | - | - | 0.300 |
| Beta 1 | -4910.65 | 28,227 | 0.0147 | 0.4098 | 11215 | 68614 | 1.668 | 2.386 | - | - | 0.305 |
| Beta 2 | -7208.33 | 29,188 | 0.0201 | 0.4622 | 24537 | 3E+09 | 2.491 | 3E+05 | - | - | 0.340 |
| GG | -4029.04 | 28,098 | 0.0119 | 0.3644 | 8859 | 3.53 | 46096 | 0.416 | - | - | 0.299 |
| Singh-Maddala | -4951.59 | 28,526 | 0.0119 | 0.3320 | 17244 | 1.96 | 2E+07 | 2E+05 | - | - | 0.298 |
| GB1 | -4029.05 | 28,099 | 0.0119 | 0.3643 | 8859 | 3.53 | 1E+06 | 0.416 | 141201 | - | 0.299 |
| GB2 | -3303.37 | 28,549 | 0.0095 | 0.2712 | 7595 | 13.84 | 36150 | 0.106 | 0.2848 | - | 0.311 |
| GB | -3291.03 | 27,729 | 0.0094 | 0.2668 | 7580 | 15.99 | 35384 | 1 | 0.0921 | 0.229 | - |

## Germany 1978

| sample mean | sample obs |
|---|---|
| 40,277 | 35731 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -6533.30 | 41,426 | 0.0198 | 0.4547 | 21103 | 17460 | 2.373 | - | - | - | 0.348 |
| lognormal | -12126.00 | 46,928 | 0.0382 | 0.6387 | 64703 | 10.43 | 0.811 | - | - | - | 0.434 |
| Weibull | -4555.74 | 40,492 | 0.0120 | 0.3298 | 15103 | 1.894 | 45627 | - | - | - | 0.306 |
| Dagum | -3213.12 | 39,907 | 0.0100 | 0.2993 | 7222 | 5.975 | 61260 | 0.241 | - | - | 0.309 |
| Beta 1 | -4516.83 | 40,069 | 0.0146 | 0.4029 | 10102 | 1E+05 | 1.618 | 2.447 | - | - | 0.313 |
| Beta 2 | -6543.12 | 41,153 | 0.0192 | 0.4421 | 23016 | 4E+11 | 2.448 | 3E+07 | - | - | 0.343 |
| GG | -3710.17 | 39,891 | 0.0118 | 0.3603 | 7986 | 3.39 | 66044 | 0.42 | - | - | 0.307 |
| Singh-Maddala | -4555.75 | 40,492 | 0.0120 | 0.3298 | 15102 | 1.894 | 3E+07 | 2E+05 | - | - | 0.306 |
| GB1 | -3710.17 | 39,891 | 0.0118 | 0.3603 | 7986 | 3.39 | 2E+06 | 0.42 | 104478 | - | 0.307 |
| GB2 | -3030.90 | 40,581 | 0.0094 | 0.2719 | 6782 | 13.11 | 51602 | 0.108 | 0.292 | - | 0.319 |
| GB | -3022.61 | 39,382 | 0.0093 | 0.2686 | 6774 | 14.81 | 50580 | 1 | 0.0961 | 0.242 | - |

## Germany 1981

| *sample mean* | *obs* |
|---|---|
| 47,117 | 2024 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -105.82 | 47,806 | 0.0028 | 0.1507 | 88 | 11408 | 4.191 | - | - | - | 0.268 |
| lognormal | -145.80 | 48,710 | 0.0045 | 0.1935 | 190 | 10.66 | 0.517 | - | - | - | 0.285 |
| Weibull | -120.42 | 47,477 | 0.0033 | 0.2036 | 120 | 2.233 | 53605 | - | - | - | 0.267 |
| Dagum | -93.07 | 48,373 | 0.0019 | 0.1336 | 63 | 4.344 | 54337 | 0.564 | - | - | 0.276 |
| Beta 1 | -105.26 | 47,731 | 0.0028 | 0.1553 | 86 | 7E+05 | 3.923 | 51.25 | - | - | 0.266 |
| Beta 2 | -105.82 | 47,805 | 0.0028 | 0.1507 | 88 | 3E+10 | 4.192 | 3E+06 | - | - | 0.267 |
| GG | -103.98 | 47,696 | 0.0027 | 0.1586 | 83 | 1.287 | 23069 | 2.651 | - | - | 0.265 |
| Singh-Maddala | -95.32 | 48,028 | 0.0021 | 0.1309 | 67 | 2.789 | 63988 | 2.267 | - | - | 0.269 |
| GB1 | -103.98 | 47,696 | 0.0027 | 0.1586 | 83 | 1.287 | 5E+08 | 2.651 | 390171 | - | 0.265 |
| GB2 | -93.00 | 48,285 | 0.0019 | 0.1313 | 63 | 4.094 | 55333 | 0.605 | 1.1111 | - | 0.274 |
| GB | -92.40 | 47,503 | 0.0019 | 0.1280 | 62 | 4.785 | 50212 | 0.983 | 0.5132 | 0.735 | - |

## Germany 1984

| *sample mean* | *obs* |
|---|---|
| 49,966 | 4086 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -170.13 | 50,071 | 0.0028 | 0.1615 | 208 | 14471 | 3.46 | - | - | - | 0.293 |
| lognormal | -299.30 | 51,516 | 0.0055 | 0.2335 | 554 | 10.68 | 0.58 | - | - | - | 0.318 |
| Weibull | -170.15 | 49,687 | 0.0027 | 0.1743 | 206 | 2.045 | 56084 | - | - | - | 0.288 |
| Dagum | -125.63 | 50,527 | 0.0015 | 0.1321 | 118 | 4.281 | 60510 | 0.482 | - | - | 0.297 |
| Beta 1 | -164.12 | 49,870 | 0.0027 | 0.1637 | 191 | 4E+05 | 3.049 | 19.52 | - | - | 0.288 |
| Beta 2 | -170.18 | 50,023 | 0.0028 | 0.1611 | 210 | 1E+11 | 3.483 | 8E+06 | - | - | 0.292 |
| GG | -156.85 | 49,835 | 0.0025 | 0.1598 | 177 | 1.496 | 37041 | 1.71 | - | - | 0.287 |
| Singh-Maddala | -139.74 | 50,148 | 0.0019 | 0.1411 | 146 | 2.411 | 81525 | 3.108 | - | - | 0.290 |
| GB1 | -156.85 | 49,836 | 0.0025 | 0.1598 | 177 | 1.495 | 9E+07 | 1.711 | 106852 | - | 0.287 |
| GB2 | -125.34 | 50,670 | 0.0015 | 0.1340 | 117 | 4.644 | 58978 | 0.439 | 0.8681 | - | 0.299 |
| GB | -124.78 | 49,740 | 0.0015 | 0.1333 | 116 | 5.122 | 55596 | 0.989 | 0.3971 | 0.675 | - |

## Germany 1989

| *sample mean* | *obs* |
|---|---|
| 59,886 | 2870 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -145.11 | 60,787 | 0.0031 | 0.1599 | 171 | 17758 | 3.423 | - | - | - | 0.294 |
| lognormal | -260.34 | 62,750 | 0.0065 | 0.2470 | 499 | 10.87 | 0.588 | - | - | - | 0.322 |
| Weibull | -125.56 | 60,267 | 0.0025 | 0.1660 | 120 | 2.063 | 68035 | - | - | - | 0.285 |
| Dagum | -105.87 | 61,143 | 0.0017 | 0.1264 | 83 | 4.557 | 76666 | 0.429 | - | - | 0.294 |
| Beta 1 | -131.71 | 60,359 | 0.0028 | 0.1614 | 134 | 3E+05 | 2.769 | 10.01 | - | - | 0.286 |
| Beta 2 | -145.11 | 60,803 | 0.0031 | 0.1602 | 171 | 6E+11 | 3.419 | 4E+07 | - | - | 0.294 |
| GG | -123.33 | 60,354 | 0.0024 | 0.1560 | 116 | 1.781 | 57718 | 1.278 | - | - | 0.285 |
| Singh-Maddala | -116.54 | 60,606 | 0.0021 | 0.1333 | 105 | 2.293 | 1E+05 | 4.845 | - | - | 0.287 |
| GB1 | -123.33 | 60,355 | 0.0024 | 0.1560 | 116 | 1.781 | 4E+07 | 1.278 | 123837 | - | 0.285 |
| GB2 | -105.86 | 61,188 | 0.0017 | 0.1272 | 83 | 4.665 | 76036 | 0.418 | 0.9585 | - | 0.295 |
| GB | -104.78 | 60,041 | 0.0016 | 0.1247 | 81 | 5.535 | 68766 | 0.984 | 0.3504 | 0.621 | - |

## Germany 1994

| sample mean | sample obs |
|---|---|
| 68,016 | 4613 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -206.95 | 67,066 | 0.0030 | 0.1966 | 297 | 27828 | 2.41 | - | - | - | 0.345 |
| lognormal | -493.35 | 71,476 | 0.0089 | 0.3333 | 1062 | 10.91 | 0.732 | - | - | - | 0.395 |
| Weibull | -146.76 | 66,354 | 0.0018 | 0.1506 | 158 | 1.735 | 74466 | - | - | - | 0.329 |
| Dagum | -104.35 | 67,045 | 0.0008 | 0.0963 | 73 | 4.413 | 94075 | 0.337 | - | - | 0.336 |
| Beta 1 | -167.61 | 66,241 | 0.0023 | 0.1741 | 199 | 3E+05 | 1.938 | 6.306 | - | - | 0.331 |
| Beta 2 | -207.08 | 66,919 | 0.0030 | 0.1935 | 301 | 3E+11 | 2.435 | 1E+07 | - | - | 0.344 |
| GG | -144.21 | 66,259 | 0.0018 | 0.1548 | 151 | 1.946 | 83904 | 0.833 | - | - | 0.329 |
| Singh-Maddala | -146.11 | 66,443 | 0.0017 | 0.1455 | 159 | 1.771 | 5E+05 | 25.12 | - | - | 0.330 |
| GB1 | -144.22 | 66,259 | 0.0018 | 0.1548 | 151 | 1.946 | 2E+07 | 0.833 | 47403 | - | 0.329 |
| GB2 | -99.84 | 67,886 | 0.0007 | 0.0926 | 64 | 6.292 | 84663 | 0.23 | 0.5604 | - | 0.343 |
| GB | -99.40 | 66,065 | 0.0007 | 0.0913 | 63 | 6.92 | 81713 | 0.996 | 0.2083 | 0.465 | - |

## Hungary 1991

| sample mean | sample obs |
|---|---|
| 264,210 | 1474 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -101.15 | 257,614 | 0.0028 | 0.1863 | 97 | 2E+05 | 1.489 | - | - | - | 0.426 |
| lognormal | -230.02 | 300,126 | 0.0106 | 0.3660 | 481 | 12.12 | 0.993 | - | - | - | 0.518 |
| Weibull | -89.16 | 254,929 | 0.0021 | 0.1577 | 70 | 1.317 | 3E+05 | - | - | - | 0.409 |
| Dagum | -72.32 | 258,117 | 0.0011 | 0.1067 | 31 | 3.671 | 4E+05 | 0.291 | - | - | 0.415 |
| Beta 1 | -91.63 | 253,375 | 0.0025 | 0.1755 | 71 | 1E+06 | 1.286 | 5.835 | - | - | 0.408 |
| Beta 2 | -101.15 | 257,614 | 0.0028 | 0.1863 | 97 | 1E+12 | 1.489 | 7E+06 | - | - | 0.426 |
| GG | -84.19 | 253,470 | 0.0020 | 0.1587 | 54 | 1.735 | 4E+05 | 0.652 | - | - | 0.405 |
| Singh-Maddala | -89.16 | 254,929 | 0.0021 | 0.1577 | 70 | 1.317 | 3E+09 | 2E+05 | - | - | 0.409 |
| GB1 | -84.19 | 253,472 | 0.0020 | 0.1587 | 54 | 1.735 | 4E+08 | 0.652 | 189315 | - | 0.405 |
| GB2 | -70.13 | 265,300 | 0.0010 | 0.0896 | 26 | 6.225 | 3E+05 | 0.166 | 0.4398 | - | 0.428 |
| GB | -69.86 | 250,674 | 0.0010 | 0.0862 | 26 | 7.421 | 3E+05 | 0.998 | 0.1385 | 0.334 | - |

## Ireland 1987

| sample mean | sample obs |
|---|---|
| 13,966 | 2482 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -116.40 | 14,412 | 0.0022 | 0.1644 | 113 | 9450 | 1.525 | - | - | - | 0.421 |
| lognormal | -309.38 | 16,536 | 0.0103 | 0.3597 | 591 | 9.243 | 0.97 | - | - | - | 0.507 |
| Weibull | -96.50 | 14,269 | 0.0014 | 0.1241 | 73 | 1.334 | 15528 | - | - | - | 0.405 |
| Dagum | -84.06 | 14,558 | 0.0009 | 0.1134 | 45 | 3.713 | 22790 | 0.287 | - | - | 0.413 |
| Beta 1 | -94.83 | 14,151 | 0.0013 | 0.1356 | 67 | 65252 | 1.268 | 4.578 | - | - | 0.400 |
| Beta 2 | -116.41 | 14,392 | 0.0022 | 0.1637 | 114 | 8E+10 | 1.531 | 9E+06 | - | - | 0.421 |
| GG | -87.64 | 14,194 | 0.0010 | 0.1184 | 53 | 1.816 | 22033 | 0.624 | - | - | 0.400 |
| Singh-Maddala | -96.50 | 14,270 | 0.0014 | 0.1241 | 73 | 1.334 | 3E+08 | 5E+05 | - | - | 0.405 |
| GB1 | -87.64 | 14,194 | 0.0010 | 0.1184 | 53 | 1.816 | 1E+07 | 0.624 | 102250 | - | 0.400 |
| GB2 | -83.34 | 14,401 | 0.0009 | 0.1082 | 44 | 3.058 | 26538 | 0.354 | 1.548 | - | 0.408 |
| GB | -80.57 | 13,973 | 0.0008 | 0.1008 | 38 | 4.425 | 17936 | 0.969 | 0.2438 | 0.526 | - |

## Israel 1979

| sample mean | sample obs |
|---|---|
| 31,013 | 1941 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -78.36 | 32,455 | 0.0010 | 0.1068 | 38 | 14447 | 2.246 | - | - | - | 0.356 |
| lognormal | -159.86 | 34,381 | 0.0049 | 0.2465 | 229 | 10.17 | 0.746 | - | - | - | 0.402 |
| Weibull | -69.97 | 32,201 | 0.0006 | 0.0846 | 21 | 1.636 | 35986 | - | - | - | 0.345 |
| Dagum | -70.62 | 32,993 | 0.0006 | 0.0913 | 22 | 3.873 | 44743 | 0.373 | - | - | 0.359 |
| Beta 1 | -71.78 | 32,173 | 0.0007 | 0.0914 | 24 | 2E+05 | 1.916 | 8.951 | - | - | 0.346 |
| Beta 2 | -78.37 | 32,490 | 0.0010 | 0.1075 | 38 | 1E+11 | 2.237 | 7E+06 | - | - | 0.357 |
| GG | -69.79 | 32,220 | 0.0006 | 0.0837 | 20 | 1.549 | 33526 | 1.094 | - | - | 0.346 |
| Singh-Maddala | -69.26 | 32,283 | 0.0005 | 0.0790 | 19 | 1.696 | 2E+05 | 13.98 | - | - | 0.347 |
| GB1 | -69.79 | 32,220 | 0.0006 | 0.0837 | 20 | 1.549 | 5E+07 | 1.094 | 93294 | - | 0.346 |
| GB2 | -68.22 | 32,448 | 0.0005 | 0.0721 | 17 | 2.471 | 63829 | 0.628 | 2.7932 | - | 0.350 |
| GB | -68.06 | 32,147 | 0.0005 | 0.0711 | 17 | 2.894 | 45039 | 0.919 | 0.5298 | 1.242 | - |

## Israel 1986

| sample mean | sample obs |
|---|---|
| 22,902 | 3942 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -76.01 | 23,179 | 0.0002 | 0.0590 | 20 | 11961 | 1.938 | - | - | - | 0.380 |
| lognormal | -139.84 | 24,486 | 0.0018 | 0.1507 | 154 | 9.792 | 0.793 | - | - | - | 0.425 |
| Weibull | -101.32 | 23,087 | 0.0009 | 0.1121 | 71 | 1.46 | 25486 | - | - | - | 0.378 |
| Dagum | -83.00 | 24,291 | 0.0004 | 0.0757 | 34 | 2.884 | 27121 | 0.53 | - | - | 0.409 |
| Beta 1 | -76.01 | 23,170 | 0.0002 | 0.0589 | 20 | 1E+11 | 1.94 | 1E+07 | - | - | 0.380 |
| Beta 2 | -73.24 | 23,309 | 0.0002 | 0.0472 | 14 | 2E+05 | 2.105 | 20.11 | - | - | 0.386 |
| GG | -73.72 | 23,260 | 0.0002 | 0.0488 | 15 | 0.828 | 6713 | 2.701 | - | - | 0.385 |
| Singh-Maddala | -73.62 | 23,518 | 0.0002 | 0.0474 | 15 | 1.725 | 41376 | 2.951 | - | - | 0.391 |
| GB1 | -73.72 | 23,260 | 0.0002 | 0.0488 | 15 | 0.828 | 8E+11 | 2.702 | 5E+06 | - | 0.385 |
| GB2 | -72.86 | 23,389 | 0.0002 | 0.0450 | 13 | 1.314 | 66987 | 1.449 | 6.2257 | - | 0.388 |
| GB | -72.83 | 23,290 | 0.0002 | 0.0455 | 13 | 1.485 | 37307 | 0.896 | 1.2527 | 2.772 | - |

## Israel 1992

| sample mean | sample obs |
|---|---|
| 66,271 | 4057 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -82.34 | 66,672 | 0.0004 | 0.0683 | 32 | 36345 | 1.834 | - | - | - | 0.389 |
| lognormal | -145.48 | 70,765 | 0.0019 | 0.1523 | 169 | 10.83 | 0.818 | - | - | - | 0.437 |
| Weibull | -110.46 | 66,437 | 0.0011 | 0.1250 | 89 | 1.411 | 72979 | - | - | - | 0.388 |
| Dagum | -77.30 | 70,110 | 0.0003 | 0.0604 | 22 | 2.761 | 76235 | 0.547 | - | - | 0.421 |
| Beta 1 | -82.34 | 66,670 | 0.0004 | 0.0683 | 32 | 4E+11 | 1.835 | 1E+07 | - | - | 0.389 |
| Beta 2 | -76.11 | 67,254 | 0.0003 | 0.0541 | 19 | 4E+05 | 2.063 | 13.67 | - | - | 0.398 |
| GG | -77.91 | 67,001 | 0.0003 | 0.0587 | 22 | 0.778 | 16374 | 2.857 | - | - | 0.396 |
| Singh-Maddala | -73.06 | 68,012 | 0.0002 | 0.0446 | 13 | 1.708 | 1E+05 | 2.593 | - | - | 0.405 |
| GB1 | -77.91 | 67,002 | 0.0003 | 0.0587 | 22 | 0.778 | 4E+12 | 2.857 | 3E+06 | - | 0.396 |
| GB2 | -73.02 | 68,115 | 0.0002 | 0.0451 | 13 | 1.79 | 1E+05 | 0.939 | 2.3396 | - | 0.405 |
| GB | -72.55 | 66,680 | 0.0002 | 0.0449 | 12 | 2.223 | 72600 | 0.955 | 0.731 | 1.139 | - |

## Israel 1997

| *sample mean* | *obs* |
|---|---|
| 116,473 | 4072 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -82.40 | 114,834 | 0.0004 | 0.0729 | 31 | 73068 | 1.572 | - | - | - | 0.416 |
| lognormal | -208.37 | 125,636 | 0.0033 | 0.2123 | 313 | 11.33 | 0.91 | - | - | - | 0.480 |
| Weibull | -91.86 | 114,320 | 0.0006 | 0.0931 | 51 | 1.306 | 1E+05 | - | - | - | 0.412 |
| Dagum | -75.98 | 119,947 | 0.0002 | 0.0572 | 19 | 2.829 | 1E+05 | 0.443 | - | - | 0.440 |
| Beta 1 | -82.40 | 114,819 | 0.0004 | 0.0729 | 31 | 3E+11 | 1.572 | 5E+06 | - | - | 0.416 |
| Beta 2 | -81.75 | 115,179 | 0.0004 | 0.0694 | 30 | 3E+06 | 1.622 | 38.88 | - | - | 0.419 |
| GG | -82.36 | 114,890 | 0.0004 | 0.0723 | 31 | 0.98 | 69585 | 1.625 | - | - | 0.417 |
| Singh-Maddala | -77.09 | 116,022 | 0.0003 | 0.0579 | 21 | 1.464 | 3E+05 | 4.389 | - | - | 0.422 |
| GB1 | -82.36 | 114,892 | 0.0004 | 0.0723 | 31 | 0.98 | 6E+11 | 1.625 | 6E+06 | - | 0.417 |
| GB2 | -74.37 | 117,662 | 0.0002 | 0.0527 | 16 | 2.185 | 2E+05 | 0.602 | 1.6439 | - | 0.430 |
| GB | -73.97 | 114,444 | 0.0002 | 0.0508 | 15 | 2.616 | 1E+05 | 0.965 | 0.4931 | 0.94 | - |

## Italy 1986

| *sample mean* | *obs* |
|---|---|
| 21,086 | 6016 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -244.65 | 21,119 | 0.0031 | 0.1987 | 349 | 6292 | 3.357 | - | - | - | 0.297 |
| lognormal | -305.49 | 21,585 | 0.0039 | 0.2251 | 492 | 9.813 | 0.578 | - | - | - | 0.317 |
| Weibull | -325.35 | 21,011 | 0.0041 | 0.2556 | 535 | 1.958 | 23698 | - | - | - | 0.298 |
| Dagum | -240.27 | 21,704 | 0.0031 | 0.2004 | 335 | 3.507 | 22273 | 0.673 | - | - | 0.317 |
| Beta 1 | -244.65 | 21,119 | 0.0031 | 0.1987 | 349 | 1E+10 | 3.357 | 2E+06 | - | - | 0.297 |
| Beta 2 | -241.02 | 21,184 | 0.0030 | 0.1928 | 341 | 2E+05 | 3.723 | 30.55 | - | - | 0.300 |
| GG | -242.69 | 21,155 | 0.0031 | 0.1954 | 345 | 0.842 | 3359 | 4.617 | - | - | 0.299 |
| Singh-Maddala | -233.17 | 21,391 | 0.0029 | 0.1866 | 322 | 2.546 | 25821 | 1.861 | - | - | 0.307 |
| GB1 | -243.59 | 21,128 | 0.0031 | 0.1972 | 347 | 0.948 | 2E+11 | 3.704 | 1E+07 | - | 0.299 |
| GB2 | -232.91 | 21,357 | 0.0029 | 0.1854 | 322 | 2.334 | 26989 | 1.128 | 2.2071 | - | 0.306 |
| GB | -227.51 | 20,945 | 0.0028 | 0.1857 | 313 | 3.393 | 19554 | 0.97 | 0.7382 | 0.797 | - |

## Italy 1995

| *sample mean* | *obs* |
|---|---|
| 33,909 | 5479 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -239.59 | 33,268 | 0.0030 | 0.1905 | 364 | 12533 | 2.654 | - | - | - | 0.330 |
| lognormal | -398.56 | 34,613 | 0.0062 | 0.2636 | 713 | 10.23 | 0.67 | - | - | - | 0.365 |
| Weibull | -254.03 | 33,051 | 0.0029 | 0.1971 | 418 | 1.762 | 37124 | - | - | - | 0.325 |
| Dagum | -215.65 | 33,960 | 0.0025 | 0.1785 | 317 | 3.598 | 40039 | 0.499 | - | - | 0.342 |
| Beta 1 | -237.85 | 33,187 | 0.0029 | 0.1926 | 364 | 5E+05 | 2.51 | 36.35 | - | - | 0.328 |
| Beta 2 | -239.59 | 33,265 | 0.0030 | 0.1905 | 364 | 1E+11 | 2.656 | 9E+06 | - | - | 0.330 |
| GG | -234.41 | 33,165 | 0.0028 | 0.1907 | 359 | 1.244 | 21372 | 1.817 | - | - | 0.326 |
| Singh-Maddala | -220.76 | 33,413 | 0.0026 | 0.1753 | 327 | 2.053 | 59523 | 3.283 | - | - | 0.331 |
| GB1 | -234.41 | 33,166 | 0.0028 | 0.1907 | 359 | 1.244 | 4E+08 | 1.817 | 192516 | - | 0.326 |
| GB2 | -214.55 | 33,717 | 0.0025 | 0.1732 | 315 | 3.11 | 43060 | 0.591 | 1.3232 | - | 0.338 |
| GB | -209.14 | 32,791 | 0.0024 | 0.1693 | 300 | 4.173 | 34151 | 0.979 | 0.4351 | 0.618 | - |

## Mexico 1984

| *sample mean* | *obs* |
|---|---|
| 507 | 4440 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -122.13 | 484 | 0.0011 | 0.1244 | 117 | 325.4 | 1.488 | - | - | - | 0.426 |
| lognormal | -217.06 | 530 | 0.0032 | 0.1965 | 329 | 5.839 | 0.932 | - | - | - | 0.490 |
| Weibull | -143.47 | 483 | 0.0016 | 0.1491 | 163 | 1.25 | 518.6 | - | - | - | 0.426 |
| Dagum | -90.73 | 513 | 0.0005 | 0.0770 | 50 | 2.536 | 562.6 | 0.516 | - | - | 0.462 |
| Beta 1 | -122.14 | 484 | 0.0011 | 0.1244 | 117 | 1E+07 | 1.488 | 34029 | - | - | 0.426 |
| Beta 2 | -109.58 | 491 | 0.0009 | 0.1081 | 89 | 2461 | 1.703 | 9.531 | - | - | 0.439 |
| GG | -115.46 | 487 | 0.0010 | 0.1178 | 102 | 0.773 | 153 | 2.314 | - | - | 0.434 |
| Singh-Maddala | -99.55 | 498 | 0.0007 | 0.0918 | 68 | 1.516 | 818.4 | 2.633 | - | - | 0.446 |
| GB1 | -115.46 | 487 | 0.0010 | 0.1178 | 102 | 0.773 | 8E+07 | 2.315 | 26247 | - | 0.434 |
| GB2 | -90.55 | 517 | 0.0005 | 0.0780 | 49 | 2.73 | 543.5 | 0.474 | 0.8834 | - | 0.466 |
| GB | -88.57 | 480 | 0.0004 | 0.0735 | 45 | 3.428 | 467.1 | 0.991 | 0.371 | 0.553 | - |

## Mexico 1989

| *sample mean* | *obs* |
|---|---|
| 10,032 | 10812 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -271.44 | 9,224 | 0.0017 | 0.1572 | 414 | 6741 | 1.368 | - | - | - | 0.441 |
| lognormal | -189.15 | 9,955 | 0.0010 | 0.1201 | 234 | 8.751 | 0.953 | - | - | - | 0.500 |
| Weibull | -352.67 | 9,230 | 0.0026 | 0.1898 | 569 | 1.167 | 9740 | - | - | - | 0.448 |
| Dagum | -108.88 | 10,317 | 0.0003 | 0.0601 | 66 | 2.085 | 8492 | 0.702 | - | - | 0.512 |
| Beta 1 | -271.47 | 9,234 | 0.0017 | 0.1579 | 413 | 9E+10 | 1.364 | 1E+07 | - | - | 0.442 |
| Beta 2 | -112.51 | 9,644 | 0.0003 | 0.0720 | 73 | 14841 | 1.955 | 4.009 | - | - | 0.479 |
| GG | -129.83 | 9,462 | 0.0005 | 0.0846 | 107 | 0.368 | 21.92 | 8.515 | - | - | 0.468 |
| Singh-Maddala | -104.60 | 9,932 | 0.0003 | 0.0586 | 58 | 1.595 | 9633 | 1.586 | - | - | 0.494 |
| GB1 | -129.85 | 9,461 | 0.0005 | 0.0846 | 108 | 0.368 | 2E+11 | 8.528 | 4955.3 | - | 0.468 |
| GB2 | -104.59 | 9,944 | 0.0003 | 0.0583 | 58 | 1.616 | 9573 | 0.981 | 1.5493 | - | 0.494 |
| GB | -104.59 | 9,944 | 0.0003 | 0.0583 | 58 | 1.616 | 9573 | 1 | 0.9809 | 1.549 | - |

## Mexico 1992

| *sample mean* | *obs* |
|---|---|
| 20,135 | 9881 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -322.84 | 15,824 | 0.0024 | 0.1778 | 532 | 12578 | 1.258 | - | - | - | 0.456 |
| lognormal | -245.56 | 17,303 | 0.0017 | 0.1459 | 358 | 9.255 | 1.003 | - | - | - | 0.522 |
| Weibull | -379.76 | 15,847 | 0.0030 | 0.2024 | 638 | 1.108 | 16465 | - | - | - | 0.465 |
| Dagum | -107.74 | 17,715 | 0.0003 | 0.0636 | 67 | 2.027 | 14529 | 0.682 | - | - | 0.529 |
| Beta 1 | -322.84 | 15,828 | 0.0024 | 0.1780 | 531 | 2E+11 | 1.257 | 1E+07 | - | - | 0.457 |
| Beta 2 | -142.88 | 16,664 | 0.0007 | 0.0974 | 138 | 24287 | 1.811 | 3.639 | - | - | 0.499 |
| GG | -176.47 | 16,257 | 0.0010 | 0.1212 | 205 | 0.38 | 67.05 | 7.279 | - | - | 0.485 |
| Singh-Maddala | -118.76 | 17,275 | 0.0004 | 0.0744 | 89 | 1.543 | 15640 | 1.524 | - | - | 0.516 |
| GB1 | -176.49 | 16,256 | 0.0010 | 0.1212 | 205 | 0.38 | 3E+12 | 7.281 | 11210 | - | 0.485 |
| GB2 | -105.26 | 18,395 | 0.0003 | 0.0619 | 62 | 2.438 | 13645 | 0.543 | 0.75 | - | 0.546 |
| GB | -103.20 | 16,005 | 0.0003 | 0.0581 | 57 | 2.831 | 12518 | 0.995 | 0.4602 | 0.566 | - |

## Mexico 1994

| *sample mean* | *obs* |
|---|---|
| 24,260 | 11931 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -357.10 | 18,518 | 0.0022 | 0.1750 | 597 | 16341 | 1.133 | - | - | - | 0.476 |
| lognormal | -358.83 | 20,856 | 0.0022 | 0.1620 | 605 | 9.362 | 1.08 | - | - | - | 0.555 |
| Weibull | -388.21 | 18,548 | 0.0025 | 0.1941 | 651 | 1.047 | 18889 | - | - | - | 0.484 |
| Dagum | -115.67 | 20,795 | 0.0003 | 0.0645 | 79 | 1.998 | 18146 | 0.61 | - | - | 0.548 |
| Beta 1 | -357.10 | 18,516 | 0.0022 | 0.1749 | 597 | 1E+11 | 1.134 | 9E+06 | - | - | 0.476 |
| Beta 2 | -168.07 | 19,519 | 0.0008 | 0.1066 | 185 | 34036 | 1.536 | 3.679 | - | - | 0.518 |
| GG | -213.59 | 18,989 | 0.0011 | 0.1324 | 277 | 0.467 | 634.8 | 4.342 | - | - | 0.503 |
| Singh-Maddala | -140.73 | 20,110 | 0.0005 | 0.0859 | 131 | 1.393 | 21134 | 1.782 | - | - | 0.532 |
| GB1 | -213.60 | 18,988 | 0.0011 | 0.1324 | 277 | 0.467 | 1E+12 | 4.341 | 22890 | - | 0.498 |
| GB2 | -108.30 | 22,262 | 0.0003 | 0.0594 | 64 | 2.68 | 16251 | 0.43 | 0.6368 | - | 0.577 |
| GB | -104.67 | 18,563 | 0.0002 | 0.0534 | 56 | 3.15 | 14939 | 0.997 | 0.3614 | 0.479 | - |

## Mexico 1996

| *sample mean* | *obs* |
|---|---|
| 31,341 | 13100 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -446.44 | 25,710 | 0.0026 | 0.1918 | 791 | 20260 | 1.269 | - | - | - | 0.455 |
| lognormal | -287.09 | 27,943 | 0.0015 | 0.1443 | 440 | 9.744 | 0.994 | - | - | - | 0.518 |
| Weibull | -529.21 | 25,754 | 0.0034 | 0.2211 | 943 | 1.111 | 26780 | - | - | - | 0.464 |
| Dagum | -137.16 | 28,929 | 0.0004 | 0.0735 | 122 | 2 | 22760 | 0.715 | - | - | 0.530 |
| Beta 1 | -446.54 | 25,659 | 0.0026 | 0.1905 | 794 | 3E+11 | 1.275 | 1E+07 | - | - | 0.454 |
| Beta 2 | -178.30 | 27,189 | 0.0007 | 0.1087 | 207 | 35170 | 1.893 | 3.449 | - | - | 0.500 |
| GG | -220.51 | 26,491 | 0.0011 | 0.1331 | 293 | 0.328 | 18.69 | 9.817 | - | - | 0.486 |
| Singh-Maddala | -148.23 | 28,296 | 0.0005 | 0.0851 | 145 | 1.579 | 23878 | 1.428 | - | - | 0.519 |
| GB1 | -220.57 | 26,488 | 0.0011 | 0.1331 | 294 | 0.327 | 3E+12 | 9.839 | 4850.3 | - | 0.486 |
| GB2 | -133.91 | 30,103 | 0.0004 | 0.0712 | 115 | 2.436 | 21471 | 0.559 | 0.7394 | - | 0.548 |
| GB | -128.21 | 25,713 | 0.0004 | 0.0639 | 103 | 3.049 | 19421 | 0.996 | 0.4331 | 0.501 | - |

## Netherlands 1983

| *sample mean* | *obs* |
|---|---|
| 54,318 | 3090 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -155.70 | 51,251 | 0.0036 | 0.1814 | 177 | 11729 | 4.37 | - | - | - | 0.262 |
| lognormal | -196.33 | 52,078 | 0.0049 | 0.1835 | 284 | 10.73 | 0.502 | - | - | - | 0.277 |
| Weibull | -201.33 | 50,922 | 0.0047 | 0.2487 | 284 | 2.247 | 57492 | - | - | - | 0.265 |
| Dagum | -129.22 | 51,957 | 0.0025 | 0.1542 | 126 | 4.109 | 54395 | 0.671 | - | - | 0.273 |
| Beta 1 | -155.71 | 51,267 | 0.0036 | 0.1814 | 177 | 6E+10 | 4.358 | 5E+06 | - | - | 0.263 |
| Beta 2 | -155.17 | 51,312 | 0.0036 | 0.1745 | 176 | 8E+05 | 4.624 | 72.52 | - | - | 0.263 |
| GG | -155.69 | 51,245 | 0.0036 | 0.1821 | 177 | 1.013 | 12276 | 4.262 | - | - | 0.262 |
| Singh-Maddala | -133.86 | 51,784 | 0.0026 | 0.1484 | 136 | 3.065 | 57678 | 1.615 | - | - | 0.269 |
| GB1 | -155.70 | 51,251 | 0.0036 | 0.1814 | 177 | 1 | 6E+10 | 4.371 | 5E+06 | - | 0.262 |
| GB2 | -128.59 | 52,222 | 0.0024 | 0.1603 | 126 | 4.729 | 52568 | 0.568 | 0.7903 | - | 0.277 |
| GB | -123.89 | 50,682 | 0.0022 | 0.1537 | 116 | 6.097 | 48013 | 0.994 | 0.437 | 0.476 | - |

## Netherlands 1987

| sample mean | obs |
|---|---|
| 57,209 | 2792 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -134.19 | 54,959 | 0.0029 | 0.1864 | 138 | 13448 | 4.087 | - | - | - | 0.271 |
| lognormal | -159.01 | 55,863 | 0.0038 | 0.1836 | 199 | 10.8 | 0.518 | - | - | - | 0.286 |
| Weibull | -183.71 | 54,643 | 0.0043 | 0.2542 | 253 | 2.156 | 61701 | - | - | - | 0.275 |
| Dagum | -113.59 | 55,955 | 0.0020 | 0.1592 | 98 | 3.848 | 56776 | 0.713 | - | - | 0.285 |
| Beta 1 | -134.19 | 54,970 | 0.0029 | 0.1862 | 138 | 5E+11 | 4.08 | 3E+07 | - | - | 0.271 |
| Beta 2 | -131.58 | 55,127 | 0.0029 | 0.1756 | 132 | 3E+05 | 4.683 | 29.53 | - | - | 0.274 |
| GG | -133.24 | 55,041 | 0.0029 | 0.1800 | 136 | 0.832 | 6555 | 5.775 | - | - | 0.272 |
| Singh-Maddala | -115.89 | 55,724 | 0.0021 | 0.1561 | 103 | 3.006 | 59474 | 1.493 | - | - | 0.281 |
| GB1 | -133.24 | 55,040 | 0.0029 | 0.1801 | 136 | 0.833 | 6E+09 | 5.761 | 96292 | - | 0.272 |
| GB2 | -113.43 | 56,124 | 0.0020 | 0.1610 | 98 | 4.156 | 55715 | 0.65 | 0.8789 | - | 0.288 |
| GB | -109.20 | 54,392 | 0.0018 | 0.1558 | 90 | 5.475 | 50197 | 0.993 | 0.4859 | 0.504 | - |

## Netherlands 1991

| sample mean | obs |
|---|---|
| 62,634 | 2971 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -500.76 | 62,002 | 0.0163 | 0.4123 | 1211 | 28260 | 2.194 | - | - | - | 0.360 |
| lognormal | -936.73 | 70,584 | 0.0343 | 0.6074 | 3512 | 10.81 | 0.839 | - | - | - | 0.447 |
| Weibull | -356.09 | 60,668 | 0.0093 | 0.2800 | 881 | 1.801 | 68223 | - | - | - | 0.319 |
| Dagum | -248.86 | 59,947 | 0.0069 | 0.2456 | 401 | 6.175 | 96338 | 0.214 | - | - | 0.321 |
| Beta 1 | -323.12 | 59,885 | 0.0099 | 0.3315 | 563 | 1E+05 | 1.472 | 2.128 | - | - | 0.321 |
| Beta 2 | -500.84 | 61,865 | 0.0162 | 0.4090 | 1235 | 5E+11 | 2.215 | 2E+07 | - | - | 0.358 |
| GG | -271.33 | 59,722 | 0.0078 | 0.2870 | 443 | 3.618 | 1E+05 | 0.364 | - | - | 0.317 |
| Singh-Maddala | -356.10 | 60,668 | 0.0093 | 0.2800 | 881 | 1.801 | 8E+07 | 4E+05 | - | - | 0.319 |
| GB1 | -271.33 | 59,722 | 0.0078 | 0.2870 | 443 | 3.618 | 3E+06 | 0.364 | 126844 | - | 0.317 |
| GB2 | -245.09 | 60,491 | 0.0068 | 0.2460 | 392 | 9.022 | 85755 | 0.145 | 0.5037 | - | 0.326 |
| GB | -243.69 | 59,106 | 0.0067 | 0.2410 | 390 | 10.88 | 80757 | 0.995 | 0.1208 | 0.346 | - |

## Netherlands 1994

| sample mean | obs |
|---|---|
| 67,120 | 3565 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -522.19 | 69,605 | 0.0137 | 0.4083 | 1163 | 35221 | 1.976 | - | - | - | 0.377 |
| lognormal | -1043.73 | 80,697 | 0.0313 | 0.6138 | 3407 | 10.9 | 0.891 | - | - | - | 0.471 |
| Weibull | -369.98 | 68,112 | 0.0077 | 0.2793 | 872 | 1.689 | 76304 | - | - | - | 0.337 |
| Dagum | -230.55 | 67,231 | 0.0049 | 0.2296 | 350 | 6.263 | 1E+05 | 0.194 | - | - | 0.335 |
| Beta 1 | -304.78 | 67,068 | 0.0072 | 0.3102 | 506 | 2E+05 | 1.337 | 1.961 | - | - | 0.335 |
| Beta 2 | -522.19 | 69,605 | 0.0137 | 0.4083 | 1163 | 6E+10 | 1.976 | 2E+06 | - | - | 0.377 |
| GG | -250.76 | 66,928 | 0.0055 | 0.2684 | 388 | 3.733 | 1E+05 | 0.325 | - | - | 0.331 |
| Singh-Maddala | -369.98 | 68,113 | 0.0077 | 0.2793 | 872 | 1.689 | 2E+08 | 5E+05 | - | - | 0.337 |
| GB1 | -250.76 | 66,928 | 0.0055 | 0.2684 | 388 | 3.733 | 3E+06 | 0.325 | 115873 | - | 0.155 |
| GB2 | -227.29 | 67,806 | 0.0048 | 0.2247 | 343 | 9.014 | 1E+05 | 0.134 | 0.5133 | - | 0.340 |
| GB | -223.88 | 66,106 | 0.0047 | 0.2124 | 336 | 12.88 | 92167 | 0.997 | 0.0936 | 0.275 | - |

## Norway 1979

| sample | |
|---|---|
| *mean* | *obs* |
| 94,793 | 8272 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -944.48 | 130,072 | 0.0121 | 0.4228 | 1762 | 1E+05 | 1.201 | - | - | - | 0.465 |
| lognormal | -1610.98 | 160,461 | 0.0190 | 0.5135 | 4166 | 11.35 | 1.131 | - | - | - | 0.576 |
| Weibull | -945.79 | 129,581 | 0.0121 | 0.4238 | 1762 | 1.118 | 1E+05 | - | - | - | 0.462 |
| Dagum | -625.37 | 128,842 | 0.0072 | 0.3232 | 1133 | 2.761 | 2E+05 | 0.39 | - | - | 0.467 |
| Beta 1 | -944.48 | 130,078 | 0.0121 | 0.4228 | 1762 | 4E+11 | 1.201 | 4E+06 | - | - | 0.465 |
| Beta 2 | -925.05 | 131,661 | 0.0118 | 0.4142 | 1740 | 1E+06 | 1.296 | 12.12 | - | - | 0.476 |
| GG | -943.57 | 129,839 | 0.0121 | 0.4226 | 1760 | 1.046 | 1E+05 | 1.118 | - | - | 0.463 |
| Singh-Maddala | -859.19 | 132,684 | 0.0103 | 0.3815 | 1691 | 1.327 | 3E+05 | 3.391 | - | - | 0.474 |
| GB1 | -943.57 | 129,837 | 0.0121 | 0.4225 | 1760 | 1.046 | 2E+11 | 1.118 | 3E+06 | - | 0.463 |
| GB2 | -385.43 | 147,016 | 0.0040 | 0.2162 | 636 | 42.08 | 1E+05 | 0.024 | 0.0415 | - | 0.528 |
| GB | -385.43 | 147,016 | 0.0040 | 0.2162 | 636 | 42.08 | 1E+05 | 1 | 0.0238 | 0.041 | - |

## Norway 1986

| sample | |
|---|---|
| *mean* | *obs* |
| 204,406 | 4250 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -540.45 | 235,147 | 0.0113 | 0.3403 | 1233 | 1E+05 | 1.97 | - | - | - | 0.378 |
| lognormal | -1136.00 | 271,334 | 0.0273 | 0.5384 | 3843 | 12.12 | 0.887 | - | - | - | 0.469 |
| Weibull | -372.86 | 230,343 | 0.0060 | 0.2292 | 889 | 1.672 | 3E+05 | - | - | - | 0.339 |
| Dagum | -219.58 | 227,747 | 0.0038 | 0.1908 | 320 | 6.184 | 4E+05 | 0.195 | - | - | 0.337 |
| Beta 1 | -306.20 | 226,973 | 0.0061 | 0.2746 | 503 | 6E+05 | 1.346 | 2.062 | - | - | 0.337 |
| Beta 2 | -540.54 | 235,143 | 0.0113 | 0.3403 | 1234 | 2E+09 | 1.97 | 13434 | - | - | 0.378 |
| GG | -245.28 | 226,677 | 0.0045 | 0.2315 | 369 | 3.598 | 4E+05 | 0.336 | - | - | 0.334 |
| Singh-Maddala | -372.86 | 230,347 | 0.0060 | 0.2291 | 889 | 1.672 | 6E+08 | 4E+05 | - | - | 0.339 |
| GB1 | -245.28 | 226,682 | 0.0045 | 0.2315 | 369 | 3.597 | 1E+07 | 0.336 | 87458 | - | 0.334 |
| GB2 | -213.57 | 230,296 | 0.0037 | 0.1925 | 305 | 10.25 | 3E+05 | 0.116 | 0.423 | - | 0.343 |
| GB | -210.84 | 224,035 | 0.0036 | 0.1860 | 300 | 14.2 | 3E+05 | 0.998 | 0.0839 | 0.252 | - |

## Norway 1991

| sample | |
|---|---|
| *mean* | *obs* |
| 262,892 | 7361 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -609.80 | 328,538 | 0.0070 | 0.2787 | 1312 | 2E+05 | 1.842 | - | - | - | 0.389 |
| lognormal | -1498.46 | 377,678 | 0.0197 | 0.4721 | 4602 | 12.43 | 0.904 | - | - | - | 0.477 |
| Weibull | -409.31 | 322,893 | 0.0038 | 0.1883 | 878 | 1.564 | 4E+05 | - | - | - | 0.358 |
| Dagum | -231.58 | 321,775 | 0.0023 | 0.1508 | 325 | 5.295 | 5E+05 | 0.219 | - | - | 0.357 |
| Beta 1 | -352.25 | 318,933 | 0.0041 | 0.2161 | 578 | 9E+05 | 1.335 | 2.539 | - | - | 0.355 |
| Beta 2 | -611.86 | 325,701 | 0.0067 | 0.2707 | 1388 | 2E+12 | 1.898 | 1E+07 | - | - | 0.384 |
| GG | -272.92 | 319,099 | 0.0029 | 0.1858 | 407 | 2.915 | 6E+05 | 0.404 | - | - | 0.352 |
| Singh-Maddala | -409.31 | 322,896 | 0.0038 | 0.1883 | 878 | 1.564 | 1E+09 | 4E+05 | - | - | 0.358 |
| GB1 | -272.92 | 319,101 | 0.0029 | 0.1858 | 407 | 2.915 | 3E+07 | 0.404 | 127778 | - | 0.352 |
| GB2 | -226.62 | 324,884 | 0.0022 | 0.1467 | 313 | 7.291 | 5E+05 | 0.157 | 0.5665 | - | 0.362 |
| GB | -225.21 | 316,781 | 0.0022 | 0.1448 | 310 | 8.56 | 5E+05 | 0.992 | 0.1336 | 0.406 | - |

## Norway 1995

| sample mean | sample obs |
|---|---|
| 285,297 | 8027 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -784.90 | 350,548 | 0.0083 | 0.3135 | 1752 | 2E+05 | 1.675 | - | - | - | 0.405 |
| lognormal | -1892.17 | 417,906 | 0.0230 | 0.5204 | 5847 | 12.47 | 0.974 | - | - | - | 0.509 |
| Weibull | -554.06 | 343,799 | 0.0047 | 0.2267 | 1321 | 1.5 | 4E+05 | - | - | - | 0.370 |
| Dagum | -258.06 | 339,505 | 0.0023 | 0.1597 | 390 | 5.817 | 6E+05 | 0.185 | - | - | 0.363 |
| Beta 1 | -408.84 | 337,845 | 0.0042 | 0.2314 | 709 | 9E+05 | 1.199 | 2.094 | - | - | 0.363 |
| Beta 2 | -785.14 | 351,650 | 0.0084 | 0.3160 | 1728 | 4E+12 | 1.659 | 2E+07 | - | - | 0.407 |
| GG | -307.56 | 337,758 | 0.0029 | 0.1939 | 489 | 3.362 | 6E+05 | 0.321 | - | - | 0.360 |
| Singh-Maddala | -554.06 | 343,802 | 0.0047 | 0.2267 | 1321 | 1.5 | 2E+09 | 5E+05 | - | - | 0.370 |
| GB1 | -307.56 | 337,760 | 0.0029 | 0.1939 | 489 | 3.363 | 2E+07 | 0.321 | 106385 | - | 0.360 |
| GB2 | -237.94 | 345,601 | 0.0021 | 0.1468 | 344 | 13.25 | 5E+05 | 0.08 | 0.2811 | - | 0.373 |
| GB | -231.54 | 332,590 | 0.0020 | 0.1360 | 331 | 20.67 | 5E+05 | 1 | 0.0512 | 0.156 | - |

## Poland 1986

| sample mean | sample obs |
|---|---|
| 477,059 | 9083 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -1135.62 | 477,689 | 0.0134 | 0.3612 | 2576 | 3E+05 | 1.489 | - | - | - | 0.426 |
| lognormal | -2668.53 | 609,728 | 0.0352 | 0.5997 | 8670 | 12.74 | 1.079 | - | - | - | 0.555 |
| Weibull | -875.59 | 467,276 | 0.0083 | 0.2653 | 2335 | 1.418 | 5E+05 | - | - | - | 0.387 |
| Dagum | -436.41 | 457,546 | 0.0049 | 0.1930 | 731 | 5.91 | 8E+05 | 0.168 | - | - | 0.377 |
| Beta 1 | -617.32 | 456,533 | 0.0073 | 0.2726 | 1096 | 1E+06 | 1.073 | 1.831 | - | - | 0.377 |
| Beta 2 | -1136.21 | 475,256 | 0.0132 | 0.3559 | 2638 | 6E+12 | 1.509 | 2E+07 | - | - | 0.423 |
| GG | -485.76 | 455,314 | 0.0055 | 0.2288 | 823 | 3.574 | 9E+05 | 0.278 | - | - | 0.373 |
| Singh-Maddala | -875.60 | 467,284 | 0.0083 | 0.2653 | 2335 | 1.418 | 7E+09 | 7E+05 | - | - | 0.387 |
| GB1 | -485.76 | 455,318 | 0.0055 | 0.2288 | 823 | 3.574 | 2E+07 | 0.278 | 79526 | - | 0.373 |
| GB2 | -421.29 | 464,383 | 0.0047 | 0.1804 | 696 | 10.93 | 7E+05 | 0.09 | 0.3637 | - | 0.384 |
| GB | -419.19 | 450,040 | 0.0046 | 0.1746 | 692 | 14.01 | 7E+05 | 0.999 | 0.0701 | 0.249 | - |

## Poland 1992

| sample mean | sample obs |
|---|---|
| 43,462 | 4179 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -299.79 | 43,445 | 0.0066 | 0.2014 | 525 | 20755 | 2.093 | - | - | - | 0.368 |
| lognormal | -644.79 | 47,533 | 0.0141 | 0.3422 | 1916 | 10.44 | 0.806 | - | - | - | 0.431 |
| Weibull | -236.14 | 42,925 | 0.0051 | 0.1666 | 345 | 1.614 | 47915 | - | - | - | 0.349 |
| Dagum | -206.72 | 43,513 | 0.0045 | 0.1599 | 264 | 4.17 | 62888 | 0.323 | - | - | 0.359 |
| Beta 1 | -251.85 | 42,759 | 0.0060 | 0.1940 | 358 | 2E+05 | 1.656 | 4.89 | - | - | 0.349 |
| Beta 2 | -299.79 | 43,445 | 0.0066 | 0.2014 | 525 | 7E+09 | 2.093 | 3E+05 | - | - | 0.368 |
| GG | -227.23 | 42,787 | 0.0053 | 0.1829 | 302 | 2.005 | 59858 | 0.711 | - | - | 0.348 |
| Singh-Maddala | -236.14 | 42,925 | 0.0051 | 0.1666 | 345 | 1.614 | 6E+07 | 1E+05 | - | - | 0.349 |
| GB1 | -227.23 | 42,788 | 0.0053 | 0.1829 | 302 | 2.005 | 2E+07 | 0.711 | 67741 | - | 0.348 |
| GB2 | -206.71 | 43,489 | 0.0045 | 0.1594 | 264 | 4.118 | 63293 | 0.327 | 1.025 | - | 0.358 |
| GB | -204.95 | 42,438 | 0.0044 | 0.1573 | 261 | 4.936 | 54185 | 0.976 | 0.2728 | 0.605 | - |

## Poland 1995

| sample mean | sample obs |
|---|---|
| 9,651 | 21091 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -245.44 | 9,435 | 0.0007 | 0.0973 | 336 | 3993 | 2.363 | - | - | - | 0.348 |
| lognormal | -252.94 | 9,747 | 0.0008 | 0.1050 | 345 | 8.942 | 0.697 | - | - | - | 0.378 |
| Weibull | -558.07 | 9,408 | 0.0024 | 0.1599 | 960 | 1.602 | 10495 | - | - | - | 0.351 |
| Dagum | -211.86 | 9,922 | 0.0006 | 0.0923 | 262 | 2.845 | 9592 | 0.693 | - | - | 0.384 |
| Beta 1 | -242.53 | 9,456 | 0.0007 | 0.0986 | 328 | 8E+11 | 2.34 | 2E+08 | - | - | 0.350 |
| Beta 2 | -148.55 | 9,560 | 0.0003 | 0.0667 | 131 | 24840 | 3.086 | 9.019 | - | - | 0.361 |
| GG | -151.15 | 9,525 | 0.0003 | 0.0683 | 136 | 0.494 | 102.8 | 8.851 | - | - | 0.359 |
| Singh-Maddala | -171.18 | 9,686 | 0.0004 | 0.0748 | 180 | 2.118 | 11171 | 1.753 | - | - | 0.370 |
| GB1 | -151.17 | 9,525 | 0.0003 | 0.0683 | 136 | 0.491 | 1E+09 | 8.928 | 3139.4 | - | 0.359 |
| GB2 | -148.54 | 9,563 | 0.0003 | 0.0666 | 131 | 1.033 | 23109 | 2.937 | 8.254 | - | 0.362 |
| GB | -148.54 | 9,563 | 0.0003 | 0.0666 | 131 | 1.033 | 23109 | 1 | 2.9374 | 8.254 | - |

## Taiwan 1981

| sample mean | sample obs |
|---|---|
| 256,816 | 14997 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -329.60 | 252,769 | 0.0016 | 0.1558 | 526 | 61915 | 4.083 | - | - | - | 0.271 |
| lognormal | -227.66 | 255,851 | 0.0010 | 0.1007 | 303 | 12.32 | 0.511 | - | - | - | 0.282 |
| Weibull | -782.11 | 251,699 | 0.0048 | 0.2645 | 1467 | 2.103 | 3E+05 | - | - | - | 0.281 |
| Dagum | -164.88 | 259,616 | 0.0006 | 0.0853 | 174 | 3.472 | 2E+05 | 0.929 | - | - | 0.293 |
| Beta 1 | -329.60 | 252,770 | 0.0016 | 0.1558 | 526 | 3E+10 | 4.082 | 6E+05 | - | - | 0.271 |
| Beta 2 | -206.66 | 255,342 | 0.0008 | 0.0964 | 260 | 3E+05 | 6.835 | 9.853 | - | - | 0.280 |
| GG | -219.94 | 254,847 | 0.0009 | 0.1033 | 287 | 0.205 | 7E-05 | 91.21 | - | - | 0.279 |
| Singh-Maddala | -164.96 | 259,241 | 0.0006 | 0.0836 | 174 | 3.296 | 2E+05 | 1.082 | - | - | 0.292 |
| GB1 | -220.19 | 254,852 | 0.0009 | 0.1034 | 287 | 0.159 | 1E+11 | 133.9 | 967.01 | - | 0.278 |
| GB2 | -164.86 | 259,466 | 0.0006 | 0.0847 | 174 | 3.416 | 2E+05 | 0.949 | 1.0256 | - | 0.293 |
| GB | -152.39 | 251,506 | 0.0005 | 0.0785 | 148 | 4.719 | 2E+05 | 0.994 | 0.641 | 0.567 | - |

## Taiwan 1986

| sample mean | sample obs |
|---|---|
| 337,994 | 15911 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -343.15 | 331,699 | 0.0017 | 0.1572 | 534 | 92971 | 3.568 | - | - | - | 0.288 |
| lognormal | -393.08 | 337,746 | 0.0020 | 0.1327 | 654 | 12.58 | 0.555 | - | - | - | 0.305 |
| Weibull | -689.02 | 330,141 | 0.0038 | 0.2406 | 1274 | 1.985 | 4E+05 | - | - | - | 0.295 |
| Dagum | -174.10 | 338,847 | 0.0006 | 0.0883 | 190 | 3.549 | 3E+05 | 0.734 | - | - | 0.306 |
| Beta 1 | -343.27 | 331,458 | 0.0017 | 0.1571 | 535 | 2E+12 | 3.587 | 2E+07 | - | - | 0.288 |
| Beta 2 | -287.08 | 333,931 | 0.0014 | 0.1285 | 414 | 1E+06 | 4.626 | 14.54 | - | - | 0.295 |
| GG | -306.47 | 333,169 | 0.0015 | 0.1376 | 452 | 0.595 | 7090 | 9.561 | - | - | 0.293 |
| Singh-Maddala | -191.90 | 338,368 | 0.0007 | 0.0919 | 227 | 2.873 | 3E+05 | 1.359 | - | - | 0.304 |
| GB1 | -306.47 | 333,170 | 0.0015 | 0.1376 | 452 | 0.595 | 2E+13 | 9.561 | 478262 | - | 0.293 |
| GB2 | -164.45 | 342,857 | 0.0005 | 0.0840 | 171 | 4.698 | 3E+05 | 0.521 | 0.6513 | - | 0.314 |
| GB | -149.09 | 328,600 | 0.0004 | 0.0690 | 140 | 5.994 | 3E+05 | 0.997 | 0.3988 | 0.43 | - |

## Taiwan 1991

| sample mean | sample obs |
|---|---|
| 592,997 | 15756 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -231.49 | 579,691 | 0.0010 | 0.1214 | 302 | 2E+05 | 3.296 | - | - | - | 0.299 |
| lognormal | -409.77 | 593,192 | 0.0020 | 0.1488 | 709 | 13.12 | 0.584 | - | - | - | 0.321 |
| Weibull | -451.66 | 576,678 | 0.0024 | 0.1927 | 762 | 1.934 | 7E+05 | - | - | - | 0.301 |
| Dagum | -140.57 | 591,983 | 0.0004 | 0.0792 | 123 | 3.607 | 6E+05 | 0.634 | - | - | 0.315 |
| Beta 1 | -231.55 | 580,037 | 0.0010 | 0.1215 | 302 | 1E+12 | 3.283 | 7E+06 | - | - | 0.300 |
| Beta 2 | -219.01 | 581,650 | 0.0009 | 0.1119 | 278 | 4E+06 | 3.683 | 28.05 | - | - | 0.303 |
| GG | -226.68 | 580,639 | 0.0010 | 0.1179 | 293 | 0.853 | 99630 | 4.418 | - | - | 0.301 |
| Singh-Maddala | -156.27 | 587,670 | 0.0005 | 0.0837 | 154 | 2.562 | 7E+05 | 1.784 | - | - | 0.309 |
| GB1 | -227.56 | 580,153 | 0.0010 | 0.1194 | 294 | 0.916 | 1E+13 | 3.877 | 2E+07 | - | 0.301 |
| GB2 | -139.96 | 593,814 | 0.0004 | 0.0784 | 122 | 3.888 | 6E+05 | 0.579 | 0.8845 | - | 0.317 |
| GB | -114.96 | 572,267 | 0.0002 | 0.0524 | 72 | 5.636 | 5E+05 | 0.995 | 0.3846 | 0.45 | - |

## Taiwan 1995

| sample mean | sample obs |
|---|---|
| 767,530 | 13619 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -374.67 | 770,304 | 0.0023 | 0.1452 | 608 | 3E+05 | 2.634 | - | - | - | 0.332 |
| lognormal | -940.01 | 806,595 | 0.0060 | 0.2510 | 2116 | 13.37 | 0.681 | - | - | - | 0.370 |
| Weibull | -338.26 | 764,176 | 0.0021 | 0.1684 | 509 | 1.774 | 9E+05 | - | - | - | 0.323 |
| Dagum | -216.03 | 778,760 | 0.0011 | 0.1086 | 272 | 3.864 | 1E+06 | 0.444 | - | - | 0.335 |
| Beta 1 | -350.70 | 766,134 | 0.0022 | 0.1572 | 540 | 6E+06 | 2.339 | 15.38 | - | - | 0.326 |
| Beta 2 | -374.67 | 770,300 | 0.0023 | 0.1452 | 608 | 3E+12 | 2.634 | 1E+07 | - | - | 0.332 |
| GG | -322.17 | 765,668 | 0.0019 | 0.1562 | 479 | 1.482 | 7E+05 | 1.354 | - | - | 0.324 |
| Singh-Maddala | -281.11 | 770,456 | 0.0016 | 0.1275 | 409 | 2.006 | 2E+06 | 4.176 | - | - | 0.327 |
| GB1 | -322.17 | 765,674 | 0.0019 | 0.1562 | 479 | 1.482 | 2E+09 | 1.354 | 158040 | - | 0.324 |
| GB2 | -211.46 | 785,772 | 0.0010 | 0.1071 | 264 | 4.702 | 9E+05 | 0.357 | 0.7148 | - | 0.341 |
| GB | -183.05 | 753,869 | 0.0008 | 0.0879 | 206 | 6.761 | 8E+05 | 0.996 | 0.2459 | 0.37 | - |

## Russia 1992

| sample mean | sample obs |
|---|---|
| 84,594 | 4851 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -136.11 | 74,951 | 0.0015 | 0.1118 | 137 | 59220 | 1.266 | - | - | - | 0.455 |
| lognormal | -460.01 | 88,787 | 0.0074 | 0.2997 | 1007 | 10.82 | 1.075 | - | - | - | 0.553 |
| Weibull | -127.35 | 74,528 | 0.0013 | 0.1102 | 117 | 1.169 | 78685 | - | - | - | 0.447 |
| Dagum | -104.93 | 77,329 | 0.0009 | 0.0883 | 71 | 2.93 | 1E+05 | 0.342 | - | - | 0.466 |
| Beta 1 | -132.53 | 74,415 | 0.0015 | 0.1159 | 126 | 1E+06 | 1.201 | 15.68 | - | - | 0.448 |
| Beta 2 | -136.36 | 75,359 | 0.0015 | 0.1155 | 136 | 1E+12 | 1.251 | 2E+07 | - | - | 0.458 |
| GG | -125.72 | 74,357 | 0.0013 | 0.1163 | 111 | 1.285 | 91217 | 0.861 | - | - | 0.445 |
| Singh-Maddala | -127.13 | 74,656 | 0.0013 | 0.1064 | 118 | 1.184 | 2E+06 | 41.85 | - | - | 0.448 |
| GB1 | -125.72 | 74,358 | 0.0013 | 0.1163 | 111 | 1.284 | 2E+09 | 0.861 | 286686 | - | 0.445 |
| GB2 | -104.93 | 77,348 | 0.0009 | 0.0883 | 71 | 2.938 | 1E+05 | 0.341 | 0.995 | - | 0.466 |
| GB | -104.01 | 73,762 | 0.0008 | 0.0843 | 69 | 3.458 | 93969 | 0.98 | 0.2877 | 0.647 | - |

## Russia 1995

| sample mean | sample obs |
|---|---|
| 61,240 | 2419 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -91.02 | 60,005 | 0.0012 | 0.1296 | 60 | 58927 | 1.018 | - | - | - | 0.497 |
| lognormal | -160.72 | 72,225 | 0.0037 | 0.2233 | 221 | 10.48 | 1.19 | - | - | - | 0.600 |
| Weibull | -91.24 | 60,059 | 0.0012 | 0.1321 | 60 | 0.999 | 60032 | - | - | - | 0.500 |
| Dagum | -79.52 | 66,192 | 0.0008 | 0.0963 | 35 | 2.166 | 75201 | 0.444 | - | - | 0.549 |
| Beta 1 | -91.02 | 60,011 | 0.0012 | 0.1296 | 60 | 5E+11 | 1.018 | 9E+06 | - | - | 0.497 |
| Beta 2 | -82.51 | 61,555 | 0.0009 | 0.1072 | 42 | 3E+05 | 1.152 | 6.537 | - | - | 0.518 |
| GG | -85.34 | 60,704 | 0.0010 | 0.1180 | 48 | 0.747 | 27849 | 1.638 | - | - | 0.510 |
| Singh-Maddala | -81.51 | 61,965 | 0.0008 | 0.1021 | 40 | 1.126 | 2E+05 | 4.203 | - | - | 0.521 |
| GB1 | -85.34 | 60,703 | 0.0010 | 0.1180 | 48 | 0.747 | 2E+12 | 1.638 | 756662 | - | 0.510 |
| GB2 | -79.05 | 64,316 | 0.0008 | 0.0906 | 35 | 1.814 | 86580 | 0.547 | 1.3876 | - | 0.537 |
| GB | -78.93 | 60,830 | 0.0008 | 0.0884 | 34 | 2.055 | 70165 | 0.975 | 0.4771 | 0.952 | - |

## Slovakia 1992

| sample mean | sample obs |
|---|---|
| 834 | 11439 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -140.79 | 824 | 0.0006 | 0.0892 | 129 | 238.8 | 3.453 | - | - | - | 0.293 |
| lognormal | -232.18 | 841 | 0.0014 | 0.1349 | 312 | 6.573 | 0.569 | - | - | - | 0.312 |
| Weibull | -275.33 | 820 | 0.0018 | 0.1447 | 399 | 2.001 | 925.7 | - | - | - | 0.293 |
| Dagum | -216.26 | 847 | 0.0012 | 0.1331 | 283 | 3.742 | 925.9 | 0.592 | - | - | 0.311 |
| Beta 1 | -140.79 | 824 | 0.0006 | 0.0892 | 129 | 9E+08 | 3.453 | 4E+06 | - | - | 0.293 |
| Beta 2 | -138.87 | 826 | 0.0006 | 0.0871 | 125 | 11902 | 3.661 | 53.75 | - | - | 0.295 |
| GG | -137.54 | 826 | 0.0005 | 0.0863 | 122 | 0.836 | 123.1 | 4.814 | - | - | 0.295 |
| Singh-Maddala | -168.45 | 830 | 0.0008 | 0.1032 | 186 | 2.433 | 1192 | 2.485 | - | - | 0.299 |
| GB1 | -134.44 | 825 | 0.0005 | 0.0832 | 116 | 0.112 | 8321 | 59.46 | 18.655 | - | - |
| GB2 | -137.54 | 826 | 0.0005 | 0.0863 | 122 | 0.836 | 4E+08 | 4.814 | 286871 | - | 0.295 |
| GB | -134.44 | 825 | 0.0005 | 0.0832 | 116 | 0.112 | 8321 | 0 | 59.464 | 18.65 | - |

## Spain 1980

| sample mean | sample obs |
|---|---|
| 669,912 | 18803 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -516.38 | 652,948 | 0.0024 | 0.1729 | 873 | 3E+05 | 2.59 | - | - | - | 0.334 |
| lognormal | -1058.45 | 680,505 | 0.0049 | 0.2322 | 2255 | 13.2 | 0.68 | - | - | - | 0.369 |
| Weibull | -611.22 | 648,783 | 0.0030 | 0.1971 | 1070 | 1.725 | 7E+05 | - | - | - | 0.331 |
| Dagum | -300.68 | 662,885 | 0.0012 | 0.1246 | 438 | 3.569 | 8E+05 | 0.506 | - | - | 0.343 |
| Beta 1 | -516.38 | 652,990 | 0.0024 | 0.1729 | 873 | 4E+11 | 2.589 | 2E+06 | - | - | 0.334 |
| Beta 2 | -516.36 | 653,035 | 0.0024 | 0.1726 | 873 | 2E+08 | 2.597 | 738.7 | - | - | 0.334 |
| GG | -509.02 | 651,640 | 0.0024 | 0.1757 | 853 | 1.145 | 4E+05 | 2.044 | - | - | 0.332 |
| Singh-Maddala | -395.14 | 658,706 | 0.0017 | 0.1385 | 637 | 2.117 | 1E+06 | 2.529 | - | - | 0.338 |
| GB1 | -509.02 | 651,648 | 0.0024 | 0.1757 | 853 | 1.144 | 2E+10 | 2.045 | 206945 | - | 0.332 |
| GB2 | -272.21 | 676,894 | 0.0010 | 0.1202 | 381 | 5.467 | 7E+05 | 0.313 | 0.518 | - | 0.356 |
| GB | -254.51 | 641,864 | 0.0010 | 0.1115 | 345 | 6.95 | 7E+05 | 0.998 | 0.2436 | 0.35 | - |

## Spain 1990

| sample mean | sample obs |
|---|---|
| 1,823,374 | 15526 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -389.47 | 1,769,559 | 0.0020 | 0.1555 | 635 | 7E+05 | 2.612 | - | - | - | 0.333 |
| lognormal | -858.98 | 1,844,144 | 0.0048 | 0.2314 | 1779 | 14.2 | 0.677 | - | - | - | 0.368 |
| Weibull | -439.59 | 1,757,885 | 0.0022 | 0.1784 | 754 | 1.742 | 2E+06 | - | - | - | 0.328 |
| Dagum | -261.50 | 1,800,116 | 0.0011 | 0.1168 | 375 | 3.604 | 2E+06 | 0.496 | - | - | 0.343 |
| Beta 1 | -387.53 | 1,766,872 | 0.0020 | 0.1588 | 630 | 4E+07 | 2.525 | 61.15 | - | - | 0.331 |
| Beta 2 | -389.47 | 1,769,676 | 0.0020 | 0.1555 | 635 | 1E+13 | 2.611 | 2E+07 | - | - | 0.333 |
| GG | -376.72 | 1,764,458 | 0.0019 | 0.1606 | 609 | 1.218 | 1E+06 | 1.854 | - | - | 0.329 |
| Singh-Maddala | -312.82 | 1,780,341 | 0.0015 | 0.1297 | 481 | 2.075 | 3E+06 | 2.924 | - | - | 0.334 |
| GB1 | -376.72 | 1,764,466 | 0.0019 | 0.1606 | 609 | 1.218 | 3E+10 | 1.855 | 220821 | - | 0.329 |
| GB2 | -258.35 | 1,813,857 | 0.0011 | 0.1192 | 369 | 4.194 | 2E+06 | 0.417 | 0.7736 | - | 0.348 |
| GB | -242.13 | 1,741,590 | 0.0010 | 0.1117 | 335 | 5.454 | 2E+06 | 0.993 | 0.3172 | 0.461 | - |

## Sweden 1967

| sample mean | sample obs |
|---|---|
| 23,531 | 4912 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -687.28 | 25,635 | 0.0108 | 0.3918 | 1830 | 17498 | 1.465 | - | - | - | 0.429 |
| lognormal | -1576.77 | 33,425 | 0.0290 | 0.6036 | 7238 | 9.81 | 1.102 | - | - | - | 0.564 |
| Weibull | -554.48 | 25,075 | 0.0068 | 0.3019 | 1740 | 1.398 | 27506 | - | - | - | 0.391 |
| Dagum | -328.11 | 24,478 | 0.0051 | 0.2323 | 609 | 5.136 | 43352 | 0.198 | - | - | 0.384 |
| Beta 1 | -479.57 | 24,576 | 0.0087 | 0.3254 | 941 | 75592 | 1.115 | 2.315 | - | - | 0.387 |
| Beta 2 | -689.83 | 25,268 | 0.0101 | 0.3812 | 2011 | 4E+11 | 1.523 | 2E+07 | - | - | 0.422 |
| GG | -387.50 | 24,460 | 0.0067 | 0.2818 | 711 | 2.953 | 47955 | 0.342 | - | - | 0.381 |
| Singh-Maddala | -554.48 | 25,076 | 0.0068 | 0.3019 | 1740 | 1.398 | 3E+08 | 5E+05 | - | - | 0.391 |
| GB1 | -387.50 | 24,461 | 0.0067 | 0.2818 | 711 | 2.954 | 2E+06 | 0.342 | 99768 | - | 0.381 |
| GB2 | -293.38 | 25,188 | 0.0045 | 0.1943 | 526 | 17.71 | 34494 | 0.056 | 0.1767 | - | 0.399 |
| GB | -293.43 | 25,202 | 0.0045 | 0.1943 | 525 | 19.34 | 34451 | 1 | 0.0515 | 0.161 | - |

## Sweden 1975

| sample mean | sample obs |
|---|---|
| 48,613 | 9535 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -277.37 | 65,898 | 0.0021 | 0.1636 | 426 | 34345 | 1.919 | - | - | - | 0.382 |
| lognormal | -840.36 | 71,734 | 0.0076 | 0.3162 | 1908 | 10.84 | 0.831 | - | - | - | 0.443 |
| Weibull | -218.12 | 65,312 | 0.0015 | 0.1440 | 288 | 1.504 | 72373 | - | - | - | 0.369 |
| Dagum | -169.93 | 66,776 | 0.0010 | 0.1172 | 192 | 3.591 | 91300 | 0.371 | - | - | 0.383 |
| Beta 1 | -242.58 | 65,236 | 0.0018 | 0.1582 | 339 | 4E+05 | 1.667 | 8.547 | - | - | 0.370 |
| Beta 2 | -292.19 | 67,307 | 0.0024 | 0.1842 | 435 | 4E+11 | 1.787 | 1E+07 | - | - | 0.394 |
| GG | -217.37 | 65,271 | 0.0015 | 0.1454 | 285 | 1.574 | 76684 | 0.93 | - | - | 0.369 |
| Singh-Maddala | -215.26 | 65,483 | 0.0015 | 0.1356 | 287 | 1.553 | 4E+05 | 15.97 | - | - | 0.371 |
| GB1 | -217.37 | 65,272 | 0.0015 | 0.1454 | 285 | 1.574 | 2E+08 | 0.93 | 191480 | - | 0.369 |
| GB2 | -169.68 | 66,997 | 0.0010 | 0.1173 | 192 | 3.799 | 88675 | 0.349 | 0.8995 | - | 0.385 |
| GB | -144.60 | 64,021 | 0.0007 | 0.0927 | 142 | 5.921 | 70943 | 0.993 | 0.2218 | 0.374 | - |

## Sweden 1981

| sample mean | obs |
|---|---|
| 82,115 | 8635 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -385.86 | 101,121 | 0.0035 | 0.2125 | 687 | 64274 | 1.573 | - | - | - | 0.416 |
| lognormal | -1246.30 | 117,758 | 0.0137 | 0.4204 | 3117 | 11.2 | 0.973 | - | - | - | 0.509 |
| Weibull | -263.15 | 99,796 | 0.0019 | 0.1444 | 436 | 1.387 | 1E+05 | - | - | - | 0.393 |
| Dagum | -142.91 | 100,468 | 0.0008 | 0.0922 | 139 | 4.38 | 2E+05 | 0.24 | - | - | 0.394 |
| Beta 1 | -223.75 | 98,628 | 0.0018 | 0.1537 | 305 | 4E+05 | 1.23 | 3.183 | - | - | 0.387 |
| Beta 2 | -386.06 | 100,823 | 0.0034 | 0.2110 | 695 | 3E+12 | 1.586 | 4E+07 | - | - | 0.415 |
| GG | -172.59 | 98,887 | 0.0012 | 0.1229 | 199 | 2.292 | 2E+05 | 0.475 | - | - | 0.385 |
| Singh-Maddala | -263.15 | 99,797 | 0.0019 | 0.1444 | 436 | 1.387 | 2E+09 | 6E+05 | - | - | 0.393 |
| GB1 | -172.59 | 98,888 | 0.0012 | 0.1229 | 199 | 2.292 | 3E+07 | 0.475 | 88473 | - | 0.385 |
| GB2 | -142.67 | 100,746 | 0.0008 | 0.0934 | 138 | 4.667 | 2E+05 | 0.225 | 0.8824 | - | 0.395 |
| GB | -138.33 | 97,803 | 0.0008 | 0.0839 | 130 | 6.104 | 1E+05 | 0.982 | 0.1713 | 0.463 | - |

## Sweden 1987

| sample mean | obs |
|---|---|
| 129,465 | 8658 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -667.13 | 146,993 | 0.0068 | 0.2987 | 1315 | 96894 | 1.517 | - | - | - | 0.422 |
| lognormal | -1754.71 | 177,371 | 0.0207 | 0.5127 | 4465 | 11.56 | 1.021 | - | - | - | 0.530 |
| Weibull | -482.61 | 144,490 | 0.0041 | 0.2210 | 980 | 1.397 | 2E+05 | - | - | - | 0.391 |
| Dagum | -173.54 | 143,126 | 0.0011 | 0.1264 | 206 | 6.194 | 3E+05 | 0.157 | - | - | 0.379 |
| Beta 1 | -270.34 | 141,584 | 0.0023 | 0.1812 | 399 | 4E+05 | 1.086 | 1.911 | - | - | 0.378 |
| Beta 2 | -667.14 | 147,077 | 0.0068 | 0.2991 | 1313 | 6E+11 | 1.515 | 6E+06 | - | - | 0.423 |
| GG | -198.93 | 141,964 | 0.0014 | 0.1477 | 256 | 3.56 | 3E+05 | 0.277 | - | - | 0.375 |
| Singh-Maddala | -482.61 | 144,492 | 0.0041 | 0.2210 | 980 | 1.397 | 3E+09 | 7E+05 | - | - | 0.391 |
| GB1 | -198.93 | 141,962 | 0.0014 | 0.1477 | 256 | 3.56 | 6E+06 | 0.277 | 49755 | - | 0.375 |
| GB2 | -166.15 | 145,199 | 0.0010 | 0.1122 | 192 | 12.91 | 2E+05 | 0.074 | 0.3292 | - | 0.384 |
| GB | -165.32 | 141,125 | 0.0010 | 0.1068 | 191 | 20.37 | 2E+05 | 1 | 0.0464 | 0.185 | - |

## Sweden 1992

| sample mean | obs |
|---|---|
| 189,352 | 10824 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -851.69 | 226,843 | 0.0076 | 0.3071 | 1661 | 2E+05 | 1.356 | - | - | - | 0.443 |
| lognormal | -2334.97 | 287,266 | 0.0236 | 0.5395 | 5827 | 11.96 | 1.107 | - | - | - | 0.566 |
| Weibull | -666.41 | 223,072 | 0.0051 | 0.2465 | 1383 | 1.304 | 2E+05 | - | - | - | 0.412 |
| Dagum | -256.10 | 220,056 | 0.0017 | 0.1395 | 361 | 5.566 | 4E+05 | 0.164 | - | - | 0.398 |
| Beta 1 | -401.83 | 217,854 | 0.0033 | 0.2072 | 645 | 6E+05 | 1.01 | 1.984 | - | - | 0.397 |
| Beta 2 | -872.85 | 235,957 | 0.0085 | 0.3339 | 1596 | 5E+13 | 1.24 | 3E+08 | - | - | 0.459 |
| GG | -298.11 | 218,205 | 0.0022 | 0.1639 | 441 | 3.239 | 5E+05 | 0.285 | - | - | 0.394 |
| Singh-Maddala | -666.41 | 223,076 | 0.0051 | 0.2465 | 1383 | 1.304 | 9E+09 | 9E+05 | - | - | 0.412 |
| GB1 | -298.11 | 218,206 | 0.0022 | 0.1639 | 441 | 3.238 | 2E+07 | 0.285 | 99236 | - | 0.394 |
| GB2 | -245.12 | 223,510 | 0.0016 | 0.1235 | 341 | 10.2 | 4E+05 | 0.088 | 0.3819 | - | 0.405 |
| GB | -245.04 | 218,252 | 0.0016 | 0.1227 | 341 | 11.04 | 4E+05 | 0.998 | 0.0811 | 0.337 | - |

## Sweden 1995

| sample mean | obs |
|---|---|
| 203,854 | 11655 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -1244.39 | 241,411 | 0.0107 | 0.3763 | 2569 | 2E+05 | 1.026 | - | - | - | 0.495 |
| lognormal | -3410.68 | 379,192 | 0.0329 | 0.6371 | 9352 | 11.88 | 1.387 | - | - | - | 0.673 |
| Weibull | -1179.37 | 238,273 | 0.0088 | 0.3359 | 2846 | 1.105 | 2E+05 | - | - | - | 0.466 |
| Dagum | -424.18 | 228,174 | 0.0031 | 0.1946 | 713 | 5.866 | 5E+05 | 0.128 | - | - | 0.436 |
| Beta 1 | -599.14 | 227,535 | 0.0048 | 0.2520 | 1059 | 7E+05 | 0.784 | 1.554 | - | - | 0.435 |
| Beta 2 | -1244.45 | 241,904 | 0.0107 | 0.3770 | 2560 | 1E+13 | 1.023 | 6E+07 | - | - | 0.496 |
| GG | -466.70 | 226,850 | 0.0035 | 0.2154 | 794 | 3.725 | 5E+05 | 0.201 | - | - | 0.432 |
| Singh-Maddala | -1179.38 | 238,281 | 0.0088 | 0.3359 | 2846 | 1.104 | 6E+10 | 1E+06 | - | - | 0.466 |
| GB1 | -466.70 | 226,849 | 0.0035 | 0.2154 | 794 | 3.724 | 1E+07 | 0.201 | 69479 | - | 0.432 |
| GB2 | -399.51 | 233,621 | 0.0028 | 0.1730 | 665 | 18.85 | 4E+05 | 0.039 | 0.1882 | - | 0.446 |
| GB | -398.88 | 223,702 | 0.0028 | 0.1705 | 664 | 24.96 | 4E+05 | 1 | 0.0296 | 0.134 | - |

## Switzerland 1982

| sample mean | obs |
|---|---|
| 56,036 | 5570 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -455.59 | 58,895 | 0.0073 | 0.3061 | 832 | 23937 | 2.46 | - | - | - | 0.342 |
| lognormal | -807.36 | 62,749 | 0.0124 | 0.4058 | 2127 | 10.79 | 0.724 | - | - | - | 0.391 |
| Weibull | -416.35 | 58,280 | 0.0069 | 0.3012 | 698 | 1.721 | 65373 | - | - | - | 0.331 |
| Dagum | -270.07 | 58,482 | 0.0037 | 0.2152 | 415 | 4.021 | 75326 | 0.412 | - | - | 0.334 |
| Beta 1 | -444.84 | 58,557 | 0.0073 | 0.3083 | 778 | 5E+05 | 2.228 | 17.09 | - | - | 0.336 |
| Beta 2 | -455.59 | 58,903 | 0.0073 | 0.3061 | 832 | 1E+11 | 2.459 | 5E+06 | - | - | 0.342 |
| GG | -413.93 | 58,362 | 0.0068 | 0.2983 | 699 | 1.573 | 58175 | 1.164 | - | - | 0.331 |
| Singh-Maddala | -371.64 | 58,901 | 0.0054 | 0.2623 | 666 | 2.015 | 1E+05 | 3.529 | - | - | 0.333 |
| GB1 | -413.94 | 58,362 | 0.0068 | 0.2983 | 699 | 1.573 | 1E+08 | 1.165 | 169119 | - | 0.331 |
| GB2 | -204.79 | 60,995 | 0.0025 | 0.1591 | 279 | 12.94 | 61597 | 0.121 | 0.2014 | - | 0.360 |
| GB | -204.79 | 60,995 | 0.0025 | 0.1591 | 279 | 12.94 | 61597 | 1 | 0.1211 | 0.201 | - |

## Switzerland 1992

| sample mean | obs |
|---|---|
| 75,080 | 4486 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -370.90 | 72,146 | 0.0068 | 0.2929 | 683 | 36580 | 1.972 | - | - | - | 0.377 |
| lognormal | -819.48 | 80,722 | 0.0172 | 0.4461 | 2098 | 10.94 | 0.853 | - | - | - | 0.453 |
| Weibull | -273.01 | 71,076 | 0.0045 | 0.2269 | 452 | 1.595 | 79250 | - | - | - | 0.353 |
| Dagum | -181.71 | 70,969 | 0.0027 | 0.1677 | 232 | 4.577 | 1E+05 | 0.28 | - | - | 0.354 |
| Beta 1 | -288.49 | 70,636 | 0.0052 | 0.2572 | 454 | 3E+05 | 1.538 | 4.004 | - | - | 0.354 |
| Beta 2 | -370.90 | 72,145 | 0.0068 | 0.2929 | 683 | 5E+11 | 1.972 | 1E+07 | - | - | 0.377 |
| GG | -240.40 | 70,585 | 0.0041 | 0.2298 | 349 | 2.306 | 1E+05 | 0.568 | - | - | 0.350 |
| Singh-Maddala | -273.01 | 71,076 | 0.0045 | 0.2269 | 452 | 1.595 | 2E+08 | 3E+05 | - | - | 0.353 |
| GB1 | -240.40 | 70,587 | 0.0041 | 0.2297 | 349 | 2.306 | 1E+07 | 0.569 | 66930 | - | 0.350 |
| GB2 | -156.25 | 73,165 | 0.0021 | 0.1447 | 179 | 12.3 | 87239 | 0.102 | 0.2379 | - | 0.373 |
| GB | -151.24 | 69,154 | 0.0020 | 0.1299 | 169 | 16.12 | 84322 | 1 | 0.0775 | 0.166 | - |

## United States 1969

| sample mean | sample obs |
|---|---|
| 9,816 | 9687 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -507.46 | 9,681 | 0.0043 | 0.2447 | 950 | 5050 | 1.917 | - | - | - | 0.382 |
| lognormal | -1382.90 | 10,795 | 0.0135 | 0.4183 | 3378 | 8.918 | 0.859 | - | - | - | 0.456 |
| Weibull | -321.88 | 9,549 | 0.0024 | 0.1721 | 542 | 1.562 | 10625 | - | - | - | 0.358 |
| Dagum | -170.59 | 9,577 | 0.0010 | 0.1091 | 193 | 4.728 | 15143 | 0.257 | - | - | 0.360 |
| Beta 1 | -322.26 | 9,473 | 0.0027 | 0.1885 | 500 | 32748 | 1.473 | 3.618 | - | - | 0.357 |
| Beta 2 | -509.88 | 9,605 | 0.0042 | 0.2380 | 985 | 7E+10 | 1.972 | 1E+07 | - | - | 0.377 |
| GG | -245.39 | 9,484 | 0.0018 | 0.1575 | 342 | 2.376 | 15373 | 0.532 | - | - | 0.354 |
| Singh-Maddala | -321.88 | 9,549 | 0.0024 | 0.1721 | 542 | 1.562 | 5E+07 | 5E+05 | - | - | 0.358 |
| GB1 | -245.39 | 9,484 | 0.0018 | 0.1575 | 342 | 2.376 | 2E+06 | 0.532 | 157264 | - | 0.354 |
| GB2 | -155.37 | 9,736 | 0.0009 | 0.0885 | 161 | 7.832 | 13103 | 0.151 | 0.4447 | - | 0.368 |
| GB | -154.32 | 9,430 | 0.0008 | 0.0866 | 159 | 8.776 | 12684 | 0.998 | 0.135 | 0.364 | - |

## United States 1974

| sample mean | sample obs |
|---|---|
| 13,465 | 9366 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -408.43 | 13,314 | 0.0034 | 0.2106 | 723 | 7308 | 1.822 | - | - | - | 0.391 |
| lognormal | -1204.81 | 14,898 | 0.0118 | 0.3896 | 2867 | 9.222 | 0.88 | - | - | - | 0.466 |
| Weibull | -264.27 | 13,146 | 0.0019 | 0.1454 | 409 | 1.504 | 14567 | - | - | - | 0.369 |
| Dagum | -144.99 | 13,230 | 0.0008 | 0.0957 | 141 | 4.446 | 20859 | 0.266 | - | - | 0.372 |
| Beta 1 | -267.81 | 13,047 | 0.0021 | 0.1693 | 390 | 49172 | 1.437 | 3.979 | - | - | 0.367 |
| Beta 2 | -408.87 | 13,265 | 0.0034 | 0.2076 | 733 | 1E+11 | 1.844 | 2E+07 | - | - | 0.389 |
| GG | -207.91 | 13,067 | 0.0015 | 0.1432 | 267 | 2.19 | 20808 | 0.566 | - | - | 0.365 |
| Singh-Maddala | -264.27 | 13,146 | 0.0019 | 0.1453 | 409 | 1.504 | 9E+07 | 5E+05 | - | - | 0.369 |
| GB1 | -207.91 | 13,067 | 0.0015 | 0.1432 | 267 | 2.19 | 4E+06 | 0.566 | 99764 | - | 0.365 |
| GB2 | -135.84 | 13,427 | 0.0007 | 0.0896 | 122 | 6.577 | 18308 | 0.176 | 0.5214 | - | 0.379 |
| GB | -134.17 | 12,974 | 0.0007 | 0.0870 | 119 | 7.646 | 17355 | 0.996 | 0.1511 | 0.394 | - |

## United States 1979

| sample mean | sample obs |
|---|---|
| 19,666 | 11609 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -369.42 | 19,526 | 0.0023 | 0.1741 | 636 | 11182 | 1.746 | - | - | - | 0.398 |
| lognormal | -1217.87 | 21,784 | 0.0093 | 0.3388 | 2802 | 9.592 | 0.891 | - | - | - | 0.471 |
| Weibull | -251.07 | 19,318 | 0.0014 | 0.1327 | 374 | 1.447 | 21299 | - | - | - | 0.381 |
| Dagum | -154.41 | 19,566 | 0.0006 | 0.0893 | 163 | 4.039 | 30075 | 0.289 | - | - | 0.386 |
| Beta 1 | -269.43 | 19,210 | 0.0016 | 0.1495 | 401 | 88273 | 1.446 | 5.199 | - | - | 0.379 |
| Beta 2 | -369.44 | 19,541 | 0.0023 | 0.1746 | 635 | 1E+11 | 1.742 | 9E+06 | - | - | 0.398 |
| GG | -220.41 | 19,240 | 0.0012 | 0.1296 | 299 | 1.874 | 28256 | 0.671 | - | - | 0.377 |
| Singh-Maddala | -251.07 | 19,318 | 0.0014 | 0.1328 | 374 | 1.447 | 1E+08 | 3E+05 | - | - | 0.381 |
| GB1 | -220.41 | 19,240 | 0.0012 | 0.1296 | 299 | 1.874 | 1E+07 | 0.671 | 113757 | - | 0.377 |
| GB2 | -148.44 | 19,825 | 0.0006 | 0.0877 | 150 | 5.484 | 26939 | 0.208 | 0.6013 | - | 0.392 |
| GB | -146.57 | 19,118 | 0.0006 | 0.0842 | 146 | 6.58 | 25154 | 0.994 | 0.1728 | 0.43 | - |

## United States 1986

| sample mean | sample obs |
| --- | --- |
| 31,538 | 9093 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| gamma | -263.29 | 31,053 | 0.0019 | 0.1662 | 408 | 18989 | 1.635 | - | - | - | 0.409 |
| lognormal | -937.39 | 35,059 | 0.0088 | 0.3453 | 2171 | 10.03 | 0.927 | - | - | - | 0.488 |
| Weibull | -186.99 | 30,738 | 0.0012 | 0.1230 | 238 | 1.388 | 33678 | - | - | - | 0.393 |
| Dagum | -109.58 | 31,185 | 0.0004 | 0.0691 | 70 | 3.811 | 47727 | 0.297 | - | - | 0.399 |
| Beta 1 | -203.47 | 30,580 | 0.0014 | 0.1406 | 261 | 2E+05 | 1.392 | 5.882 | - | - | 0.392 |
| Beta 2 | -265.95 | 31,377 | 0.0020 | 0.1742 | 402 | 3E+11 | 1.587 | 1E+07 | - | - | 0.414 |
| GG | -167.04 | 30,614 | 0.0011 | 0.1219 | 185 | 1.753 | 44232 | 0.696 | - | - | 0.390 |
| Singh-Maddala | -186.99 | 30,738 | 0.0012 | 0.1230 | 238 | 1.388 | 3E+08 | 3E+05 | - | - | 0.393 |
| GB1 | -167.05 | 30,615 | 0.0011 | 0.1219 | 185 | 1.752 | 2E+07 | 0.696 | 34305 | - | 0.390 |
| GB2 | -103.19 | 31,724 | 0.0003 | 0.0583 | 57 | 5.424 | 42021 | 0.203 | 0.5617 | - | 0.408 |
| GB | -101.34 | 30,358 | 0.0003 | 0.0523 | 53 | 6.537 | 39292 | 0.996 | 0.1679 | 0.404 | - |

## United States 1991

| sample mean | sample obs |
| --- | --- |
| 37,501 | 11544 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| gamma | -281.29 | 37,295 | 0.0018 | 0.1500 | 425 | 24464 | 1.524 | - | - | - | 0.422 |
| lognormal | -1100.98 | 42,538 | 0.0088 | 0.3307 | 2427 | 10.19 | 0.965 | - | - | - | 0.505 |
| Weibull | -206.76 | 36,952 | 0.0011 | 0.1182 | 267 | 1.326 | 40167 | - | - | - | 0.407 |
| Dagum | -137.27 | 37,674 | 0.0005 | 0.0763 | 120 | 3.628 | 58019 | 0.297 | - | - | 0.415 |
| Beta 1 | -217.60 | 36,735 | 0.0012 | 0.1331 | 284 | 2E+05 | 1.315 | 6.047 | - | - | 0.405 |
| Beta 2 | -281.29 | 37,294 | 0.0018 | 0.1500 | 425 | 1E+11 | 1.524 | 4E+06 | - | - | 0.422 |
| GG | -184.01 | 36,802 | 0.0010 | 0.1147 | 215 | 1.667 | 53156 | 0.701 | - | - | 0.403 |
| Singh-Maddala | -206.76 | 36,953 | 0.0011 | 0.1182 | 267 | 1.326 | 7E+08 | 4E+05 | - | - | 0.407 |
| GB1 | -184.01 | 36,802 | 0.0010 | 0.1147 | 215 | 1.667 | 5E+07 | 0.701 | 96096 | - | 0.403 |
| GB2 | -136.66 | 37,871 | 0.0005 | 0.0771 | 119 | 3.985 | 55379 | 0.269 | 0.8439 | - | 0.417 |
| GB | -133.86 | 36,491 | 0.0005 | 0.0734 | 113 | 4.948 | 48349 | 0.986 | 0.2148 | 0.522 | - |

## United States 1994

| sample mean | sample obs |
| --- | --- |
| 43,099 | 44255 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| gamma | -640.15 | 42,097 | 0.0012 | 0.1220 | 1160 | 28068 | 1.5 | - | - | - | 0.424 |
| lognormal | -3469.39 | 47,841 | 0.0071 | 0.3116 | 8083 | 10.31 | 0.967 | - | - | - | 0.506 |
| Weibull | -443.19 | 41,756 | 0.0008 | 0.0962 | 731 | 1.304 | 45242 | - | - | - | 0.412 |
| Dagum | -229.76 | 42,774 | 0.0003 | 0.0579 | 281 | 3.421 | 63867 | 0.318 | - | - | 0.423 |
| Beta 1 | -506.30 | 41,605 | 0.0009 | 0.1079 | 848 | 3E+05 | 1.341 | 8.265 | - | - | 0.412 |
| Beta 2 | -640.40 | 42,033 | 0.0012 | 0.1211 | 1164 | 3E+11 | 1.506 | 1E+07 | - | - | 0.424 |
| GG | -410.96 | 41,648 | 0.0007 | 0.0951 | 650 | 1.502 | 54724 | 0.8 | - | - | 0.410 |
| Singh-Maddala | -443.20 | 41,756 | 0.0008 | 0.0962 | 731 | 1.304 | 5E+08 | 2E+05 | - | - | 0.412 |
| GB1 | -410.97 | 41,648 | 0.0007 | 0.0951 | 650 | 1.502 | 1E+08 | 0.801 | 103236 | - | 0.410 |
| GB2 | -228.30 | 42,967 | 0.0003 | 0.0592 | 277 | 3.685 | 61524 | 0.293 | 0.8759 | - | 0.425 |
| GB | -226.10 | 41,678 | 0.0003 | 0.0592 | 272 | 4.093 | 56508 | 0.986 | 0.2626 | 0.672 | - |

## United States 1997

| sample mean | sample obs |
|---|---|
| 50,384 | 39815 |

| Distribution | Log-L | Mean | SSE | SAE | $\chi^2$ | Parameters | | | | | Gini |
|---|---|---|---|---|---|---|---|---|---|---|---|
| gamma | -508.82 | 46,792 | 0.0010 | 0.1153 | 868 | 31124 | 1.503 | - | - | - | 0.424 |
| lognormal | -2844.48 | 52,892 | 0.0065 | 0.2917 | 6509 | 10.41 | 0.961 | - | - | - | 0.503 |
| Weibull | -380.13 | 46,445 | 0.0007 | 0.1031 | 586 | 1.3 | 50287 | - | - | - | 0.413 |
| Dagum | -192.79 | 47,719 | 0.0003 | 0.0552 | 208 | 3.295 | 69153 | 0.336 | - | - | 0.427 |
| Beta 1 | -440.62 | 46,372 | 0.0009 | 0.1133 | 706 | 4E+05 | 1.379 | 11.22 | - | - | 0.414 |
| Beta 2 | -509.15 | 46,705 | 0.0010 | 0.1143 | 871 | 5E+11 | 1.511 | 2E+07 | - | - | 0.423 |
| GG | -372.77 | 46,384 | 0.0007 | 0.1032 | 566 | 1.396 | 55761 | 0.892 | - | - | 0.412 |
| Singh-Maddala | -376.84 | 46,530 | 0.0007 | 0.1002 | 584 | 1.321 | 7E+05 | 31.58 | - | - | 0.415 |
| GB1 | -372.77 | 46,384 | 0.0007 | 0.1032 | 566 | 1.396 | 4E+08 | 0.892 | 203027 | - | 0.412 |
| GB2 | -190.26 | 48,035 | 0.0003 | 0.0550 | 203 | 3.652 | 65797 | 0.3 | 0.8356 | - | 0.430 |
| GB | -187.77 | 46,359 | 0.0003 | 0.0538 | 198 | 4.106 | 60423 | 0.988 | 0.2657 | 0.635 | - |

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