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**Working Paper**

## Revenue ranking of optimally biased contests: The case of two players

Working Paper, No. 243

**Provided in Cooperation with:**

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*Suggested Citation:* Ewerhart, Christian (2017) : Revenue ranking of optimally biased contests: The case of two players, Working Paper, No. 243, University of Zurich, Department of Economics, Zurich, <https://doi.org/10.5167/uzh-135994>

This Version is available at:

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**University of  
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University of Zurich  
Department of Economics

Working Paper Series

ISSN 1664-7041 (print)  
ISSN 1664-705X (online)

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Working Paper No. 243

# **Revenue Ranking of Optimally Biased Contests: The Case of Two Players**

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March 2017

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# Revenue Ranking of Optimally Biased

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# Contests: The Case of Two Players\*

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Christian Ewerhart\*\*

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March 2, 2017

7 **Abstract** It is shown that the equilibrium in the asymmetric Tullock contest is unique for parameter  
8 values  $r \leq 2$ . This allows proving a *revenue ranking result* saying that a revenue-maximizing designer  
9 capable of biasing the contest always prefers a contest technology with higher accuracy.

10 **Keywords** Tullock contest · Nash equilibrium · Heterogeneous valuations · Discrimination

11 **JEL Classification** C72 · D72 · J71

12 \*) For a helpful discussion, I am indebted to Mathias Dahm.

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## 15 **1. Introduction**

16 Contests are used widely in economics and political theory. Specific applications include marketing,  
17 rent-seeking, campaigning, military conflict, and sports, for instance.<sup>1</sup> A useful contest technology,  
18 conveniently parameterized by a parameter  $r \in (0, \infty)$ , has been popularized by Tullock (1980). Pure-  
19 strategy Nash equilibria have been identified for low values of  $r$  (Mills, 1959; Pérez-Castrillo and  
20 Verdier, 1992; Nti, 1999, 2004; Cornes and Hartley, 2005), and mixed-strategy equilibria for high  
21 values of  $r$  (Baye et al., 1994; Alcade and Dahm, 2010; Ewerhart, 2015, 2016). For intermediate values  
22 of  $r$  and heterogeneous valuations, Wang (2010) has constructed additional equilibria in which only  
23 one player randomizes.

24 The present paper complements and, in a sense, completes the equilibrium analysis of Tullock's  
25 model in the important special case of two players and heterogeneous valuations. We first show  
26 that, for  $r \leq 2$ , the equilibrium is unique. This observation is useful because for  $r > 2$ , the usual  
27 equilibrium characteristics, such as expected efforts, participation probabilities, winning probabilities,  
28 expected payoffs, and revenue, are known to be independent of the equilibrium. Then, we document the  
29 properties of the equilibrium, including rent-dissipation, comparative statics, and robustness. Finally,  
30 as an application, we prove a revenue ranking result for optimally biased contests.

31 The remainder of this paper is structured as follows. Section 2 introduces the notation and reviews  
32 existing equilibrium characterizations. Section 3 presents our uniqueness result. Comparative statics  
33 are discussed in Section 4. Section 5 deals with robustness. Optimal discrimination is examined in  
34 Section 6. An Appendix contains an auxiliary result.

## 35 **2. Set-up and notation**

36 There are two players  $i = 1, 2$ . Player  $i$ 's valuation of the prize is denoted by  $V_i$ , where we assume  
37  $V_1 \geq V_2 > 0$ . Given efforts  $x_1 \geq 0$  for player 1 and  $x_2 \geq 0$  for player 2, player  $i$ 's probability of winning  
38 is specified as

$$39 \quad p_i(x_1, x_2) = \frac{x_i^r}{x_1^r + x_2^r}, \quad (1)$$

40 where  $r \in (0, \infty)$ , and the ratio is replaced by  $p_i^0 = 0.5$  should the denominator vanish.<sup>2</sup> Player  $i$ 's  
41 payoff is given by  $\Pi_i = p_i V_i - x_i$ . This defines the **two-player contest**  $\mathcal{C} = \mathcal{C}(V_1, V_2, r)$ .

42 A **mixed strategy**  $\mu_i$  for player  $i$  is probability measure on  $[0, V_i]$ . Let  $\mathcal{M}_i$  denote the set of  
43 player  $i$ 's mixed strategies. Given  $\mu = (\mu_1, \mu_2) \in \mathcal{M}_1 \times \mathcal{M}_2$ , we write  $p_i(\mu_1, \mu_2) = E[p_i(x_1, x_2) | \mu]$  and

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<sup>1</sup>Cf. Konrad (2009).

<sup>2</sup>The assumption on  $p_i^0$  will be relaxed in Section 5.

44  $\Pi_i(\mu_1, \mu_2) = E[\Pi_i(x_1, x_2)|\mu]$ , where  $E[\cdot|\mu]$  denotes the expectation operator. An **equilibrium** is a  
 45 pair  $\mu^* = (\mu_1^*, \mu_2^*) \in \mathcal{M}_1 \times \mathcal{M}_2$  satisfying  $\Pi_1(\mu_1^*, \mu_2^*) \geq \Pi_1(\mu_1, \mu_2^*)$  for any  $\mu_1 \in \mathcal{M}_1$  and  $\Pi_2(\mu_1^*, \mu_2^*) \geq$   
 46  $\Pi_2(\mu_1^*, \mu_2)$  for any  $\mu_2 \in \mathcal{M}_2$ .

47 For an equilibrium  $\mu^* = (\mu_1^*, \mu_2^*)$ , we define player  $i$ 's **expected effort**  $\bar{x}_i = E[x_i|\mu_i^*]$ , **participa-**  
 48 **tion probability**  $\pi_i = \mu_i^*(\{x_i > 0\})$ , **winning probability**  $p_i^* = p_i(\mu_1^*, \mu_2^*)$ , and **expected payoff**  
 49  $\Pi_i^* = p_i^*V_i - \bar{x}_i$ , as well as the designer's **revenue**  $\mathcal{R} = \bar{x}_1 + \bar{x}_2$ . An equilibrium  $\mu^*$  is an **all-pay auc-**  
 50 **tion equilibrium** if it shares these characteristics with the unique equilibrium of the corresponding  
 51 all-pay auction (Alcade and Dahm, 2010).

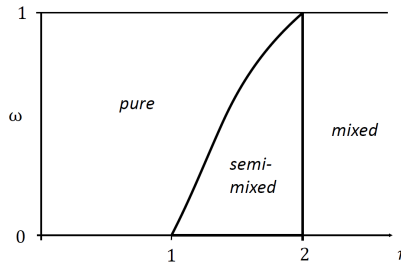
52 Let  $\omega = V_2/V_1$ . The following three propositions summarize much of the existing equilibrium  
 53 characterizations.

54 **Proposition 1.** (Mills, 1959; Pérez-Castrillo and Verdier, 1992; Nti, 1999, 2004; Cornes  
 55 and Hartley, 2005) *A pure-strategy equilibrium exists if and only if  $r \leq 1 + \omega^r$ . This equilibrium is*  
 56 *interior, and unique within the class of pure-strategy equilibria.*<sup>3</sup>

57 **Proposition 2.** (Baye et al., 1994; Alcade and Dahm, 2010; Ewerhart, 2015, 2016) *For any*  
 58  *$r \geq 2$ , there exists an all-pay auction equilibrium. Moreover, for  $r > 2$ , any equilibrium is an all-pay*  
 59 *auction equilibrium, and both players randomize.*

60 **Proposition 3** (Alcade and Dahm, 2010; Wang, 2010). *For any  $r \in (1 + \omega^r, 2]$ , there exists an*  
 61 *equilibrium in which player 1 chooses a pure strategy, while player 2 randomizes between zero and a*  
 62 *positive effort.*

63 For convenience, the cases captured by Propositions 1 through 3, respectively, will be referred to as  
 64 the pure, mixed, and semi-mixed cases. See Figure 1 for illustration.<sup>4</sup>



65  
 66 Figure 1: The parameter space.

<sup>3</sup>For homogeneous valuations and  $r \leq 2$ , the equilibrium is known to be unique even within the class of all equilibria.

<sup>4</sup>Note the overlap between the cases. Indeed, for  $r = 2$  and  $\omega = 1$ , the all-pay auction equilibrium is in pure strategies. Further, for  $r = 2$  and  $\omega < 1$ , the semi-mixed equilibrium is an all-pay auction equilibrium.

67 **3. Uniqueness**

68 The following result is key to all what follows.

69 **Proposition 4.** *For any  $r \leq 2$ , there is precisely one equilibrium.*

70 **Proof.** Assume first that  $r \leq 1 + \omega^r$ . By Proposition 1, there exists an interior pure-strategy  
 71 equilibrium  $(x_1^*, x_2^*)$ . Moreover, the only candidate for an alternative best response to  $x_1^*$  is the zero  
 72 bid (Pérez-Castrillo and Verdier, 1992; Cornes and Hartley, 2005). Since equilibria in contests are  
 73 interchangeable (cf. the Appendix), the support of any alternative equilibrium strategy must be a  
 74 subset of  $\{0, x_2^*\}$ . However, player 1's first-order necessary condition for the interior optimum,

$$75 \quad \frac{\partial p_1(x_1^*, x_2^*)}{\partial x_1} V_1 \pi_2 - 1 = 0, \quad (2)$$

76 holds for  $\pi_2 = 1$ , so that it cannot hold for  $\pi_2 < 1$ . By an analogous argument, necessarily  $\pi_1 = 1$   
 77 and, hence, the equilibrium is unique in this case. Assume next that  $r > 1 + \omega^r$ . By Proposition 3,  
 78 there exists a semi-mixed equilibrium in which player 1 uses a pure strategy  $x_1^* > 0$ , while player 2  
 79 randomizes, choosing some  $x_2 = x_2^*$  with probability  $\pi_2 \in (0, 1)$ , and  $x_2 = 0$  otherwise. As above,  
 80 it follows that player 2's best-response set is  $\{0, x_2^*\}$ . Any alternative equilibrium strategy could,  
 81 therefore, only use a different probability  $\pi_2$  of randomization across the set  $\{0, x_2^*\}$ . But this is  
 82 impossible in view of (2), which must hold also in the semi-mixed case. Moreover, by the construction  
 83 of the semi-mixed equilibrium (Alcade and Dahm, 2010; Wang, 2010), player 1's best-response set  
 84 is the same as in the associated pure-strategy equilibrium in the contest  $\hat{\mathcal{C}} = \mathcal{C}(\hat{V}_1, V_2, r)$ , with  $\hat{V}_1 =$   
 85  $V_2/(1 - r)^{1/r}$ . Hence,  $x_1^*$  is the unique best response, and uniqueness of the equilibrium follows as  
 86 above.  $\square$

87 Proposition 4 implies, in particular, that for  $r = 2$ , there does not exist any equilibrium other than  
 88 the all-pay auction equilibrium identified by Alcade and Dahm (2010, Ex. 3.3).<sup>5</sup>

89 Define **rent dissipation** as the fraction  $\phi_i = \bar{x}_i/V_i$  of the valuation spent by player  $i$ . In the pure  
 90 and mixed cases,  $\phi_i$  is known to be identical for the two players, with  $\phi \equiv \phi_1 = \phi_2$  being strictly  
 91 increasing in  $\omega$ . As noted by Wang (2010), this extends to the semi-mixed case, where

$$92 \quad \phi = \alpha(r) \frac{\omega}{2}, \quad (3)$$

---

<sup>5</sup>Unfortunately, however, the argument does not deliver uniqueness for  $r > 2$  because the best-response set is countably infinite in that case.

93 with

$$94 \quad \alpha(r) = \frac{2}{r}(r-1)^{\frac{r-1}{r}}. \quad (4)$$

95 The present analysis shows that  $\phi$  is globally strictly increasing in  $\omega$  for any  $r \in (0, \infty)$ , regardless of  
96 the equilibrium.

#### 97 4. Comparative statics

98 Table I provides an overview of the comparative statics of the equilibrium.<sup>6</sup> As can be seen, the  
99 comparative statics of the semi-mixed equilibrium with respect to  $V_1$  and  $V_2$  is identical to that of  
100 the all-pay auction. The comparative statics of the semi-mixed equilibrium with respect to  $r$  is as  
101 follows. As the contest becomes more decisive, expected efforts, player 2's participation probability,  
102 and revenue are all strictly declining towards the respective all-pay auction levels. In contrast, player  
103 1's winning probability and expected payoff are both strictly increasing towards the respective all-pay  
104 auction levels.

pure				semi-mixed				mixed		
$r \leq 1 + \frac{V_2^r}{V_1^r}$				$r \in (1 + \frac{V_2^r}{V_1^r}, 2]$				$r > 2$		
	$dV_1$	$dV_2$	$dr$		$dV_1$	$dV_2$	$dr$		$dV_1$	$dV_2$
$x_1^* = \frac{rV_1^{r+1}V_2^r}{(V_1^r+V_2^r)^2}$	+	+	$\pm$	$x_1^* = \alpha(r)\frac{V_2}{2}$	0	+	-	$\bar{x}_1 = \frac{V_2}{2}$	0	+
$x_2^* = \frac{rV_1^rV_2^{r+1}}{(V_1^r+V_2^r)^2}$	-	+	$\pm$	$\bar{x}_2 = \alpha(r)\frac{V_2^2}{2V_1}$	-	+	-	$\bar{x}_2 = \frac{V_2^2}{2V_1}$	-	+
$\pi_2 = 1$	0	0	0	$\pi_2 = \frac{V_2}{V_1(r-1)^{1/r}}$	-	+	-	$\pi_2 = \frac{V_2}{V_1}$	-	+
$p_1^* = \frac{V_1^r}{V_1^r+V_2^r}$	+	-	+	$p_1^* = 1 - \alpha(r)\frac{V_2}{2V_1}$	+	-	+	$p_1^* = 1 - \frac{V_2}{2V_1}$	+	-
$\Pi_1^* = \frac{V_1^{r+1}(V_1^r+V_2^r-rV_2^r)}{(V_1^r+V_2^r)^2}$	+	-	$\pm$	$\Pi_1^* = V_1 - \alpha(r)V_2$	+	-	+	$\Pi_1^* = V_1 - V_2$	+	-
$\Pi_2^* = \frac{V_2^{r+1}(V_1^r+V_2^r-rV_1^r)}{(V_1^r+V_2^r)^2}$	-	+	-	$\Pi_2^* = 0$	0	0	0	$\Pi_2^* = 0$	0	0
$\mathcal{R} = \frac{rV_1^rV_2^r(V_1+V_2)}{(V_1^r+V_2^r)^2}$	$\pm$	+	$\pm$	$\mathcal{R} = \alpha(r)(\frac{V_2}{2} + \frac{V_2^2}{2V_1})$	-	+	-	$\mathcal{R} = \frac{V_2}{2} + \frac{V_2^2}{2V_1}$	-	+

106 Table I: Comparative statics.

107 One can check that all the equilibrium characteristics listed in the table depend continuously on  
108 parameters. In other words, there are no jumps in the possible transitions between pure, semi-mixed,  
109 and mixed equilibria. This allows deriving global comparative statics results. For example, Yildirim  
110 (2015) has made the intuitive observation that, if the contest technology exhibits decreasing returns,  
111 the weaker player never prefers a more decisive contest. But, as  $d\Pi_2^*/dr \leq 0$  holds globally, the same  
112 conclusion holds for technologies with constant or increasing returns.

<sup>6</sup>The table summarizes and extends the results of Nti (1999, 2004), Wang (2010), and Yildirim (2015).

113 **5. Robustness**

114 So far, we assumed that  $p_1^0 = p_2^0 = 0.5$ . However, as our next result shows, this assumption is not  
 115 crucial.

116 **Proposition 5.** *The equilibrium set remains unchanged when  $p_1^0, p_2^0 \in [0, 1]$  and  $p_1^0 + p_2^0 \leq 1$ .*

117 **Proof.** Let  $\mu^* = (\mu_1^*, \mu_2^*)$  be an equilibrium under the modified rules that is not an equilibrium in  $\mathcal{C}$ .  
 118 Since mutual inactivity cannot occur with positive probability in  $\mu^*$ , some player  $i$  finds a deviation  
 119 to zero profitable in  $\mathcal{C}$ , but not profitable under the modified rules. Moreover,  $\mu_j^*$ , with  $j \neq i$ , must  
 120 have an atom at zero, and  $p_i^0 < 0.5$ . But then, player  $i$  has a profitable deviation to some small  $x_i > 0$   
 121 both in  $\mathcal{C}$  and under the modified rules. Contradiction! Conversely, suppose that  $\mu^* = (\mu_1^*, \mu_2^*)$  is  
 122 an equilibrium in  $\mathcal{C}$  that is not an equilibrium under the modified rules. Then some player  $i$  finds  
 123 a deviation to zero profitable under the modified rules, yet not profitable in  $\mathcal{C}$ . Moreover, player  $j$ 's  
 124 mixed strategy  $\mu_j^*$ , with  $j \neq i$ , necessarily has an atom at zero. Given Propositions 1 and 4, this is  
 125 feasible only if  $i = 1$  and  $r > 1 + \omega^r$ . In the semi-mixed case, however, bidding zero yields a payoff for  
 126 player 1 of

$$127 \quad \Pi_1 = p_1^0 V_1 (1 - \pi_2) \leq V_1 (1 - \pi_2) = V_1 - \frac{V_2}{(r-1)^{1/r}}, \quad (5)$$

128 which is weakly less than

$$129 \quad \Pi_1^* = \left\{ \pi_2 \frac{(x_1^*)^r}{(x_1^*)^r + (x_2^*)^r} + 1 - \pi_2 \right\} V_1 - x_1^* = V_1 - \frac{2(r-1)^{\frac{r-1}{r}}}{r} V_2, \quad (6)$$

130 because  $2(r-1)/r \leq 1$ . Similarly, in the mixed case,  $\Pi_1^* = V_1 - V_2$ , whereas a deviation to zero yields  
 131 only  $\Pi_1 = p_1^0 V_1 (1 - \pi_2) \leq V_1 - V_2$ . Contradiction!  $\square$

132 **6. Optimally biased contests**

133 Suppose now that a designer may inflate or deflate player 2's effort by a factor  $\lambda > 0$ . I.e., players'  
 134 probabilities of winning are given in the interior by

$$135 \quad p_1^\lambda(x_1, x_2) = \frac{x_1^r}{x_1^r + (\lambda x_2)^r} \quad (7)$$

136 and  $p_2^\lambda = 1 - p_1^\lambda$ . Let  $\phi(\lambda)$  denote the rent-dissipation in the contest  $\mathcal{C}^\lambda = \mathcal{C}(V_1^\lambda, V_2^\lambda, r)$ , where  
 137  $V_1^\lambda = \max\{V_1, \lambda V_2\}$  and  $V_2^\lambda = \min\{V_1, \lambda V_2\}$ . Since players act *as if* in  $\mathcal{C}^\lambda$ , the **revenue from the**



138 **biased contest** is given by

$$139 \quad \mathcal{R}(\lambda) = (V_1 + V_2)\phi(\lambda). \quad (8)$$

140 Franke et al. (2014) obtained a dominance result. Epstein et al. (2013) compared pure-strategy  
 141 equilibria directly with all-pay auction equilibria. The following result ranks a continuum of contest  
 142 technologies, explicitly taking into account the possibility of semi-mixed equilibria.

143 **Proposition 6.** *For any  $r \in (0, \infty)$ , the revenue-maximizing bias is  $\lambda^* = 1/\omega$ , with*

$$144 \quad \mathcal{R}(\lambda^*) = \min\left\{\frac{r}{2}, 1\right\} \cdot \frac{V_1 + V_2}{2} \quad (9)$$

145 *Thus, the revenue from the optimally biased contest is strictly increasing for  $r \leq 2$ , and constant for*  
 146  *$r \geq 2$ .*

147 **Proof.** Suppose first that  $r \leq 2$ . In a pure-strategy equilibrium, maximizing

$$148 \quad \mathcal{R}(\lambda) = \frac{rV_1^r(\lambda V_2)^r(V_1 + V_2)}{(V_1^r + \lambda^r V_2^r)^2} \quad (10)$$

149 yields the solution  $\lambda^* = 1/\omega$ , with  $\mathcal{R}(\lambda^*) = (r/4) \cdot (V_1 + V_2)$ . For a semi-mixed equilibrium,

$$150 \quad \mathcal{R}(\lambda) = \begin{cases} \frac{\lambda\omega}{2}\alpha(r)(V_1 + V_2) & \text{if } \lambda\omega < (r-1)^{1/r} \\ \frac{1}{2\lambda\omega}\alpha(r)(V_1 + V_2) & \text{if } \lambda\omega > (r-1)^{1/r}. \end{cases} \quad (11)$$

151 In the first case,  $\mathcal{R}(\lambda)$  is strictly increasing in  $\lambda$ . In the second case,  $\mathcal{R}(\lambda)$  is strictly declining in  $\lambda$ .  
 152 Hence, it is strictly suboptimal to implement a semi-mixed equilibrium. For  $r > 2$ , the claim has been  
 153 proved by the author in prior work (2016).  $\square$

## 154 **Appendix A. An auxiliary result**

155 Two equilibria  $(\mu_1^*, \mu_2^*)$  and  $(\mu_1^{**}, \mu_2^{**})$  are called **interchangeable** if  $(\mu_1^*, \mu_2^{**})$  and  $(\mu_1^{**}, \mu_2^*)$  are equi-  
 156 libria as well.

157 **Lemma A.1.** *Equilibria in two-player contests are interchangeable.*

158 **Proof.** The proof is a straightforward adaptation on an argument detailed in Klumpp and Polborn  
 159 (2006, p. 1104), and therefore omitted.  $\square$

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