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Global Demographic Change and Climate Policies

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Global Demographic Change and Climate Policies

Abstract

Between 1950 and 2017, world average life expectancy increased from below-50 to above-70, while the fertility rate dropped from 5 to about 2.5. We develop and calibrate an analytic climate-economy model with overlapping generations to study the effect of such demographic change on capital markets and optimal climate policies. Our model replicates findings from the OLG-demography literature, such as a rise in households' savings, and a declining rate of return to capital. We also find that demographic change raises the social cost of carbon, at 2020, from 28 euro/tCO_2 in a model that abstracts from demography, to 94 euro/tCO_2 in our calibrated model.

JEL-Codes: H230, J110, Q540, Q580.

Keywords: climate change, social cost of carbon, environmental policy, demographic trends.

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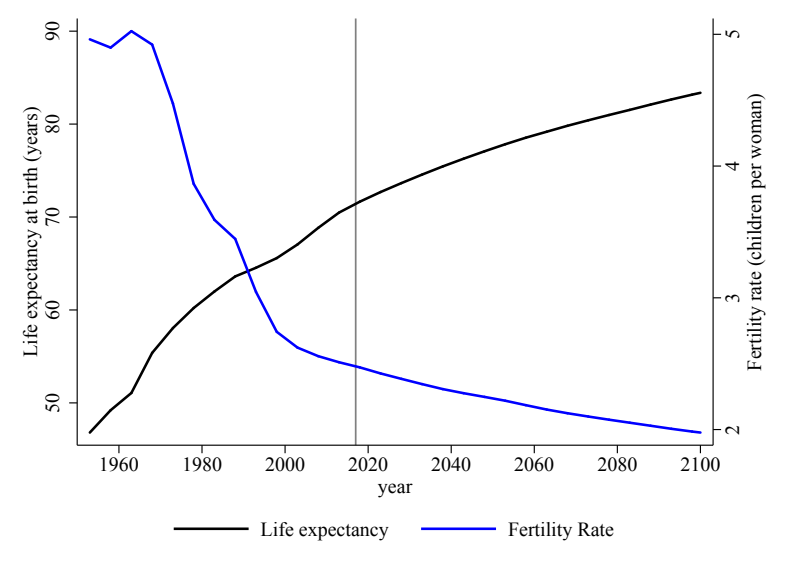
1. Introduction

Between 1950 and 2017, world average life-expectancy increased from below-50 to above-70, while the fertility rate dropped from 5 to about 2.5 children per woman (Figure 1). The worldwide rise of life-expectancy and fall of total fertility rate is expected to robustly continue for the remainder of the current century, with the emerging economies catching up the patterns typical for economies that industrialized before. The worldwide demographic trends decreases the ratio of young relative to old and change the propensity to save of the average consumer. Both supply of labour and capital will adjust, and future capital returns will most likely differ from those in the past.

Figure 1: Global demographic patterns, 1950-2100



Note: The year 2017 is indicated with a vertical line. Source: United Nations. World Population Prospects. Medium-variant projections. The projections do not take into account projected climate change, but the expected effect is small compared to other major drivers of demographic change.

Climate policies search the optimal trade-off between consumption, investments in man-made capital, and investment in natural capital through emissions reductions (Nordhaus, 1993). How should climate policies adjust in response to global demographic change? Using a stylized overlapping generations model (based on Fanti and Gori (2012); Cipriani (2014)), we analytically and quantitatively analyze the effects of an increase in life-expectancy and a decrease in fertility rates on households' savings, aggregate capital investments, and efficient climate policies. For comparison we also study demography and optimal climate policy in a Ramsey-Cass-Koopmans model (as in Nordhaus (1993)).

The overlapping generations (OLG) model is the standard approach to study demography's effects on savings and investments. The Ramsey-Cass-Koopmans (RCK) model, with an infinitely-lived representative agent, is the most common type of model to study climate policy in the context of economic growth. We compare the effects of demography on climate policy in both types of models. The OLG model suggests that falling fertility and increasing life expectancy leads to a robust increase in future households savings. In turn, these lead to a substantial tightening of *present* efficient climate policies, that is, a large increase in the (present) efficient carbon price.[1] The reason that future capital markets have such robust effects for present climate policies comes from the extreme persistence of climate change (Gerlagh and Liski, 2017b). The same demographic patterns, in the RCK model, result in a decrease in both the future savings rate and the present carbon price. We show that the OLG and RCK types of model thus provide radically different outcomes. Therefore, we call for a closer look at previous quantitative estimates for the social costs of carbon (SCC), which mostly neglected the effects of demographic patterns on capital markets and climate policies.

The intuition for our main results, derived with the OLG model, is straightforward. The two worldwide demographic trends, increasing life expectancy and decreasing fertility, both increase household savings. First, since people expect to live longer, they require more resources when old after they leave the labour market.[2] Second, a decrease in the fertility rate causes an increase in labor supply when young, and thus, a rise in income, which is partly saved to smooth consumption over the life-cycle. As capital stocks increase, the returns fall, and it becomes relatively more attractive to invest in climate change mitigation.

Our analytical model presents an analytic formula for the social cost of carbon, connecting to a large literature. A large literature on the social costs of carbon is devoted to the question of the discount rate by which future climate damages are discounted before aggregation in the Net Present Value (Weitzman, 2013).[3] Both the so-called 'pure' discount rate for utility streams and the discount rate for consumption goods feature

[1] In this paper, we use the terms (efficient) carbon price and the social costs of carbon (SCC) interchangeably. They refer to the net present value of damages associated with one extra unit of CO2 emissions.

[2] Our OLG model base set up is a fully funded pension system. We do not examine the effects of changes in institutions that affect households' transfer of resources from their young age to their old age, e.g., the pension system.

[3] See Guo et al. (2006) for an analysis of the consequences of declining discount rates on estimates of the social cost of carbon. See also Karp and Traeger (2013) and Weitzman (2013) for a discussion about discounting in cost-benefit analysis and Mastrandrea and Schneider (2001) for a better understanding of the effects of relatively low discount rates on climate policies. van den Bergh and Botzen (2015) present a recent survey of this literature.

prominently in this literature. Recently, van den Bijgaart et al. (2016) presented an alternative discount rate measure. They show that the relative value of future output compared to present output, $b_t = \frac{Y_{t+1}}{r_{t+1}Y_t}$ (labeled the effective discount factor), where Y_t is output and r_{t+1} is the marginal rate of substitution between the two periods, captures the most important discount information required to calculate the efficient carbon price.[4] We use the effective discount factor to gauge the implications of our approach for the SCC.

Table 1 reports the values of the key parameters and the implied effective annual discount factors b_t used in previous studies. Before reading the table, we note that in most models, the effective discount factor varies with demography. To the best of our knowledge, this is the first paper that makes this connection explicit. The previous studies reported in Table 1 rely on the Ramsey-Cass-Koopmans (RCK) model, in which the Ramsey rule, $r_t = \beta^{-1} m_t^{1-\xi} \gamma_t^{\eta}$, connects the marginal rate of substitution, r_t, to the pure time preference factor β,[5] population growth m_t, where $\xi \in \{0, 1\}$ is the population weight in the social planner problem,[6] the elasticity of marginal utility η and the per capita growth factor of consumption γ_t. The effective discount factor b_t then also depends on the scale of the economy through both population growth m_t and productivity growth, which approximately also measures per capita consumption growth, $Y_{t+1}/Y_t \approx m_t \gamma_t$, so that $b_t \approx m_t \gamma_t / r_t = \beta m_t^{\xi} \gamma_t^{1-\eta}$.[7]

As noted in the literature, the effective discount factor is sensitive to parameter setting procedures. For instance, whereas Stern (2007), following an ethics-based approach, chooses a rate of pure time preference of 0.1% per year, Nordhaus (2008), Nordhaus (2014) and Golosov et al. (2014), using a market-based approach, set a rate of 1.5%. Higher values for this parameter can be found in Weitzman (2007) and Nordhaus (1993) (first column entries of Table 1). Importantly, comparing the columns b_t for 2010 and 2100, we see that because population growth, m_t, is expected to decrease in the coming century, in the standard RCK model, the implied effective discount factor, b_t, decreases as well. In contrast, in the OLG model, the effective annual discount factor b_t does not

[4] See eq (18) in this paper.

[5] We can switch between rates and factors, through $\beta = 1/(1+\rho)$ where ρ is the pure time preference rate.

[6] For Benthamite welfare weighing utility by population size, $\xi = 1$ and the rate of substitution only depends on per capita consumption growth. For Millian welfare aggregating average utility, $\xi = 0$ and the rate of substitution increases with population growth. See Canton and Meijdam (1997) for more details.

[7] We use an approximate sign because equality requires that the consumption share is constant, which is approximately correct.

depend on population growth directly as in the RCK model, but only indirectly through its effects on life-cycle savings. The effective discount factor varies with demographic factors, most importantly life-expectancy, through changes in demand for capital as a retirement investment. To relate our results to the existing literature, we calibrate the model parameters such that the (time-varying) effective annual discount factor at 2010 equals the constant discount factor $b = 0.985$ used in Golosov et al. (2014). As the table shows, we find the effective discount factor to increase over the next century, rather than to decrease.

Table 1: Effective annual discount factors and social costs of carbon

	β	η	ξ	b_t		SCC_t	
				2010	2100	2010	2100
Nordhaus (1993)	0.970	1	1	0.982	0.971	14.2	110
Weitzman (2007)	0.980	2	1	0.974	0.962	10.7	84.1
Stern (2007)	0.999	1	1	1.011	1.000	-	-
Nordhaus (2008)	0.985	2	1	0.979	0.967	12.5	96.3
Nordhaus (2014)	0.985	1.45	1	0.989	0.978	18.7	140
Golosov et al. (2014)	0.985	1	0	0.985	0.985	21.2	206
This paper	n.a.	1	n.a	0.985	0.996	63.8	794

Note: β: the discount factor for utility between generations. Its value is exogenous in Ramsey-Cass-Koopmans models, and endogenous in our model. η: the elasticity of marginal utility (the inverse of the elasticity of intertemporal substitution). ξ: indicator parameter for weighting utility. $b_t \approx \beta m_t^{\xi} \gamma_t^{1-\eta}$: the effective discount factor. For per capita consumption growth we use $\gamma = 1.019$ for 2010 and $\gamma = 1.02$ for 2100. For population growth, we use $m = 1.012$ for 2010 and $m = 1.001$ for 2100. SCC_t: The social costs of carbon [EUR/tCO$_2$] tends to increase with b_t and with income, see eq (18), and Appendix C for the quantitative calculation of b_t.

The quantitative effects on the social costs of carbon, of these divergent patterns for the discount factor, are shown in the last two columns of Table 1. For instance, at our calibration year 2010, a model that features no demographics as described in Golosov et al. (2014) yields a carbon price 67% lower than the one related to our benchmark model. This difference is even higher when our OLG model is compared with the other Ramsey-Cass-Koopmans models that impose a negative effect of decreasing population growth rates on optimal savings.

2. Further literature

Our paper relates to three strands of literature.[8] Firstly, there are a few numerical Integrated Assessment Models (IAMs) that study climate policies in an economy with overlapping generations (OLG). Howarth (1998) and Gerlagh and van der Zwaan (2001) are the closest to this paper. Recently, Quaas and Bröcker (2016) developed an analytical OLG-climate model.[9] We extend these papers by providing a rich set of analytic results that characterize formally the effect of demography on climate policies. We add a formal analysis of demography in the (competing) Ramsey-Cass-Koopmans (RCK) model type. Furthermore, we set up a calibration procedure that quantifies the effects through a transparent approach, as compared to the quantitative IAMs with many moving parts in which causality is hard to gauge.

Secondly, we relate to recent economic research suggesting that demographic patterns over the last decades have increased aggregate household savings rates and decreased the capital returns, mainly through shifts in the age structure of the population (Canton and Meijdam, 1997; Bloom et al., 2003, 2007; Eggertsson and Mehrotra, 2014; Curtis et al., 2015; Carvalho et al., 2016; Imrohoroglu and Zhao, 2017). Demography also drives part of international trade and capital flows, which reflect discrepancies between savings and investments across countries (Backus et al., 2014; Fedotenkov et al., 2014; Krueger and Ludwig, 2007). Demographic change has been connected to housing prices, since lower returns to capital in the future create incentives for seeking alternative stores of value (Chen and Wen, 2017), and changes in social security systems due to population aging (Imrohoroglu and Zhao, 2017; Cipriani, 2014; Fanti and Gori, 2012). Curtis et al. (2015) show that demographic patterns are able to explain over half of the household savings rate fluctuations in China, while Sánchez-Romero (2013) finds that the demographic transition in Taiwan, during the period 1965-2005,

[8] Notice that in our modelling, demographic variables are exogenous and temperature changes only affect production. Several studies assess the effect of population aging on emissions and the environment, see Balestra and Dottori (2012), Jouvet et al. (2010), Varvarigos (2010), Mariani et al. (2010), and Ono (2005). De la Croix and Gosseries (2012) study the effect of climate policies on fertility and human capital. In a similar vein, with respect to the relationship between carbon emissions and population, Casey and Galor (2017) provide empirical evidence that a reduction in fertility rates, while increases income per capita, diminishes carbon emissions. Likewise, Dalton et al. (2008) provides an analysis of the effect of population aging on U.S energy use and carbon emissions. None of these studies point to the impact of demographic patterns on savings rates and, subsequently, on climate policies.

[9] Karp and Rezai (2014) use an OLG model for a somewhat different purpose. They show that if the environmental benefits of climate policies can be capitalized in an increased value of the capital stock, current generations also benefit from environmental policies.

substantially increased the investment rate, leading to more capital and higher GDP per capita growth.[10] Going forward, while our model formulation can reproduce the results mentioned above, we add to this literature the impact of changing savings, due to population dynamics, on the evaluation of expected future environmental damages and climate policies.

Thirdly, our theoretical framework also builds on an emerging analytical literature about optimal carbon taxes as dependent on a few parameters: pure time preferences, a carbon cycle structure, temperature adjustments, and expected damages due to climate change (Golosov et al., 2014).[11] Others have added risk updating when climate information arrives (Gerlagh and Liski, 2017a), temperature delays and general time preferences (Gerlagh and Liski, 2017b),[12] and tax distortions (Barrage, 2016). We add demographic parameters to these.

The remainder of this paper proceeds as follows. Section 3 presents the benchmark model and describes the equilibrium concept. Section 4 discusses existence issues and provides a comparative statics analysis. Section 5 introduces the RCK model with demography and outlines optimal climate policies. Section 6 includes a quantitative assessment and discusses parameter calibration, data, and the climate module, to calculate the social cost of carbon. Section 7 concludes.

3. The model

We consider a two-period OLG model with full depreciation and log utility based on Cipriani (2014) and Fanti and Gori (2012) to analyze the impact of changes in fertility and life-expectancy on savings and, therefore, through the aggregate capital market, the effect of the demographic transition on environmental policies. Time is discrete and runs to infinity. The economy's technology is based on Gerlagh and Liski (2017b), while our quantitative assessment employs the (median) climate-damage description listed in van den Bijgaart et al. (2016).

[10] It is important to mention that we model a closed economy and therefore the aggregate savings rate equals the investment rate.

[11] For a discussion of the implications and robustness of simple carbon pricing rules see e.g., Barrage (2014), van den Bijgaart et al. (2016), and Rezai and van der Ploeg (2015).

[12] See also Iverson (2012); Belfiori (2013) for discussions on time preferences and the SCC formula.

3.1. Households

Each agent lives with certainty during the first period, and faces a probability h_t of surviving to the second period. Individuals work when young and consume their savings when old. Consumption by children is not explicitly modeled, but the time that parents spend raising their children, $\chi n_t < 1$, where n_t is the number of children and χ is the time per child, is deducted from labour supply. That is, the size of each generation is denoted by N_t, with $N_{t+1} = n_t N_t$, and $M_t = N_t + h_{t-1} N_{t-1}$ is the total population. Only the young generation works, and the labour supply is inelastic, given by

$$L_t = (1 - \chi n_t) N_t. \tag{1}$$

The household chooses consumption and savings to maximize life-time utility,

$$\max_{\{c_{1,t}, c_{2,t+1}, s_t\}} u_t = u_{1,t} + \delta h_t u_{2,t+1}, \tag{2}$$

where $u_{1,t} = \ln(c_{1,t})$, $u_{2,t} = \ln(c_{2,t+1})$, subject to the budget constraints when young and old,

$$c_{1,t} + s_t + \chi w_t n_t = w_t + \tau_t, \tag{3}$$

$$h_t c_{2,t+1} = r_{t+1} s_t, \tag{4}$$

where $c_{1,t}$ and $c_{2,t+1}$ denote consumption at young and old age (when living); s_t are savings, $\delta \in (0, 1)$ refers to the discount factor; $h_t \in (0, 1)$ is the probability of surviving (longevity) which varies exogenously over time, so that $h_t c_{2,t+1}$ is expected consumption when old. The variable r_{t+1} is the return to savings; w_t represents the wage that individual receives for supplying inelastically one unit of labour; τ_t is the per capita government lump-sum transfer (recycling of carbon tax revenues, discussed below), n_t is the exogenous fertility rate and $\chi \in (0, 1)$ denotes the fraction of time that it is needed for raising children. Note that we study the effects of (exogenous) demography on climate policies. By considering endogenous fertility and health choices, as in Becker and Lewis (1973), the analysis can be extended to describe demographic policies as part of climate change policies (Harford, 1998; Murtaugh and Schlax, 2009; Bohn and Stuart, 2015).[13]

[13] These papers do not look at the effect of demography on climate policies through capital markets.

The first-order conditions give the usual Euler equation:

$$\frac{c_{2,t+1}}{c_{1,t}} = \delta r_{t+1}. \tag{5}$$

Using the Euler equation and the budget constraints (3,4), the savings of households can be derived as

$$s_t = \frac{\delta h_t}{1+\delta h_t}(w_t(1-\chi n_t)+\tau_t). \tag{6}$$

For analysis, we assume that demography converges to some long-term trend.

Assumption 1. *Fertility and life-expectancy converge to long-term values:* $n_t \to n_\infty > 0$, $h_t \to h_\infty \leq 1$.

3.2. Production and Climate

Following Gerlagh and Liski (2017b), we model the economy's technology, which is a function of capital, labour, fossil fuel energy, and the past emissions, in the following way:

$$Y_t = F(K_t, E_t, L_t; z_t), \tag{7}$$

with constant returns to scale in capital K_t , labour L_t , and emissions E_t (proximated by use of fossil fuel energy sources), and $z_t = (E_{t-1}, E_{t-2}, \dots)$ is the history of emissions. Production has constant elasticity in capital, and climate change damages are described through a multiplicative factor,

$$F(K_t, E_t, L_t; z_t) = \Omega(z_t) K_t^\alpha [A_t(E_t, L_t)]^{1-\alpha}, \tag{8}$$

with $\alpha \in (0,1)$, and $\Omega(z_t)$ refers to output loss due to past emissions. $A_t(E_t, L_t)$ has constant returns to scale, and is time dependent, accounting for effective energy-labour combination. When convenient, we write $a_t \equiv A_t/L_t$ as labour-augmenting productivity, which only depends on per labour emissions, $a_t = a_t(e_t)$, with $e_t \equiv E_t/L_t$. One can rearrange the production function into per effective labour terms to get

$$y_t = \Omega(z_t) k_t^\alpha a_t^{1-\alpha}, \tag{9}$$

where $y_t \equiv Y_t/L_t$ and $k_t \equiv K_t/L_t$ are output- and capital-labour ratios, respectively.

Assumption 2. *Effective labour productivity $a_t(e_t)$ is continously differentiable, increasing in emissions per labourer, strictly concave, emissions are not essential and become zero for a finite carbon price, $a'_t > 0$ and $a''_t < 0$ for $e_t < e_t^{\max}$, for some $e_t^{\max} > 0$; $a'_t(e_t^{\max}) = 0$, $a_t(0) > 0$, $a'_t(0) < \infty$.*

We keep a general climate impact structure. Golosov et al. (2014) relate climate impacts to atmospheric CO_2 stocks, while Gerlagh and Liski (2017a) provide an in-depth discussion of the impact function based on delays in temperature change. The general formulation is

$$\Omega(z_t) = \exp(-\sum_{i=1}^{t-1} \theta_i E_{t-i}), \tag{10}$$

where θ_i can be interpreted as a representation of the carbon cycle and temperature adjustments (see van den Bijgaart et al. (2016)).

Each period, a representative firm uses capital K, labour L and energy E to produce a final good using a Cobb-Douglas technology and solves the following problem:

$$\max_{\{K_t, L_t, E_t\}} F(K_t, E_t, L_t; z_t) - r_t K_t - w_t L_t - p_t E_t, \tag{11}$$

where p_t refers to the carbon emissions price, and the firm considers Ω_t as exogenous. The first-order conditions imply

$$r_t = \alpha \frac{Y_t}{K_t}, \tag{12}$$

$$w_t = (1-\alpha) Y_t \frac{A_{L,t}}{A_t}, \tag{13}$$

$$p_t = (1-\alpha) Y_t \frac{A_{E,t}}{A_t}. \tag{14}$$

where $A_{X,t}$ is the derivative of A_t with respect to X_t. We have that labour income plus government transfers are a constant share of output,

$$w_t L_t + p_t E_t = (1-\alpha) Y_t.$$

3.3. Carbon policies

The carbon regulator can set carbon taxes p_t, and lump-sum income transfers τ_t. The regulator is subject to an intertemporal budget condition: the net present value of car-

bon tax revenues equals lump-sum transfers,

$$\sum_{t=1}^{\infty} \frac{N_t \tau_t}{r_1^t} = \sum_{t=1}^{\infty} \frac{p_t E_t}{r_1^t}, \tag{15}$$

where r_t^{t+i} is the capital return between period t and $t+i$, $r_t^{t+i+1} \equiv r_t^{t+i} r_{t+i+1}$. As a central case, detailed in the next section, we consider equilibria where the government budget balances within each period,

$$N_t \tau_t = p_t E_t. \tag{16}$$

In Section 4.1 we derive equilibrium rules for carbon pricing p_t. Here, we define the market costs of carbon as the net present value of future marginal damages evaluated at market prices,

$$MCC_t = \sum_{i=1}^{\infty} \frac{1}{r_t^{t+i}} \frac{\partial Y_{t+i}}{\partial \Omega_{t+i}} \frac{-\partial \Omega_{t+i}}{\partial E_t}. \tag{17}$$

When we substitute (10) in (8), we find

$$MCC_t = \sum_{i=1}^{\infty} \frac{\theta_i Y_{t+i}}{r_t^{t+i}} = Y_t \sum_{i=1}^{\infty} \theta_i b_t^{t+i}, \tag{18}$$

where b_t^{t+i} is the net present value of future output, at $t+i$, relative to current output, defined recursively through $b_t^{t+1} \equiv b_t$, $b_t^{t+i} \equiv b_t b_{t+1}^{t+i}$, and

$$b_t \equiv \frac{Y_{t+1}}{r_{t+1}} \frac{1}{Y_t}. \tag{19}$$

Formula (18) for the carbon price is similar to carbon price formulas presented in Golosov et al. (2014), van den Bijgaart et al. (2016), and Gerlagh and Liski (2017b). The carbon price tends to increase proportionally with output as a larger economy values emissions more. In a larger economy, there is more at stake as damages also scale with the economy. Thus, an increase in labour supply, complemented with a proportional increase in capital, will tend to raise output, and the carbon price alongside. If we label the summation term on the RHS of (18) as g_t, then it reads $MCC_t = g_t Y_t$. In the expression, g_t depends on the value of future output, but we will provide conditions when g_t becomes a constant, that is, independent of the current capital stock or future policies. Conditional on Assumption 2, and given capital and labour factor supply, the next lemma establishes a unique static equilibrium in case $g_t Y_t$ is implemented as the

carbon price.

Lemma 1. *For any given* $L_t > 0$, *and* $0 \leq g_t \leq (1-\alpha)\frac{A_{E,t}(0,L_t)}{A_t(0,L_t)}$, *there is a unique* $E_t \geq 0$ *such that*

$$(1-\alpha)\frac{A_{E,t}(E_t, L_t)}{A_t(E_t, L_t)} = g_t.$$

In this economy, emissions versus output tend to follow a so-called Environmental Kuznets curve. For low output levels (low levels of factor supply), carbon prices are negligible and emissions increase approximately proportional with factor supply and output. For high output (factor supply), the term $\frac{A_{E,t}(0,L_t)}{A_t(0,L_t)}$ falls below g_t, and emissions become zero.

Below, we will establish closed-form solutions for b_t, and conditions under which optimal carbon prices equal the market valuation of carbon, $p_t^* = MCC_t$.[14] Novel in our analysis is that we study how the discount factor b_t^{t+i} varies with time as a result of demographic change. Therefore, optimal climate policies adjust to demographic changes as well; (18) becomes demography-dependent. We then compare our solution for the carbon price with a closed-form solution for a Ramsey-Cass-Koopmans model with Benthamite welfare as in Nordhaus (2008) and most other Integrated Assessment Models, and with Millian welfare as in Golosov et al. (2014).

3.4. Equilibrium

In equilibrium, aggregate consumption plus investments equal production,

$$N_t c_{1,t} + h_{t-1} N_{t-1} c_{2,t} + K_{t+1} = Y_t. \tag{20}$$

We can now define the competitive equilibrium as follows:

Definition 1. *Given the parameter set* $\{\chi, \delta, \alpha, (\theta_i)\}$, *a competitive equilibrium is an allocation* $\{c_{1,t}, c_{2,t}, s_t, Y_t, K_{t+1}, E_t, \tau_t\}_{t=1}^{\infty}$, *supported by prices* $\{r_t, w_t, p_t\}_{t=1}^{\infty}$ *such that (i) households maximize their life-time utility, (3-5), (ii) firms maximize profits, (12-14), (iii) the government satisfies the inter-temporal balanced budget, (15), and (iv) the aggregate goods market clearing condition holds, (20).*

The definition leaves open how carbon prices are determined; it does not impose carbon prices to reflect the social costs of carbon. Notice that the definition also does not

[14] We do not consider tax-interaction effects as in Barrage (2016).

consider conditions about income transfers between generations. The first refinement of the equilibrium concept specifies that the regulator does not transfer income between generations. We define:

Definition 2. *A competitive equilibrium is called 'representative' if the government satisfies its budget constraint period-by-period, i.e. (16) holds.*

The definition captures the idea that we want to study carbon taxes in isolation; we want carbon policies to be free from a potential preference for redistributing wealth across generations. The representative equilibrium defines what it means to exclude redistributional preferences.[15] Condition (16) determines the welfare weights, defined shortly below. They depend on demography, but are the same in a representative equilibrium with or without climate damages, and with or without climate policies.

This 'representative' equilibrium is the OLG-equivalent of the infinitely-lived representative agent approach in RCK models such as DICE (Nordhaus, 2008). The typical RCK model chooses its social preference parameters such that, in absence of a climate externality, the model reproduces realistic consumption smoothing and capital investment rates. Similarly, in our model, we calibrate preferences δ such that the competitive equilibrium without intergenerational income transfers generates realistic macro-economic savings rates.[16,17]

There is a connection between inter-generational income transfers τ_t, capital investments K_{t+1} and the value ratio b_{t+1}. We can rewrite the inter-temporal government budget (15) through a public savings variable S_t^g as

$$S_t^g = r_t S_{t-1}^g + p_t E_t - N_t \tau_t, \tag{21}$$

[15] Compare with an economy that has no climate externality, i.e. $\theta = (0, 0, ...)$. The *representative laissez-faire* competitive equilibrium, $\tau_t = p_t = 0$, is then efficient and does not redistribute income.

[16] We could say that the regulator in our economy derives its implicit welfare weights, β_t defined below, from historic macro-economic savings. As we see below in equations (22) and (23), our model also has an equivalent to 'low discounting' in the RCK model. So-called 'low' pure discount rates in RCK models are often considered fair from an ethical perspective, but discarded as inconsistent with revealed social preferences, because they would result in unrealistically high investment rates. In our economy, the equivalent of 'low discounting' is an equilibrium with positive income transfers to future generations, $S_t^g > 0$. Such transfers would result in artificial 'high' investment rates at the macro-economy level and higher implicit welfare weights for future generations.

[17] Technically, the representative equilibrium is similar to the use of Negishi welfare weights in a multi-country model. As an example, the multi-region RICE model (Nordhaus and Yang, 1996) contains an iterative algorithm to find welfare weights that are consistent with zero income transfers between regions and equal *weighted* marginal utility. A consequence of the approach in RICE is that welfare weights depend on climate policies and vary between scenarios.

where we assume $S_0^g = 0$, and we require $\lim_{t\to\infty} S_t^g / r_1^t = 0$. Public savings remain zero, $S_t^g = 0$, if and only if the period-by-period balanced budget (16) holds. It is a matter of straight verification that private plus public savings equal investments,

$$K_{t+1} = N_t s_t + S_t^g. \tag{22}$$

Furthermore, the relative value of future output depends on investments, through (12),

$$b_t = \frac{K_{t+1}}{\alpha Y_t}. \tag{23}$$

Equations (22) and (23) imply that the weight given to future output b_t corresponds one-to-one with private plus public savings. A planner who attributes high welfare weights to future generations transfers income from present generations to future generations, through positive public savings. The increased future capital stock also increases the value of future output relative to current output. In most of our analysis, we close down a preferential treatment of future generations by requiring a government balanced budget (16). A positive carbon price $p_t > 0$ can be used to address climate externalities in a market equilibrium; imposing (16) rules out other income intervention objectives.

The next definition zooms in on optimal carbon policies.

Definition 3. *A competitive equilibrium is called 'optimal' if carbon prices, p_t, and lump-sum transfers τ_t are set such that the allocation maximizes welfare*

$$W = \sum\nolimits_{t=1}^{\infty} \beta_t N_t u_t, \tag{24}$$

for some welfare weights sequence β_t, satisfying $\sum_{t=1}^{\infty} \beta_t N_t < \infty$.

The above definition does not impose restrictions to the welfare weights sequence. Below we will see that in our economy, in a representative optimal equilibrium, welfare weights β_t depend on demography.[18] We note that all optimal equilibria are Pareto-efficient.[19]

[18] In a typical infinitely-lived representative-agent (Ramsey-Cass-Koopmans) model, we have $\beta_t = \beta^t$ for some exogenous β.

[19] We do not exclude the possibility that a Pareto-efficient allocation exists that is not optimal in the above sense, e.g. when $\beta_t = 1$, so that the weight series sum to infinity and welfare is not defined.

4. Analysis

4.1. Existence

We prove existence for a representative equilibrium, and for the unique optimal representative equilibrium. We can then study the properties, specifically the effect of demography on carbon prices. The proofs are relegated to Appendix B. The propositions state both existence and properties for our OLG economy, which we compare with those for the RCK model in the next section.

The first proposition shows how the investment share varies with demography. We note that individual savings depend on both aging and fertility (eq (6)), but at the aggregate level, the savings rate only depends on life-expectancy.

Proposition 1. *For any carbon pricing rule $p_t = g_t Y_t$, for given sequence $g_t \geq 0$, a unique representative competitive equilibrium exists. The investment shares depend on demography, but not on carbon policies:*

$$\frac{K_{t+1}}{Y_t} = \frac{\delta h_t (1-\alpha)}{1+\delta h_t}, \tag{25}$$

and so does the effective discount factor (19)

$$b_t^* = \frac{\delta h_t (1-\alpha)}{(1+\delta h_t)\alpha}. \tag{26}$$

Proof. In appendix B. ■

The representative equilibrium, as such, does not require efficient carbon policies; there is no match between the carbon price and the social costs of carbon. Below, we will study efficient policies in a representative equilibrium. To ensure bounded welfare weights, we make the following assumption.

Assumption 3. *(Bounded net present value) Preference and production parameters satisfy $\delta h_t(1-\alpha) < (1+\delta h_t)\alpha$, for all t.*

The second step is that we prove uniqueness of the optimal competitive equilibrium, for given welfare weights, and we show that in such an equilibrium the market costs of carbon and social costs of carbon coincide.

Proposition 2. *For any weight sequence β_t such that $\sum_{t=1}^{\infty} \beta_t N_t < \infty$, a unique optimal competitive equilibrium exists. In this equilibrium, the carbon price equals the market costs of carbon, $p_t = MCC_t$. Carbon prices are proportional to income, and increase with weights given to*

future generations,

$$p_t^* = Y_t \sum_{i=0}^{\infty} \theta_i b_t^{t+i}, \tag{27}$$

where $b_t^{t+i} = b_t \cdot b_{t+1} ... b_{t+i}$*, and* b_t*, as in (19), is given by*

$$b_t^* = \frac{\sum_{j=0}^{\infty} \alpha^j (\beta_{t+j+1} N_{t+j+1} + \beta_{t+j} \delta h_{t+j} N_{t+j})}{\sum_{j=0}^{\infty} \alpha^j (\beta_{t+j} N_{t+j} + \beta_{t+j-1} \delta h_{t+j-1} N_{t+j-1})}. \tag{28}$$

Proof. In appendix B. ■

We have now shown existence of a unique representative equilibrium given a carbon pricing rule (Prop 1), and existence of a unique optimal equilibrium given welfare weights (Prop 2). The next proposition connects the two, proving existence of a unique optimal representative equilibrium.

Proposition 3. *A unique optimal representative competitive equilibrium exists, with welfare weights dependent on demography, but independent of climate parameters,*

$$\beta_{t+1}^* = n_t^{-1} \frac{\delta h_t (1-\alpha)}{(1+\delta h_t)\alpha} \beta_t. \tag{29}$$

The relative value of future output (19) is given by (26); the efficient carbon price is given by

$$p_t^* = Y_t \sum_{i=1}^{\infty} \theta_i \prod_{j=0}^{i-1} \frac{\delta h_{t+j} (1-\alpha)}{(1+\delta h_{t+j})\alpha}. \tag{30}$$

Proof. In appendix B. ■

The proposition reveals various important features of the optimal representative equilibrium. The weights given to future generations increase with an aging population, not because aging increases the population size per se, but because aging changes the savings decisions, and thereby changes the weights given to future output (Prop 1), and indirectly the weights given to future consumption. Welfare weights do not depend on the climate characteristics, that is, welfare weights are independent of climate policies and can be taken as input.[20] That is, a laissez-faire equilibrium with no carbon prices, $p_t = 0$, and no income transfers $\tau_t = 0$, implements the same implicit welfare weights as the optimal representative carbon policy of Prop 3.

[20] This is different from the RICE model; see also footnote 17.

4.2. Comparative demography

Here we consider, for given technologies $A_t(.)$ and $\Omega(.)$ and preferences $u(.)$, how changes in demographic parameters n_t and h_t affect equilibrium outcomes. Fertility is a determinant of future labour supply, and as such, of economic growth. Aging determines savings, and as such also affects future carbon prices. The next corollaries establish the effect of demography on output.

But before we study demographic change, the next corollary establishes the benchmark case; Golosov et al. (2014)'s result is replicated if we shut down demographic change:

Corollary 1. *Without demographic change, $n_t = 1$, $h_t = h$, optimal representative carbon prices are given by*

$$p_t^* = Y_t \sum_{i=0}^{\infty} \theta_i \beta^i,$$

for constant $\beta = \frac{\delta h(1-\alpha)}{(1+\delta h)\alpha}$, so that carbon prices are proportional to output, $\frac{p_t^*}{Y_t} = \frac{p_{t+1}^*}{Y_{t+1}}$.

Proof. Follows immediately from Proposition 3, equations (29) and (30). ■

Then we establish two results of demography for capital and output, along the lines of Canton and Meijdam (1997). These corollaries follow from Proposition 1.

Corollary 2. *In a representative equilibrium, aging (keeping fertility unchanged) increases the future capital stock, increases output, and decreases the return to investments.*

$$\text{for } i > 0; \frac{dK_{t+i}}{dh_t} > 0, \frac{dY_{t+i}}{dh_t} > 0. \frac{dr_{t+i}}{dh_t} < 0.$$

Proof. In appendix B. ■

Corollary 3. *In a representative equilibrium, decreasing fertility (keeping life-expectancy unchanged) increases the immediate next capital stock and decreases returns to investments.*

$$\frac{dK_{t+1}}{dn_t} < 0, \quad \frac{dr_{t+1}}{dn_t} > 0.$$

Proof. In appendix B. ■

The above two corollaries capture the spirit of the empirical literature, which suggests that both increasing life-expectancy and decreasing fertility over the last decades have increased aggregate household savings rates and decreased the capital returns

(Imrohoroglu and Zhao, 2017; Carvalho et al., 2016; Curtis et al., 2015; Eggertsson and Mehrotra, 2014; Bloom et al., 2007, 2003; Curtis et al., 2015; Sánchez-Romero, 2013). The last corollary remains silent on the effect of fertility on output, and the long run. An increased labour supply compensates for the decrease in capital so that the immediate output effect is ambiguous. For the long run effects, we note that emissions can either rise or fall with changing labour supply. Thus, future production may increase through increased labour supply, but it can also decrease through climate damages.

The next corollaries take the model to the domain of climate policies. The effect of fertility is ambiguous, as in the previous corollary. But for life expectancy, we have clear results. We compare a reference economy with a hypothetical alternative that has the same history, the same technology, but some different demographic future. An increase in current or future life-expectancy increases carbon prices by increasing the value associated with future damages:

Corollary 4. *In an optimal representative competitive equilibrium, present carbon prices increase with present and future increases in life-expectancy*

$$\frac{dp_t}{dh_{t+k}} > 0, \ \forall \ k \geq 0.$$

Proof. In appendix B. ■

In addition to an increase in the present carbon price, an increased life-expectancy also leads to a faster rise in carbon prices:

Corollary 5. *Assume that life-expectancy is non-decreasing, $h_t \leq h_{t+1}$, and has not reached its long-term level at time t^*, $h_{t^*} < h_\infty$, then carbon prices in an optimal representative competitive equilibrium increase faster than income for all $t < t^*$:*

$$\frac{p^*_{t+1}}{Y_{t+1}} > \frac{p^*_t}{Y_t}.$$

Proof. In appendix B. ■

The general approach in the IAM literature is that parameters are calibrated on the basis of historically observed macro variables, and then the model is used to project forwards output, investment and consumption paths. The above propositions show that demographic change creates a wedge between future projections and past observations. This result will, later on, be contrasted with the effect of demography on projections in

RCK models. We formalize the insights as a Remark, where we compare a model that abstracts from demography to a model including demographic change.

Remark 1. *Consider two economies with same technology and labour supply, calibrated to the same historic observations ($t = 0, 1$) for economic growth and returns on capital, Y_0, Y_1, r_1. In the benchmark economy, life-expectancy is assumed stationary. In the extended economy (denoted by a tilde), life expectancy is increasing, $\widetilde{h}_t \leq \widetilde{h}_{t+1}$, and has not reached its long-term level at time t^*, $\widetilde{h}_{t^*} < \widetilde{h}_\infty$. In both economies, preference parameter $\delta, \widetilde{\delta}$ is calibrated such that (25,26) hold in the calibrated period $t = 0$. Then optimal carbon prices are higher in the economy with dynamic demography. For all t*

$$\widetilde{p}_t > p_t.$$

5. The Ramsey-Cass-Koopmans model

This section presents the standard neoclassical growth model with an climate module to derive the conditions through which demography affects climate policies. We assume a logarithmic utility function, full depreciation and an identical production function as in the benchmark model. We also extend the analysis to the case where the social planner can use either a Benthamite or a Millian utility as in Canton and Meijdam (1997). The Benthamite approach suggests to maximize social welfare by weighting the utility function by the size of the dynasty M_t, while the Millian approach uses no weights. The parameter $0 \leq \xi \leq 1$ allows us to identity between those extreme cases. To certain extent, we can relate the former case to the usual assumption in the integrated assessment models $\xi = 1$ (see e.g., Nordhaus (2008)) and the latter to the modeling in Golosov et al. (2014), which implies $\xi = 0$.

5.1. Households

A representative household derives utility from aggregate consumption C_t, while population size is given by M_t,[21]

$$\max_{\{C_t, S_t \geq 0\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t M_t^{\xi} \ln\left(C_t / M_t\right), \tag{31}$$

subject to,

[21] In terms of the OLG model, we have $M_t = N_t + h_{t-1} N_{t-1}$.

$$C_t + S_t \leq r_t S_{t-1} + w_t L_t + M_t \tau_t, \quad \forall t. \tag{32}$$

The parameter $\beta \in (0,1)$ is the representative consumer's discount factor, $\xi = 0$ refers to a Millian (average) welfare function, while $\xi = 1$ refers to a Benthamite (aggregate) welfare function, labor supply L_t is inelastic and typically proportional to total population. Population growth is described through $M_{t+1} = m_t M_t$.

The government sets carbon prices and income transfers that maximize the representative household's welfare function. It is immediately clear that the distribution of income transfers over time has no effect on the consumption distribution by the households. For convenience, we consider the case where the government returns tax revenues at the same period, (16), then savings equal capital, $S_t = K_t$ and the equilibrium equals the optimal solution to maximization of (31) subject to (7) and the commodity balance,

$$C_t + K_{t+1} = Y_t.$$

5.2. Analysis

We first establish the general properties of the RCK model, both in terms of capital investments and in terms of carbon policies.

Proposition 4. *In the RCK model with dynamic population M_t, the investment share depends on demography, but not on climate parameters,*

$$\frac{K_{t+1}}{Y_t} = \frac{\sum_{j=1}^{\infty} \alpha^j \beta^j M_{t+j}^{\xi}}{\sum_{j=0}^{\infty} \alpha^j \beta^j M_{t+j}^{\xi}}. \tag{33}$$

The optimal carbon price equals the market cost of carbon and is proportional to income,

$$p_t^* = Y_t \sum_{i=1}^{\infty} \theta_i b_t^{t+i}, \tag{34}$$

where $b_t^{t+i} = b \cdot b_{t+1} \ldots b_{t+i}$, and b_t, as in (19), is given by

$$b_t^* = \frac{\sum_{j=0}^{\infty} \alpha^j \beta^{j+1} M_{t+j+1}^{\xi}}{\sum_{j=0}^{\infty} \alpha^j \beta^j M_{t+j}^{\xi}}. \tag{35}$$

Proof. In appendix B. ■

The OLG and RCK economies produce similar carbon price formulas if we neglect demographic dynamics:

Corollary 6. *For $\xi = 0$, or $\xi > 0$ without demographic change, $M_t = M$, optimal representative carbon prices are given by*

$$p_t^* = Y_t \sum_{i=0}^{\infty} \theta_i \beta^i,$$

and carbon prices are proportional to output, $\frac{p_t}{Y_t} = \frac{p_{t+1}}{Y_{t+1}}$.

Proof. In appendix B. ■

Now we consider the effects of demographic change. For the OLG model, we considered an increase in life-expectancy and fertility. The RCK model features population size, which is derived from life-expectancy and fertility. We first consider the effect on investments and output, which were established as the main factors determining the carbon price:

Corollary 7. *In the RCK economy with Benthamite welfare $\xi = 1$, population growth (assuming the future population size M_t is derived from M_1 and $(m_2, ...m_t)$) increases the future capital stock and output,*

$$\forall\, i \geq 1 : \frac{dK_{t+i}}{dm_t} > 0, \frac{dY_{t+i}}{dm_t} > 0.$$

Proof. In appendix B. ■

The above corollary provides the intuition for the next one:

Corollary 8. *For $\xi > 0$, present carbon prices increase with present and future increases in population size*

$$\frac{dp_t}{dm_{t+i}} > 0 \text{ for } i \geq 0.$$

Proof. In appendix B. ■

Evaluating the corollaries, and comparing these with their equivalent corollaries for the OLG model (1,2,3,4,5), we see that both investments and carbon price dynamics respond very differently to demographic change in the RCK model. Whereas the global trends of increasing life-expectancy leads to rising savings and fast-rising carbon prices in the OLG model, in the RCK model, the decrease in population growth leads to a decrease in the investment share, and carbon prices that increase less then with output.

Corollary 9. *Assume $\xi > 0$ and population growth is non-increasing, $m_t \geq m_{t+1}$, and has not reached its long-term level, $m_t > m_\infty$, then carbon prices increase less than income*

$$\frac{p_{t+1}}{Y_{t+1}} < \frac{p_t}{Y_t}.$$

Proof. In appendix B. ■

Comparing the corollaries with their equivalent for the OLG economy, we find sharp analytical results. Demography has opposite impacts on the paths of optimal carbon prices, dependent on the model structure. In the OLG framework, a structural increase in life-expectancy leads to higher carbon prices that grow faster than the economy. In the RCK models the outcome depends on the welfare function. On the one hand, if we apply Millian welfare ($\xi = 0$), carbon prices increase proportionally to output. On the other hand, assuming a decreasing population growth and a weighted utility function (the Benthamite case, $\xi = 1$), it turns out that carbon prices increase less than with income.

The next remark presents the effect of demographic change on calculated optimal climate policy.

Remark 2. *Consider two economies with same technology and labour supply, calibrated to the same historic observations ($t = 0, 1$) for economic growth and returns on capital, Y_0, Y_1, r_1. In the benchmark economy , population growth is assumed constant or absent, or welfare is Millian ($\xi = 0$). In the extended economy (denoted by a tilde), population growth decreases, $\widetilde{m}_t \geq \widetilde{m}_{t+1}$, and has not reached its long-term level at $t = t^*$: $\widetilde{m}_{t^*} > \widetilde{m}_\infty$. In both economies, preference parameter $\beta, \widetilde{\beta}$ is calibrated such that (33) holds in the calibrated period $t = 0$. Then optimal carbon prices are lower in the economy with dynamic demography. For all $t \leq t^*$,*

$$\widetilde{p_t} < p_t.$$

Taking the two remarks together, we can now compare the effects of demographic change in the OLG model versus the RCK model. Both models, when calibrated to the same historic output levels and returns on capital, and when abstracting from demography, will present the same optimal carbon prices. But results radically change if both models are calibrated to the same future demographic change of increasing life-expectancy, decreasing fertility, increasing population, and decreasing population growth. Then both models, when used to project future trajectories, will produce con-

trasting efficient carbon prices. The OLG economy will present high carbon prices (comparatively), that increase faster than output. The RCK model will present lower carbon prices, that increase at a rate below the growth of the economy. To assess the quantitative substance of this analytical result, we add a numerical calibration in the next section.

6. Quantitative assessment

In this section, we evaluate quantitatively carbon prices paths for the next hundred years, using our benchmark OLG model and the RCK models, with and without population dynamics. A key characteristic of our assessment is that, though the model is defined for periods that span many years, we set up our calibration and simulation such that all variables are calculated on an annual basis. First, we specify the calibration of annual values for life expectancy h_t, fertility n_t and population size and growth (M_t, m_t). Then we describe the annual climate response function θ_i. These parameters, and Golosov et al. (2014) as reference, are used to calibrate the preference parameter δ. We subsequently calculate the welfare weights β_t and output weights b_t in our model, both expressed as annualized variables. Welfare weights β_t in the RCK model are exogenous, but we calculate the RCK models' output weights b_t, which depend on economic growth if the elasticity of marginal utility is not equal to one. Finally, we bring all the variables together to determine the social costs of carbon for the various models.

6.1. Demographics calibration

Our benchmark model considers 30 years per period, excluding childhood, so that life time is maximally two periods. For low-income countries with low life-expectancy, we assume that consumers enter the labour force at age 15, so that $1 + h = (LE - 15)/30$, where LE is life-expectancy. For high-income countries with high longevity, we assume that the average consumer starts working at age 20 as usual in previous literature, so that $1 + h = (LE - 20)/30$. Assuming a smooth transition between the low and high-income countries, we calibrate our parameter h_t through

$$h_t = \min\{0.03LE_{t'} - 1.35, 1\},$$

which we calculate for each year t. In our model, h_t refers to the life-expectancy of the currently young, age 15, while our database measures life-expectancy at birth. To match the model parameter with data, we define life-expectancy in our model at time t through life-expectancy in the database at $t' = t - 15$. We use the medium-variant projection of world average data that come from the United Nations World Population Prospects, 2015 revision. Projections are available in five-year time intervals, which we linearly interpolate for annual time series. We cap the model life-expectancy at $h_t = 1$ for projected world average life-expectancies exceeding 78 years.

For fertility, the number of children per adult is half the births per women, FR_t,

$$n_t = FR_t/2.$$

The historic estimates and projections come from the same UN database.

In contrast to the OLG model, demography in the RCK models only requires a measure for population size and growth, (M_t, m_t), where M_t denotes total population (both sexes) at period t. We use the same medium-variant projections from the UN database.[22]

6.2. Preferences calibration

In order to create time series for the discount factor b_t, in the calibration of the OLG model, we determine parameter δ such that b_t, as defined in (26), equals the discount factor $b = 0.985$ used in Golosov et al. (2014) at 2010. The annualized value ratio associated with the year 2010 is then given by $b_t^{1/30}$. Notice that the parameter b_t is based on the life-expectancy at the year 1995. Thus, to calibrate our parameter δ, we set $\alpha = 0.3$, life expectancy $h_{2010} = 0.58$, output discount factor $b_{2010} = \beta^{GHKT} = 0.985^{30}$ and solve for δ as follows:

$$\delta = \frac{\alpha b_{2010}}{h_{2010}[1 - \alpha - \alpha b_{2010}]} = 0.645$$

6.3. Discount factor calculation

Using the above values for the capital share α, preferences δ, life expectancy h_t, fertility n_t, we simulate the time series for b_t from (26).

[22] Due to high uncertainty about long-run population prospects, Rozell (2017) recommends to use the International Institute for Applied Systems Analysis (IIASA) population projections in climate policy studies since that dataset implies more reliable demographic scenarios. Our calibration relies on the United Nations estimations for ease of comparison with previous literature.

For the RCK model, we apply (35) to calculate the output ratio b_t for those models with logarithmic utility, $\eta = 1$. The sequence b_t only depends on intergenerational preferences β and population growth rates m_t. For the general model with constant elasticity of substitution $\eta \neq 1$, as in Weitzman (2007); Nordhaus (2008, 2014), we conjecture that the output ratio is approximated by

$$b_t = \frac{\sum_{j=0}^{\infty} \alpha^j \beta^{j+1} (\gamma_t^{t+j+1})^{1-\eta} M_{t+j+1}^{\xi}}{\sum_{j=0}^{\infty} \alpha^j \beta^j (\gamma_t^{t+j})^{1-\eta} M_{t+j}^{\xi}} \tag{36}$$

where γ_t^{t+j} is the compounded productivity growth between period t and $t+j$. The historic estimates for productivity growth are based on World Bank data and the SSP scenarios (see Appendix A for more details). For future projections, we assume a constant annual increase of per capita income of 2 per cent per year. The formula collapses to (35) for $\eta = 1$. We assume stationary population, $n_t = m_t = 1$ and constant γ_t after 2100, and calculate the output ratios (36) backwards, as specified in detail in Appendix E. We note that, in balanced growth, the output ratio formula becomes $b = \beta \gamma^{1-\eta} m^{\xi}$. In appendix C, we show that, indeed, this formula is correct for a balanced growth path. As a static approximation, we use the formula presented in the introduction to check the numbers presented in Table 1

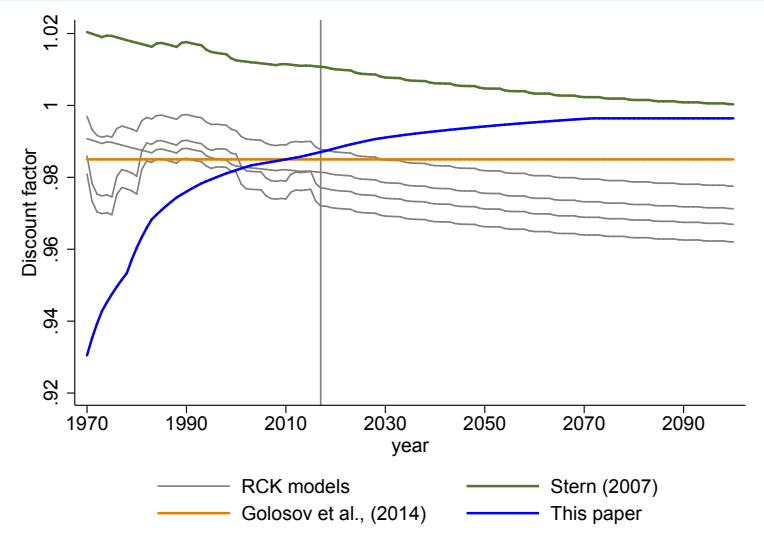
$$b_t \approx \beta m_t^{\xi} \gamma_t^{1-\eta}. \tag{37}$$

Figure 2 depicts the effective annual discount factors, calculated recursively, of our benchmark model. To relate our approximation to previous analysis, it also displays the discount factors used in Golosov et al. (2014), which abstracts from demography, and other RCK models with a Benthamite welfare function. Notice that to calculate the productivity growth factor γ_t during the period 1970-2015, we use 10-year average of GDP per capita growth rates to smooth the business cycle features of the data.

The outstanding feature of the figure is the contrasting patterns between our OLG model and the RCK models with Benthamite welfare. In our model, global demographic patterns, specially the worldwide rise of life-expectancy, lead to a rise of the value of future output b_t. In contrast, the RCK models show a discount factor that decreases over time due to declining population growth rates. As we will see shortly, these divergent patterns for the discount factor have a significant impact on carbon prices.[23]

[23] Notice also that with constant life-expectancy h and $n_t = 1$, our model would match the discount factor in Golosov et al. (2014).

Figure 2: Effective annual discount factor b_t, 1970-2100



Note: The year 2017 is indicated with a vertical line. b_t is calculated as in equation (26). The parameter δ is calibrated such that in 2010, the value of b_{2010} corresponds to 0.985, as in Golosov et al. (2014). The discount factors for the RCK models are given by (36), see Table 1 for more details and parameter values.

6.4. Climate dynamics calibration

We use a stylized climate module as in van den Bijgaart et al. (2016) and Gerlagh and Liski (2017b). The dynamics of damages θ_i are governed by carbon exchange between reservoirs, which is conveniently described through an exponential CO_2 decay function. Damages follow temperature, which slowly converges to its equilibrium level for given atmospheric CO_2; a higher atmospheric CO_2 content leads to slowly increasing damages. The reduced form ((Gerlagh and Liski, 2017b), Theorem 1), is given by a 'multi-box' representation,

$$\theta_i = \sum_j \sum_k a_j b_k \pi \varepsilon_k \frac{(1-\eta_j)^i - (1-\varepsilon_k)^i}{\varepsilon_k - \eta_j}, \tag{38}$$

where η_j are the atmospheric depreciation rates, ε_k are the temperature adjustment speeds, a_j and b_k are the shares of the relevant processes, and π is the long-run emissions-damages sensitivity. We take the parameters from van den Bijgaart et al. (2016), Table 6 and 7, for the median carbon model, $a = (0.220, 0.279, 0.278, 0.222)$, $\eta = (0, 0.0035, 0.0507, 0.2892)$, and temperature models, $b = (0.2218, 0.3306, 0.4476)$, $\varepsilon = (0.9787, 0.1980, 0.0036)$, $\pi = 0.0167$. These parameters indicate that 22 per cent of emissions remain in the atmosphere 'forever', while the same share very quickly transits to other carbon reservoirs

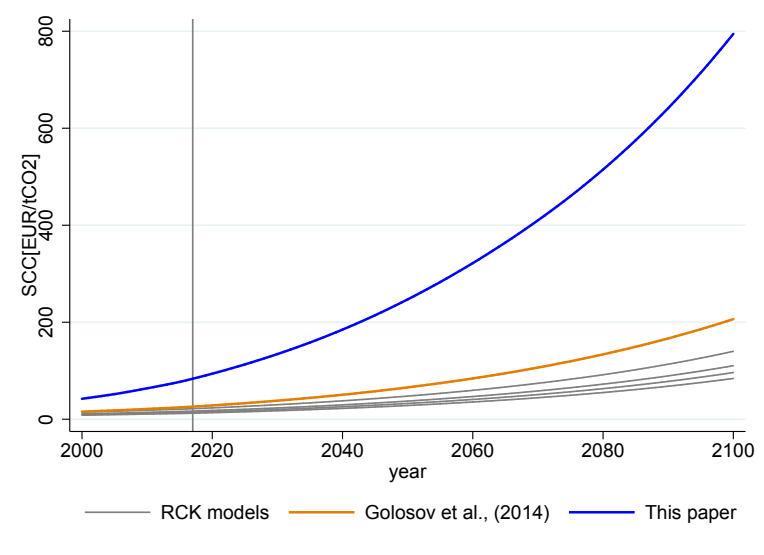
(at about 30% per year). Also, about 22% of temperature adjustment is virtually immediate, while almost half of temperature adjustment only happens after more than a century.

6.5. The social cost of carbon

We have now everything in place to determine the social costs of carbon. The carbon price factors g_t, as defined by (18), are calculated backwards from 2100 to 2000; the procedures are described in the Appendices D and E. We multiply the values for g_t by output Y_t to quantify the social costs of carbon.

Figure 3 displays the social cost of carbon for the models considered. The implied carbon price at 2017 based on the model structure of Golosov et al. (2014) is about 25 €/tCO$_2$, growing in-step with the size of the economy. Our model calculates a carbon price rising faster than the economy, but also at a much higher level, amounting to about 85 €/tCO$_2$. The RCK models with Benthamite utility, on the other hand, have carbon prices that grow less than with income, *and* start at a lower level. Notice that a worldwide rising carbon price starting at 85 €/tCO$_2$ exceeds by magnitude the objectives stated in the Paris Intended Nationally Determined Contributions (INDCs).

Figure 3: The social cost of carbon, 2000-2100



Note: The year 2017 is indicated with a vertical line.

These results suggest that demographic patterns are an important factor in the calculation of the social costs of carbon. As shown in Table 1, the carbon price in our OLG model at 2100 attains a value of approximately 800 EUR/tC02, a price that is about

four times higher than the carbon price derived from a model with no demography as in Golosov et al. (2014).

7. Discussion

In this paper we explore optimal environmental policies in a climate-economy with population dynamics that entail changes in savings patterns and capital returns. Using a stylized overlapping generations model, we show that an increase in life-expectancy and a decrease in fertility rates raise households' savings, declining the rate of return to capital due to its relative abundance. Changing savings patterns also lead to changes in the way people value future economic gains or losses, including climate damages. We propose a time-varying effective discount factor that allows climate policies to reflect demographic patterns.

Our results show that demographic factors in climate-economy models have an important effect on climate policies. We developed a stylized OLG model and Ramsey-Cass-Koopmans model with life-expectancy and fertility features. For both types of models we compared one with and one without population dynamics, assuming that they are calibrated to historic savings. We obtain outstanding divergent patterns for optimal carbon prices between OLG and RCK models. The OLG and RCK models without demography replicate the same carbon price projections. But, when we consider the expected global demographic trends as in Figure 1, we find much higher carbon prices, that also grow faster than the economy, for the OLG model, while we find lower carbon prices, that also grow below the rate of economic growth, for the RCK models.

Our study thus points to the importance of testing the long-term empirical validity of integrated assessment models, in terms of the structural connection between demography and macro-economic savings and investments, when used for climate policy advice. The OLG and RCK models, both with rigorous micro-foundation, give radically different answers, and as our quantitative assessment shows, may lead to an under- or over-reporting of the social costs of carbon by large amount.

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A. Data and Projections

The world GDP per capita data is obtained from 'World Development Indicators' of the World Bank. The GDP per capita at in constant 2010 US$ for 1960 till 2015 is available in the aforementioned dataset. The projection of this variable beyond historical data and up to 2100 is done based on a SSP[24] scenario. We picked the second scenario, i.e. SSP2, which assumes moderate challenges in the future of environmental policies. Leimbach et al. (2017) predict the GDP per capita, under SSP2 scenario, to grow on an average rate of 2% per year from 2010 till 2100.

B. Proofs

Lemma 1

Proof. We can rewrite the equality in intensive form as

$$g_t L_t a_t(e_t) = (1-\alpha) a_t'(e_t).$$

where $e_t = E_t/L_t$ is the emission intensity. The LHS is increasing in e_t, while the RHS is decreasing because of concavity. For $e_t = 0$, we have that the LHS is less than the RHS, for $e_t = e_t^{\max}$ we have that the LHS exceeds the RHS (which then equals zero). By continuity, there must be an $e_t = E_t/L_t$ for which the RHS equals the LHS. ■

Proposition 1

Proof. We prove the proposition by induction, starting to establish an allocation and prices for $t = 1$, and then continuing for increasing t.

In the first period, the capital stock, K_1 is known, labor supply $L_1 = (1 - \chi n_1)$ is inelastic, and the emission price must satisfy $p_1 = g_1 Y_1$. Using the firm's optimization on emission, (14), and Lemma 1, we establish a unique level of emissions of the firm in period one. Therefore, we have pinned down E_1, as well as L_1 and K_1. The wage rate and the rate of interest in the first period are then determined by (13) and (12). The transfer in the first period is then

$$\tau_1 = \frac{p_1 E_1}{N_1}.$$

[24] Shared Socio-economic Pathways are 5 scenarios for the path ahead of the world economy. These scenarios differ in terms of socioeconomic challenges for mitigation and adaptation activities. The scenarios are discussed in O'Neill et al. (2014).

and income of the young equals $(1-\alpha)Y_t$. Because of logarithmic utility, a fixed share is consumed when young, the remainder saved.

$$N_1 c_{1,1} = \frac{1-\alpha}{1+\delta h_1} Y_t. \tag{B.1}$$

The consumption of the old in the first period equals the value of capital

$$N_0 h_0 c_{2,1} = \alpha Y_t. \tag{B.2}$$

It follows immediately from the aggregate goods market equilibrium (20 that the remainder of output is invested. This establishes (25). Finally, (26) follows then from (12).

Now, assume that the economy is in equilibrium in period $t \geq 1$. We can invoke exactly the same equations to prove that it is in equilibrium in period $t+1$ as well. Q.E.D. ■

Proposition 2

Proof. The condition $\sum_{t=1}^{\infty} \beta_t N_t < \infty$ ensures that welfare and parameters b_t^{t+i} are well defined. To prove existence, we first solve the central planner's problem (24) given technology, (7, 8, 10), subject to the aggregate goods market clearing condition, (20). We then show that we can decentralize the optimal allocation as a competitive equilibrium.

The associated Lagrangian for the central planner's problem reads:

$$\begin{aligned} \mathcal{L} \;=\; & \beta_0 \delta h_0 N_0 u(c_{2,1}) + \sum_{t=1}^{\infty} [\beta_t \{N_t u(c_{1,t}) + \delta h_t N_t u(c_{2,t+1})\} \\ & + \lambda_t \{Y_t - N_t c_{1,t} - h_{t-1} N_{t-1} c_{2,t} - K_{t+1}\}] \end{aligned} \tag{B.3}$$

where λ_t is the shadow price for the aggregate goods market clearing condition. We consider the optimal allocation $c_{1,t}^*, c_{2,t}^*, Y_t^*, E_t^*, K_{t+1}^*$ and supporting dual variables λ_t^*, and show that we can construct prices r_t^*, w_t^*, p_t^*, savings s_t^* and transfers τ_t^* such that all conditions (3-5), (12-14), (15), hold.

We construct prices to support the competitive equilibrium, through

$$\begin{aligned} r_t^* &\equiv \frac{\partial Y_t}{\partial K_t} = \alpha \frac{Y_t^*}{K_t^*}, \\ w_t^* &\equiv \frac{\partial Y_t}{\partial L_t}, \\ p_t^* &\equiv \frac{\partial Y_t}{\partial F_t} \frac{\partial F_t}{\partial E_t}, \end{aligned}$$

so that FOCs (12-14) are satisifed by construction. The Lagrangean first-order conditions are (for $c_{1,t}, c_{2,t}, K_{t+1}, E_t$ respectively):

$$\frac{\beta_t}{c_{1,t}^*} = \lambda_t^*, \tag{B.4}$$

$$\frac{\delta \beta_{t-1}}{c_{2,t}^*} = \lambda_t^*, \tag{B.5}$$

$$r_{t+1}^* = \frac{\lambda_t^*}{\lambda_{t+1}^*}, \tag{B.6}$$

$$p_t^* = \sum_{i=0}^{\infty} \theta_i \frac{\lambda_{t+i}^*}{\lambda_t^*} Y_{t+i}^* \tag{B.7}$$

The first three FOCs give the households FOC (5). That is, all FOCs the define the competitive equilibrium are satisfied.

The savings s_t^* can subsequently be constructed from r_{t+1}^* and $c_{2,t+1}^*$ and (4). The transfers τ_t^* can subsequently be constructed from r_{t+1}^* and $c_{1,t+1}^*$, w_t^*, s_t^*, and (3). The regulator's budget (15) follows from Walras' law. Thus, we have shown that the optimal allocation is a competitive equilibrium. The last two FOCs (B.6, B.7) confirm that the optimal carbon price equals the market costs of carbon.

We now want to prove the analytical solution for the carbon prices. We guess and verify that the optimal allocation satisfies

$$\lambda_t^* Y_t^* = \sum\nolimits_{j=0}^{\infty} \alpha^j \beta_{t+j} N_{t+j} + \alpha^j \beta_{t+j-1} \delta h_{t+j-1} N_{t+j-1}. \tag{B.8}$$

We will see that this condition determines specific consumption shares to satisfy (B.4, B.5). Then, we find the investment shares, and show that this is consistent with (B.6). Finally, the carbon price rule (B.7) will be shown to produce (27).

The first two FOCs (B.4, B.5), together with our guess (B.8) imply consumption and investment shares that depend on demography, but are independent of the climate parameters:

$$\frac{N_t c_{1,t}^*}{Y_t^*} = \frac{\beta_t N_t}{\sum_{j=0}^{\infty} \alpha^j (\beta_{t+j} N_{t+j} + \beta_{t+j-1} \delta h_{t+j-1} N_{t+j-1})}, \tag{B.9}$$

$$\frac{h_{t-1} N_{t-1} c_{2,t}^*}{Y_t^*} = \frac{\beta_{t-1} \delta h_{t-1} N_{t-1}}{\sum_{j=0}^{\infty} \alpha^j (\beta_{t+j} N_{t+j} + \beta_{t+j-1} \delta h_{t+j-1} N_{t+j-1})}, \tag{B.10}$$

The market clearing (20) subsequently defines the investment share,

$$\frac{K_{t+1}^*}{Y_t^*} = \frac{\sum_{j=1}^{\infty} \alpha^j (\beta_{t+j} N_{t+j} + \beta_{t+j-1} \delta h_{t+j-1} N_{t+j-1})}{\sum_{j=0}^{\infty} \alpha^j (\beta_{t+j} N_{t+j} + \beta_{t+j-1} \delta h_{t+j-1} N_{t+j-1})}. \tag{B.11}$$

And indeed, we see that if we substitute (B.8) into the dynamic FOC (B.6), we find the same expression for the investment share. Thus, the dual variables λ_t^* defined by (B.8) implements the optimal allocation $c_{1,t}^*, c_{2,t}^*, Y_t^*, K_{t+1}^*$.

To conclude, we immediately see that (B.7) and (B.8) result in the carbon price (27) mentioned in the proposition. ■

Proposition 3

Proof. We first assume that (29) holds, and show that $\Sigma_{t=1}^{\infty} N_t \beta_t < \infty$. Note that the recursive welfare weights rule implies

$$\beta_{t+1} N_{t+1} = \frac{\delta h_t (1-\alpha)}{(1+\delta h_t)\alpha} \beta_t N_t.$$

Assumption 3 thus guarantees that the sequence $\beta_t N_t$ decreases expontentially, and the infinite sum of a geomteric series is bounded. Therefore, welfare can be maximized.

Next, we note that for the welfare weights defined by (29), we can invoke Proposition 2 and its proof. The associated Lagrangian and its FOCs are exactly the same as (B.3) and the respective FOCs in the proof of proposition 2. Combining the first two FOCs with (B.1) and (B.2) for the representative equilibrium case, we get:

$$\frac{\beta_t}{\beta_{t-1}} = \delta \frac{c_{1,t}}{c_{2,t}} = n_{t-1}^{-1} \frac{\delta (1-\alpha) h_{t-1}}{(1+\delta h_{t-1})\alpha}$$

Therefore, the weights must satisfy (29).

For the carbon price, Proposition 2 tells us that we can use (18), in which we substitute (26) to arrive at (30). ■

Corollary 2

Proof. We prove this corollary by induction. For $i = 1$, we take the derivative for (25) with respect to h_t; we have

$$\frac{dK_{t+1}}{dh_t} = \frac{\delta h_t(1-\alpha)}{(1+\delta h_t)^2} Y_t > 0.$$

Note that in a representative economy with g_t given, because of Lemma 1, E^*_{t+1} is independent of K_{t+1}, so that a rise in K_{t+1}, leads to an increased output (8). For r_{t+1}, using (12), we have

$$r_{t+1} = \alpha\Omega(z_{t+1}) K_{t+1}^{\alpha-1} [A(E_{t+1}, L_{t+1})]^{1-\alpha}. \tag{B.12}$$

According to lemma 1, E_{t+1} and L_{t+1} are not affected. Therefore, we have

$$\text{sign}(\frac{dr_{t+1}}{dh_t}) = -\text{sign}(\frac{dK_{t+1}}{dh_t}).$$

The corollary is thus correct for $i = 1$. For all future periods $i > 1$, we use forward induction. From (25), we get

$$\frac{dK_{t+i+1}}{dY_{t+i}} = \frac{\delta(1-\alpha)}{1+\delta h_t} > 0.$$

which subsequently leads to a rise in output Y_{t+i+1}, etc. Similar to above, we have

$$\text{sign}(\frac{dr_{t+i+1}}{dh_t}) = -\text{sign}(\frac{dK_{t+i+1}}{dh_t}).$$

■

Corollary 3

Proof. For capital stock of the next period, from (25), we have

$$\frac{dK_{t+1}}{dn_t} = \frac{\delta h_t(1-\alpha)}{1+\delta h_t}(1-\alpha)\frac{Y_t}{A(E_t, L_t)}[A_L(E_t, L_t) + A_E(E_t, L_t)\frac{dE_t}{dL_t}](-\chi N_t) < 0,$$

or

$$\frac{dK_{t+1}}{dn_t} = \frac{\delta h_t(1-\alpha)}{1+\delta h_t}[w_t + p_t\frac{dE_t}{dL_t}](-\chi N_t) < 0.$$

For the rate of interest,, using (B.12), we have

$$\frac{dr_{t+1}}{dn_t} = \frac{r_{t+1}}{A(E_{t+1}, L_{t+1})}[A_E(E_{t+1}, L_{t+1})\frac{dE_{t+1}}{dL_{t+1}} + A_L(E_{t+1}, L_{t+1})](1 - \chi n_{t+1})N_t > 0.$$

For Y_{t+1}, taking the derivative with respect to n_t, we get

$$\frac{dY_{t+1}}{dn_t} = \frac{dK_{t+1}}{dn_t}[\frac{dY_{t+1}}{dK_{t+1}} + \frac{dY_{t+1}}{dE_{t+1}}\frac{dE_{t+1}}{dK_{t+1}}] + (1 - \chi n_{t+1})N_t[\frac{dY_{t+1}}{dL_{t+1}} + \frac{dY_{t+1}}{dE_{t+1}}\frac{dE_{t+1}}{dL_{t+1}}],$$

or

$$\frac{dY_{t+1}}{dn_t} = \frac{dK_{t+1}}{dn_t}[r_{t+1} + p_{t+1}\frac{dE_{t+1}}{dK_{t+1}}] + (1 - \chi n_{t+1})N_t[w_{t+1} + p_{t+1}\frac{dE_{t+1}}{dL_{t+1}}].$$

The first term on the RHS is negative and the second term is positive. $dY_{t+1}/dn_t > 0$ is then equivalent to

$$\frac{\delta h_t(1-\alpha)}{1+\delta h_t}[w_t + p_t\frac{dE_t}{dL_t}][r_{t+1} + p_{t+1}\frac{dE_{t+1}}{dK_{t+1}}]\chi < (1 - \chi n_{t+1})[w_{t+1} + p_{t+1}\frac{dE_{t+1}}{dL_{t+1}}].$$

Using (25), a rise in output in $t+1$ leads to an increase in K_{t+2} and, therefore, decreases r_{t+2}. Thereby, the output and the capital stock, as of $t+2$, increases as well while the rates of interest decrease.

■

Corollary 4

Proof. Taking the derivative of (30) with respect to h_{t+k} for some $k \geq 0$, we get

$$\frac{dp_t^*}{dh_{t+k}} = Y_t\Sigma_{i=1}^{\infty}\theta_i\prod_{j=1}^{k-1}\frac{\delta h_{t+j}(1-\alpha)}{(1+\delta h_{t+j})\alpha}\prod_{j=k+1}^{i-1}\frac{\delta h_{t+j}(1-\alpha)}{(1+\delta h_{t+j})\alpha}\frac{\delta(1-\alpha)}{(1+\delta h_{t+k})^2\alpha} > 0.$$

■

Corollary 5

Proof. Using equation (30), we can write

$$\begin{aligned}
&\frac{p^*_{t+1}}{Y_{t+1}} - \frac{p^*_t}{Y_t} = \\
&\theta_1[\frac{\delta h_{t+1}(1-\alpha)}{(1+\delta h_{t+1})\alpha} - \frac{\delta h_t(1-\alpha)}{(1+\delta h_t)\alpha}] + \theta_2 \frac{\delta h_{t+1}(1-\alpha)}{(1+\delta h_{t+1})\alpha}[\frac{\delta h_{t+2}(1-\alpha)}{(1+\delta h_{t+2})\alpha} - \frac{\delta h_t(1-\alpha)}{(1+\delta h_t)\alpha}] + \\
&\theta_3 \frac{\delta h_{t+1}(1-\alpha)}{(1+\delta h_{t+1})\alpha}\frac{\delta h_{t+2}(1-\alpha)}{(1+\delta h_{t+2})\alpha}[\frac{\delta h_{t+3}(1-\alpha)}{(1+\delta h_{t+3})\alpha} - \frac{\delta h_t(1-\alpha)}{(1+\delta h_t)\alpha}] + ... = \\
&\frac{(1+\delta h_t)\alpha}{\delta h_t(1-\alpha)} \Sigma_{i=1}^{\infty} \theta_i \prod_{j=1}^{i} \frac{\delta h_{t+j-1}(1-\alpha)}{(1+\delta h_{t+j-1})\alpha} \cdot [\frac{\delta h_{t+i}(1-\alpha)}{(1+\delta h_{t+i})\alpha} - \frac{\delta h_t(1-\alpha)}{(1+\delta h_t)\alpha}].
\end{aligned} \tag{B.13}$$

Since

$$\partial \frac{\delta h_t(1-\alpha)}{(1+\delta h_t)\alpha} / \partial h_t = \frac{\delta^2(1-\alpha)}{(1+\delta h_t)^2\alpha} > 0,$$

and h_t is not decreasing in t, the last term in (B.13) is not negative, and as $h_{t^*} < h_\infty$, there is at least one future t with $h_{t+1} < h_t$, and one term is strictly positive. Therefore, we have

$$\frac{p^*_{t+1}}{Y_{t+1}} > \frac{p^*_t}{Y_t}$$

■

Proposition 4

Proof. The Lagrangian function in the government's problem is given by:

$$\mathcal{L}(\cdot) = \sum_{t=0}^{\infty} \beta^t M_t^{\xi} \ln\left(\frac{C_t}{M_t}\right) + \lambda_t \left[Y_t - C_t - K_{t+1}\right] \tag{B.14}$$

Following the same procedure as in the proof of proposition 2, and assuming that there is a unique solution for the competitive equilibrium, the first-order conditions with respect to C_t, K_{t+1} and E_t can be written as follows:

$$\beta^t \frac{M_t^{\xi}}{C_t^*} = \lambda_t^* \tag{B.15}$$

$$\frac{\alpha Y^*_{t+1}}{K^*_{t+1}} = r^*_{t+1} = \frac{\lambda^*_t}{\lambda^*_{t+1}} \tag{B.16}$$

$$p^*_t = \sum\nolimits_{t=1}^{\infty} \frac{\lambda^*_{t+i}}{\lambda^*_t} \theta_i Y^*_{t+i} \tag{B.17}$$

Likewise, we guess and verify that the sequence of optimal allocations satisfy:

$$\lambda_t^* Y_t^* = \sum_{j=0}^{\infty} \alpha^j \beta^{t+j} M_{t+j}^{\xi} \tag{B.18}$$

Using the FOC (B.15) and our guess, we get the consumption share:

$$\frac{C_t^*}{Y_t^*} = \frac{M_t^{\xi}}{\sum_{j=0}^{\infty} \alpha^j \beta^j M_{t+j}^{\xi}} \tag{B.19}$$

Hence, given the feasibility constraint, the investment share is given by (33).

Notice that this expression can be also found by replacing equation (B.18) into equation (B.16) which validates our guess. Finally, using (B.18) and (B.17), we obtain optimal carbon prices. ■

Corollary 6

Proof. For $\xi = 0$, or $\xi > 0$ with $M_t = M$, from equation (35) it follows:

$$\begin{aligned} b_t &= \frac{\sum_{j=0}^{\infty} \alpha^j \beta^{j+1}}{\sum_{j=0}^{\infty} \alpha^j \beta^j} \\ &= \frac{\beta \sum_{j=0}^{\infty} \alpha^j \beta^j}{\sum_{j=0}^{\infty} \alpha^j \beta^j} \\ &= \beta \end{aligned} \tag{B.20}$$

Finally, using (34), we get then:

$$p_t^* = Y_t \sum_{i=1}^{\infty} \theta_i \beta^i$$

The last part of the corollary follows from the above equation.

■

Corollary 7

Proof. For $i = 1$ and $\xi = 1$, using the expression for the investment share (33) and the population growth factor $M_{t+1}/M_t = m_t$, we obtain:

$$K_{t+1} = Y_t \left[\frac{m_t \sum_{j=1}^{\infty} \alpha^j \beta^j \prod_{i=2}^{j} m_{t+i}}{1 + m_t \sum_{j=1}^{\infty} \alpha^j \beta^j \prod_{i=2}^{j} m_{t+i}} \right] \tag{B.21}$$

Therefore,

$$\frac{dK_{t+1}}{dm_t} = Y_t \left[\frac{\sum_{j=1}^{\infty} \alpha^j \beta^j \prod_{i=2}^{j} m_{t+i}}{\left(1 + m_t \sum_{j=1}^{\infty} \alpha^j \beta^j \prod_{i=2}^{j} m_{t+i}\right)^2} \right] > 0 \tag{B.22}$$

From (7) it follows that $\frac{dY_{t+1}}{dK_{t+1}} > 0$, then using the previous result, we can also claim that $\frac{dY_{t+1}}{dm_t} > 0$. Using (B.21) we know that $\frac{dK_{t+i+1}}{dY_{t+i}} > 0$ and from (7) that $\frac{dY_{t+i+1}}{dK_{t+i+1}} > 0$. The corollary thus holds for all $i > 1$ as well.

■

Corollary 8

Proof. For $i = 0$, notice that from (34-35) we know that if $\frac{db_t}{dm_t} > 0$, then $\frac{dp_t}{dm_t} > 0$. Applying the same procedure as before in the proof of corollary 7, it turns out that (35) can be written as follows

$$b_t = \frac{m_t^{\xi} \sum_{j=0}^{\infty} \alpha^j \beta^{j+1} \prod_{i=1}^{j} m_{t+i}^{\xi}}{1 + m_t^{\xi} \sum_{j=1}^{\infty} \alpha^j \beta^j \prod_{i=2}^{j} m_{t+i}^{\xi}} \tag{B.23}$$

The derivative of the last expression with respect to m_t is given by:

$$\frac{db_t}{dm_t} = \frac{\xi m_t^{\xi-1} \sum_{j=0}^{\infty} \alpha^j \beta^{j+1} \prod_{i=1}^{j} m_{t+i}^{\xi}}{\left[1 + m_t^{\xi} \sum_{j=1}^{\infty} \alpha^j \beta^j \prod_{i=2}^{j} m_{t+i}^{\xi}\right]^2} > 0 \tag{B.24}$$

This implies that the corollary holds for $i = 0$. Likewise, we can show that $\frac{\partial p_t}{\partial m_{t+i}} > 0$ for all $i > 0$.

■

Corollary 9

Proof. Since the carbon tax-GDP ratio $\frac{p_t}{Y_t}$ depends on the discount factor b_t and this is a function of m_t, we just need to show that if $m_t > m_{t+1}$, then $b_t - b_{t+1} > 0$. Define $\tilde{x}_t$ as

$$\tilde{x}_t \equiv m_t^{\xi} \sum_{j=0}^{\infty} \alpha^j \beta^{j+1} \prod_{i=1}^{j} m_{t+i}^{\xi} \tag{B.25}$$

Therefore, using (B.23) we obtain

$$
\begin{aligned}
b_t - b_{t+1} &= \frac{\tilde{x}_t}{1+\alpha\tilde{x}_t} - \frac{\tilde{x}_{t+1}}{1+\alpha\tilde{x}_{t+1}} \\
&= \frac{\tilde{x}_t - \tilde{x}_{t+1}}{(1+\alpha\tilde{x}_t)(1+\alpha\tilde{x}_{t+1})} \\
&> 0
\end{aligned} \tag{B.26}
$$

as long as $\tilde{x}_t > \tilde{x}_{t+1} > 0$. Since $\tilde{x}$ is a strictly increasing function of m_t, if $m_t > m_{t+1}$, then $b_t > b_{t+1}$, and we get the result.

■

C. Balanced growth in the RCK Model with CES utility

The objective function W_t is given by:

$$
\max_{\{c_t\}} W_t = \sum_t \beta^t M_t^{\xi} \frac{(C_t/M_t)^{1-\eta}}{1-\eta} \tag{C.1}
$$

where C_t denotes consumption, M_t represents total population in this economy, β is discount factor, ξ is an indicator parameter, and η is the elasticity of marginal utility (the inverse of the elasticity of intertemporal substitution). The social planner maximizes (C.1) subject to the same economic constraints as in the main text. Let λ_t be the Lagrange multiplier associated to the goods constraint, thus

$$
\beta^t M_t^{\xi-1+\eta} C_t^{-\eta} = \lambda_t \tag{C.2}
$$

In balanced growth, population growth $m = \frac{M_{t+1}}{M_t}$ is constant, and consumption C_t, output Y_t and capital K_t grow at the same rate γm, where γ is the per capita income growth, thus:

$$
\gamma m = \frac{Y_{t+1}}{Y_t} = \frac{C_{t+1}}{C_t} = \frac{K_{t+1}}{K_t} \tag{C.3}
$$

The discount factor (26) is defined through:

$$
b_t \equiv \frac{\lambda_{t+1}}{\lambda_t} \frac{Y_{t+1}}{Y_t} \tag{C.4}
$$

Plugging (C.2) into (C.4), we get:

$$
\begin{aligned}
b_t &= \frac{\beta^{t+1} M_{t+1}^{\xi-1+\eta} C_{t+1}^{-\eta}}{\beta^t M_t^{\xi-1+\eta} C_t^{-\eta}} \frac{Y_{t+1}}{Y_t} \\
&= \beta \left(\frac{M_{t+1}}{M_t} \right)^{\xi-1+\eta} \left(\frac{C_{t+1}}{C_t} \right)^{-\eta} \frac{Y_{t+1}}{Y_t}
\end{aligned}
\tag{C.5}
$$

which becomes in balanced growth:

$$
\begin{aligned}
b &= \beta m^{\xi-1+\eta} (m\gamma)^{1-\eta} \\
&= \beta m^{\xi} \gamma^{1-\eta}
\end{aligned}
\tag{C.6}
$$

D. Calculation of g_t as in (18)

Consider the sequence θ_i as in (38), and the policy function g_t as in (18):

$$
g_t = \sum\nolimits_{i=1}^{\infty} b_t^{t+i} \theta_i. \tag{D.1}
$$

First, we decompose the response series θ_i (38) into its parts

$$
\theta_i = \sum\nolimits_k \theta_{i,k}, \tag{D.2}
$$

$$
\theta_{i,k} = \sum\nolimits_j a_j b_k \varepsilon_k \frac{(1-\eta_j)^i - (1-\varepsilon_k)^i}{\varepsilon_k - \eta_j}, \tag{D.3}
$$

and note that these parts satisfy the recursive equation

$$
\theta_{i,k} = (1-\varepsilon_k)\theta_{i-1,k} + \sum\nolimits_j (1-\eta_j)^{i-1}. \tag{D.4}
$$

We now define the auxiliary variable

$$
\begin{aligned}
x_{j,t} &\equiv \sum\nolimits_{i=0}^{\infty} b_t^{t+i} (1-\eta_j)^i \\
&= 1 + (1-\eta_j) b_{t+1} \sum\nolimits_{i=1}^{\infty} b_{t+1}^{t+i} (1-\eta_j)^{i-1} \\
&= 1 + (1-\eta_j) b_{t+1} x_{j,t+1},
\end{aligned}
\tag{D.5}
$$

which helps us to calculate $g_{k,t}$ recursively ($g_t = \sum_k g_{k,t}$)

$$\begin{aligned} g_{k,t} &\equiv \sum_{i=1}^{\infty} b_t^{t+i}\theta_{i,k} \\ &= (1-\varepsilon_k)\sum_{i=1}^{\infty} b_t^{t+i}\theta_{i-1,k} + b_{t+1}\sum_j \sum_{i=1}^{\infty} b_{t+1}^{t+i}(1-\eta_j)^{i-1} \\ &= (1-\varepsilon_k)b_{t+1}\sum_{i=1}^{\infty} b_{t+1}^{t+i}\theta_i + b_{t+1}\sum_j x_{j,t+1} \\ &= (1-\varepsilon_k)b_{t+1}g_{k,t+1} + b_{t+1}\sum_j x_{j,t+1}. \end{aligned} \tag{D.6}$$

Thus, if we assume that demography is in steady state at the final year, $t = T$, then b_t, x_t and $g_{k,t}$ are also stationary from that period onwards, and we can calculate the variables x_t and g_t, at that period, through

$$x_{j,t} = \frac{1}{1-b_t(1-\eta_j)} \tag{D.7}$$

$$g_{k,t} = \frac{b_t\sum_j x_{j,t}}{1-b_t(1-\varepsilon_k)} \tag{D.8}$$

The recursive equations are then used to calculate (x_t, g_t) backwards.

E. Calculation of b_t as defined in (36)

Notice that (36) can be written as follows:

$$b_t = \frac{\sum_{j=0}^{\infty}\alpha^j\beta^{j+1}(\gamma_t^{t+j+1})^{1-\eta}(m_t^{t+j+1})^{\xi}}{\sum_{j=0}^{\infty}\alpha^j\beta^j(\gamma_t^{t+j})^{1-\eta}(m_t^{t+j})^{\xi}} \tag{E.1}$$

Define z_t as:

$$\begin{aligned} z_t &\equiv \sum_{i=0}^{\infty}\alpha^j\beta^j(\gamma_t^{t+j})^{1-\eta}(m_t^{t+j})^{\xi} \\ &= 1 + \alpha\beta\gamma_t^{1-\eta}m_t^{\xi}\sum_{j=0}^{\infty}\alpha^j\beta^j(\gamma_{t+1}^{t+j+1})^{1-\eta}(m_{t+1}^{t+j+1})^{\xi} \\ &= 1 + \alpha\beta\gamma_t^{1-\eta}m_t^{\xi}\sum_{j=1}^{\infty}\alpha^{j-1}\beta^{j-1}(\gamma_{t+1}^{t+j})^{1-\eta}(m_{t+1}^{t+j})^{\xi} \\ &= 1 + \alpha\beta\gamma_t^{1-\eta}m_t^{\xi}z_{t+1} \end{aligned} \tag{E.2}$$

Therefore,

$$b_t = \frac{\beta\gamma_t^{1-\eta}m_t^{\xi}z_{t+1}}{z_t} \tag{E.3}$$

As shown in Appendix C, in balanced growth the last expression collapses to $b = \beta\gamma^{1-\eta}m^{\xi}$. But now, we want to solve (E.3) recursively, and we use our auxiliary variable z_t to do so. Assume that population m and productivity γ growth rates are in steady state at $t = T$, our final year, so that b_T and z_T are stationary from that period onwards. Thus, we obtain

$$z_T = \frac{1}{1 - \alpha\beta\gamma_T^{1-\eta}m_T^{\xi}} \tag{E.4}$$

And we utilize the last equation to calculate (z_t, b_t) backwards.