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Working Paper

Energy Prices, Growth, and the Channels in Between: Theory and Evidence

Economics Working Paper Series, No. 06/47

Provided in Cooperation with:

ETH Zurich, CER-ETH – Center of Economic Research

Suggested Citation: Bretschger, Lucas (2007) : Energy Prices, Growth, and the Channels in Between: Theory and Evidence, Economics Working Paper Series, No. 06/47, ETH Zurich, CER-ETH - Center of Economic Research, Zurich

This Version is available at:

<https://hdl.handle.net/10419/171491>

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Mathematical supplement

1 Cost of K

θ s denote cost shares and λ s factor shares, as in the main text. Then, \hat{c}_K is calculated as:

$$\begin{aligned}
 \hat{c}_K &= \theta_{PK}(\theta_{LP}\hat{w} + \theta_{EP}\hat{p}_E) + \theta_{NK}(\theta_{HN} \cdot (\theta_{LH}\hat{w} + \theta_{EH}\hat{p}_E) + \theta_{BN}(\theta_{LB}\hat{w} + \theta_{EB}\hat{p}_E)) \\
 &= \theta_{PK}\theta_{LP}\hat{w} + \theta_{PK}\theta_{EP}\hat{p}_E + \theta_{NK}\theta_{HN}\theta_{LH}\hat{w} + \theta_{NK}\theta_{HN}\theta_{EH}\hat{p}_E \\
 &\quad + \theta_{NK}\theta_{BN}\theta_{LB}\hat{w} + \theta_{NK}\theta_{BN}\theta_{EB}\hat{p}_E \\
 &= [\theta_{PK}\theta_{LP} + \theta_{NK}\theta_{HN}\theta_{LH} + \theta_{NK}\theta_{BN}\theta_{LB}] \hat{w} \\
 &\quad + [\theta_{PK}\theta_{EP} + \theta_{NK}\theta_{HN}\theta_{EH} + \theta_{NK}\theta_{BN}\theta_{EB}] \hat{p}_E \\
 &= \theta_{LK} \cdot \hat{w} + \theta_{EK} \cdot \hat{p}_E.
 \end{aligned}$$

with $\theta_{LK} = \theta_{PK}\theta_{LP} + \theta_{NK}\theta_{HN}\theta_{LH} + \theta_{NK}\theta_{BN}\theta_{LB} > 0$ and $\theta_{EK} = \theta_{PK}\theta_{EP} + \theta_{NK}\theta_{HN}\theta_{EH} + \theta_{NK}\theta_{BN}\theta_{EB} > 0$.

2 Proof of lemma 1

Differentiate (9) and (10) to obtain:

$$\begin{aligned}
 \hat{E} &= \lambda_{EX}(\hat{a}_{EX} + \hat{X}) + \lambda_{EK}(\hat{a}_{EK} + \hat{g}_K) \\
 0 &= \lambda_{LX}(\hat{a}_{LX} + \hat{X}) + \lambda_{LK}(\hat{a}_{LK} + \hat{g}_K) \\
 0 &= \hat{c}_K + \left(\frac{g_K}{g_K + \rho}\right)\hat{g}_K
 \end{aligned}$$

With $\tilde{\rho} = (\rho + g_K)/g_K > 1$ we can write:

$$\hat{g}_K = -\tilde{\rho} \cdot \hat{c}_K = -\tilde{\rho}\theta_{EK}\hat{p}_E - \tilde{\rho}\theta_{LK}\hat{w}$$

Use this as well as:

$$\begin{aligned}
 \hat{a}_{Eq} &= \theta_{Lq}\sigma_q(\hat{w} - \hat{p}_E) \\
 \hat{a}_{Lq} &= -\theta_{Eq}\sigma_q(\hat{w} - \hat{p}_E)
 \end{aligned}$$

for $q = X, K$ which yields:

$$\begin{aligned}
\hat{E} &= \lambda_{EX} \left[\theta_{LX} \sigma_X (\hat{w} - \hat{p}_E) + \hat{X} \right] \\
&\quad + \lambda_{EK} [\theta_{LK} \sigma_K (\hat{w} - \hat{p}_E) - \theta_{EK} \tilde{\rho} \hat{p}_E - \theta_{LK} \tilde{\rho} \hat{w}] \\
0 &= \lambda_{LX} \left[-\theta_{EX} \sigma_X (\hat{w} - \hat{p}_E) + \hat{X} \right] \\
&\quad + \lambda_{LK} [-\theta_{EK} \sigma_K (\hat{w} - \hat{p}_E) - \theta_{EK} \tilde{\rho} \hat{p}_E - \theta_{LK} \tilde{\rho} \hat{w}]
\end{aligned}$$

Multiplying gives:

$$\begin{aligned}
\hat{E} &= \lambda_{EX} \theta_{LX} \sigma_X \hat{w} - \lambda_{EX} \theta_{LX} \sigma_X \hat{p}_E + \lambda_{EX} \hat{X} \\
&\quad + \lambda_{EK} \theta_{LK} \sigma_K \hat{w} - \lambda_{EK} \theta_{LK} \sigma_K \hat{p}_E - \lambda_{EK} \theta_{EK} \tilde{\rho} \hat{p}_E - \lambda_{EK} \theta_{LK} \tilde{\rho} \hat{w} \\
0 &= -\lambda_{LX} \theta_{EX} \sigma_X \hat{w} + \lambda_{LX} \theta_{EX} \sigma_X \hat{p}_E + \lambda_{LX} \hat{X} \\
&\quad - \lambda_{LK} \theta_{EK} \sigma_K \hat{w} + \lambda_{LK} \theta_{EK} \sigma_K \hat{p}_E - \lambda_{LK} \theta_{EK} \tilde{\rho} \hat{p}_E - \lambda_{LK} \theta_{LK} \tilde{\rho} \hat{w}
\end{aligned}$$

Collecting for \hat{w} and \hat{p}_E gives:

$$\begin{aligned}
\hat{E} &= [\lambda_{EX} \theta_{LX} \sigma_X + \lambda_{EK} \theta_{LK} \sigma_K - \lambda_{EK} \theta_{LK} \tilde{\rho}] \hat{w} \\
&\quad - [\lambda_{EX} \theta_{LX} \sigma_X + \lambda_{EK} \theta_{LK} \sigma_K + \lambda_{EK} \theta_{EK} \tilde{\rho}] \hat{p}_E + \lambda_{EX} \hat{X} \\
0 &= [-\lambda_{LX} \theta_{EX} \sigma_X - \lambda_{LK} \theta_{EK} \sigma_K - \lambda_{LK} \theta_{LK} \tilde{\rho}] \hat{w} \\
&\quad + [\lambda_{LX} \theta_{EX} \sigma_X + \lambda_{LK} \theta_{EK} \sigma_K - \lambda_{LK} \theta_{EK} \tilde{\rho}] \hat{p}_E + \lambda_{LX} \hat{X}
\end{aligned}$$

For a constant w the impact of \hat{p}_E on \hat{E} is $-\lambda_{EX} \theta_{LX} \sigma_X - \lambda_{EK} \theta_{LK} \sigma_K - \lambda_{EK} \theta_{EK} \tilde{\rho} < 0$ that means it is negative as expected. Collecting further and solving the labour market for \hat{w} gives:

$$\begin{aligned}
\hat{E} &= [\lambda_{EX} \theta_{LX} \sigma_X + \lambda_{EK} \theta_{LK} (\sigma_K - \tilde{\rho})] \hat{w} \\
&\quad - [\lambda_{EX} \theta_{LX} \sigma_X + \lambda_{EK} \theta_{LK} \sigma_K + \lambda_{EK} \theta_{EK} \tilde{\rho}] \hat{p}_E + \lambda_{EX} \hat{X} \\
\hat{w} &= \frac{[\lambda_{LX} \theta_{EX} \sigma_X + \lambda_{LK} \theta_{EK} \sigma_K + \lambda_{LK} \theta_{LK} \tilde{\rho}] \hat{w}}{\lambda_{LX} \theta_{EX} \sigma_X + \lambda_{LK} \theta_{EK} \sigma_K + \lambda_{LK} \theta_{LK} \tilde{\rho}} \\
&= \frac{[\lambda_{LX} \theta_{EX} \sigma_X + \lambda_{LK} \theta_{EK} (\sigma_K - \tilde{\rho})] \hat{p}_E + \lambda_{LX} \hat{X}}{\lambda_{LX} \theta_{EX} \sigma_X + \lambda_{LK} (\theta_{EK} \sigma_K + \theta_{LK} \tilde{\rho})}
\end{aligned}$$

which shows that, for constant X , \hat{w} and \hat{p}_E have the same sign when $\sigma_K > \tilde{\rho}$ and that the opposite happens when $\lambda_{LX} \theta_{EX} \sigma_X + \lambda_{LK} \theta_{EK} (\sigma_K - \tilde{\rho}) < 0$ which

requires large values of θ_{EK} and $\tilde{\rho}$. By inserting \hat{w} in the energy equation and collecting we obtain:

$$\begin{aligned}\hat{E} = & - \{ [\lambda_{EX}\theta_{LX}\sigma_X + \lambda_{EK}\theta_{LK}(\sigma_K - \tilde{\rho}) \cdot \\ & \frac{[\lambda_{LX}\theta_{EX}\sigma_X + \lambda_{LK}\theta_{EK}(\sigma_K - \tilde{\rho})]}{\lambda_{LX}\theta_{EX}\sigma_X + \lambda_{LK}(\theta_{EK}\sigma_K + \theta_{LK}\tilde{\rho})} \\ & + [\lambda_{EX}\theta_{LX}\sigma_X + \lambda_{EK}\theta_{LK}\sigma_K + \lambda_{EK}\theta_{EK}\tilde{\rho}] \} \hat{p}_E \\ & + \left(\frac{\lambda_{LX}}{\lambda_{LX}\theta_{EX}\sigma_X + \lambda_{LK}(\theta_{EK}\sigma_K + \theta_{LK}\tilde{\rho})} + \lambda_{EX} \right) \hat{X}\end{aligned}$$

or, respectively:

$$\begin{aligned}\hat{E} = & - \left\{ \frac{[b_1\sigma_X + b_2(\sigma_K - \tilde{\rho})][b_3\sigma_X + b_4(\sigma_K - \tilde{\rho})]}{b_5} + \tilde{b} \right\} \hat{p}_E + \Gamma \cdot (\hat{Y} - \gamma) \\ = & - \left\{ \frac{b_1b_3\sigma_X^2 + \bar{b}\sigma_X(\sigma_K - \tilde{\rho}) + b_2b_4(\sigma_K - \tilde{\rho})^2}{b_5} + \tilde{b} \right\} \hat{p}_E + \Gamma \cdot (\hat{Y} - \gamma)\end{aligned}$$

with:

$$\begin{aligned}b_1 = & \lambda_{EX}\theta_{LX} > 0, \quad b_2 = \lambda_{EK}\theta_{LK} > 0, \quad b_3 = \lambda_{LX}\theta_{EX} > 0 \\ b_4 = & \lambda_{LK}\theta_{EK} > 0, \quad b_5 = \lambda_{LX}\theta_{EX}\sigma_X + \lambda_{LK}(\theta_{EK}\sigma_K + \theta_{LK}\tilde{\rho}) > 0 \\ \tilde{b} = & \lambda_{EX}\theta_{LX}\sigma_X + \lambda_{EK}\theta_{LK}\sigma_K + \lambda_{EK}\theta_{EK}\tilde{\rho} > 0 \\ \bar{b} = & b_1b_4 + b_2b_3 > 0, \quad \gamma = g_A + \frac{1-\beta}{\beta}g_K \geq 0 \\ \Gamma = & \frac{\lambda_{LX}}{\lambda_{LX}\theta_{EX}\sigma_X + \lambda_{LK}(\theta_{EK}\sigma_K + \theta_{LK}\tilde{\rho})} + \lambda_{EX} > 0\end{aligned}$$

which reveals that \hat{p}_E has an unambiguously negative impact on \hat{E} when $\sigma_K > \tilde{\rho}$. When $\sigma_K < \tilde{\rho}$ an ambiguity arises. In this (special) case, according to the expression $\lambda_{LK}\theta_{EK}(\sigma_K - \tilde{\rho})$ from above, wages decrease sharply after an energy price increase which causes a strong output effect in the capital sector possibly offsetting the direct energy price effect. This may happen even when taking into account the positive impact of \tilde{b} . In the short run, wages are not flexible; then, the impact of energy prices on energy use is unambiguous according to:

$$\hat{E} = -\tilde{b} \cdot \hat{p}_E$$

3 Proof of lemma 2

To evaluate \hat{s}_i we write:

$$\hat{s}_i = \hat{k}_i - \hat{Y} = \hat{\theta}_{ki} - \hat{p}_{ki} + \hat{p}_X - \gamma$$

We use the optimum conditions in the capital sector, i.e. $\theta_{PK}/\theta_{NK} = [p_P/p_N]^{1-\sigma_{\tilde{K}}}$ and $\theta_{HN}/\theta_{BN} = [p_H/p_B]^{1-\sigma_N}$ to derive the cost shares θ for the different capital types i where $\sigma_{\tilde{K}}$ and σ_N are the elasticities of substitution between physical and non-physical capital and between human and knowledge capital, respectively. Moreover, we express $\hat{\theta}_{ki}$ as well as \hat{p}_X and \hat{p}_{ki} in terms of input prices \hat{w} and \hat{p}_E , which yields for capital type P :

$$\begin{aligned}
\hat{s}_P + \gamma &= \theta_{NK}(1 - \sigma_{\tilde{K}})(\hat{p}_P - \hat{p}_N) + (\theta_{LX} - \theta_{LP})\hat{w} + (\theta_{EX} - \theta_{EP})\hat{p}_E \\
&= \theta_{NK}(1 - \sigma_{\tilde{K}}) [(\theta_{LP} - \theta_{LN})\hat{w} + (\theta_{EN} - \theta_{EP})\hat{p}_E] \\
&\quad + (\theta_{LX} - \theta_{LP})\hat{w} + (\theta_{EX} - \theta_{EP})\hat{p}_E \\
&= \theta_{NK}(1 - \sigma_{\tilde{K}}) [(\theta_{EN} - \theta_{EP})\hat{w} + (\theta_{EN} - \theta_{EP})\hat{p}_E] + (\theta_{EP} - \theta_{EX})(\hat{w} - \hat{p}_E) \\
&= \theta_{NK}(1 - \sigma_{\tilde{K}})(\theta_{EN} - \theta_{EP})(\hat{w} - \hat{p}_E) + (\theta_{EP} - \theta_{EX})(\hat{w} - \hat{p}_E) \\
&= [\theta_{NK}(1 - \sigma_{\tilde{K}})(\theta_{EN} - \theta_{EP}) + (\theta_{EP} - \theta_{EX})](\hat{w} - \hat{p}_E)
\end{aligned}$$

where we have used $\theta_{LX} = 1 - \theta_{EX}$, $\theta_{LP} = 1 - \theta_{EP}$ etc. Similarly, we obtain for H and B

$$\begin{aligned}
\hat{s}_H + \gamma &= [\theta_{BN}(1 - \sigma_N)(\theta_{EB} - \theta_{EH}) + (\theta_{EH} - \theta_{EX})](\hat{w} - \hat{p}_E) \\
\hat{s}_B + \gamma &= [\theta_{HN}(1 - \sigma_N)(\theta_{EH} - \theta_{EB}) + (\theta_{EB} - \theta_{EX})](\hat{w} - \hat{p}_E)
\end{aligned}$$

To find $\hat{w} - \hat{p}_E$ we use the factor market equilibria and the capital market equilibrium as well as $\hat{X} = -\theta_{LX}\hat{w} - \theta_{EX}\hat{p}_E$ to get:

$$\begin{aligned}
\hat{E} &= \lambda_{EX}(\theta_{LX}\sigma_X(\hat{w} - \hat{p}_E) - \theta_{LX}\hat{w} - \theta_{EX}\hat{p}_E) \\
&\quad + \lambda_{EK}[\theta_{LK}\sigma_K(\hat{w} - \hat{p}_E) - \theta_{EK}\tilde{\rho}\hat{p}_E - \theta_{LK}\tilde{\rho}\hat{w}] \\
0 &= \lambda_{LX}[-\theta_{EX}\sigma_X(\hat{w} - \hat{p}_E) - \theta_{LX}\hat{w} - \theta_{EX}\hat{p}_E] \\
&\quad + \lambda_{LK}[-\theta_{EK}\sigma_K(\hat{w} - \hat{p}_E) - \theta_{EK}\tilde{\rho}\hat{p}_E - \theta_{LK}\tilde{\rho}\hat{w}]
\end{aligned}$$

so that:

$$\begin{aligned}
\hat{E} &= \lambda_{EX}\theta_{LX}\sigma_X\hat{w} - \lambda_{EX}\theta_{LX}\sigma_X\hat{p}_E - \lambda_{EX}\theta_{LX}\hat{w} - \lambda_{EX}\theta_{EX}\hat{p}_E \\
&\quad + \lambda_{EK}\theta_{LK}\sigma_K\hat{w} - \lambda_{EK}\theta_{LK}\sigma_K\hat{p}_E - \lambda_{EK}\theta_{EK}\tilde{\rho}\hat{p}_E - \lambda_{EK}\theta_{LK}\tilde{\rho}\hat{w} \\
0 &= -\lambda_{LX}\theta_{EX}\sigma_X\hat{w} + \lambda_{LX}\theta_{EX}\sigma_X\hat{p}_E - \lambda_{LX}\theta_{LX}\hat{w} - \lambda_{LX}\theta_{EX}\hat{p}_E - \lambda_{LK}\theta_{EK}\sigma_K\hat{w} \\
&\quad + \lambda_{LK}\theta_{EK}\sigma_K\hat{p}_E - \lambda_{LK}\theta_{EK}\tilde{\rho}\hat{p}_E - \lambda_{LK}\theta_{LK}\tilde{\rho}\hat{w}
\end{aligned}$$

and:

$$\begin{aligned}
\hat{E} &= [\lambda_{EX}\theta_{LX}\sigma_X - \lambda_{EX}\theta_{LX} + \lambda_{EK}\theta_{LK}\sigma_K - \lambda_{EK}\theta_{LK}\tilde{\rho}]\hat{w} \\
&\quad - [\lambda_{EX}\theta_{LX}\sigma_X + \lambda_{EX}\theta_{EX} + \lambda_{EK}\theta_{LK}\sigma_K + \lambda_{EK}\theta_{EK}\tilde{\rho}]\hat{p}_E \\
0 &= [-\lambda_{LX}\theta_{EX}\sigma_X - \lambda_{LX}\theta_{LX} - \lambda_{LK}\theta_{EK}\sigma_K - \lambda_{LK}\theta_{LK}\tilde{\rho}]\hat{w} \\
&\quad + [\lambda_{LX}\theta_{EX}\sigma_X - \lambda_{LX}\theta_{EX} + \lambda_{LK}\theta_{EK}\sigma_K - \lambda_{LK}\theta_{EK}\tilde{\rho}]\hat{p}_E
\end{aligned}$$

Written in matrix form we have:

$$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \cdot \begin{bmatrix} \hat{w} \\ \hat{p}_E \end{bmatrix} = \begin{bmatrix} \hat{E} \\ 0 \end{bmatrix}$$

where

$$\begin{aligned} c_{11} &= \sum_q \lambda_{Eq} \theta_{Lq} \sigma_q - \lambda_{EX} \theta_{LX} - \lambda_{EK} \theta_{LK} \tilde{\rho} \\ c_{12} &= - \sum_q \lambda_{Eq} \theta_{Lq} \sigma_q - \lambda_{EX} \theta_{EX} - \lambda_{EK} \theta_{EK} \tilde{\rho} \\ c_{21} &= - \sum_q \lambda_{Lq} \theta_{Eq} \sigma_q - \lambda_{LX} \theta_{LX} - \lambda_{LK} \theta_{LK} \tilde{\rho} \\ c_{22} &= \sum_q \lambda_{Lq} \theta_{Eq} \sigma_q - \lambda_{LX} \theta_{EX} - \lambda_{LK} \theta_{EK} \tilde{\rho} \end{aligned}$$

By construction of the cs the determinant Δ of the system is maximum if $\sigma_q = 0$, which yields:

$$\begin{aligned} \Delta &= (\lambda_{EX} \theta_{LX} + \lambda_{EK} \theta_{LK} \tilde{\rho})(\lambda_{LX} \theta_{EX} + \lambda_{LK} \theta_{EK} \tilde{\rho}) \\ &\quad - (\lambda_{EX} \theta_{EX} + \lambda_{EK} \theta_{EK} \tilde{\rho})(\lambda_{LX} \theta_{LX} + \lambda_{LK} \theta_{LK} \tilde{\rho}) \\ &= \lambda_{EX} \theta_{LX} \lambda_{LX} \theta_{EX} + \lambda_{EX} \theta_{LX} \lambda_{LK} \theta_{EK} \tilde{\rho} + \lambda_{EK} \theta_{LK} \lambda_{LX} \theta_{EX} + \lambda_{EK} \theta_{LK} \lambda_{LK} \theta_{EK} \tilde{\rho} \\ &\quad - \lambda_{EX} \theta_{EX} \lambda_{LX} \theta_{LX} - \lambda_{EX} \theta_{EX} \lambda_{LK} \theta_{LK} \tilde{\rho} - \lambda_{EK} \theta_{EK} \lambda_{LX} \theta_{LX} - \lambda_{EK} \theta_{EK} \lambda_{LK} \theta_{LK} \tilde{\rho} \\ &= \lambda_{EX} \lambda_{LX} (\theta_{LX} \theta_{EX} - \theta_{EX} \theta_{LX}) + \lambda_{EX} \lambda_{LK} \tilde{\rho} (\theta_{LX} \theta_{EK} - \theta_{EX} \theta_{LK}) \\ &\quad + \lambda_{EK} \lambda_{LX} (\theta_{LK} \theta_{EX} - \theta_{EK} \theta_{LX}) + \lambda_{EK} \lambda_{LK} \tilde{\rho} (\theta_{EK} \theta_{LK} - \theta_{EK} \theta_{LK}) \\ &= \lambda_{EX} \lambda_{LK} \tilde{\rho} (\theta_{LX} \theta_{EK} - \theta_{EX} \theta_{LK}) + \lambda_{EK} \lambda_{LX} (\theta_{LK} \theta_{EX} - \theta_{EK} \theta_{LX}) \\ &= \lambda_{EX} \lambda_{LK} \tilde{\rho} (\theta_{LX} \theta_{EK} - \theta_{EX} \theta_{LK}) - \lambda_{EK} \lambda_{LX} (\theta_{EK} \theta_{LX} - \theta_{LK} \theta_{EX}) \\ &= (\lambda_{EX} \lambda_{LK} \tilde{\rho} - \lambda_{EK} \lambda_{LX}) (\theta_{LX} \theta_{EK} - \theta_{EX} \theta_{LK}) \end{aligned}$$

Provided that intermediates production is relatively more intensive in energy use than capital accumulation, i.e. we have $\theta_{LX} < \theta_{LK}$, $\theta_{EK} < \theta_{EX}$ and $\lambda_{LX}/\lambda_{LK} < \lambda_{EX}/\lambda_{EK}$ so that $\lambda_{EX} \lambda_{LK} \tilde{\rho} - \lambda_{EK} \lambda_{LX} > 0$ and $\theta_{LX} \theta_{EK} - \theta_{EX} \theta_{LK} < 0$, we get $\Delta < 0$. When intermediates production is relatively less energy-intensive than capital accumulation, i.e. we have $\theta_{LX} > \theta_{LK}$ and $\theta_{EK} > \theta_{EX}$ as well as $\lambda_{LX}/\lambda_{LK} > \lambda_{EX}/\lambda_{EK}$. In this case we have $\theta_{LX} \theta_{EK} - \theta_{EX} \theta_{LK} > 0$. When $\lambda_{EX} \lambda_{LK} \tilde{\rho} - \lambda_{EK} \lambda_{LX} < 0$ we get again $\Delta < 0$. For $\lambda_{EX} \lambda_{LK} \tilde{\rho} - \lambda_{EK} \lambda_{LX} > 0$ although $\lambda_{LX}/\lambda_{LK} > \lambda_{EX}/\lambda_{EK}$ we would, by the definition of $\tilde{\rho}$ and plausible values for the parameters, obtain the result that $\rho > g_K$. This, however, is not feasible in a growing economy and can be discarded. We infer that the determinant is negative, i.e. $\Delta < 0$. For the input prices we obtain:

$$\hat{w} = \frac{c_{22}}{\Delta} \hat{E} = \frac{1}{\Delta} [\lambda_{LX} \theta_{EX} (\sigma_X - 1) + \lambda_{LK} \theta_{EK} (\sigma_K - \tilde{\rho})] \cdot \hat{E}$$

$$\hat{p}_E = -\frac{c_{21}}{\Delta} \hat{E} = \frac{1}{\Delta} [\lambda_{LX}(\theta_{EX}\sigma_X + \theta_{LX}) + \lambda_{LK}(\theta_{EK}\sigma_K + \theta_{LK}\tilde{\rho})] \cdot \hat{E}$$

which means that:

$$\begin{aligned} \hat{w} - \hat{p}_E &= \{[\lambda_{LX}\theta_{EX}(\sigma_X - 1) + \lambda_{LK}\theta_{EK}(\sigma_K - \tilde{\rho})] \\ &\quad - [\lambda_{LX}(\theta_{EX}\sigma_X + \theta_{LX}) + \lambda_{LK}(\theta_{EK}\sigma_K + \theta_{LK}\tilde{\rho})]\} \cdot \frac{\hat{E}}{\Delta} \\ &= (\lambda_{LX}\theta_{EX}\sigma_X - \lambda_{LX}\theta_{EX} + \lambda_{LK}\theta_{EK}\sigma_K - \lambda_{LK}\theta_{EK}\tilde{\rho} \\ &\quad - \lambda_{LX}\theta_{EX}\sigma_X - \lambda_{LX}\theta_{LX} - \lambda_{LK}\theta_{EK}\sigma_K - \lambda_{LK}\theta_{LK}\tilde{\rho}) \cdot \frac{\hat{E}}{\Delta} \\ &= (-\lambda_{LX}\theta_{EX} - \lambda_{LK}\theta_{EK}\tilde{\rho} - \lambda_{LX}\theta_{LX} - \lambda_{LK}\theta_{LK}\tilde{\rho}) \cdot \frac{\hat{E}}{\Delta} \\ &= [-\lambda_{LX}(\theta_{EX} + \theta_{LX}) - \lambda_{LK}\tilde{\rho}(\theta_{EK} + \theta_{LK})] \cdot \frac{\hat{E}}{\Delta} = \frac{\nu}{\Delta} \hat{E} \end{aligned}$$

As we have $\Delta, \nu < 0$, it follows that a decrease in E causes an unambiguous decrease of the wage/energy price ratio. Inserting for the different capital types yields:

$$\begin{aligned} \hat{s}_P &= [\theta_{NK}(1 - \sigma_{\tilde{K}})(\theta_{EN} - \theta_{EP}) + (\theta_{EP} - \theta_{EX})] \frac{\nu}{\Delta} \cdot \hat{E} - \gamma \\ \hat{s}_H &= [\theta_{BN}(1 - \sigma_N)(\theta_{EB} - \theta_{EH}) + (\theta_{EH} - \theta_{EX})] \frac{\nu}{\Delta} \cdot \hat{E} - \gamma \\ \hat{s}_B &= [\theta_{HN}(1 - \sigma_N)(\theta_{EH} - \theta_{EB}) + (\theta_{EB} - \theta_{EX})] \frac{\nu}{\Delta} \cdot \hat{E} - \gamma \end{aligned}$$